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A Discussion on Sparse Identification for Nonlinear Dynamics (SINDy)

By:
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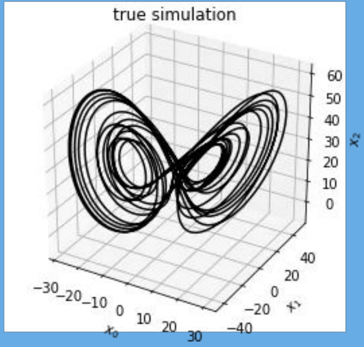
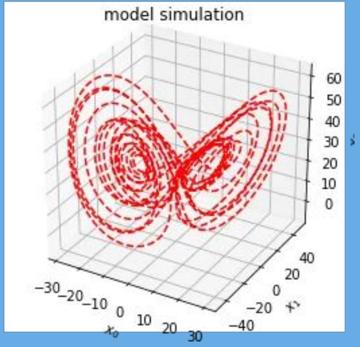
Disclaimer: Nothing I talk about here is new work. This presentation is designed as a teaching tool.

Agenda

- Agenda
- BLUF
- Introduction
- What is a Dynamical System
- Why Are Dynamical Systems Important
- SINDy
  - How it works
  - Examples
- Conclusions
- Citations



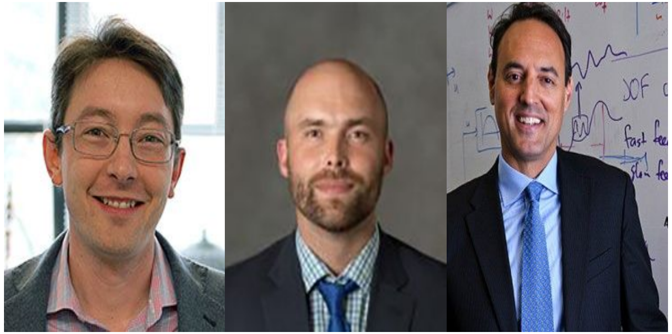


BLUF

SINDy is a regression framework for the discovery of parsimonious dynamic models and governing equations from time-series data.

Introduction

- Paper:
  - **Discovering governing equations from data by sparse identification of nonlinear dynamical systems**
- By:
  - Steven L. Brunton, Joshua L. Proctor, and J. Nathan Kutz
- Journal:
  - Proceedings of the National Academy of Sciences of the United States of America (PNAS)
- Year:
  - 2016



Brunton Proctor Kutz

What is a Dynamical System

- Systems of equations that describe physical, biological, or other phenomenon that evolve in time
- If we have access to the governing DEs, then there are lots of things you can do with them and their data
    - Evolve the system for possible future time values through integration
    - Finite-time Lypanov exponents (FTLE)
    - Uncertainty quantification- generalized polynomial chaos (gPC)

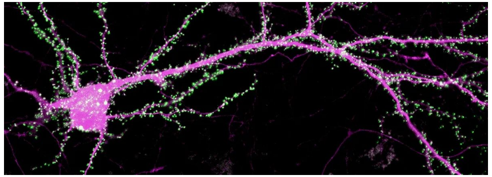
<u>**Dynamics:**</u>

$$\frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x}) \Longrightarrow \mathbf{F}_t\left(\mathbf{x}\left(t_0\right)\right) = \mathbf{x}\left(t_0 + t\right) = \mathbf{x}\left(t_0\right) + \int_{t_0}^{t_0+t} \mathbf{f}(\mathbf{x}(\tau))d\tau$$

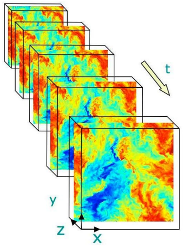
$$\Longrightarrow \mathbf{x}_{k+1} = \mathbf{F}_t\left(\mathbf{x}_k\right)$$ (Discrete-time update)

Why Are Dynamical Systems Important?

- Dynamical systems occur frequently in nature and are critical to science, engineering, and social problems.
  - I.E. guidance of satellites, turbulence of boiling water, dynamics of Earth's atmosphere, electrical activity in the brain
- **Problem:**
  - In many fields,we do not have **exact** governing equations yet to describe the data
- **Solution**
  - We need to use experimental data to discover the governing equations



JHU Neuroscience



JHU Turbulence Lab

The solution: SINDy

Suppose we have a set of measurements $x(t) \in \mathbb{R}^n$ from some physical system at various points in time t. SINDy seeks to represent the time evolution of $x(t)$ in terms of a nonlinear function f :

$$\frac{d}{dt}x(t) = f(x(t))$$

This equation constitutes a **dynamical system** for the measurements $x(t)$. The vector $x(t) = [x_1(t), x_2(t), \ldots x_n(t)]^\top$ gives the state of the physical system at time t. The function $f(x(t))$ constrains how the system evolves in time.

