CHAPTER

12
Fluid Statics

12.1 INTRODUCTION

Fluid statics deals with problems associated with fluids at rest. Fluid statics is generally referred to as ***hydrostatics*** when the fluid is a liquid and as ***aerostatics*** when the fluid is a gas. In fluid statics, there is no relative motion between adjacent fluid layers. That means, there is no shear stress acting on the fluid. The only stress acting on the fluid element is the normal stress, which is manifested in the form of pressure.

12.2 PRESSURE

A fluid will exert a force normal to a solid boundary or any plane drawn through the fluid. Consider a small area δA in a stationary fluid. Let δF be the force acting on the area δA in the normal direction. Mathematically, the pressure at a point in a stationary fluid is

$$P = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}$$

If the force is uniformly distributed over the area A, then the pressure at any point is given by

$$P = \frac{F}{A}$$

12.3 PASCAL'S LAW FOR PRESSURE AT A POINT

Pascal's law stats that pressure (or intensity of pressure) at a point in a static fluid is equal in magnitude in all directions.

To demonstrate it, let a small wedge shaped fluid element in static condition be considered. Let us assume that P_1, P_2, and P_3 are the pressures acting on the three surfaces as shown in Fig. 12.1. Since static condition, the tangential force exerted by the surrounding fluid elements is zero, the forces acting on the fluid element are pressure forces on the surfaces and the gravity forces. Let us also assume that the gravity forces are acting along negative y direction.

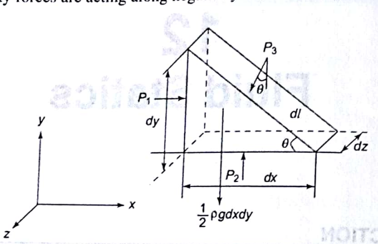

Figure 12.1 *Static equilibrium of a fluid element*

From Newton's second law, a force balance in the x and y direction gives:

$$\sum F_x = 0$$

$$P_1 dydz - P_3 dzdl \sin\theta = 0 \tag{12.1}$$

$$\sum F_y = 0$$

$$P_2 dxdz - P_3 dldz \cos\theta - \frac{1}{2}\rho g dl(dy\cos\theta)dz = 0 \tag{12.2}$$

where ρ is the density and $\frac{1}{2}\rho g dl(dy\cos\theta)dz$ is the weight of the fluid element. From the figure (right angle triangle) $dx = dl\cos\theta$ and $dy = dl\sin\theta$.

Substituting these on Eq. (12.1) and (12.2)

$$P_1 - P_3 = 0$$

or,

$$P_1 = P_3 \tag{12.3}$$

$$P_2 - P_3 - \frac{1}{2}\rho g dl \sin\theta = 0 \tag{12.4}$$

As $dl \rightarrow 0$ (the fluid element shrinks to a point), we have

$$P_2 = P_3 \tag{12.4a}$$

Therefore, from Eq. (12.3) and (12.4a) $P_1 = P_2 = P_3$ i.e., **the pressure at a point in a fluid has the same magnitude in all possible directions.**

12.4 BASIC EQUATION OF FLUID STATICS

Consider a differential fluid element at rest in rectangular Cartesian Co-ordinates with z axis vertically upward, as shown in Fig. 12.2. The forces acting on the fluid element at rest, are of surface forces and body forces. The only body force acting is the gravity force.

The body force due to gravity acting on the fluid element is $\rho g\Delta x\Delta y\Delta z$ which is acting vertically downward. Therefore, the body force acting along the z direction is $-\rho g\Delta x\Delta y\Delta z$.

Since the fluid element is at rest, the shear stress acting on the element will be zero. The only surface force is the pressure force.

Let the pressure at the center of the element be P. the surface forces acting on the different faces are shown in Fig. 12.2.

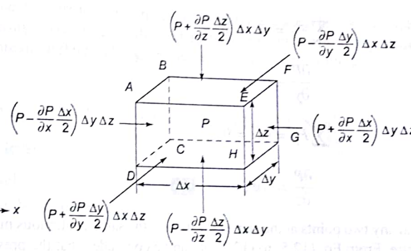

Figure 12.2 *Variation of fluid pressure*

The pressure at the bottom face (normal to z axis) of the element is $P - \frac{\partial P}{\partial z}\frac{\Delta z}{2}$

The pressure at the top face (normal to z axis) of the element is $P + \frac{\partial P}{\partial z}\frac{\Delta z}{2}$

The forces acting on faces normal to y direction are $\left(P - \frac{\partial P}{\partial y}\frac{\Delta y}{2}\right)\Delta x\Delta z$ and $\left(P + \frac{\partial P}{\partial y}\frac{\Delta y}{2}\right)\Delta x\Delta y$ respectively.

Similarly the surface forces acting on the left and right faces (normal to x direction) are $\left(P - \frac{\partial P}{\partial x}\frac{\Delta x}{2}\right)\Delta y\Delta z$ and $\left(P + \frac{\partial P}{\partial x}\frac{\Delta x}{2}\right)\Delta y\Delta z$ respectively

The forces acting on faces normal to y direction are $\left(P - \frac{\partial P}{\partial y}\frac{\Delta y}{2}\right)\Delta x\Delta z$ and $\left(P + \frac{\partial P}{\partial y}\frac{\Delta y}{2}\right)\Delta x\Delta z$ respectively.

Net forces acting on the element in the x direction is $\delta F_x = -\frac{\partial P}{\partial x}\Delta x \Delta y \Delta z$.

