CHAPTER 13

Kinematics of Fluid Flow

13.1 INTRODUCTION

In Chapter 12, fluid at rest was discussed. In this chapter, kinematics of fluid flow will be discussed. *Kinematics of fluid flow is the study of motion of fluids without considering the forces and moments that cause the motion.* We begin this chapter with the different methods of analysis of fluid flow. Different types of flows and various ways to visualise flow fields are next illustrated with examples. Finally, generalised form of continuity equation in differential form is derived.

13.2 METHODS OF DESCRIPTION/ANALYSIS

There are two different points of view in analysing problem in mechanics. The first method, which follows an individual particle moving through the flow, is called the *Lagrangian approach* after the Italian mathematician Joseph Louis Lagrange (1736–1813). In this description, the fluid motion is described by tracing the kinematic behaviour of each and every individual particle constituting the flow. The position of a particle at any instant of time becomes a function of its identity and time is given by

$$S = S(\vec{S}_0, t) \tag{13.1}$$

The second approach, appropriate to fluid mechanics, is concerned with the field of flow and is called the *Eulerian approach* named after Swiss mathematician Leonhard Euler (1707–1783). In the Eulerian method of description, the properties of a flow field are described as functions of space coordinates and time. For example, the pressure field is a *scalar field* variable. For three-dimensional unsteady fluid flow in Cartesian coordinate system, one can write

$$P = P(x, y, z, t) \tag{13.2}$$

The velocity field is a *vector field* variable and is represented by

$$\vec{V} = \vec{V}(x, y, z, t) \tag{13.3}$$

The velocity components can be expressed as

$$u = u(x, y, z, t) \tag{13.3a}$$

$$v = v(x, y, z, t) \tag{13.3b}$$

$$w = w(x, y, z, t) \tag{13.3c}$$

where u, v and w are the velocity components along x, y and z directions respectively.

The Lagrangian approach is more appropriate to solid mechanics. In the study of fluids, the Eulerian approach is preferred because it is difficult to follow a fluid particle.

13.3 CLASSIFICATIONS OF FLOWS

According to the type of variations of properties, different categories of fluid flows are as follows:

(a) Steady and unsteady flows
(b) Uniform and non-uniform flows
(c) Laminar and turbulent flows
(d) Incompressible and compressible flows
(e) One, two and three-dimensional flows
(f) Internal and external flows
(g) Inviscid and viscous flows
(h) Irrotational and rotational flows

13.3.1 Steady and Unsteady Flows

Fluid flow can be classified into steady and unsteady on the basis of variations of fluid properties and flow characteristics with time.

A flow is said to be *steady* during which the fluid properties (such as density, pressure, temperature) and flow quantities (such as velocity, acceleration, etc.) at any point does not change with time. The term steady implies no change at a point with time. Mathematically, for steady flow, one can write

$$\left(\frac{\partial \vec{V}}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in the flow field.

A flow is said to be *unsteady* during which the fluid properties and flow characteristics at any point changes with time. Mathematically, for unsteady flow, one can write

$$\left(\frac{\partial \vec{V}}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

Liquid flow through a pipe at a constant rate is steady flow, whereas liquid flow through a pipe at a variable (increasing or decreasing) rate is unsteady flow.

13.3.2 Uniform and Non-uniform Flows

Fluid flow can be classified into uniform and non-uniform on the basis of variations of fluid properties and flow characteristics with space at a given instant of time.

A *uniform flow* is defined as that type of flow in which the fluid properties and flow characteristics at any instant of time do not change with space. The term uniform implies no change with location over a specified region. Mathematically, for uniform flow, we have

$$\left(\frac{\partial \vec{V}}{\partial s}\right)_{t=t_0} = 0$$

where t_0 is any fixed instant of time during the flow.

A *non-uniform flow* is defined as that type of flow in which the fluid properties and flow characteristics at any instant of time do not change with space. Mathematically, for non-uniform flow, one can write

$$\left(\frac{\partial \vec{V}}{\partial s}\right)_{t=t_0} \neq 0$$

The liquid flow through a uniform cross-sectional area is uniform flow. Uniform flow at a section is shown in Fig. 13.1. The liquid flow through an expanding tube is non-uniform flow. Figure 13.2 shows flow through a circular pipe, which is non-uniform in nature.

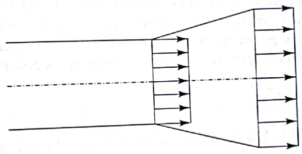

Figure 13.1 *Uniform flow at a section*

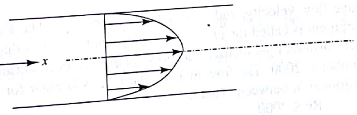

Figure 13.2 *Non-uniform flow at a section*

Note that any combinations of the above-mentioned four types of flows are possible, viz., (a) steady-uniform flow, (b) unsteady-uniform flow, (c) steady non-uniform flow, and (d) unsteady-non-uniform flow. Some of the common examples of these combinations of flows are listed in Table 13.1.

Table 13.1

Type of flow	Example
Steady-uniform flow	Flow of liquid through a long pipe of constant diameter at a constant rate
Unsteady-uniform flow	Flow of liquid through a long pipe of constant diameter at either increasing or decreasing rate
Steady non-uniform flow	Flow of liquid through a tapering pipe at a constant rate
Unsteady non-uniform flow	Flow of liquid through a tapering pipe at either increasing or decreasing rate

13.3.3 Laminar and Turbulent Flows

On the basis of flow structure, flow regimes are classified as laminar and turbulent.

A *laminar flow* is one in which the fluid particles move along smooth, regular paths which can be predicted well in advance. The fluid particles thus move in layers, gliding smoothly over adjacent layers. There is no transformation of fluid particles from one layer to another.

On the other hand, a flow is said to be *turbulent*, when the fluid particles move in very irregular paths. The velocities in turbulent flow vary from point to point in magnitude and direction as well as from instant to instant. All the fluid particles are disturbed and they mix with each other. Thus there is a continuous transfer of momentum to adjacent layers.

A familiar example is the flow of water from water tap. Whenever water is allowed to flow at a low velocity by opening the tap a little, the water flows out smoothly and the flow is laminar. However, as the tap is gradually opened to let the water velocity increase, the flow becomes turbulent.

