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# Structure Function Studies for Turbulent Interstellar Medium

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**Abstract.** We study structure functions of rotation measures in the Canadian Galactic Plane Survey (CGPS) region and the North Galactic Pole (NGP) to extract the interstellar medium (ISM) fluctuation information. The CGPS data are divided into three longitude intervals:  $82^\circ < l < 96^\circ$  (CGPS1),  $115^\circ < l < 130^\circ$  (CGPS2) and  $130^\circ < l < 146^\circ$  (CGPS3). The structure functions of all three regions have large uncertainties when the angular separation is smaller than  $\delta\theta \approx 1^\circ$ . A power law can fit the structure function well for  $\delta\theta > 1^\circ$ . The power law indices get smaller from CGPS1 to CGPS3 and the amplitudes decrease. The variations of the large-scale field and the electron density have only negligible effects on the structure function and thus cannot account for the changes, indicating that the turbulent properties of the Galactic ISM are intrinsically longitude-dependent. The Kolmogorov-like fluctuation spectrum of the electron density or the magnetic field should produce a power law structure function with an index of  $5/3$  or  $2/3$ , neither of which is consistent with our results of small indices in the three sub-CGPS regions. For the NGP region, the structure function is flat, showing that the rotation measures are mostly intrinsic to the extragalactic sources, and the ISM is very random in that part of our Galaxy. It is obvious that the ISM fluctuation is latitude-dependent when comparing the results in the NGP region and the CGPS regions.

## 1 Introduction

The electron density irregularities have been studied either by refractive scintillation or by diffractive scintillation phenomena as systematically summarized by Armstrong et al. (1995). The fluctuation spectrum from scales of  $\sim 10^6$  m to scales of  $\sim 10^{13}$  m can be fitted by a single power law with a power index of  $\sim 3.7$ , i.e. the Kolmogorov spectrum. This spectrum can be extended up to  $\sim 10^{18}$  m. The property of magnetic field fluctuations is not yet clear although they are more important than the electron density fluctuations in some circumstances such as in the solar wind (Armstrong et al. 1995).

The structure function of rotation measures (RMs) contains information on electron density and magnetic field fluctuations (Simonetti et al. 1984). More data are available now allowing a better determination of the structure function and a careful comparison of the turbulent properties of different regions. In this contribution, we show the RM structure functions of extragalactic sources in the Canadian Galactic Plane Survey (Brown et al. 2003, CGPS hereafter) and briefly discuss the results. The detailed report of this work will be given elsewhere.

## 2 The Theoretical Structure Function

The RM structure function ( $D_{\text{RM}}$ ), if assumed as a stationary random process, can be written as

$$D_{\text{RM}}(\delta\theta) = \langle [\text{RM}(\theta_0) - \text{RM}(\theta_0 + \delta\theta)]^2 \rangle, \quad (1)$$

where  $\delta\theta$  is the angular separation of two sources for RMs in degree throughout this paper, and  $\langle \cdots \rangle$  denotes the ensemble average over any  $\theta_0$  so that the result is independent of  $\theta_0$ . Both the electron density ( $n_e$ ) and the magnetic field ( $\mathbf{B}$ ) are assumed to consist of uniform backgrounds ( $n_0$  and  $\mathbf{B}_0$ ) and Gaussian fluctuations ( $\delta n$  and  $\delta \mathbf{B}$ ) with averages of zero, in the form of

$$\begin{aligned} n_e &= n_0 + \delta n, \\ \mathbf{B} &= \mathbf{B}_0 + \delta \mathbf{B}. \end{aligned}$$

