Black-body Radiation.

Wien's displacement law :-

With the increase of temp. of black body, the max. ~~radi~~ intensity of radiation shift towards the shorter wavelength region.

$$\lambda_m \cdot T = \text{constant}$$

$$\Rightarrow \boxed{\lambda_m \propto \frac{1}{T}}$$

or, $\boxed{\lambda_m \propto T}$

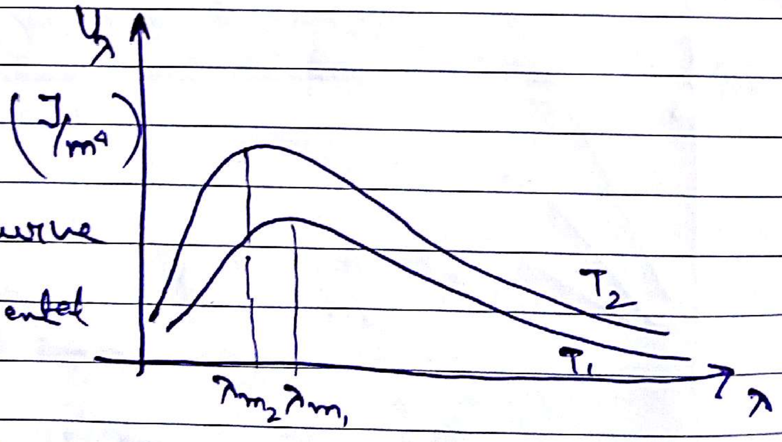
Theoretical Explanation :-

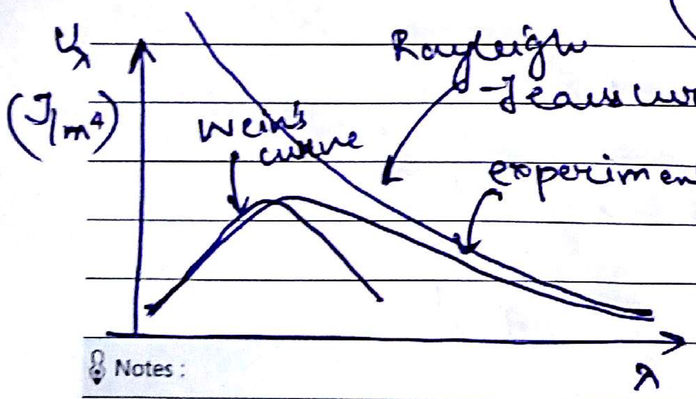
Wien's energy distribution law :-

$$\boxed{U_\lambda \, d\lambda = \frac{a}{\lambda^5} \exp\left(-\frac{b}{\lambda T}\right) d\lambda}$$

Remember

$$\boxed{E = \frac{1}{4} u \times c}$$





Notes :

Rayleigh - Jeans

$$U_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

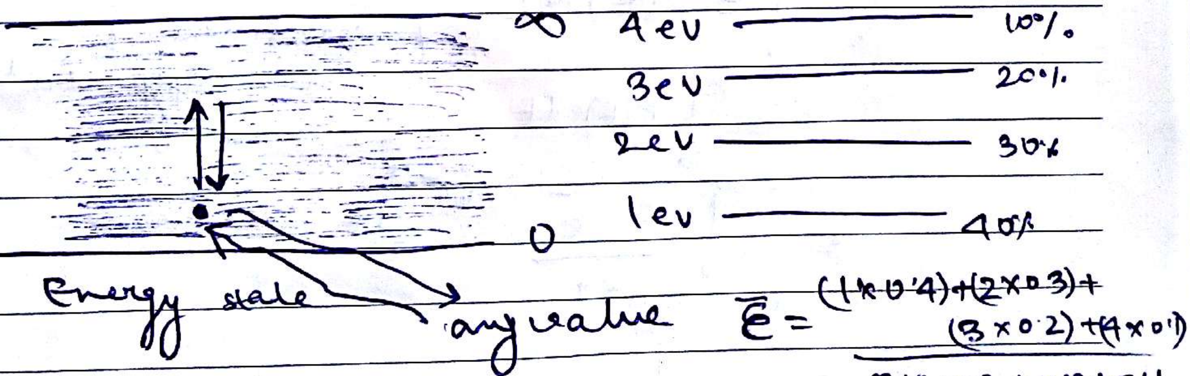
Classical explanation according to Rayleigh - Jeans

