CHAPTER

11
Properties of Fluids

11.1 INTRODUCTION

Mechanics is a subject dealing with the conditions under which a body can remain at rest or in motion. Mechanics can be classified into two: ***solid mechanics*** and ***fluid mechanics***. Fluid mechanics is that branch of science which deals with the behaviour of fluids at rest as well as in motion and the subsequent effects of fluid upon its boundaries which may be either solid surfaces or interfaces with other fluids. Both liquids and gases are classified as fluids. The number of fluids in engineering applications are enormous: breathing, blood flow, swimming, pumps, fans, blowers, turbines, ships, rivers, airplanes, missiles, rockets, engines, jets, etc. Almost everything on this planet either is a fluid or moves within or near a fluid.

The field of fluid mechanics has been divided into three branches—fluid statics, fluid kinematics and fluid dynamics. ***Fluid statics*** is concerned with the behavior of a fluid at rest. ***Fluid kinematics*** deals with the motion of fluids without reference to forces that cause the motion. ***Fluid dynamics*** involves the study of a fluid motion as a consequence of forces that causes the motion.

11.2 DEFINITION OF FLUID

From the point of view of fluid mechanics, all matter consists of only two states, fluid and solid. A solid can resist a shear stress by a static deformation, a fluid cannot. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as shear stress is applied. A fluid is a substance that deforms continuously when subjected to a shear stress, however small the shear stress may be. A fluid may be either a liquid or a gas.

11.2.1 Difference between a Solid and a Fluid

- The fundamental difference between a fluid and a solid lies in the response to a shear stress of the respective materials. For a solid, the strain is a function of the applied stress, provided that the elastic limit is not exceeded. For a fluid, the rate of strain is proportional to the applied stress.

- The strain in a solid is independent of the time over which the force is applied and, if the elastic limit is not exceeded, the deformation disappears when the force is removed. A fluid continues to flow for as long as the force is applied and will not recover its original form when the force is removed. Consider a rectangular solid element *ABCD* as shown in Fig. 11.1 (a). Under the action of a shear force *F* the element assumes the shape $ABC'D'$. If the solid is perfectly elastic, it goes back to its shape *ABCD* when the force is withdrawn. In contrast, the element of the fluid *ABCD* (refer Fig. 11.1 (b)) confined between parallel plates deforms to shapes such as $ABC'D'$ and $ABC''D''$ as long as he force *F* is maintained on the upper plate.
- Within elastic limit, a solid has perfect memory because solid always relaxes back to its preferred shape, whereas a fluid has zero memory.
- The molecules of a solid are closer together than those of a fluid. The attractive forces between the molecules of a solid are so large that a solid tends to retain its shape. However, fluids cannot retain their shape, because the attractive forces between the molecules are smaller.

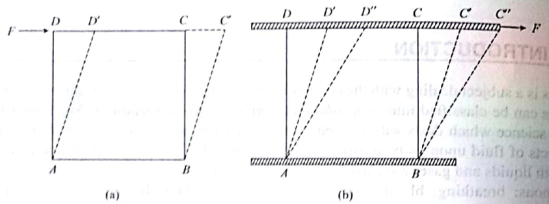

Figure 11.1 *Deformation of solid and fluid elements: (a) solid and (b) fluid*

11.2.2 Difference between Liquids and Gases

The difference between the two classes of fluids, liquids and gases, is all about the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field if unconfined from above. Since gas molecules are widely spaced with negligible cohesive forces, a gas is free to expand until it encounters confining walls. A gas has no definite volume, and when left to itself without confinement, a gas forms an atmosphere which is essentially hydrostatic.

11.3 THE CONCEPT OF CONTINUUM

The fluid flow analysis can be attempted from two different view points. One approach, popularly known as microscopic, stems from molecular point of view. It relies on the consideration that fluid essential comprises of molecules the motion of which is characterized by the laws of dynamics. The behaviour of the fluid then can be described by summing up the properties of the molecules following

statistical approach. However, it has got certain limitations in regard to its applications in gases of higher density and in case of liquids. The other approach is the macroscopic one where the gross behaviour is considered rather than an individual molecule. The macroscopic approach treats the fluid as continuous and the variations of the property values of the individual molecules are not reflected. This approach gives the concept of **continuum** where fluids can be treated as a continuous medium disregarding the discontinuity in the microscopic entities. In the continuum approach at each point of the continuous fluid there is a unique value of the field variables such as pressure, velocity, density. This continuous matter follows the conservation laws of mass, momentum and energy. These laws can be derived using a set of differential equations. In most of the engineering applications, the concept of continuum yields very good results and hence accepted well.

11.4 FLUID PROPERTIES

Certain characteristics of a continuous fluid are independent of the motion of the fluid. These characteristics are called basic properties of the fluid. Here, elaborate discussions on a few such basic properties are included.

Fluid property is defined to be a characteristic of the material structure of the fluid. ***Flow property,*** is something whose value is determined in part by how the fluid is moving. The colour of a fluid is purely a fluid property, while the velocity of a fluid is purely a flow property. The density, pressure, temperature, viscosity, etc, are actually flow properties whose precise values depend on the nature of the fluid and type of flow.

11.4.1 Density

The density of a fluid is mass per unit volume. If a fluid element enclosing a point has a volume ΔV and mass Δm then density (ρ) at that point is written as

$$\rho = \lim_{\Delta V \to V^*} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

where ΔV^* is the smallest elemental volume over which the continuum hypothesis is valid.

The dimension of density is ML^{-3} and the unit of density in SI system is kg/m³.

The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

A thermodynamic property of a fluid density has a certain value defined by the pressure and temperature of the fluid. The relationship is expressed by the characteristic equation of state. Since pressure and temperature are generally functions of position and time of flow, density must also be a function of position and time. Thus, density is flow property. The accepted value of density of air at sea level at 1.0133 bar and 288.15 K is 1.225kg/m³.

The density of most gases is proportional to pressure and inversely proportional to temperature.

The variation of density of liquids with pressure is usually negligible. For example, at 20°C, the density of water changes from 998 kg/m³ at 1 atm to 1003 kg/m³ at 100 atm (0.5% change). The density of liquids depends more strongly on temperature than it does on pressure. For example, at 1 atm, the density of water changes from 998 kg/m³ to 975 kg/m³ at 75°C (only 2.3 % change).

11.4.2 Specific Weight

The specific weight is the weight of fluid per unit volume.

$$\text{Specific weight } (\gamma) = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{\text{Mass of fluid}}{\text{Volume of fluid}} \times \text{Acceleration due to gravity}$$

$$\gamma = \rho g$$

Specific weight is $ML^{-2}T^{-2}$ and the unit of specific weight in SI system is N/m^3.

11.4.3 Specific Volume

The specific volume of a fluid is the volume occupied by unit mass of fluid.

The dimension of specific volume is L^3T^{-1} and the unit of specific volume in SI system is m^3/kg.

Specific volume is the reciprocal of density i.e. $v = \dfrac{1}{\rho}$

11.4.4 Specific Gravity (or Relative Density)

Specific gravity or relative density is defined as the ratio of the density of some standard reference fluid at a specified temperature and pressure.

The standard fluid is water for liquid and is air for gases. For liquids, the specified temperature and pressure are 4°C and 101 kN/m^2, for which the density of water is 1000 kg/m^3.

$$S_{liquid} = \frac{\rho_{liquid}}{\rho_{water}}$$

$$S_{gas} = \frac{\rho_{gas}}{\rho_{air}}$$

Specific gravity is a dimensionless parameter. Engineers find these dimensionless ratios easier to remember than the actual numerical values of density of a variety of fluids.

11.5 COMPRESSIBLE FLUID AND INCOMPRESSIBLE FLUID

Fluids can be classified into incompressible and compressible on the basis of density change with change in pressure. A fluid is usually called incompressible if its density does not change significantly with change in pressure. A fluid is compressible when it has a change in density because of change in pressure. There is no fluid in reality which is incompressible. Liquids are considered incompressible fluids, since the change in density for liquids with pressure is so small as to be negligible.

11.6 IDEAL FLUID AND REAL FLUID

An ***ideal fluid*** is a fully hypothetical fluid which is assumed to have no viscosity and no compressibility. The concept of an ideal fluid has been utilized in the analytical treatment of fluid-flow problems. The

mathematical analysis of the flow problem can be considerably simplified by assuming the fluid to be non-viscous and incompressible. Such a fluid does not exist in reality.

