ASSIGNMENT-I (1^{ST} YEAR 2^{ND} SEM)

Improper Integral & Beta Gamma Function

1) Prove that $\int_{-1}^{1}\frac{dx}{x^3}$ exists in Cauchy Principal value sense but not in general sense.
2) Evaluate (if the integral exists)
   a) $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$
   b) $\int_{-1}^{1} \frac{\sqrt{1+x}}{\sqrt{1-x}}dx$
   c) $\int_{1}^{\infty}\frac{dx}{x\sqrt{x^2-1}}$
   d) $\int_{0}^{\frac{1}{e}} \frac{dx}{x(logx)^2}$
   e) $\int_{0}^{\infty}\frac{dx}{(x+1)(x+2)}$
   f) $\int_{0}^{\infty}\frac{xdx}{(x^2+a^2)(x^2+b^2)}$
   g) $\int_{1}^{2}\frac{dx}{(x+1)\sqrt{x^2-1}}$
3) Show that:
   a) $\int_{0}^{\frac{\pi}{2}} \sqrt{cot\theta}\ d\theta=\frac{\pi}{2}$
   b) $\int_{0}^{1} x^3(1-x^2)^{\frac{5}{2}}\ dx$
   c) $\int_{0}^{1} \frac{x^2}{\sqrt{1-x^4}}\ dx \times \int_{0}^{1} \frac{1}{\sqrt{1+x^4}}\ dx = \frac{\pi}{4\sqrt{2}}$
   d) $\mathrm{B}\left(m,\frac{1}{2}\right) = 2^{2m-1}\mathrm{B}(m,m)$
   e) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^n}} = \frac{\Gamma(\frac{1}{n})\sqrt{\pi}}{\Gamma(\frac{1}{2}+\frac{1}{n})n}$
4) Assuming the convergence of the integral , prove that

$$\int_{0}^{\infty}\sqrt{x}\,e^{-x^3}dx = \frac{\sqrt{\pi}}{3}$$

5) Evaluate:
   a) $\int_{0}^{\infty} 55^{-x^2}dx$
   b) $\int_{0}^{1} x^4\{\log(\frac{1}{x})^3\}\ dx$
   c) $\int_{0}^{\frac{\pi}{2}} \frac{dx}{\sqrt{sinx}} \times \int_{0}^{\frac{\pi}{2}} \sqrt{sinx}\,dx$
   d) $\int_{0}^{1}\frac{dx}{(1-x^6)^{\frac{1}{6}}}$

Laplace Transform & Inverse Laplace Transform

1) Find the Laplace Transform of the following functions:
   a) F(t) = $(1 + t\, e^{-t})^3$
   b) F(t) = $\{(t^2 - 3t + 2) sin3t\}$
   c) F(t) = $e^{-3t}(2cos5t - 3sin5t)$
   d) F(t) = 7^t
   e) F(t) = $\dfrac{\sin^2 t}{t}$
   f) F(t) = $e^{-3t}.\frac{\sin 2t}{t}$
   g) F(t) = $\int_0^t e^u \frac{\sin u}{u}\ du$

2) Find the Laplace Transform of $\dfrac{sinat}{t}$. Hence, show that $\int_0^{\infty} \frac{sint}{t}\ dt = \frac{\pi}{2}$.

3) Evaluate $\int_0^{\infty} t\, e^{-2t}\, cost\ \ dt$.

4) Show that $\int_0^{\infty} t^2 e^{-4t} \sin 2t\, dt = \frac{11}{500}$

5) Find the Laplace Transform of the Periodic function F(t) given by

$$F(t) = \begin{cases} t\ , & for\ \ 0 < t < c \\ 2\,c - t\ \ , & for\ \ \ c < t < 2\pi \end{cases}$$

6) Express the following function in terms of unit Step Function and then find its Laplace Transform:

$$F(t) = \begin{cases} 0, & 0 < t < 1 \\ t - 1\ , & 1 < t < 2 \\ 1\ , & t > 2 \end{cases}$$

7) Find $L\ \{ \sin \sqrt{t}\ \}$,(t$>$ 0) and then obtain the value of $L\ \{ \frac{\cos \sqrt{t}}{\sqrt{t}} \}$

8) Evaluate the following:
   a) $L^{-1}\{ \frac{4s\, +\, 5}{(s-4)^2\ (s+3)} \}$
   b) $L^{-1}\{ \frac{s}{(s^2 - \ a^2)^2} \}$
   c) $L^{-1}\{ \tan^{-1} \frac{2}{s^2} \}$
   d) $L^{-1}\{ \frac{e^{4-3s}}{(s+4)^{\frac{5}{2}}} \}$
   e) $L^{-1}\{\log(1 + \frac{a^2}{s^2})\}$
   f) $L^{-1} \left\{ \log\left(\frac{s+a}{s+b}\right) \right\}$
   g) $L^{-1}\{ \frac{s^2}{(s^2+4)^2} \}$
   h) $L^{-1}\{s \log \frac{s}{\sqrt{s^2+1}} + \cot^{-1} s\}$

i) $L^{-1}\{\log(\frac{s^2-4}{s^3})^{\frac{1}{3}}\}$

9) Find the Inverse of Laplace Transform of the following by Convolution theorem:

a) $\frac{1}{(s^2+1)(s^2+9)}$

b) $\frac{s}{(s^2+9)^2}$

10) Solve the following Differential Equation using Laplace Transform:

a) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4e^{2t}$, $y(0) = -3, y'(0) = 5.$

b) $\frac{d^2y}{dt^2} + 9y = 1$, $y(0) = 1, y\left(\frac{\pi}{2}\right) = -1.$

c) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 3y = t\cos t$, $y(0) = y'(0) = 0.$

d) $y''(t) + y(t) = 8\cos t$, $y(0) = 1, y'(0) = -1.$