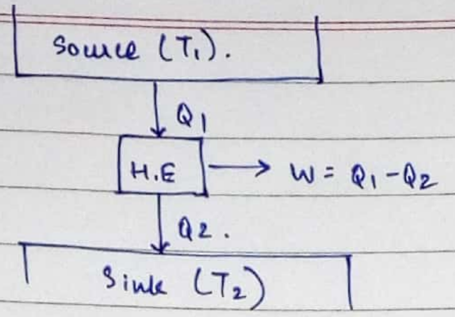
Problem-Sheet 3.

1.



Case I:

$$\frac{W}{Q_1} = \eta = \frac{Q_1 - Q_2}{Q_1} = \frac{1}{5}.$$

$$\Rightarrow 1 - \frac{Q_2}{Q_1} = \frac{1}{5}$$

$$\Rightarrow 1 - \frac{T_2}{T_1} = \frac{1}{5}.$$

Now, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\Rightarrow 5(T_1 - T_2) = T_1.$$

$$\Rightarrow 4T_1 - 5T_2 = 0 \quad \ldots\ldots(1)$$

Case II:

$$\frac{W}{Q_1} = \eta = \frac{Q_1 - Q_2'}{Q_1} = \frac{2}{5}.$$

$$\Rightarrow 1 - \frac{Q_2'}{Q_1} = \frac{2}{5} \Rightarrow \frac{T_1 - T_2'}{T_1} = \frac{2}{5}.$$

$$\Rightarrow \frac{T_1 - (T_2 - 80)}{T_1} = \frac{2}{5}.$$

$$\Rightarrow 5T_1 - 5T_2 + 400 = 2T_1$$

$$\Rightarrow 3T_1 - 5T_2 + 400 = 0 \quad \ldots\ldots(2)$$

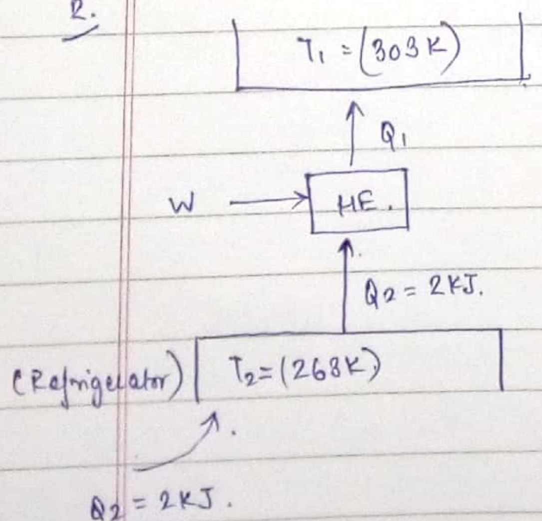
(1) − (2):

$\boxed{T_1 = 400\,K.}$

$$\Rightarrow 4(400) = 5T_2.$$

$$\Rightarrow \boxed{T_2 = 320\,K}$$

2.



Shown beside are the heat exchanges for 1 sec. only.

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}.$$

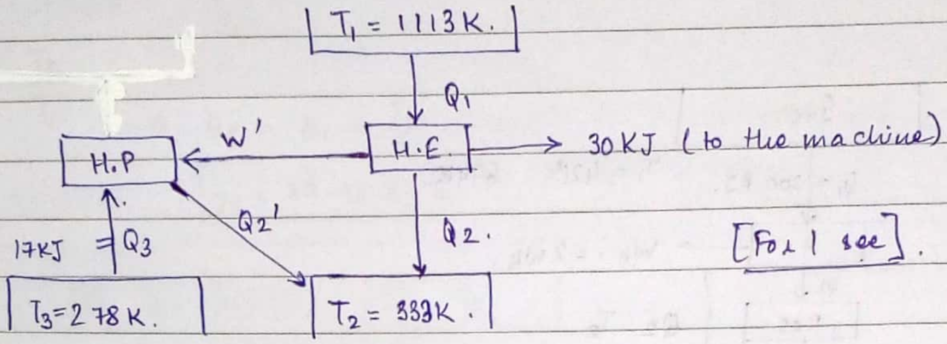
$$\Rightarrow Q_1 = \frac{2 \times 303}{263}. = 2.304\,KJ.$$

$$\therefore W_{inp} = W = Q_1 - Q_2 = 0.304\,KJ$$

∴ Work done in 1 sec = 0.304 kJ.