Similarly, net forces acting on the fluid element in the y and z direction are $\delta F_y = -\frac{\partial P}{\partial y}\Delta x \Delta y \Delta z$, and

$\delta F_z = -\frac{\partial P}{\partial z}\Delta x \Delta y \Delta z - \rho g \Delta x \Delta y \Delta z$ respectively.

From Newton's second law, a force balance in the x, y and z directions gives:

$$\sum F_x = ma_x = 0$$

or,

$$\frac{\partial P}{\partial x} = 0 \tag{12.5}$$

$$\sum F_y = ma_y = 0$$

or,

$$\frac{\partial P}{\partial y} = 0 \tag{12.6}$$

$$\sum F_z = ma_z = 0$$

or,

$$\frac{\partial P}{\partial z} = -\rho g \tag{12.7}$$

It implies that any two points at the same elevation in the same continuous mass of fluid at rest have the same pressure. From Eq. (12.5) to (12.7), it can be concluded that the pressure P is a function of z only. Therefore, Eq. (12.7) can be written as

$$\frac{dp}{dz} = -\rho g \tag{12.8}$$

Equation (12.8) relates the change of pressure to change of elevation and is applicable to both compressible and incompressible fluids.

12.4.1 Pressure Variation in an Incompressible Fluid

For most problems involving liquids, it is usual to assume that the density ρ is constant, and the same assumption can also be made for a gas if pressure differences are very small. Equation (12.8) can be written as

$$\frac{dP}{dz} = -\rho g$$

For an incompressible fluid, the density ρ does not change with change in pressure P.

If the pressure at the reference level, z_0, is designated as P_0 then the pressure P at the location z is found by integrating the Eq. (12.8),

$$\int_{P_0}^{P} dP = \int_{z_0}^{z} -\rho g dz$$

$$P - P_0 = -\rho g(z - z_0) = \rho g(z_0 - z)$$

For liquids, it is convenient to take the origin of the coordinate system at the reference level and to measure distances as positive downward from the reference level.

$$z_0 - z = h$$

$$P - P_0 = \rho g(z_0 - z) = \rho g h \tag{12.9}$$

Equation (12.9) indicates that the pressure difference between two points in a static fluid can be determined by measuring the elevation difference between the two points.

Example 12.1 Calculate the pressure at an altitude of 5 km above sea level if the atmospheric pressure and density at sea level are 101.3 kPa and 1.22 kg/m^3 respectively. Assume air as an incompressible fluid and neglect the variation of g with altitude.

Solution

Atmospheric pressure at the sea level $P_0 = 101.3\text{ kPa} = 101.3 \times 10^3\text{ N/m}^2$,

Density of air at the sea level $\rho_0 = 1.22\text{ kg/m}^3$

Altitude from the sea level $z - z_0 = 5\text{ km} = 5000\text{ m}$

Pressure at any point can be found from Eq. (12.9) as

$$P = P_0 - \rho g(z - z_0)$$

Substituting the values in the above equation, we get

$$P = 101.3 \times 10^3 - 1.22 \times 9.81 \times 5000$$

$$= 41459\text{ N/m}^2 = 41.459\text{ kN/m}^2$$

Note: *Example 12.1 illustrates that pressure decreases with an altitude from the sea level.*

12.4.2 Pressure Variation in a Compressible Fluid

So far we have considered the variation of pressure for incompressible fluid for which density variation with change in pressure is not significant and is neglected. Now, we will consider the case of pressure variation in a compressible fluid for which density varies with pressure.

For Isothermal fluid

For isothermal fluid, one can write

$$\frac{P}{\rho} = \frac{P_0}{\rho_0}$$

where P_0 and ρ_0 are the pressure and density at some arbitrary reference level z_0.

or,
$$\rho = \frac{\rho_0}{P_0} P$$

From Eq. (12.8), we have
$$\frac{dP}{dz} = -\rho g$$

Substituting the value of ρ in the above equation, we get
$$\frac{dP}{dz} = -\frac{\rho_0}{P_0} P g$$

or,
$$\frac{dP}{P} = -\frac{\rho_0}{P_0} g dz$$

Integrating, we obtain
$$\int_{P_0}^{P} \frac{dP}{P} = \int_{z_0}^{z} -\frac{\rho_0}{P_0} g dz \qquad [P = P_0 \text{ at } z = z_0]$$

or,
$$\ln \frac{P}{P_0} = \left[-\frac{\rho_0}{P_0} g(z - z_0)\right]$$

or,
$$P = P_0 \exp\left[-\frac{\rho_0}{P_0} g(z - z_0)\right] \qquad (12.10)$$

Equation (12.10) shows that the pressure decreases exponentially with the elevation (altitude) for an isothermal fluid.

Example 12.2 Calculate the pressure and density of air at an altitude of 10 km above sea level if the atmospheric pressure and density at sea level are 101.3 kPa and 1.22 kg/m^3 respectively. Assume isothermal process and neglect the variation of g with altitude.

