(1) Show that the straight lines whose d.c.s. are given by the equation $al + bm + cn = 0$, $ul^2 + vm^2 + wn^2 = 0$ are perpendicular or parallel according as $(v+w)a^2 + (w+u)b^2 + (u+v)c^2 = 0$ or $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$

Ans) Given;

$$al + bm + cn = 0 \quad \text{---(1)}$$
$$ul^2 + vm^2 + wn^2 = 0 \quad \text{---(2)}$$

Eliminating l from (1) & (2) we get.

$$\Rightarrow (ub^2 + a^2v)m^2 + 2bcumn + (uc^2 + wa^2)n^2 = 0$$

$$\Rightarrow (ub^2 + a^2v)\left(\frac{m}{n}\right)^2 + 2buc\left(\frac{m}{n}\right) + (uc^2 + wa^2) = 0 \quad \text{---(3)}$$

which is a quadratic equation in (m/n).

Let the roots be m_1/n_1 & m_2/n_2

The 2 lines are $\perp$ iff

$$l_1l_2 + m_1m_2 + n_1n_2 = 0 \quad \text{---(4)}$$

$$\Rightarrow \frac{m_1m_2}{uc^2 + wa^2} = \frac{n_1n_2}{ub^2 + va^2} = \frac{l_1l_2}{vc^2 + wb^2} = k, \text{ say}$$

from (4) we have $k(vc^2 + wb^2) + k(uc^2 + wa^2) + k(ub^2 + va^2) = 0$

$$\Rightarrow (v+w)a^2 + (u+w)b^2 + (u+v)c^2 = 0 \quad \text{(proved)}$$

Again the lines will be // iff

$$(2bcu)^2 - 4(ub^2 + va^2)(uc^2 + wa^2) = 0$$

$$\Rightarrow uwb^2 + uvc^2 + vwa^2 = 0$$

Dividing by uvw get;

$$\Rightarrow \frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0 \quad \text{--(proved)}$$

(2) Prove that the angle between ~~two diagonals~~ of cube is $\cos^{-1}(1/3)$.

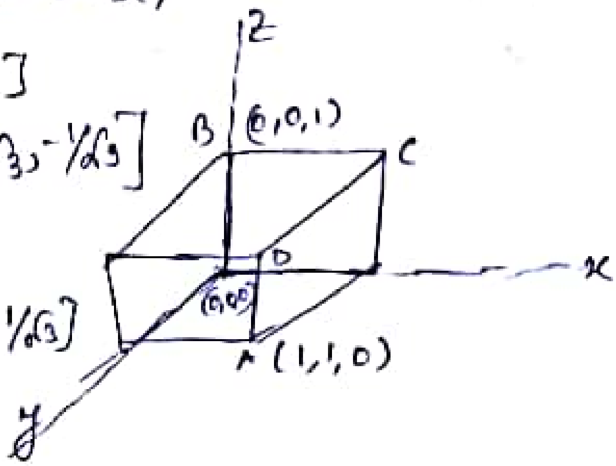
Ans) Let AD & OD be diagonals of a cube;

Now d. ratios of AB = [1, 1, -1]

d cosines of AB = $[1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3}]$

d. ratios of OD = [1, 1, 1]

d cosines of OD = $[1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}]$



$\therefore \cos\theta = \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}$

$\Rightarrow \cos\theta = (1/\sqrt{3})^2$

$\therefore \theta = \cos^{-1}\left(\frac{1}{3}\right)$ -- proved.

(3) If a line makes angles, $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$

Ans) D. cosines of the 4 diagonals of the cube are

$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ & $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

Let d. cosines be (l, m, n)

$\therefore \cos\alpha = l/\sqrt{3} + m/\sqrt{3} - n/\sqrt{3}$

$\cos\beta = \frac{l}{\sqrt{3}} - \frac{m}{\sqrt{3}} + \frac{n}{\sqrt{3}}$

$\cos\gamma = \frac{l}{\sqrt{3}} + \frac{m}{\sqrt{3}} + \frac{n}{\sqrt{3}}$

$\cos\delta = \frac{-l}{\sqrt{3}} + \frac{m}{\sqrt{3}} + \frac{n}{\sqrt{3}}$

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$

$= 4\left(\frac{l^2}{3} + \frac{m^2}{3} + \frac{n^2}{3}\right)$

after omitting the other terms we get;

$\frac{4}{3}(l^2+m^2+n^2) = \frac{4}{3}$ -- $(l^2+m^2+n^2=1)$

-- (proved)

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4) Find the point in which the perpendicular from the origin on the straight line joining the points $A(-9,4,5)$ and $B(11,0,-1)$ meets it.