∴ Power input = 0.304 kW.

3.



[For 1 sec].

For the heat pump, $\frac{Q_2'}{Q_3} = \frac{T_2}{T_3} \Rightarrow \frac{333}{278} = \frac{Q_2'}{17} \Rightarrow \boxed{Q_2' = 20.36\ kJ}.$

Now, $W' = Q_2' - Q_3 = 20.36\ kJ - 17\ kJ = \boxed{3.36\ kJ. = W'}$

∴ Nett work by heat engine = 30 kJ + 3.36 kJ.

= 33.36 kJ.

Nett Work + Q_2 = Q_1 [heat engine].

$\Rightarrow Q_1 = Q_2 + 33.36\ kJ$ ..... (i)

So, again $\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow \frac{Q_2 + 33.36}{Q_2} = \frac{1113}{333}$

$\Rightarrow 1 + \frac{33.36}{Q_2} = \frac{1113}{333}$

$\Rightarrow Q_2 = 14.20\ kJ.$

∴ $Q_1 = 14.20 + 33.26$ (putting in (i))

$Q_1 = 47.46\ kJ.$

∴ nett heat rejected to 60° reservoir

$= Q_2 + Q_2'$

$= (14.20 + 20.36) = 34.56\ kJ$

or rate of heat supply from 840°C source $= \boxed{47.46\ kW}$

∴ rate of heat rejection to the 60°C sink $= \boxed{34.56\ kW}$

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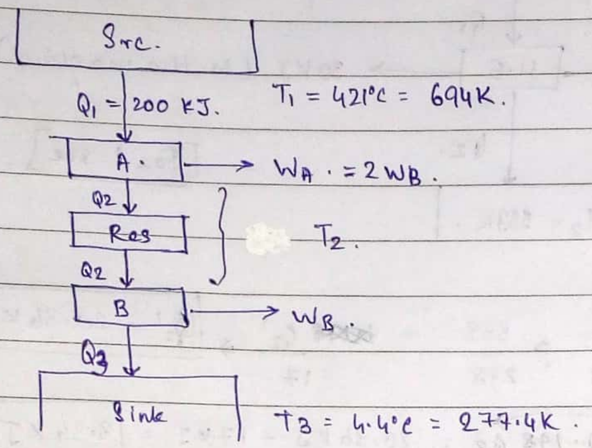
4.



Now, $\eta \propto$ work output. $(\because W_A = 2W_B)$

Now, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$. $\qquad\qquad \frac{Q_3}{Q_2} = \frac{T_3}{T_2}$.

$\frac{Q_1}{Q_2} - 1 = \frac{T_1}{T_2} - 1$ $\qquad\qquad 1 - \frac{Q_3}{Q_2} = 1 - \frac{T_3}{T_2}$.

$\Rightarrow \frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}$, $\qquad \Rightarrow \frac{Q_2 - Q_3}{Q_2} = \frac{T_2 - T_3}{T_2}$.

$\Rightarrow \frac{W_A}{Q_2} = \frac{T_1 - T_2}{T_2}$. $\qquad \Rightarrow \frac{W_B}{Q_2} = \frac{T_2 - T_3}{T_2}$ .... (2).

$\Rightarrow \frac{2W_B}{Q_2} = \frac{T_1 - T_2}{T_2}$ .... (1)

(1) ÷ (2) : $2 = \frac{T_1 - T_2}{T_2} \times \frac{T_2}{T_2 - T_3}$.

$\Rightarrow 2(T_2 - T_3) = T_1 - T_2$.

