16. A diffraction limited telescope with a 7.6 cm aperture is aimed at a target 10 km away. Find the linear limit of resolution. Take the wavelength of light = 500 nm.

[*Ans.* 8 cm]

17. A parallel beam of light of wavelength $\lambda = 600$ nm is incident normally on a converging lens of diameter 1.2 cm and focal length 50 cm. Find the linear extent of the central disk of the diffraction pattern appearing in the focal plane.

[*Ans.* 0.061 mm]

18. For sodium light (589.0 nm and 589.6 nm) incident normally on a grating having 100 lines/mm, with width of ruling 2 cm, calculate in the first order

(i) angle of diffraction,

(ii) chromatic dispersion,

(iii) resolving power.

Are the *D*-lines resolved in the first order? (*C.U.* 1998)

[*Ans.* (i) $\theta = 3.4°$ (ii) $d\theta = 0.2'$ (iii) 2000; Required

$$\text{R.P.} = \frac{\lambda}{d\lambda} = \frac{589.3}{0.6} \approx 982. \text{ So the lines are resolved.]}$$

CHAPTER 13

POLARISATION OF LIGHT

13.1 INTRODUCTION :

The phenomena of interference and diffraction of light show that the light is a wave motion which may be longitudinal or transverse in nature. It is the phenomenon of polarisation which for its explanation requires that the light must be a transverse wave. In fact in the electromagnetic theory (Chap. 14) light is considered as a transverse electromagnetic wave consisting of vibrating electric and magnetic vectors at right angles to each other and also at right angles to the

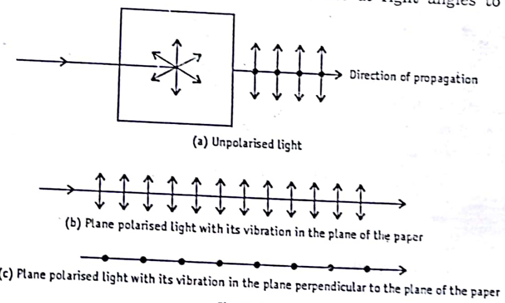

Fig. 13.1-1

direction of propagation of light. The electric vector is called light vector because it is responsible for the sensation of vision. In an ordinary beam of light with millions of waves the light vectors of component waves will remain in all possible directions on a plane drawn at right angles to the direction of propagation. This happens so due to random orientations of excited atoms or molecules in the source. Such an ordinary beam of light with the electric vectors arranged symmetrically about the direction of propagation is called an *unpolarised light.* We may assume all these vibrations to be resolved into two rectangular components of equal amplitude but having a relative phase difference which varies rapidly and randomly. Now, if by some means one of these rectangular vibrations is cut off we get vibrations of all the component waves confined in one definite direction. Such a light is said to be *plane* *polarised light* (also called *p-state light*). The pictorial representations of unpolarised and plane polarised lights are shown in Fig.13.1-1.

The electric vector of a plane polarised light is confined to a particular plane, known as the **plane of vibration**. This plane contains the $\vec{E}$-vector and the propagation vector $\vec{k}$. The plane perpendicular to the plane of vibration is known as **plane of polarisation.**

By the superposition of two plane polarised waves under suitable conditions the resultant light vector may be made to rotate in a plane perpendicular to the direction of propagation. If the magnitude of the resulting light vector remains constant the tip of the light vector appears to trace out a circle at a fixed space. Such a light is said to be **circularly polarised.** If, on looking towards the incoming light the resultant light vector appears to rotate clockwise, then the light is said to be **right circularly polarised light.** If the light vector rotates counter-clockwise then the light is said to be **left circularly polarised light.** On the other hand, if the magnitude of the resulting light vector varies periodically between a maximum and a minimum value then the tip of the light vector appears to trace out an elliptic path. Such a light is said to be **elliptically polarised.**

A mixture of polarised and unpolarised lights is called partially polarised light.

13.2 EXPERIMENT TO PROVE THAT LIGHT WAVES ARE TRANSVERSE :

When ordinary or common light from a source O is made to pass successively through two tourmaline crystals R_1 and R_2 with their axes parallel, light will be received by the eye E (Fig. 13.2-1).

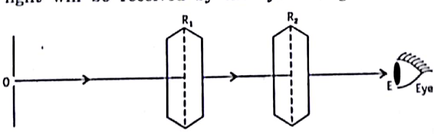

Fig. 13.2-1

If now the axis of R_2 be rotated with the incident ray as axis, the intensity of the light which is transmitted through R_2 will gradually decrease. No light will be received by the eye when the axis of R_2 becomes perpendicular to that of R_1.

If we assume that the waves of light are transverse, then the vibrations of the light falling normally on the surface of the first crystal R_1 will lie in all possible directions on its surface. These vibrations will then be resolved into two perpendicular directions, one parallel to the axis of R_1 and another perpendicular to it. The crystal R_1 will allow only those vibrations to pass through it which are parallel to its axis while the vibrations perpendicular to its axis are cut off. If the axis of R_2 be kept parallel to that of R_1, then the vibrations from R_1 will also pass through R_2 and the eye will receive light. If the axis of R_2 be kept perpendicular to that of R_1, then the vibrations from R_1 now become perpendicular to the axis of R_2. The crystal R_2, therefore, intercepts the vibrations from R_1 and the eye receives no light at all.

This experiment shows that the light waves are transverse; otherwise the light coming out of R_1 could never be extinguished by simply rotating the crystal R_2.

13.3 POLARISATION BY REFLECTION AND BREWSTER'S LAW :

If an ordinary beam of light is incident on a glass plate at a particular angle $i_p \approx 57°$ it is found that the reflected light becomes almost wholly a plane polarised light, the plane of vibration being perpendicular to the plane of incidence. For angles other than i_p the reflected light is partially polarised. This particular angle of incidence at which the reflected light is almost wholly polarised is called the **angle of polarisation** for the given surface. It depends on the nature of the reflecting medium and the wavelength of light.

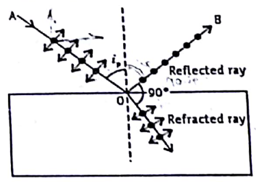

Fig. 13.3-1

Brewster discovered that at the polarising angle the reflected and the refracted rays are just 90° apart. Therefore, from Fig. 13.3-1, the angle of refaction is $90° - i_p$. Hence the refractive index of glass with respect to air is,

$$n = \frac{\sin i_p}{\sin(90° - i_p)} = \tan i_p \qquad \ldots(13.3\text{-}1)$$

Thus the tangent of the angle of polarisation is equal to the refractive index of the reflecting medium. This is known as **Brewster's law.**

Explanation :

The light vectors in the unpolarised incident light beam may be resolved into two rectangular components— one parallel to the plane of incidence (AOB) and the other perpendicular to the plane of incidence. At the polarising angle, since the reflected and refracted rays are 90° apart, at the point of incidence O the vibrations (shown by parallel arrow heads in Fig. 13.3-1) which are in the plane of incidence become parallel to the direction of reflection OB. As light is a transverse wave they cannot get reflected along OB. However, the other vibrations perpendicular to the plane of incidence (shown by dots in Fig. 13.3-1) can be reflected partly.

13.4 POLARISATION BY REFRACTION :

If a beam of ordinary light is incident on a glass plate at the polarising angle i_p for glass, the transmitted light will be partially plane polarised though the reflected light will be almost wholly plane polarised. If a number of parallel glass plates are used then at each reflection, the rays whose vibrations are perpendicular to the plane of incidence are reflected. The transmitted light contains vibrations in the plane of incidence as well as a reduced amount of vibrations perpendicular to

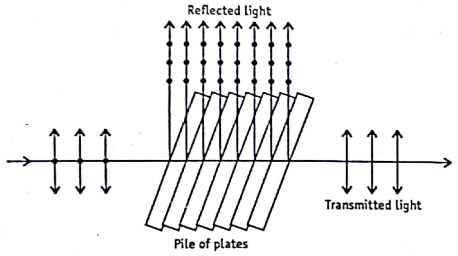

Fig. 13.4-1

the plane of incidence.Thus with a large number of parallel plates the transmitted light loses more and more of its perpendicular vibrations at each reflection and ultimately becomes wholly polarised whose vibrations remain in the plane of incidence. This arrangement is known as **piles of plates** which is shown in Fig.13.4-1.

