5.4 SOLUTION OF O. D. E. USING LAPLACE TRANSFORM

5.4.1. Solution of differential equation with constant coefficient using Laplace Transform.

A linear differential equation with constant coefficient can be solved with the help of Laplace transform.

Given a differential equation of $y(t)$, we apply Laplace transform on both side. Applying necessary property of Laplace transform, we find $L\{y(t)\}$ as function of a variable, say $\phi(s)$. Then $y(t)=L^{-1}\{\phi(s)\}$. This $L^{-1}\{\phi(s)\}$ is obtained by applying several theorems and properties of inverse Laplace transform.

The method of particular solution as well as general solution is shown in the following examples:

5.4.2. Illustrative Examples

Ex 1. *Solve, by Laplace transform, the differential equation*

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t, y(0)=0, y'(0)=1$$ [*W.B.U.T. 2008, 2012*]

Let $L\{y(t)\}=f(s)$

Taking Laplace transform on both sides of the given differential equation, we get $L\left(\frac{d^2y}{dt^2}\right)+2L\left(\frac{dy}{dt}\right)+5L(y)=L(e^{-t}\sin t)$

or, $\{s^2f(s)-sy(0)-y'(0)\}+2\{sf(s)-y(0)\}+5f(s)$

$=\frac{1}{(s+1)^2+1}$ [By Th. 1 and 2 of Art 5.2.7]

or, $\{s^2f(s)-s.0-1\}+2\{sf(s)-0\}+5f(s)=\frac{1}{s^2+2s+2}$

or, $(s^2+2s+5)f(s)=\frac{1}{s^2+2s+2}+1$

or, $f(s)=\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$

or, $L\{y(t)\}=\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$

$\therefore y(t)=L^{-1}\left\{\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right\}$

$=L^{-1}\left\{\frac{(s+1)^2+2}{[(s+1)^2+1][(s+1)^2+4]}\right\}$

$=e^{-t}L^{-1}\left\{\frac{s^2+2}{(s^2+1)(s^2+4)}\right\}$ [By 1st shifting property]

$=\frac{e^{-t}}{3}L^{-1}\left(\frac{2}{s^2+2^2}\right)+L^{-1}\left(\frac{1}{s^2+1}\right)=\frac{e^{-t}}{3}(\sin 2t+\sin t)$

Ex 2. *Solve the following differential equation using Laplace transform*

$(D^2+6D+9)y=1;$

$y(0)=y'(0)=1,\quad \left(D=\frac{d}{dx}\right)$ [*W.B.U.T. 2011*]

Taking laplace transform on both sides of the given differential equation.

$(D^2+6D+9)y=1;$

i.e., $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=1$

we, get

$L\left(\frac{d^2y}{dx^2}\right)+6L\left(\frac{dy}{dx}\right)+9L(y)=L(1)$

or, $[s^2f(s)-sy(0)-y'(0)]+6[sf(s)-y(0)]+9f(s)=\frac{1}{s}$

where $L\{y(t)\}=f(s)$

or, $f(s)\left[s^2+6s+9\right]-s-1-6=\frac{1}{s}$

or, $\left(s^2+6s+9\right)f(s)=s+7+\frac{1}{s}$

or, $f(s)=\frac{s+7}{(s+3)^2}+\frac{1}{s(s+3)^2}=\frac{(s+3)+4}{(s+3)^2}+\frac{1}{3}\frac{(s+3)-s}{s(s+3)^2}$

$$=\frac{1}{s+3}+\frac{4}{(s+3)^2}+\frac{1}{3}\left\{\frac{1}{s(s+3)}-\frac{1}{(s+3)^2}\right\}$$

$\therefore\quad L\{y(t)\}=\frac{1}{s+3}+\frac{4}{(s+3)^2}+\frac{1}{3}\left\{\frac{1}{s}-\frac{1}{s+3}-\frac{1}{(s+3)^2}\right\}$

