Module 1.

1) $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$. —(i)

Diff w.r.t to x

$$\frac{2x}{a^2+\lambda} + \frac{2y}{b^2+\lambda} y' = 0 \qquad \left[y' = \frac{dy}{dx}\right]$$

$$\Rightarrow \frac{x(b^2+\lambda) + yy'(a^2+\lambda)}{(a^2+\lambda)(b^2+\lambda)} = 0.$$

$$\Rightarrow \lambda[x + yy'] + a^2yy' + b^2x = 0$$

$$\Rightarrow \lambda = -\frac{[a^2yy' + b^2x]}{yy'+x} \quad \text{- Putting in (i)}$$

$$\Rightarrow \frac{x^2}{a^2 - \left(\frac{a^2yy'+b^2x}{yy'+x}\right)} + \frac{y^2}{b^2 - \frac{(a^2yy'+b^2x)}{(yy'+x)}} = 1$$

$$\Rightarrow \frac{x^2(yy'+x)}{a^2yy'+a^2x-a^2yy'-b^2x} + \frac{y^2(yy'+x)}{b^2yy'+b^2x-a^2yy'-b^2x} = 1$$

$$\Rightarrow \frac{x(yy'+x)}{a^2-b^2} + \frac{y(yy'+x)}{y'(b^2-a^2)} = 1$$

$$\Rightarrow (a^2-b^2)y' = (yy'+x)(xy'-y) \quad \text{[Ans]}$$

2) $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2} = r\frac{d^2y}{dx^2}$

Eqn of a circle of radius r :

$(x-a)^2+(y-b)^2 = r^2 \ldots (1)$

Diff. w.r.t to x.

$\Rightarrow 2(x-a)+2(y-b)y_1' = 0$

$\Rightarrow (x-a)+(y-b)y_1' = 0 \quad -(2)$

Diff w.r.t. x

$\Rightarrow 1+(y-b)y_2+y_1^2 = 0.$

$\Rightarrow (y-b) = -\frac{(1+y_1^2)}{y_2} \quad ---(3)$

Putting in (2)

$(x-a) = \frac{(1+y_1^2)}{y_2}\cdot y_1 \quad ---(4)$

Putting (3) & (4) in (1)

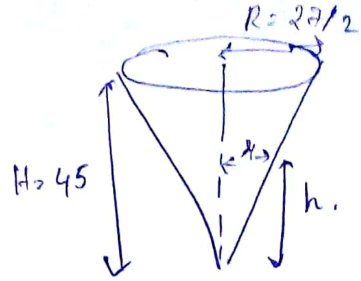
$\frac{y_1^2}{y_2^2}(1+y_1)^2 + \frac{(1+y_1^2)^2}{y_2^2} = r^2$

$\Rightarrow (1+y_1^2)^2\cdot[1+y_1^2] = r^2\cdot y_2^2$

$\Rightarrow (1+y_1^2)^3 = (r\cdot y_2)^2$

$\Rightarrow \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2} = r\cdot\frac{d^2y}{dx^2}$

3) From the problem and the diagram,



we see that.

$$\frac{r}{R} = \frac{h}{H}.$$

$$\Rightarrow r = \left(\frac{27}{2}\right) \cdot \frac{h}{45} = 0.3h.$$

$$r = 0.3h.$$

Now, volume of water ; $V = \frac{1}{3}\pi r^2 h$

$$V = \frac{1}{3}\pi (0.3h)^2 h.$$

$$\Rightarrow V = 0.03\pi h^3$$

Differentiating w.r.to t

$$\frac{dV}{dt} = 0.03\pi (3h^2)\frac{dh}{dt}.$$

$$\therefore \left.\frac{dV}{dt}\right|_{h=30} = 0.03 \times \pi \times 3 \times (30)^2 \times \frac{dh}{dt}$$

$$\Rightarrow 11 = 0.03 \times \pi \times 3 \times (30)^2 \times \frac{dh}{dt}.$$

$$\Rightarrow \frac{dh}{dt} = 0.0432 \text{ cm/min}.$$

4) $(e^x \sin y + e^{-y})dx + (e^x \cos y - xe^{-y})dy = 0.$

Let $M = e^x \sin y + e^{-y}$ and $N = e^x \cos y - xe^{-y}$

Now

$\frac{\partial M}{\partial y} = e^x \cos y - e^{-y}$, $\frac{\partial N}{\partial x} = e^x \cos y - e^{-y}.$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, hence this a exact diff. equation.