The atom of the black body, at a given temp, behaves like a simple harmonic oscillator, each having characteristic frequency of oscillation. The no. of oscillator per unit volume within the frequency range ν to $\nu + d\nu$ is:-

$$N(\nu)\, d\nu = \frac{8\pi\nu^2}{c^3} d\nu$$

$$U_\nu d\nu = N(\nu)\, d\nu \times \bar{\varepsilon}$$

According to classical concept, the energy scales are continuous i.e. each oscillator can absorb or emit any value of energy.



$$\bar{\varepsilon} = \frac{(1\times0.4)+(2\times0.3)+(3\times0.2)+(4\times0.1)}{0.4+0.3+0.2+0.1}$$

Notes :

Let $P(E)$ be the probability of finding the particle in ε energy state.

According to Boltzmann distribution $P(E) \propto e^{-\varepsilon/KT}$

$$\therefore P(\varepsilon) = ce^{-\varepsilon/KT}$$

$$\Rightarrow \int_0^\infty P(\varepsilon)\, d\varepsilon = \int_0^\infty ce^{-\varepsilon/KT}\, d\varepsilon \qquad \frac{\varepsilon}{KT} = z.$$

$$\Rightarrow 1 = c(KT)\int_0^\infty e^{-z}\, dz.$$

$$\Rightarrow 1 = c(KT)\,\Gamma(1) \qquad \int_0^\infty e^{-x} x^{n-1}\, dx = \Gamma(n)$$

$$\Rightarrow c = \frac{1}{KT} \qquad \Gamma(1) = 1$$

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(n+1) = n\Gamma(n)$$

The avg. energy of the oscillator, within the ~~energy~~ freq. range ν to $\nu + d\nu$ is

$$\bar{\varepsilon} = \frac{E_1 P(E_1) + E_2 P(E_2) + \cdots}{P(E_1) + P(E_2) + \cdots}$$

$$= \frac{\int_0^\infty E\, P(E)\, dE}{\int_0^\infty P(E)\, dE} \quad \left(\text{continuous energy state}\right)$$

$$= \int_0^\infty \frac{E\, e^{-E/KT}}{KT}\, dE$$

$$\Rightarrow \bar{\varepsilon} = \frac{1}{KT}\int_0^\infty e^{-z} z\, dz \, (KT)^2 = 1\,(KT) = KT$$

WK	S	M	T	W	T	F	S
40	1	2	3	4	5	6	7
41	8	9	10	11	12	13	14
42	15	16	17	18	19	20	21
43	22	23	24	25	26	27	28
44	29	30	31				

WK	S	M	T	W	T	F	S
44				1	2	3	4
45	5	6	7	8	9	10	11
46	12	13	14	15	16	17	18
47	19	20	21	22	23	24	25
48	26	27	28	29	30		

$\therefore \boxed{\bar{\varepsilon} = kT}$

Now the energy density of the black body radiation at a given temp. T within the frequency range ν to $\nu + d\nu$ is $U_\nu d\nu$

$$U_\nu d\nu = N(\nu)\, d\nu \times \bar{E}$$

$$= \frac{8\pi \nu^2}{c^3} d\nu \times kT$$

$$\boxed{U_\nu d\nu = \frac{8\pi kT \nu^2}{c^3} d\nu}$$

In terms of wavelength;

$$U_\lambda d\lambda = -U_\nu d\nu \qquad [d\nu \uparrow \; d\lambda \downarrow]$$

