Assignment Sheet of Ordinary Differential Equation

1. Obtain the differential equation of the system of confocal conics
$\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, $(Ans:(a^2-b^2)y'=(xy'-y)(x+yy')$
in which λ is the arbitrary parameter and a,b are given constants.
2. Show that all circles of radius r are represented by the differential equation
$\left\{1+(\frac{dy}{dx})^2\right\}^{3/2}=r\frac{d^2y}{dx^2}$.
3. Water is flowing into a right circular conical vessel, 45 cm deep and 27 cm in diameter at the rate of 11 cc per minute. How fast is the water-level rising when the water is 30 cm deep? (*Ans: 0.043 cm/min)*

Solve the following first order differential equations after finding appropriate Integrating Factor when necessary.

4. $(e^x siny+e^{-y})dx+(e^x cosy-xe^{-y})dy=0$
5. $(y^2e^{xy^2}+4x^3)dx+(2xye^{xy^2}-3y^2)dy=0$
6. $(3x^2y^4+2xy)dx+(2x^3y^3-x^2)dy=0$
7. $(x^2y-2xy^2)dx+(3x^2y-x^3)dy=0$
8. Show that e^x is an I.F. of the following differential equation :
$(x^2+y^2+2x)dx+2ydy=0$. Also, show that the particular solution passing through the point $x=1, y=1$ is : $x^2+y^2=2e^{1-x}$.
9. $\{xysin(xy)+cos(xy)\}y\,dx+\{xysin(xy)-cos(xy)\}xdy=0$
10. $(2xy^4e^y+2xy^3+y)dx+(x^2y^4e^y-x^2y^2-3x)dy=0$
11. $\left(xy^2-e^{1/x^3}\right)dx-x^2ydy=0$
12. $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$
13. $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$
14. $x^2(2ydx+3xdy)+y^2(-2ydx+2xdy)=0$
15. $(y^2e^x+2xy)dx-x^2dy=0$
16. $(x^3y^2+xy)dx=dy$
17. $\frac{dy}{dx}+\frac{ylny}{x}=\frac{y(lny)^2}{x^2}$
18. $\frac{dy}{dx}-\frac{tany}{1+x}=(1+x)e^x\sec y$
19. $\{y(1-x\tan x)+x^2cosx\}dx-xdy=0$

20. $(x+2y^3)+\frac{dy}{dx}=y$

21. $(1+x)\cos y\frac{dy}{dx}-\sin y=(1+x)^2e^x$

22. $\sin y\frac{dy}{dx}=\cos x(2\cos y-\sin^2 x)$

23. $\frac{dy}{dx}+y=y^3(\cos x-\sin x)$

24. $x\frac{dy}{dx}+y=y^2\log x$

25. $dr+(2r\cot\theta+\sin 2\theta)d\theta=0$

26. $p^2+2xp-3x^2=0 \quad \left[p \text{ denotes } \frac{dy}{dx}\right]$

27. $p-\frac{1}{p}-\frac{x}{y}+\frac{y}{x}=0$

28. $p^2+2py\cot x=y^2$

29. $xyp^2+p(3x^2-2y^2)-6xy=0$

30. $y=xp^2+p$

31. $xp^2+(y-x)p-y=0$

32. $p^3+2xp^2-y^2p^2-2xy^2p=0$

33. $e^y-p^3-p=0$

34. $y=px+\sin^{-1}p$ – find general and singular solution

35. $y=px+p-p^2$ – find general and singular solution

36. $p=\cos(y-px)$ – find general and singular solution

37. $p^2+y-x=0$

Get the general solution of the following differential equations. (by D-operator method)

38. $(D^2+2)y=x^2$

39. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+4y=e^x\sin^2 x$

40. $\frac{d^2y}{dx^2}-a^2y=\sec(ax)$

41. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x$

42. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\sin x$

43. $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x\cos 2x$

44. $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1+\sin x)^2$

45. $(D^2+2)y = x\cos x$

46. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = x + e^x\cos x$

47. $\frac{d^2y}{dx^2} - y = x^2\sin x$

48. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$

49. $(D^2-7D+6)y = (x-2)e^x$

50. $\frac{d^2y}{dt^2} + 4\frac{dx}{dt} + 4x = 3\sin 2t$, given that $x=0$, $\frac{dx}{dt} = 0$ at $t=0$

51. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}\sin 4x + 2^x$

Get the general solution by the method of variation of parameters

52. $\frac{d^2y}{dx^2} + a^2y = \operatorname{cosec} ax$

53. $\frac{d^2y}{dx^2} + y = \sec x$

54. $\frac{d^2y}{dx^2} + a^2y = \sec ax$

55. $\frac{d^2y}{dx^2} + y = \tan x$

56. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1+e^{-x}}$

57. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x\sin x$

58. $\frac{d^2y}{dx^2} + y = \frac{1}{1+\sin x}$

Solve the following (Cauchy-Euler) differential equations :

59. $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = \sin(\log x) + x\cos(\log x)$

60. $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=\log x$

61. $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=x\sin(\log x)$

62. $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$

63. $x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+4y=x\log x$

64. $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log x$

65. $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=x\log x$

66. $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=x\sin(\log x)$

64. $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\sin[2\log(1+x)]$

65. $(3x+2)^2\frac{d^2y}{dx^2}+5(3x+2)\frac{dy}{dx}-3y=x^2+x+1$

Solution of the differential equations

4. $e^x\sin y+xe^{-y}=c$

5. $e^{xy^2}+x^4-y^3=c$

6. $x^2y^3+\frac{x^2}{y}=c$

7. $\frac{x}{y}+\log\frac{y^3}{x^2}=c$

8.

9. $\log|\sec(xy)|+\log|x|-\log|y|=c$

10. $x^2e^y+\frac{x^2}{y}+\frac{x}{y^3}=c$

11. $\frac{y^2}{2x^2}-\frac{1}{3}e^{1/x^3}=c$

12. $\frac{1}{2}x^2y^4 + xy^2 + \frac{1}{3}y^6 = c$

13. $\log\frac{x^2}{y} = \frac{1}{xy} + c$

14. $\log(x^2y^3) + \frac{y^2}{x^2} = c$

15. $e^x + \frac{x^2}{y} = c$

16. $\frac{1}{y} = x^2 - 2 + ce^{-x^2/2}$

17. $\frac{1}{x\log y} = \frac{1}{2x^2} + c$

18. $Siny=(1+x)(e^x+c)$

19. $y=x^2\cos x+cx\cos x$

20. $x=y^3+cy$

21. $Siny=(1+x)\,(e^x+c)$

22. $\cos y = \frac{1}{2}\sin^2 x - \frac{1}{2}\sin x + \frac{1}{4} + ce^{-2\sin x}$

23. $y^{-2} = \frac{2}{5}(\cos x - 3\sin x) + ce^{2x}$

24. $\frac{1}{y} = (\log x + 1) + cx$

25. $x^3y^2 + \frac{x^2}{y} = c$

26. $(2y+3x^2-2c)\,(2y-x^2-2c) = 0$

27. $(xy-c)\,(y^2-x^2-2c) = 0$

28. $(y + y\cos x - c)\,(y - y\cos x - c) = 0$

29. $(y - cx^2)\,(y^2 + 3x^2 - 2c) = 0$

30. $y=xp^2+p$

    $y=p\{p\,(\log p - p + c)\,(p-1)^{-2} + 1\}$

31. $(y-x+c)\,(xy+c)=0$

32. $(y-c)\,(y+x^2-c)\,(xy+cy+1)=0$

33. $y=\log(p^3+p)$

    $x=2\tan^{-1}p - p^{-1} + c$

34. General solution $y=cx+\sin^{-1}c$

Singular solution $y=\sqrt{x^2-1}+\sin^{-1}\left(\frac{\sqrt{x^2-1}}{x}\right)$

35. General solution $y=cx+c-c^2$

Singular solution $4y=(x+1)^2$

36. General solution $y=cx+\cos^{-1}c$

$$y=\sqrt{x^2-1}+\cos^{-1}\left(\frac{1}{x}\sqrt{x^2-1}\right)$$

37. $x=c-2\{p+\log(p-1)\}$

$y=c-2\left\{\frac{1}{2}p^2+p+\log(p-1)\right\}$

38. $c_1\cos\sqrt{2}x+c_2\sin\sqrt{2}x+\frac{1}{2}(x^2-1)$

39. $e^x\left(c_1\cos\sqrt{3x}+c_2\sin\sqrt{3x}\right)+\frac{1}{6}e^x+\frac{1}{2}e^x\cos 2x$