So the solution is,

$\int M dx + \int (N \text{ without any terms of } x) dy = c$ · (where c is arbitary constant)

$\therefore \int (e^x \sin y + e^{-y}) dx + \int 0 \, dy = c$

$\Rightarrow \int e^x \sin y \, dx + \int e^{-y} dx = c.$

$\Rightarrow e^x \sin y + xe^{-y} = c.$

5). $(y^2 e^{xy^2} + 4x^3) \cdot dx + (2xy e^{xy^2} - 3y^2) dy = 0.$

Let $M = y^2 e^{xy^2} + 4x^3$ and $N = 2xy e^{xy^2} - 3y^2$

$\frac{\partial M}{\partial y} = 2y e^{xy^2} + y^2 e^{xy^2} \times 2xy$

$= 2y e^{xy^2} + 2y^3 x e^{xy^2}$

$\frac{\partial N}{\partial x} = 2y e^{xy^2} + 2xy e^{xy^2} \times y^2$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Solution is given by

$\int (y^2 e^{xy^2} + 4x^3) dx + \int -3y^2 dy = c$

$= \frac{\cancel{y^2} \times e^{xy^2}}{\cancel{y^2}} + \frac{\cancel{4} x^4}{\cancel{4}} - \frac{\cancel{3} y^3}{\cancel{3}} = c$

$= e^{xy^2} + x^4 - y^3 = c.$

6) $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$

Let $M = 3x^2y^4 + 2xy$, $N = 2x^3y^3 - x^2$

$\frac{\partial M}{\partial y} = 12x^2y^3 + 2x$, $\frac{\partial N}{\partial x} = 6x^2y^3 - 2x$.

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ ∴ Not exact.

here
$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = f(y).$$

$$\Rightarrow \frac{12x^2y^3 + 2x - 6x^2y^3 + 2x}{3x^2y^4 + 2xy}$$

$$\Rightarrow \frac{2}{y}$$

$$\therefore \text{I.F} = e^{-\int \frac{2}{y} dy} = e^{-2\log y} = 1/y^2$$

To make the equation exact. multiply the I.F on both sides.

$$\left(3x^2y^2 + \frac{2x}{y}\right)dx + \left(2x^3y - \frac{x^2}{y^2}\right)dy = 0$$

So the solution

$$\int 3x^2y^2 + \frac{2x}{y}\, dx = c.$$

$$\Rightarrow x^3y^2 + \frac{x^2}{y} = c$$

7) $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$

The above equation is homogeneous with degree 3. and not homogeneous.
So the Integrating factor will be. I.F $= \frac{1}{Mx + Ny}$

$M = x^2y - 2xy^2$, $N = 3x^2y - x^3$

$$\text{I.F} = \frac{1}{x^3y - 2x^2y^2 + 3x^2y^2 - x^3y} = \frac{1}{x^2y^2}$$

To make the equation exact.

$$\int\left(\frac{x^2y}{x^2y^2}-\frac{2xy^2}{x^2y^2}\right)dx+\int\left(\frac{3x^2y}{x^2y^2}-\frac{x^3}{x^2y^2}\right)dy=c.$$

$$\Rightarrow \int\left(\frac{1}{y}-\frac{2}{x}\right)dx+\int\frac{3}{y}\,dy=c \quad \rightarrow \text{solution}$$

$$\Rightarrow \frac{x}{y}-2\log x+3\log y=c.$$

$$\Rightarrow \frac{x}{y}+\log\frac{y^3}{x^2}=c$$

8) $(x^2+y^2+2x)dx+2y\,dy=0$

given e^x is an I.F

$\therefore$ the exact equation is

$$(e^x x^2+e^x y^2+2e^x x)\,dx+2e^x y\,dy=0.$$

for this $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

$$\Rightarrow \frac{\partial M}{\partial y}=2ye^x \;,\quad \frac{\partial N}{\partial x}=2e^x y$$

