Polarization

(OPTICS, PHYS 1001, 1st Yr By SDG)

Elementary concepts: In light and all other kinds of electromagnetic waves, the oscillating electric and magnetic fields are always directed at right angles to each other and to the direction of propagation of the wave. In other words the fields are transverse, and light is described as a **transverse wave**. (By contrast sound waves are said to be longitudinal, because the oscillations of the particles are parallel to the direction of propagation.) Since both the directions and the magnitudes of the electric and magnetic fields in a light wave are related in a fixed manner, it is sufficient to talk about only one of them, the usual choice being the electric field. Now although the electric field at any point in space must be perpendicular to the wave velocity, it can still have many different directions; it can point in any direction in the plane perpendicular to the wave's direction of travel.

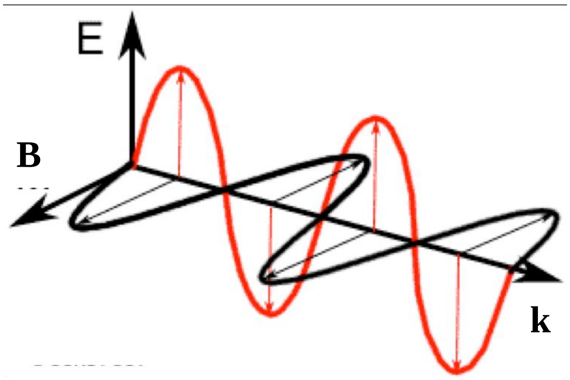

Figure 1: Representation of an electro-magnetic wave, where E is the electric vector, B is the Magnetic vector and k denotes the direction of propagation along which the wave moves with velocity v.

Mechanical demonstration of polarisation: Let us consider a rope XY, which is passing through two parallel slits S_1 and S_2 as shown in the figure 2. Suppose we hold the rope along one end and made it to vibrate perpendicular to the direction of propagation. Now as the rope is vibrating parallel to the slit S_1, a wave emerges vibrating parallel to the opening of the slit S_1. The slit S_2 allow the wave to pass through it, if S_2 is kept parallel to S_1 [figure 2A] and the wave after S_1 will not be transmitted through S_2, if S_2 is kept perpendicular to S_1 [figure 2B].

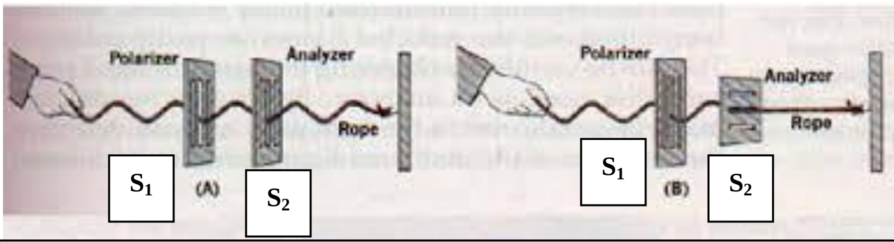

Figure 2A and 2B: Mechanical demonstration of polarisation

<u>Plane polarised light :</u> It means that the light vector is vibrating transversely to the direction of propagation in a fixed plane through this direction which does not change with time.

<u>Unpolarised light</u>:It is theoretically equivalent to two mutually perpendicular linearly polarised light having no constant phase relation to each other.

In natural light, electric vector changes direction rapidly (~10^8 times/sec) and randomly, such that due to persistency of vision, the eye would observe an average effect, that is, the wave would appear to be quite symmetrical about the direction of propagation.

Plane of Vibration: It is a plane which contains the electric field and the direction of Propagation.
Plane of Polarisation: It is defined as the plane perpendicular to the direction of propagation and to the plane of vibration in the light wave.

<u>Representation of natural/unpolarised light and polarized light</u>

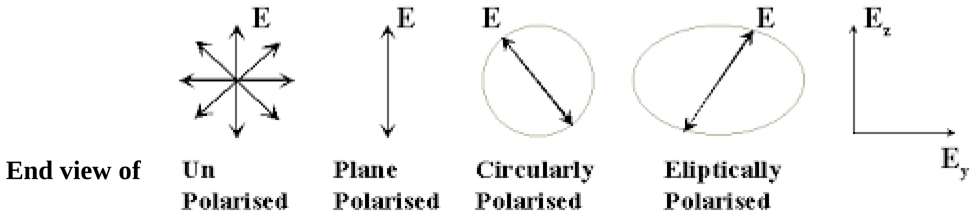

Fig - 3

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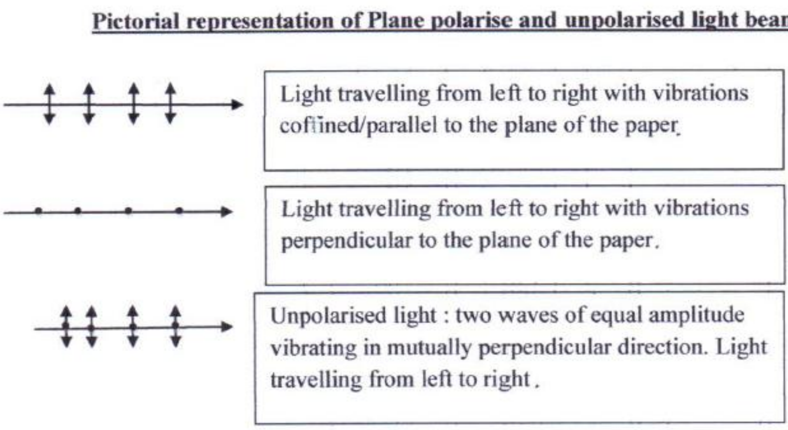


