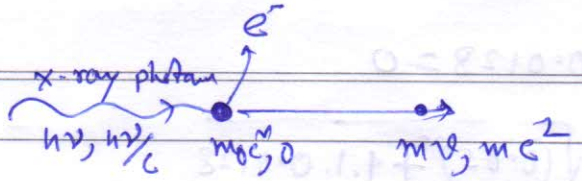
1.



Energy conservation $\rightarrow$ $h\nu + m_0c^2 = mc^2$ — (1)
momentum conservation $\rightarrow$ $h\nu/c + 0 = mv$ — (2)

$\Big\}$ We consider, a photon can give to an free electron all its energy & momentum

from (1) and (2), $mvc + m_0c^2 = mc^2$

$\Rightarrow mvc + m\sqrt{1-v^2/c^2}\cdot c^2 = mc^2$

$\Rightarrow v + c\sqrt{1-v^2/c^2} = c$

$\Rightarrow \cancel{c^2} - v^2 = \cancel{c^2} + v^2 - 2vc$

$\Rightarrow 2v^2 = 2vc \Rightarrow v(v-c) = 0 \therefore v = 0 \ \& \ v = c$

both are impossible, so It is impossible for a photon to give up all its energy & momentum to a free electron.

2. $\lambda' - \lambda = \lambda_c(1-\cos\theta)$

$\lambda = \lambda' - \lambda_c(1-\cos\theta) = 0.022\,A^0 - 2.42\times10^{-2}A^0(1-\cos45^0)$

$= (0.022 - 0.0242\times0.293)A^0$

wavelength of incident beam $\boxed{\lambda = 0.015\,A^0}$ Ans

3. $K.E = h\nu - h\nu' = 6.627\times10^{-34}\left[1.5\times10^{19} - 1.2\times10^{19}\right]$ Joule

$= 1.988\times10^{-15}$ Joule $= 1.243\times10^{4}$ eV $= 12.43$ KeV

4. $\lambda' = \lambda + \lambda_c(1-\cos\theta)$; $\lambda = \frac{c}{\nu} = \frac{3\times10^8}{3\times10^{19}} = 10^{-11}$ m. $= 10$ Pm

$= (10 + 2.42)$ Pm

$= 12.42$ Pm $\therefore \nu' = \frac{c}{\lambda'} = \frac{3\times10^8}{12.42\times10^{-12}} = 2.42\times10^{19}$ sec^{-1}

5. $(K.E)_{max} = \frac{E^2}{m_0c^2/2 + E}$ $\therefore$ ~~(K.E)max~~ $= 50\times10^{-3}$ MeV $= \frac{E^2}{0.255 + E}$

$\Rightarrow \quad E^2 = 0.0128 + 0.05E$

$\Rightarrow \quad E^2 - 0.05E - 0.0128 = 0$

$\therefore \quad E = \dfrac{0.05 \pm \sqrt{(0.05)^2 + 4.1.0.0128}}{2}$

$= \dfrac{0.05 \pm \sqrt{0.0537}}{2}$

$= \dfrac{0.05 \pm 0.232}{2}$

$\boxed{E = 0.141\ MeV}$ (taking +ve sign).

~~or E~~

6. $\lambda' = \lambda + \lambda_c (1 - \cos\theta)$

$= \left[0.558 + 0.0242 (1 - \cos 46^\circ)\right] A^\circ$

$= 0.565\ A^\circ$

7. $nh\nu = 1000$ [$n \rightarrow$ no. of photons per sec]

$n = \dfrac{1000}{6.627 \times 10^{-34} \times 880 \times 10^3}$

$= 1.71 \times 10^{30}$ photons/sec.

9. a) $\Delta\lambda = \lambda_c(1-\cos\theta)$

$= 2.42\times10^{-12}\times(1-\cos 90^\circ)$

$= 0.0242\ A^\circ$

This result is independent of incident wavelength, the same for the γ-rays as the X-rays.

b) $K.E = hc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right) = hc\cdot\frac{\Delta\lambda}{\lambda(\lambda+\Delta\lambda)}$; $\Delta\lambda = \lambda'-\lambda$.

for the X-ray beam, $\lambda = 1A^\circ$.

$$K.E = \frac{6.63\times10^{-34}\times3\times10^{8}\times2.42\times10^{-12}}{1\times10^{-10}\times(1+0.0242)\times10^{-10}} = 4.73\times10^{-17}\text{ Joule}$$

$= 295\ eV = 0.295\ KeV$.

