Slant is NP-Complete

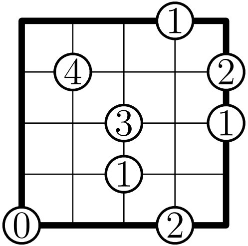
and Some Variations are in P

Jayson Lynch, Jack Spalding-Jamieson (Presenting)

July 2024

The puzzle "Slant" AKA "Gokigen Naname"

- Start with: Grid and numbers on vertices.



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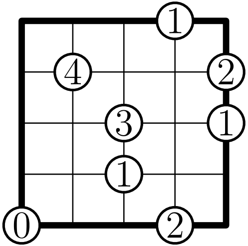

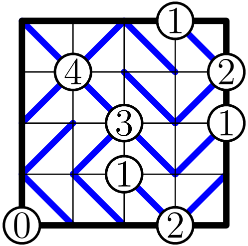

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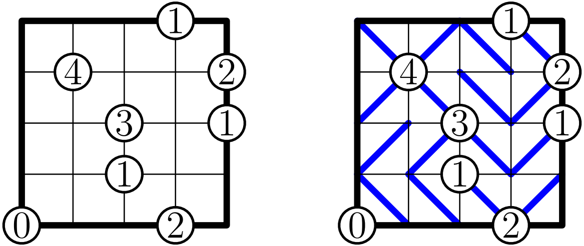

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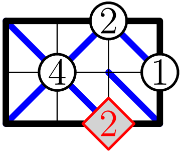
Time complexity of solving an $n \times n$ Slant puzzle?

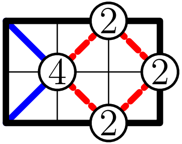
Terminology

- **Vertex constraints:** Number label = incident diagonal count
- **Cycle constraint:** No cycles

Invalid solutions:

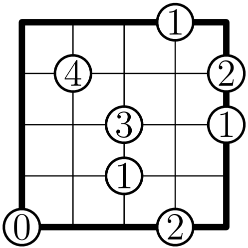


Degree 2 vertex constraint unsatisfied.



Red diagonals form a cycle.

Quick example



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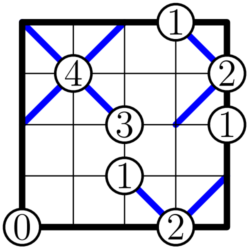

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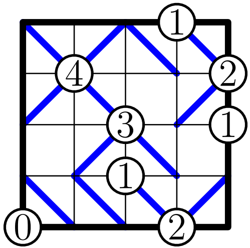

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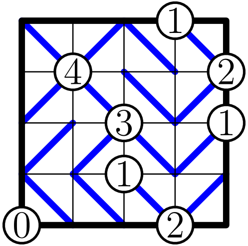

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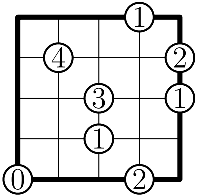
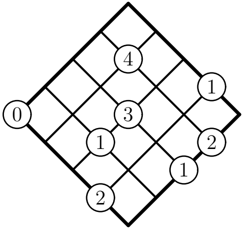
Orientation of the puzzle

Rotate slant puzzles 45° from now on.





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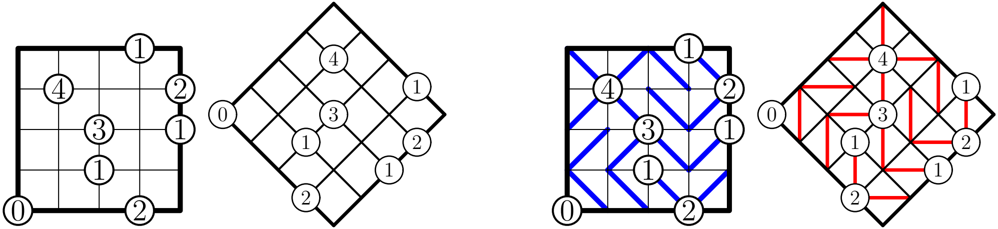
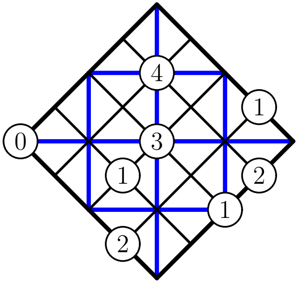


The two grid graphs forming the solution space

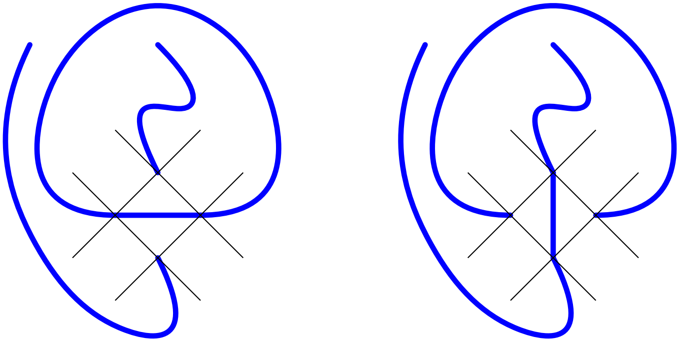


The diagonals become potential edges from two complementary (dual) grid graphs.