Fig - 4

Components of polarization

Since electric field is a vector quantity it can be described in terms of components referred to a set of coordinate directions. In the case of polarized waves we can take *any* two perpendicular directions in a plane perpendicular to the wave's direction of travel. An electric field ***E*** which makes an angle α with one of these directions can then be described completely as two components with values $\mathbf{E_x = \mathit{E} \cos\alpha}$ and $\mathbf{E_y = \mathit{E} \sin\alpha}$. We can think of these components as two independent electric fields, each with its own magnitude and direction, which are together equivalent in every respect to the original field. So any wave can be regarded as a superposition of two waves with perpendicular polarizations. [fig -5]

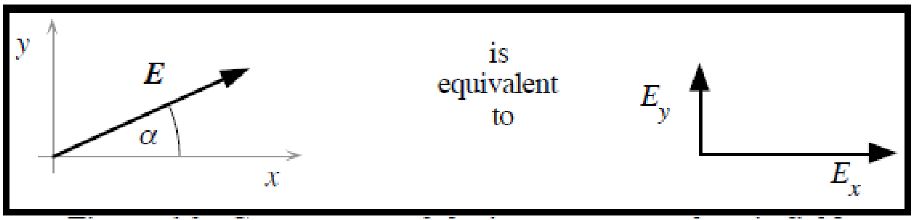


Fig - 5

<u>Polarizer and Malus's law</u>

An **ideal polariser**, or polarising filter, turns unpolarised light into completely plane polarised light. Its action can be described in terms of its effect on waves with different polarisations; waves whose polarisation is parallel to an axis in the polariser, called its **polarising axis**, are transmitted without any absorption but waves whose polarisation is perpendicular to the polarizing axis are completely absorbed. An wave whose polarisation is at some other angle to the polarising axis is partly transmitted and partly absorbed but it emerges from the other side of the polariser with a new polarisation, which is parallel to the polariser's polarization axis. This can be described in terms of components of the original wave. Since we can use any reference directions for taking components we choose one direction parallel to the polariser's axis and the other one perpendicular to it. **If the angle between the original direction of vibration and the polarization axis of the polariser is θ, then the component parallel to the polariser's axis, which gets through, has amplitude of $\boldsymbol{E_0 \cos\theta}$.** Since the other component is absorbed, the wave which emerges has a new amplitude $\boldsymbol{E_0 \cos\theta}$.
Since the **irradiance or "intensity" of light is proportional to the square of the electric field's amplitude [figure : 6]**, where E_0 is the amplitude of the electric field of the incident light making an angle θ with the polarization axis of the polarizer.

$$I = I_0 \cos^2\theta \,,$$

where, I is the transmitted intensity and I_0 is the intensity of the incident light and θ is the angle between polaristion axes of the two polarizers.

Malus Law: The intensity of Polarized light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and polariser.

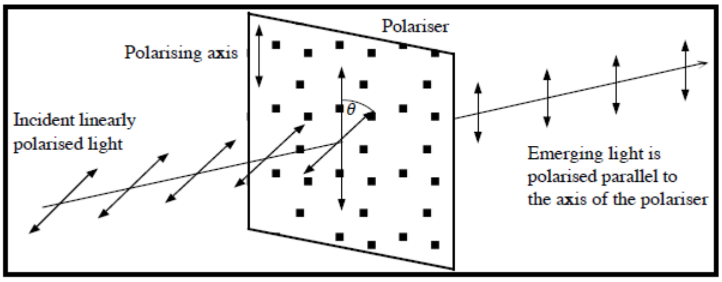


Fig - 6

Malus's law also describes the action of an ideal polariser on unpolarised light. Unpolarised light is really a vast collection of polarised elementary waves whose polarisations are randomly spread over all directions perpendicular to the wave vector (**k**). Since these elementary waves are not coherent, their intensities, rather than their amplitudes, can be added, so Malus's law works for each elementary wave. **To work out the effect of the polariser on the whole beam of unpolarised light we take the average value of $I_0 \cos^2\theta$ over all possible angles, which gives (figure 7)** $\boldsymbol{I = I_0/2}$.

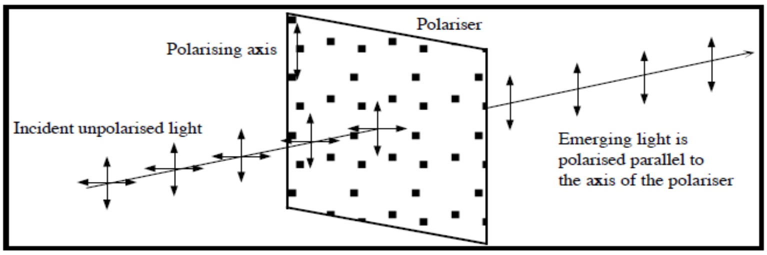


Fig - 7

<u>Optical demonstration of Polarisation:</u> If we send initially unpolarised light through two successive polarisers, the irradiance (intensity) of the light which comes out depends on the angle between the axes of the two polarisers. If one polariser is kept fixed and the axis of the other is rotated, the irradiance of the transmitted light will vary. Maximum transmission occurs when the two polarising axes are parallel. When the polarising axes are at right angles to each other the polarisers are said to be **crossed** and the transmitted intensity is a minimum. A pair of crossed ideal polarisers will completely absorb any light which is directed through them (figure 8). Note that the polarisation of the light which comes out is always parallel to the polarising axis of the last polariser.

