

b. Two beam interference pattern, Thin film

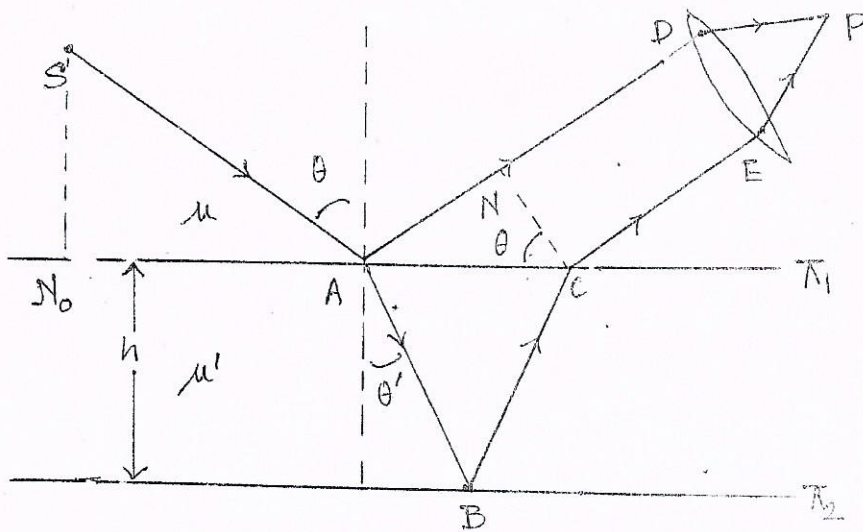


Diagram-7

π_1 and π_2 are two surfaces (upper and lower respectively) of a plate made of transparent material. π_1 and π_2 are parallel. The plate is illuminated by a point source S of quasi-monochromatic light. The point P is reached by two rays — one reflected by π_1 and other by π_2 . We get the following observations.

(i) A nonlocalized interference pattern on the same side of the plate as S .

(ii) Due to symmetry around SN_0 , the fringes in the plane parallel to the plate are circular around SN_0 , so that at any position of the point P , the ray is perpendicular to the plane SN_0P .

Explanation

The optical path difference between the rays $SADP$ and $SABCE$ is

$$\Delta = \mu'(AB + BC) - \mu AN \quad \dots (i)$$

From geometrical consideration $AB = BC = \frac{h}{\cos \theta'}$ $\dots (ii)$

and $AN = AC \sin \theta = 2h \tan \theta' \sin \theta$ $\dots (iii)$

From Snell's law of refraction

$$\mu' \sin \theta' = \mu \sin \theta \quad \dots (iv)$$

Hence from equation-(i)

$$\Delta = \mu' \frac{2h}{\cos \theta'} - \mu 2h \tan \theta' \sin \theta$$

$$\Rightarrow \Delta = 2h \left[\frac{\mu'}{\cos \theta'} - \tan \theta' \mu' \sin \theta \right]$$

$$\Rightarrow \Delta = 2\mu'h \left[\frac{1}{\cos \theta'} - \frac{\sin^2 \theta'}{\cos \theta'} \right]$$

$$\Rightarrow \boxed{\Delta = 2\mu'h \cos \theta'} \quad (\text{cosine law})$$

A phase change can occur due to reflection either from π_1 or from π_2 . Hence an extra phase difference $\pm \pi$ is to be added. Hence the net phase difference

$$\begin{aligned} \delta &= \frac{2\pi}{\lambda} \Delta \pm \pi \\ &= \frac{4\pi}{\lambda} \mu' h \cos \theta' \pm \pi \end{aligned}$$

For bright fringe: $\frac{4\pi}{\lambda} \mu' h \cos \theta' \pm \pi = 2m\pi$

$$\Rightarrow \boxed{2\mu'h \cos \theta' = (2m \pm 1) \frac{\lambda}{2}}, m = 0, 1, 2, \dots$$

For dark fringe: $\frac{4\pi}{\lambda} \mu' h \cos \theta' \pm \pi = (2n+1)\pi$

$$\Rightarrow 2\mu'h \cos \theta' = (2n \pm 2) \frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu'h \cos \theta' = p\lambda}, p = 0, 1, 2, \dots$$

Remark: 1. As θ is determined only by the position of P in the focal plane of the telescope, δ is independent of the position of the source S. It follows that fringes are as distinct with an extended source as with the point source.

2. As the fringes are characterized by the value of θ and θ' , they are called fringes of equal inclination.

3. When the telescope objective is normal to the plate, the fringes are concentric circles about the focal point for normally reflected light ($\theta = \theta' = 0$).