Solution

Atmospheric pressure at the sea level $P_0 = 101.3$ kPa

Density of air at the sea level $\rho_0 = 1.22$ kg/m^3

Altitude from the sea level $z - z_0 = 10 \text{ km} = 10000$ m

Pressure at any point can be found from Eq. (12.10) as
$$P = P_0 \exp\left[-\frac{\rho_0}{P_0} g(z - z_0)\right]$$
$$= 101.3 \times 10^3 \exp\left[-\frac{1.22}{101.3 \times 10^3} \times 9.81 \times 10000\right]$$
$$= 31.08 \times 10^3 \text{ Pa} = 31.08 \text{ kPa}$$

For isothermal process, we have
$$\frac{P}{\rho} = \frac{P_0}{\rho_0}$$

Substituting the values of p_0, ρ_0 and p, one can get
$$\frac{31.08}{\rho} = \frac{101.3}{1.22}$$

or
$$\rho = \frac{31.08 \times 1.22}{101.3} = 0.3743 \text{ kg/m}^3$$

For non-isothermal fluid

From the characteristic equation of ideal gas, one can write
$$P = \rho RT$$

where R is the characteristic gas constant.

Substituting the value of ρ in Eq. (12.8), we obtain
$$\frac{dP}{dz} = -\rho g = -\frac{P}{RT} g \qquad (12.11)$$

It has observed that up to a certain altitude temperature varies (decreases) linearly with elevation. The temperature variation can be expressed as
$$T = T_0 + \beta(z - z_0)$$

where β is the temperature lapse rate (negative), and T_0 is the temperature at $z = z_0$.

Substituting the value of T in Eq. (12.11), we have
$$\frac{dP}{dz} = -\frac{P}{R[T_0 + \beta(z - z_0)]} g$$

or,
$$\frac{dP}{P} = -\frac{g}{R[T_0 + \beta(z - z_0)]} dz \qquad (12.12)$$

Integration of Eq. (12.12) yields
$$\ln \frac{P}{P_0} = -\frac{g}{R\beta} \ln \frac{T_0 + \beta(z - z_0)}{T_0}$$

or,
$$\frac{P}{P_0} = \left(\frac{T_0 + \beta(z - z_0)}{T_0}\right)^{-\frac{g}{R\beta}} \qquad (12.13)$$

or,
$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{-\frac{g}{R\beta}} \qquad (12.14)$$

Equation (12.13) or (12.14) shows that the pressure decreases with elevation and depends on g, R and β.

Example 12.3 Calculate the atmospheric pressure at the end of troposphere, which extends up to a height of 9 km from sea level. Consider a temperature variation in the troposphere as $T = 288 - 0.006z$, where z is the elevation in metres and T is the temperature in Kelvin. The atmospheric pressure and temperature at sea level are 101.3 kN/m^2 and 288 K respectively. For air, R = 287 J/kg-K.

Solution

Altitude from the sea level	$z = 9$ km $= 9000$ m
Atmospheric pressure at the sea level	$p_0 = 101.3$ kPa
Atmospheric temperature at the sea level	$T_0 = 288$ K
Characteristic gas constant of air	$R = 287$ J/kg-K
Temperature variation	$T = 288 - 0.006z$
Temperature lapse rate	$\beta = -0.006$

Temperature at an altitude of 9 km from sea level is

$$T = 288 - 0.006 \times 9000 = 234 \text{ K}$$

Pressure at any point can be found from Eq. (12.14) as

$$P = P_0\left(\frac{T}{T_0}\right)^{-\frac{g}{R\beta}}$$

$$P = 101.3\left(\frac{234}{288}\right)^{-\frac{9.81}{287\times(-0.006)}}$$

$$= 30.998 \text{ kN/m}^2$$

The atmospheric pressure at the end of troposphere is 30.998 kN/m^2.

12.5 UNITS AND SCALE OF PRESSURE MEASUREMENT

Pressure may be expressed with reference to any arbitrary datum. The usual data are absolute zero and local atmospheric pressure. When a pressure is expressed as a difference between its value and a complete vacuum, it is called an ***absolute pressure***. When it is expressed as a difference between its value and the local atmospheric pressure, it is called a ***gauge pressure***.

Mathematically, one can write (refer Fig. 12.3)

$$P_{abs} = P_{atm} + P_{gauge} \qquad (12.15)$$

$$P_{vac} = P_{atm} - P_{abs} \qquad (12.16)$$

At sea level and 15°C, the international standard atmosphere has been chosen as 1.01325 bar (= 10.34 m of water = 760 mm of mercury). In the SI system, the unit of pressure is N/m^2, also known as ***Pascal***. To express large magnitudes, bar is also used as a unit of pressure.

$$1 \text{ bar} = 10^5 \text{ Pascal}$$

The unit mm of Hg is also called *torr* in honour of Torricelli. Therefore,

$$1 \text{ atm} = 760 \text{ torr}$$

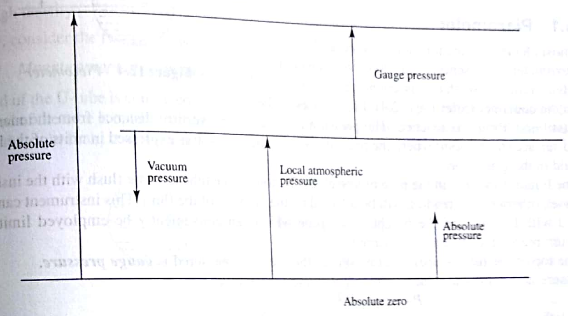

Figure 12.3 *Illustration of absolute, gauge and vacuum pressure readings*

Example 12.4 Calculate the pressure in N/m^2 corresponding to
(i) 6 m of water column
(ii) 10 cm of mercury column
(iii) 4 cm of column of a fluid of specific gravity 0.7

Solution (i) h = 6 m of water column

$$P = \rho g h$$
$$= 1000 \times 9.81 \times 6 = 58860 \text{ N/m}^2$$

(ii) h = 10 cm of mercury column = 0.1 m of mercury column

$$P = \rho g h$$
$$= 13600 \times 9.81 \times 0.1 = 13341.6 \text{ N/m}^2$$

(iii) h = 4 cm of column of a fluid of specific gravity 0.7 = 0.4 m column of fluid

Density of fluid = 0.7 × 1000 kg/m^3

$$P = \rho g h$$
$$= 0.7 \times 1000 \times 9.81 \times 0.4 = 274.68 \text{ N/m}^2$$

12.6 MEASUREMENT OF PRESSURE

The relationship between pressure and the head is utilized for pressure measurement in the manometer. *Manometers* are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere. Manometers are extensively used in flow measurements.

