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The Multi-Resolution Clean and its application to the short-spacing problem in interferometry

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Summary. A modification of the CLEAN algorithm is described which alleviates the difficulties occurring in CLEAN for extended sources. This is accomplished by appropriately combining the results of a number of conventional CLEAN operations with optimized parameters, each done at a different resolution. This algorithm can properly be called "Multi-Resolution CLEAN" or "MRC". Experiments on model sources show that this deconvolution method works well even when the source is so large that the usual CLEAN becomes impractical. Further, for observations of extended sources, MRC enhances the signal-to-noise ratio, resulting in an easier definition of the area of signal. Moreover, MRC is in principle faster than a standard CLEAN because less δ-functions are needed.

Key words: data analysis – image processing

1. Introduction

For high angular resolution studies of astronomical sources at radio wavelengths, one uses an interferometer, which measures the Fourier Transform of the sky intensity distribution. In general the data are incomplete and one has to resort to a deconvolution algorithm to remove aperture effects. For this purpose the CLEAN method (Högbom, 1974) was developed.

CLEAN is optimized for sources which are not very extended compared to the size of the synthesized beam (see also Sect. 5.1). Problems do arise if the source is well-resolved. The main adverse effects for that case are described in Sect. 2. A number of methods to deal with these have been proposed. Most involve changes to the basic CLEAN algorithm and sometimes extra assumptions are introduced. Here we describe an algorithm, the Multi-Resolution Clean (MRC), which does not modify the basic CLEAN but instead separates the process into several steps. In each step a relatively simple CLEAN with parameters optimized to give a good flux estimate is done (by adjusting the gain and search area). Two intermediate maps are constructed, the first (called the smooth map) by smoothing the data to a lower resolution and the second (called the difference map) by subtracting the smoothed map from the original data. Both these maps are then treated separately. In Sect. 3 we show that for noise-free data, if the intermediate results are properly combined, the δ-functions found by MRC converge theoretically to the same limit as those found by CLEAN.

The advantage of smoothing the map is that extended sources become more point like. By cleaning the difference map also, it is possible to retain the original resolution. For noisy maps of extended sources the MRC algorithm results in an increase of the signal-to-noise ratio during the deconvolution process, allowing more reliable definition of the parts of the map where signal is present. To test the Multi-Resolution Clean algorithm a number of experiments was done, comparing CLEAN to MRC for different source sizes and signal-to-noise ratios. The results can be found in Sect. 4. A discussion of the properties of MRC is presented in Sect. 5.

2. Applying CLEAN to an extended source

2.1. *The* CLEAN *algorithm and extended sources*

The CLEAN method is generally used to remove artifacts in interferometric maps that suffer from incomplete coverage of the aperture plane: missing baselines; missing or deleted hour-angle ranges; unmeasured short-baseline information. The calibrated uv data are Fourier transformed and give the so-called "dirty map", which CLEAN tries to decompose into a set of δ-functions. This is done iteratively by finding the point with the largest absolute brightness and subtracting the dirty beam scaled with the product of the loop gain and the intensity at that point. The resulting "residual map" is then used to repeat the process. The process is stopped when some prespecified limit is reached, resulting in a residual map and a collection of δ-functions. The convolution of the δ-functions with the dirty beam plus the residual equals the dirty map. To obtain the "clean map" one instead convolves the δ-functions with a "clean beam" which has no sidelobes and adds the residual. This "clean map" is used as the best possible approximation to the intensity distribution as a non-distorting telescope with finite resolution and sensitivity would observe it.

CLEAN not only deconvolves the dirty map but, by searching for the largest (absolute) value in the map, it also automatically defines the regions in the map where signal is present. In cases of low signal-to-noise this delineation can become difficult. CLEAN may then start to find δ-functions in places where only noise peaks are present. For sources for which the size is relatively well known (like galaxies) limiting the area in which δ-functions are searched for generally greatly improves the reliability of the result and the speed of the calculation. Using a small loop gain can also make the signal-area definition more reliable but the increased number of δ-functions required to complete the deconvolution properly can cause the calculation time to become prohibitively large.

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Furthermore, for extended sources the contribution to the integrated flux from an extended, low-level, intensity distribution can be substantial, even if the brightness in each pixel is below the noise level. Therefore one would need to apply CLEAN to very low flux levels to recover all the signal, but in practice it is difficult to use CLEAN below a few sigma. Smoothing the map to a lower resolution allows to improve the signal-to-noise ratio per pixel for extended sources. Some interesting information is then lost however.

Apart from effects associated with low signal-to-noise, a number of fundamental and practical problems arise for "extended" sources (in this context a source larger then ~ 10 synthesized beams). First, the extent of the source as measured by the number of independent synthesized beams to describe it may not exceed the number of observations (real plus imaginary values). If the uv-coverage is regular this translates into the condition that the diameter of the source may not exceed the radius of the first grating response. This can be easily shown for the one-dimensional case: suppose one samples an aperture on N regularly spaced points ranging from $-N/2$ to $+N/2$. The dirty beam will have a Full-Width-at-Half-Maximum (FWHM) of $1/N$ and the first grating response will be at a distance 1 from the center, thus there are N independent synthesized beams within the first grating radius. CLEAN may only find δ-functions on N positions, hence in some cases the equivalent system of linear equations will be underdetermined (Schwarz, 1978). For example, a source that is larger than the radius of the first grating ring can never be completely and reliably deconvolved by CLEAN.

Second, when the synthesized beam has strong side lobes, then CLEAN can introduce "corrugation", parallel stripes in the "clean map". These are artificially created because δ-functions are found at the position of spurious peaks created by the sidelobes of a dirty beam subtracted at a nearby position. The clean map that is produced is consistent with the data but not with a realistic brightness distribution. This effect was more fully described by Schwarz (1984) and can become severe for observations with incomplete uv coverage.

Third, severe problems are caused by missing short-spacing information: a) for extended sources a large part of the signal will be confined to the short interferometer spacings and thus only a small part of the Fourier Transform (FT) is sampled; b) the zero level depends on the intensity distribution of the source as the integral across the whole map must be zero. This variation of the zero level gives rise to what is commonly called the "negative bowl". The origin of the bowl can be understood by considering the following example: assume the source has a box-like intensity distribution, so that its FT is a sinc^2 function. The central part of this sinc^2 is missing from the observation. In the map this corresponds to the subtraction of the FT of the central part of the sinc^2, which results in a broad depression.