The key idea behind SINDy is that the function f is often sparse in the space of an appropriate set of basis functions. For example, the function

$$\frac{d}{dt}x = f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} = \begin{bmatrix} 1 - x_1 + 3x_1x_2 \\ x_2^2 - 5x_1^3 \end{bmatrix}$$

is sparse with respect to the set of polynomials of two variables in the sense that if we were to write an expansion of the component functions of f in this basis (e.g. $f_1(x) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{i,j}x_1^i x_2^j\Big)$, only a small number of coefficients $(a_{i,j})$ would be nonzero.

SINDy employs **sparse regression** to find a linear combination of basis functions that best capture the dynamic behavior of the physical system.

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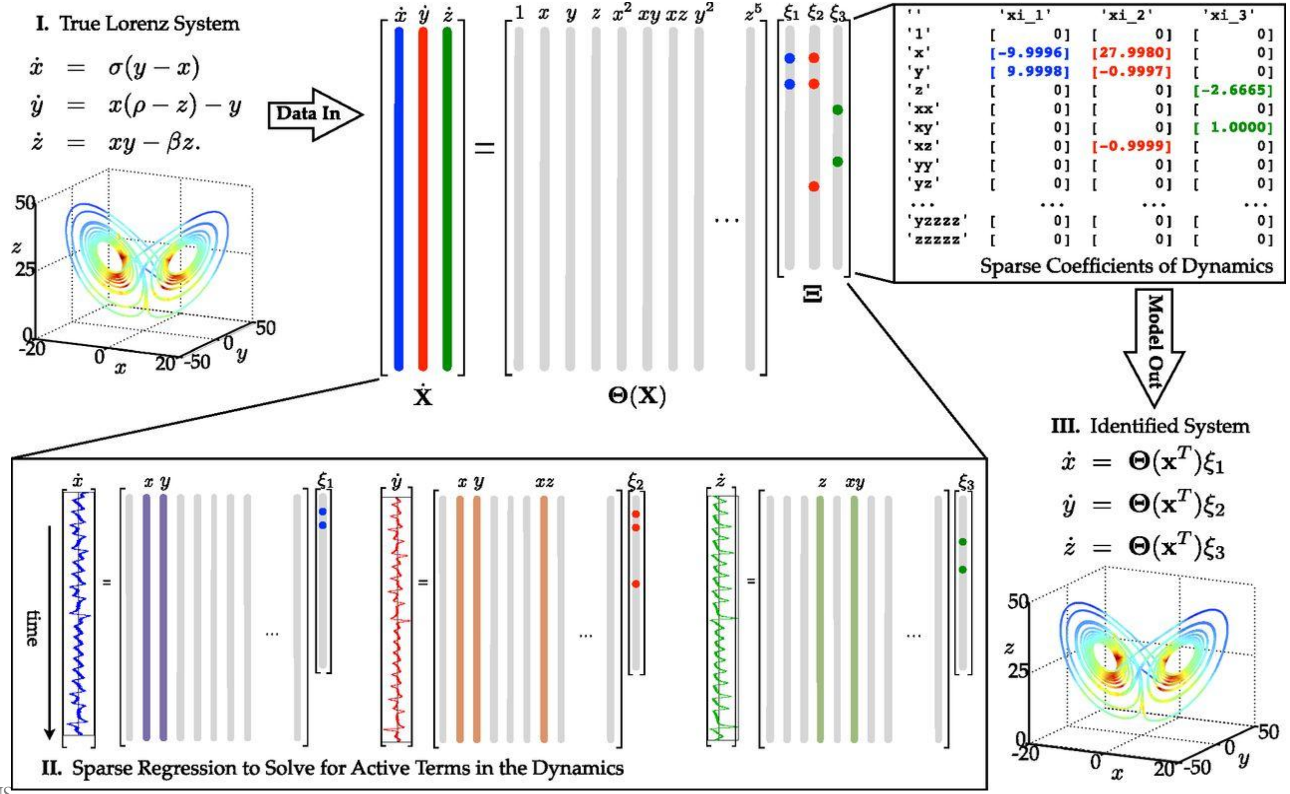
SINDy: How it Works



SINDy: Examples (Jupyter Notebook)

Conclusions

SINDy is an incredibly strong algorithm that has helped many scientists discover parsimonious governing equations from their experimental data