$\therefore$ Proved e^x is an I.F

the solution is

$$\int(e^x x^2+e^x y^2+2e^x x)\,dx=0$$

$$\Rightarrow \int e^x(x^2+y^2+2x)\,dx=0$$

$$\Rightarrow (x^2+y^2+2x)e^x-\int(2x+1)e^x=c$$

$$\Rightarrow (x^2+y^2+2x)e^x-\cancel{(x^2}+x\left(e^x(2x+1)-\int(x^2+x)e^x\right.$$

9) $\{xy\sin(xy)+\cos(xy)\}y\,dx+\{xy\sin(xy)-\cos(xy)\}x\,dy=0.$

So Let

$M = \{xy\sin(xy)+\cos(xy)\}y$

$N = \{xy\sin(xy)-\cos(xy)\}x$

$\frac{\partial M}{\partial y} = \{xy\sin(xy)+\cos(xy)\}+\{x\sin(xy)+x^2y\cos(xy)\}$

$\oplus = \sin xy \times x$

$= xy\sin(xy)+\cos(xy)+x^2y\cos(xy).$

$\frac{\partial N}{\partial x} = \{xy\sin(xy)-\cos(xy)\}+\{y\sin xy+y^2x\cos xy\}-\sin(xy)y$

$= xy\sin xy+xy^2\cos xy-\cos(xy).$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

The equation is in a form of $yf(xy)dx+xf(xy)dy=0.$

$\therefore$ the I.F is $\frac{1}{Mx-Ny}$

$I.F = \frac{1}{2xy\cos xy}$

Now the exact equation,

$y\int\frac{\tan xy}{2}dx+\int\frac{1}{2x}dx+\int\cancel{\tan(xy)\,x\,dy}-\int\frac{1}{2y}dy=c.$

$\Rightarrow \int y\tan xy\,dx+\int\frac{dx}{x}-\int\frac{dy}{y}=c'$

$xy=v.$ $\Rightarrow dv = y\,dx$

$\Rightarrow y+x\frac{dy}{dx}=\frac{dv}{dx}$

$\Rightarrow dxy+x\,dy=dv$

$\Rightarrow y\,dx = dv - x\,dy$

$\int\tan v\,dv+\int\frac{dx}{dx}-\int\frac{dy}{y}=c'$

$\Rightarrow \log|\sec v|+\log|x|-\log|y|=c'$

$\Rightarrow \log|\sec(xy)|+\log|x|-\log|y|=c'$

10) $(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$

$M = 2xy^4e^y + 2xy^3 + y$

$\frac{\partial M}{\partial y} = 2x*4y^3e^y + 2xe^y y^4 + (2x*3y^2) + 1$

$= 8xy^3e^y + 2xe^y y^4 + 6xy^2 + 1$

$N = x^2y^4e^y - x^2y^2 - 3x.$

$\frac{\partial N}{\partial x} = 2xy^4e^y - 2xy^2 - 3$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{4}{y}\frac{(2xy^3e^y + 2xy^2 + 1)}{(2xy^3e^y + 2xy^2 + 1)}$

$= 4/y.$

$I.F = e^{-\int 4/y\,dy}$

$= 1/y^4.$

The exact equation is

$\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \left(x^2e^y - \frac{x^2}{y^2} - \frac{3}{x^3}\right)dy = 0.$

The solution then can be given by.