Ans) The equation of AB is $\frac{x+9}{10} = \frac{y-4}{-2} = \frac{z-5}{-3} = r$ -- say

Let Q be the foot of the $\perp$ drawn from O to AB

$Q(10r-9, -2r+4, -3r+5)$

d·ratios of OQ is $(10r-9, -2r+4, -3r+5)$

as OQ i.e $\perp$ to AB Thus

$10(10r-4) - 2(-2r+4) - 3(-3r+5) = 0$

$\Rightarrow 113r = 113 \Rightarrow r = 1$

$\therefore$ The co-ordinates of Q is $(1,2,2)$

5) Show that the angle between the straight lines whose d.cs are given by $l+m+n=0$, $fmn+gnl+hlm=0$ is $\pi/2$. If $\frac{1}{f}+\frac{1}{g}+\frac{1}{h}=0$

Ans) Given; $l+m+n=0$ — ① $fmn+gnl+hlm=0$ — ②

$\frac{1}{f}+\frac{1}{g}+1/h=0$ — ③

Eliminating l from ① & ② we get;

$hm^2+(g+h-f)mn+gn^2=0$

$\Rightarrow h\left(\frac{m}{n}\right)^2+(g+h-f)(m/n)+g=0$ — ④

Let $\left(\frac{m_1}{n_1}\right)$ & $\left(\frac{m_2}{n_2}\right)$ be the roots of the above quadratic eqn

$\therefore \frac{m_1}{n_1}+\frac{m_2}{n_2} = \frac{f-g-h}{h}$ — ⑤

& $\frac{m_1 m_2}{n_1 n_2} = g/h$ — ⑥

from (6) $\frac{m_1 m_2}{g} = \frac{n_1 n_2}{h} = \frac{l_1 l_2}{f} = k$ (say)

from (5) $\frac{m_1 n_2 + m_2 n_1}{-f-g-h} = \frac{n_1 n_2}{2} = k$

Now; $(m_1 n_2 - m_2 n_1)^2 = k^2[f^2+g^2+h^2-2fg-2fh-2gh]$

So by symmetry;

$l_2 n_1 - l_1 n_2 = l_1 m_2 - l_2 m_1 = k^2[f^2+g^2+h^2-2fg-2fh-2gh]$

If θ be the $\angle$ b/w the lines

$\therefore \tan^2\theta = \frac{\Sigma (m_1 n_2 - m_2 n_1)^2}{(l_1 l_2 + m_1 m_2 + n_1 n_2)^2} = 3\left[\frac{f^2+g^2+h^2-2fgh\left(\frac{1}{h}+\frac{1}{g}+\frac{1}{f}\right)}{f^2+g^2+h^2+2fgh\left[\frac{1}{h}+\frac{1}{g}+\frac{1}{f}\right]}\right]$

$= 3$

$\therefore \tan^2\theta = 3$

$\Rightarrow \theta = \pi/3$ -- (proved)

(6) A variable plane which is at a constant dist. $3p$ from origin O cuts the axes A, B, C. Show that the locus of the point of intersection of the planes through A, B, C drawn parallel to the co-ordinate planes is $9(x^{-2}+y^{-2}+z^{-2}) = p^{-2}$

pro) Let a variable plane intercept the coordinate axes at $A(a,0,0)$ $B(0,b,0)$ & $C(0,0,c)$

The eqn of the plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad \text{---} (1)$$

Let N be the foot of the $\perp$ from O to (1)

$$\Rightarrow 9\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right) = \frac{1}{p^2} \quad \text{---} (2)$$

