CHAPTER

14 Dynamics of Ideal Fluids

14.1 INTRODUCTION

In Chapter 13, we dealt with the kinematics of fluid flow that includes the motion of the fluid without considering the force that causes the flow. In this chapter, the force analysis along with the motion attributes and their relationships are established. The forces that are present in fluid motion are the body forces and the surface forces. Differential equation is developed neglecting the viscous effects. It is a simplified form which is far reaching in reality. Nevertheless, it provides a lot of insight to understand the topic in its basic form. Without consideration of the viscous forces, the surface forces acting on the fluid element are the normal forces in the form of pressure.

14.2 EULER'S EQUATION OF MOTION ALONG A STREAMLINE

Consider the flow of an inviscid fluid along a streamline as shown in Fig. 14.1. The equations of motion are to be written in terms of the coordinates, distance along a streamline and the coordinate normal to the streamline. Applying Newton' second law in the streamwise direction (s-direction) to the fluid element of area , we have

$$\sum F_s = ma_s \tag{14.1}$$

where a_s is the acceleration of the fluid particle along the streamline.

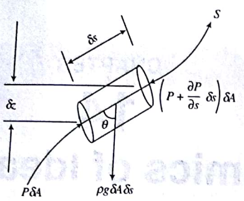

Figure 14.1

In the absence of viscous forces, forces acting in the s-direction are the pressure forces and the component of gravity force in the *s*-direction. Let the pressure at the lower face of the fluid element be *P*. The pressure acting on the other face is $P+\frac{\partial P}{\partial s}\delta s$. Therefore, the pressure force acting on the lower and upper faces are $P\delta A$ and $\left(P+\frac{\partial P}{\partial s}\delta s\right)\delta A$ respectively. Considering gravity as the only body force, the body force acting on the element is $\rho g\delta s\delta A$. Thus the component of body force along the streamline is $-\rho g\sin\theta\delta s\delta A$ (where θ is the angle between the tangent to the streamline and the direction of the gravity force). Hence, Eq. (14.1) becomes

$$P\delta A-\left(P+\frac{\partial P}{\partial s}\delta s\right)\delta A-\rho g\sin\theta\delta s\delta A=\rho a_s\delta s\delta A \quad (14.2)$$

Simplifying Eq. (14.2), we have

$$-\frac{\partial P}{\partial s}-\rho g\sin\theta=\rho a_s$$

or

$$-\frac{1}{\rho}\frac{\partial P}{\partial s}-g\frac{\partial z}{\partial s}=a_s \quad (14.3)$$

Along any streamline since the velocity *V* is a function of space and time i.e., $V=V(s,t)$, one can write

$$dV=\frac{\partial V}{\partial t}dt+\frac{\partial V}{\partial s}ds$$

or

$$\frac{dV}{dt}=\frac{\partial V}{\partial t}+\frac{\partial V}{\partial s}\frac{ds}{dt}$$

or

$$a_s=\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial s} \qquad \left[\because a_s=\frac{dV}{dt}\right]$$

Substituting the value of a_s in Eq. (14.3), we get

$$-\frac{1}{\rho}\frac{\partial P}{\partial s}-g\frac{\partial z}{\partial s}=\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial s} \quad (14.4)$$

Equation (14.4) is known as the *Euler's equation of motion* along a streamline.

For steady flow, $\frac{\partial V}{\partial t}=0$, then Eq. (14.4) becomes

$$-\frac{1}{\rho}\frac{\partial P}{\partial s}-g\frac{\partial z}{\partial s}=V\frac{\partial V}{\partial s}$$

or

$$\frac{1}{\rho}\frac{\partial P}{\partial s}+V\frac{\partial V}{\partial s}+g\frac{\partial z}{\partial s}=0$$

Since, *s* is the only independent variable; the total differential may replace the partial

$$\frac{dP}{\rho}+VdV+gdz=0 \quad (14.5)$$

Equation (14.5) is another form of Euler's equation of motion along a streamline, which is valid for steady flow.

> ***Note:*** *Eq. (14.5) is valid for inviscid, steady flow along a streamline.*

14.3 BERNOULLI'S EQUATION

Euler's equation of motion along a streamline for steady flow can be written as (Eq. 14.5)

$$\frac{dP}{\rho}+VdV+gdz=0$$

Integrating the above equation, we have

$$\int\frac{dP}{\rho}+\frac{V^2}{2}+gz=C \quad (14.6)$$

where *C* is an integration constant.

In case of an incompressible fluid (density does not change with change in pressure), Eq. (14.6) can be written as

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$$\frac{P}{\rho}+\frac{V^2}{2}+gz=C \qquad (14.7)$$

Equation (14.7) is known as *Bernoulli's equation*. The constant C is known as Bernoulli's constant which is constant for a streamline and varies from one streamline to another. Each term of Eq. (14.7) can be interpreted as a form of energy per unit mass. Here, $\frac{P}{\rho}$ represents the flow energy (or flow work) per unit mass, $\frac{V^2}{2}$ represents the kinetic energy per unit mass and gz represents the potential energy per unit mass.

Dividing by g, Eq. (14.7) becomes

$$\frac{P}{\rho g}+\frac{V^2}{2g}+z=C \qquad (14.8)$$

Equation (14.8) is another form of Bernoulli's equation. Each term of Eq. (14.8) can be interpreted as a form of energy per unit weight (also known as head in fluid mechanics). Here, $\frac{P}{\rho g}$ is the *pressure head* (flow energy per unit weight), which represents the height of a fluid column that produces the static pressure P. The term $\frac{V^2}{2g}$ is the *velocity head* (kinetic energy per unit weight) and z and is the *potential head* (potential energy per unit weight). Therefore, the Bernoulli's equation can be viewed as an expression of mechanical energy balance and can be stated as follows:

During steady, inviscid flow of an incompressible fluid along a streamline, total mechanical energy at any point is constant. The total mechanical energy consists of flow energy, kinetic energy and potential energy.

Applying Bernoulli's equation between two points 1 and 2 along the same streamline, we have

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \qquad (14.9)$$

Note: *The assumptions made in the derivation of Bernoulli's equation are*

(i) The flow is inviscid.
(ii) The flow is along a streamline.
(iii) The flow is steady.
(iv) The fluid is incompressible (density does not change with change in pressure).

Note: *Bernoulli's equation deals with the law of conservation of mechanical energy.*

Note: The equation $\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$ *holds true between any two points in the flow field provided that the flow is irrotational, inviscid, steady and fluid is incompressible.*

14.4 STATIC, DYNAMIC, STAGNATION AND TOTAL PRESSURES

Bernoulli's equation can be written as

$$\frac{P}{\rho g}+\frac{V^2}{2g}+z=\text{Constant}$$

or

$$P+\rho\frac{V^2}{2}+\rho gz=\text{Constant} \qquad (14.10)$$

The physical meaning of different terms appearing in the left hand side of Eq. (14.10) is as follows:

14.4.1 Static Pressure

The pressure P as appear in Eq. (14.10) is often referred to as the *static pressure*. It is the pressure caused by molecular collisions and can be felt at any point by an observer moving with the flow. To such an observer, the fluid appears to be static or stationary, so this pressure is often called the static pressure.

14.4.2 Dynamic Pressure

The term $\rho\frac{V^2}{2}$ in Eq. (14.10) is called the *dynamic pressure*. It represents the pressure increase that would occur if all the kinetic energy of a fluid particle in a frictionless flow were converted into a corresponding increase in pressure energy.

14.4.3 Hydrostatic Pressure

The term ρgz in Eq. (14.10) is called the *hydrostatic pressure*. It represents the change in the static pressure that would occur if the fluid moved along the streamline to an elevation of zero.

14.4.4 Stagnation Pressure

It is the pressure that could result at a point in the flow, if the flow were brought to rest in an isentropic process. Applying Bernoulli's equation between two points, one just upstream of the stagnation point and the other, the stagnation point itself, we have

$$\frac{P}{\rho g}+\frac{V^2}{2g}+z=\frac{P_s}{\rho g}+\frac{V_s^2}{2g}^{\,0}+z \quad (p_s \text{ is the stagnation pressure})$$

where V_s is the velocity at stagnation point which is zero from its definition.

or
$$P_s = P+\frac{1}{2}\rho V^2 \tag{14.11}$$

Therefore, the sum of the static pressure and dynamic pressure is called the *stagnation pressure*.

14.4.5 Total Pressure

The sum of the static, dynamic and hydrostatic pressures $\left(P+\rho\frac{V^2}{2}+\rho g z\right)$ is referred to as the *total pressure*. Therefore, *Bernoulli's equation states that the total pressure along a streamline is constant.* It is important to mention here that the sum of the pressure head $\left(\frac{P}{\rho g}\right)$ and the potential head (z) is called as *piezometric pressure head*.

