INTERFERENCE OF LIGHT

Light is a form of energy, which produces the sensation of vision in our eye.

- LIGHT AS A WAVE:

To describe the phenomena that light undergo Light has been considered to have dual character-wave and particle. The corpuscular theory of Newton(1642) failed to explain the very basic phenomena of light. The concept of light as wave was put forward by Christian Huygen in 1690. With little modification by Young, Fresnel the wave nature of light was established. It means light energy from the source is conveyed from one point to the other as a result of vibration of the particles of the medium. Later on this concept of wave for light has to be revised by Maxwell and had to put forward the concept of light as the electromagnetic wave in order to incorporate the basic property of light – the radiation which does not require any medium for its propagation.

- A FEW CONCEPTS ABOUT WAVE:

  The concept of mechanical wave is the disturbance in a medium which gets propagated from one place to the other with time by repeated vibration of the particles of the medium. The particles vibrate either along the direction of propagation (in case of longitudinal wave) or perpendicular to the direction of propagation (in case of transverse wave) . This vibration of the medium particles are simple harmonic in nature. For non dissipative medium, the amplitude of vibration of all the medium particles will be same but the attainment of the amplitude will be at different time and that depends on how far is the point from the point of disturbance. The disturbance here is the displacement of the particles vibrating simple harmonically about their mean position of rest.

  In fact any physical entity which varies both in space and time is said to constitute a wave.

  In case of light it is the basic electric field vector.

- WAVE MOTION:

  Any physical entity which varies both in space and time is said to constitute a wave.

  A common example is the displacement of water particles that are created by dropping a stone on its surface . Here displacement is a function of space and time. If we take the snap shot of the ripples that is produced on the surface of water we get the variation of the displacement function as a function of space about their mean position of rest(fig.1) . Again , if we look at a position of water surface and observe the variation of displacement with time, we get the vibration of the medium particle (fig.2) . We here will deal with periodic and harmonic wave.

Let the disturbance is a function of space and time and is represented as,

$$\Psi = f(x,t) \qquad (1)$$

The shape of the disturbance or the wave profile at any instant ,say, t=0, is given by,

$$\Psi/_{t=0} = f(x,0) = f(x) \qquad (2)$$

Let us consider the wave profile is moving along the +ve x-direction with a velocity v. So in time t, it moves through a distance vt but in all other respect the wave profile remain unchanged. Let $S^{/}$ be the co-ordinate system that is fixed with the profile and hence moving with velocity v with respect to S. So the wave profile is as a snap in $S^{/}$ always and hence can be written as,

$\Psi = f(x^{/})$ in primed system.

But, $x^{/} = x - vt$,

So, at time t and seen from S frame, the wave profile can be represented as,

$\Psi = f(x^{/}) = f(x-vt)$.

Thus the most general form of wave propagating with a velocity v along the +ve x-axis is written as, $\Psi(x,t) = f(x - vt)$ and with velocity v along –ve x-axis is written as $\Psi(x,t) = f(x + vt)$. $\Psi(x,t)$ is called wave function.

- **DIFFERENTIAL WAVE EQUATION:**

  From the one dimensional arbitrary wave function $\Psi(x,t) = f(x - vt)$, we can construct one dimensional wave equation as follows,

  with $x^{/} = x - vt$, we can write, $\Psi(x,t) = f(x^{/})$. Hence,

  $$\frac{\partial \Psi}{\partial x} = \frac{\partial f}{\partial x^{/}}\frac{\partial x^{/}}{\partial x} = \frac{\partial f}{\partial x^{/}}$$ **and**

$$\frac{\partial^2\Psi}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2}\frac{\partial x'}{\partial x} = \frac{\partial^2 f}{\partial x'^2} \qquad (3)$$

Similarly,

$$\frac{\partial\Psi}{\partial t} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial t} = \frac{\partial f}{\partial x'}(\mp v)$$

$$\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2 f}{\partial x'^2}\frac{\partial x'}{\partial t}(\mp v) = \frac{\partial^2 f}{\partial x'^2}(v)^2 \qquad (4)$$

(3) and (4) gives ,

$$\frac{\partial^2\Psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\Psi(x,t)}{\partial t^2} \qquad (5)$$

It is the one dimensional differential wave equation which is linear and homogeneous and the principle of superposition is applicable to it.

- **PRINCIPLE OF SUPERPOSITION,**

  If, Ψ_1 and Ψ_2 are the two solutions then a linear combination of them , i.e., $C_1\Psi_1 + C_1\Psi_1$is also a solution . It is allowed as the wave equation is linear.

- **HARMONIC WAVE:**

  The simplest kind of harmonic wave propagating along the +ve x-axis is,

  $$\Psi(x,t) = \text{Sin}\, k(x - vt)$$

  Or, $\Psi(x,t) = \text{Cos}\, k(x - vt)$

  The wave is periodic in both space and time. The spatial period is called wave length and temporal period is called time period.