$\therefore$ according to the question;

$$x = a,\ y = b,\ z = c.$$

Let the above plane meet at $Q(\alpha, \beta, \gamma)$

$$\Rightarrow \alpha = a,\ \beta = b,\ \gamma = c$$

Now putting the values of α, β, γ in (1)

$$9(\alpha^{-2} + \beta^{-2} + \gamma^{-2}) = p^{-2}$$

Thus the eqn holds for every point

$$\Rightarrow \text{Locus is } 9(x^{-2} + y^{-2} + z^{-2}) = p^{-2}$$

(proved)

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(7) A variable plane has intercepts on the axes, sum of whose squares is k^2. Show that the locus of the perpendicular form origin is $(x^2+y^2+z^2)^2\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right) = k^2$

Ans) $A(a,0,0)$ $B(0,b,0)$ $C(0,0,c)$

$$\Rightarrow a^2 + b^2 + c^2 = k^2 \quad \text{--- (1)}$$

eqn of plane $$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \text{--- (2)}$$

Let $N(\alpha, \beta, \gamma)$ be the foot of the $\perp^r$

$\therefore$ eqn of ON $$\frac{x}{1/a} = \frac{y}{1/b} = \frac{z}{1/c} = r \text{ (say)} \quad \text{--- (3)}$$

$$\therefore \alpha = \frac{r}{a},\ \beta = \frac{r}{b},\ \gamma = \frac{r}{c} \quad \text{--- (4)}$$

as N is a pt on (2)

$$\frac{\alpha}{a} + \frac{\beta}{b} + \frac{\gamma}{c} = 1 \quad \text{--- (5)}$$

putting the values we get;

$$\alpha^2+\beta^2+\gamma^2 = r$$

Eliminating r from above

$$(\alpha^2+\beta^2+\gamma^2)^2(\alpha^{-2}+\beta^{-2}+\gamma^{-2}) = k^2$$

$\therefore$ The required locus is

$$(x^2+y^2+z^2)^2\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right) = k^2 \text{ - (proved)}$$

(8) Show line of intersection of the planes $x+2y-z-3=0$ and $3x-y+2z-1=0$ is coplanar with line of intersection of the planes $2x-2y+3z-2=0$ & $x-y+z+1=0$. obtain eqn of plane containing the lines.

(Ans) Eqn of any plane through first line of intersection is

$$(x+2y-z-3)+\lambda(3x-y+2z-1)=0 \quad \text{—(1)}$$

plane through 2nd line of intersection has an eqn

$$2x-2y+3z-2+\mu(x-y+z+1)=0$$

If they are coplanar then;

$$\Rightarrow \frac{1+3\lambda}{2+\mu} = \frac{2-\lambda}{-2-\mu} = \frac{-1+2\lambda}{3+\mu} = \frac{-3-\lambda}{-2+\mu}$$

$$\Rightarrow 4\mu-\frac{3\mu}{2} = \frac{9}{2}+8 \Rightarrow \mu=5$$

$$\Rightarrow \lambda = -3/2$$

putting the values of μ & λ in the equations, we get the equation the plane which is

$$2x-2y+3z-2+5x-5y+5z+5=0$$

$$\Rightarrow 7x-7y+8z+3=0 \text{ (Ans)}$$

(9) Find the eqn of the plane bisecting the angle b/w the planes $x-2y+3z-5=0$ & $2x-y-z+3=0$ which contains origin.

Ans) given; $x-2y+3z-5=0$ & $-2x+y+z-3=0$

eqn in normal forms are

$$\frac{x-2y+3z-5}{\sqrt{14}}=0 \quad \text{—(I)} \quad \& \quad \frac{-2x+y+z-3}{\sqrt{6}}=0 \quad \text{—(II)}$$

∴ eqn of plane bisecting the ∠ b/w the given plane containing origin is

$$\frac{x-2y+3z-5}{\sqrt{14}}=\frac{-2x+y+z-3}{\sqrt{6}}$$

$$\Rightarrow (\sqrt{6}+2\sqrt{14})x-(2\sqrt{6}+\sqrt{14})y+(3\sqrt{6}-\sqrt{14})z-5\sqrt{6}+3\sqrt{14}=0$$

(Ans)

(10) A variable plane passes through a fixed pt (α,β,γ) and meets the axes of reference in A, B & C. Show that the locus of the point of intersection of the planes through A, B & C parallel to the coordinate planes is $\frac{\alpha}{x}+\frac{\beta}{y}+\frac{\gamma}{z}=1$.