In a ***real fluid***, shear stresses occur whenever the fluid is in motion. In other words, fluid friction exists when a real fluid is in motion. Shear stresses in a real fluid in motion are possible due to a property called the ***viscosity*** of the fluid.

11.7 VISCOSITY

The property which characterizes the resistance that a fluid offers to applied shear forces is termed viscosity. The resistance depends on the rate of deformation.

Let us consider a fluid contained between two large parallel plates, separated by a distance *L*, as shown in Fig. 11.2. The lower plate is assumed to be stationary, while the upper one is moving parallel to it with a velocity *V* under the influence of the applied shearing force *F*.

The fluid particles sticking to the moving plate move with the same velocity *V*, and the shear stress τ acting on this fluid layer is

$$\tau = \frac{F}{A}$$

where *A* is the area of contact between the plate and the fluid.

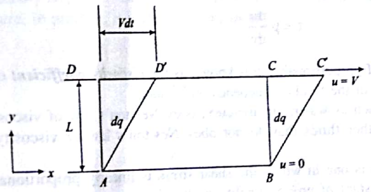

Figure 11.2

The fluid in contact with the lower plate assumes the velocity same as that of the plate, which is zero. If the gap separating the two plates is small or the velocity *V* is high, the velocity distribution will be a straight line. The equation of velocity can be written as

$$u(y) = \frac{y}{L}V$$

where *y* is the vertical distance measured from the lower plate.

The velocity gradient will be

$$\frac{du}{dy} = \frac{V}{L}$$

During a differential time interval dt, the element of the fluid deforms through a differential angle $d\theta$ while the upper plate moves a differential distance dx.

$$dx = V dt$$

The angular deformation can be expressed as

$$d\theta \approx \tan\theta = \frac{dx}{L} = \frac{V dt}{L} = \frac{du}{dy}.dt$$

The rate of angular deformation (or the shear strain) of a fluid element is equivalent to the velocity gradient $\frac{du}{dy}$.

For a well-ordered flow whereby fluid particles move in straight, parallel lines (parallel flow), Newton's law of viscosity states that the for certain fluids, called Newtonian fluids, the shear stress (τ) on an interface tangent to the direction of flow is proportional to the distance rate of charge of velocity $\left(\frac{du}{dy}\right)$, wherein the differentiation is taken in a direction normal to the interface.

Mathematically, Newton's law of viscosity can be expressed as

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy} \tag{11.1}$$

where, the constant of proportionality μ is known as the ***viscosity coefficient*** or simply the viscosity which is the property of the fluid and depends on its state.

Common fluids, such as water, air, mercury, obey Newton's law of viscosity and are known as ***Newtonian fluids***. Other fluids that do not obey Newton's law of viscosity are known as ***non-Newtonian fluids***.

A Newtonian fluid is one in which the shear stress is linearly proportional to the rate of shear deformation. The constant of proportionality is the viscosity, μ. Air would be considered a low-viscosity Newtonian fluid, while water would be a medium-viscosity Newtonian fluid. Motor oil and maple syrup are high-viscosity Newtonian fluids. Fluids that do not follow the Newtonian behaviour law include toothpaste, blood and paints. *Note that the equation 11.1 is applicable only for one-dimensional flow field and Newtonian fluid.*

11.7.1 Dimensional Formula and Units of Viscosity

The dimension of viscosity can be determined from Newton's law of viscosity (Eq. 11.1) as

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{FL^{-2}}{\frac{LT^{-1}}{L}} = FTL^{-2}$$

The dimensions of viscosity may also be expressed as $ML^{-1}T^{-1}$. The unit of viscosity if SI system is N-s/m^2, or kg/m-s.

The unit of dynamic viscosity in CGS system is Poise (P), named after Jean Louis Marie Poiseuille.

$$1 \text{ Poise} = 1 \frac{\text{g}}{\text{cm-s}} = \frac{10^{-3}\text{kg}}{10^{-2}\text{m-s}} = 0.1 \text{ kg/m-s}$$

$$1 \text{ centiPoise(cP)} = \frac{1}{100} \text{ poise} = 10^{-3} \text{ kg/m-s}$$

11.7.2 Variation of Viscosity with Temperature

The viscosity of a liquid decreases with temperature, but the viscosity of a gas increases with temperature (Fig. 11.3). The causes of viscosity are two—the intermolecular cohesive forces and the molecular momentum transfer. A liquid, with molecules much more closely spaced than a gas, has cohesive forces much larger than a gas. Although molecular momentum transfer exists, intermolecular cohesive forces predominate in the case of a liquid. Now, since intermolecular cohesive forces decrease with temperature, viscosity of liquids decreases with temperature. On the other hand, a gas has very small cohesive forces and molecular momentum transfer predominates. Since molecular momentum transfer increases with temperature, viscosity of gases also increases with temperature. According to kinetic theory of gases, viscosity of gases should be proportional to the square root of the absolute temperature. In practice, it increases more rapidly.

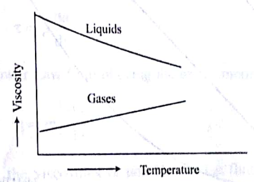

Figure 11.3 *Variation of viscosity of liquids and gases with temperature*

11.7.3 Newtonian and Non-Newtonian Fluids

Fluids for which the shear stress is proportional to the rate shear deformation are called Newtonian fluids after Sir Isacc Newton, who expressed it first in 1687. Newtonian fluids can be represented by a straight line. The slope of this line is determined by the viscosity.

The study of the response of materials to stress is called ***rheology***.

There are certain fluids where the linear relationship between the shear stress and the deformation rate (velocity gradient in parallel flow) is not valid. Because of the deviation from Newton's law of viscosity they are commonly termed non-Newtonian fluids.

Many common fluids exhibit non-Newtonian behaviour. Two familiar examples are toothpaste and paint. Toothpaste behaves as a fluid when squeezed from the tube. However, it does not run out by itself when the cap is removed. There is a yield stress below which toothpaste behaves as a solid. Paint is very thick when in the can, but becomes thin when sheared by brushing.

Non-Newtonian fluids are classified as having time-independent or time-dependent behaviour. Examples of time-independent behaviour are shown in the rheological diagram (Fig. 11.4 (a)).

The abscissa in the rheologigal diagram represents the behaviour of ideal fluids since for the ideal fluids the resistance to shearing deformation rate is always zero, and hence they exhibit zero shear stress under any condition of flow. The ordinate represents the behaviour of an ideal solid for any condition, since there is no deformation of an ideal solid for any load.

The Newtonian fluids behave according to Newton's law of viscosity that shear stress is linearly proportional to the velocity gradient for parallel flow. Thus, for Newtonian fluids, the plot of shear stress against velocity gradient is a straight line passing through the origin.

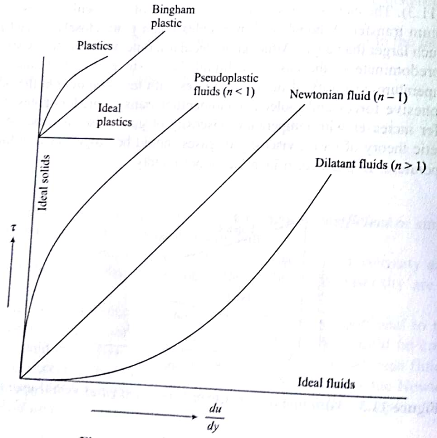

Figure 11.4 (a) *Rheological diagram*

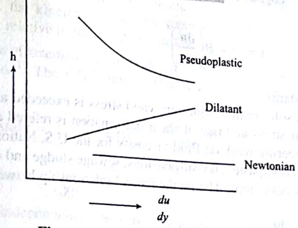

Figure 11.4 (b) *Rheological diagram*

Time-independent non-Newtonian fluids The mechanistic behaviour of a variety of non-Newtonian fluids can be described by the Power Law model.

According to Power Law model,

$$\tau = m\left|\frac{du}{dy}\right|^n = m\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy} \tag{11.2}$$

where m is known as the flow consistency index and n is the flow behaviour index.

