1.2 LINEAR AND BERNOULLI'S EQUATION

1.2.1 Linear equations.

A first order differential equation of the form

$$\frac{dy}{dx}+Py=Q \qquad \ldots \qquad (1)$$

where P and Q are functions of x alone or constants, is known as first order linear equations.

To solve such type equations, we use the integrating factor as $e^{\int Pdx}$. Multiplying both sides of (1) by $e^{\int Pdx}$ we get

$$\frac{dy}{dx}e^{\int Pdx}+Pye^{\int Pdx}=Qe^{\int Pdx}$$

or, $\dfrac{d}{dx}\left(ye^{\int Pdx}\right)=Qe^{\int Pdx}$

or, $d\left(ye^{\int Pdx}\right)=Qe^{\int Pdx}dx$.

Integrating both sides, we have

$$ye^{\int Pdx}=\int Qe^{\int Pdx}dx+c,\ c \text{ being a constant.}$$

$$\therefore \quad y=e^{-\int Pdx}\left[\int Qe^{\int Pdx}dx+c\right] \qquad \ldots \qquad (2)$$

which is the required solution of (1).

Note. Sometimes a differential equation takes linear form if we regard x as dependent variable and y as independent variable. Then the equation takes the form $\dfrac{dx}{dy}+Px=Q$. $\qquad \ldots \qquad (3)$

where P, Q are functions of y alone or constants.

In this case I.F. is $e^{\int Pdy}$ and the solution is

$$x=e^{-\int Pdy}\left[\int Qe^{\int Pdy}dy+c\right] \qquad \ldots \qquad (4)$$

Illustrative Examples.

Ex. 1. *Solve :* $\cos^2 x\dfrac{dy}{dx}+y=\tan x.$

The given equation can be written as

$$\frac{dy}{dx}+y\sec^2 x=\tan x\sec^2 x. \quad \ldots \text{(i)}$$

which is a linear equation in y.

Here I. F. $=e^{\int \sec^2 x\,dx}=e^{\tan x}.$

Multiplying both sides of (i) by $e^{\tan x}$, we get

$$\frac{d}{dx}\left(ye^{\tan x}\right)=\tan x\sec^2 x\, e^{\tan x}$$

or, $d\left(ye^{\tan x}\right)=\tan x\sec^2 x\, e^{\tan x}dx$

which on integration gives

$$ye^{\tan x}=\int \tan x\sec^2 x e^{\tan x}dx$$

$=\int ze^z dz$, by putting $\tan x=z$ i.e $\sec^2 x dx=dz$

$=ze^z-e^z+c$

$=e^{\tan x}(\tan x-1)+c$, c being a constant.

$\therefore\quad y=\tan x-1+ce^{-\tan x}$, which is the required solution.

Ex. 2. *Solve :* $(1+y^2)dx=(\tan^{-1}y-x)dy.$

The given equation can be written as

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2} \quad \ldots \text{(i)}$$

which is a linear equation in x.

Here I. F. $=e^{\int \frac{1}{1+y^2}dy}=e^{\tan^{-1}y}.$

Multiplying both sides of (i) by $e^{\tan^{-1}y}$, we get

$$\frac{d}{dy}\left(xe^{\tan^{-1}y}\right)=e^{\tan^{-1}y}.\frac{\tan^{-1}y}{1+y^2}$$

or, $d\left(xe^{\tan^{-1}y}\right)=\dfrac{\tan^{-1}y\, e^{\tan^{-1}y}}{1+y^2}dy$

which on integration gives

$$xe^{\tan^{-1}y}=\int\frac{\tan^{-1}y\, e^{\tan^{-1}y}}{1+y^2}dy$$

$=\int z e^z dz$, by putting $\tan^{-1}y=z$ i.e. $\dfrac{1}{1+y^2}dy=dz$

$=ze^z-e^z+c$, c being a constant.