for the γ-rays beam, $\lambda = 1.88\times10^{-2}A^\circ$

$$K.E = \frac{6.63\times10^{-34}\times3\times10^{8}\times2.42\times10^{-12}}{1.88\times10^{-12}\times(0.0188+0.0242)\times10^{-10}} = 5.98\times10^{-14}\text{ Joule}$$

$= 378\ KeV$

c) The incident x-ray photon energy is

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1\times10^{-10}} = 1.99\times10^{-15}\text{ Joule} = 12.2\text{KeV}$$

The energy lost by the photon equals that gained by the electron. or 0.295 KeV, So the percentage loss in energy is

$$\frac{0.295}{12.2}\times100\% = 2.4\%.$$

The incident γ-ray photon energy is

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.88\times10^{-12}} = 1.06\times10^{-13}\text{ Joule} = 660\text{ keV}$$

$\therefore$ the percentage loss in energy is –

$$\frac{378\text{ keV}}{660\text{ keV}}\times100\% = 57\%.$$

Hence, the more energetic photons (small wavelength) experience a larger percent loss in energy in Compton effect. This corresponds to the fact that the photons of smaller wavelengths experience a larger percent increase in wavelength on being scattered.

10. $K.E = hc\left[\frac{1}{\lambda}-\frac{1}{\lambda'}\right]$

$\lambda'-\lambda = \lambda_c(1-\cos\theta)$

when $\theta=\pi$, $\lambda' = \lambda_m'$ (max).

$\lambda_m'-\lambda = 2\lambda_c$

$$(K.E)_{max} = hc.\left[\frac{1}{\lambda}-\frac{1}{\lambda_m'}\right]$$

$$= hc.\frac{\lambda_m'-\lambda}{\lambda\lambda_m'}$$

$\lambda_c = \frac{h}{m_0c}$

$$= \frac{hc.2\lambda_c}{\lambda(\lambda+2\lambda_c)}$$

$h = \lambda_c.m_0c$

$$\boxed{(K.E)_{max} = \frac{2\lambda_c^2m_0c^2}{\lambda^2+2\lambda\lambda_c}}$$

11. $\Delta E = h\nu - h\nu'$; $E = h\nu$

$= (K.E)_{electron}$

$\therefore$ ~~K.E ...~~ $\therefore \frac{\Delta E}{E} = \frac{h\nu-h\nu'}{h\nu} = 1-\frac{\nu'}{\nu}$

$$\left[\frac{\cancel{\nu}}{1+\frac{h\nu}{m_0c^2}(1-\cos\theta)}\right]$$

$\therefore \cancel{\frac{\Delta E}{E} = 1 - \frac{1}{1+\frac{h\nu}{m_0c^2}(1-\cos\theta)} = \frac{\frac{h\nu}{m_0c^2}(1-\cos\theta)}{1+\frac{h\nu}{m_0c^2}(1-\cos\theta)}}$

$\because \frac{1}{\nu'}-\frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)$

$\Rightarrow 1-\frac{\nu'}{\nu} = \frac{h\nu'}{m_0c^2}(1-\cos\theta)$

$$\frac{\Delta E}{E} = \frac{h\nu'}{m_0c^2}(1-\cos\theta).$$

12. By the problem, $\frac{h\nu-h\nu'}{h\nu}\times100\% = 75\%.$

$$\Rightarrow \frac{hc\left[\frac{1}{\lambda}-\frac{1}{\lambda'}\right]}{hc/\lambda} = \frac{3}{4} \Rightarrow \frac{\lambda'-\lambda}{\cancel{\lambda}\lambda'}\times\cancel{\lambda} = \frac{3}{4}$$

$\lambda' - \lambda = \frac{3}{4}\lambda'$,

$\Rightarrow \lambda = \frac{1}{4}\lambda'$ — (1)

Now, fractional change in wavelength —

$$\frac{\lambda' - \lambda}{\lambda} \times 100\%.$$

$$= \frac{\frac{3}{4}\lambda'}{\frac{1}{4}\lambda'} \times 100\% = 300\%.$$

13. By the problem,

$$\frac{h\nu - h\nu'}{h\nu} \times 100 = 10$$

$$\Rightarrow 1 - \nu'/\nu = 0.1 \qquad \frac{\nu'}{\nu} = \frac{1}{1+\alpha(1-\cos\theta)}$$

$$\Rightarrow 1 - \frac{1}{1+\alpha(1-\cos\theta)} = 0.1 \qquad \text{where, } \alpha = \frac{h\nu}{m_0c^2}$$