Ans) Let fixed pt be (α,β,γ) and $A(a,0,0)$, $B(0,b,0)$ & $C(0,0,c)$

So, the eqn of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ —(1)

$$\therefore \frac{\alpha}{a}+\frac{\beta}{b}+\frac{\gamma}{c}=1 \quad \text{—(2)}$$

Let (x_1,y_1,z_1) be the pt. of intersection of the planes through A, B, C ∥ to coordinate planes

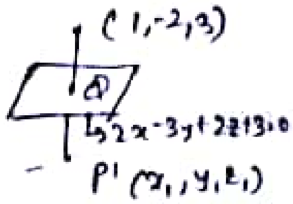
$$\Rightarrow x_1=a,\ y_1=b,\ z_1=c$$

Putting the value of a, b, c in (2) we get

$$\frac{\alpha}{x_1}+\frac{\beta}{y_1}+\frac{\gamma}{z_1}=1$$

Thus the required locus is $\frac{\alpha}{x}+\frac{\beta}{y}+\frac{\gamma}{z}=1$

(11) Find the equation of the image of the point $(1,-2,3)$ in the plane $2x-3y+2z+3=0$



Ans) Given; eqn of plane $2x-3y+2z+3=0$

D.r.s of the normal to the plane $[2,-3,2]$

Eqn of line having pt $(1,-2,3)$ & $\perp$ to given plane

$$\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z-3}{2}=r \quad \text{(say)}$$

$$\Rightarrow x=2r+1 \quad y=-3r-2, \quad z=2r+3$$

pt Q on the line (1) can be written as

$$(2r+1,\ -3r-2,\ 2r+3)$$

as Q lies on the plane, thus

$$\Rightarrow 2(2r+1)-3(-3r-2)+2(2r+3)+3=0$$

we get $r=-1$

$$\therefore Q\ (-1,+1,+1)$$

as Q is the midpoint of ~~QQ'~~ PP'.

$$\therefore \quad \frac{x_1+1}{2}=-1 \quad \frac{y_1+(-2)}{2}=1 \quad \frac{z_1+3}{2}=1$$

$$\Rightarrow x_1=-3 \quad y_1=4 \quad z_1=-1$$

$\therefore$ The image of P in the given line is

$(-3,4,-1)$ (Ans)

(13) Show that the eqn of the plane through the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and $\perp^r$ to the plane containing $\frac{x}{m} = \frac{y}{n} = \frac{z}{l}$ & $\frac{x}{n} = \frac{y}{l} = \frac{z}{m}$ is $(m-n)x + (n-l)y + (l-m)z = 0$

Ans) Eq^n of plane through the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is

$$Ax + By + Cz = 0 \quad \text{--- (1)}$$

where $\quad Al + Bm + Cn = 0 \quad \text{--- (2)}$

$[l_1, m_1, n_1]$ = D. Ratios of normal to the plane containing

$$\frac{x}{m} = \frac{y}{n} = \frac{z}{l} \quad \& \quad \frac{x}{n} = y/l = z/m$$

$$\Rightarrow \frac{l_1}{mn - l^2} = \frac{m_1}{nl - m^2} = \frac{n_1}{ml - n^2}$$

$\Rightarrow [(mn - l^2), (nl - m^2), (ml - n^2)]$ are D. ratios of the normal to the 2nd plane.

$$\therefore A(mn - l^2) + B(nl - m^2) + C(ml - n^2) = 0 \quad \text{--- (3)}$$

Eliminating A, B, C from (1), (2) & (3) we get

$$\begin{vmatrix} x & y & z \\ l & m & n \\ mn - l^2 & nl - m^2 & ml - n^2 \end{vmatrix} = 0$$

$$\Rightarrow (m-n)(lm + ln + mn)x + (n-l)(lm + ln + mn)y + (l-m)(lm + ln + mn)z = 0$$

$$\Rightarrow (m-n)x + (n-l)y + (l-m)z = 0 \quad \ldots (lm + ln + mn \neq 0)$$

(15) Find the equation of the plane through the point $(2,-1,3)$ and $\perp$ to the line $x-2y+3z-4=0=2x-3y+4z-5$