$\Rightarrow 3T_2 = T_1 + 2T_3$

$\Rightarrow \boxed{T_2 = 416.26\,K. = 143.26°C}$

$\eta_A = 1 - \frac{T_2}{T_1} = 1 - \frac{416.26}{694}$

$= 0.40$

$\boxed{\eta_A = 40\%}$

$\eta_B = 1 - \frac{T_3}{T_2} = 1 - \frac{277.4}{416.26}$

$= 0.336$

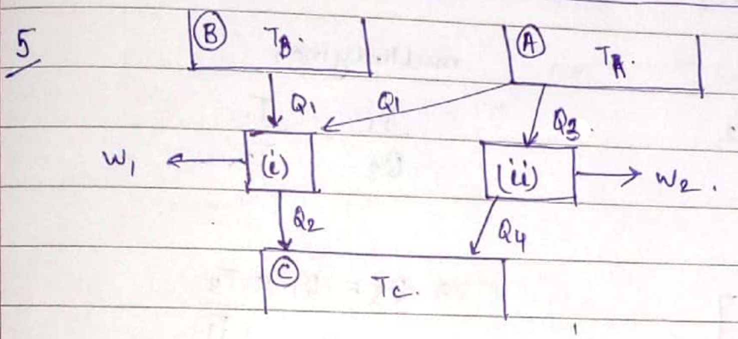
$\boxed{\eta_B = 33.6\%}$

$\frac{Q_3}{Q_1} = \frac{T_3}{T_1}$

$\Rightarrow Q_3 = Q_1 \times \frac{T_3}{T_1} = 200 \times \frac{277.4}{694} = 79.94\ kJ.$

$\boxed{Q_3 = 79.94\ kJ.}$

5



$\eta_A = \frac{W_1}{2Q_1} = \frac{1}{2}\left(\frac{2Q_1 - Q_2}{Q_1}\right)$

$= 1 - \frac{1}{2}\frac{Q_2}{Q_1}$

$\eta_B = \frac{W_2}{Q_3} = \frac{Q_3 - Q_4}{Q_3} = 1 - \frac{Q_4}{Q_3}.$

$\eta_B = \left(1 - \frac{T_C}{T_A}\right)$

Now, $\frac{Q_1}{T_A} + \frac{Q_1}{T_B} = \frac{Q_2}{T_C}.$

$\Rightarrow Q_1\left(\frac{1}{T_A} + \frac{1}{T_B}\right) = \frac{Q_2}{T_C}.$

$\Rightarrow T_C\left(\frac{T_A + T_B}{T_A \cdot T_B}\right) = \frac{Q_2}{Q_1}$

$\therefore \eta_A = 1 - \frac{1}{2} \cdot T_C \cdot \left(\frac{T_A + T_B}{T_A \cdot T_B}\right)$

Now, $\eta_A = \alpha \cdot \eta_B.$

$\Rightarrow 1 - \frac{1}{2} \cdot T_C\left(\frac{T_A + T_B}{T_A \cdot T_B}\right) = \alpha\left(1 - \frac{T_C}{T_A}\right)$

$\Rightarrow T_A T_B - \frac{1}{2}(T_A + T_B)T_C = \alpha(T_A T_B - T_B T_C)$

$\Rightarrow 2T_A T_B - (T_A + T_B)T_C = 2\alpha(T_A T_B - T_B T_C)$

$\Rightarrow -T_A T_C = T_A T_B(2\alpha - 2) + T_B T_C(1 - 2\alpha)$

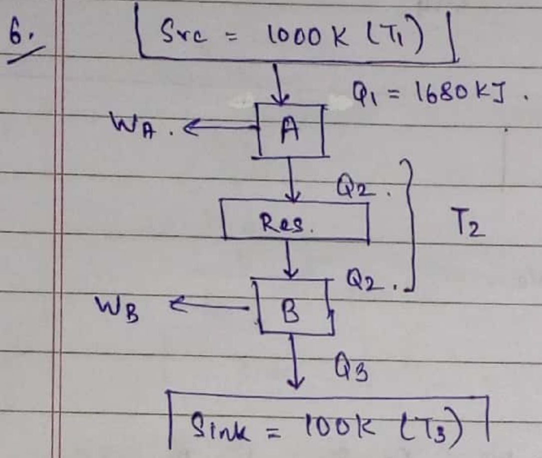
$\times \left(\frac{1}{T_B T_C}\right)$

$$\Rightarrow -\frac{T_A}{T_B} = 2\frac{T_A}{T_C}(\alpha - 1) + (1 - 2\alpha).$$

$$\Rightarrow \frac{T_A}{T_B} = 2\frac{T_A}{T_C}(1-\alpha) + (2\alpha - 1).$$

- - - - - Proved.

