1) The velocity distribution over a plate is given by $u(y) = \frac{2}{3}y - y^2$. If $\mu = 0.863$ Pa-s, find the shear stress at $y = 0$ and $y = 0.15$m.

A) $u(y) = \frac{2}{3}y - y^2$

$$\frac{du}{dy} = \frac{2}{3} - 2y$$

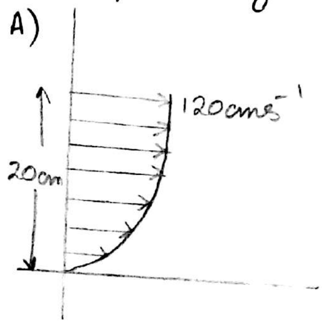
$$\tau = \mu \times \frac{du}{dy} = 0.863\left(\frac{2}{3} - 2y\right)$$

$$\tau_{y=0} = 0.863\left(\frac{2}{3} - 2(0)\right) = 0.576\ Nm^{-2}$$

$$\tau_{y=0.15} = 0.863\left(\frac{2}{3} - 2(0.15)\right) = 0.317\ Nm^{-2}$$

2) The velocity profile of a fluid over a plate is parabolic, with vertex 20 cm from the plate, where the velocity is $120 cms^{-1}$. Calculate the velocity gradient and shear stresses at a distance 0, 10, 20 cm from the plate, if the viscosity of the fluid is 8.5 Poise.

A)



General equation for a parabolic path:

$$u(y) = ay^2 + by + c$$

We have the following three conditions:

(i) $y = 0$, $u = 0$

(ii) $y = 20$, $u = 120 cms^{-1}$

(iii) $y = 20$, $\frac{du}{dy} = 0$

$$u(0) = c = 0$$

$u(20) = 400a + 20b$ i.e. ~~14400~~ $= 120 = 400a + 20b$ i.e. $6 = 20a + b$

or $b = 6 - 20a$

$$\frac{du}{dy} = 2ay + b$$

$$\left.\frac{du}{dy}\right|_{y=20} = 2a(20) + b = 40a + b = 0$$

$-40a = b$ or $6 - 20a = -40a$

Scanned by CamScanner

$u(y) = -0.3y^2 + 12y$

$$\frac{du}{dy} = -0.6y + 12$$

$$\left.\frac{du}{dy}\right|_{y=0} = 12\,s^{-1}$$

$$\left.\frac{du}{dy}\right|_{y=10} = -0.6(10) + 12 = 6\,s^{-1}$$

$$\left.\frac{du}{dy}\right|_{y=20} = -0.6(20) + 12 = 0\,s^{-1}$$

$$\tau = \mu \times \frac{du}{dy} = \frac{8.5}{10} \times \frac{du}{dy} = 0.85 \times \frac{du}{dy}$$

$$\tau_{y=0} = 0.85(12) = 10.2\ Nm^{-2}$$

$$\tau_{y=10} = 0.85(6) = 5.1\ Nm^{-2}$$

$$\tau_{y=20} = 0.85(0) = 0\ Nm^{-2}$$

3) A circular disc of diameter D is slowly rotated in a liquid of viscosity μ at a height h from a fixed surface. Derive an expression for torque T necessary to maintain an angular velocity ω of the disc.

A) $u = r\omega$, $dy = h$ i.e. $\frac{du}{dy} = \frac{\omega r - 0}{h} = \frac{\omega r}{h}$

$r = \frac{D}{2}$ i.e. $\frac{du}{dy} = \frac{\omega D}{2h}$

$\tau = \mu \times \frac{du}{dy} = \frac{\mu\omega D}{2h}$; $dF = \tau \times dA$; $dA = 2\pi \cancel{R} \times r\,dr$

$$dF = \frac{\mu\omega D}{2h} \cdot \pi R\,r\,dr$$

$$dT = dF \times r = \frac{\mu\omega D}{2h} \cdot 2\pi \cancel{R}\, r\,dr \times \frac{D}{2} = \frac{\mu\omega D^2\pi}{4h}\, r\,dr$$

$$T = \frac{\mu\omega D^2\pi}{4h}\int_0^{D/2} r\,dr = \frac{\mu\omega D^2\pi}{4h}\left.\frac{r^2}{2}\right|_0^{D/2} = \frac{\mu\omega D^2\pi}{4h} \times \frac{D^2}{8} = \frac{\mu\omega D^4\pi}{32h}$$

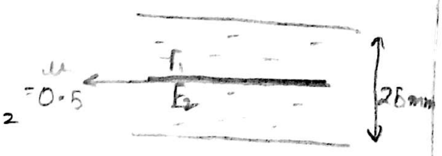
Scanned by CamScanner

4) A space 25 mm wide between two large plane surfaces is filled with glycerine. What force is required to drag a very thin plate $0.75 m^2$ in area between the surfaces at a speed of $0.5 ms^{-1}$.
(i) if this plate remains equidistant from the two surfaces.
(ii) if the plate is at a distance of 10 mm from one of the surfaces. Take $\mu = 0.785 Nsm^{-2}$.

