de-Broglie's Waves & Uncertainty principle

1. electron →

a) $P = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{2\times10^{-10}}$ kgm/s

$= 3.31\times10^{-24}$ kgm/s

This momentum is non-rel. for electron.

$E_K = \frac{P^2}{2m} = \frac{(3.31\times10^{-24})^2}{2\times9.1\times10^{-31}}$ Joule

$= 0.602\times10^{-17}$ J

$= 37.63$ eV

and rest mass energy $= m_0c^2$

$= 0.51$ MeV

$= 510000$ eV

∴ b) total energy $= 510037.63$ eV

Photon →

a) $P = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{2\times10^{-10}}$ kgm/s

$= 3.31\times10^{-24}$ kgm/s.

Photon has zero rest mass.

∴ b) Total energy =

$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{2\times10^{-10}}$ Joule

$= 9.95\times10^{-16}$ J

$= 6216$ eV

c) Electron kinetic energy $= 37.63$ eV

photon " " $= 6216$ eV.

2. Let particle velocity $= v_{particle}$

and electron " $= v_{electron}$.

Q.T.P → $v_{particle} = 3\, v_{electron}$,

again, $\frac{\lambda_{particle}}{\lambda_{electron}} = 1.1813\times10^{-4}$

$\Rightarrow \frac{h}{m_{particle}\times v_{particle}} \times \frac{m_{electron}\times v_{electron}}{h} = 1.1813\times10^{-4}$

$\Rightarrow \frac{m_{electron}}{m_{particle}} = 1.1813\times3\times10^{-4}$

$\Rightarrow m_{particle} = \frac{9.1\times10^{-31}}{1.1813\times3\times10^{-4}} = 2.57\times10^{-27}$ kg

∴ The particle is neutron.

3. for ~~a~~ relativistic speed →

$$\lambda = \frac{hc}{\sqrt{E_K(E_K+2m_0c^2)}} \; ; \quad E_K = eV \quad (V = \text{potential diff.})$$

$$\lambda = \frac{hc}{\sqrt{eV(eV+2m_0c^2)}} = \frac{hc}{\sqrt{eV \cdot 2m_0c^2\left(1+\frac{eV}{2m_0c^2}\right)}}$$

$$= \frac{h}{\sqrt{2m_0eV}}\left(1+\frac{eV}{2m_0c^2}\right)^{-1/2}$$

4. B.T.P →

$$\lambda_{electron} = \lambda_{photon} = \frac{hc}{E_K} = \frac{6.63\times10^{-34}\times3\times10^{8}}{100\times10^{3}\times1.6\times10^{-19}}\ m$$

$$\therefore \lambda_{electron} = 12.43\times10^{-12}\ m$$

using, $P = \frac{h}{\lambda_{electron}} = \frac{6.63\times10^{-34}}{12.43\times10^{-12}} = 0.533\times10^{-22}$ kgm/s.

This momentum is non-relativistic momentum for electron.

$$\therefore E_K = \frac{P^2}{2m} = \frac{(0.533\times10^{-22})^2}{2\times9.1\times10^{-31}}\ \text{Joule} = \frac{0.0156\times10^{-13}}{1.6\times10^{-19}}\ eV$$

$$E_K = 9.75\times10^{3}\ eV = 9.75\ keV.$$

5. $(E_K)_{electron} = (E_K)_{proton}$

$$\Rightarrow \frac{P_e^2}{2m_e} = \frac{P_{proton}^2}{2m_p}$$

$$\Rightarrow \frac{P_e^2}{P_{proton}^2} = \frac{m_e}{m_p} \Rightarrow \frac{\lambda_p}{\lambda_e} = \sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{9.1\times10^{-31}}{1.67\times10^{-27}}} = 2.33\times10^{-2}$$

a) $\therefore \frac{\lambda_e}{\lambda_p} = \frac{100}{2.33} = 42.9$ ✓

b) $\frac{\lambda_e}{\lambda_p} = 42.9$

$$\Rightarrow \frac{m_p v_p}{m_e v_e} = 42.9 \quad \left(P = \frac{h}{\lambda}\right).$$

$$\Rightarrow \frac{v_p}{v_e} = \frac{42.9\times9.1\times10^{-31}}{1.67\times10^{-27}} = 233.8\times10^{-4} \quad \therefore \frac{v_e}{v_p} = 42.8 \Rightarrow$$

Particle velocity = Group Velocity

$$\therefore \frac{(v_g)_{electron}}{(v_g)_{proton}} = 42.8$$

c) $$\frac{(v_e)_{phase}}{(v_p)_{phase}} = \frac{v_e/2}{v_p/2} = \frac{v_e}{v_p} = 42.8 \text{ (non-rel. case)}.$$

