Diffraction of Light

When light from a narrow linear slit is incident on the sharp edge of an obstacle, it will be found that there is illumination to some extent within the geometrical shadow of the obstacle. This shows that light can bend round an obstacle.

The bending of light around the corner of an obstacle and spreading of light wave into the geometrical shadow of that obstacle is called diffraction of light.

This effect is found to be significant when the dimension of the diffracting element becomes comparable with the wave length of light.

The Huygen's - Fresnel Principle:-

Fresnel gave a satisfactory explanation of this phenomenon by using Huygen's principle in conjunction with the principle of superposition. According to Huygen's principle each point on the wavefront acts as a source of secondary wave. The mutual interference of these secondary waves derived from a particular wavefront, produces the phenomenon of diffraction.

Thus, interference effect is due to the superposition of two distinct waves coming from two coherent sources, while diffraction is the effect of superposition of the secondary waves coming from the different parts of the same wavefront.

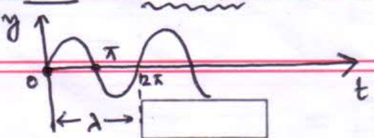
All optical instruments use only a limited portion of the incident wavefront and hence some diffraction effects are always present in the image. Diffraction effects are accordingly of great importance in the detailed understanding of optical devices.

p(2)

Interference	Diffraction.
(1) occurs between two seperate wave fronts originating from two coherent sources.	(1) It is the interference that occurs between the secondary wave-lets originating from the exposed part of the same wavefront.
(2) The interference fringes may or may not be equally spaced.	(2) The diffraction fringes are never equally spaced.
(3) Points of minimum intensity are totally dark.	(3) Points of minimum intensity are not totally dark.
(4) The maxima are of same intensity.	(4) The intensity of central maxima is maximum and decreases on either side as the order of maxima increases.

Fresnel Diffraction	Fraunhofer Diffraction
(1) Either the source or screen or both are at finite distances from the obstacle.	(1) The source of light and the screen are at infinite distances from the obstacle.
(2) Incident wave front is either spherical or cylindrical.	(2) Incident wavefront is generally plane.
(3) The centre of the diffraction pattern may be bright or dark depending upon the no. of Fresnel's zone.	(3) The centre of the diffraction pattern is always bright.

Note: Wave Eqn:-



$$y = a\sin(\omega t - \phi)$$
$$= a\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$
$$= a\sin\left(\frac{2\pi}{T}t - \frac{2\pi x}{\lambda}\right)$$
$$= a\sin\left\{(vt - x)\frac{2\pi}{\lambda}\right\}$$

$a \to$ amplitude, $\phi \to$ Phase diff. between two particles having path difference x.