4. If we consider the interference of transmitted waves, the following diagram results:-

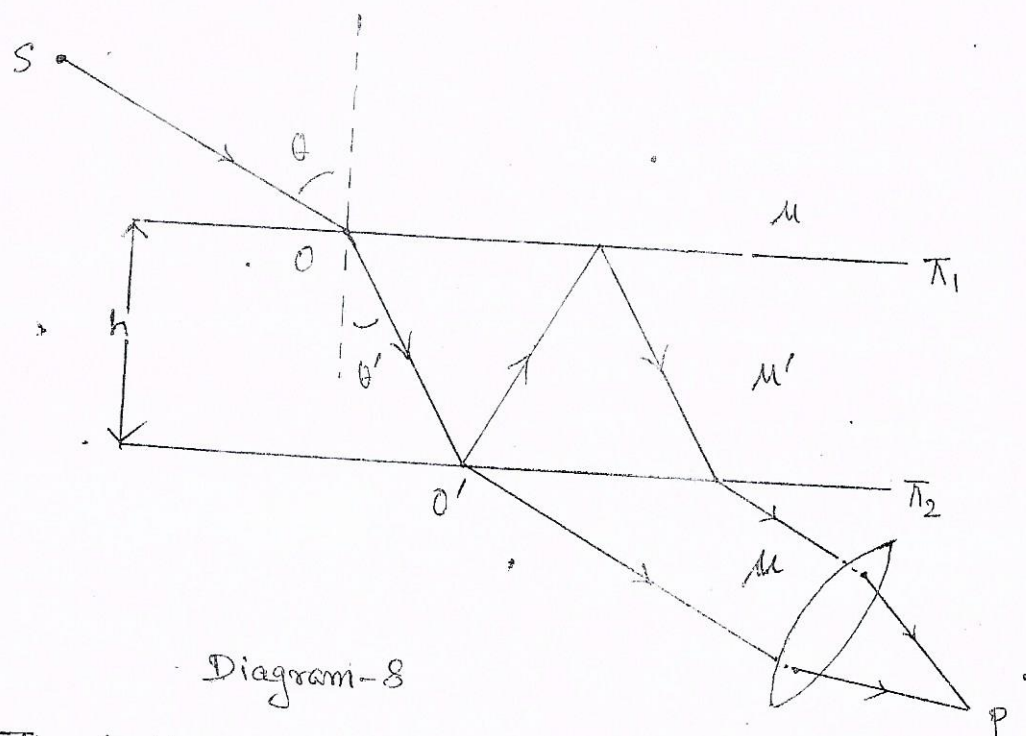


Diagram-8

The path difference can be calculated as usual giving

$$\Delta = 2\mu'h \cos \theta'$$

There is no additional phase difference from the phase change on reflection considering O' as the starting point. Hence, the phase difference

$$\delta = \frac{2\pi}{\lambda} \Delta = \frac{4\pi}{\lambda} \mu'h \cos \theta'$$

Bright fringe: $\frac{4\pi}{\lambda} \mu'h \cos \theta' = 2m\pi$

$$\Rightarrow 2\mu'h \cos \theta' = m\lambda : m = 0, 1, 2, \dots$$

Dark fringe: $\frac{4\pi}{\lambda} \mu'h \cos \theta' = (2n+1)\pi$

$$\Rightarrow 2\mu'h \cos \theta' = (2n+1)\frac{\lambda}{2} : n = 0, 1, 2, \dots$$

The conditions are just reversed as that with the reflected rays.

5. Wedge-Shaped Film: If there is a nonzero angle θ_0 between the planes π_1 and π_2 the ^{path} phase difference between the relevant reflected rays can be shown to be equal to

$$\Delta = 2\mu'h \cos(\theta' - \theta_0)$$

by considering the following diagram.

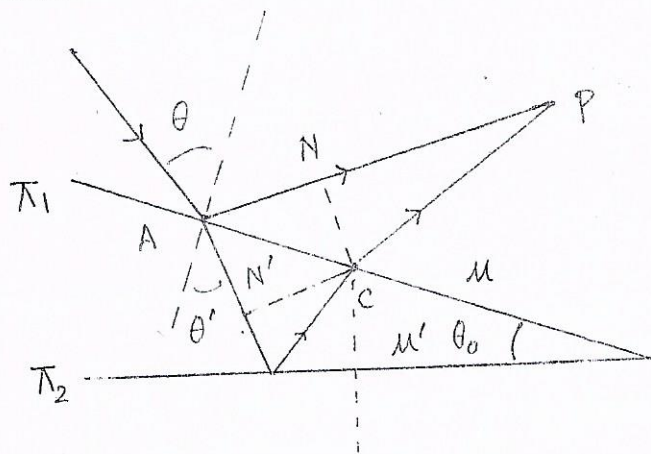


Diagram-7

Taking phase change due to reflection into account we get the following conditions:

$$\text{Bright fringe: } 2\mu'h \cos(\theta' - \theta_0) = (2m+1) \frac{\lambda}{2}$$

$$\text{Dark fringe: } 2\mu'h \cos(\theta' - \theta_0) = p\lambda$$

a. For small wedge-angle $\cos(\theta' - \theta_0)$ can be averaged over the points of the source which contribute light to P. $\langle \cos(\theta' - \theta_0) \rangle$ remains fixed in that case and the fringes are of equal thickness.

b. For $h \rightarrow 0$, the path difference $= \lambda/2$, the film surface will be perfectly dark even with the white light.

c. When the white light is incident on the film λ , θ' and μ' will be different for different colours of light. So at a particular point all the wave lengths may not satisfy the condition of maxima and minima. Hence some of the colours may be absent in the reflected beam.