6.



$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ and $\frac{Q_2}{Q_3} = \frac{T_2}{T_3}$.

multiplying,

$$\frac{Q_1}{Q_3} = \frac{T_1}{T_3}.$$

$$\Rightarrow Q_3 = Q_1 \times \frac{T_3}{T_1}$$

$$\Rightarrow \boxed{Q_3 = 168 \text{ kJ}} \quad (a)$$

Now, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$.

$\Rightarrow T_2(1680) = Q_2(1000)$.

$\Rightarrow Q_2 = 1.68\, T_2$ - - - - (i)

$\therefore Q_2 = 531.26$ kJ

$\therefore W_A = Q_1 - Q_2$

$= (1680 - 531.26)$

$\boxed{W_A = 1148.74 \text{ kJ.}}$ (c)

$W_B = Q_2 - Q_3$

$= (531.26 - 168)$

$\boxed{W_B = 363.26 \text{ kJ}}$ (c).

Also, $\eta_A = \eta_B$.

$\Rightarrow 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$.

$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2}$.

$\Rightarrow T_2^2 = (1000)(100)$.

$\Rightarrow \boxed{T_2 = 316.23 \text{ K}}$. (b).

Now, assuming $W_A = W_B$ [as given]:

$Q_1 = 1680$ kJ ; $T_1 = 1000$ K ; $T_3 = 100$ K ; $Q_3 = 168$ kJ

$W_A = W_B$.

$\Rightarrow Q_1 - Q_2 = Q_2 - Q_3$.

$\Rightarrow 2Q_2 = Q_1 + Q_3$.

$\Rightarrow \boxed{Q_2 = 924 \text{ kJ}}$ (d).

now, $\eta_A = 1 - \dfrac{Q_2}{Q_1}$ $\qquad \eta_B = 1 - \dfrac{Q_3}{Q_2}$

$\boxed{\eta_A = 0.45 \text{ or } 45\%}$ $\qquad$ (e) $\boxed{\eta_B = 0.818 \text{ or } 81.8\%}$

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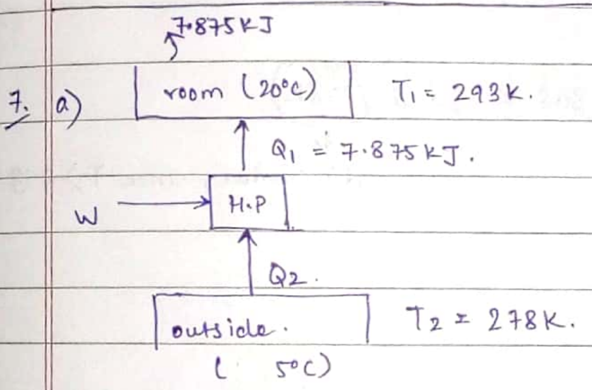
7. a)



$\Delta T = 20°C - 5°C$

$= 15°C = 15$ K.

$\therefore$ Heat escaping room /sec

$= 15 \times 0.525$ kJ.

$= 7.875$ kJ.

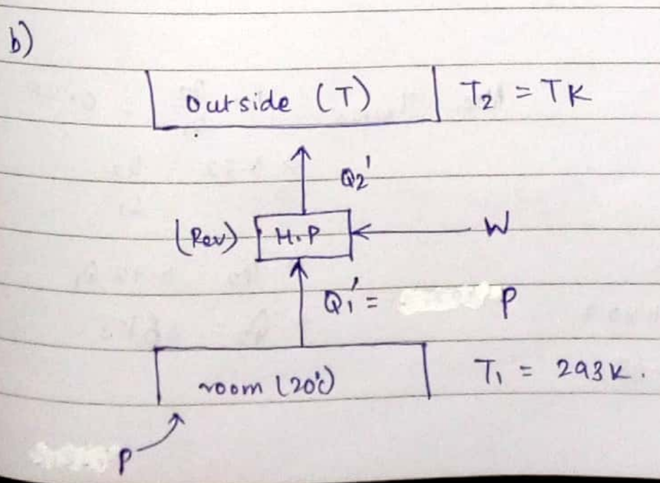
Clearly, $\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2} \Rightarrow Q_2 = Q_1 \times \dfrac{T_2}{T_1}$

$= 7.472$ kJ.