$\phi = \frac{2\pi}{\lambda}x$

$v \to$ Phase velocity

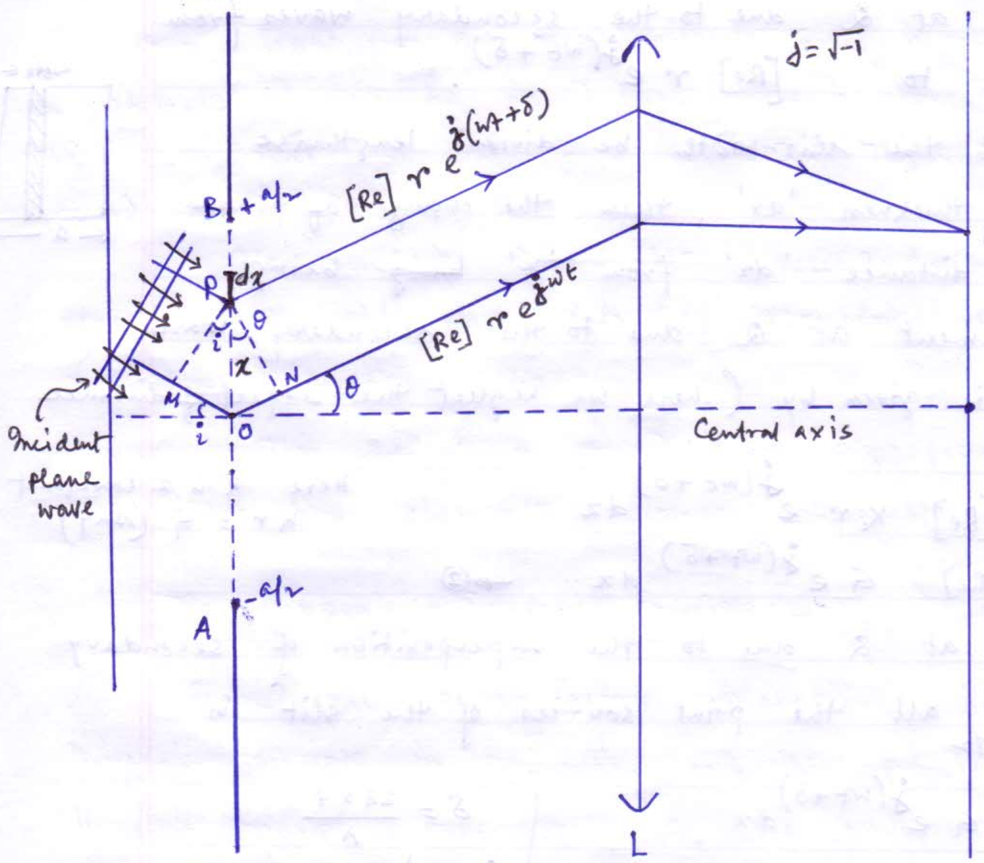
$v = \frac{\lambda}{T} = \nu\lambda$

$T \to$ time period

$\nu \to$ frequency

$\lambda \to$ wavelength.

Fraunhofer diffraction in a single slit



Let a parallel beam of monochromatic light of wavelength λ be made incident on the surface of a narrow slit of width 'a' ($=\overline{AB}$) in a direction making an angle 'i' with the normal to the slit surface, perpendicular to the plane of paper.

These rays will be diffracted in various directions.

Let us now calculate the intensity ~~distribution~~ of light at a point Q at the focal plane of the lens L.

If PM and PN are normals on the incident and diffracted beams, then by geometry $\angle OPM = i$ and $\angle OPN = \theta$.
Here PM is the incident plane wave front, PN is the diffracted plane wavefront.

Let the displacement at any instant at the point Q, due to the secondary waves from the origin 'O' (mid pt. of the slit), be proportional to the real part of $(r e^{j\omega t})$ i.e. $[Re]\, r\, e^{j\omega t}$, where, $j = \sqrt{-1}$ and r is the source strength or amplitude of the secondary wavelet and $\omega = \frac{2\pi}{T}$, T be the time period of spherical or cylindrical wavelets.

If δ be the phase difference between the secondary waves proceeding from the origin and from a point P at a distance 'x' from 'O', then

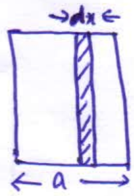
$$\delta = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$= \frac{2\pi}{\lambda} \times [MO + ON]$$

$$= \frac{2\pi}{\lambda} [\sin i \pm \sin\theta]\, x \qquad \text{Here, } \phi = \sin i \pm \sin\theta$$

$$= \frac{2\pi x}{\lambda} \phi \quad \rightarrow (1)$$

Hence, the displacement at Q due to the secondary waves from P will be proportional to $[Re]\ r\, e^{j(\omega t+\delta)}$.