Mapping the source with a single-dish telescope that is larger than the shortest measured interferometer spacing, to fill in the missing short-spacing visibilities, is the correct way to get rid of this problem. This is not always feasible in practice however and it can be difficult to find the proper calibration for combining the single-dish and interferometer maps.

If a source is smaller than the radius of the first grating ring, then it is possible in principle to use CLEAN to estimate the unmeasured visibilities for the shortest spacings, since these visibilities are directly related to those of longer spacings and spacings shorter than the longest non-measured spacing contain no extra information (c.f. the box-like intensity distribution above). On the other hand, if a source is so large that the visibilities are completely confined to the inner uv-plane (theoretically this implies a source of infinite size), it is not possible to recover any short-spacing information. In practice the presence of noise limits the size of the largest structure that can be recovered and the reliability of the estimate of the visibilities in the inner uv-plane.

2.2. *Proposed solutions for applying* CLEAN *to extended sources*

Several prescriptions for improvements to CLEAN to make it more reliable and efficient for extended sources have been published. Cornwell (1983) described a method to modify CLEAN by adding a δ-function to the beam. This method is not suitable for very extended sources, because then the solution tends to converge to the dirty map.

A paper published by Braun and Walterbos (1986) describes a way to extrapolate the data in the uv-plane. In that way they are able to rectify the zero level of a map. A disadvantage of their method is that it is only applicable when the source is strongly limited in size. Moreover good uv-coverage is needed. Side-lobe effects caused by incomplete sampling of the uv-plane are not removed and may even interfere with the process because of the way in which they define the area of signal.

Another proposal was made by Steer et al. (1984). They call an area in which the intensities are above a certain cutoff a "component" and convolve the data in that area with the beam. After the use of an appropriate scale factor the convolved map is then subtracted from the original data and the process is repeated. A large increase in speed is gained thereby and corrugation is effectively suppressed. But for low signal-to-noise it is still difficult to find the position of the δ-functions.

In 1984, Brinks and Shane described a way to use CLEAN to deconvolve a smoothed dirty map and still keep the original resolution. They did this by constructing a "correction map" from the difference of the smoothed dirty map and the corresponding smooth clean map. This correction was then added to the original map. They called this method the "Multi-Resolution Clean". In Sect. 3 we will describe a method that goes a logical final step further and is equivalent to using the original CLEAN algorithm on the full-resolution map. We think that this new method should be called "Multi-Resolution Clean" or "MRC" for short, as will become clear below.

3. Multi-Resolution CLEAN

3.1. *The smoothing idea*

One way to decrease the number of δ-functions needed and to go to lower levels, is to smooth the map and the dirty beam and use CLEAN on the smooth map because this produces improvements in three ways:

1) the ratio of source size to beam size becomes better;

2) the signal-to-noise ratio of the smoothed map will be much better than that of the original map;

3) by regridding the smoothed map so that the same sampling is kept, smaller maps can be used during cleaning.

An objection to smoothing the map is that one loses interesting information on the small-scale structure, the study of which is often the goal of the observation. With MRC the resolution problem is overcome by constructing the so-called "difference map" along with the smoothed map and using a standard CLEAN on this "difference map" also, with a "difference dirty beam". The circumstances under which this is possible will be discussed in