13.5 GEOMETRY OF CALCITE CRYSTAL AND MEANING OF ITS OPTIC AXIS AND PRINCIPAL SECTION :

Iceland spar is the transparent variety of calcite($CaCO_3$) which crytallises in many forms. Like all other crystals, it is also bounded by the plane faces which are inclined to one another by fixed angles. When this crystal is struck, it breakes obliquely in three different planes forming a rhombohedral body like $ABCDA_1B_1C_1D_1$ as showm in Fig. 13.5-1. This operation of breaking a crystal (quartz is an exception) across definite planes is known as *cleavage* and the faces produced after cleavage are known as cleavage faces. The rhomb of iceland spar is bounded by six parallelogram faces, the angles of each which are 101°55′ and 78°5′.

(a) Optic axis :

(i) On examination of the rhomb $ABCDA_1B_1C_1D_1$ of an iceland spar, we shall find that each of its eight corners is formed by the meeting of three plane faces. The two diagonally opposite corners A and C_1 of the rhomb (Fig.13.5-1), are formed by the meeting of three plane faces, each of them having an obtuse angle (101°55′) at A and C_1. If a straight line be drawn within the crystal either from A or from C_1, equally inclined to the three meeting faces at A or C_1 then this straight line or any other straight line parallel to it will be called as the *optic axis* of the crystal. Thus optic axis of a crystal is a *direction* and *not a straight line*.

(ii) *Optic axis* of a crystal is defined as the *direction* through it, along which if a ray travels then there will be no *double refraction* of the ray and both the ordinary and extraordinary rays travel with *equal velocity* along this direction.

A doubly refracting crystal possessing only one optic axis is called a *uniaxial crystal*.

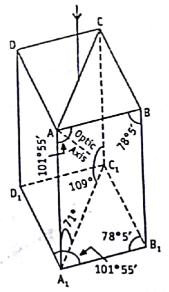

Fig. 13.5-1

(b) **Principal section :**

Principal section of a crystal is its section by a plane which passes through the optic axis of the crystal and is perpendicular to its two opposite refracting faces.

Since there are infinite number of lines parallel to the direction of the optic axis, there are also an infinite number of principal sections, of which one such section ACC_1A_1 is shown in Fig. 13.5-1.

13.6 DOUBLE REFRACTION :

Erasmus Bartholinus discovered (in 1669) that when an ordinary ray of unpolarised light is made incident on a crystal like calcite, quartz etc. it is refracted into two rays. One of these rays obeys the laws of

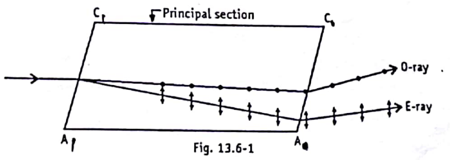

Fig. 13.6-1

refraction of light and is called the **ordinary ray** (**O-ray**) while the other ray does not, in general, obey those laws and is called the extraordinary ray (E-ray) (Fig. 13.6-1). Both of these E- and O-rays are plane polarised whose vibrations are along and at right angles to the principal section. This phenomenon in which a single incident ray is refracted into two rays is called *double refraction* or *birefringence* and the crystals which exhibit the phenomenon are called *doubly refracting crystals* or *birefringent*. This phenomenon is exhibited by other crystals excepting those which have cubic arrangements of their atoms. The state of polarisation of the refracted rays can be determined by using a tourmaline crystal.

13.7 NICOL PRISM :

Nicol prism is an optical device designed from calcite and is used for the production and analysis of plane polarised light. A beam of ordinary light entering the calcite crystal breaks up into E-rays and O-rays by double refraction. The O-ray is cut off by total internal refraction while the E-ray is allowed to pass through.

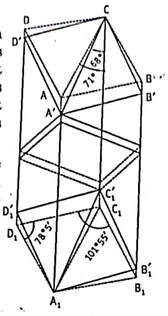

Fig. 13.7-1

To construct a Nicol prism a calcite rhombohedron of length three times its breadth is produced from natural crystal by cleavage. The end faces are then cut artificially so that the acute angles of the principal section is reduced from 71° to 68° (Fig. 13.7-1).

The resulting crystal is then cut into two portions by a plane perpendicular to both the principal section and the end faces of the rhomb. The cut surfaces are highly polished and cemented together by a layer of Canada-balsam. The sides of the prism are coated with lamp-black and are kept covered by a brass tube. The two end

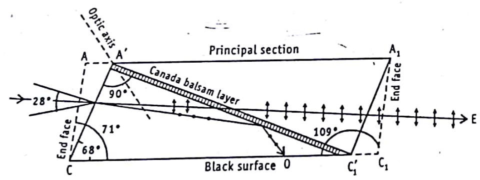

Fig. 13.7-2

faces are kept open so that the light may be incident on one end face and may emerge out of the other end face.

A section of the resulting crystal parallel to the principal section is shown in Fig 13.7-2.

Action of Nicol as polariser :

An incident ray falling on one of the end faces of the Nicol and moving in a direction nearly parallel to its length is divided into ordinary and extraordinary rays whose vibrations are respectively perpendicular and parallel to the principal section of the Nicol. Canada-balsam is optically rarer than calcite for O-ray. The geometry of Nicol prism is such that the O-ray is incident on balsam layer at an angle greater than the critical angle and is, therefore, totally reflected. This totally reflected light pass out through the side of the prism and is absorbed by the lamp-black layer on the sides of the prism.

For extraordinary ray, Canada-balsam is optically denser than calcite and this ray is freely transmitted and emerges out of the opposite end face. Thus we get a plane polarised light whose vibrations are parallel to the principal section.

For total reflection of the ordinary ray, the angle of incidence at the balsam layer must be greater than the critical angle and this will happen when the angle of incidence of the ray on the end face does not exceed 14°. Again the refractive index of calcite for E-ray depends upon the direction of propagation through the crystal. Hence for E-ray, the balsam layer will behave as an optically less dense medium than calcite, when the angle of incidence at the end face exceeds a certain limit. Thus to avoid the transmission of O-ray and total reflection of E-ray, the angle between the extreme rays of the incoming beam is limited to about 28°.

Action of Nicol as analyser :

Nicol prism can also be employed as analyser of polarised light.

Suppose a plane polarised light be made incident on one face of a Nicol so that the direction of its vibration of amplitude a, makes an angle θ with the principal section of the Nicol. This vibration of amplitude a will be resolved into two vibrations of amplitudes $a \cos \theta$ and $a \sin \theta$ lying respectively parallel and perpendicular to the principal section of the Nicol. The component $a \cos \theta$, which is parallel to the principal section, will be freely transmitted as E-ray, while the component $a \sin \theta$ which is perpendicular to the principal section of Nicol, will behave as the O-ray and will be cut off by total reflection from the balsam layer.

If now the Nicol is rotated with the incident ray as axis, the value of θ will change. When θ is zero (i.e., when the principal section of Nicol is parallel to the vibration of incident light) the amplitude of transmitted component will be a, which is maximum, and the field would be brightest. But when $\theta = 90°$ (i.e., when the principal section of Nicol is perpendicular to the vibration of incident light) the amplitude of the transmitted component is zero and the field would be dark.

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Thus we conclude that if the incident light be plane polarised, the intensity of light transmitted through the Nicol would be greatest in its one position while that would be zero in another position which is at right angle to the former. At this position of Nicol, in which the field is dark, the plane of vibration of the incident plane polarised light would be perpendicular to the principal section of the Nicol.

If the incident light be unpolarised, then for any position of the Nicol, the vibrations of the incident light can be resolved parallel and perpendicular to its principal section. The parallel component will pass through the Nicol, while the perpendicular component will be refused transmission and the field of the Nicol will always remain equally bright.