Again according to Newton's law of viscosity,

$$\tau = \mu\frac{du}{dy}$$

Hence, viscosity for the Power Law fluid obeying the above model can be described as

$$\eta = m\left|\frac{du}{dy}\right|^{n-1}$$

It is readily observed that the viscosities of non-Newtonian fluids are functions of deformation rates and are often termed ***apparent*** or ***effective viscosity*** (η). Most non-Newtonian fluids have apparent viscosities that are relatively high compared with the viscosity of water.

When $n = 1$, η equals to $m(= \mu)$, the model identically satisfies Newtonian model as a special case.

Fluids in which the apparent viscosity decreases with increasing deformation rate ($n < 1$) are called pseudoplastic (or shear thinning) fluids. Most non-Newtonian fluids fall into this group. Examples of pseudoplastic fluids are blood, milk, gelatine, paper pulp, polymer solutions, colloidal suspensions, etc.

If the apparent viscosity increases with increasing deformation rate ($n > 1$) the fluid is termed dilatant (or shear thickening). Sugars in water, suspensions of starch and of sand, and butter are examples of *dilatant fluids*.

For plastic the shear stress must reach a certain minimum value before flow commences. Thereafter, shear stress increases with the rate of shear according to the relationship

$$\tau = A + B\left(\frac{du}{dy}\right)^n$$

where A, B and n are constants.

A fluid that behaves as a solid until a minimum yield stress is exceeded and subsequently exhibits a linear relation between shear stress and rate of shear deformation is refered as a ***Bingham plastic*** after E.C. Bingham who did pioneering work on fluid viscosity for the U.S. National Bureau of Standards in the early twentieth century. Toothpaste, clay suspensions, sewage sludge and drilling muds are examples of substances exhibiting this behaviour. The mathematical relationship between shear stress and rate of shear deformation is $A + B\left(\frac{du}{dy}\right)^n$ where $n = 1$.

Time-dependent non-Newtonian fluids The study of non-Newtonian fluids is further complicated by the fact that apparent viscosity may be time dependent.

Some fluids require a gradual increasing shear stress to maintain a constant strain rate and are called ***rheopectic***. Otherwise, rheopectic fluids are those for which apparent viscosity increases with the time for which shearing forces are applied. Gypsum in water, whipping cream, bentonite clay solutions are examples of rheopectic fluids.

Thixotropic substances are those for which the apparent viscosity decreases with time (refer Fig. 11.5) for which the shearing forces are applied. Many paints, printer's ink, lipstick, enamels, crude oils are thixotropic.

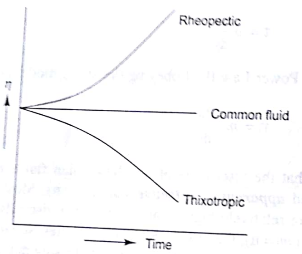

Figure 11.5 *Time-dependent non-Newtonian fluids*

11.8 KINEMATIC VISCOSITY

In fluid mechanics, the ratio of dynamic viscosity to density appears frequently. The ratio is given the name kinematic viscosity and is designated by Greek letter (nu)υ. It is expressed as $\upsilon = \frac{\mu}{\rho}$.

The kinematic viscosity is considered as a kinematic quantity, since its unit does not contain any unit of mass. Physically, the kinematic viscosity represents the ratio of the ability to diffuse a disturbance in momentum relative to the ability of sustaining the original momentum.

The dimensional formula of kinematic viscosity is L^2T^{-1}. Kinematic viscosity has an SI unit of m^2-s^{-1}, and a CGS unit of cm^2-s^{-1}. The CGS unit is also known as Stokes, in honor of the famous mathematician Stokes.

$$1\ stoke = 1\ \frac{\text{cm}^2}{\text{s}} = (10^{-2})^2\,\frac{\text{m}^2}{\text{s}} = 10^{-4}\ \text{m}^2/\text{s}$$

$$1\ centisoke = \frac{1}{100}\ stoke = 10^{-6}\ \text{m}^2/\text{s}$$

Viscosity is practically independent of pressure and depends upon temperature only. The kinematic viscosity of liquids and of gases at a given pressure is substantially a function of temperature.

Example 11.1 The specific gravity and the dynamic viscosity of a fluid are 13.6 and 0.002 N.s/m^2 respectively. Calculate its (i) density, and (ii) kinematic viscosity.

Solution

$$S_{\text{liquid}} = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}}$$

$$\rho\text{liquid} = \rho_{\text{water}} \times S_{\text{liquid}}$$

$$\rho_{\text{liquid}} = 1000 \times 13.6 = 13600\ \text{kg/m}^3$$

$$\text{Kinematic viscosity, } v = \frac{\mu}{\rho} = \frac{0.002}{13600} = 1.47 \times 10^{-7}\ \text{m}^2/\text{s}$$

Example 11.2 The velocity profile of a fluid over a plate is a parabola having a vertex of 10 cm from the boundary. The velocity at the vertex is 1.2 m/s. Calculate the velocity gradients for $y = 0$, 5 and 10 cm. Also calculate the shear stress at these points if the fluid viscosity is 0.004 Ns/m^2.

Solution Let the equation of velocity profile (parabolic) be

$$u = Ay^2 + By + C$$

where A, B and C are constants and their values are determined from boundary conditions as given by

(i) $u = 0$ at $y = 0$

(ii) $u = 1.2$ m/s at $y = 10$ cm

(iii) $\frac{du}{dy} = 0$ at $y = 10$ cm

From the first boundary condition

$$0 = A\times 0 + B\times 0 + C$$

$$C = 0$$

From the second boundary condition

$$1.2 = A\times(0.1)^2 + B\times 0.1 + 0$$

$$1.2 = 0.01A + 0.1B \quad (11.3)$$

From the third boundary condition

$$\frac{du}{dy} = 2Ay + B$$

$$0 = 2A\times 0.1 + B \quad (11.4)$$

After solving equations (11.3) and (11.4),

$$A = -120 \text{ and } B = 24$$

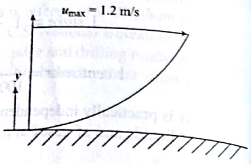

Figure 11.6

The velocity profile becomes

$$u = -120y^2 + 24y$$

$$\frac{du}{dy} = -240y + 24$$

Velocity gradient, $\frac{du}{dy} = -240y + 24$

At $y = 0$, velocity gradient $\left(\frac{du}{dy}\right)_{y=0} = -240\times 0 + 24 = 24/s.$

At $y = 0.05$ m, velocity gradient $\left(\frac{du}{dy}\right)_{y=0.05\,m} = -240\times 0.05 + 24 = 12\ /s.$

At $y = 0.1$ m, velocity gradient $\left(\frac{du}{dy}\right)_{y=0.1} = -240\times 0.1 + 24 = 0\ /s.$

Shear stress is given by $\tau = \mu\frac{du}{dy}$

Shear stress at $y = 0$ m, $\tau = \mu\left(\frac{du}{dy}\right)_{y=0} = 0.004\times 24 = 0.096\ N/m^2$

Shear stress at $y = 0.05$ m, $\tau = \mu\left(\frac{du}{dy}\right)_{y=0.05} = 0.004\times 12 = 0.048\ N/m^2$

Shear stress at $y = 0.1$ m, $\tau = \mu\left(\frac{du}{dy}\right)_{y=0.1} = 0.004\times 0 = 0$

Example 11.3 The space between two parallel plates kept 5 mm apart is filled with a fluid with a dynamic viscosity of 1 Ns/m^2. The upper plate is moving with a velocity of 2 m/s. What is the shear stress on the lower plate, which is stationary?

Solution Since the gap between the plates is very small, we can assume that the velocity distribution is a linear one.

Velocity gradient, $\frac{du}{dy} = \frac{V}{h} = \frac{2}{5\times 10^{-3}} = 400$ per second

Shear stress on the bottom plate, $\tau = \mu\frac{du}{dy} = 1\times 400 = 400\ N/m^2$

Example 11.4 A rectangular solid block of 1 m by 1 m that weighs 30 N slides down a 30° inclined plane as shown in Fig. 11.7. The plane is lubricated by a 5 mm thick film of oil of viscosity of 0.04 Ns/m^2. Calculate the terminal velocity of the block.

