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Problem Sheet-2 (Eng. Thermo).

1. $m = 8$ kg. $pV^{1.2} = c$. $P_1 = 1000 \times 10^3$ Pa $V_1 = 1 m^3$.

$= 10^6$ Pa

$P_2 = 5 \times 10^3$ Pa.

$\Delta U = 40$ kJ/kg.

nett $\Delta U = 40 \frac{kJ}{kg} \times 8$ kg.

$= -320$ kJ.

$P_1 V_1^{1.2} = P_2 V_2^{1.2}$.

$\Rightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{1.2}$

$\Rightarrow 200 = V_2^{1.2}$

$\Rightarrow V_2 = 82.70 m^3$.

Work $= \frac{P_1V_1 - P_2V_2}{n-1} = \frac{10^6(1) - 5\times10^3(82.70)}{0.2}$

$= 2932407$ J.

$= 2932.4$ kJ.

$Q = \Delta U + W$.

$= -320 + 2932.4 = 2612.4$ kJ. [Supplied to System].

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2. $u = 196 + 0.718t$ — (2) $pv = 0.287(t+273)$, — (1).

$m = 2$kg. adiabatic process. ; $Q = 0$

$pv^{1.2} = c$.

$P_1 = 10^6$ Pa $T_1 = 200°C$ $P_2 = 10^5$ Pa.

Putting P_1 and T_1 in (1).

$10^6(v_1) = 0.287(200+273)$

$\Rightarrow v_1 = 1.357 \times 10^{-4}$ m^3 kg. (s.

$V_1 = 2$kg (v_1)

$= 2.715 \times 10^{-4}$ m^3

$P_1 V_1^{1.2} = P_2 V_2^{1.2}$.

$\Rightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{1.2} \Rightarrow \frac{P_1}{P_2} \times V_1^{1.2} = V_2^{1.2}$

$\Rightarrow V_2 = 1.85 \times 10^{-3}$

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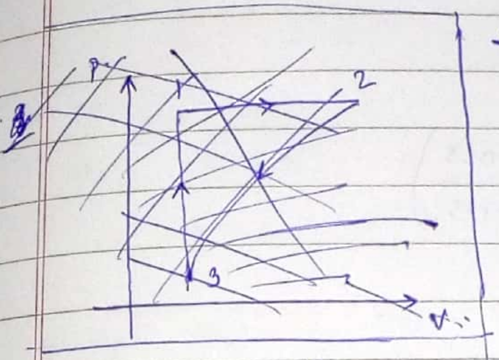
$\therefore$ Work $= \dfrac{P_1 V_1 - P_2 V_2}{n-1} = \dfrac{10^6 (2.715 \times 10^{-4}) - 10^5 (1.85 \times 10^{-3})}{0.2}$

$= 432.5$ kJ. $\rightarrow$ which is work in P.dV process

~~Now $\Delta Q = \Delta U + W$~~

~~$\therefore \Delta U = -W = -(432.5$ kJ$)$.~~ (Not applicable to this).



$P_1 v_1^{1.2} = P_2 v_2^{1.2}$

$\Rightarrow v_2^{1.2} = \dfrac{P_2}{P_1} \cdot v_1^{1.2}$

$\Rightarrow v_2 = 9.245 \times 10^{-4} m^3$

Using eqn (2)

Now $u_1 = 196 + 0.718(200)$

$= 339.6$ kJ./kg.

And, putting P_2, v_2 in (1)

$10^5 (9.245 \times 10^{-4}) = 0.287 (t_2 + 273)$.

$t_2 = 49.13°c$.

$\therefore u_2 = 196 + 0.718 (49.13)$

$= 231.27$ kJ./kg.

$\therefore \Delta U = U_2 - U_1 = -108.32$ kJ/kg.

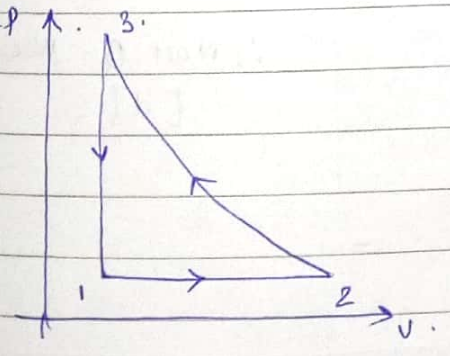
Nett $\Delta U = 2$ kg (-108.32) kJ/kg

$= -216.65$ kJ.