The behaviour of flow is governed by the magnitude of a non-dimensional *Reynolds number*, which is defined as the ratio of inertia force to viscous force and is given by

$$\mathrm{Re} = \frac{\rho VL}{\mu} \tag{13.4}$$

where ρ is the fluid density, V is the characteristic velocity of flow, L is the characteristic length, and μ is the fluid viscosity. For flow through circular pipe, Reynolds number is expressed as

$$\mathrm{Re} = \frac{\rho VD}{\mu} \tag{13.5}$$

where V is the average flow velocity, and D is diameter of the pipe. The Reynolds number at which the flow becomes turbulent is called the *critical Reynolds number*. The value of the critical Reynolds number is different for different geometries and flow conditions. For internal flow in a circular pipe, critical Reynolds number is 2000. The flow in a circular pipe is laminar for Re ≤ 2000, turbulent for Re ≥ 4000, and transitional in between. That is

Re ≤ 2000	laminar flow
2000 ≤ Re ≤ 4000	transitional flow
Re ≥ 4000	turbulent flow

Example 13.1 Water is flowing through a circular pipe of diameter 20 mm at a uniform velocity of 3 m/s. The kinematic viscosity of water is 1×10^{-6} m²/s. Determine whether the flow field is laminar or turbulent.

Solution

Diameter of pipe $D = 20\text{ mm} = 20 \times 10^{-3}\text{ m} = 0.02\text{ m}$

Velocity of flow $V = 3\text{ m/s}$

Kinematic viscosity of water $\nu = 1 \times 10^{-6}\text{ m}^2/\text{s}$

The Reynolds number is found to be (see Eq. (13.5))

$$\mathrm{Re} = \frac{\rho VD}{\mu}$$

or

$$\mathrm{Re} = \frac{VD}{\nu} \qquad \left[\because \nu = \frac{\mu}{\rho}\right]$$

$$= \frac{3 \times 0.02}{1 \times 10^{-6}} = 60000$$

Since the Reynolds number is more than 4000, the flow is turbulent.

13.3.4 Compressible and Incompressible Flows

Flows in which variations in density are negligible are termed as *incompressible flows*. When the density changes significantly within a flow, then the flow is called *compressible*. Liquid flows are considered as incompressible flows. Transmission of gases in pipelines at high pressure, flow in high-speed aircraft and missiles, fans and compressors are compressible flows.

Note: There is a subtle difference between incompressible fluid and incompressible flow. A fluid is called incompressible if its density does not change significantly with change in pressure. The change in density may not be always due to change in pressure, it may be due to change in temperature. That means there may be compressible flow of an incompressible fluid.

13.3.5 One-, Two- and Three-Dimensional Flows

All general flows such as flow around a moving car have velocity components in x, y and z directions. They are called three-dimensional flows. Thus, a *three-dimensional flow* is the one in which the velocity vector depends on three space variables and time. The velocity components can be expressed as

$$u = u(x, y, z, t) \tag{13.6a}$$

$$v = v(x, y, z, t) \tag{13.6b}$$

$$w = w(x, y, z, t) \tag{13.6c}$$

A *two-dimensional* flow is one in which the velocity vector depends on two space variables and time. Steady flow between two parallel plates close to the inlet (entrance region) as shown in Fig. 13.3 is two-dimensional. The velocity components can be expressed as

$$u = u(x, y, t) \quad (13.7a)$$
$$v = v(x, y, t) \quad (13.7b)$$

A *one-dimensional flow* is one in which velocity vector depends on only one space variable and time. The velocity can be expressed as

$$u = u(x, t) \quad (13.8)$$

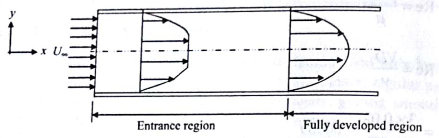

Figure 13.3 *Parallel flow between two infinite parallel plates*

Few real flows are strictly one-dimensional. Steady flow between two parallel plates in the fully developed region [$u = u(y)$] as shown in Fig.13.3 is one-dimensional.

Example 13.2 The velocity components for a flow field are given as

$$u = axy,\ v = -byzt,\ w = 0.$$

Determine
(a) whether the flow field is one-, two-, or three-dimensional.
(b) whether the flow is steady or unsteady.

Solution

(a) Since the velocity field is a function of x, y and z only, the flow field is three-dimensional.
(b) Since time t appears explicitly in the velocity, the flow is unsteady.

13.3.6 Internal and External Flows

On the basis of flow in a confined channel or over a surface, a fluid flow can be classified as internal or external flow.

The flow of fluid in a pipe or duct is *internal flow* if the fluid is completely bounded by the solid surfaces. The flow of an unbounded fluid over a surface such as flat plate is *external flow*. Water flow in a pipe is an internal flow, whereas flow over a flat plate is an external flow.

13.3.7 Inviscid and Viscous Flows

An *inviscid flow* is one in which the effect of viscosity is negligible. In inviscid flow, the fluid viscosity is assumed to be zero. Inviscid flow does not exist in reality; however, in many situations the flow can be simplified by neglecting the viscous forces. .

All fluids possess viscosity, and accordingly, *viscous flows* are important in the study of fluid mechanics.

13.3.8 Irrotational and Rotational Flows

A flow is said to be *irrotational* if the fluid particles while flowing do not rotate about their mass centres. On the other hand, a flow is said to be rotational when the fluid particles while flowing also rotate about their mass centres.

When fluid flows over a flat plate as shown in Fig. 13.4, the flow field can be separated into two regions, namely the boundary layer region (where the viscous effects are significant) and the outer region (where the viscous effects are not significant). In the boundary layer region the flow is rotational, whereas in the outer region, the flow is irrotational.

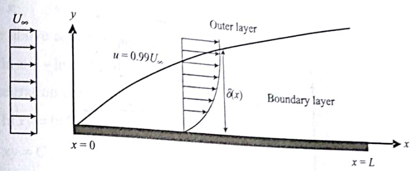

Figure 13.4 *Flow over a flat plate*

13.4 STREAMLINES, PATHINES AND STREAKLINES

The pictorial representation of fluid flow is very helpful to describe the flow characteristics, whether this be done by experimental flow visualisation or by numerical solution. Streamlines, streaklines and pathlines are widely used to describe the flow behaviour. Here, the important characteristics of the above-mentioned three lines are discussed.

13.4.1 Streamline and Streamtube

Streamline at any instant can be defined as an imaginary line in the flow field so that the tangent to the line at any point represents the direction of the instantaneous velocity of that point. For unsteady flows the streamline pattern changes with time. From the definition of streamline, it can be written

$$\vec{V} \times d\vec{S} = 0 \quad (13.9)$$

where $d\vec{S}$ is the length of an infinitesimal line segment along a streamline at a point where $\vec{V}$ is the instantaneous velocity vector.

Consider an infinitesimal arc length $d\vec{S} = dx\hat{i} + dy\hat{j}$ along a streamline in the xy-plane as shown in Fig. 13.5. From the definition of streamline, $d\vec{S}$ must be parallel to the local velocity vector $\vec{V} = u\hat{i} + v\hat{j}$. Thus, one can write

$$\vec{V} \times d\vec{S} = 0$$

or $\quad vdx - udy = 0$

or $$\frac{dx}{u} = \frac{dy}{v} \qquad (13.10)$$

Equation (13.10) represents the equation of a streamline in x-y plane.