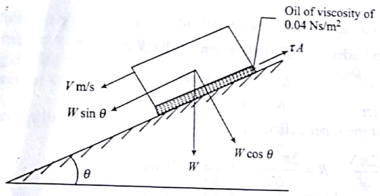

Figure 11.7

Solution

Thickness of the film $h = 5\text{ mm} = 5\times 10^{-3}\,m$

Area of the block $A = 1\times 1 = 1m^2$

Weight of the block $W = 30$ N

Viscosity of oil $\mu = 0.04\ Ns/m^2$

Component of weight along the slope is $W \sin\theta$.

Velocity gradient is found to be

$$\frac{du}{dy} = \frac{V-0}{h} = \frac{V}{h}$$

where h is the thickness of the oil film.

Viscous resistance F is given by

$$F = \text{shear stress}\times\text{area} = \tau A$$

or $$F = \mu \frac{du}{dy} A = \mu \frac{V}{h} A \qquad \left(\text{From Eq. (11.1)}, \tau = \mu \frac{du}{dy}\right)$$

At the terminal condition, equilibrium occurs. Hence, the viscous resistance to the motion should be equal to the component of the weight of the solid block along the slope. Thus,

$$\mu \frac{V}{h} A = W \sin\theta$$

or $$0.04 \times \frac{V}{5\times10^{-3}} \times 1 = 30 \sin 30^\circ$$

or $$V = 1.875 \text{ m/s}$$

Example 11.5 A liquid is filled in the annular space between two concentric cylinders 30 cm long. The inner cylinder of radius 10 cm rotates inside the outer cylinder which is stationary and has an internal radius of 10.05 cm. Determine the viscosity of the liquid if a torque of 10 N-m is required to maintain an angular velocity of 60 rpm.

Solution

Radius of the inner cylinder $R = 10 \text{ cm} = 0.1\text{m}$

Radius of the outer cylinder $R_o = 10.05 \text{ cm} = 0.1005 \text{ m}$

Length of the cylinder $L = 30 \text{ cm} = 0.3\text{m}$

Speed of inner cylinder $N = 60 \text{ rpm}$

Tangential velocity of the inner cylinder is

$$u = \frac{2\pi N}{60} \times R = \frac{2\pi \times 60}{60} \times 0.1 = 0.628 \text{ m/s}$$

Radial clearance between the cylinders is

$$h = \text{difference between radius of outer and inner cylinder}$$
$$= R_o - R = 0.1005 - 0.10 = 0.0005 \text{ m}$$

Surface area of the cylinder is

$$A = \text{circumference} \times \text{length of the pipe} = 2\pi RL$$
$$= 2\pi \times 0.1 \times 0.3 = 0.1885 \text{ m}^2$$

For the small space between the cylinders, the velocity profile may be assumed to be linear. Then, the velocity gradient is found to be

$$\frac{du}{dr} = \frac{u-0}{h} = \frac{0.628}{0.0005} = 1256 \text{ per s}$$

Let μ be the viscosity of the liquid.

The shear stress at the wall is obtained from Eq. (11.1) as

$$\tau = \mu \frac{du}{dr} = 1256\mu$$

Shear force is given by

$$F = \text{shear stress} \times \text{area} = \tau A$$
$$= 1256\mu \times 0.1885 = 236.75\mu$$

Then, the resisting torque by the fluid is

$$T = \text{Force} \times \text{radius of inner cylinder} = F \times r$$
$$= 236.75\mu \times 0.1 = 23.675\mu$$

Applied torque should be same as the resisting torque by the fluid. Therefore, one can write

$$23.675\mu = 10$$

or $$\mu = 0.42 \text{N-s/m}^2$$

Example 11.6 A 150 mm diameter shaft rotates at 1500 rpm in a 200 mm long journal bearing with 150.5 mm internal diameter. The uniform annular space between the shaft and the bearing is filled with oil of dynamic viscosity 0.8 poise. Calculate the power dissipated as heat.

Solution The arrangement is shown in Fig. 11.8.

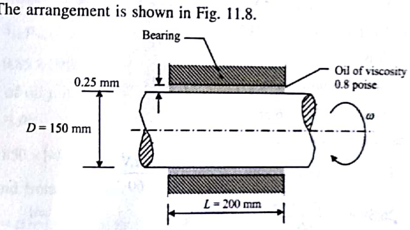

Figure 11.8

Diameter of the inner shaft $D = 150 \text{ mm} = 0.15 \text{ m}$

Radius of the shaft $R = \frac{0.150}{2} = 0.075 \text{ m}$

Radius of the bearing $R_o = \frac{150.5}{2} \text{mm} = \frac{0.1505}{2} \text{m} = 0.07525 \text{ m}$

Length of the bearing $L = 200 \text{ mm} = 0.2 \text{ m}$

Speed of shaft $N = 1500 \text{ rpm}$

Viscosity of oil $\mu = 0.8 \text{ poise} = 0.8 \times 10^{-1} \text{ N-s/m}^2$

Thickness of the oil film is

$$dy = \text{difference between radius of bearing and shaft}$$
$$= R_o - R = 0.07525 - 0.075 = 0.00025 \text{ m}$$

Surface area of the bearing is

$$A = \text{circumference of the shaft} \times \text{length of the bearing}$$
$$= \pi DL = \pi \times 0.15 \times 0.2 = 0.094 \text{ m}^2$$

Circumferential velocity of the shaft is

$$u = \text{angular velocity of the shaft} \times \text{radius of the shaft}$$
$$= \frac{2\pi N}{60} \times r = \frac{2\pi \times 1500}{60} \times \frac{0.150}{2} = 11.78 \text{ m/s}$$

Velocity gradient is found to be

$$\frac{du}{dy} = \frac{11.78 - 0}{0.00025} = 47120 \text{ per s}$$

Shear stress τ is found from Newton's law of viscosity (Eq. (11.1)) as

$$\tau = \mu \frac{du}{dy} = 0.8 \times 10^{-1} \times 47120 = 3769.6 \text{ N/m}^2$$

Shear force on the shaft is then

$$F = \text{shear stress} \times \text{area} = \tau A$$
$$= 3769.6 \times 0.094 = 354.34 \text{ N}$$

Torque T on the shaft is then

$$T = \text{Force} \times \text{radius of inner cylinder} = F \times R$$
$$= 354.34 \times 0.075 = 26.58 \text{ N-m}$$

Power dissipated as heat P is found to be

$$P = \text{Torque} \times \text{angular velocity} = T \times \omega = T \times \frac{2\pi N}{60}$$
$$= 26.58 \times \frac{2 \times \pi \times 1500}{60}$$
$$= 4175.18 \text{ W} \cong 4.175 \text{ kW}$$

Example 11.7 A hydraulic ram 200 mm diameter and 1.2 m long moves within a concentric cylinder 200.4 mm diameter. The annular clearance is filled with oil of relative density 0.85 and kinematic viscosity 400 mm²/s. What is the viscous force resisting the motion when the ram moves at a speed of 120 mm/s?

Solution

Kinematic viscosity of oil	$\nu = 400 \text{ mm}^2/\text{s} = 400 \times 10^{-6} \text{ m}^2/\text{s}$
Relative density of oil	$S_{oil} = 0.85$
Density of water	$\rho_{water} = 1000 \text{ kg/m}^3$
Speed of ram	$V = 120 \text{mm/s} = 0.120 \text{m/s}$
Length of ram	$L = 1.2 \text{ m}$
Diameter of ram	$D = 200 \text{ mm} = 0.2 \text{ m}$
Radius of ram	$R = 0.1 \text{ m}$
Diameter of cylinder	$D_o = 200.4 \text{ mm} = 0.2004 \text{ m}$
Radius of cylinder	$R_o = \frac{0.2004}{2} \text{ m} = 0.1002 \text{ m}$
Radial clearance is given by	$dy = R_o - R = 0.1002 - 0.1 = 0.0002 \text{ m}$
Velocity gradient is	$\frac{du}{dy} = \frac{V - 0}{dy} = \frac{0.120 - 0}{0.0002} = 600 \text{ per s}$

Density of the oil ρ is found as

$$\rho = \text{Specific gravity of the oil} \times \text{density of water}$$
$$= S_{oil} \rho_{water}$$
$$= 0.85 \times 1000 = 850 \text{ kg/m}^3$$

Dynamic viscosity of oil μ is found to be

$$\mu = \rho \nu$$
$$= 850 \times \left(400 \times 10^{-6}\right) = 0.34 \text{ N-s/m}^2$$

Shear stress is found from Newton's law of viscosity (Eq. (11.1)) as

$$\tau = \mu \frac{du}{dy} = 0.34 \times 600 = 204 \text{ N/m}^2$$

Shear area is

$$A = \text{circumference of the ram} \times \text{length of the ram}$$
$$= \pi DL = \pi \times 0.2 \times 1.2 = 0.754 \text{ m}^2$$

Viscous force F resisting the motion is

$$F = \text{shear stress} \times \text{area} = \tau A = 204 \times 0.754 = 153.82 \text{ N}$$

Example 11.8 The space between two large flat and parallel walls 25 mm apart is filled with a liquid of absolute viscosity 0.7 N-s/m^2. Within this space a thin flat plate is towed at a velocity of 0.15 m/s at a distance of 6 mm from one wall, the plate and its movement being parallel to the walls. Assuming linear variations of velocity between the plate and the walls, determine the force exerted by the liquid on the plate.