A) $A = 0.75 m^2$, $u = 0.5 ms^{-1}$

(i) $dy = 12.5 \times 10^{-3} m$

$du = 0.5 - 0 = 0.5 ms^{-1}$



$\tau = \mu \times \dfrac{du}{dy} = 0.785 \times \dfrac{0.5}{12.5 \times 10^{-3}} = 31.4 Nm^{-2}$

$F_1 = \tau \times A = 31.4 \times 0.75 = 23.55 N$

Similarly $F_2 = 23.55 N$

$F = F_1 + F_2 = 47.1 N \approx 47 N$

(ii) $\tau_1 = 0.785 \times \dfrac{0.5}{10 \times 10^{-3}} = 39.25 Nm^{-1}$ $(\because dy = 10 \times 10^{-3} mm)$

$F_1 = \tau_1 \times A = 39.25 \times 0.84 = 39.25 Nm^{-2}$

$\cancel{F_2 = \tau_2 \times A = 0.785 \times \dfrac{0.5}{15 \times 10^{-3}} \times 0.75 = 28.167 Nm^{-2}}$

$\tau_2 = 0.785 \times \dfrac{0.5}{15 \times 10^{-3}} = 28.167 Nm^{-2}$

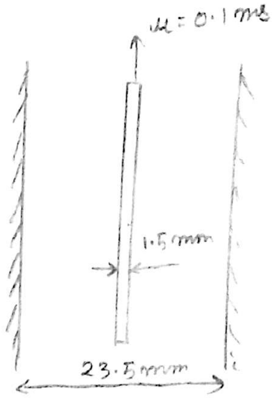
$F_2 = \tau_2 \times A = 28.167 \times 0.75 = 19.625 N$

$F = F_1 + F_2 = 49.06 N$

$\approx 49 N$

5) A vertical gap 23.5 mm wide of infinite extent contains oil of specific gravity 0.95 and viscosity 2.45 N s m^{-2}. A metal plate with dimensions (1.5 m × 1.5 m × 1.5 mm), weighing 49 N is to be lifted through the gap at a constant speed of 0.1 ms^{-1}. Estimate the force required to lift the plate.

A) $S = 0.95$



$\rho = 0.95 \times 1000 = 950 \text{ kgf m}^{-3}$

$V = 1.5 \times 1.5 \times 1.5 \times 10^{-3} = 3.375 \times 10^{-3} \text{ m}^3$

Thickness $= 1.5$ mm, $u = 0.1 \text{ ms}^{-1}$, $W = 49$ N

$dy = \dfrac{23.5 - 1.5}{2} = 11 \text{ mm}$

$\tau = 2.45 \times \dfrac{du}{dy} = \dfrac{2.45 \times 0.1}{11 \times 10^{-3}} = 22.27 \text{ Nm}^{-2}$

$F_1 = \tau \times A = 22.27 \times 2.25 = 50.11 \text{ N}$

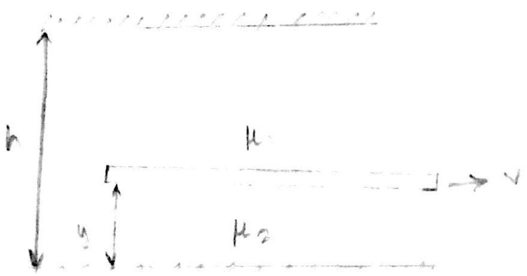
$F = 2F_1 = 100.227 \text{ N}$

Upward thrust = Wt. of fluid displaced $= 950 \times 9.81 \times 3.0375 \times 10^{-3}$
$= 31.45 \text{ N}$

Net force acting in the downward direction $= 49 - 31.45$
$= 17.55 \text{ N}$

Total force required to lift the plate up $= 100.227 + 17.55$
$= 117.77 \text{ N}$
$\approx 117.8 \text{ N}$

6) A thin plate is placed between two flat surfaces, having a very narrow gap of height h, and in such a way that the viscosity of liquid on the top and bottom of the plate are μ_1 and μ_2 respectively. Calculate the position of the thin plate such that drag force or viscous resistance to uniform motion of the thin plate is minimum.