The general differential equation for streamlines in three-dimensional flow field can be obtained as

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad (13.11)$$

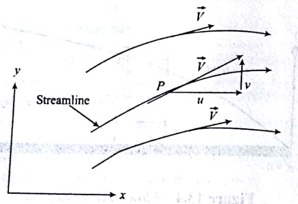

Figure 13.5 *Streamline in a two-dimensional flow field*

A bundle of neighbouring streamlines (Fig. 13.6) may be imagined to form a passage through which the fluid flows. This passage is known as a ***stream tube***. Since the stream tube is bounded on all sides by streamlines and by definition, velocity does not exist across a streamline, no fluid may enter or leave a stream tube except through its ends.

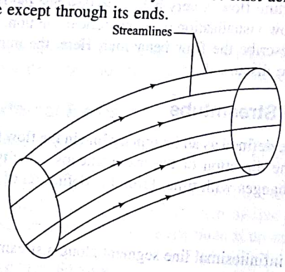

Figure 13.6 *Stream tube*

Example 13.3 For the following flow fields find the equation of streamline:

(i) $\vec{V} = x\hat{i} - y\hat{j}$ passing through the point (1, 1)

(ii) $\vec{V} = 3y\hat{i} - 2x\hat{j}$ passing through the point (2, –1)

Solution The equation of a streamline in two-dimensional flow is (see Eq. (13.10))

$$\frac{dx}{u} = \frac{dy}{v}$$

(i) x-component of velocity, $u = x$

y-component of velocity, $v = -y$

Substituting the x and y component of velocities in the equation of streamline, we have

$$\frac{dx}{x} = -\frac{dy}{y}$$

Integrating the above equation, we get

$$\ln x = -\ln y + \ln C$$

where C is integration constant

or $$\ln xy = \ln C$$

or $$xy = C$$

Streamline is passing through point (1,1)

Thus, $$C = 1 \times 1 = 1$$

The equation of streamline is $xy = 1$

(ii) x-component of velocity, $u = 3y$

y-component of velocity, $v = -2x$

Substituting the x and y component of velocities in the equation of streamline, we get

$$\frac{dx}{3y} = \frac{dy}{-2x}$$

$$2xdx = -3ydy$$

Integrating the above equation, we obtain

$$2\frac{x^2}{2} = -3\frac{y^2}{2} + \frac{C}{2}$$

where C is integration constant

$$2x^2 + 3y^2 = C$$

Streamline is passing through point (2, –1)

Thus, $$C = 2 \times 2^2 + 3 \times (-1)^2 = 11$$

The equation of streamline is $2x^2 + 3y^2 = 11$

Example 13.4 A two-dimensional flow field has velocities along the x and y directions given by $u = x^2t$ and $v = -2xyt$ respectively, where t is time. Find the equation of streamline.

Solution The equation of a streamline in two-dimensional flow is (see Eq. (13.10))

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$$\frac{dx}{u} = \frac{dy}{v}$$

Substituting $u = x^2 t$ and $v = -2xyt$ in the equation of streamline, we have

$$\frac{dx}{x^2 t} = -\frac{dy}{2xyt}$$

or
$$2\frac{dx}{x} = -\frac{dy}{y}$$

Integrating the above equation, we get

$$2\ln x = -\ln y + \ln C$$

where C is integration constant

$$\ln x^2 y = \ln C$$

or
$$x^2 y = \text{constant}$$

The equation of streamline is $x^2 y$ = constant

Example 13.5 In a flow the velocity vector is given by $V = 2x\hat{i} - 3y\hat{j} + 5z\hat{k}$. Determine the equation of the streamline passing through point (1, 4, 5).

Solution The equation of a streamline in three-dimensional flow is (see Eq. (13.11))

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

x-component of velocity, $u = 2x$

y-component of velocity, $v = -3y$

z-component of velocity, $w = 5z$

Streamline in the xy-plane is given by (see Eq. (13.10))

$$\frac{dx}{u} = \frac{dy}{v}$$

or
$$\frac{dx}{2x} = \frac{dy}{-3y}$$

or
$$\frac{dy}{3y} = -\frac{dx}{2x}$$

or
$$\frac{1}{3}\ln y = -\frac{1}{2}\ln x + \ln C_1$$

or
$$y = C_1 x^{-\frac{3}{2}}$$

Streamline is passing through point (1, 4, 5)

Thus, $C_1 = 4$

The equation of streamline passing through point (1, 4, 5) is

$$y = 4x^{-\frac{3}{2}}$$

Streamline in the xz-plane is given by (see Eq. (13.11))

$$\frac{dx}{u} = \frac{dz}{w}$$

or
$$\frac{dx}{2x} = \frac{dz}{5z}$$

or
$$\frac{1}{5}\ln z = \frac{1}{2}\ln x + \ln C_2$$

or
$$z = C_2 x^{\frac{5}{2}}$$

Streamline is passing through point (1, 4, 5).

Thus, $C_2 = 5$

The equation of streamline passing through point (1, 4, 5) is

$$z = 5x^{\frac{5}{2}}$$

13.4.2 Pathline

In experimental fluid mechanics, the concept of pathline is important. A *pathline* is the actual trajectory through space of a selected fluid article during a time of interval. Pathine and streamlines are identical in a steady flow, but not in an unsteady flow. A pathline is a Lagrangian concept because it is defined by the motion of fluid particles as shown in Fig.13.7.

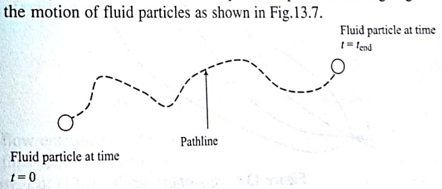

Figure 13.7 *Pathline*

Example 13.6 A two-dimensional flow field is described in the Lagrangian system as

$$x = x_0 e^{ct}, \text{ and } y = y_0 e^{-ct}$$

Find the equation of path line of the particle.

Solution The equation of pathline can be found from the equations of motion describing the flow by eliminating t.

From the given flow field, we have

$$x = x_0 e^{ct}$$

or
$$e^{ct} = \frac{x}{x_0} \qquad (13.12)$$

$$y = y_0 e^{-ct}$$

or
$$e^{ct} = \frac{y_0}{y} \qquad (13.13)$$

Eliminating t from Eqs. (13.12) and (13.13), we have

$$\frac{x}{x_0} = \frac{y_0}{y}$$

or
$$xy = x_0 y_0$$

This is the required equation of pathline.