Solution

Area of the plate $A = 25 \text{ cm} \times 25 \text{ cm} = 625 \text{ cm}^2 = 0.0625 \text{ m}^2$

Viscosity of liquid $\mu = 0.7 \text{ N-s/m}^2$

Velocity of plate $V = 0.15 \text{ m/s}$

Let F_1 and F_2 be the shear forces on the upper surface and lower surface of the thin plate respectively. Let us also consider that the distance of the thin plate from the top wall is 6 mm as shown in Fig. 11.9.

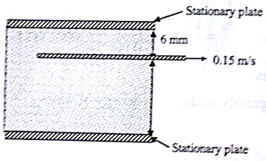

Figure 11.9

From Newton's law of viscosity (Eq. (11.1)), shear stress on the upper surface of the plate τ_1 is given by

$$\tau_1 = \mu \frac{du}{dy}$$

where dy = distance between top wall and thin plate = 6mm = 6×10^{-3} m

or $$\tau_1 = 0.7 \times \frac{0.15}{6 \times 10^{-3}} = 17.5 \text{N/m}^2$$

Shear force on the upper surface of the plate is

$$F_1 = \text{shear stress} \times \text{area} = \tau_1 A$$

$$= 17.5 \times 0.0625 = 1.094 \text{N}$$

From Newton's law of viscosity (Eq. (11.1)), shear stress on the lower surface of the plate τ_2 is given by

$$\tau_2 = \mu \frac{du}{dy}$$

where dy = distance between bottom wall and thin plate = 25 − 6 = 19mm = 19×10^{-3} m

or $$\tau_2 = 0.7 \times \frac{0.15}{19 \times 10^{-3}} = 5.526 \text{N/m}^2$$

Shear force on the bottom surface of the plate is

$$F_2 = \text{shear stress} \times \text{area} = \tau_2 A$$

$$= 5.526 \times 0.0625 = 0.345 \text{N}$$

Force exerted by the liquid on the plate is the sum of the forces on either side of the plate. Therefore, total force exerted by the liquid is

$$F = F_1 + F_2 = 1.094 + 0.345 = 1.439 \text{ N}$$

Example 11.9 A circular disc of radius R is kept at a small height h above a fixed bed by means of a layer of oil of dynamic viscosity, as shown in Fig. 11.10. If the disc is rotated at an angular velocity, ω, obtain an expression for the viscous torque on the disc. Assume linear variation of velocity within the oil film.

Solution Angular velocity of the disc = ω

Consider an element of disc at a radial distance r with width dr as shown in Fig.11.10. For linear variation of velocity with depth, the velocity gradient is given by

$$\frac{du}{dy} = \frac{u}{h} = \frac{\omega r}{h}$$

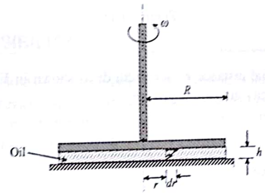

Figure 11.10

Elemental shear stress is then

$$\tau = \mu \frac{du}{dy} = \mu \frac{\omega r}{h}$$

Elemental shear force is given by

$$dF = \tau dA = \mu \frac{\omega r}{h} 2\pi r dr$$

Viscous torque acting on the element is

$$dT = dFr = \mu \frac{\omega r}{h} 2\pi r dr r = \mu \frac{\omega}{h} 2\pi r^3 dr$$

Total viscous torque on the disc is then

$$T = \int dT = \int_0^R \mu \frac{\omega}{h} 2\pi r^3 dr = \frac{2\pi\mu\omega}{h} \frac{R^4}{4} = \frac{\pi\mu\omega}{h} \frac{R^4}{2}$$

Example 11.10 A solid cone of radius R, and vertex angle 2θ is to rotate at an angular velocity, ω, as shown in Fig. 11.11. An oil of viscosity μ and thickness h fills the gap between the cone and the housing. Determine the torque T required to rotate the solid cone.

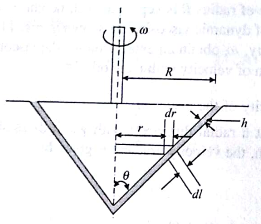

Figure 11.11

Solution

Consider an element at a radial distance r with width dr as shown in Fig. 11.11. Assuming linear variation of velocity, the velocity gradient is given by

$$\frac{du}{dy} = \frac{u}{h} = \frac{\omega r}{h}$$

From the geometry, we have

$$dl = \frac{dr}{\sin\theta}$$

Elemental area is then

$$dA = 2\pi r dl = 2\pi r \frac{dr}{\sin\theta}$$

Shear stress on the inclined wall is then

$$\tau = \mu \frac{du}{dy} = \mu \frac{\omega r}{h}$$

Elemental shear force is given by

$$dF = \tau dA = \mu \frac{\omega r}{h} 2\pi r \frac{dr}{\sin\theta}$$

Viscous torque acting on the element is

$$dT = dFr = \mu \frac{\omega r}{h} 2\pi r \frac{dr}{\sin\theta} r = \mu \frac{2\pi\omega}{h} \frac{1}{\sin\theta} r^3 dr$$

Viscous torque required to rotate the solid cone is found to be

$$T = \int dT = \int_0^R \mu \frac{2}{h} \frac{1}{\sin\theta} r^3 dr = \frac{\pi\omega\mu}{2h\sin\theta} R^4$$

11.9 NO-SLIP CONDITION

Consider the flow of fluid over a solid wall that is impervious (i.e., impermeable to the fluid). All experimental observations indicate that a fluid in motion comes to a complete stop at the wall and assumes zero velocity relative to the wall. That is, on a solid wall, the fluid can be assumed to stick to the wall and there is no slip along the solid wall the way a different solid might. This is known as the ***no-slip condition***. The physical reason for the no-slip condition is that the fluid molecules hitting the solid wall collide so frequently with the solid wall molecules that they have no average motion that is different from the wall molecules.

11.10 COMPRESSIBILITY

Compressibility of any substance is the measure of its change in volume under the action of external forces. The normal compressive stress of any fluid element at rest is known as hydrostatic pressure P and arises as a result of innumerable molecular collisions in the entire field. The degree of compressibility of a substance is characterized by the bulk modulus of elasticity E defined as

$$E = \lim_{\Delta V \to \Delta V^*} \frac{-\Delta P}{\frac{\Delta V}{V}}$$

where ΔV^* is the smallest elemental volume over which the continuum hypothesis is valid and ΔV and ΔP are the changes in the volume and pressure respectively, and V is the initial volume. The negative sign indicates that an increase in pressure is associated with a decrease in volume.

Values of E for liquids are very high compared with those of gases. Therefore, liquids are usually termed as incompressible. Density of water increases only 1 per cent if the pressure is increased by a factor of 220.

Since $\frac{\Delta V}{V}$ is a dimensionless ratio, the dimensions and units for E are the same as those for pressure i.e. FL^{-2} and N/m^2.