Citations

- S. Brunton, J. Proctor and J. N. Kutz, Discovering governing equations from data by sparse identification of nonlinear dynamical systems, Proceedings of the National Academy of Sciences (2016).
- Brian M. de Silva, Kathleen Champion, Markus Quade, Jean-Christophe Loiseau, J. Nathan Kutz, and Steven L. Brunton., (2020). *PySINDy: A Python package for the sparse identification of nonlinear dynamical systems from data.* Journal of Open Source Software, 5(49), 2104, https://doi.org/10.21105/joss.02104
- Alan A. Kaptanoglu, Brian M. de Silva, Urban Fasel, Kadierdan Kaheman, Jared L. Callaham, Charles B. Delahunt, Kathleen Champion, Jean-Christophe Loiseau,J. Nathan Kutz, and Steven L. Brunton. *PySINDy: A comprehensive Python packagefor robust sparse system identification.* arXiv preprint arXiv:2111.08481, 2021.
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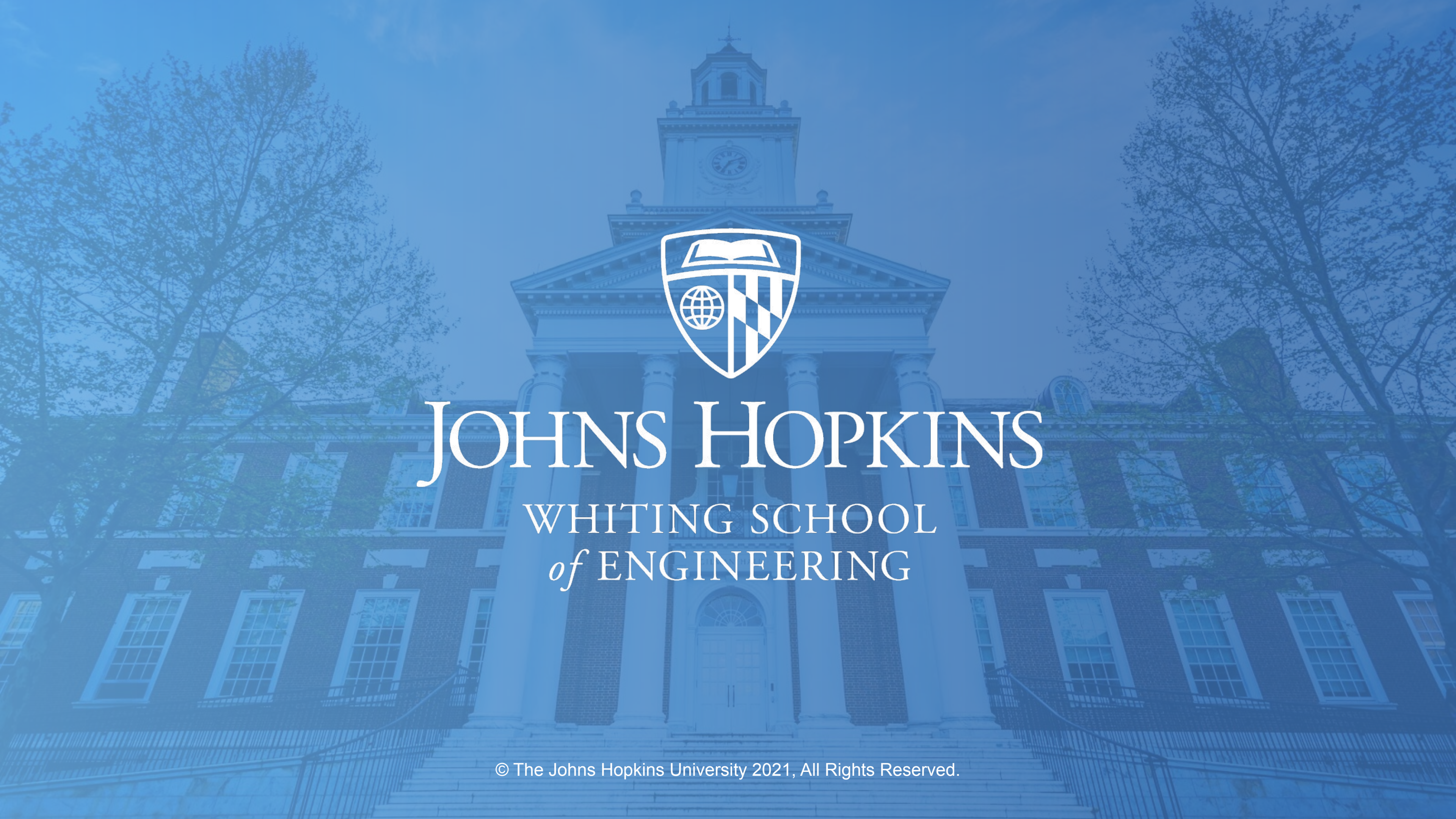

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