ELEN90052 Advanced Signal Processing

Matlab Project. Part 2.

Kalman filtering

Aim: To estimate states in a dynamical system.

Assessment: The project is assessed based on a written report. The project constitutes 15% of the overall assessment of the course.

Submission of the reports: Reports should be submitted electronically by 4.00 pm, Friday the 26th of May. Submission details will be posted on LMS.

Student groups: The students should work in groups of two.

Reports: The reports should be clearly written and explain in a logical way how the different tasks in the project have been carried out. Choices you make (e.g. state variables, inputs, initial values, covariance matrices etc) should be explained and justified. Results obtained using Matlab should be explained, i.e. it is not sufficient to copy the output of Matlab without further explanations of what the numbers or graphs mean. Figures should be included where it is appropriate. The Matlab code should be included in an appendix.

Collaboration between and within groups: It is perfectly OK to discuss problems and possible solutions with other groups. However, each group has to carry out the project independently, and e.g. copying of other groups' Matlab code is not acceptable. Both group members should do an equal amount of work on both the project itself and the writing of the report. It is not acceptable that one group member does the project and the other writes up the results.

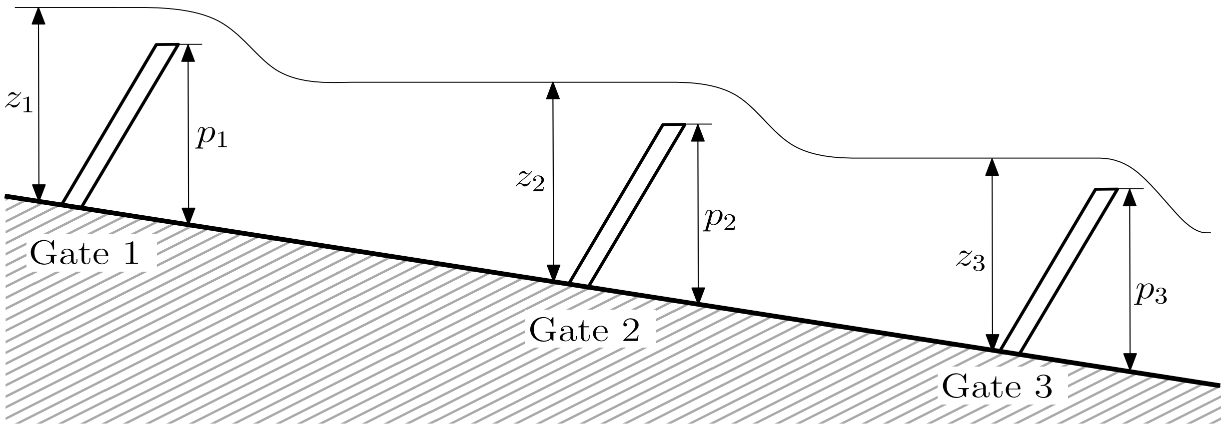

Figure 1: Open water channel.

1 System description

A simplified linearised discrete time model for the water levels in the open water channel pictured in Figure 1 can be written as

$$
\begin{aligned}
z_2(t+1) &= z_2(t) + c_1(z_1(t-1) - p_1(t-1)) + c_2(z_2(t) - p_2(t)) + d_2(t) \\
z_3(t+1) &= z_3(t) + c_3(z_2(t-1) - p_2(t-1)) + c_4(z_3(t) - p_3(t)) + d_3(t)
\end{aligned}
$$

where $z_i,\ i = 1, 2, 3$, is the water level upstream of gate i, and p_i is the position of gate i. d_2 and d_3 are disturbances. The above equations represents simple volume balances, assuming that the flow over a gate is proportional to the amount of water above the gate, i.e. proportional to $h_i = z_i - p_i$. For the flow over the upstream gate of a reach, an additional time delay has been included in order to account for the time it takes for the water to travel from the upstream to the downstream end of a reach. The sampling interval is one minute.

Under normal operations the water levels and gate positions are measured, and the gates are positioned by feedback controllers to control the water levels and flows throughout a network of open water channels. However, in large scale systems sensor and actuator failures happen from time to time, and it is important to have methods to monitor and control the system until the faulty components are replaced or repaired.

In this project we consider the situation where there is a total power loss at Gate 2, such that we have no measurements of the water level z_2, and Gate 2 cannot be moved and is in a fixed position p_2 which is known. Moreover, we also assume that Gate 3 is in a known fixed position p_3, but we do have access to measurements of the water level $z_3(t)$ via

$$y_3(t) = z_3(t) + w_3(t)$$

where $w_3(t)$ is the measurement error.

There is a large reservoir upstream of gate 1 and the water level z_1 can be assumed to be constant. Further assume that Gate 1 can be positioned without positioning error.

The fixed water level and gate positions are $z_1(t) = z_1 = 3.00$, $p_2(t) = p_2 = 1.50$ and $p_3(t) = p_3 = 1.75$, all in meters.

The model parameters are $c_1 = -c_2 = 0.04$ and $c_3 = -c_4 = 0.08$. $d_2(t)$, $d_3(t)$ and $w_3(t)$ are assumed to be mutually independent sequences of independent and identically distributed zero mean random variables with variances $\sigma^2_{d_2} = \sigma^2_{d_3} = (0.005)^2 m^2$ and $\sigma^2_{w_3} = (0.00375)^2 m^2$.