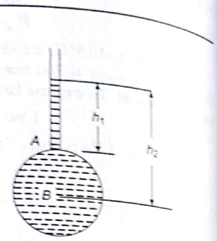

Figure 12.4 *Piezometer*

12.6.1 Piezometer

The most elementary manometer is called a piezometer. A piezometer is essentially a glass or plastic tube mounted vertically so that it is connected to the space within the container (refer Fig. 12.4). Liquid rises in the tube until equilibrium is reached. The pressure is given by the vertical distance from the meniscus (liquid surface) to the point where the pressure is to be measured. It is expressed in units of the length of liquid in the glass tube.

If the liquid is moving in the pipe or vessel, the bottom of the tube must be flush with the inside of the vessel, otherwise the reading will be affected by the velocity of the fluid. This instrument can only be used with liquids, and the height of the tube which can conveniently be employed limits the maximum pressure that can be measured.

If the top of the tube is open to atmosphere, the pressure measured is *gauge pressure*.

Pressure at A = Pressure due to column of liquid of height h_1

$$P_A = \rho g h_1$$

Similarly,

Pressure at B, $$P_B = \rho g h_2$$

There are certain drawbacks of a piezometer:

1. It measures only the positive gauge pressure (*i.e.*, the pressure in the liquid is above atmospheric pressure). The piezometer would not work for negative gauge pressures, because air would flow into the container through the tube.
2. The use of a piezometer is also impractical for measuring large pressures, since for that the vertical tube would need to be very long.
3. If the working fluid is a gas, the usage of a piezometer is not possible, because gases do not have a free surface.

The above mentioned drawbacks of the piezometer can be overcome by modifying the tube and this modified tube is known as a manometer.

12.6.2 U-tube Manometer

For the measurement of small, negative or larger gauge pressures, some modifications in the tube of a piezometer are incorporated and this modified tube is known as a manometer. The U-tube manometer is very commonly used. The lower part of the U-tube contains a liquid which is immiscible with the working fluid. This fluid is called the ***manometric fluid***.

Desirable properties of manometric fluid

(a) The fluid should have a low vapour pressure.
(b) It should have a defined meniscus at the interface for good readability.
(c) It should have low surface tension to avoid capillary rise.
(d) The fluid should be immiscible with the working fluid.

The choice of a measuring fluid is guided by the range of pressure to be measured; higher the range, heavier the fluid.

Mercury is widely used as a manometric fluid because it has properties like

(a) low vapour pressure (≈ 0.17 N/m^2 at 20 °C) and thus for all intents and purposes it can be neglected in comparison with atmospheric pressure
(b) high density

Now, consider the two cases one by one.

Case 1 Measurement of large gauge pressure

One end of the U-tube is connected to the pipe or the container whose pressure is to be measured. The other end of the tube is open to atmosphere. The level of the manometric fluid on the left limb will fall and on the right limb, it will rise (refer Fig. 12.5).

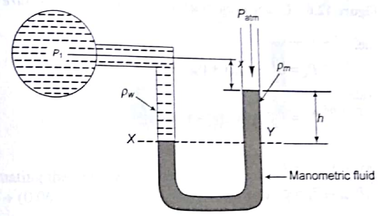

Figure 12.5 *U-tube manometer to measure gauge pressure*

Applying the fundamental law of fluid statics, the pressures at two points, X and Y, at the same elevation in the same continuous mass of fluid at rest must be equal.

For the left hand side,

$$P_X = P_1 + \rho_w g(x + h)$$

For the right hand side,

$$P_Y = P_{atm} + \rho_m gh$$

Since $$P_X = P_Y$$

$$P_1 + \rho_w g(x + h) = P_{atm} + \rho_m gh$$

$$P_1 - P_{atm} = (\rho_m - \rho_w)gh - \rho_m gx \tag{12.17}$$

where P_1 is the absolute pressure of the fluid in the pipe or the container, and P_{atm} is the local atmospheric pressure, ρ_m is the density of the manometric fluid and ρ_w is the working fluid.

Case 2 Measurement of negative gauge pressure

The level of the manometric fluid on the left limb will rise and on the right limb will fall (Fig. 12.6). After attaining the equilibrium, if we apply the fundamental law of fluid statics, the pressures at two points, X and Y, at the same elevation in the same continuous mass of fluid at rest must be equal.

Equating the pressure at the level XY (pressure at the same level in a continuous body of static fluid is equal).

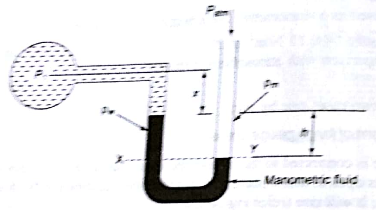

Figure 12.6 *U-tube manometer to measure vacuum pressure*

For the left hand side,

$$P_X = P_1 + \rho_w g x + \rho_m g h$$

For the right hand side,

$$P_Y = P_{atm}$$

Since

$$P_X = P_Y$$

$$P_1 + \rho_w g x + \rho_m g h = P_{atm}$$

$$P_{atm} - P_1 = \rho_w g x + \rho_m g h \qquad (12.18)$$

12.6.3 U-tube Differential Manometer

To measure the pressure difference between two points in the flow field, the manometer which is frequently used, is called the differential U-tube manometer.

