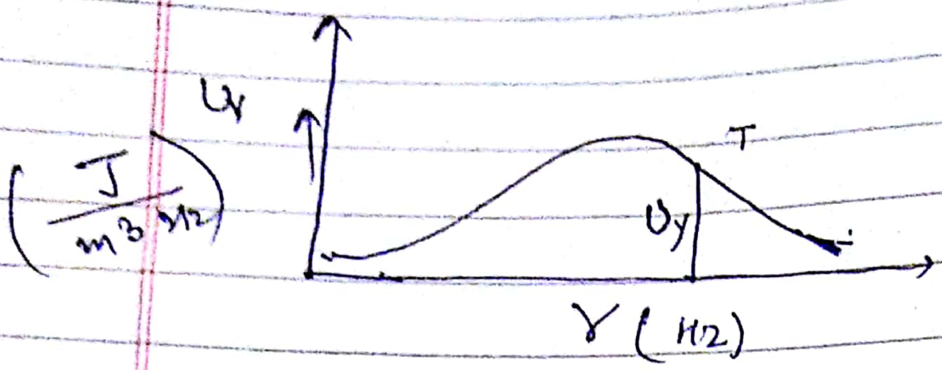


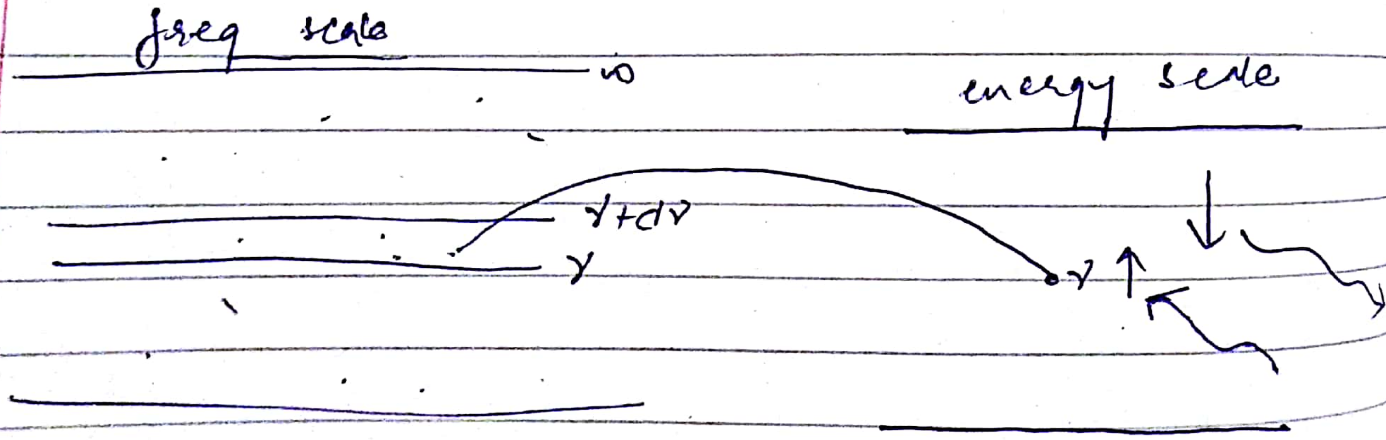
$$U_\gamma d\gamma = N(\gamma)d\gamma \times \overline{E\gamma}$$

where

$$U(\gamma)d\gamma = \frac{8\pi\gamma^2}{c^2}\, d\gamma$$

$$U = \int_0^\infty U_\gamma\, d\gamma$$

total energy density.



Imp Q. The avg energy of one oscillator having frequency $\overline{E_\gamma}$ is given by:

$$\overline{E_\gamma} = \frac{P(E_0) * E_0 + P(E_1) * E_1 + \dots}{P(E_0) + P(E_1) + \dots}$$ [from Boltzman distribution.

