Fluid Dynamics -> Problem Sheet 7:

1. Datum is 4000 mm (or 4m) below. $\Rightarrow z = 4\,m$.

$P = 20\times10^3$ Pa.

Now, diameter, $d = 0.25$ m.

$\therefore$ area of cross section $= \frac{\pi d^2}{4} = 0.049\,m^2$.

Flow rate $= \frac{2.4\,m^3}{min} = \frac{2.4}{60}\frac{m^3}{s}$

$\therefore$ Speed through a cross section $= \frac{\text{Flow rate}}{\text{Area of cross section}}$

$= \frac{2.4}{60\times0.049} = 0.815\,m/s. = V.$

$\therefore$ Total Energy head (from Bernoulli's eq^n):

$\rho = 1000\,kg/m^3$ (water)

$g = 9.81\,m/s^2$

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z$$

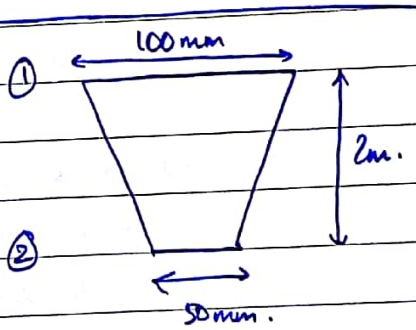
$$= \frac{20\times10^3}{10^3\times9.81} + \frac{(0.815)^2}{2\times9.81} + 4$$

$= 6.0726$ m. (Ans).

2. Flow rate = 30 L/s.

For H_2O: $[1\,L = 1\,dm^3 = 10^{-3}\,m^3]$

$= 0.03\,m^3/s$.



$\therefore A_1$ = area of cross section at 1

$= \pi\times\left(\frac{0.1}{2}\right)^2 = 7.854\times10^{-3}\,m^2$

Similarly, $A_2 = \pi\times\left(\frac{0.05}{2}\right)^2 = 1.963\times10^{-3}\,m^2$

$\therefore V_1$ = velocity of liq through 1

$= \frac{\text{Flow rate}}{A_1} = 3.82\,m/s$

and, $V_2 = \frac{\text{Flowrate}}{A_2} = 15.28\,m/s$.

From Bernoulli's eqn:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho z_1 g = P_2 + \frac{1}{2}\rho v_2^2 + \rho z_2 g.$$

$$\Rightarrow P_1 - P_2 = \frac{\rho}{2}(v_2^2 - v_1^2) + \rho g (z_2 - z_1)$$

$$= \frac{1000}{2}(15.28^2 - 3.82^2) + 1000 \times 9.81 \times (-2)$$

$$= 109443 - 19620$$

$$= 89823 \text{ Pa}$$

$$= 89.82 \text{ KPa}.$$

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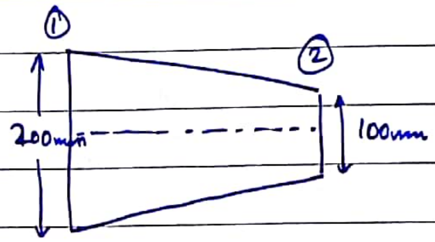
8. $z_1 - z_2 = 0$; (horizontal)



$P_1 = 400 \times 10^3$ Pa.

$P_2 = 250 \times 10^3$ Pa

A_1 = area of cross section at 1

$$= \pi \times \left(\frac{0.2}{2}\right)^2 = 0.0314 \text{ m}^2$$

$$\therefore A_2 = \pi \times \left(\frac{0.1}{2}\right)^2 = 7.854 \times 10^{-3} \text{ m}^2$$

Since it is an incompressible liq. (H_2O);

$\therefore$ from continuity eqn:

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow V_1 = \left(\frac{A_2}{A_1}\right) . V_2 = 0.25\, V_2.$$

$$\Rightarrow V_1 = 0.25\, V_2.$$

Now, from Bernoulli's eqn:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow P_1 - P_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$\Rightarrow (400-250)\times 10^3 = \frac{1000}{2}\left[V_2^2 - 0.0625 V_2^2\right]$$

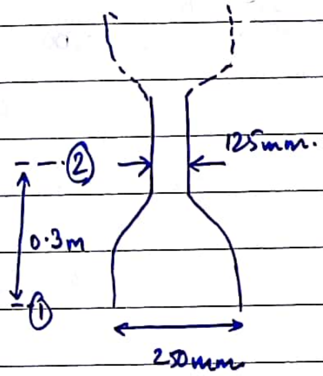
$$\Rightarrow V_2 = 17.89 \text{ m/s.}$$

$\therefore$ Flow rate = discharge = $V_2 \times A_2$ = $17.89 \times (7.854\times 10^{-3})$ m^3/s

= 0.1405 m^3/s.