The exact spectra of the electron density and magnetic field fluctuations are not known, so it is straightforward to take the power law form as

$$\begin{aligned} P_{\delta n}(q) &= \frac{C_n^2}{[l_{on}^{-2} + q^2]^{\alpha/2}}, \\ P_{\delta B}(q) &= \frac{C_B^2}{[l_{oB}^{-2} + q^2]^{\beta/2}}, \end{aligned}$$

where  $q$  is the wave number,  $C_n^2$  and  $C_B^2$  are the fluctuation intensities,  $l_{on}$  and  $l_{oB}$  are the outer scales, and  $\alpha$  and  $\beta$  are the power indices. The structure function can be reduced to the form below when  $l_{on} = l_{oB} = l_o = 1/q_o$  (Minter 1995)

$$\begin{aligned} D_{\text{RM}}(\delta\theta) &= \Psi_{\text{RM}}(\delta\theta) + 2C_{\text{RM}}^2 B_0^2 C_n^2 f(\alpha) L^{\alpha-1} \delta\theta^{\alpha-2} + 2C_{\text{RM}}^2 n_0^2 C_B^2 g(\beta) L^{\beta-1} \delta\theta^{\beta-2} \\ &\quad + 2C_{\text{RM}}^2 C_n^2 C_B^2 h(\alpha, \beta) L^{(\alpha+\beta)/2-1} q_o^{3-(\alpha+\beta)/2} \delta\theta^{(\alpha+\beta)/2-2}, \end{aligned}$$

where  $\Psi_{\text{RM}}(\delta\theta)$  is the geometrical term attributed to the variation of the large-scale electron density and the magnetic field,  $L$  is the path length,  $f(\alpha)$ ,  $g(\beta)$  and  $h(\alpha, \beta)$  are constants containing  $\Gamma$ -functions. As  $\delta\theta \ll 1$  rad, the geometrical term can be approximated as

$$\Psi_{\text{RM}}(\delta\theta) \approx C_{\text{RM}}^2 n_0^2 B_0^2 L^2 \delta\theta^2 \approx 3 \times 10^{-4} \langle \text{RM} \rangle^2 \delta\theta^2,$$

here  $\delta\theta$  is in degree. If both the electron density and the magnetic field fluctuations follow a Kolmogorov spectrum, the structure function has a simple form as below,

$$D_{\text{RM}}(\delta\theta) = \Psi_{\text{RM}}(\delta\theta) + C\delta\theta^{5/3},$$

where  $C$  is a constant related to the intensities in the regular and fluctuating components.

### 3 Analysis of Data

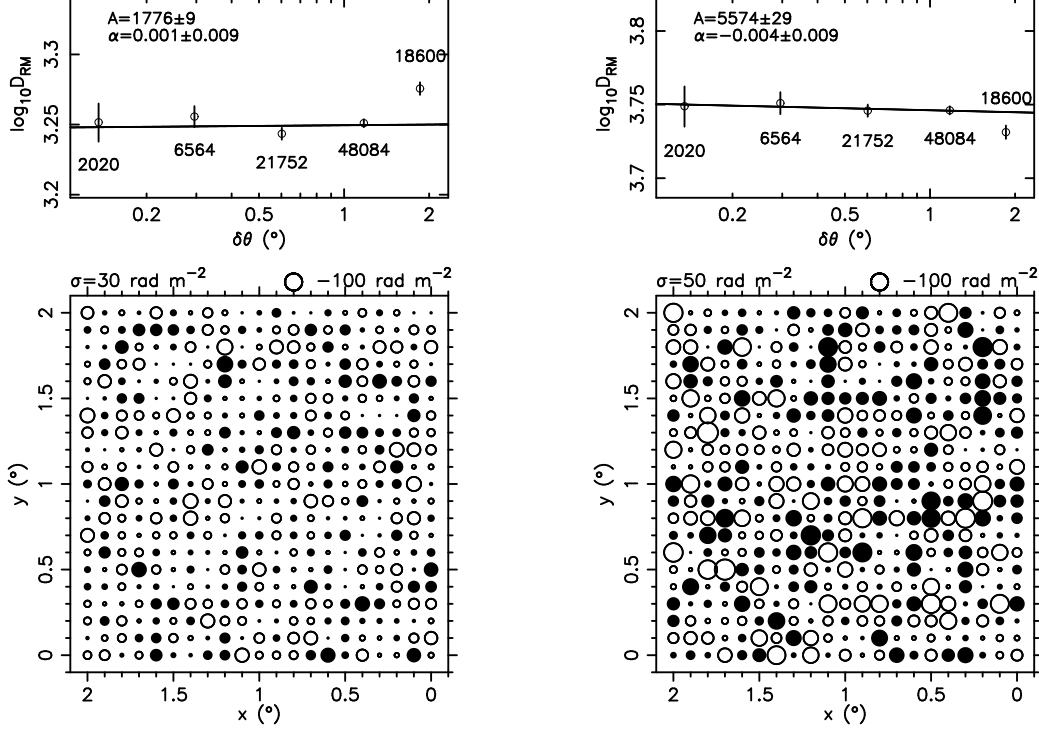
In principle, two methods can be employed to extract the fluctuation information of the ISM from RMs: the autocorrelation function and the structure function. The autocorrelation function is the Fourier transform of the power spectrum density according to the *Wiener-Khinchin theorem*, so the direct way is to derive the autocorrelation function. But this can cause problems when the data are irregularly spaced (Spangler et al. 1989). Regardless of the data sampling, it is easy to obtain the structure function, however. So we will take the structure function approach below.