$$\Rightarrow \frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)} = 0.1$$

$$\Rightarrow \alpha(1-\cos\theta) = 0.1 + 0.1\,\alpha(1-\cos\theta)$$

$$\Rightarrow \alpha(1-\cos\theta)[1-0.1] = 0.1$$

$$\Rightarrow \alpha(1-\cos\theta) = \frac{1}{9} \qquad \therefore \alpha = \frac{0.2\text{ MeV}}{0.51\text{ MeV}} = 0.392$$

$$\Rightarrow (1-\cos\theta) = \frac{1}{9\times0.392} = 0.283$$

$$\Rightarrow \cos\theta = 0.717$$

$$\Rightarrow \theta = 44.19^0$$

14. $K.E = h\nu - h\nu'$

$$= h\nu\left[1 - \nu'/\nu\right]$$

$$= h\nu\left[1 - \frac{1}{1+\alpha(1-\cos\theta)}\right]$$

$$(K.E)_{max} = h\nu\left[1 - \frac{1}{1+2\alpha}\right] \quad [\because \theta = \pi]$$

$$= \frac{h\nu \cdot 2\alpha}{1+2\alpha}$$

$$= \frac{2(h\nu)^2/m_0c^2}{1 + 2.h\nu/m_0c^2}.$$

15.

a) $$h\nu + 0 = (m_0c^2)_{e^+} + (mc^2)_{e^-}$$

$$\Rightarrow h\nu = (m_0c^2)_{e^+} + \left[(m_0c^2)_{e^-} + K_{e^-}\right]$$

$$\Rightarrow h\nu = [0.51 + 1.51]\ \text{MeV} = 2.02\ \text{MeV}$$

b) $$h\nu/c - P = 0 + mv \quad \begin{bmatrix}\text{Positron - momentum} = 0 \\ \text{electron - } \quad " \quad = \cancel{\text{one}}\end{bmatrix}$$

$$= 1.42\ \text{MeV}/c$$

$$\therefore P = 0.6\ \text{MeV}/c,\ (\text{nucleus momentum}) \quad \left(p^2c^2 = 1.51^2 - (0.51)^2\right.$$

$$pc = 1.42\ \text{MeV}$$

$$p = 1.42\ \text{MeV}/c.$$

$\therefore$ Percentage of the photon momentum is transferred to the nucleus is —

$$\cancel{\frac{h\nu/c - P}{h\nu/c} \times 100} = \frac{\cancel{0.860}}{2.02} \times 100$$

$$= 1.42$$

$$\frac{\text{photon momentum} - (e^- + e^+\ \text{momentum})}{\text{photon momentum}} \times 100\%.$$

$$= \frac{0.6}{2.02} \times 100 = 29.7\%.$$

16. $h\nu = 1.02$ MeV

$\therefore$ initial photon momentum $= 1.02$ MeV/c.

momentum of the e^- & e^+ pair $= 0$

$\therefore$ Momentum transferred to the nucleus in the process $= (1.02 - 0)$ MeV/c

$$= \frac{1.02\times10^{6}\times1.6\times10^{-19}}{3\times10^{8}}\ \frac{J}{m/s}.$$

$$= 5.44\times10^{-22}\ \text{Kg-m/sec},$$

b) K.E of the recoil nucleus

$$p^2c^2 + M_0^2c^4 = E^2$$

$$p^2c^2 = (K.E + M_0c^2)^2 - M_0^2c^4.$$

$$\Rightarrow (1.02)^2 = K.E^2 + 2K.E\,M_0c^2$$

$$(M_0)_{lead} = 82\times1837\times m_0$$

$$\Rightarrow K.E^2 + 2K.E\times76.82\times10^{3} - 1.04 = 0$$

$$M_0c^2 = 82\times1837\times m_0c^2$$

$$\Rightarrow K.E = \frac{-153.64\times10^{3} \pm \sqrt{(153.64\times10^{3})^2 - 4\cdot1\times1.04}}{2}$$

$$= \frac{-153.64\times10^{3} \pm \sqrt{23.61\times10^{9} - 4.16}}{2}$$

$$\begin{array}{r}1.04\\ 4\\ \hline 4.16\end{array}$$

b) mass of the lead atom $= 207.2\times1.661\times10^{-27}$ kg,

$\therefore$ velocity of lead atom $= 1.581\times10^{3}$ m/s.

[much less than c]

$\therefore$ K.E of the lead atom –

$$\frac{p^2}{2m} = \frac{(5.44\times10^{-22})^2}{2\times207.2\times1.661\times10^{-27}} = 2.7\ \text{eV}.$$