$\therefore\quad y(t)=L^{-1}\left(\frac{1}{s+3}\right)+4L^{-1}\left\{\frac{1}{(s+3)^2}\right\}+\frac{1}{3}L^{-1}\left(\frac{1}{s}\right)$

$$-\frac{1}{3}L^{-1}\left(\frac{1}{s+3}\right)-\frac{1}{3}L^{-1}\left\{\frac{1}{(s+3)^2}\right\}$$

$$=e^{-3t}+4e^{-3t}L^{-1}\left(\frac{1}{s^2}\right)+\frac{1}{3}\cdot 1-\frac{1}{3}e^{-3t}-\frac{1}{3}e^{-3t}L^{-1}\left(\frac{1}{s^2}\right)$$

$$=\frac{1}{3}\left(1+2e^{-3t}+11te^{-3t}\right).$$

Ex 3. *Solve the following differential equation using Laplace transform :* $y''-3y'+2y=4t+e^{3t}$

where $y(0)=1$ *and* $y'(0)=-1$. [W.B.U.T 2002, 2016]

Let $L\{y(t)\}=f(s)$

Taking Laplace transform we get

$$L(y'')-3L(y')+2L(y)=4L(t)+L\left(e^{3t}\right)$$

or, $s^2f(s)-sy(0)-y'(0)-3sf(s)+3y(0)+2f(s)=\frac{4}{s^2}+\frac{1}{s-3}$

or, $s^2f(s)-s.1-(-1)-3sf(s)+3.1+2f(s)=\frac{4}{s^2}+\frac{1}{s-3}$

or, $\left(s^2-3s+2\right)f(s)=\frac{4}{s^2}+\frac{1}{s-3}+s-4$

or, $(s-1)(s-2)f(s)=\frac{(s-2)(s+6)}{s^2(s-3)}+s-4$

$\therefore\ f(s)=\frac{s+6}{s^2(s-1)(s-3)}+\frac{s-4}{s^2(s-1)(s-2)}$

Let $\frac{s+6}{s^2(s-1)(s-3)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s-1}+\frac{D}{s-3}$

$\therefore\ s+6=As(s-1)(s-3)+B(s-1)(s-3)+Cs^2(s-3)+Ds^2(s-1)$

Putting $s=0,1,2,3$ successively we get

$A=3,\ B=2,\ C=-\frac{7}{2},\ D=\frac{1}{2}$

Again let $\frac{s-4}{(s-1)(s-2)}=\frac{E}{s-1}+\frac{F}{s-2}$

$\therefore\ s-4=E(s-2)+F(s-1)$

Putting $s=1,2$ successively we get $E=3,\ F=-2$

$\therefore\ f(s)=\frac{3}{s}+\frac{2}{s^2}-\frac{7}{2}\cdot\frac{1}{s-1}+\frac{1}{2}\cdot\frac{1}{s-3}+\frac{3}{s-1}-\frac{2}{s-2}$

or, $L\{f(t)\}=\frac{3}{s}+\frac{2}{s^2}-\frac{1}{2}\cdot\frac{1}{s-1}-\frac{2}{s-2}+\frac{1}{2}\cdot\frac{1}{s-3}$

$\therefore\ y(t)=3L^{-1}\left(\frac{1}{s}\right)+2L^{-1}\left(\frac{1}{s^2}\right)-\frac{1}{2}L^{-1}\left(\frac{1}{s-1}\right)$

$$-2L^{-1}\left(\frac{1}{s-2}\right)+\frac{1}{2}L^{-1}\left(\frac{1}{s-3}\right)$$

$$=3.1+2t-\tfrac{1}{2}e^{t}-2e^{2t}+\tfrac{1}{2}e^{3t}$$

$\therefore\ y(t)=3+2t+\frac{1}{2}\left(e^{3t}-e^{t}\right)-2e^{2t}$

Ex 4. *Using Laplace transform, solve*

$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=\sin x,\ y(0)=y'(0)=0.$ [W.B.U.T 2004]