Given a sample of RM data, the logarithm of angular separations of RM pairs are set to bins with equal lags. In each bin the differences of RM pairs are squared and then averaged to obtain the structure function. The standard deviations are taken as the uncertainty of each function value.

Around each source we can derive the local RM average and dispersion. If a RM value is different from the average by three times larger than the dispersion, it is defined as ‘‘anomalous’’. We will illustrate by simulation that such an anomalous RM can significantly affect the structure function.

Let the RMs distribute in the region  $0^\circ \leq x \leq 2^\circ$  and  $0^\circ \leq y \leq 2^\circ$  uniformly with grid intervals of  $0.1^\circ$ . Two samples of random RM values following Gaussian distributions with an average of zero and a dispersion  $\sigma$  of 30 and 50  $\text{rad m}^{-2}$ , respectively, are generated with the Monte-Carlo method. Note that the average value does not affect the results. Theoretically the structure function should be flat with an amplitude  $2\sigma^2$  and an index 0. The structure functions from simulations are displayed in Fig. 1. We then fitted the structure functions with  $D_{\text{RM}}(\delta\theta) = A\delta\theta^\alpha$  and found that the results are consistent with theoretical expectations.

As is obvious in Fig. 2, an anomalous RM value (1000  $\text{rad m}^{-2}$ , indicated by a cross) can distort the real structure function to a different extent. With regard to actual RM data, such a RM can be either due to an enhanced local medium or it is intrinsic to the extragalactic source. One should pick



**Fig. 1.** Simulated RM distributions are shown in the lower panels, with filled and open circles representing positive and negative values, respectively. The sizes of symbols are proportional to the square-root of RM values. The structure functions are plotted in the upper panels with the number of pairs in each bin marked.

**Table 1.** Fitting parameters of the structure functions in the CGPS regions and the NGP region.

Region	Range	$\langle \text{RM} \rangle$	$\sigma_{\text{RM}}$	$A$	$\alpha$
CGPS1	$82^\circ 42' < l < 96^\circ 23'$	-237	306	$118630 \pm 6901$	$0.28 \pm 0.03$
CGPS2	$115^\circ 01' < l < 129^\circ 99'$	-147	117	$23016 \pm 1241$	$0.10 \pm 0.03$
CGPS3	$130^\circ 12' < l < 146^\circ 51'$	-106	94	$17756 \pm 958$	$-0.003 \pm 0.027$
North Galactic Pole	$(b > 70^\circ)$	-3	15	$548 \pm 215$	$-0.06 \pm 0.12$

up such RMs for further identification. Below we will only show the results of real observed RM data after discarding these anomalous RMs, as identifications have not yet been made.