$$= -\frac{8\pi kT\nu^2}{c^3} d\nu$$

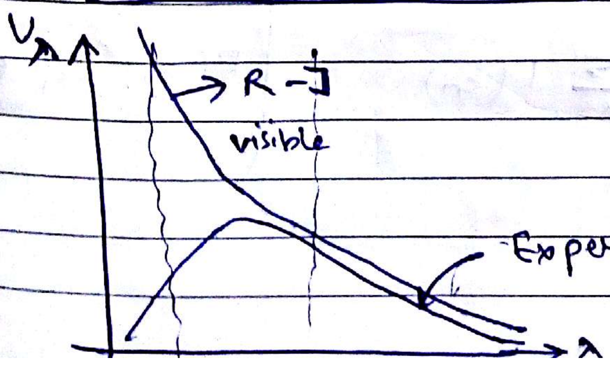
$$\nu = \frac{c}{\lambda} \Rightarrow d\nu = -\frac{c}{\lambda^2} d\lambda$$

$$= \frac{8\pi kT}{c^3} \times \frac{c^2}{\lambda^2} \times \frac{c}{\lambda^2} d\lambda$$

$$\boxed{U_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda}$$

as $\lambda \to 0$ $U_\lambda \to \infty$
but experimentally $\lambda \to 0$ $U_\lambda \to 0$

This discrepancy between theoretical and experimental value in the shorter wavelength region is known as ultraviolet catastrophe.



AUGUST							2017
WK	S	M	T	W	T	F	S
31			1	2	3	4	5
32	6	7	8	9	10	11	12
33	13	14	15	16	17	18	19
34	20	21	22	23	24	25	26
35	27	28	29	30	31		

SEPTEMBER							2017
WK	S	M	T	W	T	F	S
35						1	2
36	3	4	5	6	7	8	9
37	10	11	12	13	14	15	16
38	17	18	19	20	21	22	23
39	24	25	26	27	28	29	30

Planck's Black body Radiation law

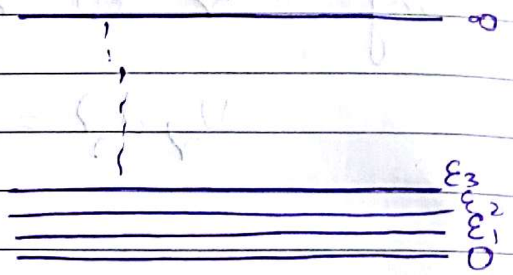
He assumed that :-

(i) The oscillator can have energies which are discrete ie. an integral multiple of finite quanta of energy.

$$\boxed{E_n = nh\nu}$$

where $n = 0, 1, 2, 3, \ldots$

$h = 6.634 \times 10^{-34}$ J/Hz



(ii) The change of energy of the oscillator due to emission and absorption of radiant energy can also take place by a discrete value $h\nu$.

Calculation: The no. of oscillator per unit volume within the frequency range ν to $\nu + d\nu$ is

$$N(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3}\,d\nu$$

~~The average energy of the oscillator having~~

frequency ν,

$$\bar{\varepsilon} = \frac{\varepsilon_0 P(\varepsilon_0) + \varepsilon_1 P(\varepsilon_1) + \varepsilon_2 P(\varepsilon_2) + \cdots}{P(\varepsilon_0) + P(\varepsilon_1) + P(\varepsilon_2) + \cdots}$$

$$= \frac{\sum_{n=0}^{n} \varepsilon_n P(\varepsilon_n)}{\sum_{n=0}^{n} P(\varepsilon_n)}$$