Parallel and crossed Nicols :

Nicol prism can be used both as polariser and analyser of light beam. Two Nicols are placed coaxially as shown in Fig. 13.7-3. The first Nicol P producing plane polarised light is called the polariser, while the second Nicol A which analyses the incident light is called analyser. When the principal sections of the two Nicols are parallel, the vibrations of E-ray coming out of the polariser are in the principal section of both the

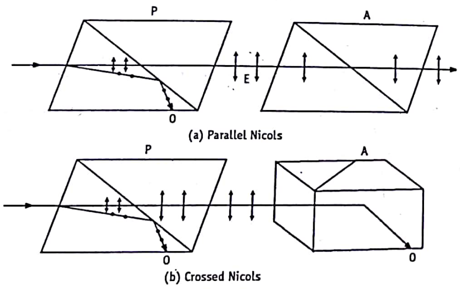

Fig. 13.7-3

polariser and the analyser. As a result it transmits light freely as E-ray through the analyser A. Such arrangement is called **parallel Nicols.**

If the analyser is gradually rotated the intensity of the transmitted light decreases and when their principal sections become mutually perpendicular, no light is transmitted. The vibrations of the E-ray which are in the principal section of polariser becomes perpendicular to principal section of the analyser. The E-ray from polariser becomes O-ray in the analyser and is totally reflected. Nicols in such arrangement are said to be **crossed Nicols.**

13.8 MALUS'S LAW :

Malus experimentally obtain a relation which shows how the intensity of light transmitted by the analyser varies with the angle that its plane of transmission makes with that of the polariser. Suppose that the angle between the two planes of transmission be θ at any instant, as shown in Fig. 13.8-1. The light vector $AP = E_0$ in the plane polarised light emerging from the polariser may be resolved into two components $AE = E_0 \cos\theta$ and $AO = E_0 \sin\theta$ respectively, along and perpendicular to the plane of transmission of the analyser. The perpendicular component $E_0 \sin\theta$ is eliminated while the parallel component is freely transmitted through the analyser. Hence the intensity of light that emerges from the analyser is given by,

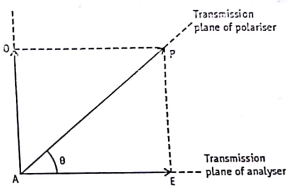

Fig. 13.8-1

$$I = E_0^2 \cos^2\theta = I_0 \cos^2\theta \qquad \text{...(13.8-1)}$$

where I_0 is the intensity of the polarised light incident on the analyser. This is, of course, one-half of the intensity of the unpolarised light incident on the polariser. Eq. (13.8-1) shows that the transmitted intensity varies as the square of the cosine of the angle between the planes of transmission of polariser and analyser. This is known as **Malus's law.**

13.9 POLARISATION BY DICHROIC CRYSTALS :

There are certain doubly refracting crystals (like tourmaline) which not only produce O-rays and E-rays but also absorb one of these two

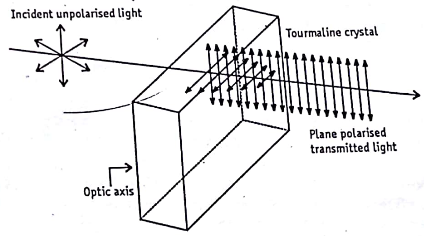

Fig 13.9-1

Thus we conclude that if the incident light be plane polarised, the intensity of light transmitted through the Nicol would be greatest in its one position while that would be zero in another position which is at right angle to the former. At this position of Nicol, in which the field is dark, the plane of vibration of the incident plane polarised light would be perpendicular to the principal section of the Nicol.

If the incident light be unpolarised, then for any position of the Nicol, the vibrations of the incident light can be resolved parallel and perpendicular to its principal section. The parallel component will pass through the Nicol, while the perpendicular component will be refused transmission and the field of the Nicol will always remain equally bright.

Parallel and crossed Nicols :

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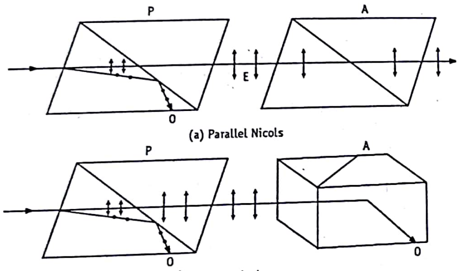

(a) Parallel Nicols

(b) Crossed Nicols

Fig. 13.7-3

polariser and the analyser. As a result it transmits light freely as E-ray through the analyser A. Such arrangement is called **parallel Nicols**.

If the analyser is gradually rotated the intensity of the transmitted light decreases and when their principal sections become mutually perpendicular, no light is transmitted. The vibrations of the E-ray which are in the principal section of polariser becomes perpendicular to principal section of the analyser. The E-ray from polariser becomes O-ray in the analyser and is totally reflected. Nicols in such arrangement are said to be **crossed Nicols**.

13.8 MALUS'S LAW :

Malus experimentally obtain a relation which shows how the intensity of light transmitted by the analyser varies with the angle that its plane of transmission makes with that of the polariser. Suppose that the angle between the two planes of transmission be θ at any instant, as shown in Fig. 13.8-1. The light vector $AP = E_0$ in the plane polarised light emerging from the polariser may be resolved into two components $AE = E_0 \cos\theta$ and $AO = E_0 \sin\theta$ respectively, along and perpendicular to the plane of transmission of the analyser. The perpendicular component $E_0 \sin\theta$ is eliminated while the parallel component is freely transmitted through the analyser. Hence the intensity of light that emerges from the analyser is given by,

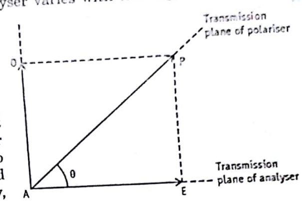

Fig. 13.8-1

$$I = E_0^2 \cos^2\theta = I_0 \cos^2\theta \qquad \text{...(13.8-1)}$$

where I_0 is the intensity of the polarised light incident on the analyser. This is, of course, one-half of the intensity of the unpolarised light incident on the polariser. Eq. (13.8-1) shows that the transmitted intensity varies as the square of the cosine of the angle between the planes of transmission of polariser and analyser. This is known as **Malus's law.**

13.9 POLARISATION BY DICHROIC CRYSTALS :

There are certain doubly refracting crystals (like tourmaline) which not only produce O-rays and E-rays but also absorb one of these two

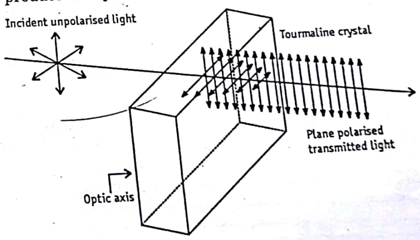

Fig 13.9-1

rays much more strongly than the other. Thus if an unpolarised light is passed through a crystal like tourmaline of proper thickness one of the components is completely absorbed and the other is transmitted in appreciable amount (Fig. 13.9-1). The emergent beam will be linearly polarised. This phenomenon is known as *dichroism* and the crystals showing this property is called *dichroic crystals*.

A thin crystal of tourmaline with its faces cut parallel to optic axis may be used as polariser of light. However, its use as a polariser is limited as the plane polarised light is coloured due to unequal absorption of light of various wavelengths.

13.10 POLAROID :

A polaroid is a thin transparent film which can porduce plane polarised light on transmission through it. It usually consists of a thin film of nitrocellulose packed with tiny dichroic crystals (herapathite) with their optic axes all parallel. Because of certain advantages it has largely replaced Nicol prism for producing and analysing polarised light in the laboratory.

At first a paste of small crystals of herapathite (Iodo-sulphate of quinine) is prepared in nitrocellulose. This paste is then squeezed out through a fine slit. As a result only those crystals pass whose axes are parallel to the length of the slit. In this way a large fine sheet is produced which contains millions of tiny crystals with their optic axes all parallel. This is mounted between two thin sheets of glass forming what is known as polaroid.

In more recent developments a better method was found in which individual molecules, rather than micro-crystals were employed to construct a polaroid known as ***H*-polaroid.** In this process a sheet of polyvinyl alcohol is heated and stretched from three to eight times its original length. By this operation the molecules are oriented in the direction of stress and when the sheet is impregnated with iodine it exhibits strong dichroism. This polaroid is colourless and transmits more light than herapathite polaroid.

Land and Rogers later discovered that when the stretched polyvinyl alcohol film is heated with a catalyst like hydrochloric acid it slightly darkens but exhibits strong dichroism. As the film does not contain any dyestuff, it is very stable and is not bleached by strong sunlight. It is called ***K*-polaroid** and is extensively employed in automobile head lights and wind screen.

Polaroids have wide applications in everyday life e.g., in sun glasses, automobile headlights, wind screen, windows of railway trains, aeroplanes, in stereoscopic motion pictures etc.

13.11 HUYGENS' THEORY OF DOUBLE REFRACTION AND THE NATURE OF WAVE SURFACES IN UNIAXIAL CRYSTAL :

Huygens with the help of his theory of secondary wavelets gave a suitable explanation of the phenomenon of double refraction in a uniaxial crystal. According to him every point of a doubly refracting crystal disturbed by the incident light becomes the source of two secondary wavelets— spherical for *O*-ray and ellipsoid of revolution about the optic axis for the *E*-ray. The two wave surfaces touch each other along the optic axis through the point of origin of the wavelets. This theory can account for the following facts regarding double refraction :

(i) There are two refracted rays— *O*- and *E*-rays.

(ii) *O*-ray travels with equal velocity in all directions and obeys the ordinary laws of refraction.

(iii) *E*-ray does not obey the laws of refraction.

(iv) Optical properties of uniaxial crystals are perfectly symmetrical about the optic axis.

(v) Along the optic axis *O*- and *E*-rays travel with equal velocity and there is no double refraction.