$=e^{\tan^{-1}y}\left(\tan^{-1}y-1\right)+c.$

$\therefore\quad x=\tan^{-1}y-1+c\,e^{-\tan^{-1}y}$, which is the required solution.

Ex. 3. *Solve :* $(x+y+1)\dfrac{dy}{dx}=1.$

The given equation can be written as

$$\frac{dx}{dy}-x=y+1 \quad \ldots \text{(i)}$$

which is a linear equation in x.

Here I. F. $=e^{\int(-1)dy}=e^{-y}.$

Multiplying both sides of (i) by e^{-y}, we get

$$\frac{d}{dy}\left(xe^{-y}\right)=(y+1)\,e^{-y}$$

or, $d\left(xe^{-y}\right)=(y+1)\,e^{-y}dy$

which on integration gives

$$xe^{-y}=\int(y+1)\,e^{-y}dy$$

$= -ye^{-y} - 2e^{-y} + c$, c being a constant.

$\therefore \quad x = -y - 2 + ce^{y}$

$\therefore \quad x + y + 2 = ce^{y}$, which is the required solution.

Ex. 4. *Show that the equation of the curve whose slope at any point is equal to* $(y + 2x)$ and which passes through the origin is $y = 2(e^{x} - x - 1)$.

The slope of the curve at any point (x, y) is $\frac{dy}{dx}$.

$\therefore$ By the given condition,

$$\frac{dy}{dx} = y + 2x$$

$$\therefore \quad \frac{dy}{dx} - y = 2x \qquad \ldots \quad \text{(i)}$$

which is a linear equation in y.

$\therefore \quad$ I.F. $= e^{\int -dx} = e^{-x}$

Mutiplying both side of (i) by e^{-x} we get,

$$e^{-x}\frac{dy}{dx} - ye^{-x} = 2xe^{-x}$$

or, $\frac{d}{dx}\left(ye^{-x}\right) = 2xe^{-x}$

$\therefore \quad d\left(ye^{-x}\right) = 2xe^{-x}dx$

which on integration gives

$$ye^{-x} = 2\int xe^{-x}dx + c$$

$$= 2\left[x\frac{e^{-x}}{-1} - \int 1.\frac{e^{-x}}{-1}dx\right]$$

$$= 2\left[-xe^{-x} - e^{-x}\right] + c$$

$\therefore \quad y = -2(x + 1) + ce^{x}$

which passes through the origin

$\therefore \quad 0 = -2(0 + 1) + c.1$

$\therefore \quad c = 2$

Thus the required equation of the curve is

$$y = -2(x + 1) + 2e^{x}$$

i.e., $y = 2\left(e^{x} - x - 1\right)$

1.2.2. Bernoulli's equation.

The equation of the type

$$\frac{dy}{dx} + Py = Q.y^{n} \qquad \ldots \quad (5)$$

where P, Q are functions of x only or constants is known as Bernoulli's equation. This equation is reducible to linear equation.

Dividing both sides of (5) by y^{n}, we have

$$y^{-n}\frac{dy}{dx} + Py^{1-n} = Q \qquad \ldots \quad (6)$$

Putting $y^{1-n} = z$ i.e. $(1 - n)y^{-n}\frac{dy}{dx} = \frac{dz}{dx}$

i.e., $y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dz}{dx}$; we get from (6)

$$\frac{1}{1-n}\frac{dz}{dx} + P.z = Q.$$

$$\therefore \quad \frac{dz}{dx}+(1-n)Pz=(1-n)Q \qquad \ldots (7)$$

which is a linear equation in z and can be solved by the method of the previous article.

Remark : (1) Putting $z=\int y^{-n}dy$, (6) can be reduced to a linear form $\frac{dz}{dx}+P(1-n)z=Q$ also.

(2) Another type Bernoulli's equation of the form

$$\frac{dx}{dy}+Px=Qx^n$$

where P, Q are function of y only or constants, can be reduced to linear equation by putting $x^{1-n}=z$

Illustrative Examples.

Ex. 1. *Solve :* $\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x^2}$.