13.4.3 Streakline

A *streakline* at any instant of time is the temporary locations of all particles that have passed through a fixed point in the flow field. Note that *smoke emitting from a lighted cigarette represents streakline.* Suppose x, y, z are the fluid particles which have passed through a reference point say A as shown in Fig. 13.8. Further, at an instant of time t, the fluid particle x, y, z are at B, C, D. Then the line joining A, B, C and D is the streakline, at time t as shown in Fig. 13.8.

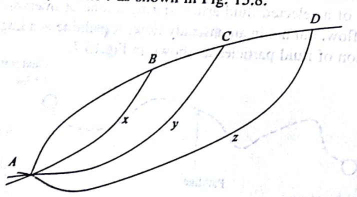

Figure 13.8 *Streakline*

Note that streamlines, pathlines and streaklines are identical in steady flow. Streamline is instantaneous line while the streakline and pathline are generated by the passage of time.

13.5 CONTINUITY EQUATION

13.5.1 Continuity Equation for Steady One-Dimensional Flow in a Conduit

Consider fluid flows steadily through a portion of the stream tube having a cross-sectional area small enough for the velocity to be considered as constant over any cross-section, for the setions 1 and 2 as shown in the Fig. 13.9. Suppose that at section 1, the area of the stream tube is A_1, the uniform velocity of the fluid is V_1 and the density is ρ_1, while at section 2 the corresponding values are A_2, V_2 and ρ_2 respectively.

According to the principle of conservation of mass for a control volume, we have

rate of mass flow entering into the control volume = rate of mass leaving from the control volume + rate of increase of mass of fluid in the control volume (13.14)

For steady flow, the mass of fluid in the control volume remains constant and Eq. (13.14) simplifies to

rate of mass flow entering into the control volume = rate of mass leaving from the control volume (13.15)

Since there cannot be any flow across the walls of a stream tube, for flow through a streamtube as shown in Fig. 13.9, Eq. (13.15) becomes

rate of mass entering at section 1 = rate of mass flow leaving at section 2 (13.16)

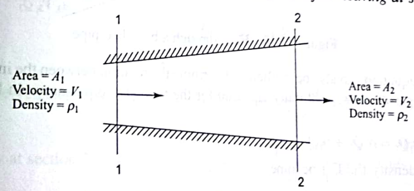

Figure 13.9 *Steady flow through a streamtube*

Rate of mass flow entering at section 1 = $\rho_1 A_1 V_1$

Rate of mass flow leaving at section 2 = $\rho_2 A_2 V_2$

For steady flow, Eq. (13.16) simplifies to

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad (13.17)$$

Equation (13.17) is the *continuity equation* for the steady flow of a compressible fluid applied to two sections along a stream tube.

For the flow of a real fluid, velocity is not uniform at any section varies from wall to wall. The *average velocity* over a cross section is given by

$$\bar{V} = \frac{\int_A V dA}{A} \tag{13.18}$$

The average velocity is physically an equivalent uniform velocity that could have given rise to the same volume flow rate as the actual one.

Using the average velocity, continuity equation for steady flow can be written as

$$\rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2 \tag{13.19}$$

For constant density, steady flow continuity equation becomes

$$A_1 \bar{V}_1 = A_2 \bar{V}_2 \tag{13.20}$$

The volume of the fluid flowing through a cross section per unit time is called the volume flow rate Q and is given by

$$Q = A\bar{V} \tag{13.21}$$

where A is the cross sectional area normal to the flow direction and $\bar{V}$ is the average velocity across the section.

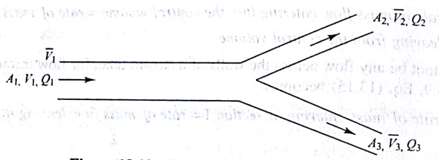

Figure 13.10 Flow through a branched pipe

The continuity equation can also be applied to determine the relation between the inflow and outflow of a junction. For steady flow, continuity equation for the branched pipe shown in Fig. 13.10 can be written as

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3 \tag{13.22}$$

For a constant density fluid, it becomes

$$Q_1 = Q_2 + Q_3$$

or

$$A_1 \bar{V}_1 = A_2 \bar{V}_2 + A_3 \bar{V}_3 \tag{13.23}$$

Note: *The expression $A_1 \bar{V}_1 = A_2 \bar{V}_2$ between any two sections holds valid only for constant density flow.*

Example 13.7 For the pipe shown in Fig. 13.11, the diameters of the pipe at sections 1-1 and 2-2 are 10 and 20 cm respectively. If the volume flow rate through the pipe is 0.005 m³/s, find the average velocity at the two sections.

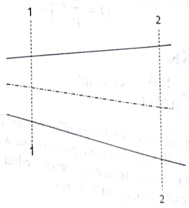

Figure 13.11

Solution

Diameter of pipe at section 1-1 $\quad D_1 = 10 \text{ cm} = 0.1 \text{ m}$

Diameter of pipe at section 2-2 $\quad D_2 = 20 \text{ cm} = 0.2 \text{ m}$

Cross-sectional area at section 1 is $\quad A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$

Cross-sectional area at section 2 is $\quad A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$

Volume flow rate $\quad Q = 0.005 \text{ m}^3/\text{s}$

From continuity equation, we have

$$Q = A_1 \bar{V}_1 = A_2 \bar{V}_2$$

Thus, the average velocity at section 1-1 is $\quad \bar{V}_1 = \frac{Q}{A_1} = \frac{0.005 \text{ m}^3/\text{s}}{0.00785 \text{ m}^2} = 0.637 \text{ m/s}$

Average velocity at section 2-2 is $\quad \bar{V}_2 = \frac{Q}{A_2} = \frac{0.005 \text{ m}^3/\text{s}}{0.0314 \text{ m}^2} = 0.16 \text{ m/s}$

Example 13.8 Air having a mass density of 1.23 kg/m³ flows in a pipe with a diameter of 20 cm at a mass flow rate of 2 kg/s. What are the mean (or average) velocity of flow in this pipe and the volume flow rate?

Solution

Density of air $\quad \rho = 1.23 \text{ kg/m}^3$

Mass flow rate $\quad \dot{m} = 2 \text{ kg/s}$

Diameter of pipe $\quad D = 20 \text{ cm} = 0.2 \text{ m}$

Cross-sectional area of pipe $\quad A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$

Volume flow rate is
$$Q = \frac{\dot{m}}{\rho} = \frac{2}{1.23} = 1.63 \text{ m}^3/\text{s}$$

Average velocity of air is found to be
$$\bar{V} = \frac{Q}{A} = \frac{1.63}{0.0314} = 51.88 \text{ m/s}$$

Example 13.9 A diffuser consists of two circular parallel plates 20 cm in diameter and 5 mm apart and connected to a 30 mm diameter pipe as shown in Fig. 13.12. If the streamlines are assumed to be radial in the diffuser, what mean velocity in the pipe will correspond to an exit velocity of 0.5 m/s.

