Assignment-I

MATH1201 CSE(SEC-A)

TOPIC: Module-IV (Three Dimensional Geometry)

1. Show that the straight lines whose $d.cs.$ are given by the equations $al + bm + cn = 0,\ ul^2 + vm^2 + wn^2 = 0$ are perpendicular or parallel according as $(v+w)a^2 + (w + ub^2 + (u+v)c^2 = 0 \ or \frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0.$

2. Prove that the angle between two diagonals of cube is $\cos^{-1}\frac{1}{3}$.

3. If a line makes angles, $\alpha, \beta\ , \gamma, \delta$ with the four diagonals of cube, prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

4. Find the point in which the perpendicular form the origin on the straight line joining the points $A(-9,4,5)\ and\ B(11,0,-1)$ meets it.

5. Show that the angle between the straight lines whose $d.cs.$ are given by $l + m + n = 0,\ fmn + gnl + hlm = 0$ is $\frac{\pi}{3}$, if $\frac{1}{f} + \frac{1}{g} + \frac{1}{h} = 0$.

6. A variable plane which is at a constant distance $3p$ from the origin O cuts the axes in A, B, C. Show that the locus of the point of intersection of the planes through *A*, *B*, *C* drawn parallel to the co-ordinate planes is $9(x^{-2} + y^{-2} + z^{-2}) = p^{-2}$.

7. A variable plane has intercepts on the co-ordinate axes, the sum of whose squares is a constant k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is $(x^2 + y^2 + z^2)^2\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) = k^2$.

8. Show the line of intersection of the planes $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$ is coplanar with the line of intersection of the planes $2x - 2y + 3z - 2 = 0$ and $x - y + z + 1 = 0$. Obtain the equation of the plane containing the lines.

9. Find the equation of the plane bisecting the angle between the planes $x - 2y + 3z - 5 = 0$ and $2x - y - z + 3 = 0$ which contains the origin.

10. A variable plane passes through a fixed point (α, β, γ) and meets the axes of reference in A, B and C. Show that the locus of the point of intersection of the planes through A, B and C parallel to the co-ordinate planes is $\frac{\alpha}{x}+\frac{\beta}{y}+\frac{\gamma}{z}=1$.

11. Find the equation of the image of the point$(1,-2,3)$ in the plane $2x-3y+2z+3=0$.

12. If the plane $3x+4y+5z=0$ be horizontal, then find the equations of the lines of greatest and least slopes on the plane $x+2y+3z=4$ through the point $(2,-2,2)$.

13. Show that the equation of the plane through the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ are perpendicular to the plane containing the lines $\frac{x}{m}=\frac{y}{n}=\frac{z}{l}$ and $\frac{x}{n}=\frac{y}{l}=\frac{z}{m}$ is $(m-n)x+(n-l)y+(l-m)z=0$.

14. Show that the equation of the plane containing the straight line $\frac{y}{b}-\frac{z}{c}=1, x=0$ and parallel to the straight line $\frac{x}{a}+\frac{z}{c}=1, y=0$ is $\frac{x}{a}-\frac{y}{b}+\frac{z}{c}+1=0$ and if $2d$ is the s.d., prove that $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$.

15. Find the equation of the plane through the point $(2,-1,3)$ and perpendicular to the line

$x-2y+3z-4=0=2x-3y+4z-5$.

16. Find the angle between the two straight lines whose direction cosines are given by $2l+2m-n=0, mn+nl+lm=0$.

17. Show that the equations of the planes through the intersection of the planes $x+3y+6=0 \text{ and } 3x-y-4z=0$ whose perpendicular distance from the origin is unity, are $2x+y-2z+3=0$ and $x-2y-2z-3=0$.

18. Show that the equation of the plane parallel to the plane $2x+4y+5z=6$ and the sum of whose intercepts on the co-ordinate axes is 19, is $2x+4y+5z=20$.

19. Show that the equation of the plane through the point $(2,3,3)$ and parallel to the straight lines $x-1=2y-5=2z$ and $3x=4y-11=3z-4 \text{ is } x-4y+2z+4=0$.

20. Find the equation of the image of the line $\frac{x-1}{2}=\frac{y-3}{5}=\frac{z-4}{2}$ in the plane $2x-y+z+3=0$.

21. Prove that the acute angle between the lines whose $d.cs.$ are given by the relations $l+m+n=0$ and $l^2+m^2-n^2=0$ is $\frac{\pi}{3}$.

22. Prove that the two lines whose direction cosines are given by the relations $2l+2m-n=0$ and $2l+2m-n=0$ and $lm+mn+nl=0$ and perpendicular to each other.

23. Find the equation of the projection of the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-4}{4}$ on the plane $x+3y+z+5=0$.

24. A point P moves on the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, which is fixed and the plane through P perpendicular to OP meets the axes in A, B, C. If the planes through A, B, C parallel to the co-ordinate planes meet in a point Q then show that the locus of Q is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}$.

25. Show that the equations of the planes through the intersection of the planes $x+3y+6=0$ and $3x-y-4z=0$ whose perpendicular distance from the origin is unity, are $2x+y-2z+3=0$ and $x-2y-2z-3=0$.

26. Show that the angle between the straight line $\frac{x-4}{7}=\frac{y-1}{4}=\frac{z+3}{4}$ and the plane $x-2y-2z=8$ is $\sin^{-1}\frac{1}{3}$.

27. Find the shortest distance and its equation between the lines:

$$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}, \quad \frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$

28. Find the shortest distance between the two skew lines $x+2y+2z-13=0=x+4y-2z-5$ and $2x-2y+z-1=0=2x-4y-z-9$.

29. Find the equation of the line through the point (1,2,4) and perpendicular to the line $3x+2y-z-4=0=x-2y-2z-5$.

30. Show that the direction cosines l, m, n of two straight lines connected by the relations $l+m+n=0$, $mn-2nl-2lm=0$ are given by $(l:m:n)=(1:1:-2)$ and $(l:m:n)=(1:-2:1)$.