If we consider that the slit-width be divided lengthwise by elementary strips of thickness 'dx', then the change of phase over the infinitesimal distance 'dx' from 'x' ~~beig~~ being negligible, the displacement at Q due to the secondary waves proceeding from 'dx' is given by (here we neglect the effect of distance and incination)

$$dy = [Re]\ K r\, e^{j(\omega t+\delta)}\, dx$$

Here K is a constant
$K r = G$ (say)

$$= [Re]\ G\, e^{j(\omega t+\delta)}\, dx \quad \rightarrow ②$$

The total displacement at Q due to the superposition of secondary waves generated from all the point sources of the slit is

$$y = [Re]\int_{-a/2}^{+a/2} G\, e^{j(\omega t+\delta)}\, dx$$

$$= [Re]\int_{-a/2}^{+a/2} G\, e^{j\frac{2\pi}{\lambda}(ct+x\phi)}\, dx$$

$$= [Re]\ G\,.\, e^{j\frac{2\pi ct}{\lambda}}\int_{-a/2}^{+a/2} e^{j\frac{2\pi\phi}{\lambda}x}\, dx$$

$\delta = \frac{2\pi x\phi}{\lambda}$

$(\omega t+\delta) = \frac{2\pi}{T}t + \frac{2\pi}{\lambda}x\phi$

$= \frac{2\pi}{\lambda}\left(\frac{\lambda}{2\pi}\cdot\frac{2\pi}{T}t + x\phi\right)$

$= \frac{2\pi}{\lambda}\left(\frac{\lambda}{T}t + x\phi\right)$

$= \frac{2\pi}{\lambda}(ct + x\phi)$ | $c = \frac{\lambda}{T}$ speed of light.

$$= [Re]\ G.e^{j.\frac{2\pi ct}{\lambda}}\cdot\frac{1}{j\frac{2\pi\phi}{\lambda}}\left[e^{j\frac{2\pi\phi}{\lambda}x}\right]_{-a/2}^{a/2}$$

Lets consider $\psi = \frac{2\pi\phi}{\lambda}$ and, $\sigma = \frac{2\pi ct}{\lambda}$

$$= [Re]\ G\, e^{j\sigma}\frac{1}{j\psi}\left[e^{j\psi a/2} - e^{-j\psi a/2}\right]$$

$$= [Re]\ G\, e^{j\sigma}\frac{1}{j\psi}\left[\cos\frac{\psi a}{2} + j\sin\frac{\psi a}{2} - \cos\frac{\psi a}{2} + j\sin\frac{\psi a}{2}\right]$$

$e^{j\theta} = \cos\theta + j\sin\theta$

$$= [Re]\ G\, e^{j\sigma}\frac{1}{j\psi}\left[2j\sin\frac{\psi a}{2}\right]$$

$$= [Re]\ \frac{2G}{\psi}\, e^{j\sigma}\sin\left(\frac{\psi a}{2}\right) \quad \rightarrow ③$$

$$\Rightarrow y = [Re] \frac{Ga}{\left(\frac{\psi a}{2}\right)} e^{j\sigma} \sin\left(\frac{\psi a}{2}\right)$$

$$= [Re]\ Ga \left\{ \frac{\sin\left(\frac{\psi a}{2}\right)}{\left(\frac{\psi a}{2}\right)} \right\} e^{j\sigma}$$

$$\Rightarrow y = [Re] \left\{ A_0 \frac{\sin\alpha}{\alpha} \right\} e^{j\sigma}$$

Space part (Amplitude) — Time part (Phase)