Let $L\{y(x)\} = f(s)$

Taking Laplace transform on both sides we get

$$s^2 f(s) - sy(0) - y'(0) + 2sf(s) - 2y(0) - 3f(s) = \frac{1}{s^2+1}$$

or, $s^2 f(s) - s.0 - 0 + 2sf(s) - 2.0 - 3f(s) = \frac{1}{s^2+1}$

or, $\left(s^2 f + 2s - 3\right) f(s) = \frac{1}{s^2+1}$

or, $f(s) = \frac{1}{\left(s^2+1\right)\left(s^2+2s-3\right)} = \frac{1}{(s-1)(s+3)\left(s^2+1\right)}$

Let $\frac{1}{(s-1)(s+3)\left(s^2+1\right)} = \frac{A}{s-1} + \frac{B}{s+3} + \frac{cs+D}{s^2+1}$

$\therefore \quad 1 = A(s+3)\left(s^2+1\right) + B(s-1)\left(s^2+1\right) + (cs+D)(s-1)(s+3)$

Putting $s = -1, -3, 0, 1$ successively we get

$$A = \frac{1}{8},\ B = -\frac{1}{40},\ C = -\frac{1}{10},\ D = -\frac{1}{5}$$

$$\therefore \quad f(s) = \frac{1}{8}\cdot\frac{1}{s-1} - \frac{1}{40}\cdot\frac{1}{s+3} - \frac{1}{10}\cdot\frac{s}{s^2+1} - \frac{1}{5}\cdot\frac{1}{s^2+1}$$

$\therefore \quad y(x) = L^{-1}\{f(s)\}$

$$= \frac{1}{8}L^{-1}\left(\frac{1}{s-1}\right) - \frac{1}{40}L^{-1}\left(\frac{1}{s+3}\right) - \frac{1}{10}L^{-1}\left(\frac{s}{s^2+1}\right) - \frac{1}{5}L^{-1}\left(\frac{1}{s^2+1}\right)$$

$$= \frac{1}{8}e^x - \frac{1}{40}e^{-3x} - \frac{1}{10}\cos x - \frac{1}{5}\sin x$$

Ex 5. *Solve* $\left(D^2-1\right)y = a\cosh\ nt$ where $y(0) = 0,\ y'(0) = 2.$ [W.B.U.T 2008]

The given equation is $\left(D^2-1\right)y = a\cosh nt$

i.e., $y'' - y = a\cosh nt$

Let $L\{y(t)\} = f(s)$

Applying Laplace transform, we get

$L(y'') - L(y) = aL(\cosh nt)$

or, $\left\{s^2 f(s) - sy(0) - y'(0)\right\} - f(s) = a\frac{s}{s^2-n^2}$

or, $s^2 f(s) - s.0 - 2 - f(s) = a\frac{as}{s^2-n^2}$

or, $\left(s^2-1\right)f(s) = a\frac{as}{s^2-n^2} + 2$

or, $f(s) = \frac{as}{\left(s^2-1\right)\left(s^2-n^2\right)} + \frac{2}{s^2-1}$

Let $\frac{s}{\left(s^2-1\right)\left(s^2-n^2\right)} = \frac{As+B}{s^2-n^2} + \frac{cs+D}{s^2+1}$

$\therefore \quad s = (As+B)\left(s^2-1\right) + (cs+D)\left(s^2-n^2\right)$

Putting $s = 1, -1, n, -n,$ successively we get

$$A = \frac{1}{n^2-1},\ B = 0,\ B = -\frac{1}{n^2-1},\ D = 0$$

$$\therefore \quad \frac{s}{\left(s^2-1\right)\left(s^2-n^2\right)} = \frac{n}{n^2-1}\cdot\frac{1}{s^2-n^2} - \frac{1}{n^2-1}\cdot\frac{1}{s^2-1}$$

$$\therefore \quad L\{y(t)\} = \frac{as}{n^2-1}\frac{1}{s^2-n^2} - \frac{a}{n^2-1}\cdot\frac{s}{s^2-1} + \frac{2}{s^2-1}$$

$$\therefore \ y(t) = \frac{a}{n^2-1}L^{-1}\left(\frac{s}{s^2-n^2}\right) - \frac{a}{n^2-1}L^{-1}\left(\frac{s}{s^2-1}\right) + 2L^{-1}\left(\frac{1}{s^2-1}\right)$$

$$= \frac{a}{n^2-1}\cdot\frac{\cosh nt}{n} - \frac{a}{n^2-1}\cosh t + 2\sinh t$$

$$\therefore \quad y(t) = \frac{a}{n^2-1}(\cosh nt - \cosh t) + 2\sinh t.$$

Ex 6. *Solve the differential equation by Laplace transform*

$$\frac{d^2y}{dx^2} = 2\frac{dy}{dt} - 3y = t\cos t,\ y(0) = y'(0) = 0. \qquad \text{[WBUT 2010]}$$