Dividing both sides by y^2, the given equation becomes

$$y^{-2}\frac{dy}{dx}+\frac{1}{x}y^{-1}=\frac{1}{x^2} \qquad \ldots (i)$$

Put $y^{-1}=z$ so that $-y^{-2}\frac{dy}{dx}=\frac{dz}{dx}$

i.e., $y^{-2}\frac{dy}{dx}=-\frac{dz}{dx}$

(student can solve by putting

$z=\int y^{-2}dy=-y^{-1}$ also)

$\therefore$ Equation (i) becomes $-\frac{dz}{dx}+\frac{1}{x}z=\frac{1}{x^2}$

i.e., $\frac{dz}{dx}-\frac{1}{x}z=-\frac{1}{x^2} \qquad \ldots (ii)$

which is a linear equation in z.

So I. F. $=e^{-\int\frac{1}{x}dx}=e^{-\log x}=x^{-1}=\frac{1}{x}$

Multiplying both sides of (ii) by $\frac{1}{x}$, we get

$$\frac{d}{dx}\left(z.\frac{1}{x}\right)=-\frac{1}{x^3}$$

or, $d\left(z.\frac{1}{x}\right)=-\frac{1}{x^3}dx$

which on integration gives

$$z.\frac{1}{x}=-\int\frac{1}{x^3}dx=\frac{1}{2x^2}+c$$

or, $y^{-1}\frac{1}{x}=\frac{1}{2x^2}+c$

$\therefore$ $\frac{1}{xy}=\frac{1}{2x^2}+c$, which is the required solution.

Ex. 2. *Solve*

$\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y$. [W.B.U.T. 2007,2016]

The given equation can be written as

$$\cos y\frac{dy}{dx}-\frac{\sin y}{1+x}=(1+x)e^x \qquad \ldots (i)$$

Putting $\sin y=z$ so that $\cos y\frac{dy}{dx}=\frac{dz}{dx}$ in (i), we get

$$\frac{dz}{dx}-\frac{z}{1+x}=(1+x)e^x \qquad \ldots (ii)$$

which is a linear equation in z.

$$\therefore \quad \text{I.F.} = e^{-\int \frac{1}{1+x}dx}$$

$$= e^{-\log(1+x)}$$

$$= (1+x)^{-1}$$

Multiplying both sides of (2) by $(1+x)^{-1}$ we get

$$(1+x)^{-1}\frac{dz}{dx} - z(1-1x)^{-2} = e^x$$

or, $\quad d\left\{z(1+x)^{-1}\right\} = e^x dx$

which on integration gives

$$z(1+x)^{-1} = e^x + c$$

or, $\quad \sin y (1+x)^{-1} = e^x + c$

$\therefore \quad \sin y = (1+x)\left(e^x + c\right)$

which is the required general solution.

Ex. 3. *Solve :* $\dfrac{dy}{dx} + \dfrac{1}{x}\sin 2y = x^3 \cos^2 y.$

Dividing by $\cos^2 y$, the given equation becomes

$$\sec^2 y \frac{dy}{dx} + \frac{1}{x} 2\tan y = x^3. \qquad \ldots \text{(i)}$$

Put $\tan y = z$, so that $\sec^2 y \dfrac{dy}{dx} = \dfrac{dz}{dx}$

Therefore the equation (i) becomes

$$\frac{dz}{dx} + \frac{1}{x} 2z = x^3 \qquad \ldots \text{(ii)}$$

which is a linear equation in z.

So I. F. $= e^{\int \frac{2}{x}dx} = e^{\log x^2} = x^2.$

Multiplying both sides of (ii) by x^2, we get

$$\frac{d}{dx}(z.x^2) = x^5$$

or, $\quad d\,(z.x^2) = x^5 dx$

which on integration gives

$$zx^2 = \frac{x^6}{6} + \frac{c}{6}, \text{ c being a constant.}$$

$\therefore \quad 6x^2 \tan y = x^6 + c$, which is the required solution.

