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Fluid Kinematics → P.S (6)

1. $\vec{V} = \underbrace{(6xt + yz^2)}_{u}\hat{i} + \underbrace{(3t + xy^2)}_{v}\hat{j} + \underbrace{(xy - 2xyz - 6tz)}_{w}\hat{k}$.

For incompressible flow, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$.

LHS $= (6t) + (2xy) + (-2xy - 6t) = 0$.

∴ Flow is possible.

(1, 1, 1), t = 1

ii) $a_x = u.\frac{\partial u}{\partial x} + v.\frac{\partial u}{\partial y} + w.\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$.

$= 6t(6xt + yz^2) + (z^2)(3t + xy^2) + (2yz)(xy - 2xyz - 6tz) + 6x$

$= 6(6+1) + 1(3+1) + (2)(-7) + 6$.

$= 38\hat{i}$

$a_y = u.\frac{\partial v}{\partial x} + v.\frac{\partial v}{\partial y} + w.\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$.

$= (6xt + yz^2)(y^2) + (3t + xy^2)(2xy) + (xy - 2xyz - 6tz)(0) + 3$

$= (6+1)(1) + (3+1)(2) + 0 + 3$

$= 18\hat{j}$

$a_z = u.\frac{\partial w}{\partial x} + v.\frac{\partial w}{\partial y} + w.\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$.

$= (6xt + yz^2)(y - 2yz) + (3t + xy^2)(x - 2xz) + (xy - 2xyz - 6tz)(-2xy - 6t) - 6z$

$= (6+1)(-1) + (3+1)(-1) + (-7)(-8) - 6$

$= 39\hat{k}$.

∴ Acceleration at A $= 38\hat{i} + 18\hat{j} + 39\hat{k}$.

2. $V = \underbrace{10x^2y}_{u}\,\hat{i} + \underbrace{15xy}_{v}\,\hat{j} + \underbrace{(25t - 3xy)}_{w}\,\hat{k}$

For steady flow, $\nabla(\rho\vec{V}) = 0$
$\Rightarrow \nabla(\vec{V}) = 0$.

But here, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \neq 0$
So, ...

Since $V = f(t)$;
$\therefore$ we have unsteady flow.

$(1, 2, -1)$ $t = 0.5s$.

$$a_x = u.\frac{du}{\partial x} + v.\frac{\partial u}{\partial y} + w.\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}.$$

$$= 10x^2y(20xy) + 15xy(10x^2) + (25t - 3xy)(0) + 0.$$

$$= 800 + 300 = 1100\,\hat{i}$$

$$a_y = u.\frac{\partial v}{\partial x} + v.\frac{\partial v}{\partial y} + w.\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}.$$

$$= (10x^2y).15y + (15xy)(15x) + (25t - 3xy)(0) + 0.$$

$$= 600 + 450 = 1050\,\hat{j}$$

$$a_z = u.\frac{\partial w}{\partial x} + v.\frac{\partial w}{\partial y} + w.\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}.$$

$$= (10x^2y)(-3y) + (15xy)(-3x) + (25t - 3xy)(0) + 25.$$

$$= \cancel{-120}\ -120 - 90 + 25 = -185.$$

$$\therefore \vec{a} = 1100\,\hat{i} + 1050\,\hat{j} - 185\,\hat{k}.$$

$$\therefore |\vec{a}| = \sqrt{(1100)^2 + (1050)^2 + (-185)^2} = 1531.9 \text{ units}.$$

Along x - dir^n only:

3. $V = 2t\left(1 - \frac{x}{2L}\right)^2$

$$a = a_x = \underbrace{u.\frac{\partial u}{\partial x}}_{\text{Convective}} + \underbrace{\frac{\partial u}{\partial t}}_{\text{Local}}.$$

$$\therefore a_{conv} = u.\frac{\partial u}{\partial x}.$$

$$= 2t\left(1 - \frac{x}{2L}\right)^2 \times \left[2t \times 2\left(1 - \frac{x}{2L}\right) \times \frac{-1}{2L}\right]$$