$\int\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \int 0\,dy = c.$

$\Rightarrow x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = c!$

11) $(xy^2 - e^{1/x^3})\,dx - x^2y\,dy = 0$

$M = xy^2 - e^{1/x^3}.$ $\quad N = -x^2y$

$\frac{\partial M}{\partial y} = 2xy, \quad \frac{\partial N}{\partial x} = -2xy$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = -\frac{4xy}{x^2 y} = -\frac{4}{x}$$

$$I.F = e^{\int -4/x\, dx}$$

$$= 1/x^4$$

∴ The exact equation,

$$\left(\frac{y^2}{x^3} - \frac{e^{1/x^3}}{x^4}\right).dx - \frac{y}{x^2}dy = 0.$$

The solution

$$\int M dx + \int N (\text{with no terms of } x)\, dy = c.$$

$$\Rightarrow \int\left(\frac{y^2}{x^3} - \frac{e^{1/x^3}}{x^4}\right)dx = c.$$

$$\Rightarrow \frac{y^2}{2x^2} - \frac{1}{3}e^{1/x^3} = c$$

2) $(xy^3+y)dx + 2(x^2y^2+x+y^4)dy = 0$

$$\frac{\partial M}{\partial y} = 3xy^2+1, \quad \frac{\partial N}{\partial x} = 2(2xy^2+1)$$
$$= 4xy^2+2.$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{-xy^2-1}{xy^3+y} = -\frac{1}{y}\frac{(xy^2+1)}{(xy^2+1)} = -1/y$$

$$I.F = e^{-\int -1/y\, dy}$$

$$= y.$$

Exact diff equation.

$$(xy^4+y^2)dx + 2(x^2y^3+xy+y^5)dy = 0$$

Solution:-

$$\int xy^4+y^2\, dx + 2\int y^5 dy = c$$

$$\Rightarrow \frac{x^2y^4}{2} + xy^2 + \frac{y^6}{3} = c$$

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13) $y(xy + 2(x^2y^2))dx + x(xy - x^2y^2)dy = 0$

$$\frac{\partial M}{\partial x} = (xy + 2x^2y^2) + y(x + 2x^2 \cdot 2y)$$
$$= 2xy + 6x^2y^2$$

$$\frac{\partial N}{\partial x} = (xy - x^2y^2) + x(y - 2xy^2)$$
$$= 2xy - 3x^2y^2$$

$$I.F = \frac{1}{Mx - Ny}$$
$$= \frac{1}{(xy)^2 + 2(xy)^3 - (xy)^2 + (xy)^3}$$
$$= \frac{1}{3(xy)^3}$$

For exact equation.

$$\int y\left(\frac{xy}{3(xy)^3} + \frac{2}{3}\frac{(xy)^2}{(xy)^3}\right)dy + x\left(\frac{xy}{3(xy)^3} - \frac{(xy)^2}{3(xy)^3}\right)dy = 0$$

$$\Rightarrow \left(\frac{y}{(xy)^2} + \frac{2}{x}\right)dx + \left(\frac{x}{(xy)^2} - \frac{1}{y}\right)dy = c'$$

Solution :-

$$\Rightarrow \int \frac{y\,dx}{(xy)^2} + \int \frac{2}{x}\,dx - \int \frac{1}{y}\,dy = c'$$

$$\Rightarrow -\frac{1}{y}\frac{1}{x} + 2\log x - \log y = c'$$

$$\Rightarrow -\frac{1}{xy} + \log \frac{x^2}{y} = c'$$

14) $x^2(2y\,dx + 3x\,dy) + y^2(-2y\,dx + 2x\,dy) = 0$

$\Rightarrow 2x^2y\,dx - 2y^3dx + 3x^3dy + 2xy^2dy = 0$

$\Rightarrow 2(x^2y - y^3)\,dx + (3x^3 + 2xy^2)\,dy$

$\frac{\partial M}{\partial y} = 2x^2 - 6y^2 \quad \Big| \quad \frac{\partial N}{\partial x} = 9x^2 + 2y^2$

$\cancel{\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{-7x^2 - 8y^2}{3x^3 + 2xy^2}}$

$\cancel{= -\frac{1}{x}\left(\frac{7x^2 + 8y^2}{3x^2 + 2y^2}\right)} \times$

I.F as homogenous and $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\therefore$ I.F $= \frac{1}{Mx + Ny} = \frac{1}{2x^3y - 2xy^3 + 3x^3y + 2xy^3}$

$= \frac{1}{5x^3y}$

The exact equation,

$\frac{2}{5}\left(\frac{1}{x} - \frac{y^2}{x^3}\right)dx + \frac{1}{5}\left(\frac{3}{y} + \frac{2y}{x^2}\right)dy = 0$

