

# *Flexible Joint Robotic Manipulator: Modeling and Design of Robust Control Law*

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**Abstract**— This paper presents modeling and sophisticated control of a single Degree Of Freedom (DOF) flexible robotic arm. The derived model is based on Euler-Lagrange approach while the first and second order (super twisting) Sliding Mode Control (SMC) is proposed as a non-linear control strategy. The control laws are subjected to various test inputs including step and sinusoids to demonstrate their tracking efficiency by observing transient and steady state behaviours. Both orders of SMC are then compared to characterize the control performance in terms of robustness, handling external disturbances and chattering. Results dictate that the super twisting SMC is more accurate and robust against the external noise and chattering phenomena compared to the first order SMC.

**Keywords**—Flexible joint; Robotic manipulator; Control techniques; Sliding Mode Control (SMC)

## I. INTRODUCTION

Since the evolution in robotics from the end of 16<sup>th</sup> century, a lot of work has been done in this research domain [1]. Technological advances in allied area like electronics [2], control [3], modeling [4], simulation [5], brain machine interface [6], cognition [7] etc. have increased the applications of robotics to an incredible extent [8]. Currently, robots are becoming an essential part in various sectors like medical [9], haptics [10, 11], nuclear power plants [12], space [13-15], automation [16, 17] and industries [18] in addition to other applications involving under-water [19], target tracking [20, 21] and tether [22]. In medical domain, technology finds enormous potential in rehabilitation [23], motion assistance [24, 25], anesthesia administration [26] and for patients' mobility [27].

The locomotion sub-system of robots mainly consist of links [28], legs [29], wheels [30-33] or their combination [34]. Consequently, the control community has been actively involved in proposing novel strategies for mobile robots [35] as well as link-based mechanisms. For the later structures, both linear [36, 37] as well as non-linear control approaches [38, 39] have been practically realized.

Robotic manipulators can be categorized as rigid and flexible, based on their make. Rigid manipulators have increased tensile strength with more precision due to the material used such as steel or aluminum frames but have high cost and weight [40]. While the advancement in material technology has enabled us to have the low cost and weight,

but less precise flexible manipulators, which requires robust or adaptive control laws to handle the trajectories. The main focus of this paper is solely the flexible joint manipulators having the rigid arm cushioned with the springs attached to the platform. Fig. 1 elaborates the flexible joint manipulator module, developed by Quanser consisting of the static base upon which the rotational platform is implemented having the flexible joint manipulator connected through springs.

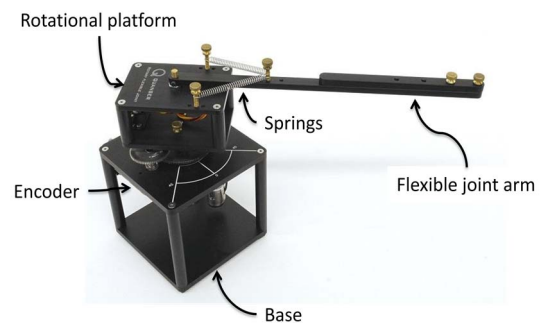


Fig. 1. Flexible joint robot arm by Quanser [41]

The complexity in controlling the vibrations due to flexibility can be undertaken by selecting the appropriate trajectories for the flexible arm. Open-loop expectations of the dynamics are used to create the trajectory profiles in order to cancel the oscillations created by previous trajectories (movements). This technique of planning the trajectories for the flexible manipulators to handle the shuddering is known as command shaping [42].

Many nonlinear control strategies have been introduced to deal with the nonlinearities present in the system [43]. The linear control fails when there is a need to tackle the external disturbances, nonlinearities and changes in the plant parameters [44]. So far, many techniques are used by scientists like; Fuzzy Logic Controller (FLC), Back Stepping (BS) Sliding Mode Control (SMC) [45] and their combinations with linear techniques for control of nonlinearities in the robotic systems and to deal with their uncertainties.

Genetic Algorithm (GA) and Input Estimation (IE) approach by are used for the design of SMC for flexible joints. The disturbances in torque are measured using Kalman Filter (KF) and Recursive Least Square Estimator (RLSE) methods, which do not require torque measurement sensors. Likewise,

the GA is used to search the optimal controller design parameters for SMC. The IE is an old approach which was used by (Tuan *et al*) and it has successfully solved the induction heat [46, 47] and many other problems.