$\therefore$ Work done /sec $= Q_1 - Q_2 = 0.4031$ kJ.

$\therefore$ Power required $= \boxed{403.1 \text{ W}}$

b)



Let outside temp (max) be T.

$\therefore$ Heat entering room /sec

$= ((T - 293) \times 0.525)$ kJ

$= P$ (say).

$$\frac{Q_1'}{Q_2'} = \frac{T_1'}{T_2'} \Rightarrow \frac{0.525(T-293)}{Q_2'} = \frac{293}{T}$$

$$\Rightarrow Q_2' = \frac{0.525(T^2 - 293T)}{293}$$

Now, $W = Q_2' - Q_1' = 0.403$

$$\Rightarrow \left(\frac{0.525T^2 - 0.525T}{293}\right) - \left((T-293)\times 0.525\right) = 0.403$$

$$\Rightarrow \frac{0.525T^2}{293} - 0.525T - 0.525T + 153.825 - 0.403 = 0$$

$$\Rightarrow \frac{0.525T^2}{293} - 1.05T + 153.422 = 0$$

$$\Rightarrow T^2 - 586T + 85624.09 = 0$$

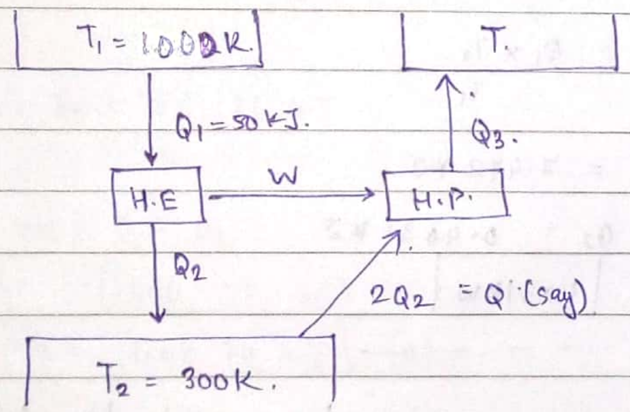
$$\Rightarrow T = \frac{586 \pm 30}{2} = 308\ K \text{ and } 278K$$

discarded, since $T > 293K$.

$\therefore T = 308K$.

$\therefore$ max temp $= 35°C$.

8.



For 1 sec

For heat engine:

$\eta = 1 - \frac{T_2}{T_1} = 1 - 0.3 = 0.7$. (max possible)

$\therefore \eta_{actual} = 0.4 \times 0.7 = 0.28$

Also, $\eta_{actual} = 1 - \frac{Q_2}{Q_1} = 0.28$

$\Rightarrow 0.72 = \frac{Q_2}{Q_1}$

$\Rightarrow Q_2 = 0.72\, Q_1$

$\Rightarrow Q_2 = 36\ KJ$.

$\therefore W = 50 - 3\cancel{8} = 14 \text{ kJ}.$

$Q = (2Q_2) = 3\cancel{8} \times 2 = 72 \text{ kJ}.$

$Q_3 = Q + W$

$= 72 + 14 = \boxed{86 \text{ kJ}}$

or rate of heat rejection by heat pump $= \boxed{86 \text{ kW}}$ [b].