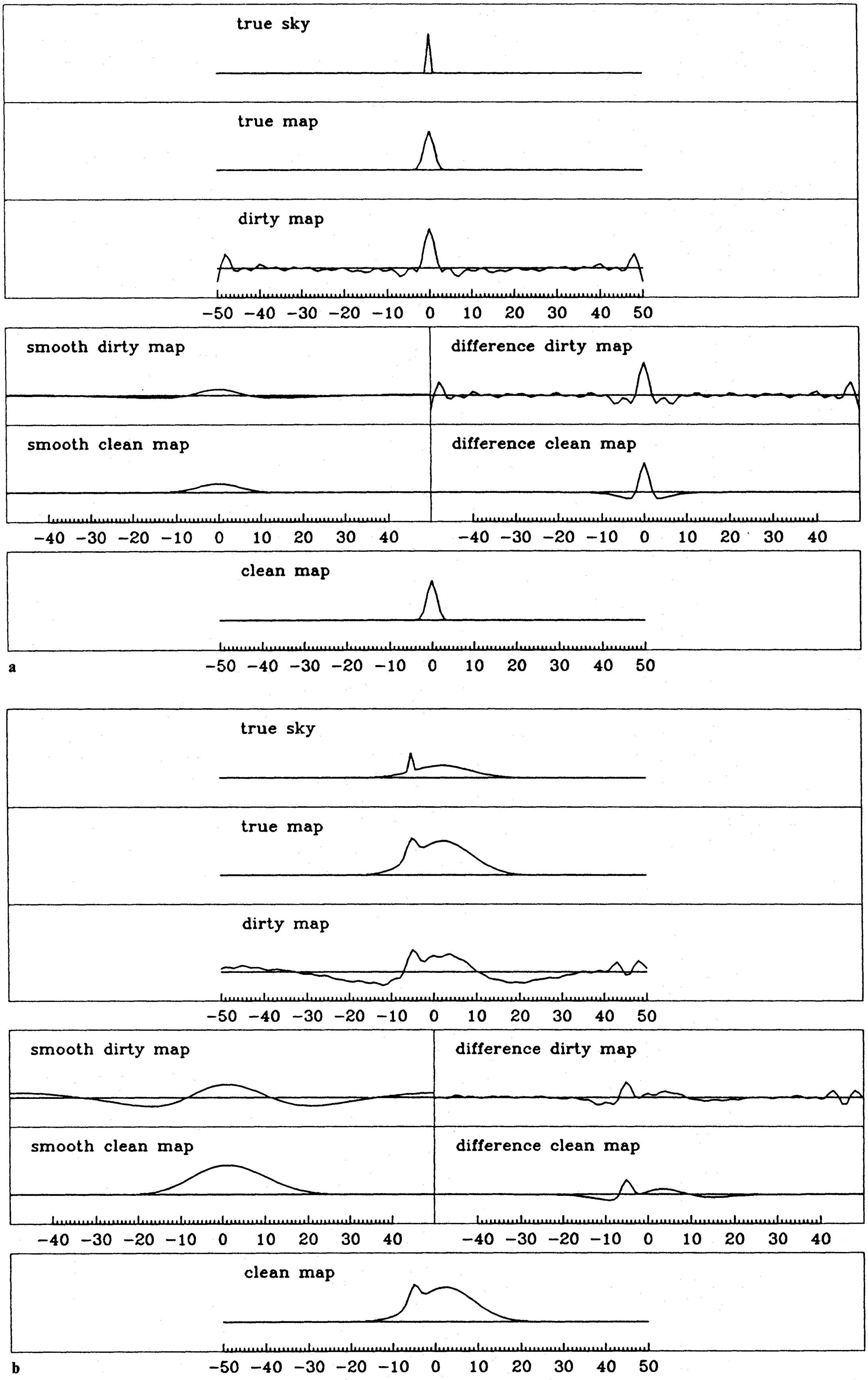

Fig. 1 a and b. Profiles showing the steps of the MRC process for **a** a point source and **b** an extended source plus point source. Drawn are the dirty and clean maps for the original, smoothed and difference case. The bowl is evident in the original and smoothed dirty maps of **b**. Also note the "sidelobes" in the difference clean map, which make sure that the contribution to the total flux from the difference map is zero

Sect. 5.1. In Sect. 3.2 we describe in detail how one has to scale and combine the intermediate results so that a clean map at the full resolution is obtained. Qualitatively the process is illustrated in Fig. 1, which shows what happens for an extended source and for a point source.

As the negative bowl is an extended structure in the map, it is hardly affected by the smoothing process and is therefore absent in the difference map. That map only suffers from side-lobe effects and it contains just the small-scale structure. Since the ratio of source size to beam size is small, CLEAN can be used effectively on that map. Also CLEAN need not be applied to low levels, because the contribution to the total flux in the final clean map is zero. Moreover, sidelobes are in general below the noise except those of strong peaks, which are usually well separated. The deep sidelobes of the difference dirty beam do not introduce extra problems. On the other hand, the removal of the bowl is done in the smoothed map for which all advantages that were mentioned above are valid. The improved signal-to-noise ratio allows to clean much deeper than was possible in the original map, therefore most of the flux can be recovered. Corrugation is also suppressed, as is discussed in Sect. 5.5.

3.2. *Combining the smoothed and difference maps*

In order to describe how the clean map at the full resolution is obtained from the smoothed and difference clean map, a number of symbols must be defined. The subscript "s" will stand for the smoothed beam or map ($F_s = F * G$), (where $*$ stands for a convolution) while the subscript "d" indicates a difference function: $F_d = F - F_s$. In the case of beams F_s and F_d are renormalized to a peak value of 1.

- $G =$ the normalized ($\int G(x)\,dx = 1$) smoothing function; the width of this function is chosen such that the FWHM of the smoothed dirty beam is f times larger then the FWHM of the original dirty beam;
- $A =$ dirty beam;
- $D =$ the dirty map i.e. the observation;
- $\delta = \delta$-functions;
- $R =$ residual after using CLEAN on the map;
- $B =$ clean beam with peak value 1;
- $C =$ the clean map:
- $s =$ the scale factor of the dirty beam needed to rescale the smoothed dirty beam back to a peak value 1;
- $r =$ the scale factor of the clean beam needed to rescale the smoothed clean beam back to a peak value of 1.

Both the smoothed and the difference dirty beam are rescaled such that the peak value is 1, i.e.: $A_s = sA * G$ and $A_d = (A - A_s/s)/(1 - 1/s)$. The same relations hold for the clean beam B with the scale factor s replaced by r. From the delta functions found by the CLEAN algorithm one can restore the dirty map by convolving with the dirty beam and adding the residuals. The dirty map is separated into two parts that are treated separately. Therefore:

$$\begin{aligned} D = D_s + D_d &= \delta_s * A_s + R_s + \delta_d * A_d + R_d \\ &= \delta_s * sA * G + \delta_d * \frac{s}{s-1}(A - A * G) + R_s + R_d \\ &= \left\{ s\delta_s * G + \frac{s}{s-1}\,\delta_d * (1 - G) \right\} * A + R_s + R_d\,. \end{aligned}$$

If one instead convolves the δ-functions with a clean beam, the clean map is obtained:

$$C = \left\{ s\delta_s * G + \frac{s}{s-1}\,\delta_d * (1 - G) \right\} * B + R_s + R_d\,.$$

This can be rewritten into two parts, corresponding to the smoothed and the difference dirty map:

$$C = \frac{s}{r}\,\delta_s * B_s + \frac{s(r-1)}{r(s-1)}\,\delta_d * B_d + R_s + R_d\,.$$

3.3. *Additional parameters of* MRC

MRC requires two more parameters than the standard CLEAN: an extra gain because the one used for the conventional CLEAN on the smooth map and that used on the difference map are independent, and the ratio f of the FWHM's of the original and the smoothed dirty beams. The latter depends on the size of the source one wants to clean. For relatively small sources, a factor of about 2 may be best, while for very extended sources larger factors are preferred. The cutoffs for the smoothed and difference map CLEAN operation can also be chosen independently in principle, but as the noise level of the map after smoothing can be estimated from the original noise, the proper cutoff for using CLEAN on the smooth and difference map can be calculated from the usual cutoff in the usual CLEAN.

3.4. *Definition of the search area*

When applying MRC to complex fields with low signal-to-noise ratio the use of a small loop gain is not sufficient in practice to determine the area of signal reliably, even after smoothing. The definition of this area can be done in a number of ways. One can use a cutoff on the data and use those points whose intensity is above a certain limit. Alternatively, a histogram of values can be constructed and all points in the upper percentiles can be used. Yet another possibility is to use CLEAN first and next use the positions of the δ-functions found to define the search area for a second CLEAN. We define the search area by further smoothing D_s and using that "extra smooth map" to define a mask consisting of those points in the extra smoothed map where the signal-to-noise ratio is higher than a certain limit. This takes most advantage of the fact that the signal-to-noise ratio improves with smoothing for extended sources.