$P(\varepsilon_n) \propto e^{-\varepsilon_n/kT}$

$P(\varepsilon_n) = C_1 e^{-\varepsilon_n/kT}$

Notes :

$$\sum P(\varepsilon_n) = c_1 \sum e^{-\varepsilon_n/kT}$$

$$\Rightarrow 1 = c_1 \sum e^{-\varepsilon_n/kT}$$

$$\Rightarrow c_1 = \frac{1}{\sum e^{-\varepsilon_n/kT}}$$

$$\left. P(\varepsilon_n) = \frac{e^{-\varepsilon_n/kT}}{\sum e^{-\varepsilon_n/kT}} \right]$$

$$\bar{\varepsilon} = \frac{\sum_{n=0}^{n} \varepsilon_n \cdot e^{-\varepsilon_n/kT}}{\sum_{n=0}^{n} e^{-\varepsilon_n/kT}}$$

$$= \frac{\sum_{n=0}^{n} nh\nu \cdot e^{-\frac{nh\nu}{kT}}}{\sum_{n=0}^{n} e^{-\frac{nh\nu}{kT}}}$$

$$\bar{\varepsilon} = \frac{0 + h\nu e^{-\frac{h\nu}{kT}} + 2h\nu e^{-\frac{2h\nu}{kT}} + 3h\nu e^{-\frac{3h\nu}{kT}} + \cdots}{1 + e^{-\frac{h\nu}{kT}} + e^{-\frac{2h\nu}{kT}} + e^{-\frac{3h\nu}{kT}} + \cdots}$$

$$= \frac{h\nu e^{-\frac{h\nu}{kT}} \left[1 + 2e^{-\frac{h\nu}{kT}} + 3e^{-\frac{2h\nu}{kT}} + \cdots \right]}{1 + e^{-\frac{h\nu}{kT}} + e^{-\frac{2h\nu}{kT}} + \cdots}$$

Notes :

$$\left[e^{-\frac{h\nu}{kT}} = x \right] \quad = \frac{h\nu x \left[1 + 2x + 3x^2 + \cdots \right.}{1 + x + x^2 + \cdots}$$

$$= \frac{h\nu \cdot x \,(1-x)^{-2}}{(1-x)^{-1}}$$

$$= h\nu x\,(1-x)^{-1}$$

$$= \frac{h\nu\, x}{1-x}$$

$$= \frac{h\nu}{\frac{1}{x}-1}$$

$$\boxed{\bar{\varepsilon} = \frac{h\nu}{e^{h\nu/KT}-1}}$$

For shorter frequency range

$$e^{h\nu/KT} - 1 \simeq \frac{h\nu}{KT}$$

$$\therefore \bar{\varepsilon} = KT$$

The energy density within the frequency range ν to $\nu+d\nu$ is given by $U_\nu d\nu = N(\nu)\,d\nu \times \bar{\varepsilon}$

$$= \frac{8\pi\nu^2}{c^3}\,d\nu \times \frac{h\nu}{e^{h\nu/KT}-1}$$

Notes:

$$\boxed{U_\nu d\nu = \frac{8\pi h\nu^3}{c^3\left[e^{h\nu/KT}-1\right]}\,d\nu}$$

in terms of wave length

$$U_\lambda d\lambda = - U_\nu d\nu$$

$$= \frac{8\pi h c^3}{c^3 \cdot \lambda^3 \left[e^{\frac{hc}{\lambda kT}} - 1\right]} \cdot \frac{c}{\lambda^2} d\lambda$$

$$\boxed{U_\lambda d\lambda = \frac{8\pi h c}{\lambda^5 \left[e^{\frac{hc}{\lambda kT}} - 1\right]} d\lambda}$$

Planck's $\longrightarrow$ Wein's ~~displacement~~ distribution law.

$$U_\lambda d\lambda = \frac{8\pi h c}{\lambda^5 \left[e^{\frac{hc}{\lambda kT}} - 1\right]} d\lambda$$

For shorter wavelength region,

$$\left[e^{\frac{hc}{\lambda kT}} - 1\right] \simeq e^{\frac{hc}{\lambda kT}}$$

$$U_\lambda d\lambda = \frac{8\pi h c}{\lambda^5 \, e^{\frac{hc}{\lambda kT}}} d\lambda = \frac{a}{\lambda^5} \exp\left(-\frac{b}{\lambda T}\right) d\lambda$$

$$a = 8\pi hc \,;\; b = \frac{hc}{k}$$

Planck's $\longrightarrow$ R-J law

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left[e^{\frac{hc}{\lambda kT}} - 1\right]} d\lambda$$