$t = 3$, $x = 0.5$, $L = 0.8$

$$\therefore a_{conv} = 2(3) \cdot \left(1 - \frac{0.5}{2(0.8)}\right)^2 \left[2(3)\left(1 - \frac{0.5}{2(0.8)}\right) \times \frac{-1}{(0.8)}\right]$$

$$= -14.623 \ m^2/s.$$

$$\therefore a_{local} = \frac{\partial u}{\partial t} = 2\left(1 - \frac{x}{2L}\right)^2$$

$$= 2\left(1 - \frac{0.5}{2(0.8)}\right)^2 = 0.945 \ m^2/s$$

$$\therefore \text{Total } a = a_{conv} + a_{local} = -13.678 \ m^2/s.$$

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4. $\vec{V} = u_0\left(1 + \frac{2x}{L}\right)$ [Along x-direction only:]

$$\therefore a = a_x = \frac{\partial u}{\partial x} \cdot u + \frac{\partial u}{\partial t}.$$

$$= u_0\left(1 + \frac{2x}{L}\right)\left[\frac{2u_0}{L}\right]$$

$$= \frac{2u_0^2}{L}\left(1 + \frac{2x}{L}\right)$$

$$v = \frac{dx}{dt} = u_0\left(1 + \frac{2x}{L}\right).$$

$$\Rightarrow \int_0^L \frac{dx}{\left(1 + \frac{2x}{L}\right)} = \int_0^t u_0 \, dt$$

(neglecting 'c' for definite integral)

$$\Rightarrow \left[\frac{\ln\left(1 + 2x/L\right)}{2/L}\right]_0^L = u_0 t$$

$$\Rightarrow u_0 t = \left[\ln 3 - \ln 1\right] \cdot \frac{L}{2}$$

$$\Rightarrow t = \frac{L}{2u_0} \cdot \ln 3.$$

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5. $\vec{v} = 3x\hat{i} + 4y\hat{j} - 7z\hat{k}$ $M(1,4,5)$.

For streamline flow, $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

$\therefore \int \frac{dx}{3x} = \int \frac{dy}{4y}$.

$\Rightarrow \frac{1}{3}\ln x = \frac{1}{4}\ln y + \ln c^{1/4}$

$\Rightarrow x^{1/3} = (y.c)^{1/4}$

$\Rightarrow x^{4/3} = y.c$

for $x = 1$, $y = 4$

$\Rightarrow c = 1/4$

$\therefore \boxed{4.x^{4/3} = y.}$

$\int \frac{dx}{3x} = \int \frac{dz}{-7z}$.

$\Rightarrow \frac{1}{3}\ln x = -\frac{1}{7}\ln z + \ln c^{-1/7}$

$\Rightarrow x^{1/3} = (z.c)^{-1/7}$.

$\Rightarrow x^{7/3} = \frac{1}{z.c}$.

for $x = 1$, $z = 5$.

$\therefore c = 1/5$

$\therefore \boxed{z = \frac{5}{x^{7/3}}}$

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6. $\vec{v} = \underbrace{\left(\frac{A}{x}\right)}_{u}\hat{i} + \underbrace{\left(\frac{Ay}{x^2}\right)}_{v}\hat{j}$

$A = 2m^2/s$. $(x, y) = (1, 3)$

Through

For a streamline; $\frac{dx}{u} = \frac{dy}{v}$

$\Rightarrow \frac{dx}{A/x} = \frac{dy}{Ay/x^2}$.

$\Rightarrow \frac{\cancel{x}}{\cancel{A}}.dx = \frac{x^{\cancel{2}}}{\cancel{A}}.\frac{dy}{y}$.

$\Rightarrow \int \frac{dx}{x} = \int \frac{dy}{y}$.

$\Rightarrow \ln x = \ln y + \ln c$.

$\Rightarrow x = y.c$

$(x = 1;\ y = 3)$.

$\Rightarrow c = 1/3$.

$\therefore \boxed{y = 3x}$

x - comp. of flow:

$\vec{u} = (A/x)\hat{i}$.

$\Rightarrow \frac{dx}{dt} = \frac{A}{x}$.

$\Rightarrow \int_1^3 x\,dx = \int_0^t A.dt$.

(neglecting constant for definite int).

$\Rightarrow \left[\frac{x^2}{2}\right]_1^3 = At$.

$\Rightarrow \frac{9-1}{2} = 2(t)$.

$\Rightarrow \boxed{t = 2sec}$

7. $x = x_0 e^{-kt} + y_0(1 - e^{-2kt})$; $y = y_0 e^{kt}$

$e^{kt} = y/y_0 \Rightarrow e^{-kt} = y_0/y$. $\therefore e^{-2kt} = (y_0/y)^2$

$\therefore x = x_0 \cdot \frac{y_0}{y} + y_0\left(1 - \left(\frac{y_0}{y}\right)^2\right)$

$\Rightarrow xy^2 = x_0 y_0 y + y_0(y^2 - y_0^2)$

$\Rightarrow y^2(x - y_0) - x_0 y_0 y + y_0^3 = 0$ (eq^n of pathline).

Vel. comp, $u = \frac{dx}{dt} = \cancel{-k x_0 e^{-kt} + 2k y_0 e^{-2kt}}$

$\cancel{x = x_0 \cdot e^{-kt} + y_0 - y_0 \cdot e^{-2kt}}$.