A) $F_1 = \mu \times \frac{du}{dy} \times A$, $F_2 = \mu_2 \times \frac{du}{dy} \times A$

$= \mu_1 \times \frac{u}{y} \times A$ $\qquad = \mu_2 \times \frac{u}{(h-y)} \times A$

$$F = F_1 + F_2 = \frac{Av\mu_1}{y} + \frac{Av\mu_2}{h-y} = Av\left(\frac{\mu_1}{y} + \frac{\mu_2}{h-y}\right)$$

$$\frac{dF}{dy} = Av\left\{\frac{-\mu_1}{y^2} + \frac{\mu_2}{(h-y)^2}\right\} = 0$$

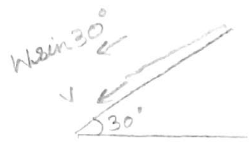
$$-\mu_1(h-y)^2 + \mu_2 y^2 = 0$$

or $\frac{\mu_1}{y^2} = \frac{\mu_2}{(h-y)^2}$ or $\sqrt{\frac{\mu_1}{\mu_2}} = \frac{y}{h-y}$

$$y = \frac{h}{1+\sqrt{\frac{\mu_1}{\mu_2}}}$$

7) A 90 N rectangular solid block slides down a 30° inclined plane. The plane is lubricated by a 3 mm thick oil of viscosity 0.8 Pa-s. If the contact area is 0.3 m², estimate the terminal velocity of the block.

A) Effective weight $= 90 \sin 30 = 90 \times \frac{1}{2} = 45N$



$dy = 3\times10^{-3} m$, $\mu = 0.8\ Nsm^{-2}$

$$\tau = 0.8 \times \frac{du}{3\times10^{-3}} = \frac{F}{A}$$

$$\frac{45}{0.3} = 0.8 \times \frac{du}{3\times10^{-3}} \quad \text{or } du = 0.5625\ ms^{-1}$$

i.e. $u = 0.5625\ ms^{-1}$

8) A 150 mm diameter shaft rotates at 1500 rpm in a 200 mm long journal bearing, with an internal bearing diameter 150.5 mm. The uniform annular space between the shaft and the bearing is filled with oil of dynamic viscosity 0.8 Poise. Calculate the power required to rotate the shaft.

A: D = 150 mm , N = 1500 rpm , L = 200 mm , D_2 = 150.5 mm

$\mu = \frac{0.8}{10} = 0.08\ Nsm^{-2}$

$du = \frac{\pi DN}{60} = \frac{\pi \times 150\times10^{-3}\times1500}{60} = 11.78\ ms^{-1}$

$A = \pi DL = \pi \times 150\times10^{-3}\times200\times10^{-3} = 0.094\ m^2$

$dy = \frac{150.5-150}{2} = 0.25\times10^{-3}\ m$

$\tau = \mu \times \frac{du}{dy} = 0.08 \times \frac{11.78}{0.25\times10^{-3}} = 3769.6\ Nm^{-2}$

$F = \tau\times A = 3769.6\times0.094 = 354.34\ N$

$T = F\times\frac{D}{2} = 354.34\times\frac{150\times10^{-3}}{2} = 26.57\ Nm$

$W = T\omega = T\times\frac{2\pi N}{60} = 26.57\times\frac{2\pi\times1500}{60} = 4.174\ kW$

9) A shaft 80 mm diameter is being pushed through a bearing sleeve 80.2 mm in diameter and 0.3 m long. The clearance, assumed uniformed is filled with lubricating oil of viscosity 0.1 kgms^{-1} and specific gravity 0.9.

a) If the shaft moves axially at 0.8 ms^{-1}, estimate the resistance force exerted by the oil on the shaft.

b) If the shaft is axially fixed, and rotated at 1800 rpm, estimate the resisting torque exerted by the oil and the power required to rotate the shaft.