- Wave length: If x is changed by $\pm\lambda$, the function should remain unaltered.

  i.e., $\Psi(x,t) = \text{Sin}\,[k\{(x \pm \lambda) - vt\}] = \text{Sin}\,[k(x - vt) \pm 2\pi]$

  thus, $k\lambda = 2\pi$ or, $k = \frac{2\pi}{\lambda}$ called the wave number . writing ϕ = k(x-vt) called the phase of the wave, we can see that the wave profile $\Psi(x)$ is zero at ϕ = 0,π, 2 π,... etc. And x= 0, $\lambda/2$, λ, ...etc.

- TIME PERIOD:

  If t is replaced by (t+T) , the function should remain unaltered then T is called time period.

i.e., $\Psi(x,t) = \text{Sin}\,[k\{x - v(t+T)\}] = \text{Sin}\,[k(x - vt) - 2\pi]$

thus, $kvT = 2\pi$ or, $T = \frac{2\pi}{kv} = \frac{2\pi}{2\pi\upsilon} = \frac{2\pi}{\omega}$ called the Time period.

Problem 1.

Show that $y(x,t) = \sin(bx - at + \alpha)$ represents a harmonic wave propagation. Identify the phase velocity of the wave.

Superposition of waves

We know that each field of electro-magnetic wave satisfies the scalar 3-D wave equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{.........} \quad (A)$$

Since this a linear and homogeneous equation and derivatives appear only to the first power, consequently, if $\psi_1(\mathbf{r}, t), \psi_2(\mathbf{r}, t), \ldots\ldots, \psi_n(\mathbf{r}, t)$ are individual solutions of the wave equation, then any linear combination of them will also be the solution of the wave equation.

i.e., $\psi(\mathbf{r}, t) = \sum_{i=1}^{n} C_i\psi_i(\mathbf{r}, t)$ will satisfy equation (A).

Principle of superposition states that the resultant disturbance at any point in a medium is the algebraic sum of the separate constituent waves.

Superposition of two waves having same frequency

A solution of the differential wave equation can be written in the form

$$E(x,t) = E_0 \text{ Sin } [\omega t - (kx + \varepsilon)]$$

where E_0 is the Amplitude of the harmonic disturbance propagating along +ve x-axis, k is the wave vector, ε is the initial phase of the wave and ω is the frequency of the wave. **The phase of the wave is represented by the argument of the *sin* term, i.e [ωt – (kx + ε)]**. Position of the point of interest is denoted by x and t represents the time.

Considering the superposition of two waves moving in the same direction (same k) having same frequency (same ω) but different initial phases and amplitudes, the two superposing waves can be represented by,

$$E_1 = E_{01} \sin(\omega t+\alpha_1) \text{ and } E_2 = E_{02} \sin(\omega t+\alpha_2),$$

Here, the space and time part of the phase of the wave is separated as follows,

$$\alpha(x,\varepsilon) = -(kx+\varepsilon)$$

The resultant wave thus becomes, $E = E_1 + E_2 = E_{01} \sin(\omega t+\alpha_1) + E_{02} \sin(\omega t+\alpha_2)$

$= E_{01}[\sin \omega t \cos \alpha_1 + \cos \omega t \sin \alpha_1] + E_{02} [\sin \omega t \cos \alpha_2 + \cos \omega t \sin \alpha_2]$

$= \sin \omega t\,[E_{01} \cos \alpha_1 + E_{02} \cos \alpha_2] + \cos \omega t\,[E_{01} \sin \alpha_1 + E_{02} \sin \alpha_2]$

$= E_0 \cos \alpha \sin \omega t + E_0 \sin \alpha \cos \omega t = E_0 \sin (\omega t+\alpha)$ [another wave with same frequency but with different amplitude and phase]

The resultant amplitude is, $E_0^2 = [E_{01} \cos \alpha_1 + E_{02} \cos \alpha_2]^2 + [E_{01} \sin \alpha_1 + E_{02} \sin \alpha_2]^2 =$

$E_{01}^2 \cos^2\alpha_1 + E_{02}^2 \cos^2\alpha_2 + 2E_{01}E_{02} \cos \alpha_1 \cos \alpha_2 + E_{01}^2 \sin^2\alpha_1 + E_{02}^2 \sin^2\alpha_2 + 2E_{01}E_{02} \sin \alpha_1 \sin \alpha_2 = E_{01}^2 [\cos^2\alpha_1 + \sin^2\alpha_1] + E_{02}^2 [\cos^2\alpha_2 + \sin^2\alpha_2] + 2E_{01}E_{02}[\sin \alpha_1 \sin \alpha_2 + \cos \alpha_1 \cos \alpha_2]$

$= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} \cos (\alpha_2 - \alpha_1)$

or, and the resultant phase is, $\tan \alpha = (E_{01} \sin \alpha_1 + E_{02} \sin \alpha_2)/ (E_{01} \cos \alpha_1 + E_{02} \cos \alpha_2)$

If the difference in phase between the two interfering waves E_1 and E_2 is δ,

Then, $\delta = (\alpha_2 - \alpha_1) = - (kx_2 - \varepsilon_2) - [-(kx_1 - \varepsilon_1)] = k (x_1 - x_2) + (\varepsilon_1 - \varepsilon_2)$

or, $\boldsymbol{\delta = (2\pi/\lambda)(x_1 - x_2) + (\varepsilon_1 - \varepsilon_2)}$.

$E= E_0 \sin (\omega t+\alpha)$ Resultant wave of two superposing waves having same frequency ω

$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} \cos (\alpha_2 - \alpha_1)$ Resultant Amplitude

$\tan \alpha = (E_{01} \sin \alpha_1 + E_{02} \sin \alpha_2)/ (E_{01} \cos \alpha_1 + E_{02} \cos \alpha_2)$ Resultant Phase

$\delta = (2\pi/\lambda)(x_1 - x_2) + (\varepsilon_1 - \varepsilon_2)$ Phase difference between the interfering waves

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amplitude/intensity at a position, we need to maximize or minimize the **$\cos \delta$** term, which means for a particular position, **δ should not vary with time such that δ is only dependent on x_1 and x_2**. Since, $(2\pi/\lambda)(x_1 - x_2)$ term will not vary with time for a particular position. **Therefore, the initial phase of the interfering waves either has to be same or should have a constant phase difference**. These type of sources are known as **coherent source**.

If the initial phases are same, $\varepsilon_1 = \varepsilon_2$, then the phase difference will depend only on the path difference of the two interfering waves.

Therefore, the amplitude of the resultant wave at a point varies with time. For an optical wave $\omega \approx 10^{15}$ sec^{-1}, thus the amplitude of the resultant wave changes 10^{15} times per second, but the detection limit of our eye is 1/10 sec. As the intensity of light is proportional to the square of the amplitude of the electric field vector, thus to calculate the intensity of the superposed wave at a particular position we have to consider the time average over the detection period.

Interference by division of wavefront

Young Double Slit Experiment

Thomas Young in 1802 devised an way to produce two coherent sources by dividing the primary wave-front of a source (S) into secondary wave-fronts as if they emanated from two sources (S_1 and S_2 respectively) having a constant phase relationship (i.e. coherent) and thus they will produce a stationary interference pattern.

In the actual experiment, a monochromatic light source illuminates the pinhole S. Light diverging from this pinhole fell on a barrier which contained two closely placed pinholes $S_1$and S_2 and were located equidistant from S. Spherical waves emanating from S_1 and S_2 are coherent and fringes will be observed on the screen GG'. This is an interference effect, because if we cover either S_1 or S_2, the fringe pattern will disappear.

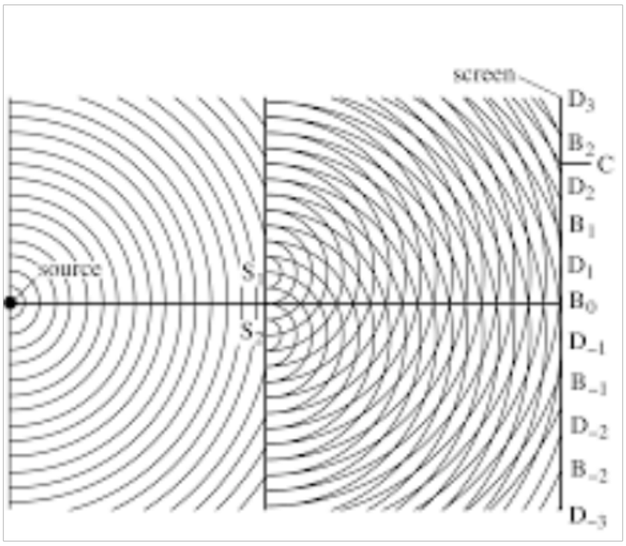

Fig:1 Intereference by divison of wavefront

B_0 is the central bright spot, $B_{\pm n}$ and $D_{\pm n}$ are the n^{th} Bright or Dark Fringes on both side of the central spot respectively. (n =0,1,2…)

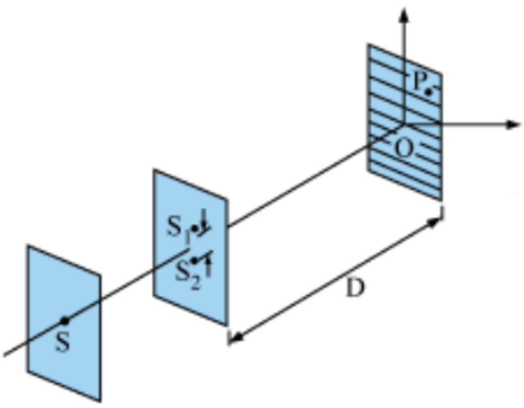

Fig:2 Set up of Young' Double Slit experiment

From the source S, two coherent sources S_1 and S_2 are produced by division of wavefront, which in turn will give interfernce pattern on the screen kept at distance D.