The solution

$\frac{2}{5}\int\left(\frac{1}{x} - \frac{y^2}{x^3}\right)dx + \frac{3}{5}\int\frac{1}{y}\,dy = c$

$= \frac{2}{5}\log|x| + \frac{y^2}{5x^2} + \frac{3}{5}\log|y| = c'$

$\Rightarrow \log(x^2 * y^3) + \frac{y^2}{x^2} = c'$

$\Rightarrow \log(x^2y^3) + \frac{y^2}{x^2} = c'$

15) $(y^2e^x + 2xy)\,dx - x^2 dy = 0$

$$\frac{\partial M}{\partial y} = 2ye^x + 2x, \quad \frac{\partial N}{\partial x} = -2x.$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

Now,

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{2(ye^x + 2x)}{y(ye^x + 2x)} = 2/y.$$

$$I.F. = e^{-\int 2/y\, dy}$$

$$= 1/y^2.$$

Exact equation

$$\left(e^x + \frac{2x}{y}\right)dx - \frac{x^2}{y^2}dy = 0$$

Solution,

$$\int\left(e^x + \frac{2x}{y}\right)dx = c$$

$$\Rightarrow e^x + \frac{x^2}{y} = c$$

16) $(x^3y^2 + xy)\,dx - dy = 0.$

$M = x^3y^2 + xy.$ | $N = -1$

$\frac{\partial M}{\partial y} = 2x^3y + x$ | $\frac{\partial N}{\partial x} = 0.$

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{2x^3y + x}{x^3y^2 + xy}$$

$$= \frac{2x^2y + 1}{x^2y^2 + y}$$

$$= \frac{2x^2y + 1}{y(x^2y + 1)}$$

17) $\frac{dy}{dx} + \frac{y \ln y}{x} = y\frac{(\ln y)^2}{x^2}$

$\Rightarrow \frac{1}{y(\ln y)^2}y_1 + \frac{1}{\ln y}\frac{1}{x} = \frac{1}{x^2}$

$z = \frac{1}{\ln y}$

$\frac{dz}{dx} = -\frac{1}{(\ln y)^2}\frac{1}{y}\frac{dy}{dx}$

$\Rightarrow -\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$

$\Rightarrow \frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$

$I.F = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = 1/x$

Solution is

$\frac{z}{x} = \int -\frac{1}{x^2} \times \frac{1}{x}\,dx + c$

$\Rightarrow \frac{z}{x} = -\int \frac{dx}{x^3} + c.$

$\Rightarrow \frac{z}{x} = \frac{1}{2x^2} + c$

$\Rightarrow \frac{1}{(\ln y)x} = \frac{1}{2x^2} + c.$

18) $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$

$\Rightarrow \cos y\frac{dy}{dx} - \frac{\sin y}{1+x} = (1+x)e^x$

$z = \sin y.$

$\frac{dz}{dx} = \cos y\frac{dy}{dx}$

$\Rightarrow \frac{dz}{dx} - \frac{z}{1+x} = (1+x)e^x$

$I.F = e^{\int -\frac{dx}{1+x}} = e^{-\log(1+x)} = \frac{1}{1+x}$

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Solution

$$\frac{z}{1+x} = \int \frac{(1+x)e^x}{(1+x)} dx + C$$

$$\Rightarrow \frac{z}{1+x} = e^x + C.$$

$$\Rightarrow \frac{\sin y}{1+x} = e^x + C$$

19) $\{y(1-x\tan x)+x^2\cos x\}dx - xdy = 0$

$$\Rightarrow y(1-x\tan x)+x^2\cos x = x\frac{dy}{dx}$$

$$\Rightarrow y - xy\tan x + x^2\cos x = \frac{xdy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - y\tan x + x\cos x$$