6. B.T.P →

$$v_e = v_p$$

$$\therefore \frac{\lambda_e}{\lambda_p} = \frac{m_p v_p}{m_e v_e} = \frac{1.67\times10^{-27}}{9.1\times10^{-31}} = 1835$$

Similarly, $$\frac{(v_g)_{electron}}{(v_g)_{proton}} = \frac{v_e}{v_p} = 1$$

and $$\frac{(v_e)_{phase}}{(v_p)_{phase}} = 1,$$

7. $$\lambda_e = \lambda_p$$

$$\Rightarrow \frac{h}{m_e v_e} = \frac{h}{m_p v_p} \Rightarrow m_e v_e = m_p v_p$$

Now, $$\frac{E_e}{E_p} = \frac{\frac{1}{2} m_e v_e^2}{\frac{1}{2} m_p v_p^2} = \frac{v_e}{v_p} = \frac{m_p}{m_e} = 1835.$$

$$\therefore E_e > E_p$$

8. electron → $E_k = 1 \text{ MeV}$ (greater than rest mass energy)

$$\therefore \lambda = \frac{hc}{\sqrt{E_k(E_k + 2m_0c^2)}}$$

$[m_0c^2 = 0.51 \text{ MeV}]$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{\sqrt{1\times1.51\,(\text{MeV})^2}}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{1.23 \text{ MeV}} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.23\times10^{6}\times1.6\times10^{-19}} = 10.11\times10^{-13} \text{ m.}$$

neutron →

$E_K = 1\,MeV$ (less than rest mass energy)

$$\lambda = \frac{h}{\sqrt{2mE_K}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{2\times1.68\times10^{-27}\times1\times10^{6}\times1.6\times10^{-19}}}\ m$$

$$= \frac{6.63}{2.32}\times10^{-14}\ m = 2.86\times10^{-14}\ m$$

Photon

$E_K = 1\,MeV$

$$\lambda = \frac{hc}{E_K} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1\times10^{6}\times1.6\times10^{-19}}\ m = 12.43\times10^{-13}\ m.$$

9.

de-Broglie wave length

$$\lambda_d = \frac{h}{mv}$$

$$= \frac{h\sqrt{1-v^2/c^2}}{m_0 v} \qquad \left[m = \frac{m_0}{\sqrt{1-v^2/c^2}}\right]$$

$$= \frac{h\sqrt{1-\frac{1}{2}}}{m_0 \cdot c/\sqrt{2}}$$

$$= \frac{h}{m_0 c} \qquad \lambda_c = \text{compton wave length}$$

$$\boxed{\lambda_d = \lambda_c}$$ Proof

10.

$$v_{Phase} = \sqrt{\frac{2\pi S}{\lambda \rho}}$$

We know, $v_g = v_p - \lambda \frac{dv_p}{d\lambda}$

$$= v_p - \lambda \times \sqrt{\frac{2\pi S}{\rho}} \cdot \left(-\frac{1}{2}\right)\frac{1}{\lambda^{3/2}}$$

$$= v_p + \frac{1}{2}\sqrt{\frac{2\pi S}{\lambda \rho}}.$$

$$= v_p + \frac{1}{2}v_p = \frac{3}{2}v_p$$

11. $v_p = \sqrt{g\lambda/2\pi}$

$$\therefore v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

$$= v_p - \lambda \sqrt{\frac{g}{2\pi}} \cdot \frac{1}{2} \frac{1}{\sqrt{\lambda}}$$

$$= v_p - \frac{1}{2}\sqrt{\frac{g\lambda}{2\pi}}$$

$$= v_p - \frac{1}{2} v_p = v_p/2$$

12. $v = 0.900c$

$\therefore v_g$ (group velocity) $= 0.900c$

again, $v_p = \frac{c^2}{v} = \frac{c^2}{0.9c} = \frac{c}{0.9} = \frac{3\times10^8}{.9}$

$$= 3.33\times10^8 \text{ m/s}.$$

13. $v_g = \frac{d\omega}{dk}$

$$= \frac{d(2\pi\nu)}{d(2\pi/\lambda)} = \frac{d\nu}{d(1/\lambda)}.$$

14. a) $v_{Phase} = \frac{\omega}{k} = \frac{E}{p} = \frac{\sqrt{p^2c^2+m_0^2c^4}}{p} = \sqrt{c^2+m_0^2c^4/p^2}$

$$v_{Phase} = c\sqrt{1+\left(\frac{m_0c^2}{h}\lambda\right)^2}.$$

b) $\lambda = 1\times10^{-13}$

$$\boxed{\begin{aligned} p &= \frac{h}{\lambda} \\ \Rightarrow \cancel{v} &= \cancel{\frac{6.63\times10^{-34}}{9.1\times10^{-31}\times1\times10^{-13}} = 0.73\times10^{10}} \end{aligned}}$$