We took the RMs of the extragalactic sources (EGSs) in the CGPS region but excluded sources containing separate or unresolved components (marked by a, b and c in the catalog of Brown et al. 2003). The data are divided into three longitude intervals:  $82^\circ < l < 96^\circ$  (CGPS1),  $115^\circ < l < 130^\circ$  (CGPS2) and  $130^\circ < l < 146^\circ$  (CGPS3). We also collected the RMs of EGSs in the North Galactic Pole (NGP) region with latitudes larger than  $70^\circ$ . Then we calculated the structure functions for these four regions, as shown in Figs. 3, 4, 5, and 6. The fitting results using  $D_{\text{RM}}(\delta\theta) = A\delta\theta^\alpha$  are listed in Table 1, where Column 1 refers to the region, Columns 2 and 3 to the average and dispersion of the RMs. The amplitude and power index of each power law structure function are listed in Columns 4 and 5.

## 4 Discussion

The electron density fluctuations are better known than the magnetic field fluctuations. Armstrong et al. (1995) have shown that the electron density fluctuation spectrum is Kolmogorov-like, but the

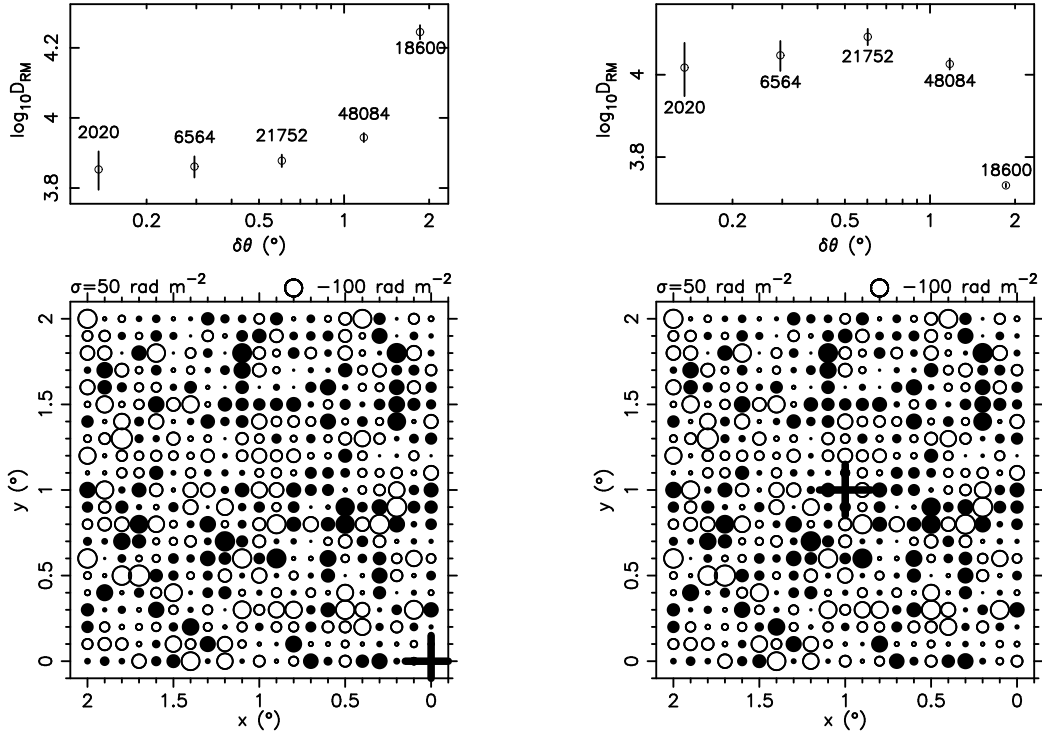


Fig. 2. Same as Fig. 1 for  $\sigma = 50 \text{ rad m}^{-2}$  but with an anomalous RM of  $1000 \text{ rad m}^{-2}$  in the corner and in the center as indicated by a cross.