Ans) Let (l,m,n) be the d.cs of the given line.

$$\left.\begin{array}{l} l-2m+3n=0 \\ 2l-3m+4n=0 \end{array}\right\} \Rightarrow \frac{l}{1}=\frac{m}{2}=\frac{n}{1}$$

$\Rightarrow (1,2,1)$ are the d. ratios of the given line which is $\perp^r$ the plane with point $(2,-1,3)$

Hence the required eqn of the plane is

$$1(x-2)+2(y+1)+1(z-3)=0$$

$$\Rightarrow x+2y+z-3=0 \text{ (Ans)}$$

(17) Show that the equations of the planes through the intersection of the planes $x+3y+6=0$ & $3x-y-4z=0$ whose $\perp^r$ distance from the origin is unity, are $2x+y-2z+3=0$ & $x-2y-2z-3=0$

Ans) Here the plane through the line of intersection of the given plane can be taken as;

$$x+3y+6+\lambda(3x-y-4z)=0$$

$$\Rightarrow (1+3\lambda)x+(3-\lambda)y-4\lambda z+6=0 \quad \text{--- (1)}$$

The $\perp$ dist from origin to the plane (1) is

$$\left|\frac{6}{[(1+3\lambda)^2+(3-\lambda)^2+16\lambda^2]^{1/2}}\right|=1 \text{ (given)}$$

$$\Rightarrow 36=9\lambda^2+6\lambda+1+\lambda^2-6\lambda+9+16\lambda^2$$

$$\Rightarrow \lambda=\pm 1$$

$\therefore$ The required eqn's of the planes are

$$\left.\begin{array}{l} x-2y-2z-3=0 \\ \& \ 2x+y-2z+3=0 \end{array}\right] \text{(Ans)}$$

(18) Show that the equation of the plane parallel to the plane $2x + 4y + 5z = 6$ and the sum of whose intercepts on the co-ordinate axes is 19 is $2x + 4y + 5z = 20$.

Ans) The eqn of any plane // to the plane $2x + 4y + 5z = 6$ can be taken as

$$\frac{x}{\lambda/2} + \frac{y}{\lambda/4} + \frac{z}{\lambda/5} = 1 \quad \ldots (\lambda \neq 0)$$

$$\Rightarrow \frac{\lambda}{2} + \frac{\lambda}{4} + \frac{\lambda}{5} = 19$$

$$\Rightarrow \lambda = 20$$

$\therefore$ The req. eqn of the plane is $2x + 4y + 5z = 20$ — (Ans)

(19) Show that the equation of the plane through the pt $(2,3,3)$ and // to st lines $x - 1 = 2y - 5 = 2z$ & $3x = 4y - 11 = 3z - 4$ is $x - 4y + 2z + 4 = 0$

Ans) eqn of plane through the pt $(2,3,3)$ can be taken as

$$A(x-2) + B(y-3) + C(z-3) = 0 \quad \text{—(1)}$$

Let this plane be // to the given lines

$$\therefore \frac{x-1}{2} = \frac{y-5/2}{1} = \frac{z}{1} \quad \text{—(2)}$$

$$\Rightarrow \frac{x}{4} = \frac{y - 11/4}{3} = \frac{z - 4/3}{4} \quad \text{—(3)}$$

$\Rightarrow 2A + B + C = 0$ —(4) & $4A + 3B + 4C = 0$ —(5)

eliminating A, B, C from (3) (4), (5) we get;

$$\begin{vmatrix} x-2 & y-3 & z-3 \\ 2 & 1 & 1 \\ 4 & 3 & 4 \end{vmatrix} = 0 \Rightarrow (x-2)(4-3) - (y-3)(8-4) + (z-3)(6-4)$$

$$\Rightarrow x - 2 - 4y + 12 + 2z - 6 = 0$$

$$\Rightarrow x - 4y + 2z + 4 = 0 \quad \text{— (Proved)}$$

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(20) Find the eqn of the image of the line $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z-4}{2}$ in the plane $2x-y+z+3=0$

Ans) Let P' be the image of the pt P on the plane;

eqn of PP' is $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = r$ (say)