40. $c_1\cos ax+c_2\sin ax+\frac{x\sin ax}{a}+\frac{\cos ax\log\cos ax}{a^2}$

41. $(c_1+c_2x)e^x+\frac{1}{6}x^3e^x$

42. $c_1+c_2e^{2x}-\frac{1}{2}e^x x\cos x$

43. $c_1\cos\sqrt{2}x+c_2\sin\sqrt{2}x+\frac{e^{3x}}{11}\left(x^2-\frac{12}{11}x+\frac{50}{121}\right)+\frac{e^x}{17}(4\sin 2x-\cos 2x)$

44. $e^{-\frac{x}{2}}\left(A\cos\frac{\sqrt{3}x}{2}+B\sin\frac{\sqrt{3}x}{2}\right)+\frac{1}{26}(3\cos 2x-2\sin 2x)-2\cos x+\frac{3}{2}$

45. $c_1\cos\sqrt{2}x+c_2\sin\sqrt{2}x+x\cos x+2\sin x$

46. $e^x(c_1\cos x+c_2\sin x)+\frac{1}{2}(x+1)+\frac{1}{2}xe^x\sin x$

47. $c_1e^x+c_2e^{-x}-\frac{1}{2}x^2\sin x-x\cos x$

48. $c_1e^x+c_2e^{2x}+\frac{e^{3x}}{2}\left(x-\frac{3}{2}\right)+\frac{1}{20}(3\cos 2x-\sin 2x)$

49. $c_1e^x+c_2e^{6x}+\left(\frac{9}{25}x-\frac{1}{10}x^2\right)e^x$

50. $x = \frac{1}{8}(3+6t)e^{-2t} - \frac{3}{8}\cos 2t$

51. $e^{3x}(c_1\cos 2x + c_2\sin 2x) \frac{-2}{3} e^{3x}\sin 4x + \frac{2^x}{(\log 2)^2 - 6\log 2 + 13}$

52. $y=(c_1 - x/a)\cos ax + \left[c_2 + \left(1/a^2\right)\right]\log \sin ax \Big] \sin ax$

53. $y=c_1 \cos x + c_2 \sin x + \cos x \log(\cos x) + x\sin x$

54. $y=c_1 \cos ax + c_2 \sin ax - \frac{1}{a^2}\cos ax \log(\sec ax) + \frac{1}{a} x\sin ax$

55. $y=c_1 \cos x + c_2 \sin x - \cos x \log(\sec x + \tan x)$

56. $y=\left(e^x + e^{2x}\right)\log(1+e^x) + (c_1 - 1 - x)e^x + (c_2 - x)e^{2x}$

57. $y=c_1 + c_2 e^{2x} - \frac{1}{2} e^x \sin x$

58. $y=c_1 \cos x + c_2 \sin x + \sin x \log(1+\sin x) - x\cos x - 1$

59. $y=c_1 x + \frac{c_2}{x} - \frac{1}{2}\sin(\log x) + \frac{x}{5}\left[2\sin(\log x) - \cos(\log x)\right]$

60. $c_1 x^{-1} + c_2 x^{-2} + \frac{1}{2}\log x - \frac{3}{4}$

61. $y=x\left\{c_1 \cos(\sqrt{3}\log x) + c_2 \sin(\sqrt{3}\log x)\right\} + \frac{1}{2} x\sin(\log x)$

62. $y=\frac{1}{x}(c_1 + c_2 \log x) + \frac{1}{x}\log\frac{x}{1-x}$

63. $y=x^{-2}(c_1 + c_2 \log x) + \frac{x}{9}\left(\log x - \frac{2}{3}\right)$

64. $y=\frac{c_1}{x} + c_2 x^3 - \frac{1}{9} x^2 (3\log x + 2)$

65. $y=x(c_1 \cos\log x + c_2 \sin\log x) + x\log x$

66. $y=x[a\cos(\log x) + b\sin(\log x)] - \frac{1}{2}\log x \cos(\log x)$

67. $y=c_1 \cos\log(1+x) + c_2 \sin\log(1+x) - \frac{1}{3}\sin[2\log(1+x)]$

68. $y=c_1(3x+2)^{1/3} + c_2(3x+2)^{-1} + \frac{1}{27}\left[\frac{1}{15}(3x+2)^2 + \frac{1}{4}(3x+2)^{-7}\right]$