Intensity distribution

Let us assume that $\mathbf{E_1}$ and $\mathbf{E_2}$ be the electric fields at point P due the sources S_1 and S_2 respectively. If S_1P and S_2P are very large in comparison to S_1S_2, then $\mathbf{E_1}$ and $\mathbf{E_2}$ will be in same direction for practical purpose. We will consider the frequency and the initial phases of the two fields to be equal. Therefore, we can write,

$\mathbf{E_1} = \mathbf{i}\, E_{01}\cos[(2\pi/\lambda)S_1P - \omega t]$ and $\mathbf{E_2} = \mathbf{i}\, E_{02}\cos[(2\pi/\lambda)S_2P - \omega t]$, where $\mathbf{i}$ is the direction of the electric field. Due to the superposition of $\mathbf{E_1}$ and $\mathbf{E_2}$, the resultant field at P is

$$\mathbf{E} = \mathbf{E_1}+\mathbf{E_2} = \mathbf{i}\, E_{01}\cos[(2\pi/\lambda)S_1P - \omega t] + \mathbf{i}\, E_{02}\cos[(2\pi/\lambda)S_2P - \omega t]$$

Since intensity is proportional to the square of amplitude and if we consider the proportionality constant to be a, the intensity at P will be $I = aE^2$.

$$I = \langle a\, [E_{01}\cos[(2\pi/\lambda)S_1P - \omega t] + E_{02}\cos[(2\pi/\lambda)S_2P - \omega t]]^2 \rangle$$

$$= \langle a[\, [E_{01}\cos[(2\pi/\lambda)S_1P - \omega t]]^2 + [E_{02}\cos[(2\pi/\lambda)S_2P - \omega t]]^2$$

$$+ 2E_{01}E_{02}\cos[(2\pi/\lambda)S_1P - \omega t]\cos[(2\pi/\lambda)S_2P - \omega t]\,] \rangle$$

[Since, $2\cos A \cos B = \cos(A+B) + \cos(A-B)$]

Therefore, $I = \langle a\, [E_{01}^2\cos^2[(2\pi/\lambda)S_1P - \omega t] + E_{02}^2\cos^2[(2\pi/\lambda)S_2P - \omega t]$

$$+ E_{01}E_{02}\,[\cos[(2\pi/\lambda)(S_2P-S_1P)] + \cos[(2\pi/\lambda)(S_2P+S_1P) - 2\omega t]] \rangle$$

The time average over a time period (T) of a time varying function is defined by,

$$\langle f(t) \rangle = \frac{1}{T}\int_0^T f(t)\,dt.$$

Thus, $\left\langle \cos^2\left[\left(\frac{2\pi}{\lambda}\right)S_1P - \omega t\right] \right\rangle = \frac{1}{T}\int_0^T \cos^2\left[\left(\frac{2\pi}{\lambda}\right)S_1P - \omega t\right]dt = \frac{1}{T}\int_0^T \cos^2[\vartheta - \omega t]\,dt$

$$= \frac{1}{T}\int_0^T \frac{(1 + \cos 2[\vartheta - \omega t])}{2}dt = \frac{1}{2T}\int_0^T dt + \frac{1}{2T}\int_0^T \cos 2(\vartheta - \omega t)\,dt = \frac{1}{2} + \frac{1}{2T}\left(\frac{1}{-2\omega}\right)\sin 2(\vartheta - \omega t)\Big|_0^T$$

$$= \frac{1}{2} + \frac{1}{2T}\left(\frac{1}{-2\omega}\right)[\sin 2(\vartheta - \omega T) - \sin 2\vartheta] = \frac{1}{2} + \frac{1}{2T}\left(\frac{1}{-2\omega}\right)[\sin 2(\vartheta - 2\pi) - \sin 2\vartheta]$$

$= \frac{1}{2} + \frac{1}{2T}\left(\frac{1}{-2\omega}\right)[\sin 2\vartheta - \sin 2\vartheta] = \frac{1}{2}$ [as for a particular point/position S_1P is constant, thus $(2\pi/\lambda)S_1P$ is also constant (denoted by ϑ)].