<u>Analyser :</u> So far we have considered a polariser as something which produces polarised light. It can also be considered as a device for detecting polarised light. When it is used that way it may be called an **analyser**. For example, in the case of crossed polarising filters above, you can think of the first filter as the polariser, which makes the polarised light, and the second filter as the analyser which reveals the existence of the polarised light as it is rotated.

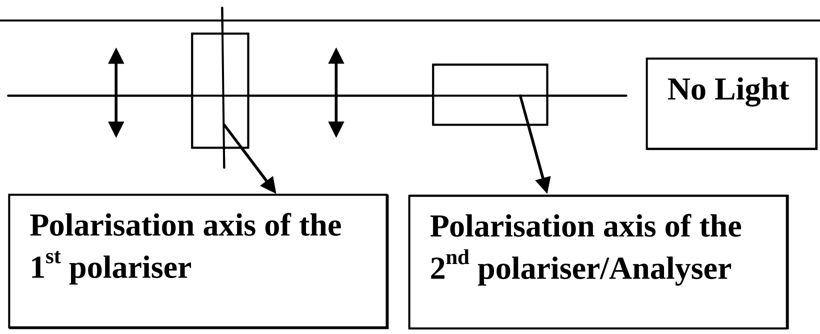


Fig: 8

Numerical: 1) A polarizer and an analyzer are oriented so that the maximum intensity is achieved. To what fraction of its maximum value is the intensity reduced when the analyzer is rotated through 60^0?

Ans. If θ be the angle between the polarizer and analyzer and I_0 be the maximum original intensity, then according to Malus Law, the transmitted intensity through the analyzer is $I=I_0 \cos^2\theta$ or, $I/I_0 = \cos^2\theta$.
The reduction in intensity is $(I_0 – I)$ and the fraction of the intensity reduced is $(I_0 – I)/I_0 = 1 – I/I_0 = 1 – \cos^2\theta = 1 – \cos^2 60^0 = 1 – (1/2)^2 = 3/4$.
Therefore, percentage reduction in intensity, = ¾ *100 =75%.

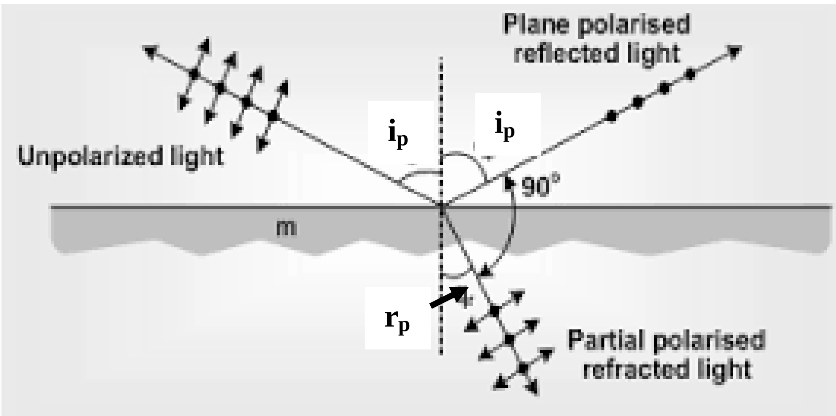
2) Two polarizing sheets have their polarizing directions parallel so that the intensity of the transmitted light is a maximum. Through what angle must either sheet be turned so that intensity becomes one half of the initial value?

Ans. If, $I = I_0 /2$, then $I_0/2 = I_0 \cos^2\theta$

$$\text{Or, } \cos^2\theta = ½ \text{ or, } \theta = \cos^{-1} (\pm 1/\sqrt{2}) = \pm 45^0 , \pm 135^0$$

Different ways of Production of plane polarised light: 1) Reflection, 2) Refraction, 3) double refraction, 4) selective absorption and 5) scattering .

Production of plane polarised light by reflection: In 1808, the French physicist Malus discovered that when a unpolarised light is incident at a particular angle about 57^0, on a glass plate, the reflected light is plane polarised (vibrating perpendicular to the plane of incidence) and refracted light is partially polarised. [Figure : 9]



Plane of incidence is the plane of the paper

Fig - 9

Brewster's Law: The tangent of the angle of polarisation is numerically equal to the refractive index of the medium.

$$\mu = \tan i_p$$

The angle of incidence i_p, at which the reflected light is plane polarised is called the polarising/ Brwester's angle and it varies with the nature of the refracting medium and frequency of light.

From Snell's Law we can say that, $\mu = \sin i_p / \sin r_p$ and Brewster's Law gives,

$$\mu = \sin i_p / \cos i_p = \sin i_p / \sin (90^0 - i_p)$$

Therefore, $\mathbf{i_p + r_p = \pi/2}$

Important corollary: At polarising angle, the reflected and refracted rays are perpendicular to each other. Therefore, at polarising angle (i_p), the corresponding angle of refraction $r_p = (90 - i_p)$

Numerical: A ray of light is incident on the surface of a glass plate of refractive index 1.732 at polarizing angle. Calculate the corresponding angle of refraction of the ray.

Ans. According to Brewsters' law, $\mu = \tan i_p$.
$i_p = \tan^{-1} \mu = \tan^{-1} (1.732) = 60^0$.
If r be the angle of refraction, then since $i_p + r_p = 90^0$, $r_p = 30^0$.

Production of Plane polarised light by selective absorption: The production of plane polarised light by selective absorption is exhibited by certain class of doubly refracting crystals, which not only produce two internal beams polarised at right angles to each other but also absorb one of the polarised components if cut off to the appropriate thickness. This phenomenon is known as dichroism and the crystals exhibiting this property are said to be dichoric. A good example is Tourmaline. Since they are able to produce a plane polarised light from a unpolarised light they are called as polariser. And the emergent polarised light can be analysed by a second tourmaline called as an analyser.