Let $L\{y(t)\} = f(s)$

Taking Laplace transform of both sides of the given differential equation, we get

$$L(y'') - 2L(y') - 3L(y) = L(t\cos t)$$

or, $s^2 f(s) - sy(0) - y'(0) - 2\{sf(s) - y(0)\} - 3f(s)$

$$= -\frac{d}{ds}\{L(\cos t)\}$$

or, $\left(s^2 - 2s - 3\right)f(s) = -\dfrac{d}{ds}\left(\dfrac{s}{s^2+1}\right)$

or, $(s+1)(s-3)f(s) = -\dfrac{1}{s^2+1} + \dfrac{2s^2}{\left(s^2+1\right)^2} = \dfrac{s^2-1}{\left(s^2+1\right)^2}$

$\therefore \quad f(s) = \dfrac{s^2-1}{(s+1)(s-3)\left(s^2+1\right)^2} = \dfrac{s-1}{(s-3)\left(s^2+1\right)^2}$

Let $\dfrac{s-1}{(s-3)\left(s^2+1\right)^2} = \dfrac{A}{s-3} + \dfrac{Bs+C}{s^2+1} + \dfrac{Ds+E}{\left(s^2+1\right)^2}$

$\therefore \quad s - 1 = A\left(s^2+1\right)^2 + (Bs+C)(s-3)\left(s^2+1\right) + (Ds+E)(s-3)$

Putting $s = 0, 1, -1, 2, 3$ successively we get

$$A = \frac{1}{50},\; B = -\frac{1}{50},\; C = -\frac{3}{50},\; D = -\frac{1}{5},\; E = \frac{2}{5}$$

$\therefore \quad f(s) = \dfrac{1}{50}\cdot\dfrac{1}{s-3} - \dfrac{1}{50}\dfrac{s}{s^2+1} - \dfrac{3}{50}\dfrac{1}{s^2+1} - \dfrac{1}{5}\dfrac{s}{\left(s^2+1\right)^2}$

$$+\frac{2}{5}\frac{1}{\left(s^2+1\right)^2}$$

$\therefore \quad y(t) = \dfrac{1}{50}L^{-1}\left(\dfrac{1}{s-3}\right) - \dfrac{1}{50}L^{-1}\left(\dfrac{s}{s^2+1}\right) - \dfrac{3}{50}L^{-1}\left(\dfrac{1}{s^2+1}\right)$

$$-\frac{1}{5}L^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\} + \frac{2}{5}L^{-1}\left\{\frac{1}{\left(s^2+1\right)^2}\right\}$$

$$= \frac{e^{3t}}{50} - \frac{1}{50}\cos t - \frac{3}{50}\sin t - \frac{1}{5}L^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\}$$

$$+\frac{2}{5}L^{-1}\left\{\frac{1}{\left(s^2+1\right)^2}\right\}$$

As $L^{-1}\left(\dfrac{1}{s^2+1}\right) = \sin t = f(s)$, say, so we have

$$L^{-1}\left\{\frac{d}{ds}\left(\frac{1}{s^2+1}\right)\right\} = -t f(t)$$

i.e., $L^{-1}\left\{\dfrac{-2s}{\left(s^2+1\right)^2}\right\} = -t\sin t$

$\therefore \quad L^{-1}\left\{\dfrac{s}{\left(s^2+1\right)^2}\right\} = \frac{1}{2}t\sin t$