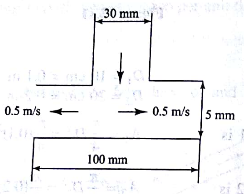

Figure 13.12

Solution

Diameter of the plates	$d = 20$ cm $= 0.2$ m
Diameter of the pipe	$D = 30$ mm $= 0.03$ m
Distance of separation of plates	$t = 5$ mm $= 0.005$ m
Exit velocity from plates	$U = 0.5$ m/s

From continuity equation, we have
$$\pi dt \times U = \frac{\pi}{4} D^2 \times \bar{V}$$

Substituting the respective values, we get
$$\pi \times 0.2 \times 0.005 \times 0.5 = \frac{\pi}{4} \times (0.03)^2 \times \bar{V}$$

or $\bar{V} = 2.222$ m/s

Example 13.10 A pipe 40 cm in diameter branches into two pipes of diameters 25 cm and 20 cm respectively as shown in Fig. 13.13. The average velocity in 40 cm diameter pipe is 4 m/s. Find

(a) the discharge through 40 cm diameter pipe, and

(b) the average velocity in 25 cm diameter pipe if the average velocity in 20 cm pipe is 2 m/s.

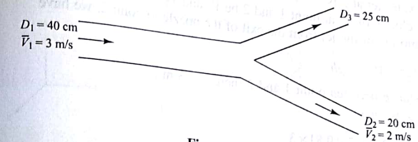

Figure 13.13

Solution

(a) Diameter of main pipe, $D_1 = 40$ cm $= 0.4$ m

Average velocity of flow in main pipe, $\bar{V}_1 = 3$ m/s

Diameter of branched pipe 2, $D_2 = 20$ cm $= 0.2$ m

Average velocity of flow in branched pipe 2, $\bar{V}_2 = 2$ m/s

Diameter of branched pipe 3, $D_3 = 25$ cm $= 0.25$ m

The discharge through 40 cm diameter pipe is given by

Q = Area of main pipe × average velocity of flow in main pipe

$$= A_1\bar{V}_1 = \frac{\pi}{4} D_1^2 \bar{V}_1 = \frac{\pi}{4} \times 0.4^2 \times 3 = 0.377 \text{ m}^3/\text{s}$$

(b) For incompressible steady flow continuity equation becomes

Area of main pipe × average velocity in main pipe = Area of pipe 2 × average velocity in pipe 3 + Area of pipe 2 × average velocity in pipe 3

or $A_1\bar{V}_1 = A_2\bar{V}_2 + A_3\bar{V}_3$

or $\frac{\pi}{4} D_1^2 \bar{V}_1 = \frac{\pi}{4} D_2^2 \bar{V}_2 + \frac{\pi}{4} D_3^2 \bar{V}_3$

or $\frac{\pi}{4} \times 0.4^2 \times 3 = \frac{\pi}{4} \times 0.2^2 \times 2 + \frac{\pi}{4} \times 0.25^2 \bar{V}_3$

or $\bar{V}_3 = 6.4$ m/s

Example 13.11 A jet of water issuing from a 20 mm diameter nozzle is directed vertically upwards. The diameter of the water jet at a point 3 m above nozzle is 40 mm. Find the velocity of jet at point 3 m above nozzle. Assume that jet remains steady and there is no loss of energy.

Solution: Let the nozzle exit and the point 3 m above nozzle be designated by point 1, and 2 respectively, as shown in Fig. 13.14.

Diameter at nozzle exit is D_1 = 20 mm = 0.02 m

Diameter of the water jet at 2 is D_2 = 40 mm = 0.04 m

Let the average velocity of jet at point 1 and 2 be V_1 and V_2, respectively.

Considering the motion of the jet from the exit of the nozzle to point 2, we have

$$V_2^2 = V_1^2 - 2gh$$

where h is the distance between point 1 and 2 (here h = 3 m)

Putting the value h, we get

$$V_2^2 = V_1^2 - 2\times 9.81\times 3$$

or
$$V_2^2 = V_1^2 - 58.86 \tag{13.24}$$

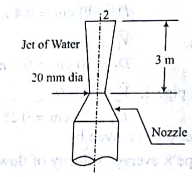

Figure 13.14

Applying the continuity equation between the exit of the nozzle 1 and the point 2, we have

$$A_1V_1 = A_2V_2$$

where A_1 and A_2 are the cross-sectional area at sections 1 and 2 respectively.

or
$$\frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}D_2^2V_2$$

or
$$V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{0.04}{0.02}\right)^2 V_2 = 4V_2 \tag{13.25}$$

From Eqs. (13.24) and (13.25), we have

$$V_2^2 = (4V_2)^2 - 58.86$$

or,
$$V_2 = 1.98 \text{ m/s}$$

13.5.2 Continuity Equation-Differential Form

A rectangular parallelepiped with sides dx, dy and dz in the x, y and z directions, respectively, is considered as the control volume in three-dimensional Cartesian coordinate system as shown in Fig. 13.15.

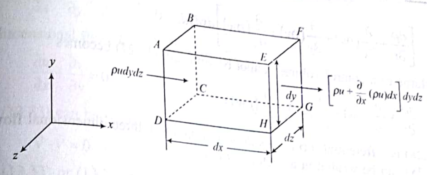

Figure 13.15 *Differential control volume in rectangular cartesian coordinate system*

Let the fluid enter through the surface $ABCD$ (normal to the x-axis) with a velocity u and a density ρ.

Rate of mass inflow through the surface $ABCD$ (normal to x axis) = $\rho u dydz$

Rate of mass outflow through the surface $EFGH$ (normal to x-axis) = $\left[\rho u + \frac{\partial}{\partial x}(\rho u)dx\right]dydz$

Net rate of mass outflow in x-direction $= \left[\rho u + \frac{\partial}{\partial x}(\rho u)dx\right]dydz - \rho u dydz = \frac{\partial}{\partial x}(\rho u)dxdydz$

Similarly,

Net rate of mass outflow in y-direction $= \frac{\partial}{\partial y}(\rho v)dx\,dy\,dz$

Net rate of mass outflow in z-direction $= \frac{\partial}{\partial z}(\rho w)dxdydz$

Therefore, total net rate of mass outflow in x-, y- and z-direction

$$= \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]dxdydz \tag{13.26}$$

The effect of mass loss in Eq. (13.26) is to cause the time rate of decrease of mass encompasses by the volume.

Since $\frac{\partial \rho}{\partial t}$ is the rate of change of mass density, the rate of change of mass in control volume $= -\frac{\partial \rho}{\partial t}dxdydz$.