• C. Newton's Ring

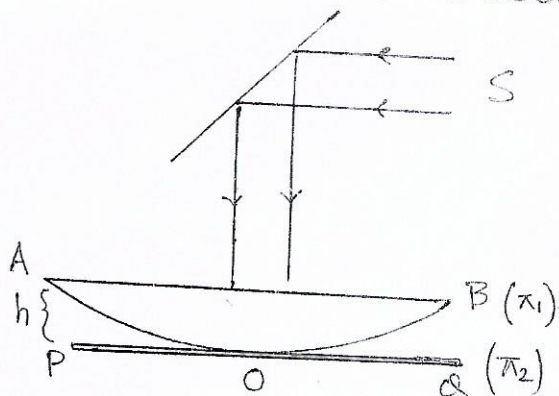


Diagram-10

A plano-convex lens AOB is placed on a glass plate PQ. We are considering the interference due to thin film (air) captured between the surface π_1 (curved) surface of the lens AOB and the surface π_2 (glass plate PQ) is a wedge-shaped region and interference is achieved by division of

amplitude. The light reflected from the surface AOB and that reflected from the surface PQ will therefore interfere as path difference develops due to

(i) Extra path traversed by the ray reflected by the surface PQ.

(ii) A phase difference that develops due to the reflection from optically denser medium.

Neglecting the wedge angle, the path difference

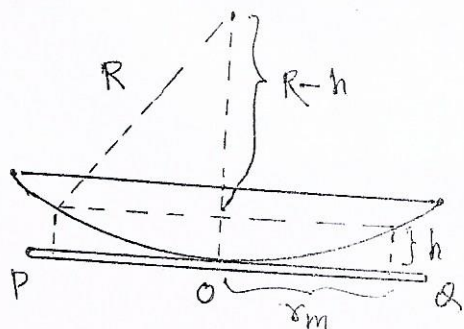
$$\text{Bright fringe} \Rightarrow 2\mu'h = (2m+1) \frac{\lambda}{2} \quad | m = 0, 1, \dots$$

$$\text{Dark fringe} \Rightarrow 2\mu'h = m\lambda \quad | m = 0, 1, 2$$

• Remark : 1. As the thickness at the point of contact is zero, the central fringe will be ~~zero~~ dark.

2. Since, the convex side of the lens is a spherical surface, the thickness of the air film will be constant over a circle. As a result concentric dark and bright fringes will be available.

3. The radius of the m -th fringe can be calculated by the following consideration.



Let, r_m be the radius of the m -th fringe and h be the thickness of the film at a distance r_m from the point of contact O. From geometrical consideration,

$$r_m^2 = R^2 - (R-h)^2 = h(2R-h)$$

$$\Rightarrow r_m \approx \sqrt{2Rh}$$

Now for m -th bright fringe $2\mu' h_c = (2m+1) \frac{\lambda}{2}$

$$\Rightarrow 2\mu' \frac{r_m^2}{2R} = (2m+1) \frac{\lambda}{2}$$

$$\Rightarrow r_m^2 = \frac{2m+1}{2\mu'} \lambda R$$

Similarly for m -th dark fringe

$$r_m^2 = \frac{m \lambda R}{\mu'}$$

a. The above calculation shows that the radii of the ring vary as square root of natural numbers. This fact reflects into the observation that the rings appear close to each other with radius increasing.

b. The difference in square of the radii of m -th and $(m+p)$ -th fringes is

$$r_{m+p}^2 - r_m^2 = p \lambda R, (\mu' = 1)$$

$$\Rightarrow \lambda = \frac{r_{m+p}^2 - r_m^2}{pR}$$

$$\Rightarrow \lambda = \frac{D_{m+p}^2 - D_m^2}{4pR} \quad (D = \text{diameter})$$

c. Gradual lifting of the lens along the upward direction will result in the collapse of the fringes. An upward shift by an amount $\lambda/4$ will result cause the next fringe to occupy the position of the previous fringe of smaller radius.

d. There can be available a pattern due to the transmitted light but their visibility status is very low.

e. If white light is used in Newton's ring experiment the central fringe will be dark as usual, but away from centre, the pattern from different monochromatic components of the source become increasingly out of step — coloured ring will be observed in a characteristic sequence known as Newton's colour.

1, 2, 3, 4, 5, 7, 8, 9
11, 12, 13, 14, 15,
17, 18, 20, 21, 21,
23, 24, 25, 26,
28, 29, 30, 31, 33
35, 36, 39, 41, 42,
43, 44, 45, 46, 47,
49, 50, 51, 54,
53, 54, 56, 57,
58, 59, 60, 61, 62,
63, 64, 65, 68