For longer wavelengths

$$e^{\frac{hc}{\lambda kT}} - 1 \simeq \frac{hc}{\lambda kT}$$

$$\therefore U_\lambda d\lambda = \frac{8\pi \cancel{hc}}{\lambda^{\cancel{5}\,4}\left(\frac{\cancel{hc}}{\cancel{\lambda} kT}\right)} d\lambda$$

$$= \frac{8\pi kT}{\lambda^4} d\lambda \rightarrow \text{R-J's law}$$

Q] K.E = 1 MeV of an electron. Calculate momentum.

$$KE = E - m_0c^2$$

$$\Rightarrow (1 + 0.59)\, MeV = E$$

$$\therefore E^2 = p^2c^2 + (0.59)^2$$

$$\Rightarrow p = 1.47\, MeV/c$$

Notes :

Planck's → Wein's displacement law.

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left[e^{\frac{hc}{\lambda KT}} - 1\right]} d\lambda$$

At $\lambda = \lambda_m$; U_λ is max.

So the denominator will be min.

Let $y = \lambda^5 \left(e^{\frac{hc}{\lambda KT}} - 1\right)$.

$$\left.\frac{dy}{d\lambda}\right|_{\lambda=\lambda_m} = 0 .$$

$$\Rightarrow 5\lambda_m^4 \left(e^{\frac{hc}{\lambda KT}} - 1\right) + \lambda^{\cancel{5}\,3} e^{\frac{hc}{\lambda KT}} \left(-\frac{hc}{\cancel{\lambda^2} KT}\right) = 0 .$$

$$\Rightarrow 5\lambda_m^4 \left(e^{\frac{hc}{\lambda_m KT}} - 1\right) - \lambda^3 e^{\frac{hc}{\lambda_m KT}} \cdot \frac{hc}{KT} = 0 .$$

$$\Rightarrow 5\left(e^{\frac{hc}{\lambda_m KT}} - 1\right) = \frac{hc}{\lambda_m KT} e^{\frac{hc}{\lambda_m KT}}$$

Let $\frac{hc}{\lambda_m KT} = x$.

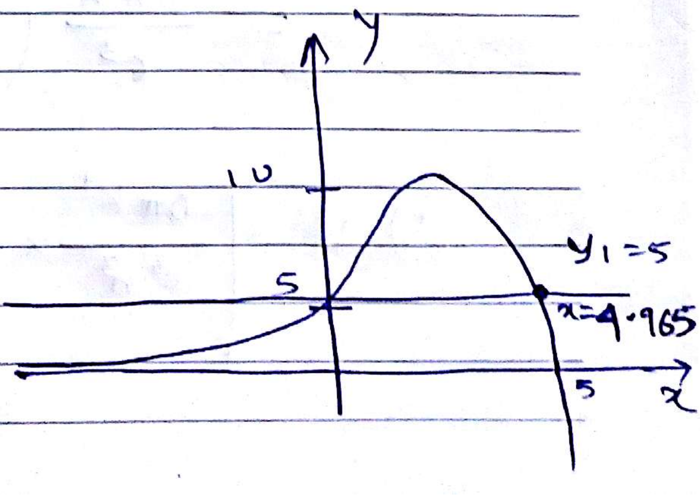
$$\therefore 5(e^x - 1) = x \cdot e^x$$

Notes :

$$\Rightarrow e^x(5 - x) = 5$$

Let $y_1 = 5$

$y_2 = e^x(5 - x)$



$$x = 4.965 = \frac{hc}{\lambda_m kT}$$

$$\Rightarrow \boxed{\lambda_m T = \frac{hc}{4.965 \times k} = 2.9 \times 10^{-3} \, mK}$$

④ Planck's – Stefan's law.