A differential U-tube manometer is very handy to measure the pressure difference directly and is basically similar to the U-tube manometer discussed above. What the open end was before is now connected to a different pressure P_2 in the flow field as shown in Fig. 12.7.

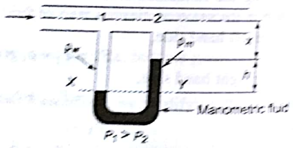

Figure 12.7 *U-tube differential manometer*

$$P_X = P_1 + \rho_w g(x + h)$$

$$P_Y = P_2 + \rho_w g x + \rho_m g h$$

Equating the pressure at the level XY (pressure at the same level in a continuous body of static fluid is equal).

$$P_X = P_Y$$

$$P_1 + \rho_w g(x + h) = P_2 + \rho_w g x + \rho_m g h$$

$$P_1 - P_2 = (\rho_m - \rho_w) g h \qquad (12.19)$$

In forming the connection from a manometer to a pipe or vessel in which a fluid is flowing, care must be taken to ensure that the connection is perpendicular to the wall and flush internally. Any burr or protrusion on the inside of the wall will disturb the flow and cause a local change in pressure so that the manometer reading will not be correct.

Example 12.5 Determine the pressure difference between points A and B, as shown in Fig. 12.8.

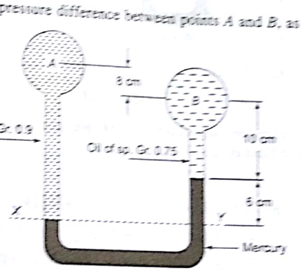

Figure 12.8

Solution Equating the pressures of both the limb along the horizontal plane XY

$$P_A + (0.06 + 0.1 + 0.08) \times 0.9 \times 1000 \times 9.81 = P_B + 0.06 \times 13600 \times 9.81 \times 0.1 \times 0.75 \times 1000 \times 9.81$$

$$P_A - P_B = 4502.79 \text{ Pa} = 4.503 \text{ kPa}$$

Example 12.6 A differential U-tube mercury manometer is used to measure the pressure difference between points 1 and 2 in a pipeline conveying water. The point 1 is 0.5 m lower than the point 2. The difference in the level of manometric fluid on the two limbs is 0.8 m. Calculate the pressure difference between points 1 and 2.

Solution The problem is shown in Fig. 12.9.

Here,

$$z_2 - z_1 = 0.5 \text{ m}$$

$$h = 0.8 \text{ m}$$

$$P_X = P_1 + \rho_w g(z_1 - z)$$

$$P_Y = P_2 + \rho_w g(z_2 - z - h)x + \rho_m g h$$

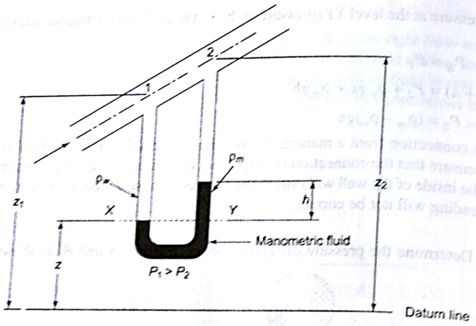

Figure 12.9

Equating the pressure at the level XY (pressure at the same level in a continuous body of static fluid is equal),

$$P_X = P_Y$$

$$P_1 + \rho_w g(z_1 - z) = P_2 + \rho_w g(z_2 - z - h)x + \rho_m gh$$

$$P_1 - P_2 = (\rho_m - \rho_w)gh + \rho_w g(z_2 - z_1)$$

$$P_1 - P_2 = (13600 - 1000)9.81 \times 0.8 + 1000 \times 9.81 \times 0.5$$

$$= 103789.8 \text{ N/m}^2 = 103.79 \text{ kN/m}^2$$

12.6.4 U-Tube Manometer with One Leg Enlarged

The disadvantage of the simple U-tube manometer is that movement of the liquid in both limbs must be read. By making the diameter of one leg very large as compared with the other, it is possible to make the movement in the large leg very small, so that it is only necessary to read the movement of the liquid in the narrow leg. Let M–N be the level of the liquid surface, when the pressure difference is zero and the working fluid is a gas.

Then when pressure is applied ($P_1 > P_2$), the level in the right-hand limb will rise a distance h vertically as shown in Fig. 12.10.

Volume of liquid transferred from left-hand leg to right-hand leg $V = \frac{\pi}{4} d^2 h$.