4. $P_1 = 60$ KPa = 60×10^3 Pa

$P_2 = 20\times 10^3$ Pa.



A_1 = area of cross section at ①:

$$= \pi\left(\frac{0.25}{2}\right)^2 = 0.0491 \text{ m}^2$$

$$\therefore A_2 = \pi\left(\frac{0.125}{2}\right)^2 = 0.0123 \text{ m}^2$$

For an incompressible liq.; we apply continuity eqn.

$$V_2 = \left(\frac{A_1}{A_2}\right).V_1$$

$$\Rightarrow V_2 = 3.992 V_1$$

Now, from Bernoulli's equ:

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$$

$$\Rightarrow P_1 - P_2 + \rho g(z_1 - z_2) = \frac{\rho}{2}\left[(3.992V_1)^2 - V_1^2\right]$$

$$\Rightarrow (60-20)\times 10^3 + 1000\times 9.81(-0.3) = \frac{1000}{2}(14.936)V_1^2$$

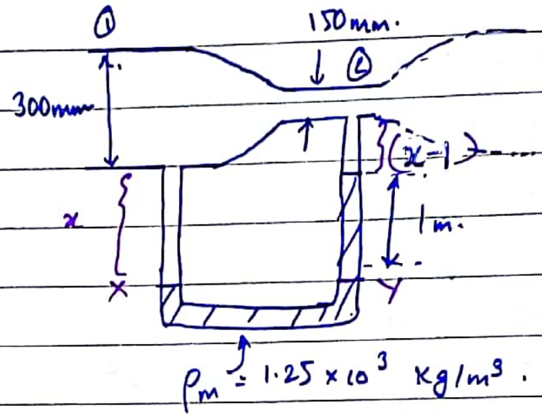
$$\Rightarrow V_1 = 2.227 \text{ m/s.}$$

$\therefore$ Vol. flowrate $= A_1 V_1$
$= 0.10937 \ m^3/s.$

or, $V_{th} = 0.10937 \ m^3/s.$

$\therefore$ $V_{actual} = C_d \times V_{th} = 0.98 \times 0.10937$
$= 0.1072 \ m^3/s.$ [$1 \ Lit = 10^{-3} m^3$]
$= 107.2$ Lit/s.

5. A_1
$= \pi \left(\frac{0.3}{2}\right)^2$
$= 0.0707 \ m^2$



$A_2 = \pi \left(\frac{0.15}{2}\right)^2$

$= 0.0177 m^2$

Now, from Bernoulli's equ:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2.$$

$$\Rightarrow v_2^2 - v_1^2 = 2g\left[\frac{P_1 - P_2}{\rho g} + z_1 - z_2\right]$$

Now, $v_2 = \frac{A_1}{A_2} v_1 \Rightarrow v_2 = 3.994 \ v_1$

$$\Rightarrow 14.954 \ v_1^2 = 2 \times 9.81 \left[\frac{P_1 - P_2}{\rho g} + z_1 - z_2\right] \quad \text{--- (1)}$$

For manometer:

~~P_1 x~~

$$P_x = P_y$$

$$\Rightarrow P_1 + x\rho.g = P_2 + (x-1)\rho.g + 1.\rho_m g.$$

$$\Rightarrow P_1 - P_2 = \rho_m g - \rho.g.$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = 1\left[\frac{\rho_m}{\rho} - 1\right] = \left[\frac{1250}{1000} - 1\right] = 0.25$$

Putting in (1), and $\because$ it is horizontal, $z_1 - z_2 = 0$.

$\Rightarrow 14.954\, V_1^2 = 2 \times 9.81\, (0.25)$

$\Rightarrow V_1 = 0.5727$ m/s.

$\therefore$ Vol. flow (theoretical) $= Q_{th} = A_1 V_1 = 0.0405\ m^3/s$.