Since $Q = 0$.

$W = -(\Delta U) = 216.65$ kJ.

3.



[a]

$W_{1-2} = 10.5$ kJ, (given).

$P_2 = P_1 = 1.4$ bar

Now. $W_{1-2} = P\,dV$.

$\Rightarrow 10.5 \times 10^3 = 1.4 \times 10^5 \times dV$.

$\Rightarrow dV = 0.075 m^3$.

$\Delta V = V_2 - V_1 = 0.075$.

$\Rightarrow V_2 = 0.075 + (0.028) = 0.103\ m^3$

$V_3 = V_1 = 0.028\ m^3$.

$\therefore$ For $2 \to 3$: $P_2 V_2 = P_3 V_3$

$\Rightarrow (1.4 \times 10^5) \times (0.103) = P_3 (0.028)$

$\Rightarrow P_3 = 5.15 \times 10^5\ Pa$.

$\therefore W_{2-3} = P_2 V_2 \ln V_3/V_2$.

$= (1.4 \times 10^5) \times (0.103) . \ln \left(\frac{0.028}{0.103} \right)$

$= -18.782\ kJ$.

And $W_{3-1} = 0$.

$\therefore \Sigma W = 10.5 - 18.782 = -8.282\ kJ$. [b]

Now, $U_1 - U_3 = -26.4\ kJ$. and $U_3 = U_2$.

$\Rightarrow U_1 - U_2 = -26.4\ kJ$.

or $U_2 - U_1 = \Delta U = 26.4\ kJ$

Now, for 1-2: $Q_{(1-2)} = \Delta U + W_{1-2}$

$= 26.4 + 10.5$

$= 36.9\ kJ$. [c].

Also, $Q_{2-3} = \Delta U + W_{2-3}$

$= 0 + (-18.782) = -18.782\ kJ$

$\Delta U = U_1 - U_3 = -26.4\ kJ$.

$\therefore$ For 3-1: $Q_{(1-3)} = \Delta U + W_{3-1}$

$= -26.4 + 0$.

$= -26.4\ kJ$.

Nett Q

$= 36.9 - 18.782 - 26.4 =$

$= -8.282\ kJ$

$\therefore$ Nett Q = Nett W

[d].

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4. $p = a + bV$ $\quad m = 1.5\,kg.$ $\quad P_1 = 10^6\,Pa$ (10^3 kPa.) $\quad P_2 = 2\times10^5\,Pa$ (200 kPa.)

$V_1 = 0.20\,m^3 \qquad V_2 = 1.20\,m^3$

$u = (1.5pv - 85)\ kJ/kg. \quad \boxed{p \to kPa\ \&\ v \to m^3/kg.}$

$u_1 = \left(\frac{1.5\times10^3(0.20)}{1.5} - 85\right)\frac{kJ}{kg} \qquad u_2 = \left(\frac{1.5\times200\times(1.20)}{1.5} - 85\right)$

$= 115\ KJ/kg. \qquad\qquad = 155\ KJ/kg.$

$U_1 = 172.5\ KJ. \qquad\qquad U_2 = 232.5\ KJ.$

$\therefore\ \Delta U = U_2 - U_1 = 60\ KJ.$

Now, $1000 = a + b(0.2)$ and $200 = a + b(1.2)$

$\Rightarrow$ $(-)$: $800 = -b.$

$\Rightarrow b = -800.$

$\therefore\ 200 = a - 800(1.2)$

$\Rightarrow a = 1160.$

Now, $W = \int P.\,dV$

$= \int (a+bV)\,dV. = \left[aV + \frac{bV^2}{2}\right]_{0.2}^{1.2}$

$\Rightarrow \left[1160V - \frac{\overset{400}{\cancel{800}}V^2}{\cancel{2}}\right]_{0.2}^{1.2}$

$= 816 - (216) = 600\,KJ.$

$\therefore\ Q = \Delta U + W = 660\ KJ.$

Now $U = (1.5pv - 85)\ KJ/kg.$

$= \left(\frac{1.5PV}{1.5} - 85\right)\times1.5\ KJ.$

$= (PV - 85)\,1.5\ KJ. \quad = \{(a+bV)V - 85\}\,1.5$

U is max when $\frac{dU}{dV} = 0 \Rightarrow (a + 2bV) = 0.$

$\Rightarrow V = \frac{-a}{2b} = 0.725\,m^3$

$U_{max} = \{(1160 - 800V)V - 85\}\,1.5 = 503.25\ KJ.$

(putting $V = 0.725$)