$\therefore$ By convolution theorem

$$L^{-1}\left\{\frac{1}{\left(s^2+1\right)^2}\right\} = L^{-1}\left\{\frac{1}{s^2+1}\cdot\frac{1}{s^2+1}\right\}$$

$$= \int_0^t f(u)f(t-u)\,du$$

$$= \int_0^t \sin u \sin(t-u)\,du$$

$$= \tfrac{1}{2}\int_0^t [\cos(2u-t) - \cos t]\,du$$

$$= \tfrac{1}{2}\left[\tfrac{1}{2}\sin(2u-t) - u\cos t\right]_{u=0}^{t}$$

$$= \tfrac{1}{2}\left(\tfrac{1}{2}\sin t - t\cos t + \tfrac{1}{2}\sin t\right)$$

$$= \tfrac{1}{2}(\sin t - t\cos t)$$

$$\therefore \quad y(t) = \frac{e^{3t}}{50} - \frac{1}{50}\cos t - \frac{3}{50}\sin t - \frac{1}{10}t\sin t + \frac{1}{5}(\sin t - t\cos t)$$

$$= \frac{e^{3t}}{50} - \frac{1}{50}(1+10t)\cos t + \frac{1}{50}(7-5t)\sin t$$

Ex 7. *Solve* $\frac{d^3y}{dt^3} + y = 1, t > 0$.

Given $y = Dy = D^2y = 0$ *when* $t = 0$.

Let $L\{y(t)\} = f(s)$

Taking Laplace transform on both sides of the given differential equation, we get $L\left(\frac{d^3y}{dt^3}\right) + L(y) = L(1)$

or, $s^3 f(s) - s^2 y(0) - sy'(0) - y''(0) + f(s) = \frac{1}{s}$

or, $s^3 f(s) - s^2.0 - s.0 - 0 + f(s) = \frac{1}{s}$

or, $(s^3+1)f(s) = \frac{1}{s}$

or, $f(s) = \frac{1}{s(s^3+1)}$

or, $L\{y(t)\} = \frac{1}{s(s^3+1)}$

or, $y(t) = L^{-1}\left\{\frac{1}{s(s^3+1)}\right\} = L^{-1}\left\{\frac{1}{s(s+1)(s^2-s+1)}\right\}$

Let $\frac{1}{s(s+1)(s^2-s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2-s+1}$

or, $1 = A(s+1)(s^2-s+1) + Bs(s^2-s+1) + (Cs+D)(s+1)s$

From this we get, after equating the coefficients from both side,

$$A = 1,\ B = \frac{1}{3},\ C = -\frac{2}{3},\ D = \frac{1}{3}$$

$$\therefore \frac{1}{s(s+1)(s^2-s+1)} = \frac{1}{s} + \frac{1}{3}\cdot\frac{1}{s+1} - \frac{2}{3}\cdot\frac{s-\frac{1}{2}}{s^2-s+1}$$

$$\therefore L^{-1}\left\{\frac{1}{s(s+1)(s^2-s+1)}\right\}$$

$$= L^{-1}\left\{\frac{1}{s}\right\} + \frac{1}{3}.L^{-1}\left\{\frac{1}{s+1}\right\} - \frac{2}{3}L^{-1}\left\{\frac{s-\frac{1}{2}}{\left(s-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= 1 + \frac{1}{3}e^{-t} - \frac{2}{3}e^{\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t$$

$\therefore$ From (1), we get $y = 1 + \frac{1}{3}e^{-t} - \frac{2}{3}e^{\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t$

EXERCISE

[I] SHORT ANSWER QUESTIONS

Solve the differential equation using Laplace Transform :

1. $2\frac{dy}{dt} = e^{-t}$, when $y = 0$, at $t = 0$.

2. $\frac{dy}{dt} = 2t$, when $y(0) = 0$.

3. $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 0$, when $y(0) = 0,\ y'(0) = 0$

4. $\frac{d^2x}{dt^2} + x = t,\ x(0) = 1, x'(0) = -2$

ANSWERS

1. $y(t) = \frac{1}{2}(1 - e^{-t})$ **2.** $y = t^2$

3. $y = 0$ **4.** $x = t + \cos t - 3\sin t$.