11.11 COHESION, ADHESION AND SURFACE TENSION

11.11.1 Cohesion

A definite amount of fluid mass is envisaged as an aggregation of several molecules of the fluid in close association with each other. Any particular molecule is attracted in all the directions by an equal amount of force exerted by the surrounding molecules. ***Cohesion is the property of the fluid by virtue of which liquid molecules are connected with each other so as to form a continuous mass.***

11.11.2 Adhesion

Adhesion is the property of the fluid by virtue of which liquid adheres another body that comes in its contact. For instance, if a rod is immersed in water and subsequently taken out, it is found that the rod becomes wet since the water molecules adhere to the rod. The same phenomenon occurs when a liquid is contained in a vessel and it is emptied.

It is noteworthy that some fluids may not exhibit adhesion. Example: mercury.

11.11.3 Surface Tension

A molecule (molecule A in Fig. 11.12) within the liquid and below the free surface is attracted equally in all directions by the other molecule surrounding it by virtue of cohesion. This ensures that such molecules remain in equilibrium. However, the same situation does not exist for the molecules in the free surface (molecules B and C in Fig. 11.12). This is due to the fact that such molecules although are attracted towards the inside mass by its neighboring molecules (lower molecules), but no liquid molecules is available on the upper side to counteract the pull force generated by the former molecules. This causes imbalances and thus molecules lying on the surface, experience a net downward pull force towards the interior of the liquid. An amount of energy/work is therefore required to bring the molecule to the free liquid surface which acts like an elastic membrane. This is called surface tension. It represents the surface energy per unit area, and is denoted by Greek letter σ (sigma). It has dimensions of FL^{-1} or MT^{2} and units of N/m or kg/s^2 in SI units. Surface tension decreases slightly with increasing temperature.

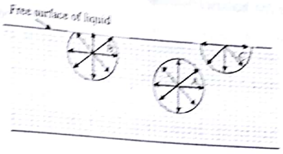

Figure 11.12

Surface tension on liquid droplet

Let the liquid droplet is cut into two halves. Half of the droplet is shown in Fig. 11.13 (a).

Tensile force due to surface tension acting around the circumference of the cut portion as shown in Fig. 11.13 (a) is

= Circumference × Surface tension

$= \pi d\sigma$

Pressure force acting on the half of the droplet (Fig. 11.13 (a)) is

= Area × pressure

$= \frac{\pi d^2}{4}\Delta P$

The pressure force in the droplet is balanced by the surface tension force around the circumference. Hence, equating the above two forces, we have

$$\frac{\pi d^2}{4}\Delta P = \pi d\sigma$$

or,
$$\Delta P = \frac{4\sigma}{d} \tag{11.5}$$

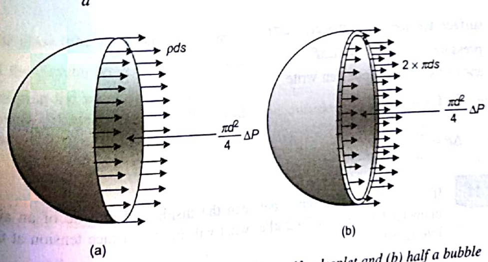

Figure 11.13 *Free body diagram of (a) half a droplet and (b) half a bubble*

Surface tension on a soap (or hollow) bubble

In a soap bubble, there are two interfaces as shown in Fig. 11.14 (b). Therefore, the tensile force due to surface tension acting around the circumference of the cut portion as shown in Fig. 11.14 (b) is

$$= \text{Circumference} \times \text{Surface tension}$$

$$= \pi d\sigma$$

Pressure force acting on the half of the droplet (Fig. 11.14 (b)) is

$$= \text{Area} \times \text{pressure}$$

$$= \frac{\pi d^2}{4}\Delta P$$

The pressure force is balanced by the surface tension force on the two circumferences. Hence

$$\frac{\pi d^2}{4}\Delta P = 2 \times \pi d\sigma$$

or, $$\Delta P = \frac{8\sigma}{d} \tag{11.6}$$

Surface tension on a liquid jet

Consider a liquid jet of diameter d and length L. Semi circular jet is shown in Fig. 11.15.

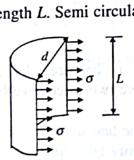

Figure 11.14 *Forces on liquid jet*

Force due to surface tension $= \sigma \times 2L = 2\sigma L$

Force due to pressure $= dL\Delta P$

Equating the above two forces, one can write

$$2\sigma L = dL\Delta P$$

or, $$\Delta P = \frac{2\sigma}{d} \tag{11.7}$$

Example 11.11 If the pressure difference between the inside and outside of an air bubble of diameter 0.01 mm is 29.2 kPa, what will be the surface tension at the air-water interface?

Solution Pressure difference between the inside and outside of an air bubble

$$\Delta P = 29.2 \text{ kPa} = 29.2 \times 10^3 \text{ N/m}^2$$

Diameter of the bubble $d = 0.01\text{mm} = 0.01 \times 10^{-3}$ m

The pressure difference between the inside and outside of an air bubble can be found from Eq. (11.5) as

$$\Delta P = \frac{4\sigma}{d}$$

$$\sigma = \frac{\Delta Pd}{4} = \frac{29.2 \times 10^3 \times 0.01 \times 10^{-3}}{4} = 0.073 \text{ N/m}$$

Example 11.12 Find the internal pressure in a soap bubble of 4 cm diameter, when the surface tension at the soap-air interface is 0.08 N/m.

Solution

Diameter of the soap bubble $d = 4\text{cm} = 4 \times 10^{-2}$ m

Surface tension at the soap-air interface $\sigma = 0.08$ N/m

Since for a soap bubble, there are two interfaces, the pressure difference is (Eq. (11.6))

$$\Delta p = \frac{8\sigma}{d}$$

$$= \frac{8 \times 0.08}{4 \times 10^{-2}} = 16 \text{ N/m}^2 \text{ above atmospheric pressure}$$

Example 11.13 A circular jet of water 0.5 mm in diameter issues from an opening. What is the pressure difference between the inside and outside of the jet. The surface tension at the water-air interface is 0.073 N/m.

Solution

Radius of the glass tube $r = 1 \text{ mm} = 0.001$ m

Diameter of the circular jet $d = 0.5 \text{ mm} = 0.5 \times 10^{-3}$ m

Surface tension at the water-air interface $\sigma = 0.073$ N/m

The pressure difference between the inside and outside of a circular jet can be found from Eq. (11.7) as

$$\Delta P = \frac{2\sigma}{d}$$

Substituting the values of σ and r, we have

$$\Delta P = \frac{2 \times 0.073}{0.5 \times 10^{-3}} = 292 \text{ N/m}^2$$

11.12 CAPILLARITY

When a glass tube of small diameter is dipped into a vessel or container that contains water, it is found that water rises in the tube to a level higher than that of the level of water in the container. Conversely, when mercury is used instead of water, the liquid level falls as compared to the level of the container. This phenomenon of rise or fall of a liquid surface in a small diameter tube relative to the adjacent general level of liquid when the tube is held vertically in liquid is called ***capillary*** or ***meniscus effect*** or ***capillarity***. Such narrow tubes are called ***capillaries***.

Rise of the liquid surface is called ***capillary rise (Fig. 11.14)*** and fall of the liquid surface is called ***capillary depression (Fig. 11.14)***.

Capillarity is due to both force of cohesion and adhesion. When the force of adhesion predominates, the liquid will wet a solid surface with which it is in contact and rise at the point of contact. If force of cohesion predominates the liquid surface will be depressed at the point of contact. The curved free surface of a liquid in a capillary tube is called the ***meniscus***. The strength of the capillary effect is quantified by the area wetting the contact angle, θ, which is defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact. A liquid is said to wet the surface when $\theta < 90°$ and not to wet the surface when $\theta > 90°$.

Weight of column of liquid h is found to be

$$= \text{mass of liquid} \times \text{acceleration due to gravity}$$

$$= \text{density of liquid } \times \text{volume of liquid having height } h \times g$$

$$= \rho \times \frac{\pi}{4} d^2 h \times g = \frac{\pi}{4} d^2 \rho g h$$

where d is the diameter of the capillary tube, ρ is the density of liquid and g is the acceleration due to gravity.

Vertical component of the surface tension force is

$$= (\text{Surface tension} \times \text{Circumference}) \cos\theta$$

$$= \sigma \pi d \cos\theta$$

where σ is the surface tension coefficient and θ is the area wetting contact angle.