The CLEAN operation is then separated into several iterations: first the area in which δ-functions are searched for is defined and a normal CLEAN is used with a certain cutoff. Next the residual map is used to redefine the search area. This residual map replaces the dirty map for the next iteration. The process is repeated several times with decreasing cutoff levels. In this manner CLEAN does not try to find δ-functions at those places where it should not. For high signal-to-noise this method is effectively equal to using a very small loop gain in a conventional CLEAN. The iterative approach is much faster however.

4. Numerical experiments on MRC

4.1. *Selection of models*

The MRC algorithm was tested by constructing a model map, convolving it with a dirty beam and using both CLEAN and MRC on the resulting dirty map after the addition of a noise map. As a model a two-dimensional gaussian intensity distribution was taken with a number of different values for its FWHM. In most cases the FWHM's in x and y were chosen to be equal. The dirty beam was taken from a full-synthesis observation done with the Westerbork Synthesis Radio Telescope. The noise level was chosen by taking a number of different signal-to-noise ratios

Table 1. Results of deconvolving model sources with RMC and with CLEAN; the calculations were done on the VAX 8600 computer of the Rekencentrum of the University of Groningen. The unit of the flux is arbitrary

Source size ('/beams)		0/0	2/6	5/15	10/30	15/45	20/60	5×15/ 15×45
Model	flux	1.00	160	1002	4010	9012	15683	3007
	peak	1.00	5.37	5.52	5.55	5.55	5.55	5.54
Flux measured at (')		2	4	10	15	20	20	20
S/N = ∞								
MRC	flux	0.99[f]	158[f]	939[f]				
	peak	1.00	5.40	5.55				
	# δ-funct.	11	165	6082				
		5	1357	10000				
CLEAN	flux	1.00[f]	160[f]	946				
	peak	1.00	5.60	5.76				
	# δ-funct.	14	1787	10000				
S/N = 20								
MRC	flux	1.06[r]	156[f]	962[f]	2788[f]	402[r]	417[r]	2680[f]
	peak	1.00	5.37	5.43	4.81	1.45	0.25	5.25
	# δ-funct.	301	201	1205	15829	3171	1510	2759
		3	554	448	723	925	901	449
CLEAN	flux	0.88[d]	151[d]	563[c]	653[c]			
	peak	1.00	5.41	5.28	3.97			
	# δ-funct.	1434	1925	7964	11709			
S/N = 5								
MRC	flux		152[f]	1232[f]	3237[f]			
	peak		5.44	561	5.13			
	# δ-funct.		149	1052	9022			
			441	3653	1597			
CLEAN	flux		125[d]	162[c]	327[c]			
	peak		5.14	5.22	3.17			
	# δ-funct.		304	6501	6773			

Notes: f: flat cumulative flux profile; r: still rising cumulative flux profile; d: decreasing cumulative flux profile (i.e. not yet converged); c: very strongly changing cumulative flux profiles (i.e. little flux found)

(strictly speaking the ratio of the peak value to the noise was used). Three different values for this signal-to-noise were taken: ∞, 20 and 5. The smoothing factor f (the ratio of the FWHM of the smoothed dirty beam to that of the original dirty beam) was chosen to be 4 in all tests.

A dirty beam with a FWHM of 20″ was used, so for the chosen source sizes of 0′, 2′, 5′, 10′, 15′ and 20′ the ratios of source size to beam size are 0, 6, 15, 30, 45 and 60, respectively. Also a model source of size 5′ × 15′ (15 × 45 beams) was cleaned. The results are listed in Table 1. In Figs. 2–4 a sample of the results is shown. All plots in these figures are for a signal-to-noise ratio of 20. Each figure consists of four parts correponding to 4 different source sizes. In each part four profiles can be seen: that of the model map without noise, the dirty map, the clean map produced by MRC and the clean map produced by CLEAN. In Fig. 2 the map profiles are shown, while Fig. 3 gives the profiles in the aperture plane. Figure 4 contains the cumulative sum of integrating the maps in rings. For both MRC and CLEAN the parameters were optimized, i.e. for CLEAN a gain of 0.25 and a 1σ cutoff were used, while limiting the search area to that part of the map where signal was expected. The latter was also done for MRC in which case gains of 0.4 and 0.7 were used for the cleaning of the smoothed and difference maps, respectively.

4.2. *Results for gaussian sources*

It is immediately clear that the clean map produced by CLEAN can look reasonably good at first sight, though the Fourier Transform and the integral show that it is still considerably influenced by the dirty beam. MRC gives a much better result, especially where the flattening of the zero level is concerned. For not so extended sources (< 15 beams) CLEAN and MRC are about comparable, as was to be expected, though MRC is already superior for a source with a FWHM of 6 beams.

The total time used by MRC to CLEAN a map does not increase rapidly when the source gets larger. The smoothing and combining steps take a much larger amount of time than what is needed for the step of applying CLEAN to the smooth dirty map. For CLEAN on the other hand, as the source gets larger, the time required may become prohibitively large. Of course, in a realistic case, the comparison might be somewhat more favorable for