[A polariser absorbs the perpendicular vibration with respect to its polarising/ optic axis and transmits the parallel vibration with respect to its polarising/ optic axis]

Double Refraction / Birefringence

In some materials incident light split up in two refracted rays, which in general travel at different speeds within the crystal. One of the refracted rays obeys the laws of refraction and known as the Ordinary Ray (O-ray) and the other one doesn't obey the laws of refraction and known as the Extra-ordinary Ray (E-ray).

Thus, in this case the material has different refractive indices for light of the same frequency, one for O-ray and the other for E-ray. Such materials are said to be **doubly refracting** or **birefringent**. [Figure -11]

Examples are crystals such as the minerals **calcite (calcium carbonate) and quartz (silicon dioxide)** or materials like Cellophane when it is placed under stress. As usual we can regard any beam of light as a superposition of two linearly polarised components at right angles to each other. It is found that one component wave, called the **ordinary wave**, travels at the same speed in all directions through the crystal, thus the wave-surface for O-ray is spherical. But the speed of the other polarization component, called the **extraordinary wave**, depends on its direction of travel, having an symmetry about the Optic axis, thus the wave-surface of E-ray is ellipsoidal.

Optic axis: There are some propagation directions in which all polarisations of light travel at the same speed and a line within the crystal parallel to one of those directions is called an optic axis. Along the optic axis, $v_e = v_o$, where v_e and v_o are the velocities of E-ray and O-ray respectively.

Principal Section: It is defined as a plane in the crystal which contains the optic axis and perpendicular to two end faces of the crystal. (Figure: 10) A principal section cuts the crystal surface in a parallelogram with angles 71^0 and 109^0.

Geometry of Calcite: It is known as Iceland spar (chemically hydrated $CaCo_3$) exists in several forms all of which give a rhombohedron on cleavage. The six faces of the rhombohedron are parallelograms each having angles of $101^0 55'$ and $78^0 5'$. The shorter diagonal of this rhombohedron will denote the direction of the Optic axis.

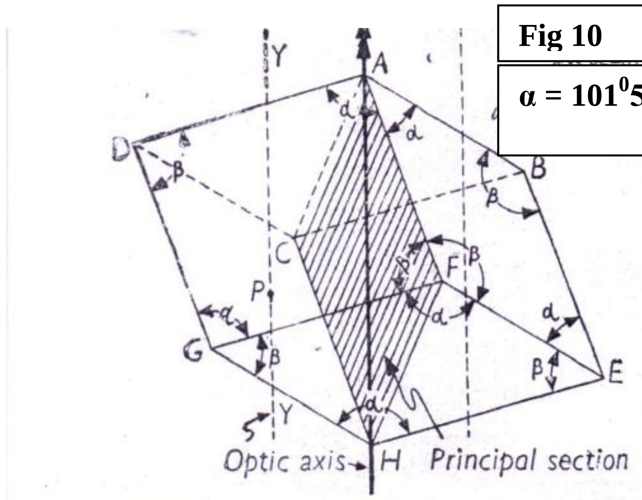
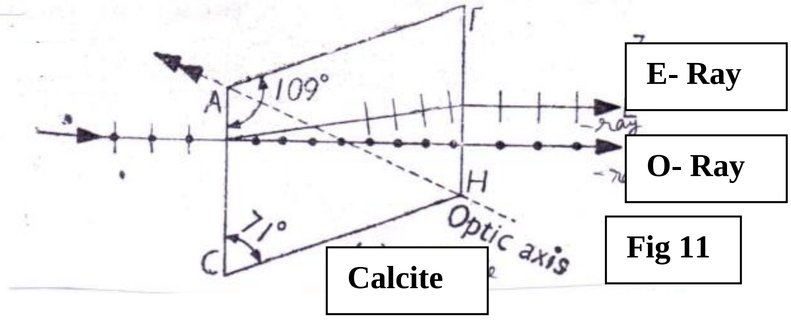


Fig 10

$\alpha = 101^0 55'$, $\beta = 78^0 5'$

Geometry of Calcite

(Showing Principal Section and Optic Axis)



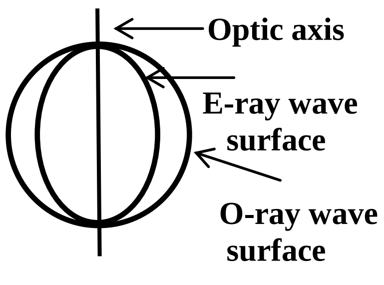
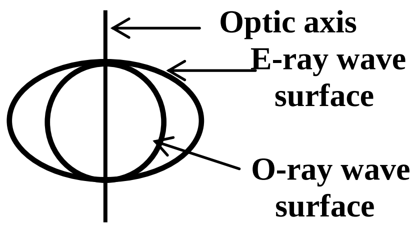
Demonstration of Double Refraction in the principal section of a calcite crystal

Fig 11

Differences between O-ray and E-ray

Property	Ordinary Ray (O-ray)	Extraordinary Ray (E-ray)
Obeys laws of refraction	**Yes**	**No**
Equal velocity along any direction of the doubly refracting crystal	**Yes**	**No**
Direction of polarisation	**Plane Polarised in the principal section**	**Plane Polarised perpendicular to the principal section**
Direction of vibration	**Perpendicular to the principal section**	**Parallel to the principal section**
3-D Wave surface	**Spherical**	**Ellipsoidal**