A) a) $du = 0.8\ ms^{-1}$

$\tau = \mu\times\frac{du}{dy} = 0.1\times\frac{0.8}{\frac{(80.2-80)\times10^{-3}}{2}} = 800\ Nm^{-2}$

$F = \tau\times A$, Now $A = \pi DL = \pi\times80\times10^{-3}\times0.3$

$= 800\times\pi\times80\times10^{-3}\times0.3$

$= 60.32\ N$

b) $N = 1800\, rpm$

$$du = \frac{\pi D N}{60} = \frac{\pi \times 80 \times 10^{-3} \times 1800}{60} = 7.539\, ms^{-1}$$

$$\tau = \mu \times \frac{du}{dy} = 0.1 \times \frac{7.539}{1 \times 10^{-3}} = 7539.8\, Nm^{-2}$$

$$A = \pi D L = \pi \times 80 \times 10^{-3} \times 0.3$$

$$F = \tau \times A = 7539.8 \times \pi \times 80 \times 10^{-3} \times 0.3 = 568.48\, N$$

$$T = F \times \frac{D}{2} = 568.48 \times \frac{80 \times 10^{-3}}{2} = 22.74\, Nm$$

$$W = T\omega = \frac{T \times 2\pi N}{60} = 22.74 \times \frac{2\pi \times 1800}{60} = 4.28\, kW$$

10) A hydraulic ram of 200mm diameter and 1.2m long moves within a concentric cylinder 200.2mm diameter. The annular clearance is filled with oil of specific gravity 0.85 and kinematic viscosity $400\, mm^2 s^{-1}$. What is the viscous force resisting the motion when the ram moves at a speed of $120\, mms^{-1}$.

A) $D = 200 \times 10^{-3} m$, $l = 1.2m$, $dy = \frac{200.2 - 200}{2} = 0.1 \times 10^{-3} m$

$$\nu = \frac{\mu}{\rho} \Rightarrow 400 \times 10^{-6} = \frac{\mu}{0.85 \times 10^{3}} \quad \text{or } \mu = 0.34\, Nsm^{-2}$$

$$du = 120 \times 10^{-3}\, ms^{-1}$$

$$\tau = \mu \times \frac{du}{dy} = \frac{0.34 \times 120 \times 10^{-3}}{0.1 \times 10^{-3}} = 408\, Nm^{-2}$$

$$A = \pi D L = \pi \times 200 \times 10^{-3} \times 1.2 = 0.7539\, m^2$$

$$F = \tau \times A = 408 \times 0.7539$$

$$= 307.6N$$

11) A vertical shaft has a hemispherical bottom of radius R which rotates inside a bearing of identical shape at its ends. An oil film of thickness h and viscosity μ is maintained over the curved surface in the bearing. Estimate the viscous torque in the shaft when it rotates with an angular velocity ω.

A) $\frac{du}{dy} = \frac{\omega R \sin\theta}{h}$

$d\tau = \mu \times \frac{du}{dy} = \frac{\mu \omega R \sin\theta}{h}$

$dF = d\tau \times dA$

$= \frac{\mu \omega R \sin\theta}{h} \times 2\pi R \sin\theta \times R d\theta$

where $dA = 2\pi R \sin\theta \, R d\theta$

$dT = dF \times r$

$= \frac{\mu \omega R \sin\theta}{h} \times 2\pi R \sin\theta \times R d\theta \times R \sin\theta$

$T = \int_0^{\pi/2} dT = \int_0^{\pi/2} \frac{\mu \omega R^4 \sin^3\theta}{h} \times 2\pi \, d\theta$

$= \frac{\mu \omega R^4 2\pi}{h} \int_0^{\pi/2} \sin^3\theta \, d\theta$

$= \frac{\mu \omega R^4 2\pi}{h} \int_0^{\pi/2} \frac{3\sin\theta - \sin 3\theta}{4} d\theta$

$= \frac{2\pi \mu \omega R^4}{4h} \left(3 - \frac{1}{3}\right)$

$= \frac{4\pi \mu \omega R^4}{3h}$

12) An increase in pressure of a liquid from 8 MPa to 17 MPa results in 0.3% decrease in its volume. Determine the bulk modulus of elasticity and coefficient of compressibility of the liquid.

A) $\Delta V = \frac{-0.3}{100} V$

$\Delta p = (17 - 8) = 9 \text{ MPa}$

Scanned by CamScanner

$$\beta = \frac{-dp}{\Delta V / V}$$

$$= \frac{-9 \times 10^{6} \times 100 \times V}{-0.3 V}$$

$$= 3 \times 10^{9} \, Pa$$

$$= 3000 \, MPa$$

$$K = \frac{1}{\beta}$$

$$K = \frac{1}{3 \times 10^{9}} = 0.33 \times 10^{-9} \, m^{2} N^{-1}$$