Ex. 4. *Solve,*

$$\frac{dy}{dx} + \frac{y\log y}{x} = \frac{y(\log y)^2}{x^2}$$

[W.B.U.T. 2006]

The given equation can be written as

$$\frac{1}{y(\log y)^2}\frac{dy}{dx} + \frac{1}{x}\cdot\frac{1}{\log y} = \frac{1}{x^2} \qquad \ldots \text{(i)}$$

Putting $z = \dfrac{1}{\log y}$ so that

$$\frac{dz}{dx} = -\frac{1}{(\log y)^2}\cdot\frac{1}{y}\cdot\frac{dy}{dx}$$

i.e., $\quad \dfrac{1}{y(\log y)^2}\dfrac{dy}{dx} = -\dfrac{dz}{dx}$

in (i) we get,

$$-\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$$

$$\therefore \frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$$

which is a linear equation in z

$$\therefore \text{I.F.} = e^{-\int \frac{1}{x}dx}$$

$$= e^{-\log x}$$

$$= x^{-1}$$

Multiplying both sides of (i) by x^{-1}, we get,

$$x^{-1}\frac{dz}{dx} - zx^{-2} = -x^{-3}$$

or, $\frac{d}{dx}\left(zx^{-1}\right) = -x^{-3}$

$$\therefore d\left(zx^{-1}\right) = -x^{-3}dx$$

which on integration gives

$$zx^{-1} = -\frac{x^{-2}}{-2} + c$$

or, $\frac{z}{x} = \frac{1}{2x^2} + c$

or, $\frac{1}{x\log y} = \frac{1}{2x^2} + c$

or, $\frac{1}{\log y} = \frac{1+2cx^2}{2x}$

$$\therefore \log y\left(1+2cx^2\right) = 2x$$

which is the required general solution.

Ex. 5. *Solve :*

$$x\frac{dy}{dx} + y = y^2 \log x$$ [WBUT 2008]

The given equation can be written as

$$\frac{dy}{dx} + y\cdot\frac{1}{x} = y^2\frac{\log x}{x} \quad \ldots \text{(i)}$$

It is of Bernoulli's (i) by y^2 we get,

Diving both sides of (i) by y^2 we get

$$y^{-2}\frac{dy}{dx} + y^{-1}\frac{1}{x} = \frac{\log x}{x} \quad \ldots \text{(ii)}$$

Put $y^{-1} = z$

so that $-y^{-2}\frac{dy}{dx} = \frac{dz}{dx}$

$$\therefore y^{-2}\frac{dy}{dx} = -\frac{dz}{dx}$$

$\therefore$ Then (i) takes the form

$$-\frac{dz}{dx} + z.\frac{1}{x} = \frac{\log x}{x}$$

$$\therefore \frac{dz}{dx} - z.\frac{1}{x} = -\frac{\log x}{x} \quad \ldots \text{(iii)}$$

which is a linear equation in z

$$\therefore \text{I.F.} = e^{\int -\frac{1}{x}dx}$$

$$= e^{-\log x} = x^{-1}$$

Multiplying both sides of (iii) by x^{-1} we get,

$$x^{-1}\frac{dz}{dx} - zx^{-2} = -x^{-2}\log x$$

or, $\frac{d}{dx}\left(zx^{-1}\right) = -x^{-2}\log x$

or, $d\left(zx^{-1}\right) = -x^{-2}\log x\, dx$

which on integration gives

$$zx^{-1} = -\int x^{-2}\log x\, dx$$

or, $y^{-1}x^{-1} = -\left\{\log x\frac{x^{-1}}{-1} - \int \frac{1}{x}\frac{x^{-1}}{-1}dx\right\}$

or, $\frac{1}{xy} = \frac{\log x}{x} - \frac{x^{-1}}{-1} + c$

$\therefore$ $\frac{1}{xy} = \frac{\log x + 1}{x} + c$

which is the required general solution.