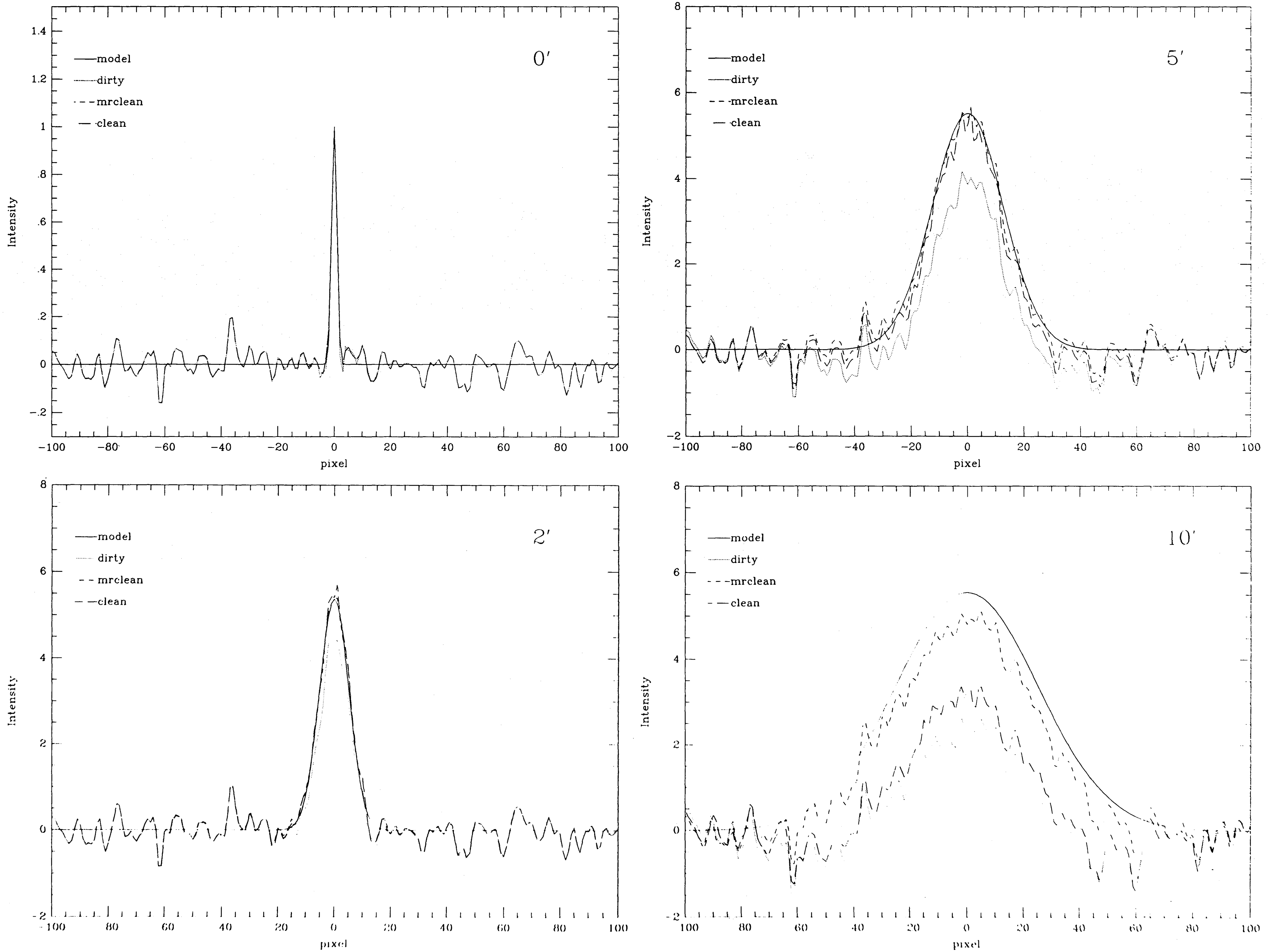

Fig. 2. One-dimensional cuts through the map plane for the model, dirty, CLEAN and MRC-clean maps at 4 different resolutions as calculated from the experiments described in Sect. 4 with a signal-to-noise ratio of 20. The FWHM of the model gaussian is shown in the upper right corner. The dirty beam had a FWHM of 20″. Except in the 10′ case both clean maps follow the model profile closely