Here, T is the time period of optical wave, which is of the order of 10^{-15} second. Thus, within the eye detection limit, τ (1/10 th of a second), 10^{14} time periods will be covered for an optical wave, resulting the term

$$\frac{1}{\tau}\int_0^{\tau}\cos 2(\vartheta - \omega t)dt = 0, \text{ therefore, } \left\langle \cos^2\left[\left(\frac{2\pi}{\lambda}\right)S_1P - \omega t\right]\right\rangle = \frac{1}{2}$$

Similar arguments will show that $\left\langle \cos^2\left[\left(\frac{2\pi}{\lambda}\right)S_2P - \omega t\right]\right\rangle = \frac{1}{2}$

$$\left\langle \cos\left[\left(\frac{2\pi}{\lambda}\right)(S_2P + S_1P) - 2\omega t\right]\right\rangle = \frac{1}{2T}\int_0^T \cos\left[\left(\frac{2\pi}{\lambda}\right)(S_2P + S_1P) - 2\omega t\right]dt = \frac{1}{2T}\int_0^T \cos[\beta - 2\omega t]dt$$

$$= \frac{1}{2T}\left(\frac{1}{-2\omega}\right)\sin(\beta - 2\omega t)\Big|_0^T = \frac{1}{2T}\left(\frac{1}{-2\omega}\right)[\sin(\beta - 4\pi) - \sin(\beta)]$$

$$= \frac{1}{2T}\left(\frac{1}{-2\omega}\right)[\sin\beta - \sin\beta] = 0$$

[as for a particular point/position $S_1P + S_2P$ is constant, thus $(2\pi/\lambda)(S_1P+S_2P)$ is also constant (denoted by β)].

Therefore, Intensity at a point $I = \frac{aE_{01}^2}{2} + \frac{aE_{02}^2}{2} + 2E_{01}E_{02}[\cos\left[\left(\frac{2\pi}{\lambda}\right)(S_2P - S_1P)\right]]$

Since, (S_2P-S_1P) is the path difference between the two interfering waves, therefore $[(2\pi/\lambda)(S_2P-S_1P)]$ is the corresponding phase difference δ.

Thus, $\mathbf{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$, where **$I_1$ and I_2 are intensities of the constituent waves .**

Coherent Superposition

a) Condition of Constructive Interference: for cos δ = +1, $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

Thus, δ = 2nπ, where n = 0, 1, 2,…….

If we define path difference $(S_2P - S_1P) = \Delta$, then,

$(2\pi/\lambda)\Delta = 2n\pi$ gives, $\Delta = 2n\lambda/2$, where n = 0,1,2…..

b) Condition of Destructive Interference: for cos δ = -1, $I_{max} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

Thus, δ = (2n+1)π, where n = 0, 1, 2,…….

If we define path difference $(S_2P - S_1P) = \Delta$, then,

$(2\pi/\lambda)\Delta = (2n+1)\pi$ gives, $\Delta = (2n+1)\lambda/2$, where n = 0,1,2.....

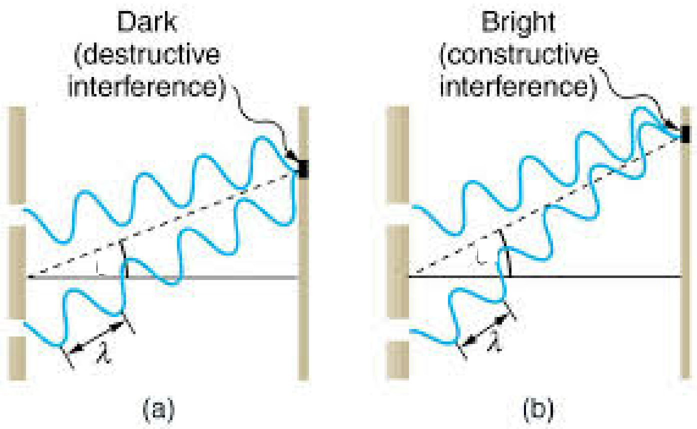

Fig 3: Schematic Representation of Constructive and Destructive Interference

c) If S_2P and S_1P are extremely large compared to S_1S_2, then we can consider,

$I_1 \approx I_2 \approx I_0$ (say), therefore, the resultant intensity

$$\mathbf{I = 2I_0 + 2I_0\cos\delta = 2I_0\,(1+\cos\delta) = 4I_0\cos^2(\delta/2)}$$