Ex. 6. *Solve :* $y\,(2xy + e^x)\,dx - e^x dy = 0.$

The given equation can be written as

$$\frac{dy}{dx} - y = 2y^2xe^{-x}.$$

Dividing both sides by y^2, we get

$$y^{-2}\frac{dy}{dx} - y^{-1} = 2xe^{-x}. \quad \ldots \quad \text{(i)}$$

Put $y^{-1} = z$, so that $-y^{-2}\frac{dy}{dx} = \frac{dz}{dx}$

i.e., $y^{-2}\frac{dy}{dx} = -\frac{dz}{dx}$.

The equation (i) becomes

$$-\frac{dz}{dx} - z = 2xe^{-x}$$

$$\therefore \quad \frac{dz}{dx} + z = -2xe^{-x} \quad \ldots \quad \text{(ii)}$$

which is a linear equation in z.

So I. F. $= e^{\int 1.dx} = e^x$.

Multiplying both sides of (ii) by e^x, we get

$$\frac{d}{dx}(z\,e^x) = -2x$$

or, $d\,(z\,e^x) = -2x\,dx$

which on integration gives

$z\,e^x = -x^2 + c,$ c being a constant.

$\therefore$ $y^{-1}e^x = c - x^2$, which is the required solution.

Ex. 7. *Solve*

$$\frac{dy}{dx} + y = y^3(\cos x - \sin x)$$ [W.B.U.T. 2009, 1010]

The given equation is of Bernoulli's form.

Dividing the given equation by y^3 we get,

$$y^{-3}\frac{dy}{dx} + y^{-2} = \cos x - \sin x \quad \ldots \quad \text{(i)}$$

Put $y^{-2} = z$

or, $\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$

$\therefore$ $y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dz}{dx}$

$\therefore$ (i) takes the form

$$-\frac{1}{2}\frac{dz}{dx}+z=\cos x-\sin x$$

$\therefore$ $$\frac{dz}{dx}-2z=2(\sin x-\cos x) \qquad \ldots \text{(ii)}$$

which is a linear equation in z.

$\therefore$ I.F. $=e^{\int -2dx}=e^{-2x}$

Multiplying both sides of (ii) by e^{-2x} we get,

$$e^{-2x}\frac{dz}{dx}-2ze^{-2x}=2(\sin x-\cos x)e^{-2x}$$

or, $d\left(ze^{-2x}\right)=2(\sin x-\cos x)e^{-2x}dx$

which on integration gives

$$ze^{-2x}=2\int(\sin x-\cos x)e^{-2x}dx+c \qquad \ldots \text{(iii)}$$

Let I $=\int(\sin x-\cos x)e^{-x}dx$

$$=(\sin x-\cos x)\frac{e^{-2x}}{-2}-\int(\cos x+\sin x)\frac{e^{-2x}}{-2}dx$$

$$=-\frac{(\sin x-\cos x)e^{-2x}}{2}+$$

$$\frac{1}{2}\left\{(\cos x+\sin x)\frac{e^{-2x}}{-2}-\int(-\sin x+\cos x)\frac{e^{-2x}}{-2}dx\right\}$$

$$=-\tfrac{1}{2}(\sin x-\cos x)e^{-2x}-\tfrac{1}{4}(\cos x+\sin x)e^{-2x}-$$

$$\tfrac{1}{4}\int(\sin x-\cos x)e^{2x}dx$$

$\therefore$ $$I=-\tfrac{1}{2}(\sin x-\cos x)e^{-2x}-\tfrac{1}{4}(\cos x+\sin x)e^{-2x}-\tfrac{1}{4}I$$

or, $I\left(1+\tfrac{1}{4}\right)=-\tfrac{1}{2}(\sin x-\cos x)e^{-2x}-\tfrac{1}{4}(\cos x+\sin x)e^{-2x}$