Short Note on Positive and Negative Crystal

Positive Crystal	Negative Crystal
The velocity of E-Ray (v_e) is direction dependent but the velocity of O-Ray (v_o) is constant in all directions of the crystal.	**The velocity of E-Ray (v_e) is direction dependent but the velocity of O-Ray (v_o) is constant in all directions of the crystal.**
Along the optic axis, $v_e = v_o$ but in any other direction $v_e < v_o$. **v_e is minimum along the perpendicular direction of optic axis**	**Along the optic axis, $v_e = v_o$ but in any other direction $v_e > v_o$.** **v_e is maximum along the perpendicular direction of optic axis**
The refractive index of E-ray (μ_e) is greater than that of the O-ray (μ_o) **i.e $\mu_e > \mu_o$**	The refractive index of E-ray (μ_e) is less than that of the O-ray (μ_o) **i.e $\mu_e < \mu_o$**
O-ray wave surface lies outside the E-ray wave surface 	**O-ray wave surface lies inside the E-ray wave surface** 
Example: Quartz	**Example: Calcite**

Nicol Prism

Since calcite is colourless and absorbs very little of either extraordinary or ordinary light, very pure calcite (called 'Iceland spar') was once used to make a very good kind of polariser, called a Nicol prism. A crystal of calcite is carefully shaped and cut in two. The two parts are then rejoined using a thin layer of transparent glue **(Canada Balsam)** whose refractive index lies between those for the e and o rays. For a suitable direction of incident unpolarised light, the ordinary rays are totally internally reflected at the boundary with the glue, while the extraordinary rays pass through. This gives a separation of the light into two components with different polarisations, travelling in quite different directions.

A Nicol prism has the advantage that the light coming out is completely plane polarized.

μ_o = 1.658 (O-ray refractive index for calcite crystal)
μ_{CB} = 1.55 (refractive index of Canada Balsam)
μ_e = 1.486 (E-ray refractive index for calcite crystal)

Since **μ_o<μ_{CB}<μ_e**, Canada Balsam acts as a rarer medium for ordinary ray and denser medium for E-ray. $S_{o,}$ when the O-ray passes from a portion of the calcite into the layer of Canada Balsam it passes from denser to a rarer medium, **when the angle of incidence is greater than the critical angle, the O-ray is totally internally reflected** and is absorbed by the lamp black pasted in the bottom of the Nicol Prism.

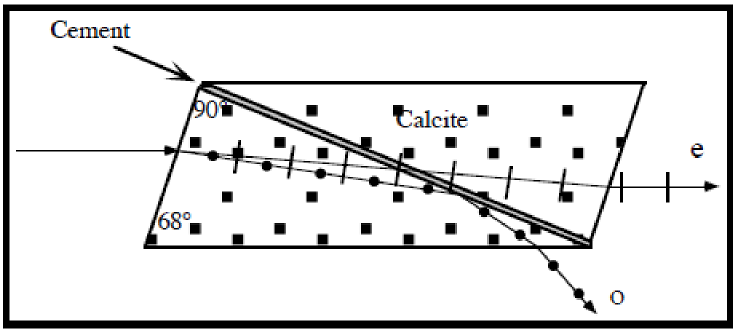


Fig - 12

States of Polarisation

Let us consider the coherent superposition of two perpendicularly plane polarised light moving in the same direction (here, z-axis, and the electric fields are vibrating along x-axis and y-axis respectively), having same frequency and a phase difference φ. Then the resultant wave can be calculated as,

$$\mathbf{E_x = E_{0x}\ \sin(kz - \omega t)}$$

$$\mathbf{E_y = E_{oy}\ \sin(kz - \omega t + \varphi) = E_{0y}\ [\sin(kz - \omega t)\ \sin\varphi + \cos(kz - \omega t)\ \cos\varphi]}$$

$$\Rightarrow {E_y}/{E_{0y}} = {E_x}/{E_{0x}} \cos\varphi + \sqrt{1 - \left({E_x}/{E_{0x}}\right)^2}\ \sin\varphi$$

$$\Rightarrow \left[{E_y}/{E_{0y}} - {E_x}/{E_{0x}} \cos\varphi\right]^2 = \left[1 - \left({E_x}/{E_{0x}}\right)^2\right] sin^2\varphi$$

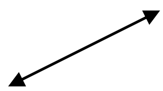
$$\boxed{\Rightarrow \left({E_y}/{E_{0y}}\right)^2 + \left({E_x}/{E_{0x}}\right)^2 - 2\,{E_y}/{E_{0y}}\ {E_x}/{E_{0x}} \cos\varphi = sin^2\varphi}$$

<u>**Special cases**</u>

1) $\varphi = 2n\pi$ (even multiple of π),

State of polarisation of the resultant wave will be plane/linearly polarised (LP), and will follow the equation, thus will vibrate in 1st and 3rd quadrant in a right-handed coordinate system.

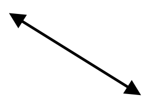
$$\left[{E_y}/{E_{0y}} - {E_x}/{E_{0x}}\right]^2 = 0, \Rightarrow E_y = E_x\,{E_{0y}}/{E_{0x}}$$



2) $\varphi = (2n+1)\pi$ (odd multiple of π),

State of polarisation the resultant wave will be plane/linearly polarised (LP), and will follow the equation, thus will vibrate in 2nd and 4th quadrant in a right-handed coordinate system.

$$\left[{E_y}/{E_{0y}} + {E_x}/{E_{0x}}\right]^2 = 0, \Rightarrow E_y = -\,E_x\,{E_{0y}}/{E_{0x}}$$



Linear Polarization: We have already seen that the resultant of two linear polarisations with zero phase difference is also a linear polarisation. Another special case is the combination of two elementary linearly polarised waves whose phase difference is exactly π. The resultant is a linear polarisation but its orientation is perpendicular to the linear polarisation when the component waves have no phase difference.