Therefore, according to the principle of conservation of mass,

Total net rate of mass outflow in x-, y- and z-direction = Rate of change of mass in control volume

$$\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]dxdydz = -\frac{\partial \rho}{\partial t}dxdydz$$

or $$\left[\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]dxdydz=0 \quad (13.27)$$

Since the volume of a control volume cannot be zero, Eq. (13.27) becomes

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \quad (13.28)$$

Equation (13.28) is differential form of *continuity equation* in three-dimensional flow field. Equation (13.28) can be written in a vector form as

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\vec{V})=0 \quad (13.29)$$

where $\vec{V}$ represents the velocity vector and $\nabla\cdot(\rho\vec{V})$ represents divergence of $(\rho\vec{V})$.

For steady flow, $\frac{\partial \rho}{\partial t}=0$. Then, Eq. (13.28) simplifies to

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \quad (13.30)$$

For two-dimensional flow, Eq. (13.30) simplifies to

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0 \quad (13.31)$$

Equation (13.30) can be written in a vector form as

$$\nabla\cdot(\rho\vec{V})=0 \quad (13.32)$$

From Eq. (13.28), we get

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}+\rho\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=0$$

$$\frac{D\rho}{Dt}+\rho\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=0$$

$$\frac{1}{\rho}\frac{D\rho}{Dt}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

For incompressible flow, the rate of volumetric dilation per unit volume $\left(\frac{1}{\rho}\frac{D\rho}{Dt}\right)$ of a fluid element in motion is zero. Then the above equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (13.33)$$

For two-dimensional flow, Eq. (13.33) simplifies to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (13.34)$$

Equation (13.33) can be written in a vector form as

$$\nabla\cdot\vec{V}=0 \quad (13.35)$$

Equation (13.33) or (13.35) holds for incompressible (both steady as well as unsteady) flow.

> *Note: Any velocity field representing the motion of a fluid should satisfy the continuity equation.*

> *Note:*
>
> *The equation* $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ *(equivalently* $\nabla\cdot\vec{V}=0$ *) is applicable for incompressible flow.*
>
> *The equation* $\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$ *is applicable for steady flow.*
>
> *The equation* $\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$ *holds for steady and two-dimensional flow.*
>
> *The equation* $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ *holds for two-dimensional and incompressible (both steady as well as unsteady) flow.*

Example 13.12 The velocity field in a two-dimensional flow field is given by

$$\vec{V}=(x^2y+y^2)\hat{i}-xy^2\hat{j}$$

Check whether the velocity field describes the motion of an incompressible flow or not?

Solution For a two-dimensional, incompressible flow, the continuity equation can be written in differential form as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Given that $u=x^2y+y^2$, and $v=-xy^2$

Hence, $\frac{\partial u}{\partial x}=2xy$, and

$$\frac{\partial v}{\partial y} = -2xy$$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation, we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2xy - 2xy = 0$$

This shows that the above velocity field satisfies the continuity equation.

Example 13.13 Check whether the following sets of velocity components satisfy the continuity equation of steady, incompressible flow:

(a) $u = x^2 - y^2,\ v = x - 2xy$

(b) $u = -\ln xy,\ v = \frac{y}{x}$

(c) $u = 2x^2 - xy + z^2,\ v = x^2 - 4xy + y^2$ and $w = 2xy - yz + y^2$

Solution For a two-dimensional, incompressible flow the continuity equation can be written in differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(a) Given that $u = x^2 - y^2$ and $v = x - 2xy$

Hence, $\frac{\partial u}{\partial x} = 2x$ and $\frac{\partial v}{\partial y} = -2x$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation, we find

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2x - 2x = 0$$

Hence the continuity equation is satisfied.

(b) Here, $u = -\ln xy$ and $v = \frac{y}{x}$

Hence, $\frac{\partial u}{\partial x} = -\frac{1}{xy}y = -\frac{1}{x}$ and $\frac{\partial v}{\partial y} = \frac{1}{x}$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation, we find

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{x} + \frac{1}{x} = 0$$

Hence the continuity equation is satisfied.

(c) Here, $u = 2x^2 - xy + z^2$, $v = x^2 - 4xy + y^2$ and $w = 2xy - yz + y^2$

Hence, $\frac{\partial u}{\partial x} = 4x - y$, $\frac{\partial v}{\partial y} = -4x + 2y$, and $\frac{\partial w}{\partial z} = -y$

For a three-dimensional, incompressible flow the continuity equation can be written in a differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation, we find

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 4x - y - 4x + 2y - y = 0$$

Hence the continuity equation is not satisfied.

Example 13.14 The velocity components for a two-dimensional, incompressible flow are given as

$$u = -\frac{x}{x^2 + y^2},\ v = -\frac{y}{x^2 + y^2}$$

Show that the velocity field satisfies the continuity equation.

Solution For a two-dimensional, incompressible flow the continuity equation can be written in differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Given that $u = -\frac{x}{x^2 + y^2}$, and $v = -\frac{y}{x^2 + y^2}$

Hence, $\frac{\partial u}{\partial x} = -\frac{1}{x^2 + y^2} + \frac{2x^2}{\left(x^2 + y^2\right)^2}$, and $\frac{\partial v}{\partial y} = -\frac{1}{x^2 + y^2} + \frac{2y^2}{\left(x^2 + y^2\right)^2}$

Substituting $\frac{\partial u}{\partial x}$, and $\frac{\partial v}{\partial y}$ in the continuity equation, we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

This shows that the above velocity field satisfies the continuity equation.

Example 13.15 For a two-dimensional incompressible flow, the x-component of velocity is $u = 2xy$. What is the y component that will satisfy continuity equation?

Solution Given that $u = 2xy$

Hence, $$\frac{\partial u}{\partial x} = 2y$$

For a two-dimensional, incompressible flow the continuity equation can be written in a differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

or $$2y + \frac{\partial v}{\partial y} = 0$$

or $$\frac{\partial v}{\partial y} = -2y$$

or $$v = -y^2 + f(x)$$

where $f(x)$ is a constant of integration.

Example 13.16 In a three-dimensional incompressible fluid flow, the flow field is given by expression $V = (x+y+z)\hat{i} - (xy+yz+zx)\hat{j} + (w)\hat{k}$. Find the w component of velocity so that the case is possible for an incompressible fluid flow.

Solution For an incompressible flow the continuity equation can be written in differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Given that $u = x + y + z$, and $v = -xy - yz - zx$

Hence, $$\frac{\partial u}{\partial x} = 1, \text{ and}$$

$$\frac{\partial v}{\partial y} = -x - z$$

Substituting $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in the continuity equation, we get

$$1 - x - z + \frac{\partial w}{\partial z} = 0$$

or $$\frac{\partial w}{\partial z} = x + z - 1$$

Integrating with respect to z, we have

$$w = xz + \frac{z^2}{2} - z + f(x, y)$$

SUMMARY

- In Lagrangian description, the fluid motion is described by tracing the kinematic behaviour of each and every individual particle constituting the flow. The position of a particle at any instant of time becomes a function of its identity and time and is given by

$$S = S(\vec{S}_0, t)$$

The Lagrangian approach is more appropriate to solid mechanics.