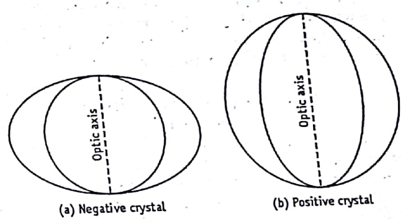

Fig. 13.11-1

The nature of the secondary wave surfaces is shown in Fig. 13.11-1.

Crystals like calcite in which *O*-ray travels with smaller velocity than *E*-ray (i.e., $n_e < n_o$) in the direction normal to the optic axis are called *negative crystals*. In this case the spherical wave surface is considered within the ellipsoid of revolution. Crystals like quartz in which *O*-ray travels with greater velocity than *E*-ray (i.e., $n_e > n_o$) in the direction normal to the optic axis are called *positive crystals* and here the ellipsoid of revolution is considered within the sphere.

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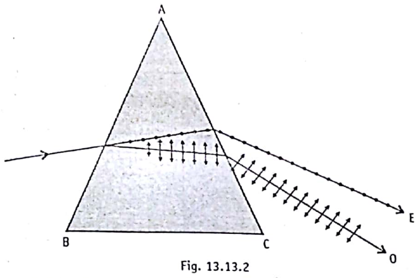


Fig. 13.13.2

angles to the optic axis by using the formula,

$$n = \frac{\sin(A + D_m)/2}{\sin A/2}.$$

13.14 METHODS OF SEPARATING ORDINARY AND EXTRAORDINARY REFRACTED RAYS BY (A) ROCHON'S PRISM AND (B) WOLLASTON'S PRISM :

Sometimes it becomes necessary to have E-rays and O-rays widely separated from each other. For this purpose Rochon's prism and Wollaston's prism are used.

(a) By Rochon's prism :

It consists of two right angled prisms ABD and ACD of iceland spar which are cemented together at their hypotenuse AD to form a rectangular slab. The optic axis in ABD is perpendicular to the face AB and also to the edge at A, while that in ACD is parallel to the face CD and also to the edge at D (Fig. 13.14-1).

An incident ray PQ falling normally on the face AB is proceeding along the axis of the prism ABD and hence both the ordinary and extraordinary rays will have the same velocity and same refractive index (n_o). Refraction will occur at the interface AD. As the refractive indices of both the prisms for ordinary ray are same ($= n_o$) the ordinary ray will not be deviated at the interface AD but will go straight along $PQRO$. As calcite is a negative crystal its refractive index for ordinary light (n_o) is greater than that for extraordinary light (n_e). Hence extraordinary ray will bend away from the normal at the point of incidence R on the interface AD. This extraordinary ray will be further refracted at the point S in going from prism ACD to air and thus it gets separated from O-ray.

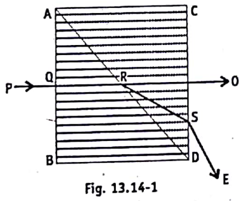


Fig. 13.14-1

(b) By Wollaston's prism :

This also consists of two right-angled prisms of iceland spar, cemented together at their hypotenuse to form a rectangular slab. The optic axis of the prism ABD is parallel to the face AB but perpendicular to the edge A, while the optic axes of the prism ACD is parallel to the face CD and also to the edge D (Fig. 13.14-2). As the edges at A and D are parallel we infer that the optic axis in the two prisms are at right angles to each other. Hence the ordinary and extraordinary rays in the prism ABD, on entering the prism ACD, will be extraordinary and ordinary rays respectively.

A ray PQ, incident on the face AB normally, will be broken up into ordinary and extraordinary rays, which will travel in the same straight line (for incidence is normal) with unequal speed. The ordinary ray in ABD will be extraordinary ray in ACD and hence this ray will be deviated away from normal (for $n_o > n_e$). Again the extraordinary ray in ABD will enter the prism ACD as ordinary ray and hence it will be deviated towards the normal. Thus the two emerging ordinary and extraordinary rays will be separated from each other by an angle which will be double the angle of separation in Rochon's prism.

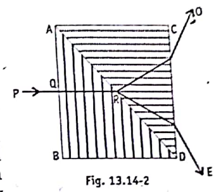


Fig. 13.14-2

13.15 SUPERPOSITION OF TWO PLANE POLARISED WAVES VIBRATING IN TWO MUTUALLY PERPENDICULAR PLANES :

Let a plane polarised monochromatic light be incident normally on a calcite plate cut with faces parallel to optic axis (Fig. 13.15-1). Suppose the electric vector makes an angle θ with the optic axis. On entering the crystal the amplitude 'A' of the incident wave may be assumed to be resolved into two components one along the optic axis forming E-wave of amplitude $a = A \cos\theta$ and the other normal to the optic axis forming O-wave of amplitude $b = A \sin\theta$. If the incident wave is represented by $E = A\cos(\omega t - kz)$ then the component waves may be represented as

$$x = a\cos(\omega t - kz) \quad \text{and} \quad y = b\cos(\omega t - kz)$$

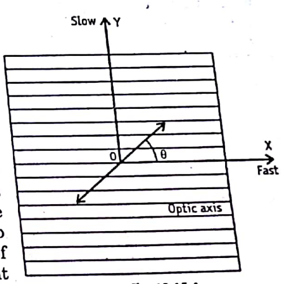


Fig. 13.15-1

Now the E- and O-rays travel through the crystal with different velocities ($v_e > v_o$ in calcite) and therefore, they emerge from the crystal with certain relative phase difference, say, δ. So the emergent rays may be represented as

$$x = a\cos(\omega t - kz + \delta) \qquad ...(13.15\text{-}1)$$

$$y = b\cos(\omega t - kz) \qquad ...(13.15\text{-}2)$$

From Eq. (13.15-1)

$$\frac{x}{a} = \cos(\omega t - kz).\cos\delta - \sin(\omega t - kz)\sin\delta$$

From Eq. (13.15-2), $\cos(\omega t - kz) = y/b$. Hence,

$$\frac{x}{a} = \frac{y}{b}\cdot\cos\delta - \sqrt{1-\frac{y^2}{b^2}}\cdot\sin\delta$$

or, $$\left[\frac{x}{a}-\frac{y}{b}\cos\delta\right]^2 = \left(1-\frac{y^2}{b^2}\right)\sin^2\delta$$

or, $$\frac{x^2}{a^2}-\frac{2xy.\cos\delta}{ab}+\frac{y^2}{b^2} = \sin^2\delta \qquad ...(13.15\text{-}3)$$

This equation, in general, represents an ellipse confined within a rectangle of sides $2a$ and $2b$. The exact nature of the resultant emerging light, however, depends upon the value of δ.

Special cases :

(i) If $\delta = 2m\pi$, $m = 0, 1, 2, 3...$, Eq. (13.15-3) simplifies to

$$y = \frac{b}{a}x = \tan\theta.x \qquad ...(13.15\text{-}4)$$

In this case the emergent light is plane polarised, the plane of vibration making an angle θ with the x-axis.

(ii) If $\delta = (2m+1)\pi$, $m = 0, 1, 2, 3, ...$ then Eq. (13.15-3) simplifies to

$$y = -\frac{b}{a}x = -\tan\theta.x \qquad ...(13.15\text{-}4)$$

The emergent light is again plane polarised with the direction of vibration making an angle $-\theta$ with the x-axis.

(iii) If $\delta = (2m+1)\pi/2$, $m = 0, 1, 2, 3, ...$, Eq. (13.15-3) reduces to

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ...(13.15\text{-}5)$$

In this case the emergent light is elliptically polarised.

If $\delta = \pi/2, 5\pi/2, 7\pi/2$ ... etc. then from Eqs. (13.15-1) and (13.15-2) we can write

$$x = -a\sin\omega t,$$
$$y = b\cos\omega t \qquad ...(13.15\text{-}6)$$

where we assume $z = 0$ as the plane of observation.

Eqs. (13.15-6) describe an ellipse in counter-clockwise direction (left-handed) with respect to an observer towards whom the wave is travelling. (Note that the vector $\vec{r} = \hat{i}x + \hat{j}y$ rotates counter-clockwise with increasing time).

If $\delta = 3\pi/2, 7\pi/2,...$ etc. then from Eqs. (13.15-1) and (13.15-2) we get at $z = 0$,

$$x = a\sin\omega t,$$
$$y = b\cos\omega t \qquad ...(13.15\text{-}6)$$

It describes an ellipse in clockwise direction (right-handed). Here the vector $\vec{r} = \hat{i}x + \hat{j}y$ rotates clockwise with increasing time.