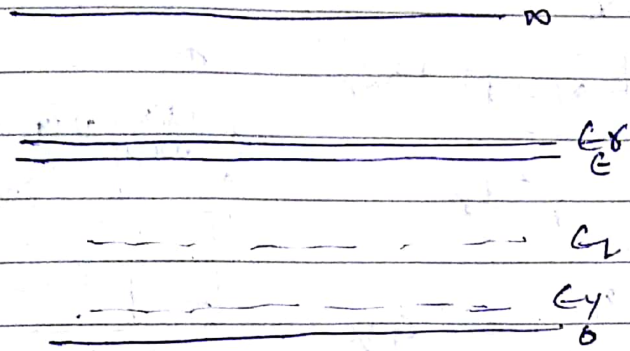
$P(E) \propto e^{-E/KT}$

$P(E) = C_1 e^{-E/KT}$

$$\int_0^\infty P(E)\, dE = C_1 \int_0^\infty e^{-E/KT} dE$$

$$\Rightarrow 1 = C_1 (KT) \int_0^\infty e^{-x} dx$$

$C_1 = 1/KT$



$\cancel{P(E) \propto e^{-E/KT}}$

$\cancel{P(E_1) = C_1 e^{-E/KT}}$

$$\frac{\int_0^\infty P(E) * E\, dE}{\int_0^\infty P(E)\, dE} = \frac{1}{KT}\int_0^\infty e^{-E/KT} \cdot E\, dE$$

$\int_0^\infty e^{-x} \cdot x^{n-1} dx = \Gamma(n)$

- $\Gamma(1) = 1$
- $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
- $\Gamma(n+1) = n\,\Gamma(n)$

$$U_\gamma d\gamma = N(\gamma)\,d\gamma \times E_\gamma$$

$$U_\gamma d\gamma = \frac{8\pi\gamma^2}{c^3} \times KT\,d\gamma$$

→ R.J & D. law in term of frequency

In terms of wavelength

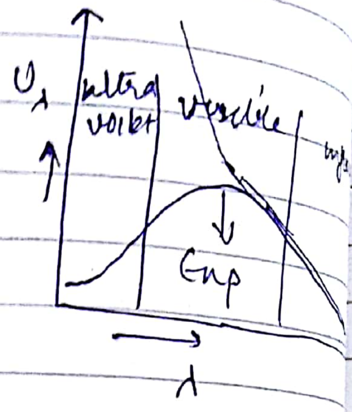
$U_\lambda d\lambda = -U_\gamma d\gamma$

↳ [$d\gamma \uparrow$ $d\lambda \downarrow$]

$$= -\frac{8\pi KT}{c^3}\gamma^2 d\gamma$$

$$= \frac{8\pi KT c^2}{c^3 \lambda^2} \cdot \frac{c}{\lambda^2}\, d\gamma$$

$$U_\lambda d\lambda = \frac{8\pi KT}{\lambda^4}\, d\lambda$$



For Rayleigh & Jeans law

Ultraviolet Castrophe

$$\gamma = c/\lambda$$

$$d\gamma = -\frac{c}{\lambda^2}\, d\lambda$$

This discrepancy in the experimental & theoretical value in the shorter wavelength region (UV region) is known as ultraviolet catastrophe.

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Max Planck's B B Radiation law

~~Each~~ The oscillator with frequency ν can have energy which are discrete ie are integral multiple of finite quanta of energy.

$E_n = nh\nu$ $\quad$ $n = 0, 1, 2, 3$

h = Planck const

ν

$E_0 = 0$

$E_1 = h\nu$

$E_2 = 2h\nu$

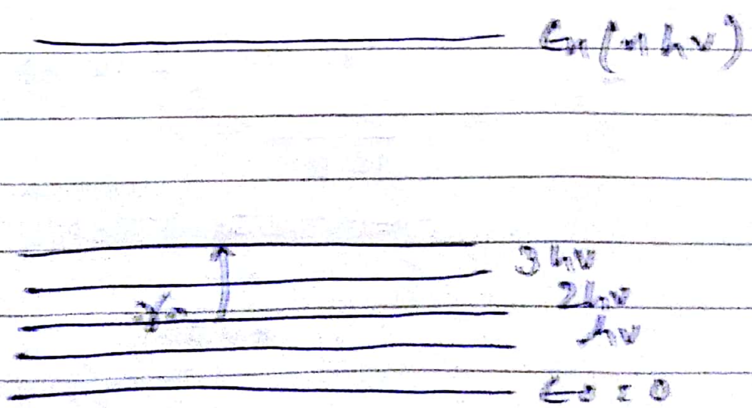
$E_3 = \cdot$

$\cdot$

$\cdot$

$E_n = nh\nu$

The change of energy of the oscillator due to emission & adsorption of radiant energy can also take place by a discrete value $h\nu$.