or, $\frac{5I}{4}=e^{-2x}\left\{-\tfrac{1}{2}\sin x+\tfrac{1}{2}\cos x-\tfrac{1}{4}\cos x-\tfrac{1}{4}\sin x\right\}$

$$=e^{-2x}\left(-\tfrac{3}{4}\sin x+\tfrac{1}{4}\cos x\right)$$

$\therefore$ $$I=\frac{e^{-2x}}{5}(\cos x-3\sin x)$$

$\therefore$ (iii) gives,

$$ze^{-2x}=\tfrac{2}{5}e^{-2x}(\cos x-3\sin x)+c$$

or, $y^{-2}e^{-2x}=\tfrac{2}{5}e^{-2x}(\cos x-3\sin x)+c$

$\therefore$ $$\frac{1}{y^2}=\tfrac{2}{5}(\cos x-3\sin x)+ce^{2x}$$

which is the required general solution.

EXERCISE

[I] Short Answer Questions

1. Find the general solution of the differential equation $\frac{dy}{dx}+Py=0$ where P is a function of x only.

2. Find the general solution of $\frac{dy}{dx}-y=Q$ where Q is a function of x only.

3. Convert the equation $\frac{dy}{dx}+\frac{x}{1-x^2}y=x\sqrt{y}$ into a linear equation.

4. Show that the equation $\frac{dy}{dx}+\frac{4x}{x^2+1}y=\frac{1}{\left(x^2+1\right)^3}$ is not exact. Find the factor which makes it exact after mulitplication.

5. Arrange the differential equation $1+y^2+\left(x-e^{-\tan^{-1}y}\right)\frac{dy}{dx}=0$ as a linear form.

6. Reduce the equation $\frac{dy}{dx}+y\cos x=y^n\sin 2x$ to a linear form.

7. Find an IF of the differential equation
$\frac{dx}{dy}+(1-n)x\cos y=(1-n)\sin 2y$.

8. Show that the equation $\{p(x)+q(y)\}dx+\{r(x)+s(y)\}dy=0$ is exact if and only if $q(y)dx+r(x)dy=0$ be exact.

9. Show the equation $p(x)dx+q(x)r(y)dy=0$ is exact if and only if $q(x)$ be constant.

10. If $Mdx+Ndy=0$ and $Pdx+Qdy=0$ are exact, is
$(M+P)dx+(N+Q)dy=0$ exact ?

11. If $F(x,y)=C$ is the general solution of a differential equation, then find a particular solution satisfying the initial condition $y(a)=b$.

Answers

1. $y=ce^{-\int Pdx}$ 2. $y=e^x\left(\int e^{-x}Qdx+C\right)$

3. $\frac{dz}{dx}+\frac{x}{2(1-x^2)}z=\frac{x}{2}$ 4. $(x^2+1)^2$

5. $\frac{dx}{dy}+\frac{1}{1+y^2}\cdot x=\frac{e^{-\tan^{-1}y}}{1+y^2}$; this is linear in x.

6. $\frac{dz}{dx}+(1-n)z\cos x=(1-n)\sin 2x$ 7. $e^{(1-n)\sin y}$

[II] Long Answer Questions

Solve the following equations (1-33)

1. $\frac{dy}{dx}+2xy=e^{-x^2}$.

2. $(x^2+1)\frac{dy}{dx}+2xy=4x^2$.

3. $(x^3y^2+xy)dx=dy$.

4. $\frac{dy}{dx}+y\tan x=y^3\cos x$.

5. $\left(x+2y^3\right)\frac{dy}{dx}=y$

6. $(1+x)\frac{dy}{dx}-xy=1-x$

7. $x(x-y)\,dy+y^2dx=0$.

8. $\frac{dy}{dx}=\frac{y}{x+\sqrt{xy}}$.

9. $xy(1+xy^2)\frac{dy}{dx}=1$.

10. $(x^2y^3+2xy)\,dy=dx$.

11. $(1-x^2)\frac{dy}{dx}-xy=1$.