$\phi = \sin i \pm \sin\theta$

Here,

$\psi = \frac{2\pi\phi}{\lambda}$, $\sigma = \frac{2\pi ct}{\lambda}$

$A_0 = Ga$

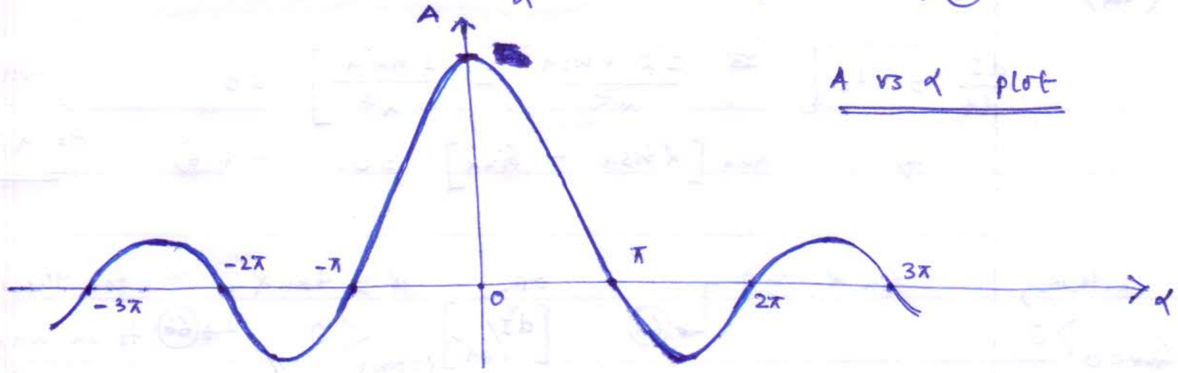
and $\alpha = \frac{\psi a}{2} = \frac{2\pi\phi}{\lambda}\cdot\frac{a}{2} = \frac{\pi\phi a}{\lambda}$

Finally $$\boxed{y = [Re]\ A\ e^{j\cdot\frac{2\pi ct}{\lambda}}} \rightarrow (4)$$

where, $A = A_0 \frac{\sin\alpha}{\alpha}$ is the amplitude of the wave at Q.

Hence, the intensity of the illumination at Q is:

$$I = A^2 = A_0^2 \frac{\sin^2\alpha}{\alpha^2} = I_0 \frac{\sin^2\alpha}{\alpha^2} \rightarrow (5)$$ where, $I_0 = A_0^2 = G^2a^2$



Intensity distribution:-

Intensity $I = I_0 \frac{\sin^2\alpha}{\alpha^2}$ where, $I_0 = G^2a^2$

$\alpha = \frac{\pi a\phi}{\lambda}$

$\phi = \sin i \pm \sin\theta$

It is important to remember that α is a function of θ [angle of defraction]. So we can write

$I \equiv I(\alpha) \equiv I(\theta)$

Assume: normal incidence $(i=0) \Rightarrow \phi \approx \sin\theta$

When $\theta \to 0$, $\alpha \left(= \frac{\pi a}{\lambda}\sin\theta\right) \to 0$ and $\frac{\sin\alpha}{\alpha} \to 1$.

$\Rightarrow$ So, when $\boxed{\alpha \to 0 \Rightarrow I = I_0}$

We now see the significance of the constant (Ga). For $\alpha = 0$, $A_0 = Ga$ — it represents the amplitude when all the wavelets arrive in phase. $G^2 a^2$ $(= I_0)$ is then the value of the maximum intensity, at the centre, Q_0 of the pattern.

I_0 is known as the principal maximum (intensity). From the principal maximum the intensity falls to zero at $\alpha = \pm\pi$ ($\because$ then $\sin\alpha = 0$ & $I = 0$), then passes through several secondary maxima with equally spaced points of zero intensity at $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$, or $\alpha = n\pi$ $(n \neq 0)$. These are discussed systematically below.

Conditions for Minima $(I = 0)$ and secondary maxima

$$I(\alpha) = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

The extrema of $I(\alpha)$ $\left[\alpha = \frac{\pi a}{\lambda}\sin\theta\right]$ occur at values of α that cause $\left(\frac{dI}{d\alpha}\right)$ to be zero, i.e.,

$$\frac{dI}{d\alpha} = I_0\left[\frac{2\sin\alpha\cos\alpha}{\alpha^2} - \frac{2\sin^2\alpha}{\alpha^3}\right] = 0$$

$$\Rightarrow \sin\alpha\left[\alpha\cos\alpha - \sin\alpha\right] = 0 \longrightarrow (6) \qquad \text{As } \alpha \neq 0.$$

Hence either, $\sin\alpha = 0$ $\longrightarrow (6a)$ $\left[d^2I/d\alpha^2\right]_{\sin\alpha=0} > 0$

OR, $\alpha = \tan\alpha$ $\longrightarrow (6b)$ $\left[d^2I/d\alpha^2\right]_{\alpha=\tan\alpha} < 0$

for the intensity to be an extremum.