(iv) If $\delta = (2m+1)\pi/2$ and $\theta = 45°$ then $a = b$ and the ellipse reduces to a circle,

$$x^2 + y^2 = a^2$$

The emergent light is then said to be circularly polarised.

As above, for $\delta = \pi/2, 5\pi/2...$ etc. we get left circularly polarised light and for $\delta = 3\pi/2, 7\pi/2,...$ etc. we get right circularly polarised light.

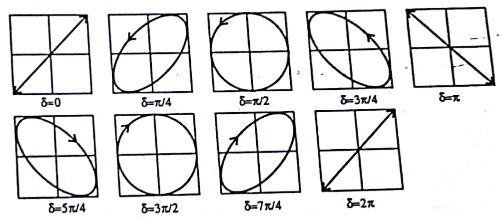


Fig. 13.15-2

The states of polarisation corresponding to different values of δ are shown in Fig. 13.15-2.

u the

13.16 RETARDATION PLATES :

A plate made of dubly refracting crystal with its refracting faces cut parallel to the optic axis and employed to introduce a given phase difference between E-ray and O-ray is called a *retardation plate.* If d be the thickness of the plate, n_o and n_e be the refractive indices for O-ray and E-ray then the path difference introduced by the plate is given by,

$$(n_o \sim n_e).d \qquad \ldots(13.16\text{-}1)$$

Corresponding phase difference is given by

$$\delta = \frac{2\pi}{\lambda}(n_o \sim n_e)d \qquad \ldots(13.16\text{-}2)$$

If the thickness 'd'of the plate is such that a path difference of $\lambda/4$ or a phase difference of $\pi/2$ is introduced between O-ray and E-ray then the plate is called a **quarter-wave plate.** Thus the thickness of a quarter wave plate is given by

$$(n_o \sim n_e)\, d = \lambda/4; \text{ or, } d = \frac{\lambda}{4(n_o \sim n_e)} \qquad \ldots(13.16\text{-}3)$$

On the other hand, if the thickness d of the plate is such that a path difference of $\lambda/2$ or a phase difference of π is introduced between O- and E-waves then the plate is called a **half-wave plate.** The thickness of a half-wave plate is given by,

$$(n_o \sim n_e).d = \lambda/2; \text{ or, } d = \frac{\lambda}{2(n_o \sim n_e)} \qquad \ldots(13.16\text{-}4)$$

13.17 PRODUCTION OF ELLIPTICALLY AND CIRCULARLY POLARISED LIGHT :

Elliptically polarised light :

An elliptically polarised light can be produced by superposing two mutually perpendicular coherent linear vibrations of unequal amplitudes but differing in phase by $\pi/2$. For example, the linear optical vibrations (at $z = 0$),

$$x = a\cos(\omega t + \pi/2),\ y = b\cos\omega t$$

yield an elliptic vibration given by,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Two such linear vibrations can be obtained by allowing a beam of plane polarised light to be incident normally on a quarter-wave plate with the direction of vibration making an angle θ other than 45°(about 30°) with the optic axis of the plate. The incident light of amplitude

'A' can be resolved into two components, one along the optic axis forming E-wave of amplitude $a = A\cos\theta$ and the other normal to the optic axis forming O-wave of amplitude $b = A\sin\theta$. Let these two components (at $z = 0$) be represented by,

$$x = a\cos\omega t \text{ and } y = b\cos\omega t.$$

On passing through the $\lambda/4$ plate a relative phase difference of $\pi/2$ will be introduced between these two waves. As a result the emergent light will be elliptically polarised as discussed above.

Circularly polarised light :

A circularly polarised light can be produced by superposing two mutually perpendicular coherent linear vibrations of equal amplitudes but differing in phase by $\pi/2$. For example, the linear optical vibrations (at $z = 0$),

$$x = a\cos(\omega t + \pi/2) \text{ and } y = a\cos\omega t$$

yield a circular optical vibration $x^2 + y^2 = a^2$

Two such linear vibrations can be obtained by allowing a beam of plane polarised light to be incident normally on a $\lambda/4$ plate with the direction of vibration making an angle $\theta = 45°$ to the optic axis of the plate. The incident light of amplitude 'A' can be resolved into two components, one along the optic axis forming E-wave of amplitude $A\cos 45° = A/\sqrt{2}$ and the other normal to the optic axis forming O-wave of amplitude $A\sin 45° = A/\sqrt{2}$. Let these two components (at $z = 0$) be represented as,

$$x = \frac{A}{\sqrt{2}}\cos\omega t \text{ and } y = \frac{A}{\sqrt{2}}\cos\omega t$$

Now on passing through the $\lambda/4$ plate a relative phase difference of $\pi/2$ will be introduced between these two waves. As a result the emergent light will be elliptically polarised as discussed above.

13.18 DETECTION AND ANALYSIS OF POLARISED LIGHT BY USING NICOL PRISM (OR POLAROID) AND $\lambda/4$ PLATE :

Detection :

If an ordinary light or a circularly polarised light or a mixture of these two is incident on a Nicol (or polaroid) and the Nicol (or a polaroid) is rotated slowly through 360° no variation in intensity is obtained. If a plane polarised light is incident on a Nicol and it is rotated slowly through 360° two maxima and two complete extinctions are obtained. If an elliptically polarised light or a mixture of unpolarised and elliptically polarised light or a mixture of unpolarised and linearly polarised light is incident on a Nicol prism and it is rotated slowly through 360° two maxima and two minima (not complete extinctions) are obtained.

A pictorial representation of the above detection scheme is shown in Fig. 13.18-1.

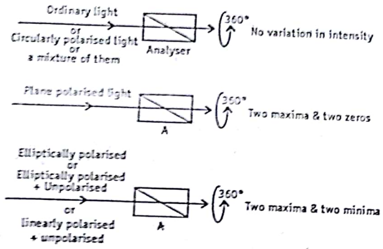


Fig 13.18-1

Distinction between circularly polarised light, unpolarised light and a mixture of these two :

The incident light is first passed through a $\lambda/4$ plate and then through a Nicol prism. If no variation in intensity is observed on rotating the Nicol the incident light would be unpolarised. The unpolarised light remains unpolarised on passing through the $\lambda/4$ plate and hence no variation in intensity is observed through the Nicol prism. If two maxima and two complete extinctions are obtained the incident light

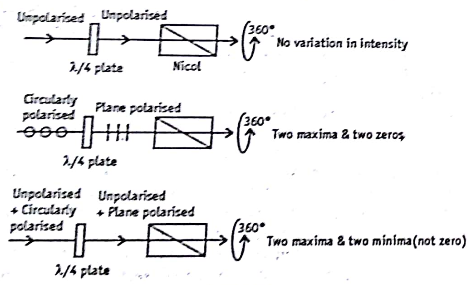


Fig 13.18.2

must be circularly polarised. The circularly polarised light on passing through the $\lambda/4$ plate becomes plane polarised which can be extinguished by the analysing Nicol at two positions during one complete rotation.

If two maxima and two minima (not zero) are obtained during one complete rotation of the Nicol, then the incident light must be a mixture of unpolarised and circularly polarised light. These processes of analysis are explained pictorially in Fig. 13.18-2.

Distinction between elliptically polarised light, mixture of elliptically polarised and unpolarised light and mixture of plane and unpolarised light :

The light in question is first made to fall on a $\lambda/4$ plate and then passed through an analyser. The $\lambda/4$ plate is now rotated in small steps of 1° or so and for each setting of the $\lambda/4$ plate the analyser is given a complete rotation. Then any one of the following observations will be obtained :

(i) If for one setting of the $\lambda/4$ plate zero intensity is obtained for one position of the analyser with its principal section inclined to the optic axis of the plate. Then we conclude that the incident light is elliptically polarised.

The reason for our conclusion is as follows : When the axes of elliptic vibration of incident light are parallel and perpendicular to the axis of the $\lambda/4$ plate, the elliptic vibration will be resolved into two rectangular components of unequal amplitudes having a phase difference of $\pi/2$. On passing through the $\lambda/4$ plate an additional phase difference of $\pi/2$ will be introduced between them. As a result the two component vibrations combine to form a plane polarised light which can be completely extinguished when the principal section of the analyser is perpendicular to the plane of vibration.

(ii) If for one complete rotation of the analyser two position of minimum intensity (not zero) are obtained with the principal section of the analyser inclined to the axis of the $\lambda/4$ plate. Then we conclude that the incident light is a mixture of unpolarised and elliptically polarised light.