SMC is one of the most robust nonlinear techniques which can handle the changes in the plant very efficiently as compare to the linear controllers. The Simulink model shown in fig. 2 has been implemented using the derived model and control law equations. The plant model block has the dynamic model, which provides angular accelerations of rotational platform and flexible joint manipulator as an output. Integrator blocks are used to get the angular positions from accelerations. The controller block is deciding the control input 'u' based on the magnitude of the error computed by comparing the output trajectory with the desired input. The controller is making its decisions based on sliding surface which is the linear combination of errors and their higher derivatives.

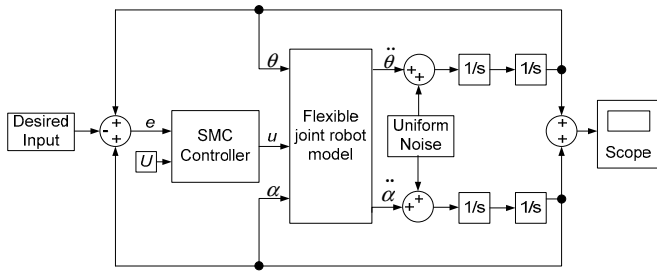


Fig. 2. Overall block diagram of the proposed feedback control system.

## II. MODELING OF FLEXIBLE JOINT ROBOTIC MANIPULATOR

The arm used here is the single link revolute joint and has two Degree Of Freedom (DOF). The base is joined with the set of springs which provides the essence of flexible joint. This type of arm with flexibility is the latest focus of modern research and teaching because of its applications in the industry. Fig. 3 illustrates the top view of the arm operating on a vertical plane.

In the hardware, the deflection/vibrations in the flexible manipulator are recorded by the encoder placed in the joint. The deflections in the flexible manipulator can be reduced by controlling the applied torque to the servo motor and catering the abrupt changes in the input, which is achieved by the robust controllers and are explained in section III.

The Euler-Lagrange equations of motion are used to obtain the mathematical model of the arm. The parameters of flexible joint robot that are used for the derivation of the model are shown in table I. The coordinates that is present in the platform for the modeling [41] are the angular motion of the rotational platform ( $\theta$ ) and the angular displacement of the flexible joint ( $\alpha$ ) as shown in figure 4. The Euler-Lagrange's equation (3) 'L' requires the total kinetic and potential energies. The total potential energy ' $P_{Total}$ ' is the sum of the spring's stored energy at the joint and gravity given by (1). The sum of kinetic energies of rotational platform and flexible link manipulator constitutes ' $K_{Total}$ ', which is given by (2).

$$P_{Total} = \frac{1}{2}K_s\alpha^2 + mgh \cos(\theta + \alpha) \quad (1)$$

$$K_{Total} = \frac{1}{2}J_h(\dot{\theta}^2) + \frac{1}{2}J_l(\dot{\theta} + \dot{\alpha})^2 \quad (2)$$

$$L = K_{Total} + P_{Total} \quad (3)$$

The Euler-Lagrange's motion equation (4) is then used to acquire the required results of rotational accelerations of rotational platform and flexible joint given by (5) and (6) respectively. In (4) the torque is represented by ' $\tau$ ' and ' $q_i$ ' is the variable of differentiation i.e. ' $\theta$ ' or ' $\alpha$ '.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau \quad (4)$$

$$\ddot{\theta} = \frac{1}{J_h}(\tau + K_s\alpha) \quad (5)$$

$$\ddot{\alpha} = -K_s\alpha\left(\frac{1}{J_h} + \frac{1}{J_l}\right) + \frac{1}{J_l}mgh \sin(\theta + \alpha) - \tau \quad (6)$$

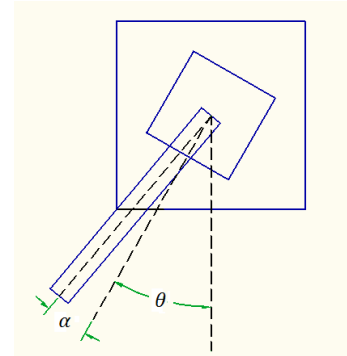


Fig. 3: Schematic diagram of flexible joint robotic arm.

TABLE I. PARAMETERS OF FLEXIBLE JOINT-ROBOT

Symbol	Description	Value	Units
$K_s$	Spring stiffness	5.468	N/m
$m$	Link mass	0.1	kg
$J_h$	Inertia of rotational platform	0.00035	kgm <sup>2</sup>
$J_l$	Inertia of flexible manipulator	0.003882	kgm <sup>2</sup>
$h$	Distance of center of gravity of rotational platform	0.06	m
$g$	Gravitational acceleration	-9.81	N/m

## III. NON-LINEAR CONTROLS TECHNIQUES

Linear controllers such as PID's have a problem that they cannot handle the random changes, external noise and nonlinearities in the system. So, we need to have some robust, accurate and easy to tune controller which can deal with all these issues.