> ***Note:*** *Fluid flows from higher piezometric pressure to lower piezometric pressure (not necessarily from higher static pressure to lower static pressure).*

Example 14.1 Water is flowing in a pipe of 200 mm diameter with an average velocity of 5 m/s. At a particular section 1, the pressure is measured to be 250 kN/m². If the section 1 is 7 m above the datum, determine total head of water.

Solution

Average velocity of flow $V = 5$ m/s

Pressure at section 1 $P = 250 \text{ kN/m}^2 = 250\times10^3 \text{ N/m}^2$

Pressure head at section 1 is $\frac{P}{\rho g} = \frac{250\times10^3}{1000\times9.81} = 25.484$ m of water

Velocity head at section 1 is $\frac{V^2}{2g} = \frac{5^2}{2\times9.81} = 1.274$ m of water

Datum head at section 1 is $z = 7$ m

Total head of water at section 1 is $\frac{P}{\rho g}+\frac{V^2}{2g}+z = 25.484+1.274+7 = 33.758$ m

Example 14.2 Water is flowing through a pipe having diameters 30 cm and 20 cm at section 1 and 2 respectively. The average velocity of water at section 1 is 4 m/s. Find the velocity head at the section 1 and 2 and also rate of discharge.

Solution

Diameter of pipe at section 1 $D_1 = 30$ cm $= 0.3$ m

Diameter of pipe at section 2 $D_2 = 20$ cm $= 0.2$ m

Cross-sectional area at section 1 is $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.0707$ m²

Cross-sectional area at section 2 is $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314$ m²

Average velocity of water at section 1 $V_1 = 4$ m/s

Velocity head at section 1 is $\frac{V_1^2}{2g} = \frac{4^2}{2\times9.81} = 0.815$ m of water

The rate of discharge is found to be $Q = A_1V_1 = 0.0707\times4 = 0.2828$ m³/s

Let the average velocity at section 2 be V_2

From continuity equation, we have

$$Q = A_1V_1 = A_2V_2$$

or
$$V_2 = \frac{A_1V_1}{A_2} = \frac{0.0707\times4}{0.0314} = 9 \text{ m/s}$$

Velocity head at section 2 is $\frac{V_2^2}{2g} = \frac{9^2}{2\times9.81} = 4.128$ m of water

Example 14.3 A vertical tapering pipe is 2 m long. The diameter of the pipe is 20 cm at the top end and 10 cm at the bottom end. If 30 litres/sec of water flows through the pipe, find the difference in pressure between the two ends of the pipe. Neglect losses.

Solution

The pipeline is schematically shown in Fig. 14.2. Let 1 and 2 respectively designate the bottom and top end of the pipeline.

Diameter of pipe at section 1 $D_1 = 10$ cm $= 0.1$ m

Diameter of pipe at section 2 $D_2 = 20$ cm $= 0.2$ m

Cross-sectional area at section 1 is $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.1)^2 = 0.00785$ m²

Cross-sectional area at section 2 is $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314$ m²

Discharge $Q = 30 \text{ litres/s} = 30\times10^{-3} \text{ m}^3/\text{s} = 0.03 \text{ m}^3/\text{s}$

Difference in datum head between sections 1 and 2 $z_2 - z_1 = 2$ m

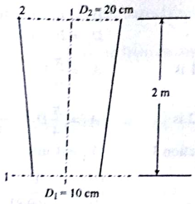

Figure 14.2

Average velocity at section 1 is $\quad V_1 = \dfrac{Q}{A_1} = \dfrac{0.03 \text{ m}^3/\text{s}}{0.00785 \text{ m}^2} = 3.82$ m/s

Average velocity at section 2 is $\quad V_2 = \dfrac{Q}{A_2} = \dfrac{0.03 \text{ m}^3/\text{s}}{0.0314 \text{ m}^2} = 0.955$ m/s

Applying Bernoulli's equation between sections 1 and 2 along a streamline, one can write

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + z_2 - z_1$$

or $$\frac{P_1 - P_2}{\rho g} = \frac{0.955^2 - 3.82^2}{2 \times 9.81} + 2 = -0.697 + 2 = 1.303 \text{ m of water}$$

or $$P_1 - P_2 = 1.303\rho g = 1.303 \times 1000 \times 9.81 \text{N/m}^2$$

or $$P_1 - P_2 = 12.78 \times 10^3 \text{ N/m}^2 = 12.78 \text{ kN/m}^2$$

Example 14.4 An oil of density 900 kg/m^3 is flowing through a vertical pipe having diameters 30 cm and 20 cm at section 1 and 2 respectively. The rate of flow through pipe is 50 litres/s. The section 1 is 9 m above datum and section 2 is 5 m above datum. If the pressure at section 1 is 300 kN/m^2, find the intensity of pressure at section 2. Neglect friction.

Solution

The pipeline is schematically shown in Fig. 14.3.

Density of oil $\quad \rho = 900$ kg/m^3

Diameter of pipe at section 1 $\quad D_1 = 30$ cm $= 0.3$ m

Diameter of pipe at section 2 $\quad D_2 = 20$ cm $= 0.2$ m

Cross-sectional area at section 1 is $\quad A_1 = \dfrac{\pi}{4} D_1^2 = \dfrac{\pi}{4}(0.3)^2 = 0.0707 \text{ m}^2$

Cross-sectional area at section 2 is $\quad A_2 = \dfrac{\pi}{4} D_2^2 = \dfrac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$

Discharge $\quad Q = 50$ litres/s $= 50 \times 10^{-3}$ m^3/s $= 0.05$ m^3/s

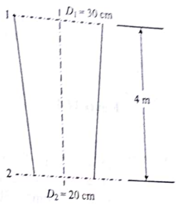

Figure 14.3

Difference in datum head between sections 1 and 2 $\quad z_1 - z_2 = 9 - 5 = 4$ m

Pressure at 1 $\quad P1 = 300$ kN/m$^2 = 300 \times 10^3$ N/m^2

Average velocity at section 1 is $\quad V_1 = \dfrac{Q}{A_1} = \dfrac{0.05}{0.0707} = 0.707$ m/s

Average velocity at section 2 is $\quad V_2 = \dfrac{Q}{A_2} = \dfrac{0.05}{0.0314} = 1.592$ m/s

Applying Bernoulli's equation between sections 1 and 2 along a streamline, one can write

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{P_1}{\rho g} + \frac{V_1^2 - V_2^2}{2g} + z_1 - z_2 = \frac{P_2}{\rho g}$$

or $$\frac{300 \times 10^3}{1000 \times 9.81} + \frac{0.707^2 - 1.592^2}{2 \times 9.81} + 4 = \frac{P_2}{\rho g}$$

or $$33.979 - 0.1037 + 4 = \frac{P_2}{\rho g}$$

or $\dfrac{P_2}{900\times 9.81} = 37.8753$

or $P_2 = 900\times 9.81\times 37.8753 = 336232\text{N/m}^2 = 336.23\text{ kN/m}^2$

Example 14.5 Water flows through a tapering pipe as shown in Fig. 14.4. The diameter at sections 1 and 2 are 10 cm and 20 cm respectively, and the heights above a horizontal datum are 3 and 5 m respectively. The pressure at 1 is 30 kN/m². Water flow rate is 0.03 m³/s. Estimate the pressure at section 2.

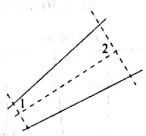

Figure 14.4

Solution

Diameter of pipe at section 1	D_1 = 10 cm = 0.1 m
Diameter of pipe at section 2	D_2 = 20 cm = 0.2 m
Pressure at section 1	$P_1 = 10\text{ kN/m}^2 = 30\times 10^3\text{ N/m}^2$
Height of section 1 above datum	$z_1 = 3$ m
Height of section 2 above datum	$z_2 = 5$ m
Volume flow rate of water	$Q = 0.03\text{ m}^3\text{/s}$

Cross-sectional area at section 1 is $A_1 = \dfrac{\pi}{4}D_1^2 = \dfrac{\pi}{4}(0.1)^2 = 0.00785\text{ m}^2$

Cross-sectional area at section 2 is $A_2 = \dfrac{\pi}{4}D_2^2 = \dfrac{\pi}{4}(0.2)^2 = 0.0314\text{ m}^2$

From continuity equation, we have

$$Q = A_1V_1 = A_2V_2$$

Thus, the average velocity at section 1 is $V_1 = \dfrac{Q}{A_1} = \dfrac{0.03\text{ m}^3\text{/s}}{0.00785\text{ m}^2} = 3.82\text{ m/s}$

Average velocity at section 2 is $V_2 = \dfrac{Q}{A_2} = \dfrac{0.03\text{ m}^3\text{/s}}{0.0314\text{ m}^2} = 0.955\text{ m/s}$

Applying Bernoulli's equation between sections 1 and 2 along a streamline, one can write

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

or $$\frac{30\times 10^3}{1000\times 9.81}+\frac{3.82^2}{2\times 9.81}+3 = \frac{P_2}{1000\times 9.81}+\frac{0.955^2}{2\times 9.81}+5$$

or $$3.058+0.744+3 = \frac{P_2}{1000\times 9.81}+0.046+5$$

or $$\frac{P_2}{1000\times 9.81} = 1.756$$

or $$P_2 = 1.756\times 1000\times 9.81 = 17226.36\text{ N/m}^2 = 17.226\text{ kN/m}^2$$

Example 14.6 A 500 m long pipe has a slope of 1 in 100 and tapers from 1 m diameter at higher end to 0.5 m diameter at the lower end. It carries an oil of specific gravity 0.85 at a rate of 100 litres/s. If the pressure at high end is 250 kN/m², find the pressure at the lower end. Neglect losses due to friction.