The reason for our conclusion is as follows : When the axes of elliptic vibration are parallel and perpendicular to the optic axis of $\lambda/4$ plate, then on passing through the $\lambda/4$ plate, the elliptically polarised light changes into a plane polarised light whose direction of vibration is inclined to the optic axis of the $\lambda/4$ plate. The unpolarised light remains unchanged on transmission through the $\lambda/4$ plate. Therefore, when the principal section of the analyser is perpendicular to the plane of vibration of the plane polarised part of the emergent light, the intensity becomes minimum. This time the principal section of the analyser is inclined to the optic axis of the $\lambda/4$ plate.

(iii) If in the setting of minimum intensity (not zero) the principal section of the analyser is found to be either parallel or perpendicular to the optic axis of the $\lambda/4$ plate, then our conclusion is that the incident light is a mixture of unpolarised and plane polarised light.

The reason for our conclusion is as follows : The unpolarised part is transmitted without any modification. The plane polarised part is converted into elliptically polarised light on passing through the $\lambda/4$

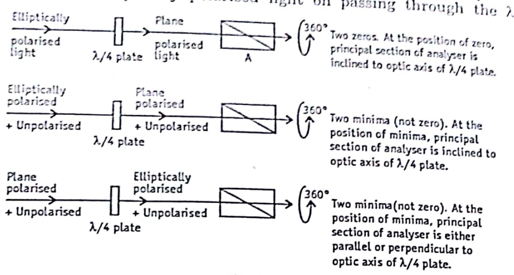


Fig 13.18-3

plate. The principal axes of the elliptic vibration are parallel and perpendicular to the optic axis of the $\lambda/4$ plate. Now minimum intensity is obtained when the principal section of the analyser is parallel to the minor axis of the ellipse. Hence the principal section of the analyser must be either parallel or perpendicular to the optic axis of the $\lambda/4$ plate. The above processes of analysis are explained pictorially in Fig. 13.18-3.

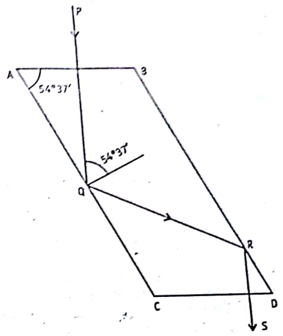
13.19 FRESNEL RHOMB AND ITS USE TO PRODUCE AND DETECT ELLIPTICALLY AND CIRCULARLY POLARISED LIGHT :

Fresnel rhomb is a parallelopiped of glass whose opposite acute angles are 54°37′ (Fig. 13.19-1).

(i) Production of elliptically polarised light :

Suppose a ray PQ of plane polarised light is incident normally on the end face AB of the rhomb, so that the plane of its vibration is making an angle θ with the plane of incidence. The incident vibration of amplitude A may be resolved into two components of amplitudes $A\cos\theta$ and $A\sin\theta$ along and perpendicular to the plane of incidence respectively. These two components will suffer total reflections at Q and R inside the rhomb, at each of which the angle of incidence is 54°37′, which is greater than the critical angle between glass and air. At each internal reflection from the surface of glass of refractive index 1.51, a phase difference of 45° will be set up between the two rectangular components. Thus when these two rectangular components will come

out of the rhomb after two internal reflections at Q and R, a total phase difference of 90° will be set up between them. As the amplitudes of



13.19-1

these two components are unequal they will combine to form an *elliptically polarised* light.

(ii) Production of circularly polarised light :

If the plane of vibration of the incident plane polarised light makes an angle of 45° with the plane of incidence then the amplitudes of the two rectangular components will be $A\cos 45°$ and $A\sin 45°$, which are, therefore, equal. In this case, the emergent rectangular components, having a phase difference of 90°, will combine to produce *circularly polarised* light.

(iii) Detection of elliptically polarised light :

If an elliptically polarised light be made incident on the face AB of the rhomb in such a way that the axes of the elliptical vibration are along and perpendicular to the plane of incidence, then this elliptical vibration will be resolved into two rectangular components having a phase difference of 90°. The rhomb will introduce an additional phase difference of 90°. Hence the net phase difference of the two emergent rectangular components will be either zero or 180°. In both the cases they will combine to form a linear vibration which can be cut off by a Nicol. But this is not possible in the case of a partially polarised light.

(iv) Detection of circularly polarised light :

If a circularly polarised light be made incident on the rhomb, then this circular vibration will be resolved into two rectangular components of equal amplitudes but differing in phase by 90°. The rhomb will introduce an additional phase difference of 90° and hence the two rectangular vibrations after emergence will combine to form a linear vibration which can be cut off by a Nicol. But this is not possible for an unpolarised light.

13.20 ANALYSIS OF CIRCULARLY POLARISED LIGHT BY USING QUARTER-WAVE PLATE :

Fast and slow axes of $\lambda/4$ plate :

A quarter-wave plate ($\lambda/4$ plate) has two principal directions, one parallel and another perpendicular to the optic axis which is lying on its suface. The vibrations of the E-ray are along the optic axis while those of O-ray are perpendicular to the optic axis. In the case of a positive crystal like quartz, the O-ray travels faster than E-ray in the direction perpendicular to the optic axis and it is customary to call the direction perpendicular to the optic axis of positive crystal as the *fast axis* while the axis of positive crystal as the *slow axis*. In the case of a negative crystal like calcite, O-rays travel slower than E-ray in the direction perpendicular to the optic axis and for this the axis perpendicular to the optic axis of a negative crystal is called slow axis while the axis of the negative crystal is called fast axis.

Principle of analysis :

Suppose a right-handed circularly polarised light represented at $z = 0$ by $x = a \cos \omega t$, $y = a \cos (\omega t + \pi/2)$ is made incident normally on the $\lambda/4$ plate (made of +*ve* crystal) whose fast axis is vertical and slow axis is horizontal (Fig. 13.20-1). This circular vibration on incidence, will be resolved into two rectangular components in which the phase of y-component will be in advance of that of x-component by $\pi/2$. During their passage through the $\lambda/4$ plate there will be an additional phase advancement of y-component relative to x-component by $\pi/2$. So the emergent light will combine to form a linear vibration R_1R_2 making an angle $-45°$ with the slow axis;

$$x = a \cos \omega t, \quad y = a \cos(\omega t + \pi) = -a \cos \omega t, \quad \text{or,} \quad y = -x$$

Hence to cut off this vibration, the principal section of analysing Nicol should be along N_1N_2 which makes an angle of $+45°$ with the slow axis (Fig. 13.20-1).

If left-handed circularly polarised light represented at $z = 0$ by $x = a \cos(\omega t + \pi/2)$, $y = a \cos \omega t$ is made to fall on this $\lambda/4$ plate normally, then it will be resolved into two rectangular components, in which the phase of x-component will exceed that of y-component by $\pi/2$. On passing through the $\lambda/4$ plate there will be a phase advancement of y-component relative to the x-component by $\pi/2$. So the emergent light can be represented by,

$$x = a \cos(\omega t + \pi/2), \quad y = a \cos(\omega t + \pi/2) \quad \text{or,} \quad y = x.$$

It represents a linear vibration along R_1R_2 making an angle of $+45°$ with the slow axis (Fig. 13.20-2). To cut off this vibration the principal section of the analysing Nicol should be along N_1N_2 which makes an angle of $-45°$ with the slow axis.

Experiment :

In the experimental arrangement a parallel beam of circularly polarised light falls normally on the quarter-wave plate whose slow axis is kept horizontal. The analysing Nicol A, placed behind the $\lambda/4$ plate, is now rotated to make the field dark. If the principal section of analyser makes an angle of $+45°$ with the slow axis of the $\lambda/4$ plate then the

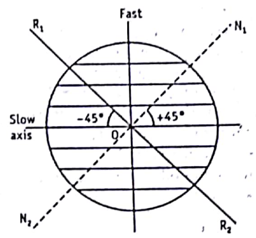


Fig. 13.20-1

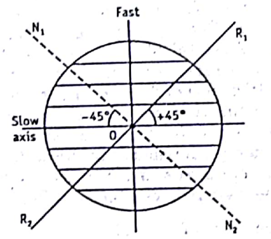


Fig. 13.20-2

given circularly polarised light is right-handed (Fig. 13.20-1). On the other hand, if the shorter diagonal of analyser (A) makes an angle of $-45°$ with the slow axis of $\lambda/4$ plate then the circularly polarised light is left-handed (Fig. 13.20-2).

13.21 ANALYSIS OF ELLIPTICALLY POLARISED LIGHT BY QUARTER-WAVE PLATE :

Principle of analysis :

If an elliptically polarised light be made incident normally on a quarter-wave plate so that the axes of the elliptical vibration may be coincident with the principal directions of $\lambda/4$ plate then the elliptic vibration at incidence will be resolved into two rectangular vibrations having a phase difference of $\pi/2$. The $\lambda/4$ plate will introduce a further relative phase change of $\pi/2$. Hence the phase difference of the two rectangular vibrations emerging from the $\lambda/4$ plate will be either zero or π. In both cases the resultant vibration will be linear which can be cut off by the analysing Nicol A.