$$\Rightarrow \frac{dy}{dx} - y\left(\frac{1}{x} - \tan x\right) = x\cos x$$

$$\text{I.F} = e^{-\int\left(\frac{1}{x} - \tan x\right)dx}$$

$$= e^{\ln\left|\frac{\sec x}{x}\right|}$$

$$= \frac{\sec x}{x}$$

Solution

$$y \times \frac{\sec x}{x} = \int 1\,dx + C.$$

$$\Rightarrow \frac{y}{x}\sec x = x + C.$$

$$\Rightarrow y\sec x = x^2 + Cx.$$

$$\Rightarrow y = x^2\cos x + Cx\cos x$$

20) $(x+2y^3)\frac{dy}{dx} = y.$

$\Rightarrow \left(\frac{x}{y} + 2y^2\right)\frac{dy}{dx} = 1.$

$\Rightarrow \frac{x}{y} + 2y^2 = \frac{dx}{dy}$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y^2.$

$I.F = e^{\int -\frac{1}{y}dy}$

$= e^{-\ln y}$

$= 1/y.$

Solution

$\frac{x}{y} = \int 2y\,dy + C.$

$\Rightarrow x/y = y^2 + c.$

21) $(1+x)\cos y\frac{dy}{dx} - \sin y = (1+x)^2 e^x$

$\Rightarrow \frac{dy}{dx} - \frac{\tan y}{1+x} = e^x \sec y\,(1+x)$

$\Rightarrow \cos y\frac{dy}{dx} - \frac{\sin y}{1+x} = e^x(1+x)$

$z = \sin y$

$\frac{dz}{dx} = \cos y\frac{dy}{dx}.$

$\frac{dz}{dx} - \frac{z}{1+x} = \frac{e^x}{\cancel{1+x}}(1+x)$

$I.F = e^{-\int\frac{dx}{1+x}}$

$= e^{-\log(1+x)}$

$= \frac{1}{1+x}$

$\frac{z}{1+x} = \int e^x\,dx + C,$

$\sin y = (1+x)(e^x + c).$

22) $\sin y \frac{dy}{dx} = \cos x \left(2\cos y - \sin^2 x \right)$

$\Rightarrow \sin y \frac{dy}{dx} - 2\cos x \cos y = -\sin^2 x \cos x$

$z = \cos y$

$\frac{dz}{dx} = -\sin y \frac{dy}{dx}$

$\Rightarrow -\frac{dz}{dx} - 2\cos x \, z = -\sin^2 x \cos x$

$\Rightarrow \frac{dz}{dx} + 2\cos x \, (z) = \sin^2 x \cos x$

$I.F = e^{\int 2\cos x \, dx}$

$= e^{2\sin x}$

Solution

$z e^{2\sin x} = \int e^{2\sin x} \sin^2 x \cos x$

Let $2\sin x = t$

$\Rightarrow 2\cos x \, dx = dt$

$\Rightarrow z e^{2\sin x} = \int e^t \frac{t^2}{4} \frac{dt}{2}$

$\Rightarrow z e^{2\sin x} = \frac{1}{8} \int e^t t^2 dt + C$

$\Rightarrow 8 z e^{2\sin x} = \left[t^2 e^t - \int 2t e^t dt \right] + C$

$\Rightarrow 8 z e^{2\sin x} = \left[t^2 e^t - \left[2t e^t + 2\int e^t dt \right] \right] + C$

$\Rightarrow 8 z e^{2\sin x} = \left[4\sin^2 x \, e^{2\sin x} - \left[4\sin x \, e^{2\sin x} - 2 e^{2\sin x} \right] \right] + C'$

$\Rightarrow 4 yiny = 2\sin^2 x - 2\sin x + 1 + C' e^{-2\sin x}$

23) $\frac{dy}{dx} + y = y^3(\cos x - \sin x)$

$\Rightarrow y^{-3}\frac{dy}{dx} + \frac{1}{y^2} = \cos x - \sin x$

Let

$z = \frac{1}{y^2}$

$\Rightarrow -2\frac{1}{y^3}\frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow -\frac{1}{2}\frac{dz}{dx} + z = \cos x - \sin x$