Solution The pipe is schematically shown in Fig. 14.5. Let 1 and 2 respectively designate the lower and higher end of the pipe.

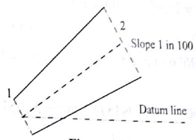

Figure 14.5

Specific gravity of oil	$S = 0.85$
Density of oil	$\rho = 0.85\times 1000 = 850\text{ kg/m}^3$
Diameter of pipe at lower end	$D_1 = 0.5$ m
Diameter of pipe at higher end	$D_2 = 1$ m
Pressure at higher end	$P_2 = 250\text{ kN/m}^2 = 250\times 10^3\text{ N/m}^2$
Volume flow rate of oil	$Q = 100\text{ lit/s} = \dfrac{100}{1000}\text{m}^3\text{/s} = 0.1\text{ m}^3\text{/s}$
Cross-sectional area at lower end is	$A_1 = \dfrac{\pi}{4}D_1^2 = \dfrac{\pi}{4}(0.5)^2 = 0.1963\text{ m}^2$
Cross-sectional area at higher end is	$A_2 = \dfrac{\pi}{4}D_2^2 = \dfrac{\pi}{4}(1)^2 = 0.7854\text{ m}^2$
Length of pipe	$L = 500$ m
Slope of pipe	= 1 in 100

Let the datum line passes through the centre of the lower end. Then

Height of lower end above datum $z_1 = 0$

Height of higher end above datum $z_2 = \dfrac{1}{100}\times 500 = 5\text{ m}$

From continuity equation, we have

$$Q = A_1V_1 = A_2V_2$$

Thus, the average velocity at lower end is $V_1 = \dfrac{Q}{A_1} = \dfrac{0.1}{0.1963} = 0.509$ m/s

Average velocity at higher end is $V_2 = V_2 = \dfrac{Q}{A_2} = \dfrac{0.1}{0.7854} = 0.127$ m/s

Applying Bernoulli's equation between sections 1 and 2 along a streamline, we have

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{P_1}{\rho g} + \frac{0.509^2}{2\times 9.81} + 0 = \frac{250\times 10^3}{1000\times 9.81} + \frac{0.127^2}{2\times 9.81} + 5$$

or $$\frac{P_1}{\rho g} + 0.0132 + 0 = 29.98 + 0.00082 + 5$$

or $$\frac{P_1}{850\times 9.81} = 34.9676 \text{ m of oil}$$

or $$P_1 = 34.9676\times 850\times 9.81 = 291577 \text{ N/m}^2 = 291.577 \text{ kN/m}^2$$

Example 14.7 Water flows through a conical tube fixed vertically with its smaller end upwards. The average velocities at the smaller and larger end are 4.5 m/s and 1.5 m/s respectively. Length of the conical tube is 1.5 m. The pressure at the upper end is equivalent to a head of 10 m of water. Neglecting losses, determine the pressure at the lower end of the tube. Also find the piezometric head at both the ends.

Solution The conical tube is schematically shown in Fig. 14.6. Let 1 and 2 respectively designate the smaller and larger end of the pipe.

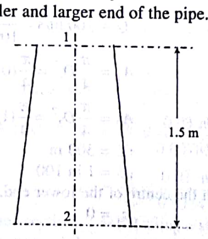

Figure 14.6

Length of the pipe $L = z_1 - z_2 = 1.5$ m

Pressure head at smaller end $\dfrac{P_1}{\rho g} = 10$ m of water

Average velocity at smaller end $V_1 = 4.5$ m/s

Average velocity at larger end $V_2 = 1.5$ m/s

Applying Bernoulli's equation between sections 1 and 2 along a streamline, we have

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 - z_2 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

or $$10 + \frac{4.5^2}{2\times 9.81} + 1.5 = \frac{P_2}{\rho g} + \frac{1.5^2}{2\times 9.81}$$

or $$\frac{P_2}{1000\times 9.81} = 12.42 \text{ m of water}$$

or $$P_2 = 12.42\times 1000\times 9.81 = 121.84\times 10^3 \text{ N/m}^2 = 121.84 \text{ kN/m}^2$$

Let us consider that the larger end (section 2) represents the reference datum.

Datum head at 1 $z_1 = 1.5$ m

Datum head at 2 $z_2 = 0$ m

Piezometric head at 1 is then $\dfrac{P_1}{\rho g} + z_1 = 10 + 1.5 = 11.5$ m of water

Piezometric head at 2 is then $\dfrac{P_2}{\rho g} + z_2 = 12.42 + 0 = 12.42$ m of water

14.5 ENERGY EQUATION FOR REAL FLUID

For flow of real fluids, there is a viscous force that resists the flow. Because of the resistive force there is a loss of energy during the flow of fluids. This energy loss is to be taken into consideration during the derivation of energy equation. Applying energy equation between 1 and 2 with consideration of loss due to fluid friction, one can write

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f \qquad (14.12)$$

where h_f is the loss of head between 1 and 2. Detailed analysis on loss of energy due to fluid friction is not included in this text. Equation (14.12) looks similar to Bernoulli's equation (Eq. (14.8)) with an additional term h_f on the right-hand side which represents the loss of head between points 1 and 2.

Example 14.8 Water flows vertically upwards through a pipe of 1 m diameter and 10 m length. The pressure at the upper end of the pipe is 5 m of water and the head loss due to friction is 1 m of water column. When water flows at an average velocity of 5 m/s, find the pressure at the lower end of the pipe.

Solution The flow arrangement is schematically shown in Fig. 14.7.

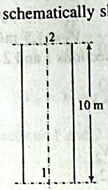

Figure 14.7

Applying energy equation between sections 1 and 2, one can write

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f$$

or $$\frac{P_1}{\rho g}+\frac{V^2}{2g}+0=5+\frac{V^2}{2g}+10+1 \qquad [\because V_1=V_2=V]$$

or $$\frac{P_1}{\rho g}=16 \text{ m of water column}$$

or $$P_1=1000\times 9.81\times 16=156.96\times 10^3 \text{ N/m}^2=156.96 \text{ kN/m}^2$$

Example 14.9 Water is flowing through a pipeline at a rate of 0.04 m^3/s as shown in Fig. 14.8. The pipeline is 10 cm in diameter and it is at an elevation of 80 m at section *A* (Fig. 14.8). At section *B* it is at an elevation of 82 m and has diameter of 20 cm. The pressure of water at *A* is 40 kN/m^2 and the energy loss in pipe between section *A* and *B* is 0.3 m of water. Calculate pressure at *B* if flow is from *A* to *B*.

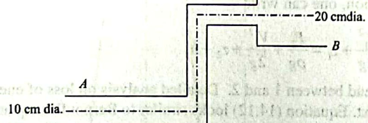

Figure 14.8

Solution

Diameter of pipe at section *A*	D_A = 10 cm = 0.1 m
Diameter of pipe at section *B*	D_B = 20 cm = 0.2 m
Pressure at section *A*	P_A = 40 kN/m^2 = 40 × 1000 N/m^2
Height of section *A* above reference datum	z_A = 80 m
Height of section *B* above reference datum	z_B = 82 m
Volume flow rate of water	Q = 0.04 m3/s
Loss of energy between *A* and *B*	h_{A-B} = 0.3 m of water

Cross-sectional area at section *A* is $$A_A=\frac{\pi}{4}D_A^2=\frac{\pi}{4}(0.1)^2=0.00785 \text{ m}^2$$

Cross-sectional area at section *B* is $$A_B=\frac{\pi}{4}D_B^2=\frac{\pi}{4}(0.2)^2=0.0314 \text{ m}^2$$

Average velocity at section *A* is $$V_A=\frac{Q}{A_A}=\frac{0.04}{0.00785}=5.096 \text{ m/s}$$

Average velocity at section *B* is $$V_B=V_B=\frac{Q}{A_B}=\frac{0.04}{0.0314}=1.274 \text{ m/s}$$

Applying energy equation between sections *A* and *B*, we have

$$\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+z_A=\frac{P_B}{\rho g}+\frac{V_B^2}{2g}+z_B+h_{A-B}$$

or $$\frac{40\times 1000}{1000\times 9.81}+\frac{5.096^2}{2\times 9.81}+80=\frac{P_B}{\rho g}+\frac{1.274^2}{2\times 9.81}+82+0.3$$

or $$\frac{P_B}{1000\times 9.81}=3.018$$

or $$P_B=1000\times 9.81\times 3.018 \text{ N/m}^2=29.61\times 10^3 \text{ N/m}^2=29.61 \text{ kN/m}^2$$

Example 14.10 Water is flowing steadily in a 30 cm diameter pipe at an average velocity of 4 m/s. At points *A* and *B* measurements of pressure and elevation are 200 kN/m^2 and 150 kN/m^2 and 12 m and 17 m respectively. Find the loss between the two points.