$\therefore P' \equiv (2r+1, 3-r, r+4)$

mid point of $PP' = (r+1, -\frac{r}{2}+3, \frac{r}{2}+4)$

as R lies on plane

$2(r+1) + \frac{r}{2} - 3 + \frac{r}{2} + 4 + 3 = 0$

$\Rightarrow r = -2$

$\therefore P'(-3, 5, 2)$

Let Q be the pt of intersection of line & the plane given

$\therefore Q(2r_1+1, 5r_1+3, 2r_1+4)$

$\therefore 2(2r_1+1) - (5r_1+3) + 2r_1 + 4 + 3 = 0$

$\Rightarrow r_1 = -6$

$\therefore Q \equiv (-11, -27, -8)$

$\therefore$ The required image of the line is

$$\frac{x+3}{8} = \frac{y-5}{32} = \frac{z-2}{10}$$

$$\Rightarrow \frac{x+3}{4} = \frac{y-5}{16} = \frac{z-2}{5}$$ Ans

21) Prove that the acute angle b/w the lines whose d.cs are given by $l+m+n=0$ & $l^2+m^2-n^2=0$ is $\pi/3$.

Ans) Given;

$$l+m+n=0 \quad -(1)$$
$$l^2+m^2-n^2=0 \quad -(2)$$

eliminating l we have

$$(m+n)^2+m^2-n^2=0$$
$$\Rightarrow 2m^2+2mn=0$$
$$\Rightarrow m(m+n)=0$$
$$m=0 \text{ or } m=-n$$

(i) when $m=0$, $\Rightarrow l=-n \Rightarrow \frac{l}{1}=\frac{m}{0}=\frac{n}{-1}$

(ii) when $m=-n \Rightarrow l=0 \Rightarrow \frac{l}{0}=\frac{m}{1}=\frac{n}{-1}$

d.cs case(i) are $[1/\sqrt{2}, 0, -1/\sqrt{2}]$

d.cs case(ii) are $[0, 1/\sqrt{2}, -1/\sqrt{2}]$

$$\theta=\cos^{-1}\left(\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\right) \Rightarrow \cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3} \text{ (proved)}$$

23) Find the eqn of the projection of line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-4}{4}$ on the plane $x+3y+z+5=0$

Ans) Eqn of any plane through the given line is

$$A(x-1)+B(y-2)+C(z-4)=0 \quad -1$$

where $2A+3B+4C=0$

Let this plane be $\perp$ to the plane

$$\therefore A+3B+C=0$$

Eliminating A, B, C

$$\begin{vmatrix} x-1 & y-2 & z-4 \\ 2 & 3 & 4 \\ 1 & 3 & 1 \end{vmatrix}=0 \Rightarrow -9(x-1)-(y-2)(-2)+(z-4)\cdot 3=0$$

$\therefore$ The required eqn of projection is

$$x+3y+z+5=0=9x-2y-3z+7$$

(Ans)

(24) A point P moves on the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, which is fixed and the plane through P perpendicular to OP meets axes in A, B, C. If the planes through A, B, C ∥ to the co-ordinate planes meet in a pt Q then show that the locus of Q is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}$$

Ans) Let $P \equiv (\alpha, \beta, \gamma)$

$$\therefore \frac{\alpha}{a}+\frac{\beta}{b}+\frac{\gamma}{c}=1 -$$

d.cs of OP $[\alpha, \beta, \gamma]$ hence plane ⊥ to OP is

$$\alpha x+\beta y+\gamma z=\lambda$$

If it passes thru $P(\alpha, \beta, \gamma)$

$$\therefore \alpha^2+\beta^2+\gamma^2=\lambda$$

hence plane ⊥ to OP & thru P is

$$\alpha x+\beta y+\gamma z=\alpha^2+\beta^2+\gamma^2$$

A.T.Q $A\left(\frac{\alpha^2+\beta^2+\gamma^2}{\alpha}, 0, 0\right)$, $B\left(0, \frac{\alpha^2+\beta^2+\gamma^2}{\beta}, 0\right)$ & $C\left(0, 0, \frac{\alpha^2+\beta^2+\gamma^2}{\gamma}\right)$

from (2)