Circular polarisation

3) $\varphi = (2n+1)\pi/2$ (odd multiple of $\pi/2$) and $E_{0x} = E_{0y}$, State of polarisation the resultant wave will be circularly polarised (CP),

Thus, a circularly polarised elementary wave can be described as the superposition of two plane polarised waves with the same amplitude which are out of phase by a quarter of a cycle ($\pi/2$) or three quarters of a cycle ($3\pi/2$) etc.

In case of **circular polarisation the electric field vector at a point in space rotates in the plane perpendicular to the direction of propagation, instead of oscillating in a fixed orientation, and the magnitude of the electric field vector remains constant**.
Looking into the oncoming wave the electric field vector can rotate in one of two ways. If it **rotates clockwise** the wave is said to be **right-circularly polarised (RCP)** and if it rotates **anticlockwise** the light is **left-circularly polarized (LCP)**.

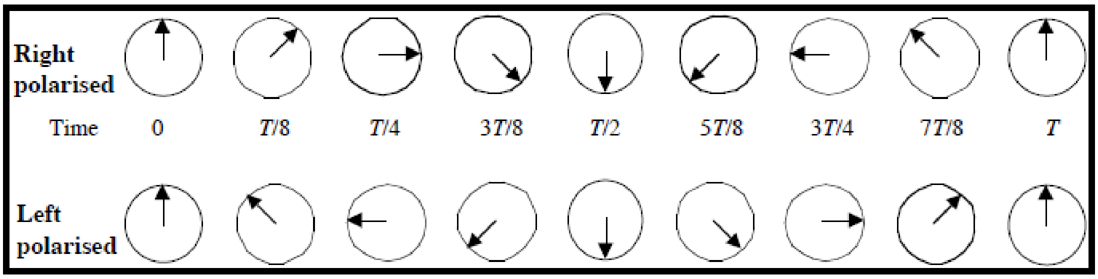


Fig – 13

T is the time period

Elliptical polarization

4) $\varphi = (2n+1)\pi/2$ (odd multiple of $\pi/2$) and $E_{0x} \neq E_{0y}$, State of polarisation of the resultant wave will be elliptically polarised (EP), whose semi-major and semi-minor axis will coincide with the E_x and E_y axes.

In general the combination of two linearly polarised waves with the same frequency but having unequal amplitudes and an arbitrary value of the phase difference, produces a resultant wave whose electric vector both rotates and changes its magnitude. The tip of the electric field vector traces out an ellipse so the result is called elliptical polarisation (figure 14).
Circular polarisation is thus a special case of elliptical polarization and we can define Right Elliptically polarized (REP) light and Left-elliptically polarized (LEP) light using similar convention of circular polarization case.

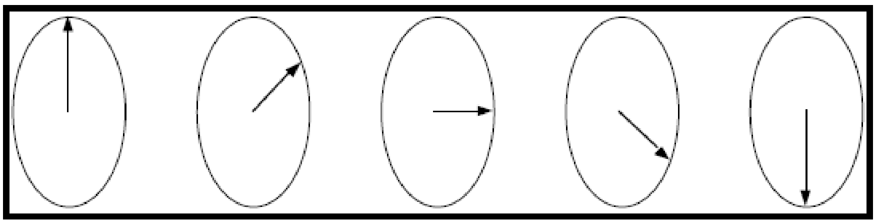


Fig - 14

Different states of Polarisation as a function of phase difference between the superposing perpendicular plane polarized light.