From Planck's law,

$$U_\nu d\nu = \frac{8\pi h \nu^3}{c^3 \left[e^{h\nu/kT} - 1 \right]} d\nu$$

$$U = \int_0^\infty U_\nu d\nu$$

$$= \int_0^\infty \frac{8\pi h \nu^3}{c^3 \left[e^{h\nu/kT} - 1 \right]} d\nu$$

$$= \frac{8\pi h}{c^3} \int_0^\infty \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$$

$$\frac{h\nu}{kT} = x$$

$$= \frac{8\pi h}{c^3} \left(\frac{kT}{h} \right)^3 \times \frac{kT}{h} \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

$$\frac{\pi^4}{15}$$

Notes :

$$\Rightarrow U = \left[\frac{8\pi k^4}{c^3 h^3} \times \frac{\pi^4}{15} \right] T^4$$

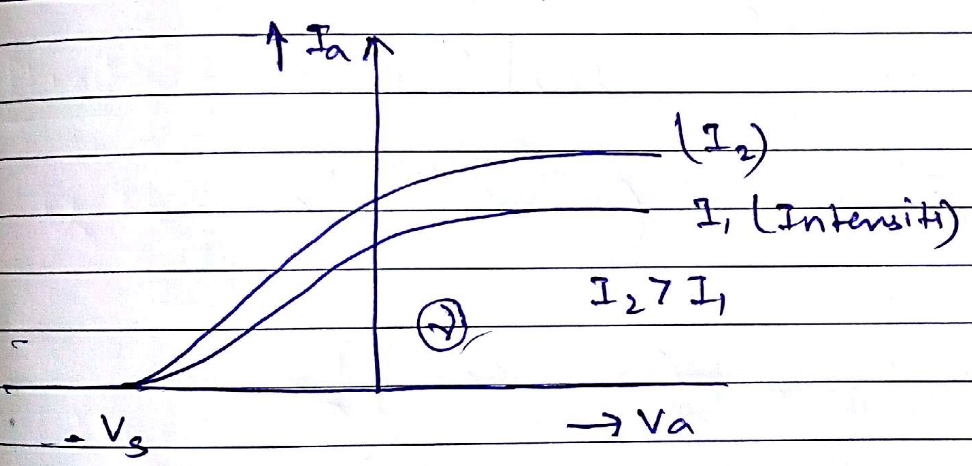
$$E = \frac{c}{4} \times U$$

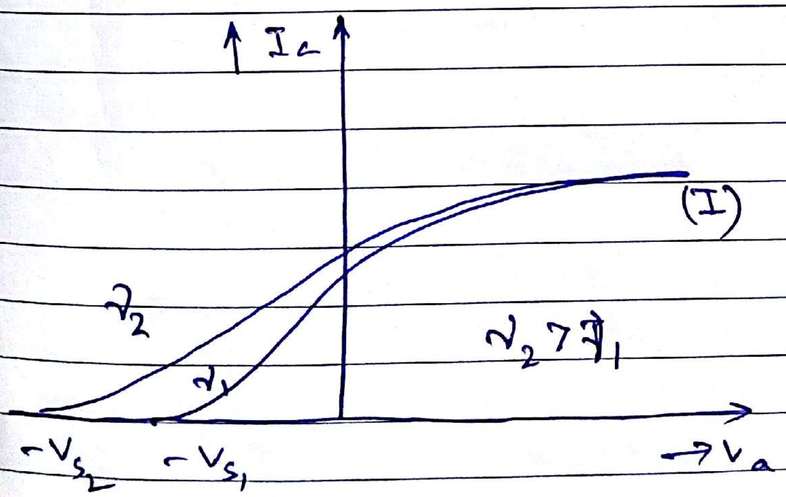
$$E = \left[\frac{2\pi^5 k^4}{15 c^2 h^3}\right] T^4$$

$$\Rightarrow E = (5{\cdot}6 \times 10^{-8})\, T^4$$

$$E = \sigma T^4 \qquad \sigma = \text{Watt}/m^2\text{-}K^4$$

Photo electric effect (Experimental curve)





Notes :