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$$\bar{E}_\gamma = \frac{P(\epsilon_0)\times \epsilon_0 + P(\epsilon_1)\times \epsilon_1}{P(\epsilon_0)+P(\epsilon_1)}$$

$$= \sum_{n=0}^{n} \frac{P(\epsilon_n)\times \epsilon_n}{\sum P(\epsilon_n)}$$

from boltzmann distribution

$$\rightarrow \left[P(\epsilon_n) \propto e^{-\epsilon_n/KT} \right.$$

$$\rightarrow P(\epsilon_n) = C_2 e^{-\epsilon_n/KT}$$

$$\Rightarrow \sum_{n=0}^{n=\infty} P(\epsilon_n) = C_2 \sum_{n=0} e^{-\epsilon_n/KT}$$

$$\Rightarrow 1 = C_2 \sum e^{-\epsilon_n/KT}$$

$$C_2 = \frac{1}{\sum e^{-\epsilon_n/KT}}$$

$$P(\epsilon_n) = \frac{e^{-\epsilon_n/KT}}{\sum e^{-\epsilon_n/KT}}$$

$$\bar{\epsilon}_\gamma = \frac{\sum_{n=0}^{n} e^{-\epsilon_n/KT}\cdot \epsilon_n}{\sum_{n=0}^{n} e^{-\epsilon_n/KT}}$$

$$= \frac{\sum e^{-nh\gamma/KT}\cdot nh\gamma}{\sum e^{-nh\gamma/KT}}$$

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$$\bar{E_\gamma} = \frac{0 + h\gamma e^{-h\gamma/KT} + 2h\gamma e^{-2h\gamma/KT} + \cdots}{1 + e^{-h\nu/KT} + e^{-2h\nu/KT} + \cdots\cdots}$$

$$\left[e^{-h\gamma/KT} = x\right]$$

$$= \cancel{h\gamma x + 2e\,x\,e^{2x}}$$

$$\frac{h\gamma e^{-h\gamma/KT}\left[1 + 2e^{-h\gamma/KT} + 3e^{-2h\gamma/KT} + \cdots\right]}{1 + e^{-h\gamma/KT} + e^{-2h\gamma/KT} + \cdots}$$

$$\bar{E_\gamma} = \frac{h\gamma x\left[1 + 2x + 3x^2 + x^3 + \cdots\right]}{1 + x + x^2 + \cdots}$$

$$\bar{E_\gamma} = \frac{h\gamma x(1-x)^{-2}}{(1-x)^{-1}}$$

$$= \frac{h\gamma x}{1-x}$$

$$\bar{E_\gamma} = \frac{h\gamma}{\frac{1}{x} - 1}$$

$$\boxed{\bar{E_\gamma} = \frac{h\gamma}{e^{h\gamma/KT} - 1}}$$

for low freq region
γ is very small

$$e^{h\gamma/KT} - 1 \cong 1 + \left(\frac{h\gamma}{KT}\right) + \frac{(\ )^2}{2!}$$

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The energy density with frequency interval

$$\gamma = \gamma + d\gamma$$

$$U_\gamma d\gamma = N(\gamma)\, d\gamma \times \overrightarrow{E_\gamma}$$

$$= \frac{8\pi\gamma^2}{c^2} \times \frac{h\nu}{e^{h\gamma/KT} - 1}\, d\gamma$$

$$U_\gamma d\gamma = \frac{8\pi h\gamma^3}{c^3\left[e^{h\gamma/KT} - 1\right]}\, d\gamma$$

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left[e^{hc/\lambda KT} - 1\right]}\, d\lambda$$

(I) Planck's $\longrightarrow$ Wein ~~dist~~ distribution law.

From Planck's Law we know

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left(e^{hc/\lambda KT} - 1\right]}\, d\lambda$$

for shorter wavelength region
λ is very small.

$$e^{h\gamma/\lambda KT} - 1 \simeq e^{hc/\lambda KT}$$

$$U_\lambda d\lambda = \frac{8\pi hC \; d\lambda}{\lambda^5 \, e^{hc/\lambda KT}} = \frac{a}{\lambda^5}\exp\left(\frac{-b}{\lambda}\right)$$