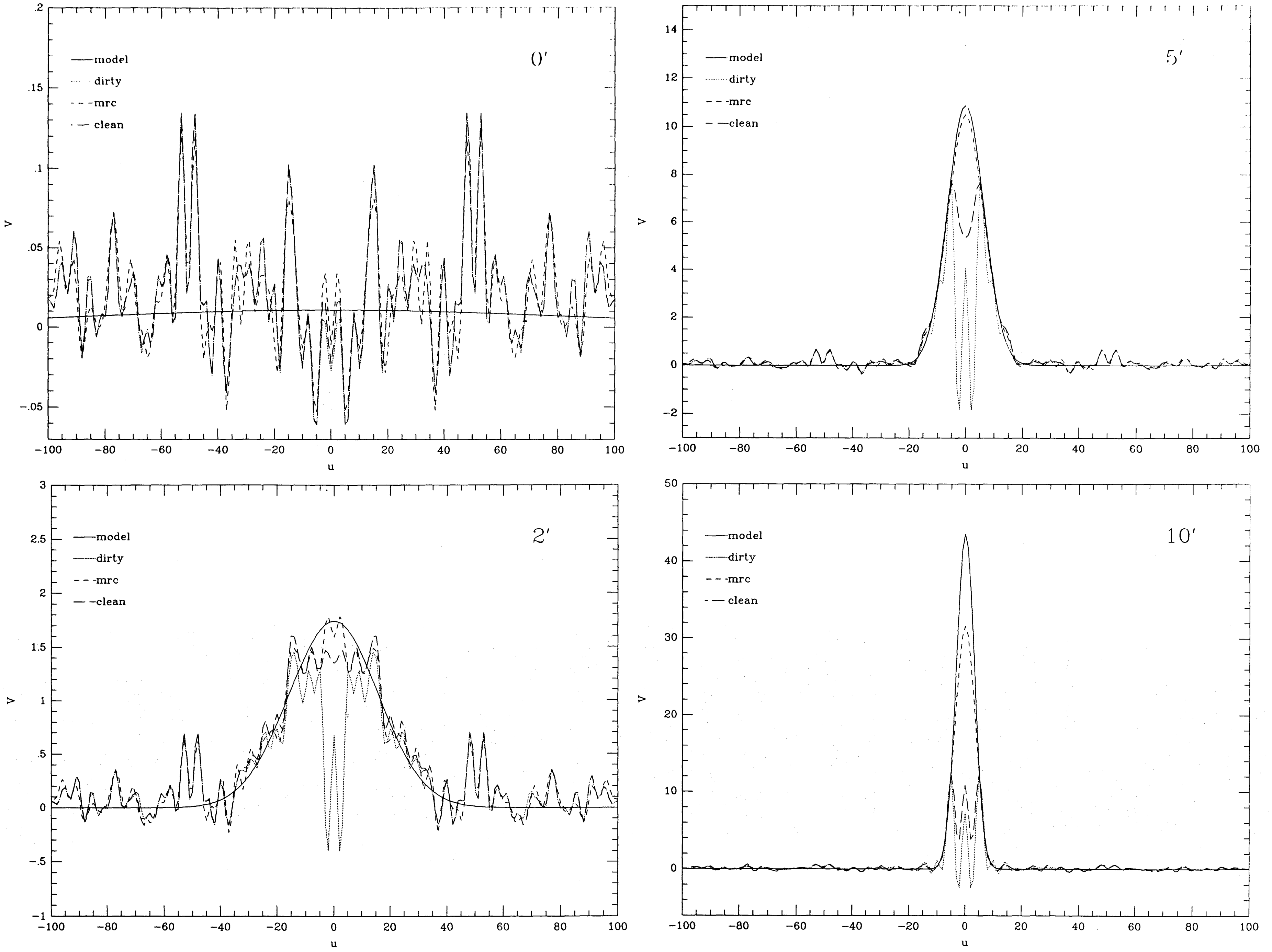

Fig. 3. One-dimensional cuts through the Fourier plane for the model, dirty, CLEAN and MRC-clean maps at 4 different resolutions and signal-to-noise of 20. For $u \sim 0$ the MRC profile more closely follows the model profile than the CLEAN profile. Especially in the 10′ case the CLEAN profile does not differ much from the dirty profile

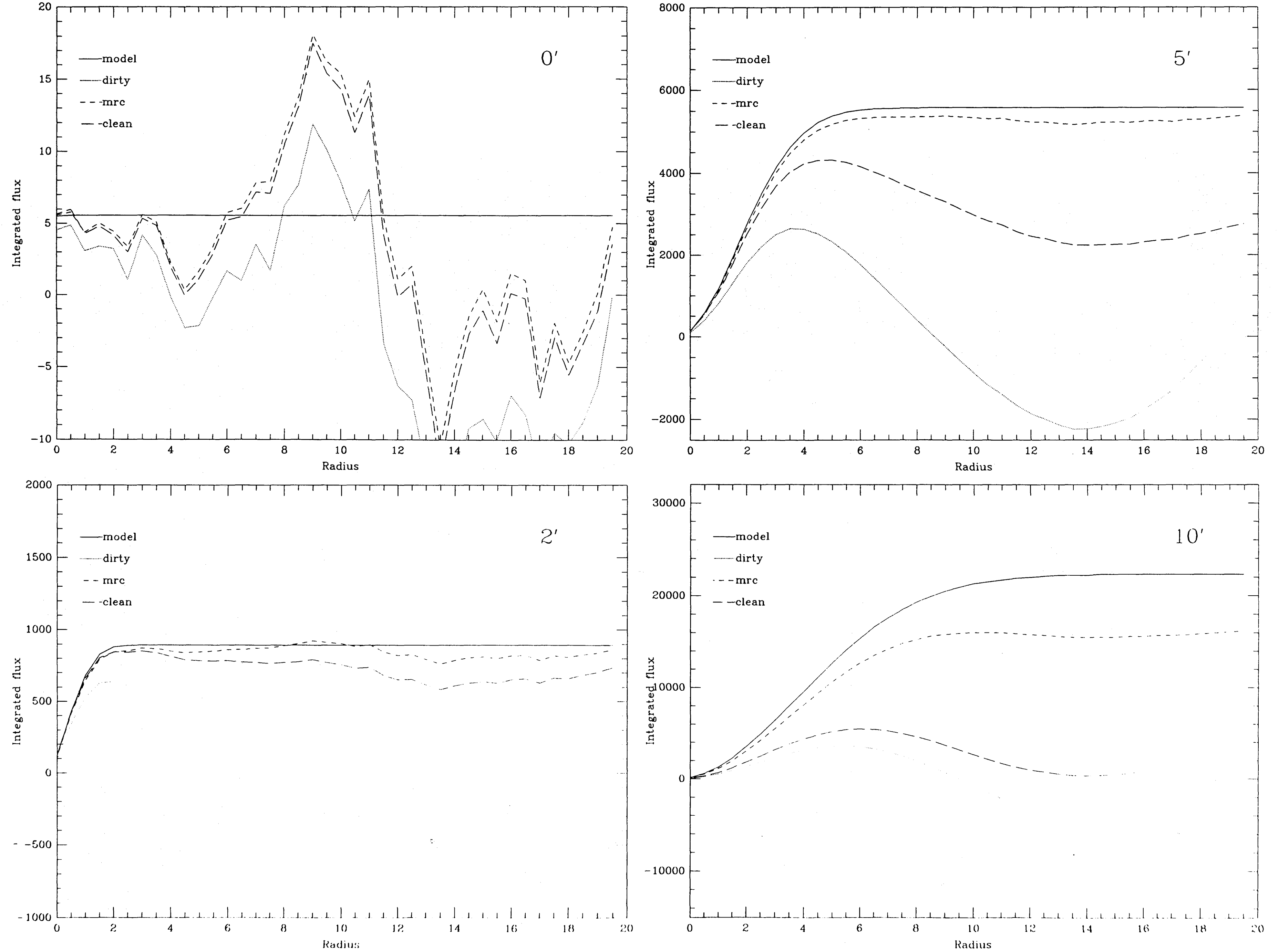

Fig. 4. Integrated flux as a function of distance to the map center for the model, dirty, CLEAN and MRC-clean maps at 4 different resolutions with signal-to-noise ratio of 20. For the point source the noise is rather large. The MRC profiles lie close to the model profile in all cases, while the CLEAN profile still shows the uncleaned remnant of the bowl in the 5′ and 10′ cases