Solution

Diameter of pipe	D = 30 cm = 0.3 m
Average velocity	V = 4 m/s
Pressure at point *A*	P_A = 200 kN/m^2 = 200 × 1000 N/m^2
Pressure at point *B*	P_B = 150 kN/m^2 = 150 × 1000 N/m^2
Elevation of point *A* above reference datum	z_A = 12 m
Elevation of point *B* above reference datum	z_B = 17 m

Total mechanical energy at point A is

$$=\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+z_A$$

$$=\frac{200\times10^3}{1000\times9.81}+\frac{4^2}{2\times9.81}+12=20.387+0.815+12=33.202 \text{ m}$$

Total mechanical energy at point B is

$$=\frac{P_B}{\rho g}+\frac{V_B^2}{2g}+z_B$$

$$=\frac{150\times10^3}{1000\times9.81}+\frac{4^2}{2\times9.81}+17=15.29+0.815+17=33.105 \text{ m}$$

Loss of head between points A and B is then

$= 33.202 - 33.105 = 0.097$ m of water column

Example 14.11 At a certain location A of a pipeline carrying an oil of density 850 kg/m³, the diameter is 80 cm, the pressure is 180 kN/m² and the average velocity is 5 m/s. At another section B which is 3 m higher than A, the diameter is 50 cm and the pressure is 100 kN/m². What is the direction of flow?

Solution

Density of oil	$\rho = 850$ kg/m³
Diameter of pipe at location A	$D_A = 80$ cm $= 0.8$ m
Diameter of pipe at location B	$D_B = 50$ cm $= 0.5$ m
Average velocity at location A	$V_A = 5$ m/s
Pressure at location A	$P_A = 180$ kN/m² $= 180 \times 1000$ N/m²
Pressure at location B	$P_B = 100$ kN/m² $= 100 \times 1000$ N/m²

Let the datum line passes through the location A.

Height of location A above reference datum	$z_A = 0$
Height of location B above reference datum	$z_B = 3$ m

Cross-sectional area at location A is $\quad A_A=\frac{\pi}{4}D_A^2=\frac{\pi}{4}(0.8)^2=0.5026 \text{ m}^2$

Cross-sectional area at location B is $\quad A_B=\frac{\pi}{4}D_B^2=\frac{\pi}{4}(0.5)^2=0.1963 \text{ m}^2$

From continuity equation, we have

$$Q=A_AV_A=A_BV_B$$

Thus, the average velocity at location B is

$$V_B=\frac{A_AV_A}{A_B}=\frac{0.5026\times5}{0.1963}=12.8 \text{ m/s}$$

Total mechanical energy at location A is

$$=\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+z_A$$

$$=\frac{180\times10^3}{1000\times9.81}+\frac{5^2}{2\times9.81}+0=18.349+1.274+0=19.623 \text{ m}$$

Total mechanical energy at location B is

$$=\frac{P_B}{\rho g}+\frac{V_B^2}{2g}+z_B$$

$$=\frac{100\times10^3}{1000\times9.81}+\frac{12.8^2}{2\times9.81}+3=10.194+8.35+3=21.544 \text{ m}$$

Since the total mechanical energy at location B is higher than that at location A, the flow takes place from location B to location A.

> *Note: Example 6.11 illustrates that fluid flows from higher mechanical energy to lower mechanical energy (not necessarily from higher pressure to lower pressure).*

Example 14.12 A pipeline carrying water changes in diameter from 20 cm at section 1 to 40 cm diameter at section 2 which is 6 m at higher level. If the pressure at section 1 and 2 are 120 kN/m² and 80 kN/m² respectively and the discharge is 200 litres/s, determine the loss of head and the direction of flow.

Solution

Diameter of pipe at section 1	$D_1 = 20$ cm $= 0.2$ m
Diameter of pipe at section 2	$D_2 = 40$ cm $= 0.4$ m
Pressure at section 1	$P_1 = 120$ kN/m² $= 130 \times 1000$ N/m²
Pressure at section 2	$P_2 = 80$ kN/m² $= 80 \times 1000$ N/m²

Let us consider the reference datum for elevation is at section 1.

Height of section 1 above reference datum	$z_1 = 0$ m
Height of section 2 above reference datum	$z_2 = 6$ m
Volume flow rate of water	$Q = 200$ litres/s $= 0.2$ m³/s

Average velocity at section1 is $\quad V_1=\frac{Q}{A_1}=\frac{0.2}{\frac{\pi}{4}(0.2)^2}=6.366 \text{ m/s}$

Average velocity at section 2 is $\quad V_2=\frac{Q}{A_2}=\frac{0.2}{\frac{\pi}{4}(0.4)^2}=1.59 \text{ m/s}$

Total mechanical energy at section 1 is

$$= \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1$$

$$= \frac{120 \times 10^3}{1000 \times 9.81} + \frac{6.366^2}{2 \times 9.81} + 0 = 14.298 \text{ m}$$

Total mechanical energy at section 2 is

$$= \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$= \frac{80 \times 10^3}{1000 \times 9.81} + \frac{1.59^2}{2 \times 9.81} + 6 = 14.284 \text{ m}$$

Loss of head between sections 1 and 2 is then

$= 14.298 - 14.284 = 0.014$ m of water column

Since the total mechanical energy at section 1 is higher than that at section 2, the flow takes place from section 1 to section 2.

Example 14.13 Water is flowing vertically upwards through a pipeline having diameter 1 m and 0.5 m at the base and top respectively. The pressure at the lower end is 450 mm of mercury, while the pressure at the upper end is 20 kN/m^2. If the loss of head is 20% of difference in velocity head, calculate the discharge. The difference in the elevation is 4 m. The density of mercury is 13600 kg/m^3.

Solution

Let 1 and 2 respectively designate the lower end and upper end of the pipe.

Diameter of pipe at lower end $D_1 = 1$ m

Diameter of pipe at upper end $D_2 = 0.5$ m

Cross-sectional area at lower end is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ m}^2$

Cross-sectional area at upper end is $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$

Pressure at the lower end $P_1 = 450$ mm of Hg $= 0.45 \times 13600 \times 9.81 = 60037.2 \text{N/m}^2$

Pressure at the upper end $P_2 = 20 \text{kN/m}^2 = 20 \times 1000 \text{N/m}^2$

Difference in datum head between the upper end and lower end $z_2 - z_1 = 4$ m

Loss of energy head $h_{1-2} = 0.2 \frac{V_2^2 - V_1^2}{2g}$

Applying continuity equation between the lower end and upper end, we have

$$A_1 V_1 = A_2 V_2$$

or $$V_2 = \frac{A_1}{A_2} V_1 = \frac{0.7854}{0.1963} V_1 = 4V_1$$

Applying energy equation between the lower end and upper end, one can write

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{1-2}$$

or $$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + (z_2 - z_1) + 0.2 \frac{V_2^2 - V_1^2}{2g}$$

or $$1.2 \frac{V_2^2 - V_1^2}{2g} = \frac{P_1 - P_2}{\rho g} - (z_2 - z_1)$$

or $$1.2 \frac{(4V_1)^2 - V_1^2}{2g} = \frac{P_1 - P_2}{\rho g} - (z_2 - z_1) \qquad [\because V_2 = 4V_1]$$

or $$1.2 \frac{15V_1^2}{2 \times 9.81} = \frac{60037.2 - 20000}{1000 \times 9.81} - 4 = 4.08 - 4 = 0.08$$

or $$V_1^2 = \frac{0.08 \times 2 \times 9.81}{1.2 \times 15} = 0.0872$$

or $$V_1 = \sqrt{0.0872} = 0.295 \text{ m/s}$$

The discharge is then found to be

$$Q = A_1 V_1 = 0.7854 \times 0.295 = 0.2317 \text{ m}^3/\text{s}$$

14.6 APPLICATION OF BERNOULLI'S EQUATION FOR MEASUREMENT OF FLOW RATE THROUGH PIPES

Flow rate through a pipe is usually measured by providing a coaxial area contraction within the pipe and by recording the pressure drop across the contraction. Determination of the flow rate from the measurement of the concerned pressure drop depends on the straight forward application of Bernoulli's equation. Three different flow meters primarily operate on this principle. These are: (i) Venturimeter, (ii) Orificemeter and (iii) Flow nozzle. Here, we discuss the working principle of Venturimeter and Orificemeter. Further, the working principle of Pitot tube, commonly used to measure the velocity at a point in the flow field is discussed.