∴ Fall in level of the left-hand leg

$$= \frac{\text{Volume transferred}(V)}{\text{Area of left hand leg}(A)} = \frac{\frac{\pi}{4} d^2 h}{\frac{\pi}{4} D^2} = \left(\frac{d}{D}\right)^2 h.$$

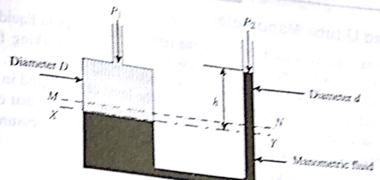

Figure 12.10 *U-tube with one leg enlarged*

The pressure difference, $P_1 - P_2$, is represented by the height of the manometric liquid corresponding to the new difference of level.

$$\therefore \quad P_1 - P_2 = \rho_m g\left[h + \left(\frac{d}{D}\right)^2 h\right] = \rho_m gh\left[1 + \left(\frac{d}{D}\right)^2\right]$$

If $D >> d$ then neglecting $(d/D)^2$, the above equation yields to

$$P_1 - P_2 = \rho_m gh \tag{12.20}$$

12.6.5 Inclined Tube Manometer

If the pressure difference to be measured is small, the leg of the U tube may be inclined. The movement of the meniscus along the inclined leg, read off on the scale, is considerably greater than the change in level h as shown in Fig. 12.11.

∴ pressure difference,

$$P_1 - P_2 = \rho gh = \rho gs \sin\theta \tag{12.21}$$

The manometer can be made as sensitive as may be required by adjusting the angle of inclination of the leg and selecting a liquid with an appropriate value of density ρ to give a scale reading s of the desired size for a given pressure difference.

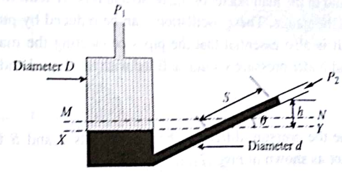

Figure 12.11 *Inclined manometer*

12.6.6 Inverted U-tube Manometer

The inverted U-tube manometer is used for measuring pressure differences in liquids. The top of the U-tube is filled with a fluid which has a density less than that of the working fluid as depicted in Fig. 12.12. For an inverted U-tube manometer, the manometric fluid is usually air. Air can be admitted or expelled through the tap on the top, in order to adjust the level of the liquid in the manometer.

If the density of manometric fluid in the top of the tube is very close to that of the working fluid then the result will be very sensitive giving a large value of h for a small pressure difference.

$$P_X = P_1 - \rho_w g(x + h)$$

$$P_Y = P_2 - \rho_w g x - \rho_m g h$$

Equating the pressure at the level XY (pressure at the same level in a continuous body of static fluid is equal),

$$P_X = P_Y$$

$$P_1 - \rho_w g(x + h) = P_2 - \rho_w g x - \rho_m g h$$

$$P_1 - P_2 = (\rho_w - \rho_m) g h \tag{12.22}$$

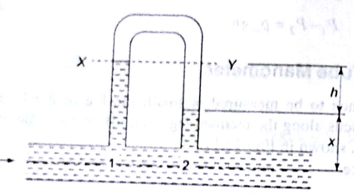

Figure 12.12 *Inverted U-tube manometer*

A major disadvantage of the manometer is its slow response, which makes it unsuitable for measuring fluctuating pressures. Even under comparatively static conditions, slight fluctuations of pressure can make the liquid in the manometer oscillate, so that it is difficult to get a precise reading of the levels of the liquid in the gauge. These oscillations can be reduced by putting restrictions in the manometer connections. It is also essential that the pipes connecting the manometer to the pipe or vessel containing the liquid under pressure should be filled with this liquid and that there should be no air bubbles in the liquid.

Example 12.7 Determine the pressure difference between points A and B for the inverted U-tube manometer as shown in Fig. 12.13.

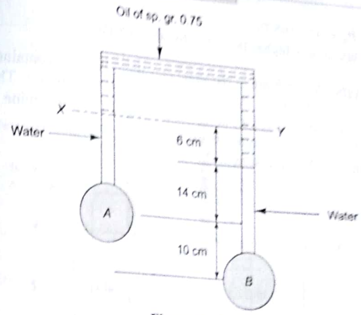

Figure 12.13

Solution Equating the pressures of both the limbs along the horizontal plane XY

$$P_A - 1000 \times 9.81(0.14 + 0.06) = P_B - 1000 \times 9.81 \times (0.1 + 0.14) - 0.06 \times 0.8 \times 1000 \times 9.81$$

$$P_A - P_B = -863.28 \text{ Pa} = -0.863 \text{ kPa}$$

Pressure at B is higher than at A by 0.863 kPa.

Example 12.8 Determine the pressure difference between the pipes A and B for the inverted U-tube manometer as shown in Fig. 12.14.

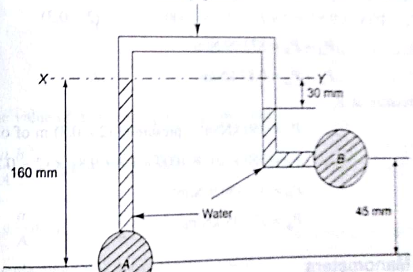

Figure 12.14

Solution Equating the pressures of both the limb along the horizontal plane XY

$$P_A - 1000 \times 9.81 \times 0.16 = P_B - 1000 \times 9.81 \times (0.16 - 0.03 - 0.045) - 0.85 \times 1000 \times 0.03 \times 9.81$$

$P_A - P_B = 485.595$ Pa

Pressure at A is higher than at B by 485.595 Pa.

Example 12.9 Two pipes A and B are in the same elevation. Water is contained in A and rises to a level of 2 m above it. Pipe B contains an oil of sp. gr. 1.7. The inverted U-tube is filled with compressed air at 350 kN/m² and 20 °C. Determine

(i) the pressure difference between A and B and

(ii) the absolute pressure in B

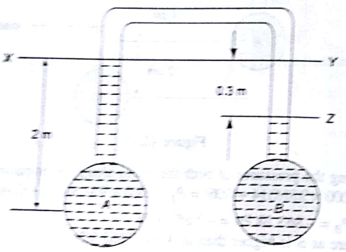

Figure 12.15

Solution Pressure at Y is the same as that at Z.