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[II] LONG ANSWER QUESTIONS

1. Solve : $\frac{d^2y}{dt^2}+8\frac{dy}{dt}+8y=1, y=0, \frac{dy}{dt}=1$ at $t=0$.

2. Solve : $y''+y'-2y=2(1+t-t^2), y(0)=0, y'(0)=3$.

3. Solve : $y''+a^2y=\phi(t), y(0)=1, y'(0)=-2$.

4. Solve $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=4t+12e^{-t}, y(0)=6, y'(0)=-1$.

5. Solve $y''(t)+y(t)=8\cos t, y(0)=1, y'(0)=-1$.

[W.B.U.T. 2015, 2005]

6. Solve $y''-y'-2y=4x^2$; $y(0)1, y'(0)=4$

7. Solve $y''(t)+y(t)=\sin 2t,\ y(0)=0,\ y'(0)=1$ [W.B.U.T.2003]

8. Solve $Y''(t)+9Y(t)=18t$ if $Y(0)=0, Y\left(\frac{\pi}{2}\right)=0$.

9. Solve $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=f(t)$ where $y=\frac{dy}{dt}=0$ at $t=0$ and

$$f(t)=1, \quad 0<t<1$$
$$=0, \quad t>1.$$

10. Solve $(D^2+6D+9)y=\sin x$, where $y(0)=1, y'(0)=0$.

11. Solve $(D^2+m^2)y=a\cos nt$ where $y(0)=0, Dy(0)=0$.

12. Solve $\frac{d^2y}{dt^2}+a^2\frac{dy}{dt}=F(t), y(0)=\alpha, y'(0)=\beta$.

13. Solve $\frac{d^3y}{dt^3}-\frac{dy}{dt}=e^t, y(0)=0, y'(0)=0, y''(0)=0$.

14. Find the solution of the differential equation

$$\frac{d^3y}{dt^3}-3\frac{d^2y}{dt^2}+3\frac{dy}{dt}-y=t^2e^t, y=1, \frac{dy}{dt}=0, \frac{d^2y}{dt^2}=-2 \text{ at } t=0.$$

15. Solve $\frac{d^4y}{dt^4}+2\frac{d^2y}{dt^2}+y(t)=\sin t$, where $y=\frac{dy}{dt}=\frac{d^2y}{dt^2}=\frac{d^3y}{dt^3}=0$ at $t=0$.

ANSWERS

1. $\frac{1}{8}\left(1-e^{-4t}\cos 2\sqrt{2}t+\sqrt{2}\,e^{-4t}\sin 2\sqrt{2}-1\right)$

2. $y=t^2+e^{2t}-e^{-t}$

3. $y=\cos at-\frac{2\sin at}{a}+\frac{1}{a}\int_0^t F(u)\sin a(t-u)\,du$

4. $y=3e^t-2e^{2t}+2t+3+2e^{-t}$

5. $y=4\sin t-4\cos t+5e^{-t}$

6. $y=2e^{2x}+2e^{-x}-2x^2+2x-3$

7. $y=\frac{1}{3}(5\sin t-\sin 2t)$

8. $Y(t)=2t+\pi\sin 3t$

9. $y=\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}-\left\{\frac{1}{2}-e^{-(t-1)}+\frac{1}{2}e^{-2(t-1)}\right\}u(t-1)$

10. $y=\frac{1}{50}(53+155x)e^{-3x}-(3\cos x-4\sin x)$

11. $y=\frac{a}{m^2-n^2}(\cos nt-\cos mt)$

12. $y=\alpha\cos at+\frac{\beta}{\alpha}\sin at+\frac{1}{\alpha}\int_0^t \sin a(t-u)F(u)\,du$

13. $y=\frac{te^t}{2}-\frac{3e^t}{4}+1-\frac{e^{-t}}{4}$.

14. $y=c_1t^2+c_2te^t+c_3e^t+\frac{t^5e^t}{60}$

15. $y(t)=\frac{1}{8}\left\{(3-t^2)\sin t-3t\cos t\right\}$