Minima. The intensity in the diffraction pattern is zero when

$\sin\alpha = 0$ $\left[\frac{d^2I}{d\alpha^2}\text{ is found to be +ve when } \sin\alpha = 0\right]$.

$$\left.\begin{aligned} &\Rightarrow \boxed{\alpha = n\pi}, \quad n = \pm 1, \pm 2, \pm 3, \ldots \\ &\Rightarrow \frac{\pi a}{\lambda}\sin\theta = n\pi \;\Rightarrow\; \boxed{\sin\theta = \frac{n\lambda}{a}} \end{aligned}\right\} \longrightarrow (7)$$

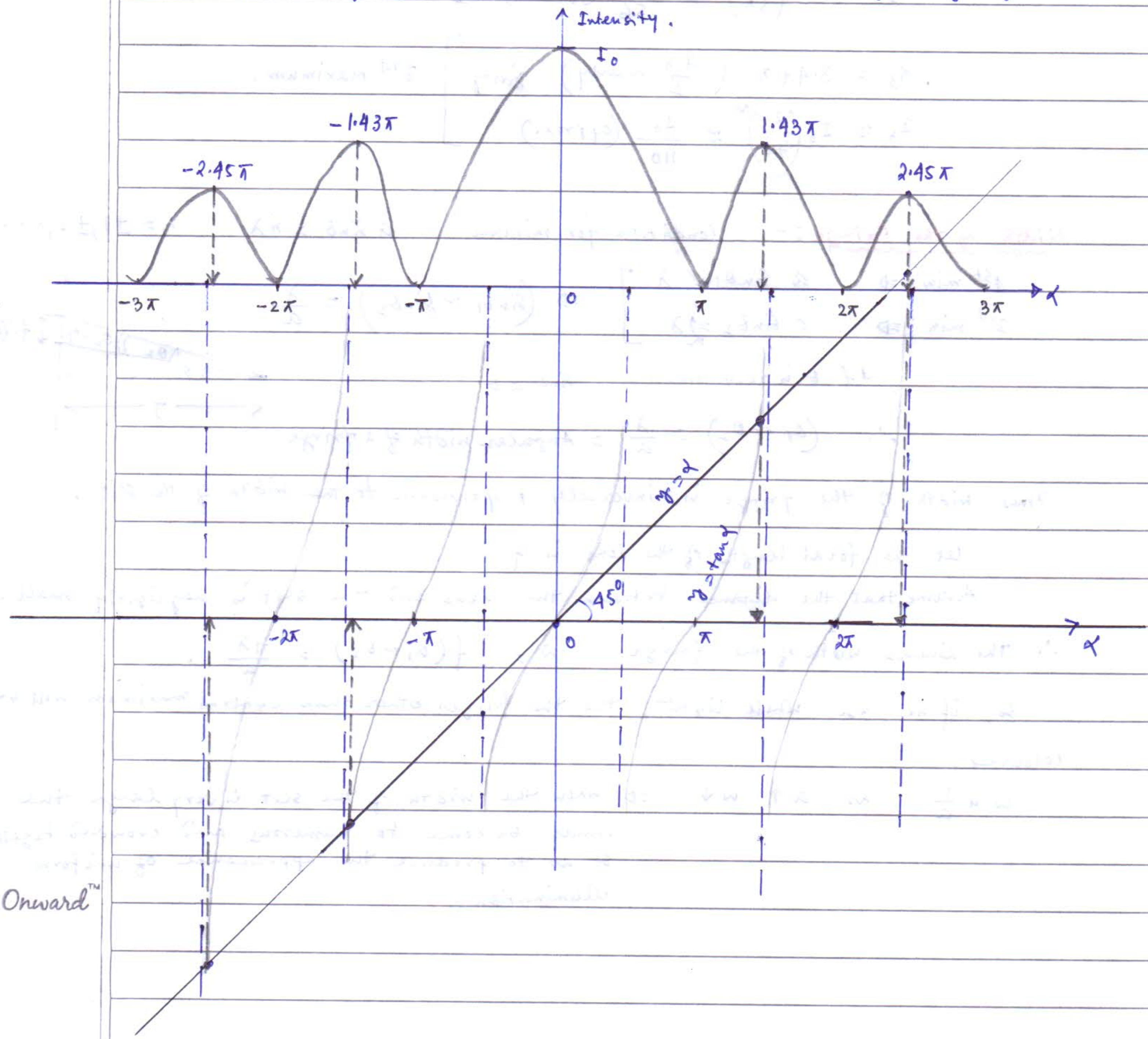
In other words, $I = I_0 \frac{\sin^2\alpha}{\alpha^2} = 0$ when $\sin\theta = \frac{n\lambda}{a}$, $n = \pm1, \pm2, \pm3, \ldots$ $\longrightarrow (7a)$