Extending partial solutions

Theorem

A partially-filled Slant board w/o vertex constraints and no cycles can be extended to a solution.



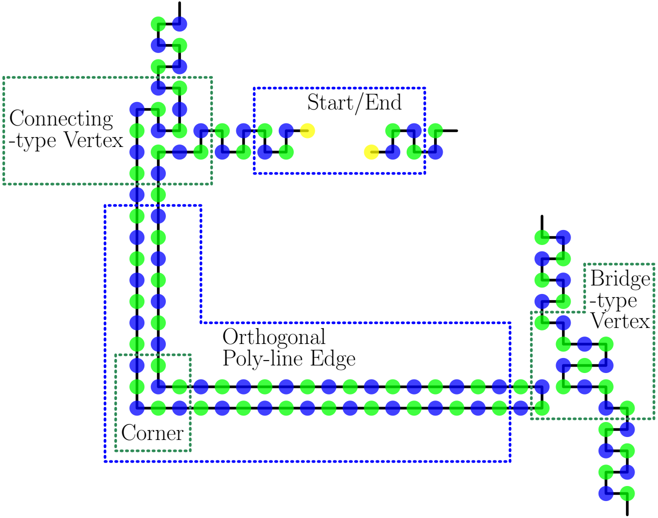
Greedily fill: If one choice for a cell induces a cycle, the other cannot.

NP-Hardness: Two-step reduction idea

Facts:

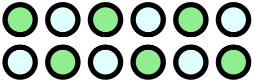
- Hamiltonian Cycle is NP-complete for planar, bipartite, 3-regular graphs.
- Hamiltonian Cycle is NP-complete for grid graphs. Reduction from planar, bipartite, 3-regular graphs.

Additional gadget: Forced edge



NP-Hardness: Backbone

Goal: Mimic behaviour of Hamiltonian Cycle in each gadget



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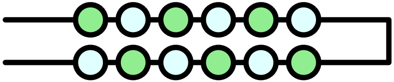

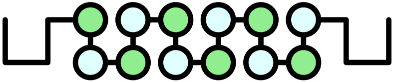

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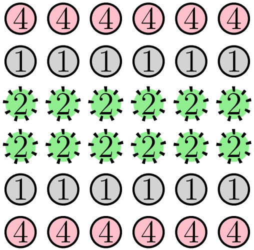

Green vertices will emulate the grid graph in the Hamiltonian Cycle problem.

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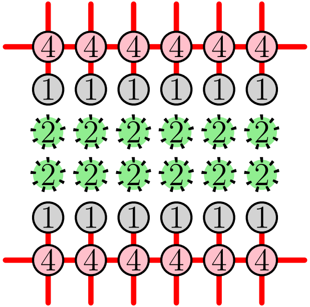

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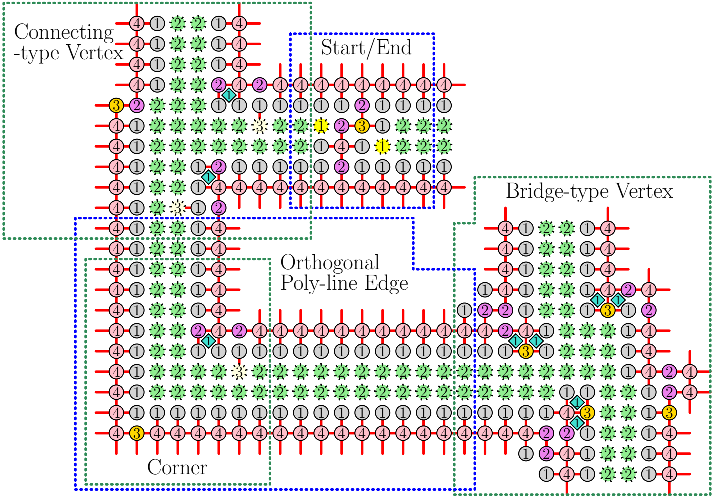
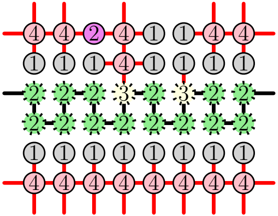
NP-Hardness: Emulating the gadgets

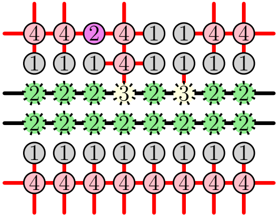


NP-Hardness: Filling the void

Problem 1: Backbone around a face would create a cycle
Problem 2: Vertices in face itself are not connected to rest of graph

Solution: One incision along each face + partial fill result.





Results not talked about here + Open questions

In the paper:

- The problem's complexity remains essentially the same for both dense and sparse representations.
- The constraints break into a specific 5-way matroid intersection—two partition matroid pairs representing *b*-matchings in the dual grid graphs, and one planar matroid.

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Fin.