$\cancel{x = x_0 \cdot e^{-kt} + y \cdot e^{-kt} - y \cdot e^{-3kt}}$.

$\cancel{\Rightarrow x - y \cdot e^{-kt} + y \cdot e^{-3kt} = x_0 \cdot e^{-kt}}$.

$x = x_0 \cdot e^{-kt} + y_0 - y_0 \cdot e^{-2kt}$

$\therefore u = \frac{dx}{dt} = -k \cdot x_0 \cdot e^{-kt} + 2k \cdot y_0 \cdot e^{-2kt}$. — ①

Now, from original equ. of x:

$x_0 = \left[x - y_0(1 - e^{-2kt})\right] \cdot e^{kt}$.

$= \left[x - y \cdot e^{-kt} + y \cdot e^{-3kt}\right] \cdot e^{kt}$.

$= x \cdot e^{kt} - y + y \cdot e^{-2kt}$. → Putting in ①.

$u = -k(x \cdot e^{kt} - y + y \cdot e^{-2kt}) \cdot e^{-kt} + 2k \cdot y \cdot e^{-3kt}$.

$= -kx + ky \cdot e^{-kt} - ky\, e^{-3kt} + 2ky \cdot e^{-3kt}$.

$= -kx + ky\, e^{-kt} + ky\, e^{-3kt}$.

$\Rightarrow \boxed{u = -kx + ky(e^{-kt} + e^{-3kt})}$

$y = y_0 e^{kt}$. $\therefore v = \frac{dy}{dt} = k \cdot y_0 \cdot e^{kt}$.

$\Rightarrow \boxed{v = ky}$

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8. $v = y^2 - 2x + 2y$. $\quad u = 0$ at $x = 0$.

For a steady, incompressible flow, (in x, y).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

$$\Rightarrow \frac{du}{dx} + (2y + 2) = 0.$$

Clearly, $c = f(y)$.

$$\Rightarrow \int du = \int (-2y - 2)\, dx.$$

$$\Rightarrow u = -2xy - 2x + c.$$

$$\Rightarrow u = -2xy - 2x + f(y).$$

at $u = 0$; $x = 0$.

$$\Rightarrow 0 = 0 + 0 + f(y) \Rightarrow f(y) = 0.$$

$$\therefore \boxed{u = -2yx - 2x}$$

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9. $$V = \left(\frac{x}{x^2 + y^2}\right)\hat{i} + \left(\frac{y}{x^2 + y^2}\right)\hat{j}$$

For an incompressible flow, $\frac{d\rho}{dt} = 0$. (Since it is in two dimensions,

$$\therefore \frac{du}{\partial x} + \frac{\partial v}{\partial y} = 0$$ (the continuity equation; diff. form).

Now, $$\frac{\partial u}{\partial x} = \frac{(x^2 + y^2) - x(2x)}{(x^2 + y^2)^2} = \frac{y^2 - x^2}{(x^2 + y^2)^2}.$$

$$\frac{\partial v}{\partial y} = \frac{(x^2 + y^2) - y(2y)}{(x^2 + y^2)^2} = \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{y^2 - x^2 + x^2 - y^2}{(x^2 + y^2)^2} = 0$$

$\therefore$ Continous.

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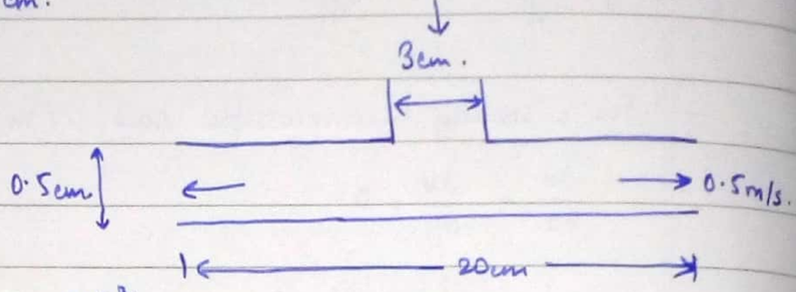
10. $D = 3$ cm $\quad d = 20$ cm.

$t = 0.5$ cm.



(A_2) Area of inlet portion = $\frac{\pi D^2}{4}$

(V_2) Let inlet velocity (avg) = v.

(A_1) Area of outflow = $\pi . d . t$.

(V_1) Vel. of outflow = 0.5 m/s.

$\because$ Streamlines are radial in diffuser, and considering incompressibility of working fluid $\left(\frac{dP}{dt} = 0\right)$, the continuity eqn. gives:

$A_2 V_2 = A_1 V_1$

$\Rightarrow \frac{\pi (0.03)^2}{4} \times V = \pi (0.2)(0.005) \times 0.5$

$\Rightarrow \boxed{V = 2.22 \text{ m/s.}}$