We want to implement a Kalman filter so that we can monitor the water level z_2 in order to avoid flooding or draining of the first reach.

1.1 Project tasks

The tasks should be carried out using Matlab. You are **not** allowed to use inbuilt Matlab routines for state estimation. (You are allowed to use routines for solving the algebraic Riccati equation.)

Note: Students can introduce additional assumptions which do not violate the main specifications.

Part 1

a) Derive a state space model of the system. The initial water levels $z_2(1)$ and $z_3(1)$ should be above the gate positions p_2 and p_3 respectively. Hint: It is not necessary that z_2 and z_3 are states, but you should be able to compute them from the states and given information.

b) Simulate the system over a six hours period. Let the gate position $p_1(t)$ be a signal which varies between two levels between 2.5 and 2.7m, e.g. a binary signal like the one shown in Figure 2.

c) Implement the Kalman filter both in predictor and filter form using the true values of the covariance matrices. Plot the true, measured and estimated water levels and the Kalman gain. Compare the squared estimation errors for the water levels z_2 and z_3 for the two estimators. Also compare the covariance matrices for the estimation errors. Any comments.

d) Implement the time invariant Kalman filter and compare with the results in **c)**.

e) Implement the Kalman filter, but with incorrect values for the variance of the measurement noise. Compare with **c)**.

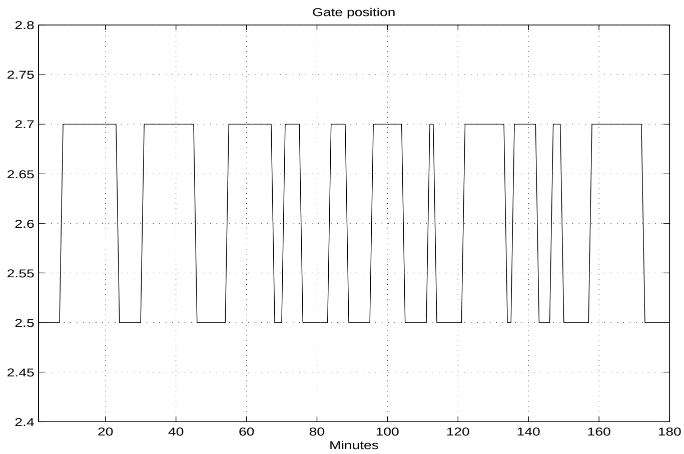

Figure 2: Example of gate position signal

f) Implement the Kalman filter with different covariance matrices for the initial state. Compare with **c)**. You may want to run simulations with different values of the initial state in order to better see the effect of changing the covariance matrix.

g) Let all random variables be jointly Gaussian. Run the Kalman filter from **c)** (using the correct values for all variances and covariance matrices), and compute a 90% confidence interval for the water level $z_2(360)$, i.e the final water level. Describe and implement a test in Matlab for checking that your confidence interval contains the true water levels with the desired probability.

h) As in **g)**, let all random variables be jointly Gaussian, and run the Kalman filter from **c)** with the correct values. Compute a 90% confidence ellipsoid for the water levels $[z_2(360)\ z_3(360)]^T$.

Part 2

The channel operators suspect that water is leaking out of the channel between Gate 1 and 2. The model for the water level z_2 is modified to

$$\begin{aligned} z_2(t+1) &= z_2(t) + c_1(z_1(t-1) - p_1(t-1)) + c_2(z_2(t) - p_2(t)) - q_{\text{out}}(t) + d_2(t) \\ q_{\text{out}}(t+1) &= q_{\text{out}}(t) + d_4(t) \end{aligned}$$

where $d_4(t)$ represents random perturbations to the outflow.

In the simulations of the true system, let the outflow $q_{\text{out}}(t)$ be a constant q_c for all t. The q_c value should be chosen such that the water level $z_2(t)$ remains higher than p_2. Increase the simulation time to 12 hours.

a) By experimenting with different values used in the Kalman filter for the mean and variance of the initial $q_{\text{out}}(1)$ and the variance of $d_4(t)$ find a Kalman filter which manage to give reasonable estimates the water levels z_2 and the outflow q_{out} for a range of q_c

values.

b) Assume now that leaks occur both between Gate 1 and 2 and between Gate 2 and 3. Modify the equations, and repeat **a)**. From physical considerations explain why it is now more difficult to estimate the water level z_2 and the outflow q_{out}.

Part 3

A more realistic model is

$$
\begin{aligned}
z_2(t+1) &= z_2(t) + \bar{c}_1(z_1(t-1) - p_1(t-1))^{3/2} + \bar{c}_2(z_2(t) - p_2(t))^{3/2} + d_2(t) \\
z_3(t+1) &= z_3(t) + \bar{c}_3(z_2(t-1) - p_2(t-1))^{3/2} + \bar{c}_4(z_3(t) - p_3(t))^{3/2} + d_3(t)
\end{aligned}
$$

with $\bar{c}_1 = -\bar{c}_2 = 0.025$ and $\bar{c}_3 = -\bar{c}_4 = 0.05$.

a) Implement and simulate an Extended Kalman Filter for the above system and compare with the Kalman filter from Part 1 **c)**.

b) Repeat **a**), but let the variations in $p_1(t)$ be larger.

1.2 Approximate weighting (Total: 15 marks)

Part a):	9 marks
Part b):	3 marks
Part c):	3 marks