For equilibrium, weight of column of liquid h should be equal to vertical component of the surface tension force. Thus, equating the above two forces, we get

$$\frac{\pi}{4} d^2 \rho g h = \sigma \pi d \cos\theta$$

or

$$h = \frac{4\sigma\cos\theta}{\rho g d} \tag{11.8}$$

Capillary rise is inversely proportional to the diameter of the tube. Therefore, thinner the tube is, the greater the rise or fall of the liquid in the tube. The capillary effect is usually negligible in tubes whose diameter is greater than 1 cm.

The density of the fluid should be high to avoid the capillary rise. In a tube of 5 mm diameter, the capillary rise of water will be approximately 4.5 mm, while for mercury the capillary depression will be 1.4 mm.

For pure water in contact with air in a clean glass tube, the capillary rise takes place $\theta = 0$. The value of θ may be different from zero in practice where cleanliness of a high order is seldom found. Mercury causes capillary depression with an angle of contact of about 130° in a clean glass in contact with air.

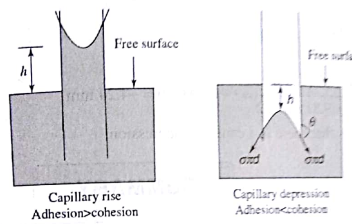

Figure 11.15 *Capillary effects*

Example 11.14 Calculate the capillary depression in a glass tube of 1 mm radius when immersed vertically in water. Take the surface tension of water in contact with air as 0.073 N/m and the area wetting contact angle as 0°. Density of water is 1000 kg/m³.

Solution

Radius of the glass tube	$r = 1 \text{ mm} = 0.001 \text{ m}$
Diameter of the glass tube	$d = 2 \text{ mm} = 0.002 \text{ m}$
Surface tension of water in contact with air	$\sigma = 0.073 \text{ N/m}$
Density of water	$\rho = 1000 \text{ kg/m}^3$
Area wetting contact angle	$\theta = 0°$

From Eq. (11.8), capillary rise is found to be

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

$$= \frac{4 \times 0.073 \times \cos 0°}{1000 \times 9.81 \times 0.002} = 0.0149 \text{ m} = 1.49 \text{ cm}$$

Example 11.15 Calculate the capillary depression in a glass tube of 1 mm radius when immersed vertically in mercury. Take the surface tension of mercury in contact with air as 0.44 N/m and the area wetting contact angle as 130°. Density of mercury is 13600 kg/m³.

Solution

Radius of the glass tube $r = 1$ mm $= 0.001$ m

Diameter of the glass tube $d = 2$ mm $= 0.002$ m

Surface tension of mercury in contact with air $\sigma = 0.44$ N/m

Density of mercury $\rho = 13600$ kg/m^3

Area wetting contact angle $\theta = 130°$

From Eq. (11.8), capillary rise is found to be

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

$$= \frac{4 \times 0.44 \times \cos 130°}{13600 \times 9.81 \times 0.002} = -0.0042 \text{ m} = -4.24 \text{ mm}$$

The minus sign indicates that there is a capillary depression.

SUMMARY

- A fluid is a substance that deforms continuously when subjected to a shear stress, however small the shear stress may be. A fluid may be either a liquid or a gas.
- A solid can resist a shear stress by a static deformation, a fluid cannot. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as shear stress is applied.
- The density of a fluid is mass per unit volume.
- The specific weight is the weight of fluid per unit volume.
- The specific volume of a fluid is the volume occupied by unit mass of fluid.
- Specific gravity or relative density is defined as the ratio of the density of some standard reference fluid at a specified temperature and pressure.
- An *ideal* fluid is a fully hypothetical fluid which is assumed to have no viscosity and no compressibility.
- In a *real* fluid, shear stresses occur whenever the fluid is in motion.
- The property which characterizes the resistance that a fluid offers to applied shear forces is termed *viscosity.*
- According to Newton's law of viscosity, the shear stress is proportional to the rate of shear strain.

  Mathematically, for one-dimensional flow, Newton's law of viscosity can be expressed as $\tau = \mu \frac{du}{dy}$

  where, the constant of proportionality μ is known as the viscosity coefficient or simply the viscosity which is the property of the fluid and depends on its state. Common fluids, such as water, air, mercury, obey Newton's law of viscosity and are known as Newtonian fluids. Other fluids that do not obey Newton's law of viscosity are known as non-Newtonian fluids.
- The viscosity of a liquid decreases with temperature, but the viscosity of a gas increases with temperature.
- The study of the response of materials to stress is called *rheology.*
- The ratio of dynamic viscosity to density is known as kinematic viscosity.

- Compressibility of any substance is the measure of its change in volume under the action of external forces. The degree of compressibility of a substance is characterized by the bulk modulus of elasticity E defined as

  $$E = \lim_{\Delta V \to \Delta V'} \frac{-\Delta P}{\frac{\Delta V}{V}}$$
- Cohesion is the property of the fluid by virtue of which liquid molecules are connected with each other so as to form a continuous mass.
- Adhesion is the property of the fluid by virtue of which a liquid adheres to another body that comes in its contact.
- When a glass tube of small diameter is dipped into a vessel or container that contains water, it is found that water rises in the tube to a level higher than that of the level of water in the container. Conversely, when mercury is used instead of water, the liquid level falls as compared to the level of the container. This phenomenon of rise or fall of a liquid surface in a small diameter tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid is called *capillary* or *meniscus effect* or *capillarity.* Such narrow tubes are called *capillaries.*Rise of the liquid surface is called *capillary rise* and fall of the liquid surface is called *capillary depression.*
- *Capillary rise* or *depression* is given by

  $$h = \frac{4\sigma\cos\theta}{\rho g d}$$

  where σ is the surface tension coefficient, θ is the area wetting contact angle and d is the diameter of tube.

REVIEW QUESTIONS

11.1 What is a fluid? How does it differ from a solid?

11.2 Differentiate between liquids and gases.

11.3 Define density, specific weight, specific volume and specific gravity.

11.4 State and explain Newton's law of viscosity.

11.5 What is the effect of temperature on viscosity of water and that of air?

11.6 Distinguish between ideal fluids and real fluids.

11.7 Why does the viscosity of a liquid decreases with increase in temperature while that of a gas increases with increase in temperature.

11.8 What is the difference between dynamic viscosity and kinematic viscosity?

11.9 Define Newtonian and non-Newtonian fluids.

11.10 Explain the no-slip condition of a viscous fluid.

11.11 Define compressibility. How is it related to bulk modulus of elasticity?

11.12 Discuss the shear characteristics of different fluids. Give at least one example of each type of fluid. What is the Ostwald-de Waele equation?

11.13 What do you mean by surface tension? What are the factors that affect surface tension?

11.14 What is bulk modulus of elasticity?

NUMERICAL PROBLEMS

11.1 The specific gravity and the kinematic viscosity of a fluid are 0.8 and 2×10^{-6} m^2/s respectively. Calculate its (i) density and (ii) dynamic viscosity.

11.2 A hot plate of area 0.125 m^2 is pulled at 0.25 m/s with respect to another parallel plate 1 mm distant from it the space between the plates containing water of viscosity 0.001 Ns/m^2. Find the force necessary to maintain this velocity and also the power required.

11.3 A 10 cm diameter shaft rotates at 1000 rpm in a 10 cm long journal bearing of 10.05 cm internal diameter. The annular space in the bearing is filled with oil having dynamic viscosity of 0.1 $N.s/m^2$. Estimate the power dissipated as heat.

11.4 Two large plane surfaces are 15 mm apart and the gap contains a liquid of viscosity 0.8 $N\text{-}/m^2$. Within the gap a thin plate of cross-sectional area 0.5 m^2 is to be pulled at a velocity of 0.5 m/s at a distance of 5 mm from one surface. Determine the force required for pulling the plate.

11.5 A piston of 20 cm diameter and 40 cm length works in a cylinder of 20.50 cm diameter. The annular space of the piston is filled with an oil of viscosity 0.5 $N\text{-}s/m^2$. If an axial load of 25 N is applied to the piston, calculate the speed of movement of the piston.

11.6 A vertical shaft has a hemispherical bottom of radius R which rotates inside a bearing of identical shape at its end. An oil film of thickness h and viscosity μ is maintained in the bearing. Estimate the viscous torque in the shaft when it rotates with an angular velocity ω.