14.6.1 Venturimeter

The venturimeter, invented by the American Engineer Clemans Herschel (1842–1932) and named by him after Italian Giovanni Venturi (1746–1822). The venturimeter is one of the popular devices for measuring rate of flow in a pipe. It consists of a short converging conical tube leading to a cylindrical portion, called the throat, of smaller diameter than that of the pipeline, which is followed by a diverging section in which the diameter increases again to that of the main pipeline (Fig.14.9). The inlet and outlet diameters are the same as the diameter of the pipe in which it is to be installed. The velocity of flow increases in course of flow from pipe to throat (converging cone) and the pressure correspondingly decreases. The velocity reaches the maximum value at the throat and the pressure is minimum. The throat is followed by a diffuser which restores the pressure as nearly as possible to the original value. *The expansion angle of the diffuser is very small (usually 5° to 7°) to reduce the possibility of flow separation.* The size of a venturimeter is specified by the pipe and throat diameter, e.g., a 300 by 150 mm venturimeter fits a 300 mm diameter pipe and has a 150 mm diameter throat. For accurate results the venturimeter should be preceded by at least 10 diameters of straight pipe. The pressure difference from which the volume flow rate can be determined is measured between the entry section 1 and the throat section 2, by means of a differential U-tube manometer (Fig. 14.9).

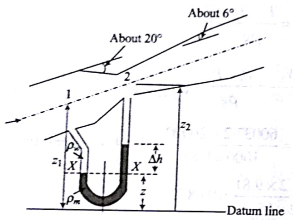

Figure 14.9 *Venturimeter*

Assuming that there is no loss of energy and applying Bernoulli's equation across sections 1 and 2, along a streamline, we get

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

$$V_2^2-V_1^2=2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right] \tag{14.13}$$

Applying continuity equation between sections 1 and 2, one can write

$$A_1V_1=A_2V_2$$

or $$V_2=\frac{A_1}{A_2}V_1$$

Substituting the expression of V_2 in Eq. (14.13), we have

$$V_1^2\left[\left(\frac{A_1}{A_2}\right)^2-1\right]=2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]$$

$$V_1=\frac{A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]}$$

Volume flow rate then can be found to be

$$Q_{th}=A_1V_1=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]} \tag{14.14}$$

or $$Q_{th}=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gH} \tag{14.15}$$

where $$H=\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right] \tag{14.16}$$

The value of H in Eq. (14.16) can be found from the reading of the U-tube differential manometer (Fig. 14.9). Assuming that the connections to the gauge are filled with the fluid flowing in the pipeline, which has a density ρ_w and that the density of manometric fluid is ρ_m. Then, since pressures at level XX must be same in both limbs

$$p_X=P_1+\rho_w g(z_1-z)=P_2+\rho_w g(z_2-z-\Delta h)+\rho_m \Delta h g$$

or $$H=\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]=\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h \tag{14.17}$$

From Eqs. (14.16) and (14.17), one can write

$$Q_{th}=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h} \tag{14.18}$$

In practice, some loss of energy will occur between section 1 and 2. The value of Q_{th} given by Eq. (14.18) is a theoretical value which will be slightly greater than the actual value. A coefficient of discharge C_d is, therefore, introduced, which is defined as the ratio of actual discharge (Q) to that of theoretical discharge (Q_{th}) and is given by

$$C_d=\frac{Q}{Q_{th}}$$

The usual value of C_d for venturimeter varies from 0.95 to 0.99.

Actual discharge is then given by

$$Q=C_d\times Q_{th}$$

or $$Q=\frac{C_dA_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h} \tag{14.19}$$

> ***Note:*** *Manometer connected across two sections of a venturimeter actually measures the piezometric pressure differences, not the static pressure difference.*

> ***Note:*** *The angle of the converging cone is steeper than the diffuser angle to minimize the loss due to flow separation.*

Example 14.14 A venturimeter having throat diameter of 150 mm is set in a vertical pipe of 300 mm diameter to measure the discharge of an oil of specific gravity 0.85 which is flowing through the pipe in upward direction. The difference in elevations of the throat section and entrance section of the venturimeter is 4 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. If the coefficient of discharge of the venturimeter is 0.95, calculate the discharge of oil flowing through the pipe.

Solution

Diameter at inlet	$D_1 = 300$ mm $= 0.3$ m
Diameter at throat	$D_2 = 150$ mm $= 0.15$ m
Differential manometer reading	$\Delta h = 25$ cm $= 0.25$ m
Coefficient of discharge	$C_d = 0.95$

Cross-sectional area of pipe is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.07068 \text{ m}^2$

Cross-sectional area of venturimeter at throat is $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.15)^2 = 0.01767 \text{ m}^2$

The discharge is given by Eq. (14.19) as

$$Q = \frac{C_d A_1 A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

$$= \frac{0.95 \times 0.07068 \times 0.01767 \times \sqrt{2 \times 9.81 \times \left(\frac{13.6}{0.85} - 1\right) \times 0.25}}{\sqrt{(0.07068)^2 - (0.01767)^2}}$$

$$= 0.1487 \text{ m}^3/\text{s}$$

Example 14.15 A vertical venturimeter has an area ratio of 5. It has a throat diameter of 1 cm. When oil of specific gravity 0.85 flows through it the mercury in the differential gauge indicates a difference in height of 20 cm. Find the discharge through the venturimeter. Take coefficient of discharge of the venturimeter as 0.98.

Solution

Diameter at throat	$D_2 = 1$ cm $= 0.01$ m
Differential manometer reading	$\Delta = 20$ cm $= 0.2$ m
Coefficient of discharge	$C_d = 0.98$

Cross-sectional area of venturimeter at throat is $A_2 = \frac{\pi}{4} \times (0.01)^2 = 7.854 \times 10^{-5} \text{ m}^2$

The discharge through the venturimeter is given by Eq. (14.19) as

$$Q = \frac{C_d A_1 A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

$$= \frac{C_d A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

$$= \frac{0.98 \times 7.854 \times 10^{-5} \times \sqrt{2 \times 9.81 \times \left(\frac{13.6}{0.85} - 1\right) \times 0.2}}{\sqrt{1 - \left(\frac{1}{5}\right)^2}}$$

$$= 0.00059 \text{ m}^3/\text{s}$$

Example 14.16 Water flows through a 300 mm × 150 mm venturimeter at the rate of 0.065 m³/s and the differential gauge is deflected 1.2 m. Specific gravity of the manometric liquid is 1.6. Determine the coefficient of discharge of the venturimeter.

Solution

Diameter at inlet	$D_1 = 300$ mm $= 0.3$ m
Diameter at throat	$D_2 = 150$ mm $= 0.15$ m
Differential manometer reading	$\Delta h = 1.2$ m
Specific gravity of manometric liquid	$S_m = 1.6$
Volume flow rate	$Q = 0.065 \text{ m}^3/\text{s}$

Cross-sectional area of pipe is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.07068 \text{ m}^2$

Cross-sectional area of venturimeter at throat is $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}\times(0.15)^2 = 0.01767\ \text{m}^2$

The discharge through the venturimeter is given by Eq. (14.19) as

$$Q = \frac{C_d A_1 A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

or
$$0.065 = \frac{C_d \times 0.07068 \times 0.01767 \times \sqrt{2\times 9.81\times\left(\frac{1.6}{1}-1\right)\times 1.2}}{\sqrt{(0.07068)^2 - (0.01767)^2}}$$

or
$$C_d = 0.95$$

Example 14.17 An oil of relative density 0.8 flows through a vertical pipe of diameter 24 cm. The flow is measured by a 24 cm × 12 venturimeter. The throat is 30 cm above the inlet section. A differential mercury U-tube manometer is connected to the inlet and throat. The manometer shows a deflection of 12 cm. Calculate the flow rate through the pipe. Take coefficient of discharge of the venturimeter as 0.98.