Fig 4: Fringe Pattern in Young's Double Slit Experiment

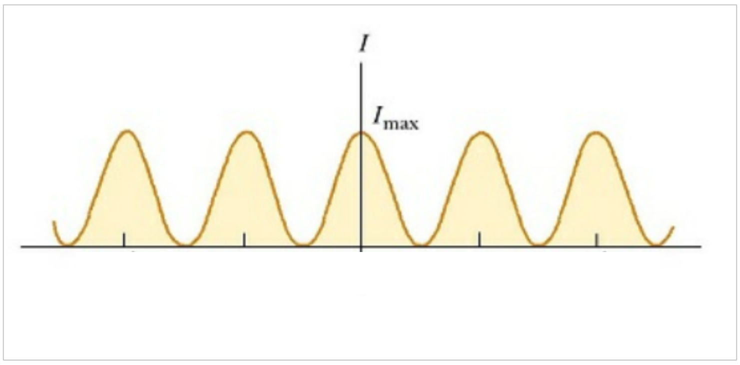

Fig 5: Intensity Distribution when the intensity of interfering waves are equal, $4I_0\cos^2(\delta/2)$

<u>Incoherent superposition</u>: In this case the phase of the two interfering waves will change rapidly with time (10^8 times per second, as in reality nearly 10^8 wave trains are emitted per second from a source and different wave trains have random phases.), the phase difference between the two waves is now a function of time and as we will measure the intensity at a point, we have to consider the time average over the detection period $\langle \cos\delta(t)\rangle = 0$

Therefore, the average intensity in interference is **$I = I_1 + I_2$ which is same as that of before interference. Thus, energy is globally conserved in interference and** redistributed among bright and dark fringes.

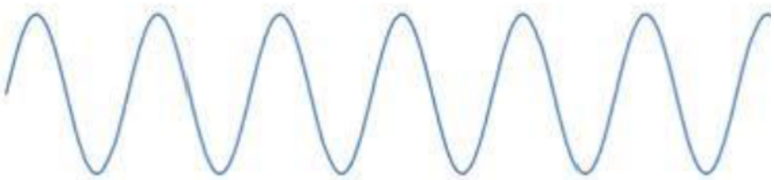

Fig 6: Representation of Coherent waves

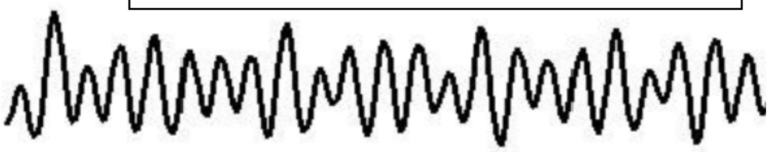

Fig 7: Representation of Incoherent

In terms of	Condition of	
	Constructive Interference	**Destructive Interference**
Phase Difference (δ)	**$\delta = 2n\pi$, n = 0, 1,2,…….**	**$\Delta = 2n\lambda/2$, n = 0,1,2…..**
Path Difference (Δ)	**$\Delta = 2n\lambda/2$, n = 0,1,2…..**	**$\Delta = (2n+1)\lambda/2$, n = 0,1,2…..**

Type of Superposition	Expression of Intensity		Inference
	Constructive Interference	**Destructive Interference**	
Coherent ($I_1 \neq I_2$)	$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$	$I_{max} = (\sqrt{I_1} - \sqrt{I_2})^2$	**Amplitudes added up**
Coherent ($I_1 \approx I_2 \approx I_0$)	**$4I_0$**	**Zero**	**Amplitudes added up**
Incoherent	**No interference pattern, uniform illumination** **$I = I_1 + I_2$**		**Intensities added up**

Conservation of energy in Interference

When two monochromatic waves are interfering, they will produce alternative bright (constructive interference) and dark fringes (destructive interference). In order to check the energy conservation, we have to consider the full fringe system. At a particular position on the screen, one will observe either a bright or a dark fringe depending on the path difference/phase difference at that point. But this will vary from one position to the other position. Therefore, if we take the average over all phases, we will obtain the mean intensity after interference.

Since, $\frac{1}{2\pi}\int_0^{2\pi}\cos\delta \, d\delta = 0$, therefore, the average intensity after interference is **$I = I_1 + I_2$**, which equal to the intensity before interference. Thus, energy is globally conserved in interference. Or, in other words, in interference phenomena energy is redistributed within the bright and the dark fringes.

Independence of the phase with respect to time

Important Point: If $\mathbf{E_1}$ and $\mathbf{E_2}$ are two interfering waves, then the resultant intensity at any point will be

$$I = a\langle(\mathbf{E}_1 + \mathbf{E}_2).(\mathbf{E}_1 + \mathbf{E}_2)\rangle = a\langle|\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\mathbf{E}_1.\mathbf{E}_2\rangle$$

where, a is the proportionality constant. The $2\mathbf{E}_1.\mathbf{E}_2$ **term is responsible for interference and thus two perpendicularly plane polarised light will not produce any interference effect even if they are monochromatic and coherent, because in that case,**

$$2\mathbf{E}_1.\mathbf{E}_2 = 2|\mathbf{E}_1||\mathbf{E}_2|\cos 90^0 = 0$$

Conditions of sustained Interference

1) The two sources should be **coherent** (either having same initial phase or a constant phase difference).