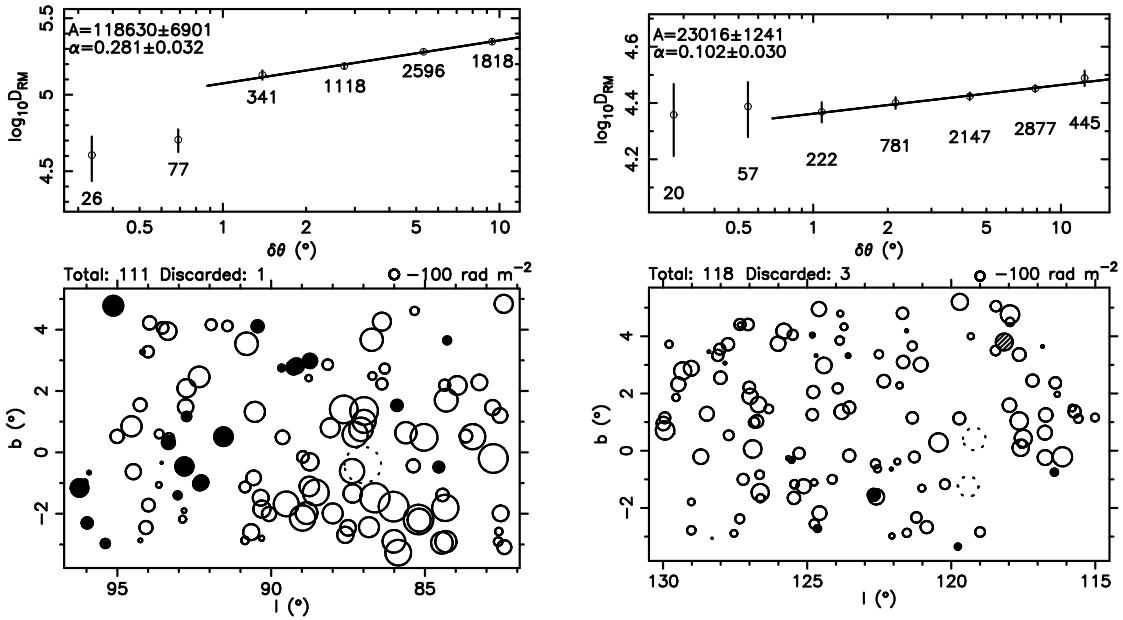


Fig. 3. The RM distribution in the CGPS1 region (lower panel) and the calculated structure function (upper panel). The symbols are plotted as in Fig. 1, but anomalous RMs are plotted as hatched circles when positive or dotted circles when negative.

Fig. 4. Same as Fig. 3 but for the CGPS2 region.

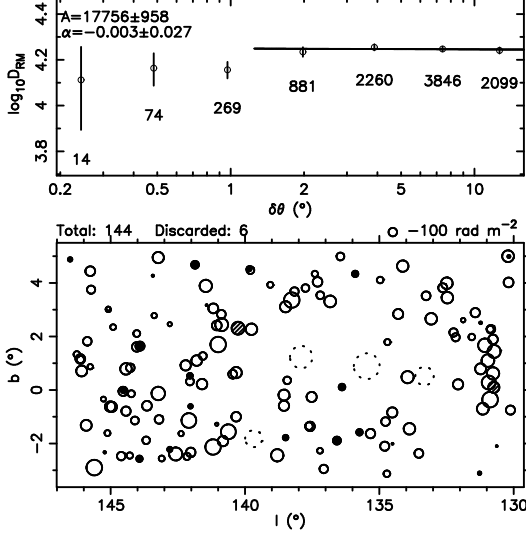


Fig. 5. Same as Fig. 3 but for the CGPS3 region.

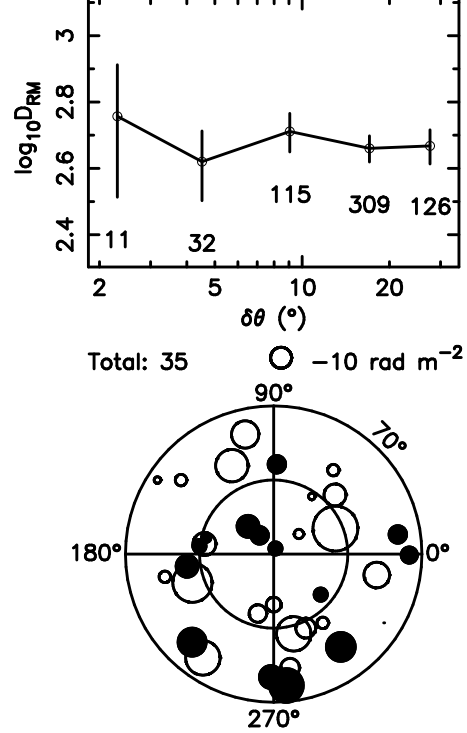


Fig. 6. Same as Fig. 3 but for the NGP region.