- In *Eulerian description*, the properties of a flow field are described as functions of space coordinates and time. For example, the pressure field is a scalar field variable. For three-dimensional unsteady fluid flow in Cartesian coordinates, the velocity field is given by

$$\vec{V} = \vec{V}(x, y, z, t)$$

In fluid mechanics, the Eulerian approach is preferred because it is difficult to follow a fluid particle.

- A *steady flow* is that type of flow in which the fluid properties (such as density, pressure, temperature) and flow characteristics (such as velocity, acceleration, etc.) at a point do not change with time. If they change with time, the flow is called *unsteady flow.*

$$\left(\frac{\partial \vec{V}}{\partial t}\right)_{x_0, y_0, z_0} = 0, \text{ for steady flow}$$

$$\left(\frac{\partial \vec{V}}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \text{ for unsteady flow}$$

- A *uniform flow* is that type of flow in which the fluid properties (such as density, pressure, temperature) and flow characteristics (such as velocity, acceleration, etc.) at any instant of time do not change with space. If they change with space, the flow is called *non-uniform flow.*

$$\left(\frac{\partial \vec{V}}{\partial s}\right)_{t=t_0} = 0, \text{ for uniform flow}$$

$$\left(\frac{\partial \vec{V}}{\partial s}\right)_{t=t_0} \neq 0, \text{ for non-uniform flow}$$

- A *laminar flow* is one in which the fluid particles move along smooth, regular paths which can be predicted well in advance. The fluid particles thus move in layers, gliding smoothly over adjacent layers. In turbulent flow, the fluid particles move in very irregular paths. In the *turbulent regime*, the flow structure is characterised by random three-dimensional motions of fluid particles in addition to the mean motion. The flow in a circular pipe is laminar for Re ≤ 2000, turbulent for Re ≥ 4000, and transitional in between. That is

Re ≤ 2000	laminar flow
2000 ≤ Re ≤ 4000	transitional flow
Re ≥ 4000	turbulent flow

- Flows in which variations in density are negligible are termed as *incompressible flows*. When the density changes significantly within a flow, then the flow is called compressible.
- A *one-dimensional flow* is one in which velocity vector depends on only one space variable and time. A *two-dimensional flow* is one in which the velocity vector depends on two space variables and time, that is, $\vec{V} = \vec{V}(x, y, t)$. A *three-dimensional flow* is the most general flow in which the velocity vector depends on three space variables and time, that is, $\vec{V} = \vec{V}(x, y, z, t)$
- The flow of fluid in a pipe or duct is *internal flow* if the fluid is completely bounded by the solid surfaces. The flow of an unbounded fluid over a surface such as flat plate is *external flow*.
- An *inviscid flow* is one in which the effect of viscosity is negligible. In inviscid flow, the fluid viscosity is assumed to be zero. All fluids possess viscosity, and accordingly, all real flows are *viscous*.
- A flow is said to be *irrotational* if the fluid particles while flowing do not rotate about their mass centres. On the other hand, a flow is said to be *rotational* when the fluid particles while flowing also rotate about their mass centres.
- *Streamline* at any instant can be defined as an imaginary line in the flow field so that the tangent to the line at any point represents the direction of the instantaneous velocity of that point. For unsteady flows the streamline pattern changes with time. From the definition of streamline, it can be written

$$\vec{V} \times d\vec{S} = 0$$

A bundle of neighbouring streamlines may be imagined to form a passage through which the fluid flows. This passage is known as a *stream tube*.
- A *pathline* is the actual trajectory through space of a selected fluid article during a time of interval.
- A *streakline* at any instant of time is the temporary locations of all particles that have passed through a fixed point in the flow field.
- In a steady flow, the streamlines, path lines and streak lines are identical.

- Three-dimensional continuity equation in differential form is given by

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

For steady flow, the continuity equation is given by

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

For incompressible (*both steady as well as unsteady*) flow, the continuity equation is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

For two-dimensional incompressible (*both steady as well as unsteady*) flow continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

REVIEW QUESTIONS

13.1 Explain Lagrangian and Eulerian methods of describing fluid flow.
13.2 Distinguish between:
  (a) Steady flow and unsteady flow
  (b) Uniform flow and non-uniform flow.
  (c) Incompressible and compressible flow.
13.3 Give examples of laminar flow, turbulent flow, steady flow, unsteady flow, uniform flow and non-uniform flow.
13.4 What do you mean by one-, two- and three-dimensional flows?
13.5 Define Reynolds number. State its significance regarding the determination of type of flow-laminar and turbulent.
13.6 Explain the terms:
  (a) Streamline
  (b) Streakline
  (c) Pathline
13.7 Define streamline. What do streamlines indicate?
13.8 Distinguish between a pathline and a streak line.
13.9 What does the smoke emitting from a lighted cigarette represent, streamline or pathline or streakline? Why?
13.10 Derive an expression for continuity for three dimensional flow and reduce it for steady, incompressible two dimensional flow.

NUMERICAL PROBLEMS

13.1 A 30 cm diameter pipe 50 km long transport oil from a tanker to the shore at 0.02 m^3/s. Find the Reynolds number and comment on the type the flow. The dynamic viscosity and density of oil are 0.1 Ns/m^2 and 850 kg/m^3 respectively.

13.2 A fluid flow is represented by the velocity field $\vec{V} = ax\hat{i} + ay\hat{j}$, where is a constant. Find the equation of streamline passing through a point (1, 2).

13.3 For the following flows find the equation of streamline:

(i) $\vec{V} = 2y\hat{i} - x^2\hat{j}$ passing through the point (1, 2)

(ii) $\vec{V} = 4x\hat{i} + 3y\hat{j}$ passing through the point (1, 4).

13.4 Obtain the equation of the streamlines for the velocity field given as $\vec{V} = 2x^3\hat{i} - 6x^2y\hat{j}$

13.5 In a flow the velocity vector is given by $\vec{V} = 4x\hat{i} - 3y\hat{j} - 5z\hat{k}$. Determine the equation of the streamline passing through a point (1, 1, 1).

13.6 A three-dimensional velocity field is given by $u = -x$, $v = 2y$, $w = 5 - z$. Find the equation of streamline through (1, 2, 1).

13.7 A two-dimensional flow is described in the Lagrangian system as

$$x = x_0 e^{-kt} + y_0(1 - e^{-2kt}) \text{ and } y = y_0 e^{kt}$$

Find the equation of path line of the particle.

13.8 A 40 cm diameter pipe is in series with a 30 cm diameter pipe. The volume flow rate of water in the system is 4 m^3/s. What is the average velocity of flow in each pipe?