$\text{COP}_{(\text{actual})} = \dfrac{Q_3}{W} = \dfrac{86 \text{ kJ}}{14 \text{ kJ}}$

$= 6.1428$

[actual, since calculated directly from heat transfer]

$\text{COP}_{(\text{max poss})} = 2 \times 6.1428 = 12.29$

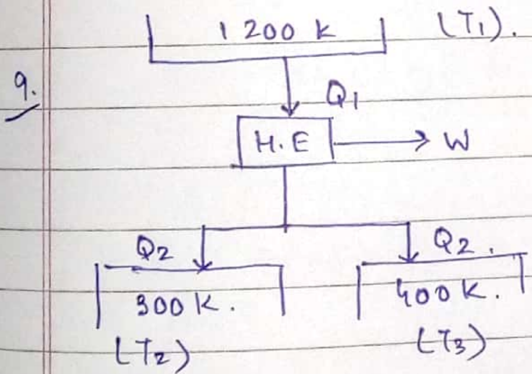
Now $\text{COP} = \dfrac{T}{T - T_2} = 12.29$

$\cancel{\Rightarrow T = 3.07T - 921}$

$\Rightarrow T = 12.29T - 3687.$

$\Rightarrow \boxed{T = 326.57 \text{ K}.}$ [a].

9.



Now, $\dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2} + \dfrac{Q_2}{T_3}$

$\Rightarrow \dfrac{Q_1}{1200} = Q_2 \left[\dfrac{1}{300} + \dfrac{1}{400}\right]$

$\Rightarrow Q_1 = \dfrac{700}{120000} \times 1200 \times Q_2$

$\Rightarrow Q_1 = 7Q_2$

Now, $\eta = 1 - \dfrac{2Q_2}{Q_1}$

$= 1 - \dfrac{2Q_2}{7Q_2}$

$= 1 - \dfrac{2}{7} = 5/7$ [Ans].

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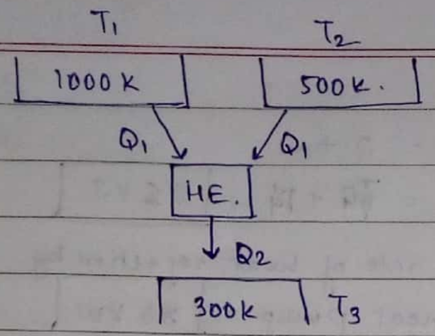
10. 

$$\frac{Q_1}{T_1} + \frac{Q_1}{T_2} = \frac{Q_2}{T_3}.$$

$$\Rightarrow Q_1\left[\frac{1}{1000} + \frac{1}{500}\right] = \frac{Q_2}{300}.$$

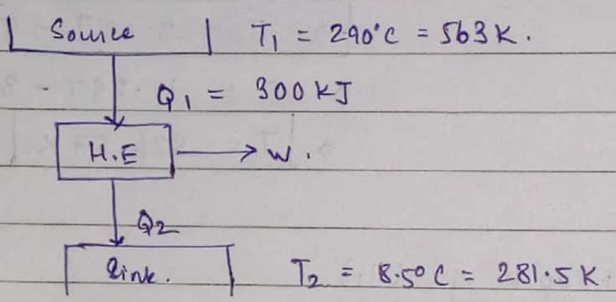
$$\Rightarrow Q_1 \times \frac{1500}{500000} \times 300 = Q_2.$$

$$\Rightarrow Q_2 = 0.9\, Q_1$$

$$\therefore \eta = 1 - \frac{Q_2}{2Q_1}$$

$$= 1 - \frac{0.9 Q_1}{2 Q_1} = 0.55 \quad \text{or } \boxed{55\%}$$

11. (For 1 sec):



Max efficiency (reversible mode) $= 1 - \frac{T_2}{T_1}$

$= 1 - \frac{281.5}{563}$

$= 0.5$ or 50%

a) $Q_2 = 215$ kJ.

$\therefore \eta = 1 - \frac{Q_2}{Q_1} = 0.283$ or 28.3%

$\eta < \eta_{max}$ $\therefore$ possible; irreversible.

b) $Q_2 = 150$ kJ.

$\therefore \eta = 1 - \frac{Q_2}{Q_1} = 0.5$ or 50%.

$\therefore \eta = \eta_{max} \Rightarrow$ possible; reversible.

c) $Q_2 = 75$ kJ.

$\therefore \eta = 1 - \frac{Q_2}{Q_1} = 0.75$ or 75%

$\therefore \eta > \eta_{max}$ $\therefore$ not possible.