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{1\times10^{-13}} = 6.63\times10^{-21} \text{ kgm/s}.$$

This momentum is relativistic momentum for electron,

$$\therefore mv = 6.63\times10^{-21}$$

$$\frac{m_0 v}{\sqrt{1-v^2/c^2}} = 6.63\times10^{-21}$$

$$\Rightarrow \frac{v}{\sqrt{1-v^2/c^2}} = \frac{6.63\times10^{-21}}{9.1\times10^{-31}} = 0.73\times10^{10}$$

$$\therefore \frac{v^2}{1-v^2/c^2} = 0.53\times10^{20}$$

$$\Rightarrow c^2 v^2 = 0.53\times10^{20}(c^2-v^2)$$

$$\Rightarrow v^2(9\times10^{16}+0.53\times10^{20}) = 9\times10^{16}\times0.53\times10^{20}$$

$$\Rightarrow v^2\times5.309\times10^{19} = 4.77\times10^{36}$$

$$\Rightarrow v^2 = 0.898\times10^{17} = 8.98\times10^{16}$$

$$\Rightarrow v = 2.99\times10^{8}\ m/s.$$

$$\therefore v_g = v = 2.99\times10^{8} = 0.99\ c$$

and $$v_p = \frac{c^2}{v} = \frac{c^2}{0.99\ c} = \frac{c}{0.99} = 1.01\ c$$

14.

$$v = 1 km/s = 1000\ m/s.$$

$$\Delta v = 1000\times\frac{0.05}{100} = 0.5$$

$\therefore$ Using Heisenberg Uncertainty principle –

$$\Delta x.\Delta p \geq \hbar$$

$$\therefore \Delta x.\ m.\Delta v \geq \hbar$$

$$\Rightarrow \Delta x \geq \frac{\hbar}{m.\Delta v} = \frac{6.63\times10^{-34}}{2\times\pi\times9.1\times10^{-31}\times0.5} = 0.232\times10^{-3}\ m.$$

15. $\Delta x = 10^{-9}\ m.$

$\therefore$ Using H.U.P.

$$\Delta v = \frac{\hbar}{\Delta x.\ m} = \frac{6.63\times10^{-34}}{10^{-9}\times2\times\pi\times9.1\times10^{-31}} = 0.116\times10^{6}\ m/s.$$

Using H.U.P —

$$\Delta E \, \Delta t \geq \hbar$$

$$\Delta E \geq \frac{6.63\times10^{-34}}{2\pi\times10^{-8}}$$

$$\therefore (\Delta E)_{min} = \frac{6.63\times10^{-34}}{2\pi\times10^{-8}}$$

$$= 1.056\times10^{-26} \text{ Joule}$$

$$= 6.59\times10^{-8} \text{ eV.}$$

17. $\Delta x = \lambda$

$$\therefore \Delta x = \frac{h}{mv}$$

Using H.U.P —

$$\Delta x \, \Delta p \geq \hbar$$

$$\therefore \Delta x . m \Delta v \geq \hbar$$

$$\Rightarrow \Delta v \geq \frac{h}{2\pi\times\Delta x . m}$$

$$\geq \frac{\cancel{h} \, \cancel{m} v}{2\pi\times\cancel{h} . \cancel{m}}$$

$$\boxed{\Delta v \geq v/2\pi}$$

if we consider, $\Delta x \, \Delta p \geq h$

then $\boxed{\Delta v \geq v.}$

$\Delta v = \frac{300\times0.01}{100} = 0.03 \quad \therefore \Delta x = \frac{h}{2\pi\times m\Delta v} = \frac{6.63\times10^{-34}}{2\pi\times50\times10^{-3}\times0.03}$

$$= 7.07\times10^{-32} \text{ m,}$$

$\Delta L = 2\hbar\times\frac{5}{100} = \frac{\hbar}{10}$

Using H.U.P — $\Delta L \, \Delta\theta \geq \hbar \Rightarrow \Delta\theta = 10$ radian,

... $(\Delta\theta)_{max} = 2\pi$ radian, $\therefore$ cannot be specified at all.