Solution

Diameter at inlet	$D_1 = 24$ cm $= 0.24$ m
Diameter at throat	$D_2 = 12$ cm $= 0.12$ m
Differential manometer reading	$\Delta h = 12$ cm $= 0.12$ m
Coefficient of discharge	$C_d = 0.98$

Cross-sectional area of pipe is $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}\times(0.24)^2 = 0.0452\ \text{m}^2$

Cross-sectional area of venturimeter at throat is $A_1 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}\times(0.12)^2 = 0.0113\ \text{m}^2$

The discharge through the venturimeter is given by Eq. (14.19)

$$Q = \frac{C_d A_1 A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

$$= \frac{0.98\times 0.0452\times 0.0113\times\sqrt{2\times 9.81\times\left(\frac{13.6}{0.8}-1\right)\times 0.12}}{\sqrt{(0.0452)^2-(0.0113)^2}}$$

$$= 0.07\ \text{m}^3/\text{s}$$

Example 14.18 A venturimeter with inlet and throat diameters are 150 mm and 75 mm respectively, is mounted in a vertical pipe carrying water, the flow being upwards. The throat section is 250 mm above the inlet of the venturimeter. The discharge of the venturimeter is 40 litres/s and the coefficient of discharge is 0.96. Calculate (a) the static pressure difference between inlet and throat, and (b) the difference in levels of mercury in a vertical U-tube manometer connected between these points.

Solution

The schematic of the venturimeter is shown in Fig. 14.10.

Diameter at inlet	$D_1 = 150$ mm $= 0.15$ m
Diameter at throat	$D_2 = 75$ mm $= 0.075$ m
Volume flow rate	$Q = 40$ litres/s $= 40\times 10^{-3}$ m³/s
Coefficient of discharge	$C_d = 0.96$

Cross-sectional area of pipe is $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}\times(0.15)^2 = 0.01767\ \text{m}^2$

Cross-sectional area of venturimeter at throat is $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}\times(0.075)^2 = 0.0044\ \text{m}^2$

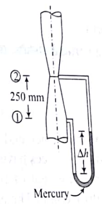

Figure 14.10

The discharge through the venturimeter is given by Eq. (14.19)

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]}$$

Substituting the values, we have

$$40\times 10^{-3} = \frac{0.96\times 0.01767\times 0.0044\times\sqrt{2\times 9.81\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]}}{\sqrt{(0.01767)^2-(0.0044)^2}}$$

or $$\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right] = 4.28$$

or $$\left[\frac{(P_1 - P_2)}{\rho g} - 0.25\right] = 4.28$$

or $$\frac{(P_1 - P_2)}{1000 \times 9.81} = 4.28 + 0.25 = 4.53 \text{ m}$$

or $$P_1 - P_2 = 4.53 \times 1000 \times 9.81 = 44439 \text{N/m}^2 = 44.439 \text{ kN/m}^2$$

Static pressure difference between inlet and throat section is 44.439 kN/m^2.

(b) From Eq. (14.17), we have

$$\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right] = \Delta h\left(\frac{\rho_m}{\rho_w} - 1\right)$$

or $$4.28 = \Delta h\left(\frac{13600}{1000} - 1\right)$$

or $$\Delta h = 0.3397 \text{ m} = 33.97 \text{ cm}$$

The difference in levels of mercury in a vertical U-tube manometer connected between these points is 33.97 cm.

14.6.2 Orifice Meter

The venturimeter is relatively complex to construct and hence expensive. Especially, for small pipelines, its cost seems prohibitive, so simpler devices have been invented, such as orificemeter. The orifice meter is a simple device for the measurement of flow. The orificemeter consists of a thin circular plate with sharp edge circular hole drilled in it. The orifice plate produces a constriction of the flow as shown in Fig. 14.11.

The streamlines continue to converge short distance downstream of the plane of the orifice, where the vena contract is formed, and then expand. Hence the minimum flow area is actually smaller than the area of the orifice. Applying energy equation between section 1 and 2(vena contracta), we have

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$V_2^2 - V_1^2 = 2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right] \tag{14.20}$$

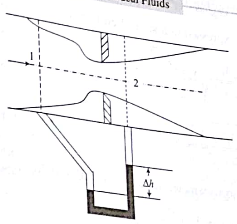

Figure 14.11 *Orificemeter*

From continuity equation, we have

$$A_1 V_1 = A_2 V_2$$

or $$V_1 = \frac{A_2}{A_1} V_2$$

Substituting the value of in Eq. (14.20), we have

$$V_2^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = 2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]$$

or $$V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}$$

Area of vena contracta (A_2) is less than that of orifice (A_o) and coefficient of contraction (C_c) is defined as

$$C_c = \frac{A_2}{A_o}$$

or $$V_2 = \frac{A_1}{\sqrt{A_1^2 - C_C^2 A_o^2}}\sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}$$

Volume flow rate can be expressed as

$$Q = A_2 V_2 = C_c A_o V_2$$

or $$Q = \frac{C_C A_o A_1}{\sqrt{A_1^2 - C_C^2 A_o^2}}\sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]} \tag{14.21}$$

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Equation (14.21) is also expressed in a simplified form as

$$Q=\frac{C_d A_o A_1}{\sqrt{A_1^2-A_o^2}}\sqrt{2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]} \tag{14.22}$$

where $C_d=\dfrac{C_C\sqrt{A_1^2-A_o^2}}{\sqrt{A_1^2-C_C^2A_o^2}}$

The usual value of C_d for orificemeter varies from 0.60 to 0.70.

$$\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)=\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h$$

From Eq. (14.17), the piezometric pressure difference can also be expressed in terms of the manometer reading as

$$\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)=\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h$$

Then, Eq. (14.22) can be written as

$$Q=\frac{C_d A_o A_1}{\sqrt{A_1^2-A_o^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h} \tag{14.23}$$

Example 14.19 An orificemeter with orifice diameter 12 cm is inserted in a pipe of 24 cm diameter through which an oil of density 850 kg/m^3 is flowing. A differential mercury U-tube manometer is connected to the two sides of the orificemeter to measure the pressure difference. The manometer shows a deflection of 20 cm. If the coefficient of discharge for the orificemeter is 0.6, find the discharge through the pipe.

Solution

Diameter of orifice	D_o = 12 cm = 0.12 m
Diameter of pipe	D_1 = 24 cm = 0.24 m
Coefficient of discharge	C_d = 0.6
Density of oil	ρ_w = 850 kg/m^3
Differential manometer reading	Δh = 20 cm = 0.20 m

Cross-sectional area of orifice is $A_o=\frac{\pi}{4}D_o^2=\frac{\pi}{4}\times(0.12)^2=0.0113\text{ m}^2$

Cross-sectional area of pipe is $A_1=\frac{\pi}{4}D_1^2=\frac{\pi}{4}\times(0.24)^2=0.0452\text{ m}^2$

The discharge through the orificemeter is given by Eq. (14.23) as

$$Q=\frac{C_d A_o A_1}{\sqrt{A_1^2-A_o^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h}$$

$$=\frac{0.6\times0.0113\times0.0452}{\sqrt{0.0452^2-0.0113^2}}\sqrt{2\times9.81\times\left(\frac{13600}{850}-1\right)\times0.2}$$

$$=\frac{0.0003}{0.0438}\times7.672=0.0525\text{ m}^3/\text{s}$$

Example 14.20 A horizontal orificemeter with orifice diameter 20 cm is inserted in a pipe of 30 cm diameter through which water is flowing. Coefficient of discharge for the orificemeter is 0.62. If the pressure gauges fitted upstream and downstream of the orificemeter show pressure and respectively, find the discharge through the pipe.

Solution

Diameter of orifice	D_o = 20 cm = 0.2 m
Diameter of pipe	D_1 = 30 cm = 0.3 m
Coefficient of discharge	C_d = 0.62
Density of water	ρ_w = 1000 kg/m^3
Pressure at upstream of the orificemeter	P_1 = 290 kN/m^2 = 290 × 10^3 N/m^2
Pressure at downstream of the orificemeter	P_2 = 195 kN/m^2 = 195 × 10^3 N/m^2

Cross-sectional area of orifice is $A_o=\frac{\pi}{4}D_o^2=\frac{\pi}{4}\times(0.20)^2=0.0314\text{ m}^2$

Cross-sectional area of pipe is $A_1=\frac{\pi}{4}D_1^2=\frac{\pi}{4}\times(0.3)^2=0.0707\text{ m}^2$

The discharge through the orificemeter is given by Eq. (14.22) as

$$Q=\frac{C_d A_o A_1}{\sqrt{A_1^2-A_o^2}}\sqrt{2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]}$$

$$=\frac{0.62\times0.0314\times0.0707}{\sqrt{0.0707^2-0.0314^2}}\sqrt{2\times9.81\left[\frac{290\times10^3-195\times10^3}{1000\times9.81}+0\right]}$$

$$=\frac{0.001376}{0.0633}\times13.784=0.2996\text{ m}^3/\text{s}$$

14.6.3 Pitot Tube

Pitot tube is commonly used to measure the velocity at a point in the flow field. The working principle of Pitot tube is based on the Bernoulli's equation. The simplest Pitot tube consists of a right angled transparent tube with one vertical leg projecting out of the flow and another leg pointing directly upstream in the flow as shown in Fig 14.12. At location 1, the flow is practically undisturbed by the presence of the tube. At location 2, the flow has been completely stopped by the tube which has been inserted i.e., velocity at 2 is zero.