13.9 A pipe 30 cm diameter is carrying oil of density 900 kg/m^3 with an average velocity of 2 m/s. Calculate the discharge. If the pipe bifurcates into two pipes of 15 cm each, find the average velocity of oil in the 15 cm diameter pipe.

13.10 Fluid flows through a pipeline which contracts from 45 cm diameter at A to 30 cm diameter at B and then branches into two pipes C and D (Fig. 13.16). The diameter of the pipe C is 15 cm and diameter of the pipe D is 20 cm. If the velocity at A be 1.8 m/s and that at D be 3.6 m/s. Determine

(a) Velocity at B, and

(b) Discharge at C and D

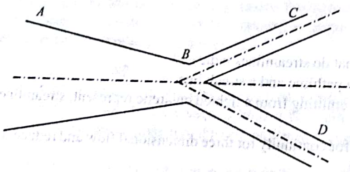

Figure 13.16

13.11 In a two-dimensional incompressible low, the velocity component in the x-direction is given by $u = \frac{2x}{x^2 + y^2}$. Evaluate the velocity field, if $v = 0$ at $y = 0$.

13.12 If $u = x^2 + y^2z^2$, $v = -xy - yz - zx$, determine the velocity component w, which will satisfy continuity for incompressible flow.

13.13 Check whether the following sets of velocity components satisfy the continuity equation of incompressible flow

(i) $u = x + y$, $v = x - y$

(ii) $u = 3xy$, $v = x^3 - xy^3$

(iii) $u = 2x^2 - xy + z^2$, $v = x^2 - 4xy + y^2$, $w = 2xy - yz + y^2$

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer.

13.1 The necessary condition for the flow to be steady is that
(a) the velocity does not change from place to place at any instant
(b) the velocity is constant at a point with respect to time
(c) the velocity changes at a point with respect to time
(d) the velocity changes with location at any instant

13.2 Uniform flow occurs when
(a) the velocity does not change from place to place at any instant
(b) the velocity is constant at a point with respect to time
(c) the velocity changes at a point with respect to time
(d) the velocity changes with location at any instant

13.3 During the opening of a valve in a pipeline, the flow is
(a) steady (b) unsteady (c) uniform (d) laminar

13.4 One-dimensional flow is
(a) uniform flow
(b) steady flow
(c) restricted to flow in a straight line
(d) one which neglects changes in fluid properties in a transverse direction

13.5 For pipes, turbulent flow occurs when Reynolds number is
(a) less than 2000 (b) between 2000 and 4000
(c) more than 4000 (d) less than 4000

13.6 For an ideal fluid flow the Reynolds number is
(a) 2100 (b) 100 (c) zero (d) infinity

13.7 In laminar flow
(a) Newton's law of viscosity applies
(b) fluid particles move in irregular and haphazard paths
(c) the viscosity is unimportant
(d) All of the above

13.8 Laminar flow generally occurs for cases involving
(a) very slow motions (b) highly viscous fluids
(c) very narrow passages or capillary tubes (d) All of the above

13.9 Velocity vector of a flow field is given as $\vec{V} = 2xy\vec{i} - x^2z\vec{j}$. The velocity vector at (1, 2, 1) is
(a) $4\vec{i} - \vec{j}$ (b) $4\vec{i} - \vec{k}$ (c) $\vec{i} - 4\vec{j}$ (d) $\vec{i} - 4\vec{k}$

13.10 A streamline is a line
(a) which is along the path of a particle
(b) across which there is no flow
(c) on which tangent drawn at any point gives the direction of velocity
(d) Both (b) and (c)

13.11 The velocity field is given by $\vec{V} = 3xy\vec{i} + \frac{3}{2}\left(x^2 - y^2\right)\vec{j}$. What is the relevant equation of a streamline?
(a) $\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$
(b) $\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$
(c) $\frac{dy}{dx} = \frac{x^2 - y^2}{3xy}$
(d) $\frac{dy}{dx} = \frac{xy}{x^2 - y^2}$

13.12 The streamline shapes of the following 2-D velocity field: $u = -y$, $v = x$ will be
(a) circle
(b) parabola
(c) ellipse
(d) rectangular hyperbola

13.13 A fluid flow is represented by the velocity field $\vec{V} = ax\hat{i} + ay\hat{j}$, where a is a constant. The equation of streamline passing through a point (1, 2) is
(a) $x - 2y = 0$
(b) $2x - y = 0$
(c) $2x + y = 0$
(d) $x + 2y = 0$

13.14 A two-dimensional flow field has velocities along the x and y directions given by $u = x^2t$ and $v = -2xyt$ respectively, where t is time. The equation of streamlines is
(a) xy = constant
(b) xy^2 = constant
(c) x^2y = constant
(d) Not possible to determine

13.15 The equation of a streamline passing through the origin in a flow field $u = \cos\theta$; $v = \sin\theta$ for a constant is
(a) $y = x^3$
(b) $y = x\cos^2\theta$
(c) $y = x\tan\theta$
(d) $y = \sin\theta$

13.16 A path line describes
(a) the velocity direction at all points on the line
(b) the path followed by particles in a flow
(c) the path over a period of times of a single particle that has passed out at a point
(d) the instantaneous position of all particles that have passed a point

13.17 Streamline, pathline and streakline are identical when
(a) the flow is uniform
(b) the flow is steady
(c) the flow velocities do not change steadily with time
(d) the flow is neither steady nor uniform.

13.18 The continuity equation is the result of application of the following law to the flow field
(a) Conservation of momentum
(b) Conservation of energy
(c) Conservation of force
(d) Conservation of mass

13.19 The continuity equation (at two sections 1 and 2) for an incompressible fluid is given as
(a) $\rho_1 A_1 V_1^2 = \rho_2 A_2 V_2^2$
(b) $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$
(c) $A_1 V_1 = A_2 V_2$
(d) $\rho_1^2 A_1 V_1 = \rho_2^2 A_2 V_2$

13.20 An ideal fluid flow must satisfy
(a) continuity equation
(b) Newton's law of viscosity
(c) Pascal's law
(d) None of the above

13.21 For two-dimensional incompressible flow, if the x component of velocity is $u = Ae^x$, then what is the y-component of velocity?
(a) $-Ae^{-x}$
(b) $-Ae^{-x}y$
(c) Ae^y
(d) $-Ae^x y$

13.22 For the continuity equation given by $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ ($\nabla . \vec{V} = 0$, where $\vec{V}$ is the velocity vector) to be valid, which one of the following is a necessary condition?
(a) steady flow
(b) incompressible flow
(c) inviscid flow
(d) irrotational flow

13.23 The general form of expression for the continuity equation in a Cartesian coordinate system for incompressible or compressible flow is given by
(a) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$
(b) $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$
(c) $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{az} = 0$
(d) $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{az} = 1$