Now, $C_d = \dfrac{Q_{actual}}{Q_{th}} = \dfrac{0.037}{0.0405}$

$\Rightarrow C_d = 0.914$

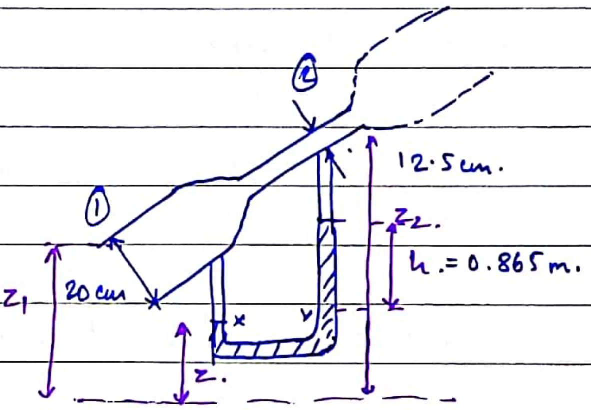
6. Manometer:

$P_x = P_y$

$\Rightarrow P_1 + \rho g (z_1 - z)$

$= P_2 + \rho g (z_2 - z - h)$

$+ \rho_m g (h)$



$\Rightarrow P_1 - P_2 = \rho g (z_2 - z_1) + h(\rho_m g - \rho g)$.

$\Rightarrow \left[\dfrac{P_1 - P_2}{\rho g} + (z_1 - z_2)\right] = h\left[\dfrac{\rho_m}{\rho} - 1\right] = 0.865\left[\dfrac{13600}{1000} - 1\right]$

$= 10.9$. (units).

$A_1 = \pi\left(\dfrac{0.2}{2}\right)^2 = 0.0314\ m^2$

$A_2 = \pi\left(\dfrac{0.125}{2}\right)^2 = 0.0123\ m^2$

Now, using Bernoulli eqn. for venturimeter:

$V_2^2 - V_1^2 = 2g\left[\dfrac{P_1 - P_2}{\rho g} + z_1 - z_2\right] = 2 \times 9.81 \times (10.9)$

$= 213.84$ (units).

From continuity, $V_1 = \dfrac{A_2}{A_1} . V_2 \Rightarrow V_1 = 0.391\, V_2$

$\Rightarrow 0.847\, V_2^2 = 213.84$

$\Rightarrow \boxed{V_2 = 15.88 \text{ m/s}}$

$\therefore Q = A_2 V_2$

$\Rightarrow \boxed{Q = 0.195 \text{ m}^3/\text{s}.}$

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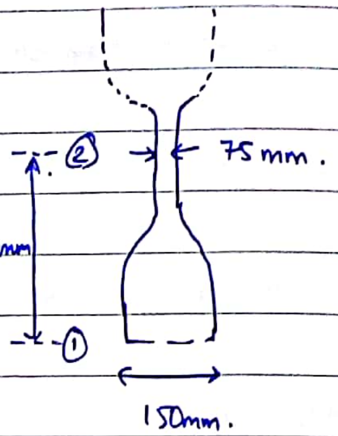
7. $\rho = 1000 \text{ kg/m}^3$



$Q_{actual} = 40 \text{ lit/s}$; $C_D = 0.96$.

$\therefore Q_{th} = 40/0.96 = 41.67 \text{ lit/s}.$

$= 0.04167 \text{ m}^3/\text{s}$

$= A_1 V_1$

$\Rightarrow V_1 = \dfrac{0.04167}{A_1}$

$= \dfrac{0.04167}{\pi \times \left(\frac{0.15}{2}\right)^2} \rightarrow V_1 = 2.358 \text{ m/s}.$

From continuity, $V_2 = \dfrac{A_1}{A_2} \cdot V_1$

$= \dfrac{\pi \times (0.15/2)^2}{\pi \times (0.075/2)^2} \times 2.358 = 9.432 \text{ m/s}.$

Now, by Bernoulli's eq^n:

$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$

$\Rightarrow P_1 - P_2 = \frac{\rho}{2}(V_2^2 - V_1^2) + \rho g (z_2 - z_1)$

$= \frac{1000}{2}\left[9.432^2 - 2.358^2\right] + 1000 \times 9.81 \times (0.25)$

$= 44,153 \text{ Pa}$

$= 44.2 \text{ kPa}.$

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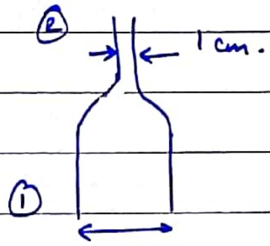
8. For the manometric reading,

$$H = \left[\frac{P_1 - P_2}{\rho g} + z_1 - z_2\right] = h\left[\frac{\rho_m}{\rho} - 1\right] = 0.12\left[\frac{13600}{800} - 1\right]$$

$$= 1.92 \text{ (units).}$$

Now, $\frac{A_1}{A_2} = 5$ (given).