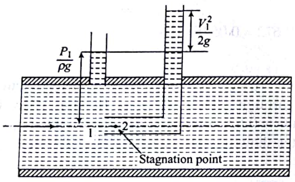

Figure 14.12 *Pitot tube*

Applying Bernoulli's equation between points 1 and 2 along a streamline, we have

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_{stag}}{\rho g}$$

or

$$\frac{V_1^2}{2g}=\frac{P_{stag}}{\rho g}-\frac{P_1}{\rho g}$$

or

$$\frac{V_1^2}{2g}=h_{stag}-h_{stat}$$

or

$$V_1=\sqrt{2g\left(h_{stag}-h_{stat}\right)} \tag{14.24}$$

With consideration of loss, the velocity is given by

$$V_1=C\sqrt{2g\left(h_{stag}-h_{stat}\right)} \tag{14.25}$$

where C is the coefficient of the tube.

Example 14.21 A Pitot tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 8 m and static pressure head is 6 m. Calculate the velocity of flow, assuming the coefficient of the tube equal to 0.98.

Solution

Stagnation pressure head h_{stag} = 8 m

Static pressure head h_{stat} = 6 m

Coefficient of tube C = 0.98

Velocity of flow is given by Eq. (14.25) as

$$V=C\sqrt{2g(h_{stag}-h_{stat})}$$

$$= 6.14 \text{ m/s}$$

Pitot Static Tube The main limitation associated with the Pitot tube as described above is the necessity of two piezometers. Connecting the pizometers to a manometer would simplify things. Pitot static tube consists of a slender double-tube aligned with the flow and connected to a differential manometer as shown in Fig.14.13. The inner tube is fully open to flow at the nose, and thus it measures the stagnation pressure at that location. The outer tube is sealed at the nose, but it has holes on the side of the outer wall and thus it measures the static pressure.

Using the theory of manometer, we have

$$P_X=P_2+\rho_w gx$$

$$P_Y=P_1+\rho_w g(x-\Delta h)+\rho_m g\Delta h$$

Equating the pressures of both the limb along the horizontal plane XY, we obtain

$$P_X=P_Y$$

$$P_2+\rho_w gx=P_1+\rho_w g(x-\Delta h)+\rho_m g\Delta h$$

From Eq. (14.11), we know that

$$P_2=P_1+\frac{1}{2}\rho_w V_1^2$$

Substituting the value of P_2 to the above equation, we have

$$P_1+\frac{1}{2}\rho_w V_1^2+\rho_w gx=P_1+\rho_w g(x-\Delta h)+\rho_m g\Delta h$$

or

$$\frac{1}{2}\rho_w V_1^2=(\rho_m-\rho_w)g\Delta h$$

or

$$V_1^2=2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h$$

or

$$V_1=\sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h} \tag{14.26}$$

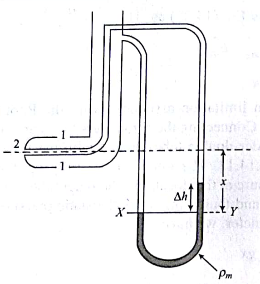

Figure 14.13 *Pitot static tube*

The Pitot static tube is a simple, inexpensive and highly reliable device since it has no moving parts. It also causes very small pressure drop and usually does not disturb the flow appreciably. However, it is important that it be properly aligned with the flow to avoid significant errors that may be caused by misalignment.

Example 14.22 An oil of specific gravity 0.85 flows through a horizontal pipe of diameter 20 cm. A Pitot static tube is inserted at the center of a pipe and its leads are filled with the same oil and attached to a U-tube containing water. The reading on the manometer is 15 cm. Determine the quantity of oil flowing through the pipe. The coe?cient of Pitot static tube is unity.

Solution

Diameter of pipe	$D = 20$ cm $= 0.2$ m
Specific gravity of oil	$Sw = 0.85$
Density of oil	$\rho_w = 0.85 \times 1000 = 850$ kg/m^3
Density of manometric fluid (water)	$\rho_m = 1000$ kg/m^3
Manometer reading	$\Delta h = 15$ cm $= 0.15$ m
Coefficient of tube	$C = 1$
Cross-sectional area of pipe is	$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}\times(0.2)^2 = 0.0314$ m^2

Since the coefficient of the tube is unity, the velocity of flow is given by Eq. (14.26) as

$$V = \sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h}$$

$$= \sqrt{2\times 9.81\left(\frac{1000}{850}-1\right)\times 0.15}$$

$$= 0.72 \text{ m/s}$$

The quantity of oil flowing through the pipe is

$$Q = AV = 0.0314 \times 0.72 = 0.0226 \text{ m}^3/\text{s}$$

SUMMARY

- Euler's equation of motion along a streamline is given by

$$\frac{dP}{\rho} + VdV + gdz = 0$$

which is valid for steady and inviscid flow.

- Bernoulli's equation along a streamline is given by

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = C$$

Here, $\frac{P}{\rho g}$ is the pressure head (flow energy per unit weight), $\frac{V^2}{2g}$ is the velocity head (kinetic energy per unit weight) and z is the potential head (potential energy per unit weight).

- Bernoulli equation can be stated as follows:
  During steady, inviscid flow of an incompressible fluid along a streamline, total mechanical energy at any point is constant. The total mechanical energy consists of flow energy, kinetic energy and potential energy.
- Bernoulli's equation deals with the law of conservation of mechanical energy. The equation holds true between any two points in the flow field provided that the flow field is irrotational and the flow is inviscid, steady and incompressible.
- The assumptions made in the derivation of Bernoulli's equation are
  - i) The flow is inviscid
  - ii) The flow is along a streamline
  - iii) The flow is steady
  - iv) The fluid is incompressible (density does not change with change in pressure)
- The discharge through a venturimeter is given by

$$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h}$$

where A_1 and A_2 are the cross-sectional areas of the venturimeter at its inlet and throat respectively, ρ_m and ρ_w are the density of the manometric fluid and the working fluid respectively, Δh is the difference in height of the manometric fluid, and C_d is the co-efficient of discharge of the venturimeter.

- The discharge through an orificemeter is given by

$$Q = \frac{C_d A_o A_1}{\sqrt{A_1^2 - A_o^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w}-1\right)\Delta h}$$

where Ao and A_1 are the cross-sectional areas of the orifice and pipe respectively, and are the density of the manometric fluid and the working fluid respectively, is the difference in height of the manometric fluid, and is the coefficient of discharge of the orificemeter.

- The velocity at a point in the flow filed measured by a Pitot tube given by

$$V = C\sqrt{2g\left(h_{stag} - h_{stat}\right)}$$

where h_{steg} is the stagnation pressure head and h_{stat} is the static pressure head and is the coefficient of the tube.

REVIEW QUESTIONS

14.1 Name the different forces present in a fluid flow. For the Euler's equation of motion, which forces are taken into consideration?

14.2 Derive Euler's equation of motion along a streamline.

14.3 Bernoulli's theorem is based on which principle? Give its statement. Name three devices where Bernoulli's equation is applied.

14.4 State the Bernoulli equation. List out the assumptions and limitations of Bernoulli's equation. How is it modified while applying in practice?

14.5 Define potential head, velocity head and datum head.

14.6 Draw a neat sketch of venturimeter. State why the length of divergent cone is made longer?

14.7 Starting with the Continuity and Bernoulli's equations, derive the following expression that can be used to measure flow rate with a venturimeter:

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}$$

Also show that when the pressure difference is measured using a manometer the following expression can be used:

$$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}$$

14.8 Derive an expression for the rate of flow through an inclined venturimeter and show that, if a U-tube gauge is used to measure the pressure difference, the gauge reading will be same for a given discharge irrespective of the inclination of the meter.

14.9 Why is the angle of the converging cone in a venturimeter steeper than the diffuser angle?

14.10 Is it possible that the flow in a converging section of a vertical venturimeter takes place in a direction from lower pressure to higher pressure, if (i) the flow is in the direction of gravity, and (ii) the flow is opposite to the direction of gravity? Give reasons. Neglect viscous effects.

14.11 Coefficient of discharge of venturimeter is always greater than orificemeter. Why?

14.12 Derive an expression for the volumetric flow rate of a fluid flowing through an orificemeter. Write down the advantages and disadvantages of using orificemeter over a venturimeter.

14.13 How does a venturimeter differ from an orificemeter?

14.14 Explain in brief the working principle of a Pitot-tube with the help of a neat sketch.

14.15 Differentiate between Pitot tube and Pitot static tube

NUMERICAL PROBLEMS

14.1 An oil of relative density 0.9 is flowing in a pipe of 10 cm diameter with an average velocity of 3 m/s. At a particular section 1 the pressure is measured to be 300 kN/m^2. If the section 1 is 6 m above the datum, determine total head of oil.