12. $\{y(1-x\tan x)+x^2\cos x\}dx-xdy=0$

13. $\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$

14. $\frac{dy}{dx}+\frac{y\ln y}{x-\ln y}=0$

15. $y'+y=\frac{1}{1+e^{2x}}$

16. $\left(1+x^2\right)dy+2xydx=\cot x\,dx$

17. $dx+(3y-x)dy=0$

18. $y^2dx+\left(xy-2y^2-1\right)dy=0$

19. $ydx - (x + 2y^3)dy = 0$

20. $\cos x dy = y(\sin x - y)dx$

21. $(xy^3 + y)dx - dy = 0$

22. $xy'' - 3y' = 4x^2$

23. $(4e^{-y}\sin x - 1)dx - dy = 0$

24. $xy - \frac{dy}{dx} = y^3 e^{-x^2}$

25. $(x^2+1)\frac{dy}{dx} - 2xy = (x^4 + 2x^2 + 1)\cos x$

26. $x\frac{dy}{dx} + y = y^2x^3\cos x$

27. $\frac{dy}{dx} + \frac{4x}{x^2+1}y = \frac{1}{(x^2+1)^3}$

28. $y - \cos x\frac{dy}{dx} = y^2(1-\sin x)\cos x$, given that $y = 2$ when $x = 0$.

29. $x(1-4y)dx - (x^2+1)dy = 0$ with $y(2) = 1$

30. $2xyy' = y^2 - 2x^3, y(1) = 2$

31. $y' + \frac{y}{2x} = \frac{x}{y^3}, \ y(1) = 2$

32. Reduce the equation $\sin y \cdot \frac{dy}{dx} = \cos x(2\cos y - \sin^2 x)$ to a linear equation and hence solve it.

33. Solve : $\sin x\frac{dy}{dx} + 4y\cos x = \cot x$, given that $y = \frac{4}{3}$ when $x = \frac{\pi}{2}$.

Answers

1. $ye^{x^2} = x + c.$

2. $3(x^2+1)y = 4x^3 + c.$

3. $\frac{1}{y} = x^2 - 2 + c\,e^{-\frac{1}{2}x^2}.$

4. $y^2(\sin 3x + 9\sin x + c) + 6\cos^2 x = 0$

5. $x = y(y^2 + c)$

6. $y(1+x) = x + ce^x$

7. $y/x = \log y + c.$

8. $\sqrt{x} = \sqrt{y}\,(\log\sqrt{y} + c).$

9. $\frac{1}{x} = 2 - y^2 + c\,e^{-\frac{1}{2}y^2}.$

10. $\frac{2}{x} = 1 - y^2 + c\,e^{-y^2}.$

11. $y\sqrt{1-x^2} = c + \sin^{-1}x.$

12. $y = x^2\cos x + cx\cos x$

13. $ye^{\tan^{-1}x} = \frac{1}{2}\left(e^{\tan^{-1}x}\right)^2 + c$

14. $2x\ln y = (\ln y)^2 + 2c$

15. $y = \tan^{-1}x + ce^{-x}$

16. $y(1+x^2) = \log\sin x + c$

17. $x - 3y - 3 = ce^y$

18. $xy = y^2 + \ln y + c$

19. $x = y^3 + cy$

20. $\sec x = y(\tan x + c)$

21. $\frac{1}{y^4} + x = \frac{1}{4} + ce^{-4x}$

22. $y = c_1x^4 - \frac{4}{3}x^3 + c_2$

23. $e^y = 2(\sin x - \cos x) + ce^{-x}$

24. $y^2(2x + c) = e^{x^2}$

25. $y = (x^2+1)(\sin x + c)$

26. $xy(x\sin x + \cos x + c) + 1 = 0$

27. $y(x^2+1)^2 = \tan^{-1}x + c$

28. $2(\tan x + \sec x) = y(2\sin x + 1)$

29. $4y(x^2+1)^2 = x^4 + 2x^2 + 76$

30. $y^2 = x(5 - x^2)$

31. $x^2y^4 = x^4 + 15$

32. $\cos y = \frac{1}{2}\sin^2 x - \frac{1}{2}\sin x + ce^{-2\sin x} + \frac{1}{4}$