SMC is one of the nonlinear control techniques, of which the purpose is to force the systems trajectories to the particular surface, called sliding surface through its control law. Hence,

this technique can be divided into two phases namely; sliding surface design and control input design.

As already discussed, for SMC we need to define a sliding surface which would be same for both first and second order SMCs. The surface consists of the linear combination of errors and their higher derivatives as described in (7). Where, 'S' is the sliding surface, 'e' is the error between desired and actual output, 'a' is the constant and 'k' is one less than the relative degree between input and output which is unity in our case.

$$S = \left( \frac{d}{dt} + a \right)^k e \quad (7)$$

The distance of the sliding trajectories from the sliding surface ( $S = 0$ ) is determined by the Euclidian norm which is obtained from the Lyapunov function (8). This function is half the product of sliding surface 'S' and its transpose.

$$V = \frac{1}{2} \times S \times S^T \quad (8)$$

The sufficient condition (9) for the existence of the mode is that the product of sliding surface and its rate of change must be less than zero. The feedback control should be picked such that the relation for  $\dot{V}$  [48] provided in (9) always exists. In short the control law 'u' will make 'S' positive if 'S' is negative and vice versa.

$$S^T \times \dot{S} < 0 \quad (9)$$

#### A. First order SMC design

The design of control input 'u' for first order SMC is relatively simple as defined in (10-11). The control effort is dependent on the sign of the sliding surface, i.e. if the magnitude of sliding surface would be positive then the negative pulse of magnitude 'U' would be provided and vice versa. This phenomenon can be seen in fig. 4, where the control input is plotted against the sliding surface, in which the 'U' is tuned at the value of 0.05 and the value of constant 'a' given in (7) is 0.14. The graph with solid line describes the sliding surface, which is fluctuating around the time axis. This fluctuation represents the chattering problem which is higher and dominant in first order SMC and can be reduced using higher order SMC by modifying the control law.

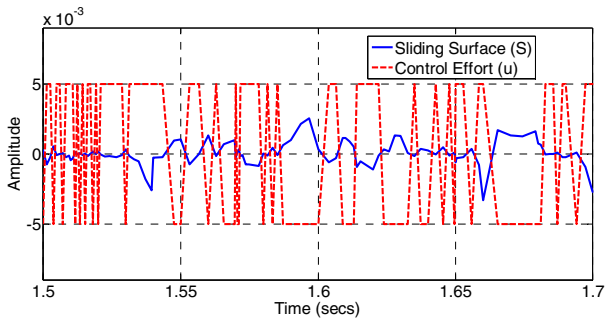


Fig. 4. Performance of the control effort illustrated by plotting control input alongside sliding surface(linear combination errors and higher derivatives) (First order SMC)

$$u = -U \operatorname{sign}(S) \quad (10)$$

$$u = \begin{cases} -U & S > 0 \\ U & S < 0 \end{cases} \quad (11)$$

#### B. Super twisting control

The super twisting control possesses the continuous control input (12) which is comparable to the PI controller, as it also has two components; the nonlinear gain (13) 'w' and the integral part (14) 'y'. Where 'U' is the positive constant, of which the value is set by progressively increasing its magnitude until the desired trajectory is obtained. This type of "trial and error" practice is used to get the desired response. The 'sgn()' function in (13) and (14) returns the sign (positive or negative) of the sliding function S and '| |' return the absolute value of a function placed inside it.

$$u = w + y \quad (12)$$

$$w = -\sqrt{U} \times \sqrt{|S|} \times \operatorname{sgn}(S) \quad (13)$$

$$\dot{y} = -1.1 \times U \times \operatorname{sgn}(S) \quad (14)$$

The second order SMC solved the chattering problem to a high extent as shown in fig. 5. Here the control effort is shown through the green colored dotted line, which is the continuous function and in result the chattering is reduced by the factor of 17.1 than the first order case. The constant 'U' here again is tuned at the value of 0.05 and the value of constant 'a' given in (7) is selected to be 0.08.