$\therefore$ by continuity; $V_2 = \frac{A_1}{A_2} \cdot V_1$

$\Rightarrow V_2 = 5V_1$

Now, by Bernoulli eqn.

$$P_1 + \frac{1}{2}\rho v_1^2 + z_1 \rho g = P_2 + \frac{1}{2}\rho v_2^2 + z_2 \rho g.$$

$$\Rightarrow P_1 - P_2 + \rho g (z_1 - z_2) = \frac{\rho}{2}(v_2^2 - v_1^2)$$

$$\Rightarrow 2g\left[\frac{P_1 - P_2}{\rho g} + z_1 - z_2\right] = v_2^2 - v_1^2.$$

$$\Rightarrow (5v_1)^2 - v_1^2 = 2 \times 9.81 \times (1.92) \Rightarrow 24 V_1^2 = 2 \times 9.81 \times 1.92$$

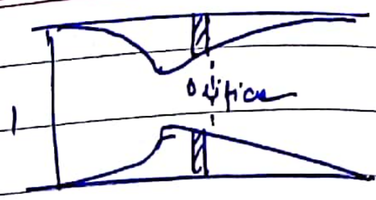
$\Rightarrow v_1 = 1.253$ m/s

$\therefore V_2 = 5V_1 = 6.264$ m/s.

$A_2 = \pi \left(\frac{0.01}{2}\right)^2 = 7.854 \times 10^{-5}$

$\therefore$ Discharge $= Q = A_2 \times V_2 = 4.92 \times 10^{-4}$ m³/s

$= 0.492$ lit/s.



9. $d_0 = 0.1 m.$

$d_1 = 0.2 m$

$\therefore A_1 = \pi \left(\frac{0.2}{2}\right)^2 = 3.1416 \times 10^{-2} m^2$

$A_0 = \pi \times \left(\frac{0.1}{2}\right)^2 = 7.86 \times 10^{-3} m^2$

Now, by Bernoulli's eqn:

$P_1 + \frac{1}{2}\rho v_1^2 = P_0 + \frac{1}{2}\rho v_0^2$ $[z_1 - z_2 = 0]$

$\Rightarrow (P_1 - P_2) = \frac{\rho}{2}(v_0^2 - v_1^2)$

Now, by continuity, $V_0 = \frac{A_1}{A_0} \cdot V_1$

$\Rightarrow V_0 = 4 V_1$

$\Rightarrow$

$\Rightarrow (19.62 - 9.81) \times 10^4 Pa = \frac{1000}{2}(15 v_1^2)$

$\Rightarrow V_1 = 3.62 m/s.$

$\therefore$ Discharge $Q = A_1 V_1 = 0.11362 m^3/s.$

$\therefore Q_{actual} = Q_{th} \times C_d = (0.1132 \times 0.6) m^3/s.$

$= (0.1132 \times 0.6) \times (10^2)^3 cm^3/s.$

$= 68235.54 cm^3/s.$

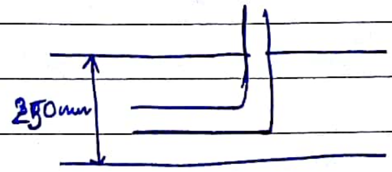
$= 68.23 lit/s.$

10. Dia of pipe = 0.25 m.

$\therefore$ Area of cross section (A)

$= \pi \times \left(\frac{0.25}{2}\right)^2$

$= 0.0491 \ m^2$



Static pressure (vacumm) $= -40$ mm Hg $= -0.04$ m Hg.

$= -0.04 \times 13600 \times 9.81$

$\therefore$ Static pressure head $= -\frac{(0.04 \times 13600 \times 9.81)}{1000 \times 9.81}$

$= -0.544$ m of water.

Stagnation pressure $= 7.85$ kPa.

Stagnation pressure head $= \frac{7.85 \times 10^3}{\rho g} = \frac{7.85 \times 10^3}{1000 \times 9.81}$

$= 0.8002$ m of water.

$h = \frac{\Delta P}{\rho} =$ Stagnation pressure head $-$ Static Pressure head.

$= 0.8002 - (-0.544)$.

$= 1.3442$ m of water.

$\therefore$ Velocity at centre $= \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.3442}$

$= 5.135$ m/s.

or, $V_{actual} = C_v \times V_{th}$.

$= 0.98 \times 5.135$..

$= 5.033$ m/s. { Velocity at centre $\because$ pitot tube is at centre?

$\therefore$ mean $V = 0.8 \times 5.033 = 4.026$ m/s.

$\therefore$ Discharge $= AV = 0.0491 \times 4.026 = \underline{0.1977 \ m^3/s}$.