5. $P_1 = 100\,kPa$ $t_1 = 20°C$. $KE \to 0$. $V \to 100\,m/s$

$P_2 = 1000\,kPa$ $t_2 = 400°C$

$P_i = 4000\,kW \Rightarrow \frac{\partial w_x}{\partial T} = -4000\,kW$.

$$\frac{\partial m}{\partial t}\left[h_1 + \cancel{\frac{V_1^2}{2}}^{0} + \cancel{z_1 g}^{0}\right] + \cancel{\frac{\partial Q}{\partial T}}^{0} = \frac{\partial m}{\partial t}\left[h_2 + \frac{V_2^2}{2} + \cancel{z_2 g}^{0}\right] + \frac{\partial w_m}{\partial t}$$

$$\Rightarrow 4000 \times 10^3 = \frac{\partial m}{\partial T}\left[h_2 + \frac{V_2^2}{2} - h_1\right]$$

Now, $(h_2 - h_1) = u_2 + P_2 V_2 - (u_1 + P_1 V_1)$

$= \Delta U + P_2V_2 - P_1V_1$

$= \Delta U + 1.0\,R\,[T_2 - T_1]$.

$= (272.84 + 107.16)$

$= (380)\,kJ$.

Now, $\Delta U = C_v \cdot dT$

$= 0.718\,[380]$

$= 272.84\,kJ$

$$\Rightarrow 4000 \times 10^3 = \frac{\partial M}{\partial T}\left[380 \times 10^3 + \frac{10000}{2}\right]$$

$$\Rightarrow 4000 \times 10^3 = \frac{\partial M}{\partial T}\left[385 \times 10^3\right]$$

$$\Rightarrow \frac{\partial M}{\partial T} = 10.39\,\frac{kg}{s}$$

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6. $\frac{\partial M}{\partial T} = 2.5\,\frac{kg}{s}$ $h_1 = 2700\,kJ/kg$ $h_2 = 1800\,kJ/kg$.

$V_1 = 35\,m/s$ $V_2 = 250\,m/s$.

$\frac{\partial Q}{\partial T} = -40\,kW$.

$$\cancel{\frac{\partial M}{\partial T}\left[h_1 + \frac{V_1^2}{2} + z_1 g\right] - h_2 - \frac{V_2^2}{2} - z_2 g = \frac{\partial w_m}{\partial t}}$$

$$\frac{\partial M}{\partial T}\left[h_1 + \frac{V_1^2}{2} + \cancel{z_1 g}^{0}\right] + \frac{\partial Q}{\partial T} = \frac{\partial M}{\partial T}\left[h_2 + \frac{V_2^2}{2} + \cancel{z_2 g}^{0}\right] + \frac{\partial w_x}{\partial T}.$$

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$$\Rightarrow \frac{dW_x}{\partial T} = \frac{\partial M}{\partial T}\left[h_1 + \frac{V_1^2}{2} - h_2 - \frac{V_2^2}{2}\right] + \frac{\partial Q}{\partial T} \rightarrow -40$$

$\frac{\partial M}{\partial T} = 2.5$, $h_1 = 2700\times10^3$, $V_1 = 35$, $h_2 = 1800\times10^3$, $V_2 = 250$.

$= 2173.4\ kW - 40\ kW$

$= 2133.4\ kW.$

~~*7. $\frac{\partial W_x}{\partial T} = 5000\ kW$ $\frac{\partial Q}{\partial M} = 4700\ \frac{kJ}{kg}$ (supplied) $\frac{\partial Q}{\partial M} = 2200\ \frac{kJ}{kg}$~~

*7. $\frac{\partial W_x}{\partial T} = 5000\ kW.$ $h_1 = 4700\ \frac{kJ}{kg}$ $h_2 = 2200\ \frac{kJ}{kg}.$

$\frac{\partial Q}{\partial T} = 10\ kW.$

Now, $\frac{\partial M}{\partial T}\left[h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{Z_1 g}\right] + \frac{\partial Q}{\partial T} = \frac{\partial M}{\partial T}\left[h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{Z_2 g}\right] + \frac{\partial W_x}{\partial T}.$