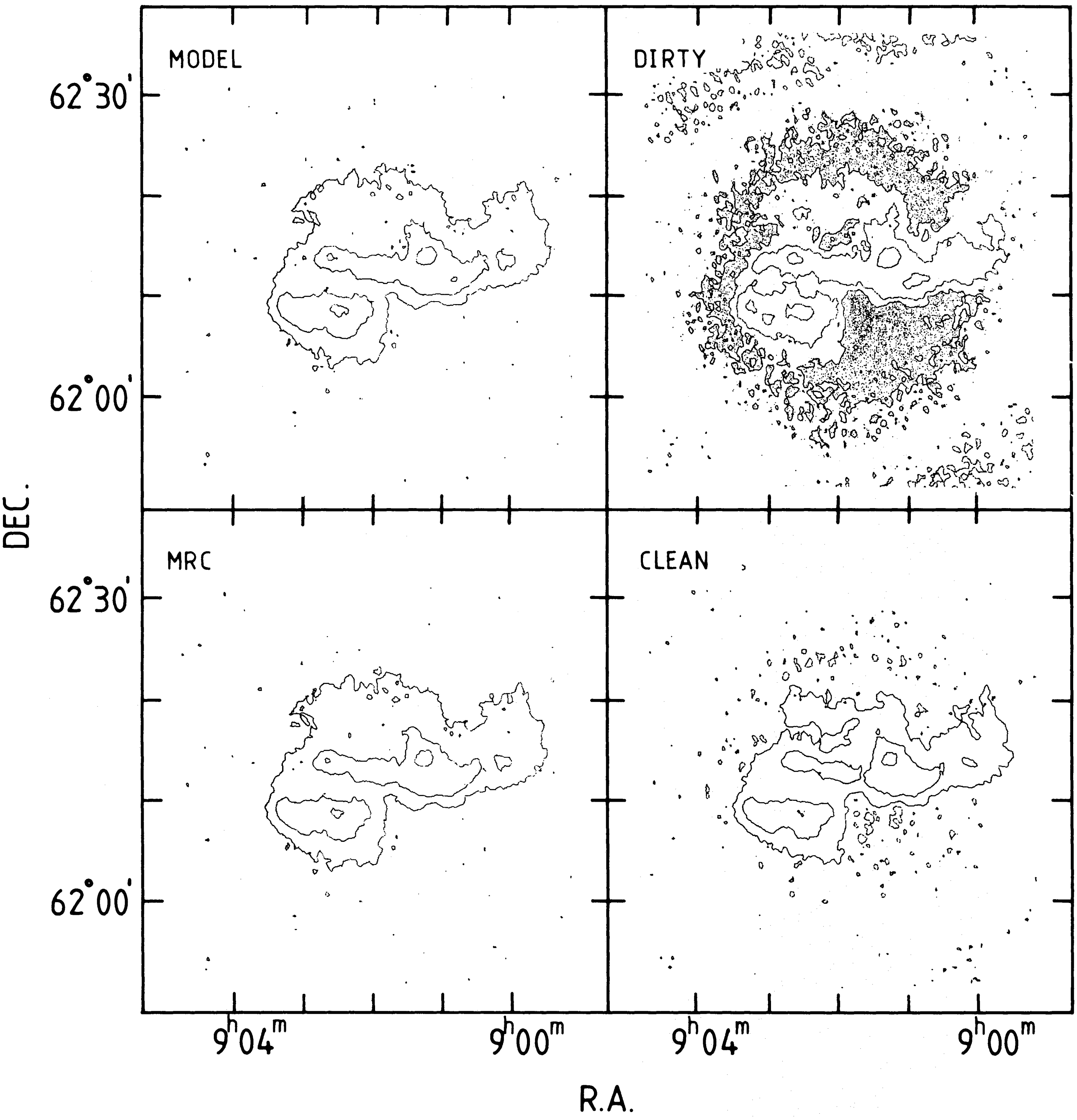

Fig. 5. The model, dirty, CLEAN and MRC-clean map for a realistic source mode. Contour levels are −3, 3, 15 and 30σ. Areas of negative intensities are shaded. The CLEAN clean map took 90 minutes to construct; 15000 δ-functions (1σ cutoff) were found. In the MRC clean map the equivalent of 50000 δ-functions was found in 15 minutes

CLEAN because real sources are not as smooth as a gaussian, so that high spatial frequencies are relatively stronger.

For CLEAN the flux that is recovered in the case of large extended sources becomes a meaningless number, because it is not possible to go deep enough to neglect the flux left in the residual, so that the zero level still varies and the answer depends on how large the area is over which the flux is measured. One might try to use the sum of the flux in the δ-functions instead, but this may give a wrong answer if CLEAN did not go far enough. The fact that in Table 1 the percentage of flux recovered by MRC decreases rapidly for the very large model sources can be understood by looking at the aperture plane. There the FT of a gaussian with 60 beams FWHM falls completely within the shortest observed spacing so that no information is left. In the 15×45 beams case however, enough information is present so that MRC still can get a reasonable answer.

4.3. *Results for a realistic source*

Another test was done using a clean map of a real source prepared with MRC as the model. In principle there is no signal at short baselines in this model that does not correspond to information on the long baselines. Therefore MRC should be able to recover the map exactly. The size of the source is about 30′ or 90 beams, though it appears to be only about 15′ in diameter in the dirty map. The result is given in Fig. 5. Again CLEAN can not obtain a useful result, while MRC can. To produce the maps shown, MRC needed only one third of the time. The original flux in the model map was 10000 units, MRC recovered 9358 of them in $16*2822+1049=46201$ δ-functions, while with CLEAN (after 15606 δ-functions) only 6910 units were found and the bowl is still present.

5. Discussion

5.1. Theoretical limitations

When Högbom (1974) introduced the CLEAN method, the justification given was that the sky is essentially empty with a few small sources scattered around. We extend this assumption by the following: there is a resolution at which the sky appears essentially empty. This implies a hierarchical source structure, which can occur for a collection of similar objects at varying distance or objects at the same distance but caused by different physical processes (like lobes and peak sources in radiogalaxies). Then, if a map shows details and is smoothed the details become lost but other structures turn up. If the latter cover only part of the sky we can CLEAN these, describing them with fewer parameters, consistent with the smaller amount of information present.

Schwarz (1978) showed that in the absence of noise, CLEAN can in principle fit the observed visibilities exactly. The same will be true for the Multi-Resolution Clean method. But in the presence of noise it is in practice not possible to obtain the exact result, especially for extended sources where a number of theoretical limitations increase the difficulties. For these sources MRC can approach the exact result better than a standard CLEAN.

5.2. Total flux estimation

The results of the experiments with model sources show that MRC is able to recover most of the flux in a dirty map. The reason is that MRC finds the flux in the smoothed map, so that the signal-to-noise ratio is much better and more δ-functions can be found before the limit imposed by the noise level is reached. The only part of the flux that can never be found in principle is that corresponding to structures whose visibility function is completely confined to the unmeasured spacings. This implies for instance that one can recover an elongated structure that is too large in one dimension but small enough in the other, as is shown in the experiment with the 15×45 beams gaussian model source. Therefore, in the case of galaxies, it should in general be possible to do without single-dish observations. On the other hand, for observations if interstellar matter in our own galaxy, it is conceivable that structure with too large extent in all directions is present.