33. $y\sin^4 x = \frac{1}{3}\sin^3 x + 1.$

MULTIPLE CHOICE QUESTIONS

[III]

1. The general form of a first order linear equation in x is

$\frac{dx}{dy}+Px=Q$ where

(a) P and Q are both functions of x
(b) P and Q are both functions of y
(c) P is a function of x and Q is a function of y
(d) P is a function of y and Q is a function of x

2. The IF of $\cos x\frac{dy}{dx}+y\sin x=1$ is

(a) $\cos x$
(b) $-\sec^2 x$
(c) $\sec x$
(d) $\log\sec x$

3. The integration factor of the differential equation

$2x^2\frac{dy}{dx}+4x^3y=x^2$ is

(a) e^{x^3}
(b) e^{x^2}
(c) e^x
(d) $2e^x$

4. Integration factor of $x\frac{dy}{dx}-y=xe^x$ is

(a) x
(b) e^x
(c) e^{-x}
(d) $\frac{1}{x}$

5. The IF of $\frac{dy}{dx}+2xy=x^3$ is

(a) x^2
(b) e^{x^2}
(c) x^3
(d) e^{2x}

6. An integrating factor of $\frac{dy}{dt}+y=1$ is

(a) e^t
(b) $\frac{e}{t}$
(c) et
(d) $\frac{t}{e}$

[W.B.U.T. 2007, 2008]

7. The IF of the equation $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{-\tan^{-1}y}}{1+y^2}$ is

(a) $\tan^{-1}y$
(b) $e^{\tan^{-1}y}$
(c) $e^{\cot^{-1}y}$
(d) none

[W.B.U.T. 2014]

8. The differential equation $x\frac{dy}{dx}+y=y^2\sin x$ is a first order

(a) linear equation in x
(b) linear equation in y
(c) Bernoulli's equatiion
(d) homogeneous equation

9. The integrating factor of $\frac{dx}{dy}+\frac{2x\log y}{y}=2$ is

(a) $e^{-(\log y)^2}$
(b) $e^{2\log y}$
(c) $e^{(\log y)^2}$
(d) none of these

10. The IF of $\frac{dy}{dx}+\frac{x}{2(1-x^2)}y=\frac{x}{2}$ is

(a) $(1-x^2)^{-\frac{1}{4}}$
(b) $\sqrt{1-x^2}$
(c) $\log(1-x^2)$
(d) none

11. The equation $\frac{dy}{dx}+x\sin 2y = x^3\cos^2 y$ can be reduced to one of the following linear forms

(a) $\frac{dz}{dx}+xz = x^3$ (b) $\frac{dz}{dx}-2xz = x^3$

(c) $\frac{dz}{dx}+2xz = x^3$ (d) none [W.B.U.T 2012]

12. The differential equation

$(ax+by+c)dx+(Ax+By+C)dy=0$ is exact if and only if

(a) $a = B$ (b) $b = A$

(c) $a = b, A = B$ (d) none

13. The I.F. of the differential equation

$\frac{dy}{dx}-3y=\sin 2x$ is

(a) e^{3x} (b) e^{-3x}

(c) e^{x} (d) none of these [W.B.U.T 2011]

14. The integrating factor of $\cos x\frac{dy}{dx}+y\sin x=1$ is

(a) $\tan x$ (b) $\cos x$

(c) $\sec x$ (d) $\sin x$

15. I.F. of the differential equation

$x\log x\frac{dy}{dx}+y=2\log x$

(a) $\log x$ (b) x

(c) $\log(\log x)$ (d) e^x

ANSWERS

1.b	2.c	3. b	4.d	5.b	6.a	7.b
8.c	9.c	10.a	11.c	12.b	13.b	14.c
15.a						