$\Rightarrow \frac{\partial M}{\partial T}\left[h_1 - h_2\right] = \frac{\partial W_x}{\partial T} - \frac{\partial Q}{\partial T}.$

$\Rightarrow \frac{\partial M}{\partial T} = 1.996\ \frac{kg}{s}.$

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8. $\frac{\partial M}{\partial T} = 1\ \frac{kg}{s}.$ $T_1 = 293\ K$ $\frac{\partial W_x}{\partial T} = -15\ kW$

$V_1 = 100\ m/s$ $V_2 = 150\ m/s$ $\frac{\partial Q}{\partial T} = 0$ $C_p = 1.005\ kJ/kg.K.$

$= 1005\ J/kg.K.$

$$\frac{\partial M}{\partial T}\left[h_1 + \frac{V_1^2}{2} + \cancel{Z_1 g}\right] + \cancel{\frac{\partial Q}{\partial T}} = \frac{\partial M}{\partial T}\left[h_2 + \frac{V_2^2}{2} + \cancel{Z_2 g}\right] + \frac{\partial W_x}{\partial T}$$

$$15\ kW = \frac{\partial M}{\partial T}\left[h_2 - h_1 + \frac{V_2^2}{2} - \frac{V_1^2}{2}\right]$$

$$\Rightarrow 15000 = 1\left[\Delta h + 6250\right]$$

$\Rightarrow \Delta h = 8750 \ J/kg.$

Now $\Delta h = C_p \Delta T.$

$\Rightarrow \Delta T = \frac{8750}{1005} K = 8.70 K.$

$\therefore T_2 = 8.70 K + 293 K = 301.70 K = 28.70 K.$

$h_1 = 3000 \frac{KJ}{kg}$ $\quad V_1 = 60 \frac{m}{s}$ $\quad h_2 = 2762 \frac{KJ}{kg}$ $\quad \frac{\partial Q}{\partial T} = 0, \quad \frac{\partial W_x}{\partial T} = 0.$

9.

(i) $\frac{\partial M}{\partial T}\left[h_1 + \frac{V_1^2}{2} + \cancel{Z_1 g}\right] + \cancel{\frac{\partial Q}{\partial T}} = \frac{\partial M}{\partial T}\left[h_2 + \frac{V_2^2}{2} + \cancel{Z_2 g}\right] + \cancel{\frac{\partial W_x}{\partial T}}.$

$\Rightarrow h_1 + \frac{V_1^2}{2} = \frac{V_2^2}{2} + h_2$

$\Rightarrow (238 \times 10^3) + \frac{(60)^2}{2} = \frac{V_2^2}{2} \quad \Rightarrow V_2 = 692.53 \ m/s.$

(ii) $v_1 = 0.187 \ m^3/kg \quad A_1 = 0.1 m^2$

Mass flow rate $= \frac{\partial M}{\partial T} = \frac{A_1 V_1}{v_1} = \frac{0.1 \times 60}{0.187} = 32.085 \frac{Kg}{s}.$

(iii) $v_2 = 0.498 \ m^3/Kg$

$\frac{\partial M}{\partial T} = \text{const.} = \frac{A_2 V_2}{v_2} = 32.085$

$\Rightarrow \frac{A_2 (692.53)}{0.498} = 32.085$

$\Rightarrow A_2 = 0.023 \ m^2$

$P_1 = 1 \text{ bar} = 10^5 \text{ Pa}$ $\quad T_1 = 15°C = 288K.$ $\quad P_2 = 27.59 \times 10^5 \text{ Pa}$

10. a) Isothermal:

$$W = RT_1 \ln P_1/P_2$$

$$= 0.287 [\cancel{T_1}\ 288] . \ln\left(\frac{1}{27.59}\right)$$

$$= -274.20 \text{ KJ/Kg.} \quad [m \to 1].$$

$\Delta q = w \quad [\because \Delta U = 0].$

$\Rightarrow \Delta q = -274.20 \text{ KJ/kg.}$

Isothermal. $\therefore T_2 = 15°C.$

$\Delta u = 0 \quad [\because \text{Isothermal}]$

b) Polytropic, with $n = 1.3$

$$P_1(V_1)^{1.3} = P_2(V_2)^{1.3}$$

$$\Rightarrow P_1\left(\frac{RT_1}{P_1}\right)^{1.3} = P_2\left(\frac{RT_2}{P_2}\right)^{1.3}$$

$$\Rightarrow \left(\frac{P_2}{P_1}\right)^{0.3} = \left(\frac{T_2}{T_1}\right)^{1.3}.$$

$$\Rightarrow T_2 = 619.26 K.$$

$$\therefore W = \frac{P_1V_1 - P_2V_2}{1.3 - 1}$$

$$= \frac{R(T_1 - T_2)}{0.3}$$

$$= \cancel{0.287}[= -316.9 \text{ KJ/kg}$$

$C_p = 1.005 \text{ KJ/kg K.}$ (for air)

