安徽中考(2017)压轴题,虽然条件很少,但考点明确、 考题经典

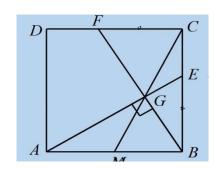
如图,已知正方形 ABCD,点 M 为边 AB 的中点.点 G 为线段 CM 上的一点,且 $\angle AGB = 90^{\circ}$,延长 AG , BG 分别与边 BC , CD 交于点 E , F .

(1) 求证: BE = CF

(2) 求证:
$$BE^2 = BC \cdot CE$$

证明:

$$(1) \angle AEB + \angle CBF = \angle CFB + \angle CBF = 90^{\circ}$$



得

$$\angle AEB = \angle CFB$$

 $\angle ABC = \angle BCD$
 $BE = CF$
 $\Rightarrow \Delta ABE \cong \Delta BCF \Rightarrow BE = CF$

(2)

$$BE^2 = BC \cdot CE \Rightarrow BE^2 = BC(BC - BE) \Rightarrow \frac{BE}{BC} = \frac{\sqrt{5} - 1}{2} \Rightarrow \tan \angle BAE = \frac{BE}{AB} = \frac{BE}{BC} = \frac{\sqrt{5} - 1}{2}$$

利用二倍角公式

$$\angle CMB = 2\angle EAB \Rightarrow \tan \angle CMB = \tan 2\angle EAB \xrightarrow{\text{ = GhAct}} \tan \angle CMB = \frac{2\tan \angle EAB}{1-(\tan \angle EAB)^2} = 2$$

从而解得

$$\tan \angle EAB = \frac{\sqrt{5} - 1}{2}$$

满足题意,得证.