$$\frac{\alpha}{a(\alpha^2+\beta^2+\gamma^2)}+\frac{\beta}{b(\alpha^2+\beta^2+\gamma^2)}+\frac{\gamma}{c(\alpha^2+\beta^2+\gamma^2)}=\frac{1}{\alpha^2+\beta^2+\gamma^2}$$

$$=\frac{1}{ax_1}+\frac{1}{by_1}+\frac{1}{cz_1}=\frac{1}{\alpha^2+\beta^2+\gamma^2}$$

$$\Rightarrow \frac{1}{ax_1}+\frac{1}{by_1}+\frac{1}{cz_1}=\frac{1}{x_1^2}+\frac{1}{y_1^2}+\frac{1}{z_1^2}$$

∴ The required locus is

$$\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}=\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}$$

(26) Show that the angle b/w the st. line $\frac{x-4}{7} = \frac{y-1}{4} = \frac{z+3}{4}$ and plane $x-2y-2z=8$ is $\sin^{-1}(1/3)$

Ans) d.r.s of $g\left[\frac{x-4}{7} = \frac{y-1}{4} = \frac{z+3}{4}\right]$ are $\left[\pm\frac{7}{9}, \pm\frac{4}{9}, \pm\frac{4}{9}\right]$ taking all ~~+ve~~ (-ve) values

The d.c.s of the normal to the given plane are $\left[\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\right]$

If ϕ be the $\angle$ b/w them

$$\cos\phi = \frac{1}{9\cdot 3}\left[-7\cdot 1 + (-4)(-2) + (-4)(-2)\right]$$

Let θ be the acute $\angle$ b/w them

$$\therefore \cos(90-\theta^\circ) = 1/3$$

$$\sin\theta = \frac{1}{3} \Rightarrow \theta = \sin^{-1}(1/3) \text{ (proved)}$$

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(27) Find the shortest distance and its equation b/w the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ ①, $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ ②

Ans) Let $P_1(3,-15,9)$ is a pt on ① & $Q(-1,1,9)$ on ②

there PQ = orthogonal projection of P_1Q_1 on line PQ

$$= \lambda(-1,-3) + \mu(1+15) + \gamma(9-9)$$

$$\Rightarrow -4\lambda + 16\mu \quad .. \quad (\lambda, \mu, \gamma) .. \text{dcs of PQ}$$

and they are given by

$$\left.\begin{array}{l} 2\lambda - 7\mu + 5 = 0 \\ 2\lambda + \mu - 3 = 0 \end{array}\right\} \Rightarrow \frac{\lambda}{16} = \frac{\mu}{16} = \frac{\gamma}{16}$$

$$\Rightarrow \frac{\lambda}{1} = \frac{\mu}{1} = \frac{\gamma}{1} = \frac{1}{\sqrt{3}}$$

$\therefore$ The required dist is $\frac{4}{\sqrt{3}}$ units (Ans)

(30) Show that the direction cosines l, m, n of two straight lines connected by the relations $l+m+n=0$, $mn-2nl-2lm=0$ are given by $(l:m:n) = (1:1:-2)$ and $(l:m:n) = (1:-2:1)$

Ans) Given;

$$l+m+n=0 \quad \text{---} (1)$$
$$mn-2nl-2lm=0 \quad -(2)$$

Eliminating l from (1) & (2)

$$mn-(2n+2m)(-m-n)=0$$
$$\Rightarrow (2m+n)(m+2n)=0$$
$$\Rightarrow 2m=-n \quad \& \quad m=-2n$$
$$\Rightarrow \frac{m}{1}=\frac{n}{-2} \quad \& \quad \frac{m}{2}=\frac{n}{-1}$$

Case 1: When $n=-2m$, $l-m=0$

$$\Rightarrow \frac{l}{1}=\frac{m}{1}=\frac{n}{-2} \quad \therefore [1:1:-2] \text{ (proved)}$$

Case 2: When $m=-2n$ $l-n=0 \Rightarrow \frac{l}{1}=\frac{n}{1}$

$$\frac{l}{-1}=\frac{m}{2}=\frac{n}{-1} \quad \therefore [-1,2,-1]$$

i.e. $[1:-2:1]$ (proved)

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