* Now, $C_v = C_p - R. = 0.718 \text{ KJ/kg.K.}$

$$\therefore \Delta U = C_v \Delta T.$$

$$= 0.718(619.26 - 288) = 237.84 \text{ KJ/kg.}$$

$$\therefore \Delta Q = \Delta U + W$$

$$= -79.05 \text{ KJ/kg.}$$

11) $P_1 = 0.7$ bar. $T_1 = 57°C$. $V_1 = 200$ m/s.

$P_2 = 1$ bar.

$A_2 = A_1 + \frac{20}{100} \times A_1 = \frac{6}{5} A_1$

$P_1 v_1 = RT_1$

$\Rightarrow v_1 = \frac{0.287 \xrightarrow{\times 10^3} [330]}{0.7 \times 10^5} = 1.353 \ m^3/kg.$

Now, $P_2 v_2 = RT_2$.

$\Rightarrow v_2 = \frac{0.287 \xrightarrow{\times 10^3} (T_2)}{1 \times 10^5} = 2.87 \times 10^{-3} \times T_2$

So, $\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$

$\Rightarrow \frac{\cancel{A_1}(200)}{1.353} = \frac{6/5 \cancel{A_1} (V_2)}{(2.87 \times 10^{-3}) T_2} \rightarrow \frac{V_2}{T_2} = 0.3535 \Rightarrow T_2 = 2.83 V_2$

Now, $\frac{\partial M}{\partial T}\left[h_1 + \frac{V_1^2}{2} + \cancel{Z_1 g}\right] + \cancel{\frac{\partial Q}{\partial T}} = \frac{\partial M}{\partial T}\left[h_2 + \frac{V_2^2}{2} + \cancel{Z_2 g}\right] + \cancel{\frac{\partial W_x}{\partial T}}^{0}$

$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

$\Rightarrow V_1^2 - V_2^2 = 2(h_2 - h_1)$

$\Rightarrow 4 \times 10^4 - V_2^2 = 2 \times C_p (T_2 - T_1)$

$\Rightarrow 4 \times 10^4 - V_2^2 = 2 \times 1005 (T_2 - 330)$

$\Rightarrow 4 \times 10^4 - V_2^2 = 2 \times 1005 (2.83 V_2 - 330)$ 5903.4

$\Rightarrow \cancel{V_2^2 + 2844.15 V_2 - 291650 = 0}$

$\Rightarrow V_2^2 + 5688.3 V_2 - 623300 = 0.$

$\Rightarrow V_2 = 110.51$ m/s.

imperfection in quadratic, gives (nearly) answer. xD.

$\therefore T_2 = 312.74$ K.

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12. volume flow rate = $0.05\ m^3/s$. $\rho = 10^3\ kg/m^3$.

$\therefore$ mass flow rate $= \frac{\partial M}{\partial T} = 0.05 \times 10^3\ \frac{kg}{s}$.

$= 50\ \frac{kg}{s}$. $Z_2 = Z_1 + 100$.

Now $\frac{\partial M}{\partial T}\left[h_1 + \cancel{\frac{v_1^2}{2}} + Z_1 g\right] \cancel{+ \frac{\partial Q}{\partial T}} = \frac{\partial M}{\partial T}\left[h_2 + \cancel{\frac{v_2^2}{2}} + Z_2 g\right] + \frac{\partial W_x}{\partial T}$

Now $h = U + PV$

$\Delta U = 0$ [$\because T \to$ const].

$P, V \to$ const. [given].

$\therefore h_2 - h_1 = 0$.

$\Rightarrow \frac{\partial M}{\partial T}\left[\cancel{(h_1 - h_2)} + (Z_1 - Z_2) g\right] = \frac{\partial W_x}{\partial T}$.

$\Rightarrow \frac{\partial W_x}{\partial T} = 5\left[-100 \times 9.81\right] = -4905\ W$.

$= -4.905\ kW$.

—x—