Equating the pressures of both the limbs along the horizontal plane XY,

$$P_A - 1000 \times 9.81 \times 2 = P_B - 1.7 \times 1000 \times 9.81 \times (2 - 0.3)$$

$$P_A - P_B = 873.09 \text{ N/m}^2$$

$$P_A - P_B = 0.87 \text{ kN/m}^2$$

Pressure at B,

$$P_B = 350 \text{ kN/m}^2 + \text{presure of } (2 - 0.3) \text{ m of oil column}$$

$$P_B = 350 \times 10^3 + 1000 \times 1.7 \times 9.81 \times (2 - 0.3) \text{ N/m}^2$$

$$P_B = 378350.9 \text{ N/m}^2$$

$$P_B = 378.35 \text{ kN/m}^2$$

12.6.7 Micro Manometers

A micro manometer is used for measuring small differences of pressures in the order of 0.001 mm of Hg. It utilizes two liquids which are immiscible with each other. Let ρ_m and ρ_g be the density of the two manometric fluids (let $\rho_m > \rho_g$) the denser liquid will fill the bottom of the U-tube.

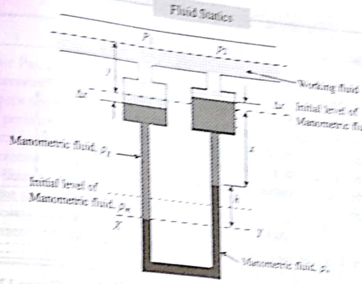

Figure 12.16 *Micro manometer*

Let

A = area of each enlarged end

a = area of the tube

Equating the pressure at the level XY (pressure at the same level in a continuous body of static fluid is equal).

$$P_X = P_Y$$

$$P_1 + \rho_w g(y + \Delta x) + \rho_g g(x + h - \Delta x) = P_2 + \rho_w g(y - \Delta x) + \rho_g g(x + \Delta x) + \rho_m g h \quad (12.23)$$

The volume of the manometric liquid of density ρ_g displaced in the enlarged section equals to the displacement in the U-tube.

$$A\Delta x = a\frac{h}{2}$$

$$\Delta x = \frac{a}{A}\frac{h}{2}$$

Substituting the value of Δx in the Eq. (12.23), we have

$$P_1 + \rho_w g\left(y + \frac{a}{A}\frac{h}{2}\right) + \rho_g g\left(x + h - \frac{a}{A}\frac{h}{2}\right) = P_2 + \rho_w g\left(y - \frac{a}{A}\frac{h}{2}\right) + \rho_g g\left(x + \frac{a}{A}\frac{h}{2}\right) + \rho_m g h$$

$$P_1 - P_2 = -\rho_w g\frac{a}{A}h - \rho_g g\left(h - \frac{a}{A}h\right) + \rho_m g h \quad (12.24)$$

If $D >> d$ then neglecting (a/A), above equation yields to

$$P_1 - P_2 = \rho_m g h - \rho_g g h = (\rho_m - \rho_g) g h \quad (12.25)$$

If the densities of the two manometric fluids are close to each other then for a small pressure difference we can achieve a reasonable value of h.

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12.6.8 Barometer

The barometer is a special manometer used for measuring atmospheric air pressure. Mercury is used as the manometric fluid. The tube is evacuated of all gas so that no atmospheric pressure acts on the top of the mercury column. Because atmospheric pressure acts on the bottom of the mercury, the height to which the mercury column is lifted represents atmospheric pressure.

The pressure at A is the local atmospheric pressure and is expressed by the following equation:

$$P_{atm} - P_v = \rho g h \qquad (12.26)$$

where P_v is the vapour pressure of mercury. Since mercury has a low vapour pressure ($\approx$ 0.17 N/m^2 at 20 °C) it can be neglected in comparison with atmospheric pressure for all intents and purposes.

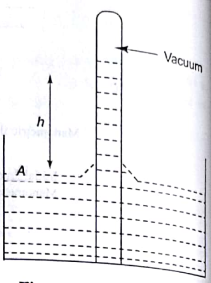

Figure 12.17 *Barometer*

SUMMARY

- Pascal's law states that pressure (or intensity of pressure) at a point in a static fluid is equal in magnitude in all directions.
- Two points at the same elevation in the same continuous mass of fluid at rest have the same pressure.

$$\frac{dp}{dz} = -\rho g$$

- When a pressure is expressed as a difference between its value and a complete vacuum, it is called an absolute pressure. When it is expressed as a difference between its value and the local atmospheric pressure, it is called a gauge pressure.

$$P_{abs} = P_{atm} + P_{gauge}$$

$$P_{vac} = P_{atm} - P_{abs}$$

- Manometers are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere.
- A simple U-tube manometer is used to measure pressure at a point.
- A differential manometer is used to measure difference of pressure between two points.
- Micro manometers are used for measuring small differences of pressures in the order of 0.001 mm of Hg.
- The barometer is a special manometer used for measuring atmospheric air pressure.

REVIEW QUESTIONS

12.1 State and prove Pascal's law of hydrostatics.
12.2 What is a manometer? How are manometers classified?
12.3 Differentiate between the following:
(i) Absolute pressure and gauge pressure
(ii) Piezometer and simple manometer
(iii) Simple manometer and differential manometer
(iv) U-tube differential manometer and inverted U-tube differential manometer
12.4 Explain how the static pressure is isotropic at any horizontal cross section in a fluid.
12.5 State the hydrostatic law and derive the expression for the same.
12.6 Prove that pressure varies exponentially with elevation for isothermal condition.