11.7 A thin plate of very large area is placed in a gap of height h with oils of viscosities μ_1 and μ_2 on the two sides of the plate. The plate is pulled at a constant velocity V. Calculate the position of the plate so that

(i) the shear force on the two sides of the plate is equal
(ii) the force required to drag the plate is minimum

11.8 During the flow of a non-Newtonian fluid it is observed that the velocity distribution within the fluid film can be expressed by

$$\frac{u}{u_{max}} = 2\left(\frac{y}{h}\right) - \frac{1}{2}\left(\frac{y}{h}\right)^3$$

where h is the film thickness and u_{max} the maximum velocity, y is measured from the solid surface. The viscosity of the fluid is 0.5 $N\text{-}s/m^2$ and $n = 1.3$. Calculate the shear stress at the solid surface when $u_{max} = 0.4$ m/s and $h = 10$ mm. What should be the viscosity of a Newtonian fluid to induce the same shear stress value for similar velocity profile and the same maximum velocity?

11.9 Calculate the capillary depression in a glass tube of 1 mm radius when immersed vertically in mercury. Take the surface tension of mercury in contact with air as 0.44 N/m and the area wetting the contact angle as 130°.

11.10 A space of 2.5 cm wide between two large plane surface is filled with a liquid of absolute viscosity of 0.785 $N\text{-}s/m^2$. What force is required to drag a very thinplate 0.75 m^2 in area with a speed of 0.5 m/s if the plate remains equidistant fromthe two surfaces?

11.11 The space between two square flat parallel plates is filled with oil. Each side of the plate is 30 cm long the thickness of oil film is 14 mm. The upper plate whichmoves at 2.5 m/s requires a force of 120 N to maintain the speed. Determine (a) dynamic viscosity of oil, and (b) kinematic viscosity of oil, if the sp. gravity of oil is 0.8.

11.12 A thin plate is placed between two flat surfaces h apart such that the viscosity of liquids on the top and bottom of the plate are μ_1 and μ_2 respectively. Determinethe position of the plate such that the viscous resistance to uniform motion of theplate is minimum.

11.13 Calculate the capillary depression in a glass tube of 1 mm radius when immersed vertically in mercury. Take the surface tension of mercury in contact with air as 0.44 N/m and the area wetting contact angle as 130°.

11.14 Find the pressure difference between inside and outside of an air bubble of diameter 0.02 mm if the surface tension at air-water interface is 0.073 N/m.

MULTIPLE-CHOICE QUESTIONS

11.1 An ideal fluid is defined as a fluid which
(a) is incompressible (b) is compressible
(c) is incompressible and non-viscous (d) has negligible surface tension

11.2 Newton's law of viscosity states that
(a) shear stress is directly proportional to the velocity
(b) shear stress is directly proportional to the velocity gradient
(c) shear stress is directly proportional to shear strain
(d) shear stress is directly proportional to the viscosity

11.3 A Newtonian fluid is defined as a fluid which
(a) is incompressible and non-viscous (b) obeys Newton's law of viscosity
(c) is highly viscous (d) is compressible and non-viscous

11.4 Kinematic viscosity is defined as equal to
(a) dynamic viscosity × density (b) dynamic viscosity / density
(c) dynamic viscosity × pressure (d) pressure × density

11.5 Dynamic viscosity has the dimensions
(a) MLT^{-2} (b) $ML^{-1}T^{-1}$ (c) $ML^{-1}T^{-2}$ (d) $M^{-1}L^{-1}T^{-1}$

11.6 Poise is the unit of
(a) density (b) kinematic viscosity (c) viscosity (d) velocity gradient

11.7 The increase in temperature
(a) increases the viscosity of a liquid (b) decreases the viscosity of a liquid
(c) increases the viscosity of a gas (d) decreases the viscosity of a gas
(e) both (b) and (d)

11.8 Stoke is the unit of
(a) surface tension (b) kinematic viscosity
(c) viscosity (d) none of the above

11.9 Surface tension has the unit of
(a) force per unit area (b) force per unit length
(c) force per unit volume (d) none of the above

11.10 Fluid is a substance that
(a) cannot be subjected to shear stress
(b) always expands until it fills any container
(c) has the same shear stress at a point regardless of its motion
(d) cannot remain at rest under action of any shear stress

11.11 Practical fluids
(a) are viscous
(b) possess surface tension
(c) are compressible
(d) possess all the above properties

11.12 Property of a fluid by which its own molecules are attracted is called
(a) adhesion (b) cohesion (c) surface tension (d) viscosity

11.13 Property of a fluid by which molecules of different kinds of fluids are attracted is called
(a) adhesion (b) cohesion (c) surface tension (d) viscosity

11.14 The conditions of no-slip at rigid boundaries is applicable to
(a) flow of Newtonian fluids only
(b) flow of ideal fluids only
(c) flow of all real fluids
(d) flow of non-Newtonian fluids only

11.15 Typical example of a non-Newtonian fluid of pseudoplastic variety is
(a) water (b) air (c) blood (d) printing ink

11.16 The relationship between the shear stress τ and the rate of shear strain $\frac{du}{dy}$ is expressed as $\tau = m\left[\frac{du}{dy}\right]^n$. The fluid with the exponent $n < 1$ is known as
(a) Pseudoplastic fluid (b) Bingham fluid (c) Dilatant fluid (d) Newtonian fluid

11.17 Shear stress for a general fluid motion is represented by $\tau = \mu\left(\frac{du}{dy}\right)^n + A$, where n and A are constants. A Newtonian fluid is given by
(a) $n > 1$ and $A = 0$ (b) $n = 1$ and $A = 0$ (c) $n > 1$ and $A \neq 0$ (d) $n < 1$ and $A = 0$

11.18 A fluid which obeys the relation $\mu = \dfrac{\tau}{\frac{du}{dy}}$
(a) real fluid (b) perfect fluid (c) Newtonian fluid (d) dilatant fluid

11.19 Newton's law of viscosity depends upon the
(a) viscosity and shear stress
(b) stress and strain in a fluid
(c) shear stress and rate of strain
(d) shear stress, pressure and velocity

11.20 The coefficient of viscosity is a property of the
(a) fluid
(b) boundary condition
(c) flow velocity
(d) body over which flow occurs

11.21 Paper pulp can be regarded as
(a) Dilatant (b) Newtonian fluids (c) Bingham plastic (d) Pseudoplastic fluids

11.22 Which of the following is the bulk modulus of elasticity K of fluid?
(a) $\rho\frac{dP}{d\rho}$ (b) $\frac{dP}{\rho d\rho}$ (c) $\frac{d\rho}{\rho dP}$ (d) $\frac{\rho d\rho}{dP}$

11.23 The dimension of surface tension is
(a) $FL^{-1}T^{-1}$ (b) FL^{-1} (c) FL^2T^{-1} (d) FLT^{-2}

11.24 The dimension of viscosity is
(a) L^2T^{-1} (b) $ML^{-1}T^{-1}$ (c) MT^{-2} (d) L^2T^{-1}

11.25 The unit of surface tension is
(a) J/m (b) J/m^2 (c) W/m (d) N/m^2

11.26 If p is the gauge pressure within a spherical droplet, the gauge pressure within a bubble of the same fluid and of same size will be
(a) $P/4$ (b) $P/2$ (c) $2P$ (d) P

11.27 Spherical shape of droplets of mercury is due to
(a) high density
(b) high surface tension
(c) high adhesion
(d) low vapour pressure

11.28 Which fluid does not experience shear sress during flow?
(a) Pseudoplastic (b) Dilatant (c) Inviscid (d) Newtonian

11.29 The bulk modulus of elasticity
(a) is independent of temperature
(b) increases with the pressure
(c) has the dimensions of $\frac{1}{\rho}$
(d) is larger when the fluid is more compressible
(e) is independent of pressure and viscosity

11.30 The bulk modulus of elasticity for a gas at a constant temperature T is given by
(a) $\frac{P}{\rho}$ (b) RT (c) $P\rho$ (d) ρRT
(e) none of these

11.31 The property of a fluid by virtue of which it offers resistance to shear strain is called
(a) surface tension (b) viscosity (c) adhesion (d) cohesion