When this happens, the principal directions of $\lambda/4$ plate will represent the positions of the axes of elliptic vibrations. If the principal section of the analysing Nicol (A) makes an angle θ with any one of the two principal directions of $\lambda/4$ plate (at the position of extinction) then $\tan\theta$ = ratio of the amplitudes of two rectangular vibrations forming elliptically polarised light.

Suppose a left-handed elliptical vibration is incident normally on $\lambda/4$ plate such that the axes of elliptic vibration coincide with the axes of the $\lambda/4$ plate. The incident wave then can be resolved into two linear vibrations differing in phase by $\pi/2$. Let

$$x = a\cos(\omega t + \pi/2),\quad y = b\cos\omega t$$

where x-axis is along the optic axis and y-axis is perpendicular to it. Now, if we assume a $+ve$ crystal then $v_o > v_e$ i.e., there will be a phase advancement of y-component (O-ray) by $\pi/2$ relative to the x-component. So the emergent vibrations become

$$x = a\cos(\omega t + \pi/2),\quad y = b\cos(\omega t + \pi/2)\quad \text{or,}\quad y = \frac{b}{a}x$$

It represents a linear vibration making an angle $\theta = \tan^{-1}\frac{b}{a}$ with the x-axis. It can be extinguished when the principal section of the

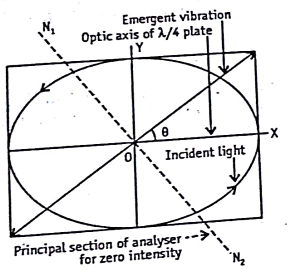


Fig. 13.21-1

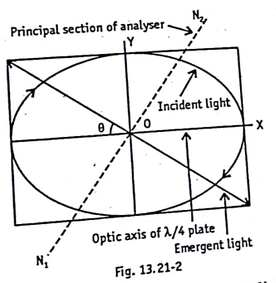


Fig. 13.21-2

analyser is perpendicular to the plane of vibration of the emergent light (Fig. 13.21-1).

If the incident light is right-handed elliptic vibration, then incident light can be resolved as,

$$x = a\cos\omega t,\quad y = b\cos(\omega t + \pi/2)$$

The emergent vibrations become

$$x = a\cos\omega t,\quad y = b\cos(\omega t + \pi) = -b\cos\omega t$$

Therefore, $$y = -\frac{b}{a}x$$

which represents a linear vibration making an angle $\theta = -\tan^{-1}\frac{b}{a}$ with the x-axis (Fig. 13.21-2).

Experiment :

A parallel beam of elliptically polarised light is incident normally on a $\lambda/4$ plate. The analysing Nicol is placed behind the $\lambda/4$ plate to receive the transmitted light. The $\lambda/4$ plate is now rotated in small steps and at each step the analyser is rotated to the position of minimum intensity. Proceeding in this way we get zero intensity for one position of the analyser. This time the principal directions of $\lambda/4$ plate will coincide with the axes of elliptical vibration.

The tangent of the angle between the principal section of the analyser and the axes of $\lambda/4$ plate will give the ratio of the semi-axes of the elliptical vibration.

13.22 BABINET'S COMPENSATOR :

In the study of optical phenomenon it is found to be convenient to have a crystal plate of variable thickness. Babinet's compensator is such a plate. The use of $\lambda/4$ plate in the production and analysis of elliptically polarised light is limited to a narrow range of wavelength. Babinet's compensator has no such limitation.

Construction :

It consists of two slender right-angled quartz prisms ABD and ACD placed together with their hypotenuse AD in contact with each other

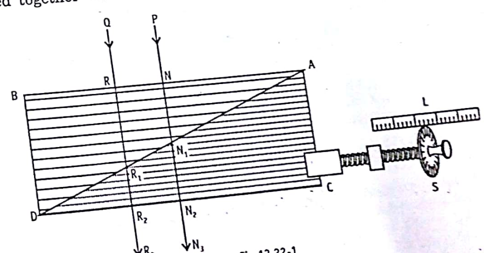


Fig.13.22-1

so that they together form a rectangular slab (Fig. 13.22-1). The axis of ABD is on the face AB and parallel to AB of the section ABD while

2. What is Verdet's constant? Describe an experiment for its determination for a transparent medium.

3. What are the differences between natural rotation and magnetic rotation of the plane of polarisation of a plane polarised light?

4. What is Faraday effect? Discuss its theory on the basis of interaction of electrons with the electromagnetic field of incident light and the applied magnetic field.

5. What is Kerr effect? What is a Kerr cell? Describe a method of determining the velocity of light by using Kerr cell.

6. Discuss the importance of the velocity of light in physics. Describe Anderson's method of the determination of the velocity of light by using a Kerr cell. Which factor limits the accuracy obtainable in this method?

7. What is Zeeman effect? Describe an experimental arrangement by which you can study Zeeman effect. Discuss how you would proceed to find the ratio of the charge to the mass of an electron with the above experimental set up.

8. What is normal Zeeman effect? Give a theoretical explanation of this effect on the basis of classical electron theory of matter.

9. Write short notes on (i) Faraday effect, (ii) Zeeman effect, (iii) Kerr effect and (iv) Stark effect.

PROBLEMS

1. A plane polarised light is passed through a tube of length 10 cm containing water and placed in an axial magnetic field of 10^4 gauss. Calculate the rotation of the plane of vibration of the polarised light. Given Verdet constant $V = 0.0131$ min.gauss^{-1}.cm^{-1}.

[*Ans.* 21° 50′]

2. A block of crown glass 5 cm long is placed between the pole pieces of an electromagnet producing a magnetic field of 1 tesla. Calculate the angle of rotation of the plane of vibration of a plane polarised sodium light of wavelength $\lambda = 589.3$ nm when sent through the glass parallel to the magnetic field. Given Verdet's constant $V = 0.0161$ min.gauss^{-1}.cm^{-1}.

[*Ans.* 13° 25′]

3. Calculate the wavelength difference between two component lines in the normal longitudinal Zeeman effect when a magnetic field of 0.8 tesla is employed. Given wavelength of the original line = 600 nm.

[Hints : $2|\Delta\lambda| = \dfrac{\lambda^2}{c}\cdot\dfrac{eB}{2\pi m} = 0.0269$ nm]

4. Find the value of specific charge of an electrn if the Zeeman shift for a field of 1 tesla is $\Delta\upsilon = 1.4 \times 10^{10}$ Hz.

[*Ans.* 1.7592×10^{11} C.kg^{-1}]

CHAPTER 16

FIBRE OPTICS AND WAVEGUIDES

16.1 INTRODUCTION :

In recent times transparent dielectric fibres have been developed to carry efficiently optical signals from one place to another. These are known as *optical fibres*. The study of the properties of such optical fibres is known as *fibre optics*. Usually the diameter of these fibres is large compared to the small wavelength of optical signals. So the inherent wave nature of light may be neglected and we can apply the known laws of geometrical optics to study the properties of optical fibres. The importance of fibre optics arises from the fact that an optical fibre can carry a huge amount of information by using a light beam as a carrier wave. Moreover, because of low cost, high reliability and extremely low losses optical fibres are gradually being used more and more.

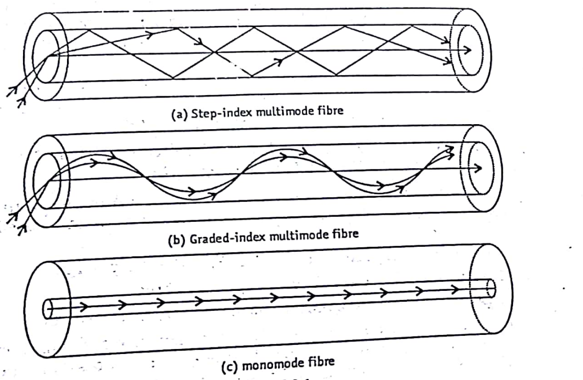
If the diameter of the fibre becomes of the order of wavelength, the wave nature of light cannot be neglected (e.g., in fibres of thin variety). In this case light travels as an electromagnetic wave along the fibre in a manner similar to that of microwave propagation along a waveguide. In order to have an idea of such mode of propagation we present at the end of this chapter a brief discussion on microwave propagation along a waveguide.