2) The two sources must emit **monochromatic** light (same frequency (ω) and for a given medium same wavelength (λ) because wavelength changes in refraction)

3) The amplitudes of the interfering beams should be equal to get completely dark fringes.

4) The separation between the two sources **d should be small** and the distance between the source and the screen **D should be large** in order to observe distinct fringes.

5) For **constructive interference** (i.e. for maximum intensity) the path difference should be **even multiple of λ/2** and for **destructive interference** (i.e. for minimum intensity) the path difference should be **odd multiple of λ/2**.

6) The two sources **should not be perpendicularly polarized.**

<u>Calculation of Fringe width in Young's Double Slit Experiment</u>

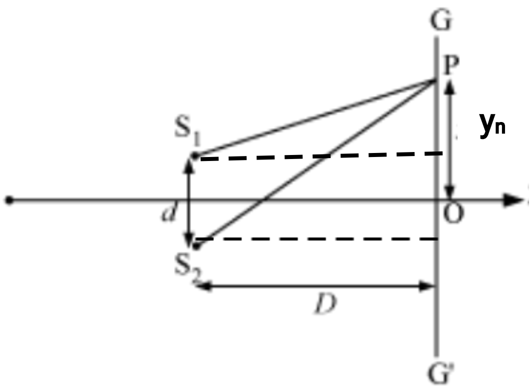

Fig 8: Arrangement of Young's Double Slit Experiment

The path difference (Δ) between the waves emitted from S_1 and S_2 is $(S_1P \sim S_2P)$, where P is the position of the n^{th} fringe (y_n) on the screen from the central spot O. In order to calculate the fringe width let us first draw two perpendiculars from S_1 and S_2 on the screen, where the foot of the perpendiculars on the screen are A and B respectively. If the distance between the sources (S_1 and S_2) is denoted by d and the distance between the screen and the source by D, then in practice **D>>>d**, so that $S_1P + S_2P \approx 2D$

Then, AP = (y_n −d/2) and BP = (y_n + d/2)

Thus, $(S_1P)^2 = (S_1A)^2 + (AP)^2 = D^2 + (y_n - d/2)^2$ and $(S_2P)^2 = (S_2B)^2 + (BP)^2 = D^2 + (y_n + d/2)^2$

Therefore, $(S_2P)^2 - (S_1P)^2 = D^2 + (y_n + d/2)^2 - D^2 + (y_n - d/2)^2$

or, $(S_2P - S_1P)(S_1P + S_2P) = 2y_n d = 2D\Delta$

or, $\boldsymbol{\Delta = y_n d/D}$

Now, the position of the **n^{th} bright fringe** is,

$\Delta = y_n d/D = 2n\lambda/2$, where λ is the wavelength of the monochromatic light used.

or, $y_n = nD\lambda/d$, where, n = 0,1,2,……….

Thus, the position of the $(n+1)^{th}$ bright fringe is $y_{n+1} = (n+1)D\lambda/d$

Therefore, the fringe width (distance between two consecutive bright fringes)

$\beta = y_{n+1} - y_n = (n+1)D\lambda/d - nD\lambda/d = D\lambda/d$

Now, the position of the **n^{th} dark fringe** is,

$\Delta = y_n d/D = (2n+1)\lambda/2$, where λ is the wavelength of the monochromatic light used.

Or, $y_n = (2n+1)D\lambda/2d$, where, n=0,1,2,……….

Thus, the position of the $(n+1)^{th}$ dark fringe is $y_{n+1} = (2n+3)D\lambda/2d$

Therefore, the fringe width (distance between two consecutive dark fringes)

$\boldsymbol{\beta} = y_{n+1} - y_n = (2n+3)D\lambda/2d - (2n+1)D\lambda/2d = \boldsymbol{D\lambda/d}$

Note that the fringe width in this case is independent of the order of the fringe and same for both bright and dark fringe. That is why these fringes are known as fringes of equal width. And the interference pattern (around point O) consists of a series of dark and bright lines perpendicular to plane (fig 4), O being the foot of the perpendicular from the point S on the screen. The central spot is bright because at O, there will be z

Fringe width β = Dλ/d, independent of the order of bright or dark fringe, fringes of equal width.

Displacement of fringes

One will observe the fringe pattern to be displaced because of the insertion of a thin transparent sheet of thickness t in front of one of the interfering beam (say S_1).