The experiments (see Fig. 4) show that, while for MRC the integrated flux in the clean map converges, for CLEAN there is still a remnant of the bowl present which causes the "flux" to vary with radius. The main reason for this difference is that one can not go as far with CLEAN as with MRC in obtaining δ-functions because the signal-to-noise ratio is much worse in the CLEAN case. The sum of all δ-functions might be used to measure the flux instead, however any residual flux that was not recovered is then not taken into account.

5.3. Aspects of MRC in the uv-domain

The smoothing of the dirty map can also be done in the uv domain, in the map-making stage, by applying a taper to the uv-data. The difference map is then created by using a taper that is the difference of two taper functions with different widths, such that they cancel each other at the zero-spacing. The clean beams should then be computed from the same tapers.

An obvious application of MRC that can be understood by looking at the uv-plane is to try to make use of the full resolution information available by applying uniform weighting to the uv-data. The smoothed map can be constructed by using a strong taper. This will suppress "ringing" in the map plane, allowing a more reliable CLEANing. The sharp frequency cutoff introduces distortions in the difference dirty beam, but these are relatively easy to handle, because the structures in the difference dirty map are mainly on small scales.

In the cleaning of the difference map short spacings are also recovered. By applying the difference clean beam to the δ-functions they are suppressed when constructing the difference clean map, so that no spurious large-scale structure is introduced.

5.4. The full Multi-Resolution Clean

The combination of smoothed and difference map described in Sect. 3.2 is strictly speaking only a "Double-Resolution Clean". A genuine *Multi*-Resolution Clean can be constructed by repeating the process of making smooth and difference maps. The smoothed map of the stated formulation then replaces the original dirty map in a subsequent iteration. The smoothed maps at different resolutions will correspond to different scales in the map. The process can of course be repeated many times and it is possible in principle to obtain only a few δ-functions per dirty map so that every CLEAN step is fully optimized. In practice this is not necessary and the "Double-Resolution Clean" suffices to get a good result as is shown in Sect. 4. Making many different maps will increase the time needed for the process, however. With only a two step process the method is faster than CLEAN for extended sources, while it is slower for small sources because of the overhead needed for the many combinatory steps. So for small sources there is a trade-off between accuracy and speed, while for extended sources MRC is faster as well as better than CLEAN.

5.5. Corrugation

Corrugation can occur for extended sources as a modulation of the source; in the uv-domain this corresponds to a δ-function occurring at the edge of the sampling or in small holes in the visibility sampling (Schwarz, 1984). With MRC the smoothing corresponds to a taper in the uv plane so that only the shorter spacings are used. Then the holes generally lie outside the radius of the longest used spacing and the spatial frequency of the sidelobes of the smoothed dirty beam is well within the sampling region, so that corrugation no longer occurs. If the holes responsible for corrugation are at short spacings, special convolution functions could be used to enforce suppression of corrugation.

5.6. The speed of MRC

In general the speed of CLEAN or MRC depends on many external parameters, e.g. whether or not smoothing is done by means of FT methods and how many I/O operations are needed by the computer. The computations given below illustrate the relative speed of CLEAN and MRC based on the simplifying assumption that all operations are done in the map domain and in the core of the computer. The number of calculations per pixel that is needed can be estimated as follows: two convolutions, two subtractions and two decimation steps (reprojecting to a f times larger gridspacing) are needed for constructing both the smooth and difference dirty beam and map; for each of the n iterations in the definition of the search area there is one convolution-, one masking- and one CLEAN-operation, all $1/f^2$ smaller; the CLEAN operation takes 3 arithmetic operations per δ-function (find the maximum, multiply by the gain and subtract); for the combination of the smooth and difference clean maps 6 operations (scaling and adding) are needed. So, with f typically of the order of 2–4 and n about 3–6, the ratio of the number of arithmetic operations per pixel S needed by MRC to that needed by CLEAN becomes:

$$R=\frac{S_{\mathrm{MRC}}}{S_{\mathrm{CLEAN}}}=\frac{25f^2}{N}+\frac{N_s+N_d}{N}.$$

(where N is the number of δ-functions found). For an extended source: $N_s \gg N_d$ and $N_s \simeq N/f^2$, so that $R \simeq 25/N_s + 1/f^2$, which for a realistic number of δ-functions (a few 100–1000) is $\ll 1$. For a concentrated source however: $N_s \ll N_d$ and $N_d \simeq N$, so $R \simeq 25f^2/N_d + 1$. When $f=4$ this is less than e.g. 1.5 only when $N \geqq 800$. This is much higher than can be expected for concentrated sources. A source which has as much small-scales as large-scales structure will result in $N_s \simeq N_d$ and so, with $N \simeq f^2 N_s + N_d$: $R \simeq 25/N_s + 2/f^2 \ll 1$. Therefore, the fact that one δ-function in the smoothed map is equivalent to f^2 δ-functions in the map at the original resolution leads in general to a substantial increase in speed for MRC compared to CLEAN. In the testcase for a S/N of 20 and a small source (2′) R was $\simeq 0.6$, but with MRC a much lower cutoff could be used. The resulting fluxes for MRC and CLEAN were about equal. For a more extended source (5′) R was $\simeq 0.25$ and CLEAN found only half of the flux that MRC could recover.

6. Conclusion

We have shown how a variant of the CLEAN algorithm can be used to circumvent some of the problems of CLEAN for low signal-to-noise maps and in particular for extended sources. The MRC approach can use a much lower cutoff in the deconvolution process, and therefore more fully realizes the potential that CLEAN has. Furthermore, the parameters of the several intermediate CLEAN steps can be optimized more than is possible in general for the standard CLEAN. This includes the better signal-to-noise ratio in the smoothed maps, the better source to beam size ratio in both the smoothed and the difference maps, the smaller amount of pixels needed in applying CLEAN to the smoothed map and the smaller number of δ-functions needed in the difference map.

Especially for extended sources, MRC is much faster and more reliable than CLEAN, without the need to use a priori information on the flux distribution of the source. An advantage of MRC above several other proposed modifications of CLEAN is that no extra assumptions are introduced to those made for CLEAN.

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