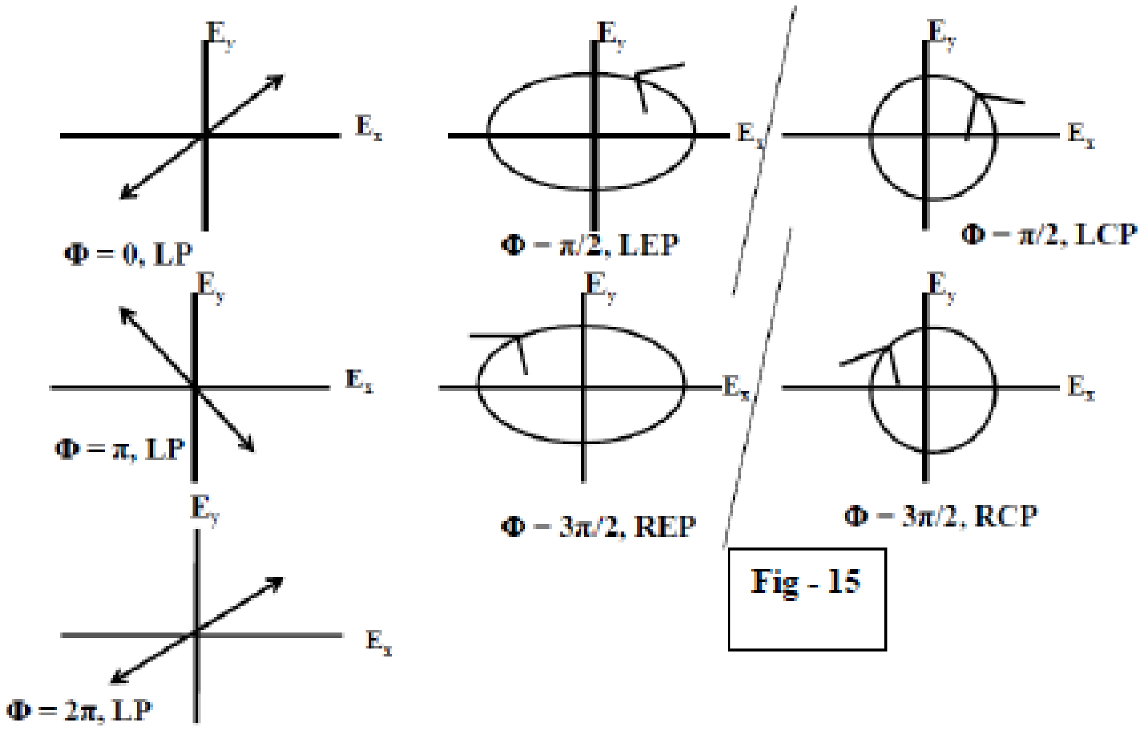


Fig - 15

Retardation Plate

It is a plate of doubly refracting crystal of calcite or quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optic axis and the optic axis lies in the plane of incidence. At normal incidence of the incident linearly/plane polarised light, the O-ray and E-ray travels along the same direction with different velocities. If the thickness of the plate is t and the refractive indices of O-ray and E-ray are μ_o and μ_e respectively, then the optical path difference (Δ) introduced between the two rays at the emergence from the retardation plate is given by:

1) $(\mu_o - \mu_e)t = \Delta$, for –ve crystal (**calcite**)

and 2) $(\mu_e - \mu_o)t = \Delta$, for +ve crystal (**quartz**).

Plane of incidence is the plane of the paper and the refracting surface is perpendicular to plane of the paper.

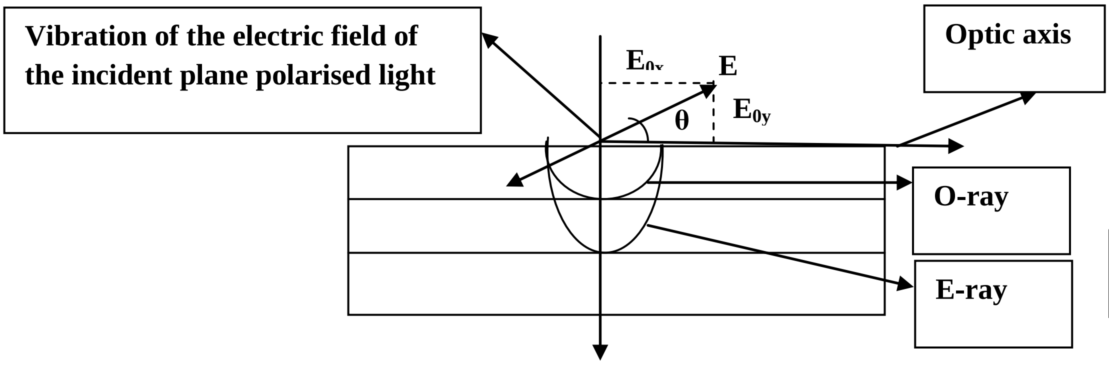


Fig - 16

Here, θ is the angle between the optic axis and the electric field vibration of the incident light Such, that E_{0x} = E cos θ and E_{0x} = E sin θ, where E is amplitude of the electric field of incident light.

Thickness of a Quarter Wave Plate (QWP): It introduces at least λ/4 path difference (Δ) between O-ray and E-ray, thus minimum of π/2 phase difference (φ), where λ is the wavelength of the incident light.

Thus, the **minimum thickness of a QWP**, made up of **Calcite** is**:**

$$\boxed{\Delta = \lambda/4, \text{ thus, } \quad t_{min} = \lambda/4(\mu_o - \mu_e)}$$

and the general expression is, t = $(4n + 1)\,\lambda/4(\mu_o - \mu_e)$,where n = 0,1,2,3 ......

Thickness of a Half Wave Plate (HWP) : It introduces at least λ/2 path difference (Δ) between O-ray and E-ray, thus minimum of π phase difference (φ), where λ is the wavelength of the incident light.

Thus, the **minimum thickness of a HWP,** made up of **Calcite** is**:**

$$\boxed{\Delta = \lambda/2, \text{ thus, } \quad t_{min} = \lambda/2(\mu_o - \mu_e)}$$

and the general expression is, t = $(2n + 1)\,\lambda/4(\mu_o - \mu_e)$,where n = 0,1,2,3 ......

Production of polarised light using retardation plate

1) LP $\longrightarrow$ QWP ($\theta = 45^0$) $\longrightarrow$ CP

2) LP $\longrightarrow$ QWP ($\theta \neq 0^0, 45^0, 90^0$) $\longrightarrow$ EP

3) CP $\longrightarrow$ QWP $\longrightarrow$ LP

4) EP $\longrightarrow$ QWP $\longrightarrow$ LP

5) RCP $\longrightarrow$ HWP $\longrightarrow$ LCP

6) LCP $\longrightarrow$ HWP $\longrightarrow$ RCP

7) REP $\longrightarrow$ HWP $\longrightarrow$ LEP

8) LEP $\longrightarrow$ HWP $\longrightarrow$ REP

9) LP $\longrightarrow$ HWP $\longrightarrow$ LP

10) LP $\longrightarrow$ HWP $\longrightarrow$ LP

Important: A quarter wave plate (QWP) can convert LP either to CP or EP and CP or EP to LP, but a half-wave plate (HWP) changes the sense of rotation/vibration.

An unpolarised light will remain unpolarised after passing through a QWP or HWP.

Important: Two consecutive QWP acts as a HWP and two consecutive HWP plate acts as a Full-wave Plate, when oriented for maximum intensity.