In this case, the optical path from S_1 to P has been modified. In order to calculate the modified path difference, first consider the time taken for light to reach P from S_1, which is:

$(S_1P – t)/c + t/v$, where c and v are the velocities of light in vacuum and the transparent sheet respectively. Now,

$(S_1P – t)/c + t/v = (S_1P – t)/c + \mu t/c$, where μ is the refractive index of the transparent sheet.

Therefore, the modified path difference between the two waves (emitted from S_1 and S_2) at point P on the screen will be $= S_2P – \{S_1P + (\mu-1)t\} = (S_2P – S_1P) - (\mu-1)t = y_n d/D - (\mu-1)t$.

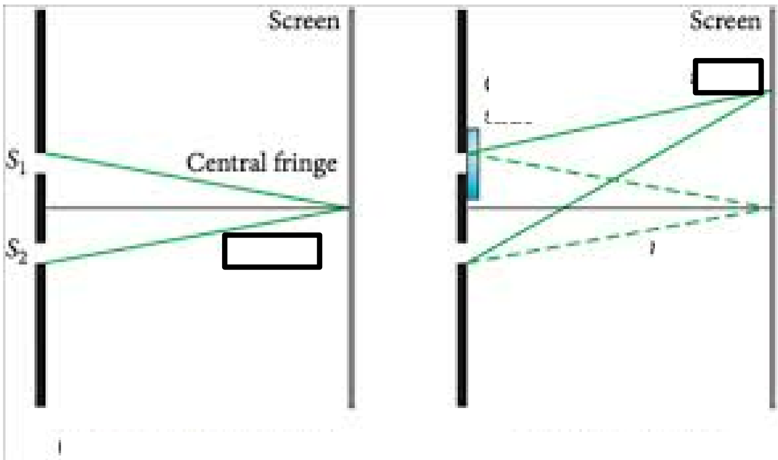

Fig 9: Fringe Shift by inserting a thin transparent sheet

Therefore, the position of the n^{th} bright fringe in this case is,

$y_n d/D - (\mu-1)t = 2n\lambda/2$, where λ is the wavelength of the monochromatic light used.

or, $y_n = [n\lambda + (\mu-1)t]D/d$, where, n = 0,1,2,……….

Thus, the displacement of the n^{th} bright fringe will be,

$\delta = [n\lambda + (\mu-1)t]D/d – n\lambda D/d = (\mu-1)tD/d = (\mu-1)t\, \beta/\lambda$, which is not dependent on the order of the fringe. [The same result can be derived by starting with the n^{th} dark fringe].

Therefore, the whole fringe pattern will be shifted towards the path of thin transparent sheet.

Fringe shift, $\delta = (\mu-1)\, t D/d = (\mu-1)t\, \beta/\lambda$,

which is not dependent on the order of the fringe. Experimentally, by measuring the shift of the central spot (O, which is easy to determine), one can find the thickness of an extremely thin transparent sheet.

Numerical :

1) In an experiment using Young's double slits, the distance between the centre of the interference pattern and tenth bright fringe on either side is 3.44cm and the distance between the slits and the screen (D) is 200cm. If the wavelength of light (λ) used is $5.89*10^{-5}$ cm, determine the separation between the slits (d).

Ans. The distance between the nth bright fringes from the centre of interference pattern is

y_n=D n λ/d, In our case n=10.

Therefore, separation between the slits, d = D$n\lambda/y_n$

= $200*10*5.89*10^{-5}/3.44$ cm = **0.0342 cm**

2) A double slit of 0.5mm separation is illuminated by light of λ=4800Å. How far behind the slits must go to obtain fringes that are 0.1 cm apart?

Ans. We have to find the distance between the slits and the screen, where fringe width β=0.1 cm, separation between the slits d=0.05cm and λ=4800Å.

Therefore, $\beta=D\lambda/d$ gives,

$D=\beta d/\lambda$ =$0.1*0.05/4800*10^{-8}$ cm =**104.17 cm**

3) In a Young's double slit experiment, the slits are (d) 0.2 mm apart and the screen is 1.5 m (D) away. It is observed that the distance between the central bright fringe and the fourth dark fringe is (y_4) 1.8 cm, Find the wavelength of light.

4) The path difference between the two interfering rays at a point on the screen is 1/8 th of a wave length . Find the ratio of the intensity at that point to that of the central bright fringe.

Ans. The distances of the 4th dark fringe from the central fringe is

$y_n = (2n+1)D\lambda/2d$, here n=3

Therefore, $\lambda = y_n * 2 * d/(2n+1) * D$ = 1.8*2*0.02/9*150 cm = **5333Å**

5) **To do:** In an experiment of Young's double slit, the slits are 2mm apart and are illuminated with a mixture of two wavelengths λ_1=5000Å and λ_2=6000Å. At what minimum distance from the common central bright fringe on a screen, 2m away from the slits, will a bright fringe of one interference pattern coincide with a bright fringe from the other?