NUMERICAL PROBLEMS

12.1 Calculate the pressure and density of air at an altitude of 6 km from the sea level. The pressure, temperature and density of the air at sea level are 101.3 kPa, 288 K and 1.22 kg/m^3 respectively. The temperature lapse rate is 0.00065 K/m.

12.2 Calculate the pressure at an altitude of 2 km above sea level if the atmospheric pressure and density at sea level are 101.3 kPa and 1.22 kg/m^3 respectively. Assume air as an incompressible fluid and neglect the variation of g with altitude.

12.3 Determine the pressure difference between points A and B, as shown in Fig. 12.18.

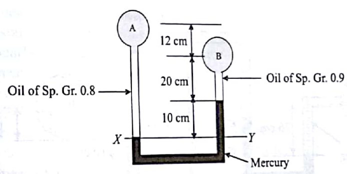

Figure 12.18

12.4 A differential U-tube mercury manometer is used to measure the pressure difference between points 1 and 2 in a pipeline conveying water. The point 1 is 0.5 m lower than the point 2. The difference in the level of manometric fluid on two limbs is 0.8 m. Calculate the pressure difference between points 1 and 2.

12.5 Water is flowing through two different pipes A and B to which an inverted differential manometer having an oil of sp.gr. 0.9 is connected. The pressure in the pipe A is 2.5 m of water. Find the pressure in the pipe B for the manometer readings as shown in Fig. 12.19.

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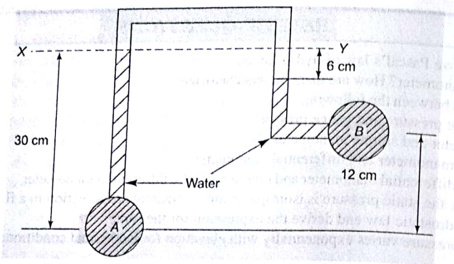

Figure 12.19

12.6 While one end of a U-tube mercury manometer is connected to a horizontal pipe in which water is flowing, its other end is open to the atmosphere. If the difference of mercury levels in the two limbs of this U-tube manometer is found to be 25 cm and the vertical height of water above mercury remains 10 cm below the pipe axis, find the pressure in the pipe.

12.7 A multi-tube manometer is used to determine the pressure difference between points A and B as shown in Fig.12.20. For the given values of heights, determine the pressure difference between points A and B. Specific gravities of benzene, kerosene and mercury are 0.88, 0.82 and 13.6 respectively.

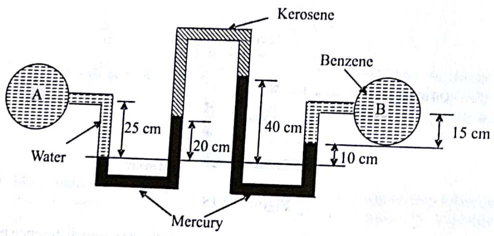

Figure 12.20

12.8 A differential U-tube mercury manometer is used to measure the pressure difference between two sections of a vertical pipe through which water flows upwards and shows a deflection of 10 cm. The distance between the two sections is 30 cm. Determine the difference of pressure between the two sections. Assume density of mercury as 13600 kg/m^3 and density of water as 1000 kg/m^3.

12.9 The pressure and density of air at an altitude of 2000 m above sea level are 79.98 kPa and 0.963 kg/m^3 respectively. Find the atmospheric pressure and density of air at sea level. Assume an isothermal process and neglect the variation of g with altitude.

MULTIPLE-CHOICE QUESTIONS

12.1 The intensity of pressure at a point in a fluid is the same in the directions, only when
(a) the fluid is frictionless and incompressible
(b) the fluid is frictionless
(c) there is no motion of one fluid layer relative to an adjacent layer
(d) the fluid has zero viscosity and is at rest

12.2 In a static fluid, with y as the vertical direction, the pressure variation is given by
(a) $\frac{dp}{dy} = \rho$ (b) $\frac{dp}{dy} = -\rho$
(c) $\frac{dp}{dy} = \gamma$ (d) $\frac{dp}{dy} = -\gamma$

12.3 In an isothermal atmosphere, the pressure
(a) remains constant
(b) decreases linearly with elevation
(c) decreases exponentially with elevation
(d) varies in the same way as the density

12.4 The piezometric head in a static liquid
(a) remains constant at all points in the liquid
(b) increases linearly with depth below a free surface
(c) decreases linearly with depth below a free surface
(d) remains constant only on a horizontal plane

12.5 Local atmospheric pressure is measured by
(a) thermometer (b) manometer
(c) barometer (d) hydrometer

12.6 In a barometer, mercury is preferred over water because
(a) it has higher vapour pressure and bulk modulus of elasticity
(b) it has higher thermal conductivity
(c) it has higher density and lower vapour pressure
(d) its surface is easier to read

12.7 A differential manometer is used to measure
(a) atmospheric pressure (b) very low pressure
(c) difference of pressure between two points (d) velocity in pipes

12.8 Gauge pressure is equal to
(a) absolute pressure − atmospheric pressure (b) atmospheric pressure − absolute pressure
(c) atmospheric pressure + absolute pressure (d) atmospheric pressure + vacuum

12.9 Mercury is used in barometers on account of its
(a) negligible capillary effect (b) high density
(c) very low vapour pressure (d) low compressibility