16.2 CLASSIFICATION OF OPTICAL FIBRES :

Optical fibres or fibre guides are hair thin flexible structures that can guide optical signal from one place to another. An optical fibre consists of an inner cylinder made of glass or plastic, called the *core*. The core is surrounded by a glass or plastic coating of lower refractive index called the *cladding*. The function of the cladding is to add mechanical strength to protect the core from environment and to reduce scattering losses at the core surface. Optical fibres can be classified on the basis of refractive index profile of the core and the number of modes propagating in the fibre.If the core has a uniform refractive index then it is called a *step-index fibre*. On the other hand, if the core has a non-uniform refractive index that gradually decreases from the centre toward the core-cladding interface, the fibre is classified as *graded-index fibre*.

Fig.16.2-1(a) shows a step-index fibre with relatively large core. Depending on the launch angle of the incident light into the fibre, there can be large number of different zig-zag ray paths or modes by which energy can travel down the core. Then it is called a *step-index multimode* fibre. The rays travelling along different paths take different time to reach the other end of the fibre. This is known as *intermodal dispersion*. This difference in transit time leads to a distortion in the signal being transmitted.

Fig. 16.2-1(b) shows a *multimode graded index fibre* in which the problem of difference in transit times can be greatly reduced. Here the rays do not follow zig-zag paths but are refracted gradually. The refractive index is higher and hence velocity of light is smaller at the central region. On the other hand, refractive index is smaller and velocity of light is greater near the cladding. So the rays spiralling around near the axis take shorter paths and travel slowly, whereas the rays spiralling around near the cladding take longer paths and travel at faster rate. The result is that all the rays tend to recross the axis at the same instant of time.



(a) Step-index multimode fibre

(b) Graded-index multimode fibre

(c) monomode fibre

Fig. 16.2-1

Fig. 16.2-1(c) shows a *monomode optical fibre*. Its core is made so narrow that only one mode of propagation is possible. The rays travel only parallel to the axis and the problem of intermodal dispersion is practically eliminated.

The multimode step-index fibre is the oldest of the three types. It is least expensive, rugged and can be easily infused with light. But it has the serious drawback of *intermodal dispersion* and is not suitable for long distance communication.

The multimode graded-index fibre has medium price. Intermodal dispersion problem is not very serious here. It is widely used in medium distance communications.

The monomode fibre is relatively expensive and requires laser sources. But here there is no intermodal dispersion problem. This type of fibre is most suitable for long distance communication.

If a large number of uncladded fibres are packed together to form bundle then light can leak through from one fibre to another. This is known as *cross-talk*. For this in today's fibre cladding is used. However, if a fibre is bent too sharply, the angle of incidence at the core-cladding interface may become less than the critical angle, a part of light escapes and may cause *cross-talk*.

If the fibres in a bundle are not aligned the bundle is said to be an *incoherent bundle*. On the other hand, if the fibres are properly aligned the bundle is said to be a *coherent bundle*.

16.3 ADVANTAGES OF OPTICAL FIBRES OVER COAXIAL CABLES OR TWISTED WIRE PAIRS :

Main application of optical fibres is in communication. It has the following advantages over the conventional coaxial cable or twisted wire pairs.

(i) The channel capacity of optical fibres is very large.

(ii) Losses are extremely low in optical fibres.

(iii) Optical fibres are small in size and light in weight.

(iv) Optical fibres are non-conductive, non-radiative and non-inductive. Thus there is practically no signal leakage, no cross-talk and the communication is very secured and free from extraneous electromagnetic disturbances.

(v) The basic raw material of fibres is silica which is abundantly available in nature whereas the basic raw material of coaxial cables is copper which is being increasingly costly.

(vi) Optical fibres are expected to be relatively cheap in near future.

16.4 APPLICATIONS OF OPTICAL FIBRES :

Optical fibres are nowadays being used in various fields. Some of the common applications are mentioned below.

(i) *Communication* : Most important application of optical fibres is in communication. Because of high bandwidth, low loss, small size, light weight and certain other attractive features optical fibres are being extensively used in both long distance communication and local area networks that include telephone, television, computers etc.

(ii) *Transmission of light*: Optical fibres are used for transmission of light to illuminate hard-to-reach places or to conduct light out of such places.

(iii) *Transmission of images*: A flexible bundle of optical fibres is called a *fibrescope*. It is extensibly used in carrying images. Light is passed down the outer fibres from the source and the reflected light is piped along the inner fibres. It is used in medicine to look into internal organs of human body and in non-destructive testing in industry.

(iv) *Sensors*: Optical fibres can be used in the field of fibre optic sensors which can sense and measure acoustic fields, magnetic fields, pressure, temperature etc.

(v) *Coupler*: Optical fibres can be used to couple two electrical circuits without introducing a direct link.

16.5 ACCEPTANCE ANGLE AND NUMERICAL APERTURE OF AN OPTICAL FIBRE :

We consider a step-index optical fibre with its core and cladding having refractive indices n_1 and n_2 respectively. Let n_0 be the refractive

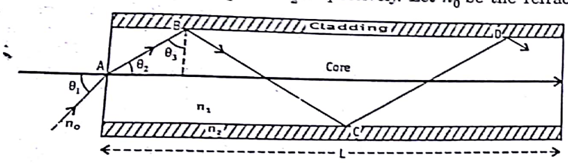


Fig. 16.5-1

index of the outside medium. Let a ray is incident on the entrance aperture of the fibre at an angle θ_1 with the axis. If θ_2 be the angle of refraction we get from Snell's law

$$n_0 \sin\theta_1 = n_1 \sin\theta_2 \qquad ...(16.5\text{-}1)$$

In order to keep the light inside the core the angle of incidence θ_3 at the core-cladding interface must not be less than the critical angle θ_C. From Fig. 16.5-1,

$$\theta_3 = 90^\circ - \theta_2 \qquad ...(16.5\text{-}2)$$

If θ_1 is increased θ_2 increases and hence θ_3 decreases. So there is a maximum value θ_A of θ_1 for which θ_3 is not less than θ_C and the ray undergoes total internal reflection at the core-clading interface. This angle θ_A is known as *acceptance angle*. Thus the acceptance angle is the maximum angle of incidence for which any ray is totally internally reflected at the interface and, therefore, transmitted without loss. A cone of light of semiangle θ_A is known as acceptance cone. To determine we write from Eqs. (16.5-1) and (16.5-2)

$$n_0 \sin\theta_1 = n_1 \sin\theta_2 = n_1 \sin(90^\circ - \theta_3) = n_1 \cos\theta_3$$

For $\theta_1 = \theta_A$, $\theta_3 = \theta_C$, therefore,

$$n_0 \sin\theta_A = n_1 \cos\theta_C \qquad ...(16.5\text{-}3)$$

Since $\sin\theta_C = \dfrac{n_2}{n_1}$ we can write from Eq.(16.5-3)

$$n_0 \sin\theta_A = n_1\sqrt{1-\sin^2\theta_C} = n_1\sqrt{1-\frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2} \qquad ...(16.5\text{-}4)$$

The quantity $n_0 \sin\theta_A$ is defined as the **numerical aperture** (NA) of the fibre. All the rays entering inside a solid angle $\pi \sin^2\theta$ will be transmitted and hence the square of the numerical aperture gives a measure of the capacity of the fibre to transmit light power. If NA is large it would be easier to launch power into the fibre.

16.6 INTERMODAL DISPERSION IN A STEP-INDEX FIBRE :

Fig.16.5-1 shows a step-index fibre with the core and the cladding having uniform refractive indices n_1 and n_2 respectively. Depending on the launch angle there can be large number of different zig-zag paths along which light energy can travel down the fibre. The ray travelling along the axis takes the shortest time. If L is the length of the fibre then minimum time of travel is

$$t_{min} = \frac{\text{axial length}}{\text{Velocity of light in the fibre}} = \frac{L}{(c/n_1)} \qquad ...(16.6\text{-}1)$$

If a ray refracted into the fibre makes an angle θ_2 with the axis then path length l traversed by the ray will be $l = \dfrac{L}{\cos\theta_2}$. The path length will be the longest when the ray is incident on the core-cladding interface at the critical angle i.e, $\theta_3 = \theta_C$. Since $\theta_2 = 90^\circ - \theta_3$, the maximum time of travel is given by

$$t_{max} = \frac{l}{(c/n_1)} = \frac{Ln_1}{c\cos\theta_2} = \frac{Ln_1}{c\cos(90^\circ - \theta_C)} = \frac{Ln_1}{c\sin\theta_C}$$

Now $\sin\theta_C = \dfrac{n_2}{n_1}$ and therefore, $t_{max} = \dfrac{Ln_1^2}{cn_2}$ ...(16.6-2)

Thus the maximum time interval between the rays at the output end will be