magnetic field fluctuation remains mysterious and totally unclear as mentioned above. We know that many physical processes in the ISM affect the electron density and the magnetic field at the same time, such as supernova explosions. The magnetic field is often assumed to be frozen into the interstellar medium. So we can assume that there is no distinct discrepancy between the fluctuations of the electron density and the magnetic field. In this case the structure function can be written as  $D_{\text{RM}}(\delta\theta) = \Psi(\delta\theta) + C\delta\theta^{\alpha-2}$  where  $C$  is a constant.

In the three sub-CGPS regions, the average rotation measures  $\langle \text{RM} \rangle = -237, -147$  and  $-106$   $\text{rad m}^{-2}$ , respectively, corresponding to the geometrical term contributions  $\Psi_{\text{RM}}(\delta\theta) = 17\delta\theta^2, 6\delta\theta^2$  and  $3\delta\theta^2$ , which are definitely negligible when compared to the obtained amplitudes ( $A$ ) of the structure functions in these regions. In fact, the indices from our results are much smaller than 2, which also means that the geometrical term plays a minor role in the structure function. Therefore the structure function can be simplified as  $D_{\text{RM}}(\delta\theta) \approx C\delta\theta^{\alpha-2}$ .

The structure function  $D_{\text{RM}}(\delta\theta) \approx C\delta\theta^{5/3}$  holds for 3D Kolmogorov fluctuations and  $D_{\text{RM}}(\delta\theta) \approx C\delta\theta^{2/3}$  for 2D Kolmogorov fluctuations (Minter & Spangler 1996). *It is clear that the Kolmogorov spectrum cannot account for our results.* Actually, the indices of the structure functions in the three sub-CGPS regions are very small, close to zero. Let us try to discuss the nearly flat structure functions we got.

From the results (Table 1) it is evident that the amplitudes of the structure function  $A$  decrease from CGPS1 to CGPS3. This is probably caused by the extent of the ISM in our Galaxy. From our simulation, we know that  $A \sim 2\sigma_{\text{RM}}^2$ , where  $\sigma_{\text{RM}}$  is the dispersion of RM along the line of sight. Assuming that there are many fluctuation cells with typical scale  $l$  and a RM dispersion  $\sigma_l$  along the line of sight, the total RM dispersion can be represented by  $\sigma_{\text{RM}}^2 = \frac{L}{l}\sigma_l^2$ ; where  $L$  is the path length to the edge of our Galaxy. Because  $L$  decreases from CGPS1 to CGPS3, it is understandable that the fluctuation intensity gets smaller.

Due to the disk structure of the Galactic electron distribution and the Sun's location at 8 kpc distance from the Galactic center, the RMs of EGSs near the pole region are little affected by the

Galactic interstellar medium, so that the structure function for the EGSs in the pole region should be very flat. Our result in Fig. 6 and the fitting parameters in Table 1 are consistent with Simonetti et al. (1984) and strengthen the fact that the RMs of EGSs in the pole region are very random, indeed mainly of intrinsic origin. We can also see that the fluctuation intensity in the NGP region is much smaller than that in the CGPS region, indicating a trend of a latitude-dependence of the ISM turbulence.

## 5 Conclusion

We have used the structure function to study ISM fluctuations. Simulation shows that an anomalous RM can significantly influence the structure function, so we suggest that these large RMs should be carefully checked before performing a structure function analysis, or otherwise should be used with caution. The structure functions of the RMs of EGSs in the CGPS region have been calculated. The structure functions at  $\delta\theta > 1^\circ$  can be fitted by a power law. The power indices are all nearly zero which cannot be interpreted by Kolmogorov-like ISM fluctuations. The fluctuation intensity is longitude-dependent. The flat structure function in the NGP region shows that the RMs in this region are almost intrinsic to EGSs.

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