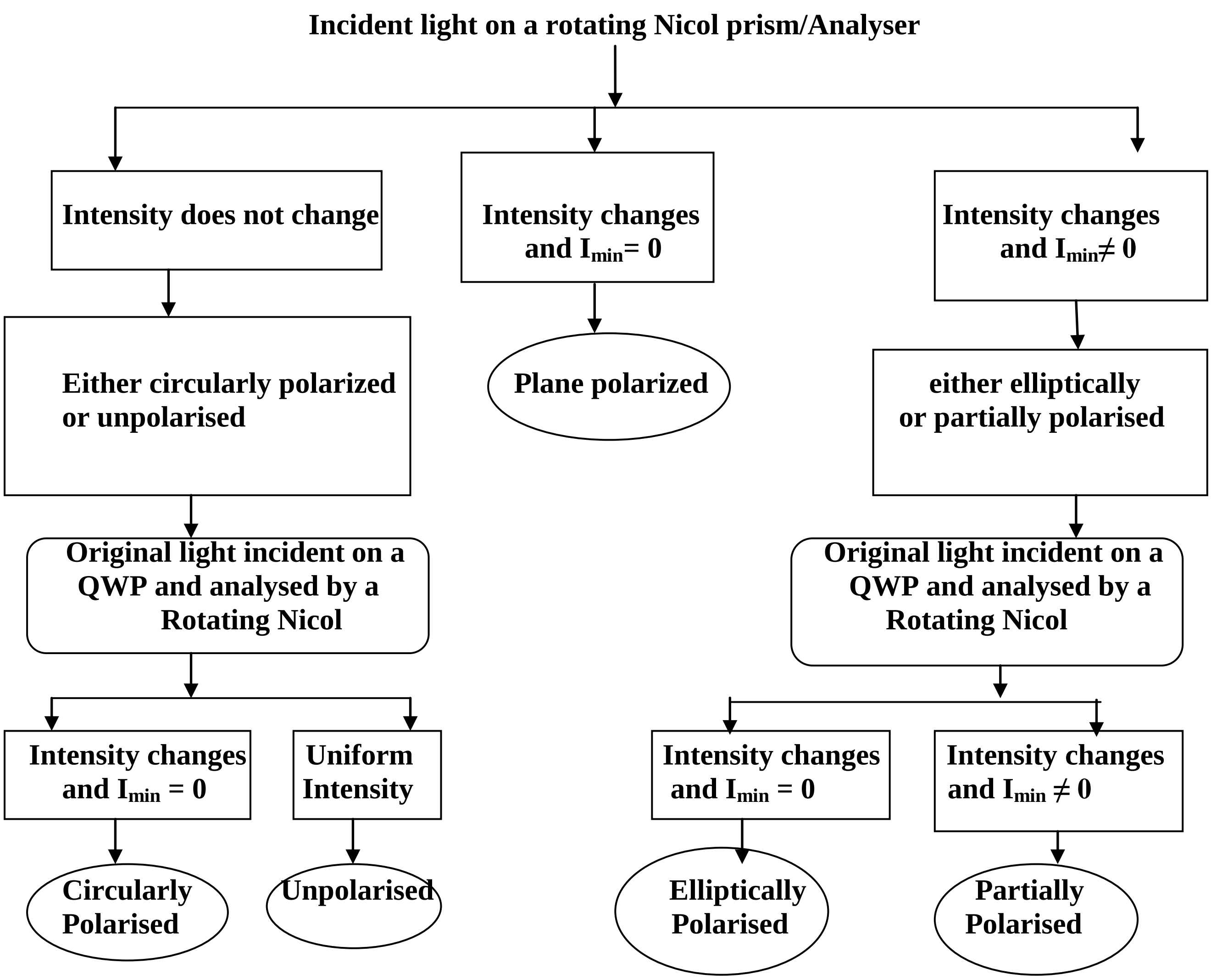
Numerical

1) Calculate the minimum thickness of quarter-wave plate with $\lambda = 5.8 * 10^{-7}$ m, $\mu_e = 1.553$ and $\mu_o = 1.544$.

Ans. The minimum thickness of a quarter wave plate of quartz ($\mu_e > \mu_o$), the required thickness of the plate is given by,

$t = \lambda / 4(\mu_e - \mu_o) = 5.8*10^{-7}/4*(1.553 – 1.544)$ m $= 1.61 * 10^{-5}$m.

Systematic Analysis of Polarised light



Miscellaneous Applications of Polarisation

1) A pair of polarisers can be used to control the intensity of light by varying the angle between their polarising axes.

2) Polarising sunglasses are used to reduce glare. Since light scattered from the sky and light reflected from shiny surfaces such as water or hot roads is partially plane polarized, the appropriately oriented polarising material reduces the intensity of such light and the associated glare.

3) When a thin slice of rock is placed between crossed polarisers in a petrological microscope the appearance of the mineral grains depends on their crystal shape, their light absorbing properties and birefringence. This aids in their identification.

4) Birefringence can be induced in some materials by high electric fields (a phenomenon known as the Kerr effect). This effect can be used to make fast shutters for high speed photography.

5) Birefringence can be induced in glass and some plastics by mechanical stress. This phenomenon is called **photoelasticity**. Photoelasticity can be used to study stress patterns in loaded engineering structures and other objects. A Perspex model of the object (for example an engine part, a bridge or a bone) is constructed and placed between crossed polarisers. When external forces are applied to the model, the internal strains cause birefringence, so that some of the light now gets through and the light patterns reveal the patterns of the internal strains. Since the refractive indices also depend on the frequency of the light the resulting patterns are brightly coloured when incident white light is used.