14.2 A vertical pipeline 10 cm diameter at the top tapers uniformly to 20 cm at bottom. The length of the pipeline is 2 m. If the discharge through the pipeline is 30 litres/s, find the difference in pressure. Neglect friction.

14.3 An oil of specific gravity 0.85 is flowing through a vertical pipe having diameter 30 cm and 15 cm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 200 kN/m^2 and at the upper end is 98 kN/m^2. If the rate of flow through pipe is 50 lit/s, find the difference in datum head. Neglect friction.

14.4 A 300 m long pipe has a slope of 1 in 100 and tapers from 1.2 m diameter at higher end to 0.6 m diameter at the lower end. It carries water at a rate of 100 litres/s. Find the average velocities at the higher and lower end. If the pressure at high end is 150 kN/m^2, find the pressure at the low end. Neglect the losses due to friction.

14.5 At a certain location X of a pipe line carrying water, the diameter is 70 cm, the pressure is 200 kN/m^2 and the average velocity is 6 m/s. At another section Y which is 3 m higher than X, the diameter is 40 cm and the pressure is 120 kN/m^2. What is the direction of the flow?

14.6 An oil with density 900 kg/m^2 and viscosity 0.18 Ns/m^2 flows through a pipeline. The diameter of the pipe changes from 20 cm at a position A to 50 cm at a position B which is 4 m above A. If the pressure at A and B are 300 kPa and 200 kPa respectively and the discharge is 0.3 m^3/s, determine the direction of flow and the loss of head.

14.7 In a smooth inclined pipe of uniform diameter 250 mm, a pressure of 50 kPa was observed at section 1 which was at elevation 10 m. At another section 2 at elevation 12 m, the pressure was 20 kPa and the velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in the pipe is water. The density of water is 998 kg/m^3.

14.8 A venturimeter of throat diameter 6 cm is fitted into a 12 cm diameter pipeline carrying water. Calculate the discharge in the pipeline when the reading on a U-tube mercury manometer connected to the upstream and throat sections shows a reading of 25 cm. Take coefficient of discharge of the venturimeter as 0.96.

14.9. A venturimeter is used for the measurement of discharge of water in a horizontal pipeline. The pipe diameter is 250 mm and the throat diameter is 125 mm. The difference in pressure between the inlet and the throat is 3 m of head of water. Calculate the discharge in the pipe. Take coefficient of discharge of the venturimeter as 0.98.

14.10 An oil of relative density 0.8 flows through a vertical pipe of diameter 24 cm. The flow is measured by a 240 mm × 120 mm venturimeter. The throat is 200 mm above the inlet section. A differential mercury U-tube manometer is connected to the inlet and throat. The manometer shows a deflection of 130 mm. Take coefficient of discharge of the venturimeter as 0.98. Calculate the flow rate through the pipe.

14.11 A vertical venturimeter measures the flow of oil of specific gravity 0.82 and has an entrance diameter of 125 mm and a throat diameter of 50 mm. There are pressure gauges at the entrance and at the throat, which is 50 mm above the entrance. If the coefficient of the meter is 0.97, find the flow in m^3/sec when the pressure difference is 3 kPa.

14.12 A horizontal venturimeter of 24 cm × 12 cm is used to measure the discharge of an oil of density 850 kg/m^3. A differential mercury manometer is connected to the inlet and throat for the purpose. If the difference of mercury level in between two limbs of manometer is 0.98. The discharge is 100 litres/s, find the coefficient of discharge of the venturimeter is 0.98.

14.13 A venturimeter of throat diameter 50 mm is fitted into a 125 mm diameter water pipeline. The coefficient of discharge is 0.96. Calculate the flow and velocity in the pipeline when the reading on a mercury-water differential U-tube manometer connected to the upstream and throat sections shows a reading of 200 mm. The piezometer head difference H is related to the gauge reading Δh by

$$H = \Delta h\left(\frac{\rho_m}{\rho_w} - 1\right)$$

where ρ_m and ρ_w are the densities of mercury and water respectively.

14.14 An orificemeter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure gauges fitted upstream and downstream of the orificemeter gives readings of N/cm^2 and 19.62 N/cm^2 respectively. Coefficient of discharge for the meter is given as 0.64. Find the discharge of the water through the pipe.

14.15 A pitot-static tube is used to measure air velocity. If a manometer connected to the instrument indicates a difference in pressure head between the tappings of 4 mm of water. Calculate the air velocity assuming the coefficient of the pitot tube to be unity. Density of air = 1.2 kg/m^3.

14.16 In a pitot - static tube stagnation pressure and static pressure are 6 kPa ad 1 kPa respectively. Calculate the velocity of flow of air assuming the coefficient of the tube equal to 0.98 and the water density as 1000 kg/m^3.

14.17 A pitot-static tube at the centre of a 10 cm diameter pipe is aligned in the direction of flow. When air flows through the pipe, the differential manometer across the pitot tube reads 6 mm of water gauge. It is known that for the air flow under consideration, the centre line velocity is 18 % higher than the average. Calculate the flow rate of air considering the coefficient of the pitot tube as unity and air density 1.2 kg/m^3.

MULTIPLE-CHOICE QUESTIONS

14.1 Bernoulli's theorem deals with the law of conservation of
(a) mass (b) momentum (c) energy (d) concentration

14.2 All the terms of energy in Bernoulli's equation $\left(\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}\right)$ have dimension of
(a) mass (b) energy (c) length (d) work

14.3 Bernoulli's equation relates
(a) various forms of mechanical energy
(b) various forces involved in fluid flow
(c) torque to change in angular momentum
(d) various forces with change in momentum

14.4 Each term of Bernoulli's equation stated in the form $\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$, has units of
(a) N (b) Nm/s (c) Nm/kg (d) Nm/N

14.5 In the most general form of Bernoulli's equation $\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$, each term represents
(a) energy per unit mass (b) energy per unit weight
(c) energy per unit volume (d) none of these

14.6 Euler's equation is written as
(a) $\frac{dP}{\rho} + V^2 dV + gdz = 0$ (b) $\frac{dP}{\rho} + VdV + gdz = 0$
(c) $\frac{dP}{\rho} + V^2 dV + gdz = 0$ (d) $\frac{dP}{\rho^2} + V^2 dV + gdz = 0$

14.7 A stagnation point is a point in fluid flow where
(a) total energy is zero (b) pressure is zero
(c) velocity of flow is zero (d) total energy is maximum.

14.8 Bernoulli equation can be derived from
(a) momentum balance only (b) energy balance only
(c) either momentum balance or energy balance (d) conservation of mass only

14.9 Pitot tube is used for the measurement of
(a) flow (b) velocity at a point
(c) discharge (d) pressure

14.10 The range for co-efficient of discharge for a venturimeter is
(a) 0.6 to 0.7 (b) 0.7 o 0.8 (c) 0.8 to 0.9 (d) 0.95 to 0.99

14.11 The range for co-efficient of discharge for a venturimeter is
(a) 0.6 to 0.7 (b) 0.7 o 0.8 (c) 0.8 to 0.9 (d) 0.95 to 0.99

14.12 A venturimeter is a device used to measure
(a) pressure in a fluid (b) velocity at a point
(c) flow rate (d) temperature of the fluid

14.13 A pitot tube measures in a pipe
(a) the average velocity of the fluid (b) the local velocity of flow
(c) the maximum velocity of flow only (d) the flow rate in the pipe

14.14 Which of the following instrument is used to measure flow by application of Bernoulli's theorem
(a) Venturimeter (b) Orificemeter (c) Pitot tube (d) all of the above

14.15 When is Bernoulli's equation applicable between any two points in a flow field?
(a) Steady, inviscid and irrotational flow of a compressible fluid
(b) Unsteady, inviscid and irrotational flow of a compressible fluid
(c) Steady, inviscid and rotational flow of an incompressible fluid
(d) Steady, inviscid and irrotational flow of an incompressible fluid

14.16 It is recommended that the diffuser angle should be kept less than 6° because
(a) pressure decreases in flow direction and flow separation may occur
(b) pressure decreases in flow direction and flow may become turbulent
(c) pressure increases in flow direction and flow separation may occur
(d) pressure increases in flow direction and flow may become turbulent

14.17 The difference of pressure head, H measured by a mercury-oil differential manometer is expressed as
(a) $H = \left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h$ (b) $H = \left(1 - \frac{\rho_m}{\rho_w}\right)\Delta h$
(c) $H = (\rho_m - \rho_w)\Delta h$ (d) $H = (\rho_w - \rho_m)\Delta h$
where Δh = manometer reading; ρ_m and ρ_w are the densities of mercury and oil respectively.

14.18 A pitot-static tube ($C = 1$) is used to measure air flow. With water in the differential manometer and a gauge difference of 75 mm, what is the value of air speed if the density of air is 1.16 kg/m^3?
(a) 1.21 m/s (b) 16.2 m/s (c) 5.6 m/s (d) 71.2 m/s