$\Rightarrow \frac{dz}{dx} - 2z = -2(\cos x - \sin x)$

$I.F = e^{\int -2dx}$
$= e^{-2x}$

Solution,

$z\, e^{-2x} = \int -2 \times e^{-2x}(\cos x - \sin x)\,dx + c.$

$\Rightarrow z\, e^{-2x} = -2\int e^{-2x}(\cos x - \sin x)\,dx + c$

Let $I = -2\int e^{-2x}(\cos x - \sin x)\,dx$

$\Rightarrow I = -2\left[(\cos x - \sin x)\frac{e^{-2x}}{-2} - \int(-\sin x - \cos x)\frac{e^{-2x}}{-2}\,dx\right]$

$\Rightarrow I = e^{-2x}(\cos x - \sin x) + \int e^{-2x}(\sin x + \cos x)\,dx$

$I = \frac{2}{5}e^{-2x}(\cos x - 3\sin x)\ \cancel{+c}.$

$\therefore$ Putting values of z and I

$\frac{e^{-2x}}{y^2} = \frac{2}{5}e^{-2x}(\cos x - 3\sin x) + c$

$\Rightarrow \frac{1}{y^2} = \frac{2}{5}(\cos x - 3\sin x) + Ce^{2x}$

24) $x\frac{dy}{dx} + y = y^2 \log x$

$\Rightarrow y^{-2}\frac{dy}{dx} + \frac{1}{yx} = \frac{\log x}{x}$

$\Rightarrow z = 1/y$

$\Rightarrow \frac{dz}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$

$-\frac{dz}{dx} + \frac{z}{x} = \frac{\log x}{x}$

$\Rightarrow \frac{dz}{dx} - \frac{z}{x} = -\frac{\log x}{x}$

$I.F = e^{\int -\frac{1}{x}dx}$

$= 1/x$

$\frac{z}{x} = -\int \frac{\log x}{x} dx$

$\Rightarrow \frac{z}{x} = -\int t e^{-t} dt$

$\Rightarrow \frac{z}{x} = -\left[t\frac{e^{-t}}{-1} - \int \frac{e^{-t}}{-1} dt \right]$

$\Rightarrow \frac{z}{x} = -\left[-t e^{-t} + \int e^{-t} dt \right]$

$\Rightarrow \frac{z}{x} = \left[t e^{-t} + e^{-t} \right] + c$

$\Rightarrow \frac{1}{xy} = \frac{1}{x}\left[\log x + 1 \right] + c$

$\Rightarrow y^{-1} = [\log x + 1] + cx$

25) $dr + (2r\cot\theta + \sin 2\theta)d\theta = 0.$

$\Rightarrow (2r\cot\theta + \sin 2\theta) = -\frac{dr}{d\theta}.$

$\Rightarrow \frac{dr}{d\theta} + 2r\cot\theta = -\sin 2\theta.$

$I.F = e^{\int 2\cot\theta\, d\theta}$

$= e^{2\log_e \sin\theta}.$

$= \sin^2\theta.$

Solution,

$r\sin^2\theta = -\int \sin^2\theta \sin 2\theta\, d\theta. + C.$

$\Rightarrow r\sin^2\theta = -2\int \sin^3\theta\cos\theta\, d\theta + C.$

$\Rightarrow r\sin^2\theta = -2\int t^3 dt + C.$

$\Rightarrow r\sin^2\theta = -2\frac{t^4}{2} + C.$

$\Rightarrow r\sin^2\theta = -\frac{\sin^4\theta}{2} + C.$

26) $p^2 + 2xp - 3x^2 = 0$

$p^2 + 3xp - xp - 3x^2 = 0$

$\Rightarrow p(p+3x) - x\cdot(p+3x) = 0$

$\Rightarrow (p-x)(p+3x) = 0$

$p = x, p = -3x$

$\frac{dy}{dx} = x$

$\Rightarrow \int dy = \int x\,dx$

$y = \frac{x^2}{2} + c$

$\frac{dy}{dx} = -3x$

$\int dy = -3\int x\,dx$

$y = -3\frac{x^2}{2} + c$

Solution,

$\therefore (2y - x^2 - 2c)(2y + 3x^2 - 2c) = 0.$