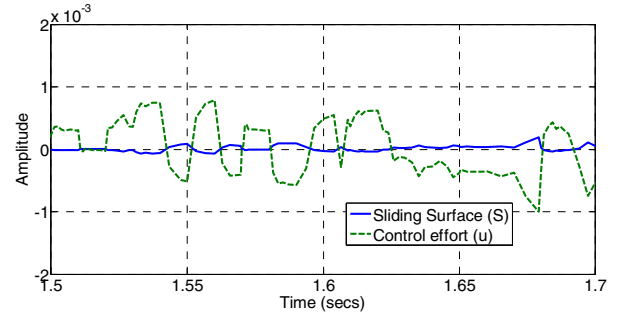


Fig. 5. Performance of the control effort illustrated by plotting control input alongside sliding surface(linear combination errors and higher derivatives) (Second order SMC)

## IV. RESULTS AND DISCUSSION

The major problem with the first order SMC is the chattering phenomena due to discontinuous control input. This can be studied by observing the sliding surface fluctuating around zero or the steady state of the step response in which the output vibrates across the desired input trajectory. Fig. 6 (a-b) demonstrates the chattering phenomena for first and second order SMCs. It is shown that the chattering effect close to the origin is higher than the second order due to the use of discontinuous control law.

The first and second order (Super twisting) SMCs are tuned and subjected to the step and sinusoidal inputs. The dynamic characteristics have been studied and the capability of tracking the inputs is observed using the simulations. The comparisons of these controllers are studied by examining

different transient and steady state parameters and their effects on the performance. The chattering phenomena is major issue in lower order SMCs, which is challenged and controlled using the higher order Super twisting controller. Fig. 7(a-c) illustrates the step response, steady state error and sinusoidal responses of the system.

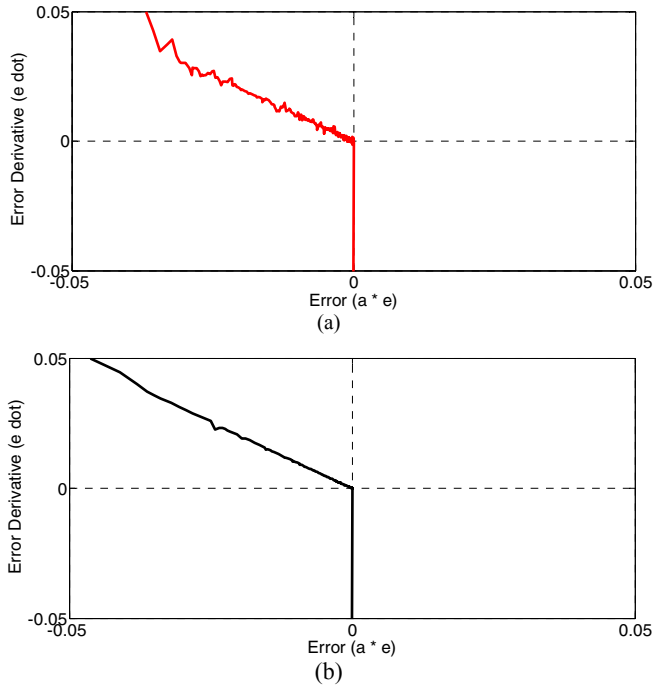


Fig. 6: Trajectories of the system at  $S = 0$  stabilized by (a). First order SMC and (b). Super Twisting second order SMC

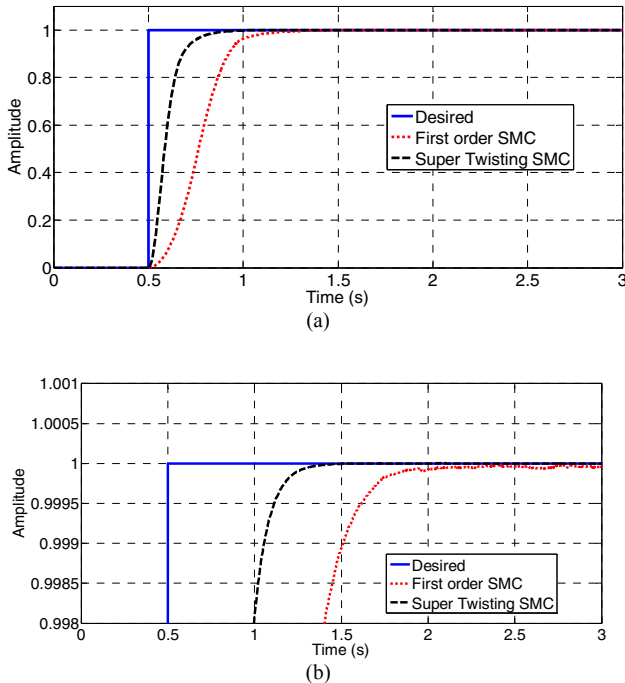


Fig. 7. Performance of first order and super twisting (second order) SMC demonstrated by (a) Step response (b) Zoomed step response (Steady-state error) and (c) Sinusoidal response of a flexible joint robotic manipulator