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(2) Planck → RJ distribution law

for longer wavelength eg
λ is very large

$$e^{hc/\lambda kT} - 1 \simeq \left\{ \left(1 + \frac{hc}{\lambda kT} + \frac{(\ )^2}{2!} + \right) - 1 \right\}$$

$$= \frac{hc}{\lambda kT}$$

$$u_\lambda d\lambda = \frac{8\pi hc \times \lambda kT}{\lambda^5 hc} d\lambda$$

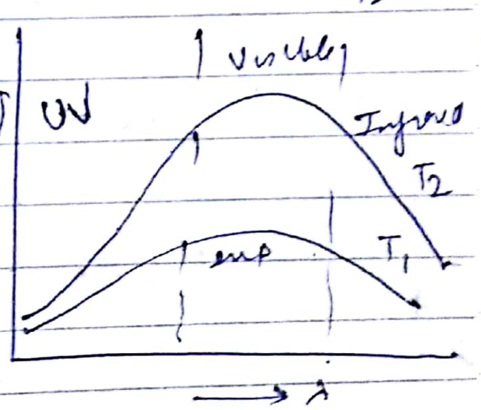
$$= \frac{8\pi kT}{\lambda^4} d\lambda$$

(3) Planck's → Wein's displacement law:

$\lambda_m T = const$

$T_2 > T_1$

$$u_\lambda = \frac{8\pi hc}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$



when $\lambda = \lambda_m$, u_λ is max

$$\left.\frac{du_\lambda}{d\lambda}\right|_{\lambda=\lambda_m} = 0$$

$$= a\lambda^{-5}(e^{b/\lambda T} - 1)^{-1}$$

]al

→ Wein's

$$\Rightarrow -5a\lambda_m^{-6}(e^{b/\lambda_m T} - 1)^{-1} + a\lambda_m^{-5} \cdot \frac{-1 \cdot e^{\frac{b}{\lambda T}}}{(e^{b/\lambda T}-1)^2} \cdot \left(\frac{-b}{\lambda_m^2 T}\right) = 0$$

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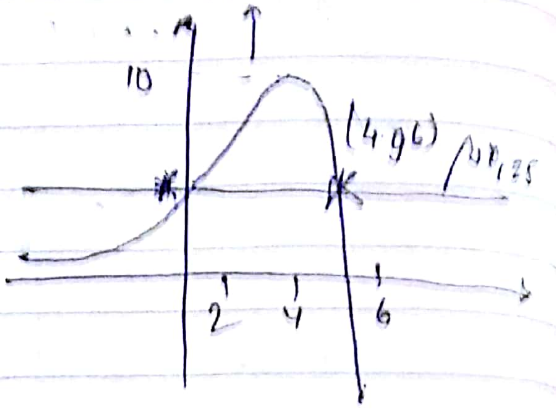
$$\frac{a}{\lambda_m^6 (e^{b/\lambda T} - 1)^2} \left[-5 (e^{b/\lambda_m T} - 1) + \frac{b}{\lambda_m T} \cdot e^{b/\lambda_m T} \right] = 0$$

$$5 [e^x - 1] = x e^x \qquad y_2 = e^x (5 - x)$$

$$5 = e^x (5 - x)$$

$$Y_1 = 5$$

$$Y_2 = e^x (5 - x)$$



$$x = 4.96$$

$$\frac{b}{\lambda_m T} = 4.96$$

$$\lambda_m T = \frac{b}{4.96} = \frac{hc}{k \times 4.96}$$

$$\lambda_m T = 2.9 \times 10^{-3} \text{ m-k}$$

Planck's Stefan's Law :-

$$U_\gamma d\gamma = \frac{8\pi h \gamma^3 \, d\gamma}{c^3 [e^{h\gamma/kT} - 1]}$$

The total energy density at a given temp T of one black body radiation is :

$$U = \int_0^\infty U_\gamma d\gamma = \frac{8\pi h}{c^3} \int_0^\infty \frac{\gamma^3 d\gamma}{e^{h\gamma/kT} - 1}$$

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now $\frac{h\nu}{KT} = x$

$$\frac{8\pi h}{c^3}\left(\frac{KT}{h}\right)^4 \left(\int_0^\infty \frac{x^3\,dx}{e^x - 1}\right) \rightarrow \frac{\pi^4}{15}$$

$$U = \frac{8\pi h}{c^3}\left(\frac{KT}{h}\right)^4 \frac{\pi^4}{15}$$

$$U = \left[\frac{8\pi^5 K^4}{15\, c^3 h^3}\right] T^4$$

The emissive power $E = \frac{C U}{4}$

$$\left[\frac{2}{15}\frac{\pi^5 K^4}{c^2 h^3}\right] T^4$$

$$E = \sigma T^4$$

$$\sigma = 5.68 \times 10^{-8} \frac{\text{Watt}}{m^2 - K^4}$$