Secondary Maxima :-

The Positions of maxima can be obtained by solving Eqn. (6b)

$$\alpha = \tan\alpha$$

It is a trancendental eqn. and can be solved graphically by plotting the curves $y = \alpha$ and $y = \tan\alpha$ and finding the point of intersections as shown in the following figure.



Intersecting points between $y = \alpha$ and $y = \tan\alpha$ curves.

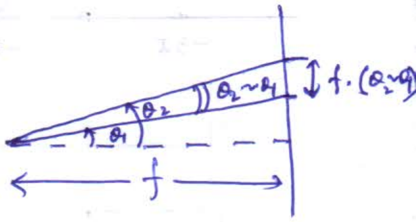
$$\left.\begin{array}{l} \alpha_1 = 1.43\pi \ \left(\frac{3\pi}{2} \text{ nearly}\right) \text{ giving} \\ I_1 \approx I_0\left(\frac{2}{3\pi}\right)^2 \approx \frac{I_0}{20} \ (\text{approx.}) \end{array}\right] \ 1^{st} \text{ Maximum.}$$

$$\left.\begin{array}{l} \alpha_2 = 2.45\pi \ \left(\frac{5\pi}{2} \text{ nearly}\right) \text{ giving} \\ I_2 \approx I_0\left(\frac{2}{5\pi}\right)^2 \approx \frac{I_0}{56} \ (\text{approx.}) \end{array}\right] \ 2^{nd} \text{ Maximum.}$$

$$\left.\begin{array}{l} \alpha_3 = 3.47\pi \ \left(\frac{7\pi}{2} \text{ nearly}\right) \text{ giving} \\ I_3 \approx I_0\left(\frac{2}{7\pi}\right)^2 \approx \frac{I_0}{110} \ (\text{approx.}) \end{array}\right] \ 3^{rd} \text{ Maximum.}$$

Width of the Fringe:- Condition for minima $a \sin\theta = n\lambda$ $n = \pm 1, \pm 2, \cdots$

$$\left.\begin{array}{ll} 1^{st} \text{ min} \Rightarrow & a \sin\theta_1 = \lambda \\ 2^{nd} \text{ min} \Rightarrow & a \sin\theta_2 = 2\lambda \end{array}\right] \Rightarrow (\sin\theta_1 \sim \sin\theta_2) = \frac{\lambda}{a}$$



If θ is very small $\sin\theta \simeq \theta$

$\therefore$ $(\theta_1 \sim \theta_2) = \frac{\lambda}{a}$ = Angular width of a fringe

Thus width of the fringe is inversely proportional to the width of the slit.

Let the focal length of the lens is f

Assume that the distance between the lens and the slit is negligibly small.

$\therefore$ The linear width of the fringe $w = f(\theta_1 \sim \theta_2) = \frac{f\lambda}{a}$.

So, if we use white light, the the fringes other than central maximum will be coloured.

$w \propto \frac{1}{a}$ as $a \uparrow$ $w \downarrow$ $\Rightarrow$ when the width of the slit is very large the bands become to numerous and crowded together, so as to produce the appearence of uniform illumination.