Considering the step response of Figure 8(a), the results of performance comparison of the transient behavior for first order and super twisting SMC is listed in Table II. The comparison shows that the results of super twisting (second order) is far better than the first order SMC in terms of each parameter under study. Table II also illustrates the Integral Time-weighted Absolute Error (ITAE) integrates the absolute error multiplied by time over time, Integral Absolute Error (IAE) integrates the absolute error over time and Integral Squared Error (ISE) which integrates the square of the error over time as shown in (15-17) respectively. All these parameters show that their value for second order is much less than the first order. Hence, the performance of Super Twisting second order is highly efficient than the first order SMC.

$$ISE = \int \varepsilon^2 dt \quad (15)$$

$$IAE = \int |\varepsilon| dt \quad (16)$$

$$ITAE = \int t|\varepsilon| dt \quad (17)$$

Fig. 7(b) demonstrates that the steady state error for second order super twisting is almost zero or minimally small, which shows the advantage of using continuous control effort instead of discontinuous pulses, as in case of first order SMC. Thus, the chattering effect is considerable reduced.

TABLE II. PERFORMANCE COMPARISON OF FIRST ORDER AND SUPER TWISTING SMC

Step	First Order SMC	Super Twisting
Rise Time (s)	0.29	0.12
Settling Time (2 %) (s)	0.48	0.21
Over shoot	0	0
Peak time (s)	1.6743	1.1375
ITAE	0.1754	0.0523
IAE	0.2674	0.0939
ISE	0.2054	0.0666

Both the controllers have also been assessed in the presence of external disturbances (noise) as shown in fig. 8 (a). The step response is plotted in the presence of external noise added to the model as shown in fig. 8 (b-c), which



shows that both the controllers especially the super twisting is highly robust in tackling the external disturbances. The steady state error has been evolved for the first order SMC and its graph has also been distorted due to noise. But, the second order super twisting performed almost the same as in the absence of external noise.

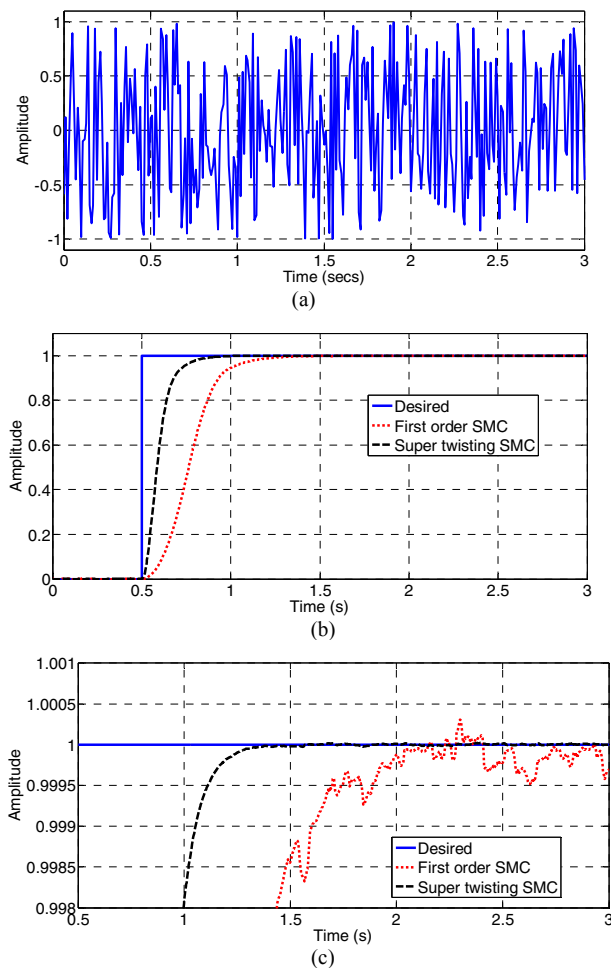


Fig. 8. Disturbance rejection (a) Bounded external disturbances to the model (b) Step Tracking performance and (c) Zoomed step response (Steady-state error) using first order and Super Twisting SMC in the presence of external noise

## V. CONCLUSIONS:

This research provides the study of the non-linear behavior of the flexible joint robot manipulator. The model derived using Euler-Lagrange method was used to study the effect of the non-linear control techniques on the resulting output trajectories. It is witnessed that both techniques; first and second order SMC gave optimal performances. But, the chattering phenomenon in first order SMC was more domineering than the Super twisting (second order) SMC. Also, the capability of handling external disturbances (the noise in the model) by super twisting was much higher than the first order case. Thus, the second order super twisting SMC is more robust, accurate and desirable than the first order SMC in terms of handling disturbances and nonlinearities in the system.

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