1. True or false: If every vertex of a graph G has degree 2, then it must be a cycle. Explain.
2. Prove that among any six people, there are three mutual acquaintances or three mutual strangers.
3. The complete bipartite graph whose partite sets have m and n elements is Eulerian if and only if m and n are both even or one of them is 0.
4. Prove that every Eulerian bipartite graph has an even number of edges.
5. True or false: Every Eulerian simple graph having an even number of vertices has an even number of edges. Explain.
6. A graph having exactly two vertices of odd degree must contain a path from one to the other.
7. If every vertex of G has even degree, then G has no cut-edge.
8. True or false: Every graph with fewer edges than vertices has a component that is a tree.
9. Prove that a graph is a tree if and only if it is connected and every edge is a cut-edge.
10. Show that the complete graph having four vertices has (i) a walk that is not a trail and (ii) a trail that is not closed and is not a path.
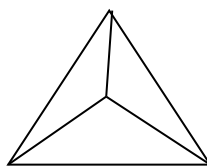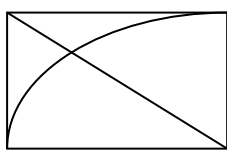
11. Determine the adjacency matrix of the following graph:
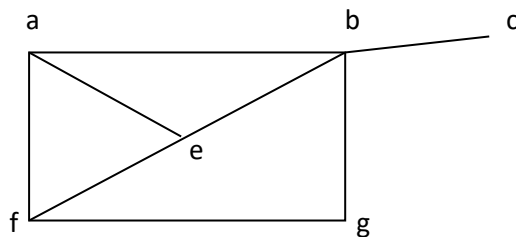
12. Construct the graph G, whose adjacency matrix is given below:

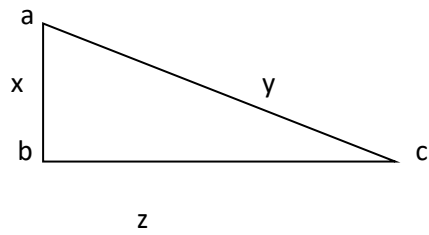$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

13. Draw the graph whose incidence matrix is
$$\begin{bmatrix} 0 & 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$
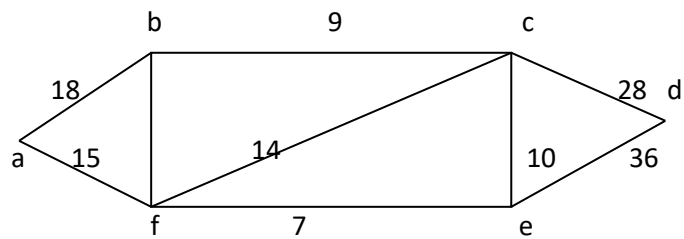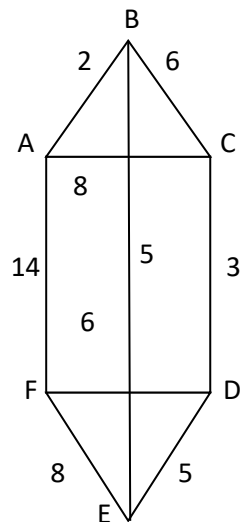
14. Are the given graphs isomorphic?

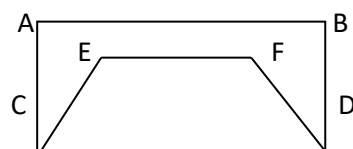15. Represent the given graph using incidence matrix:

16. Apply Dijkstra's Algorithm to find the shortest path from a to d in the given graph:

17. By Dijkstra's Algorithm, find the length of the shortest path from the vertex A to D in the following graph. Show the shortest path.

18. Is the given graph bipartite? Justify your logic.

19. Justify whether the matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ is an adjacency matrix of a graph or not.

20. The adjacency matrix of s graph is a symmetric matrix. True or false?

21. Does there exist a tree with 5 vertices and two vertices of degree 3?

22. Are these following trees possible to draw?
    (i)   A tree having 9 vertices and 9 edges.
    (ii)   A tree with 6 vertices where the sum of all the degrees of the vertices is 14.
    (iii)   A tree with all the vertices of degree 2.
    (iv)   A tree with 6 vertices with degree sequence {1,1,1,1,3,3}.

23. Find the following answers:
    (i)   How many vertices a binary tree should have that has 15 vertices?
    (ii)   How many total vertices a binary tree has if it has 18 internal vertices?
    (iii)   How many internal vertices a binary tree should have with 20 pendant vertices?

24. If a simple graph G and its complement G' both are tree then find the number of vertices in it.

25. A tree has 4 vertices of degree 2, 3 vertices of degree 3, 3 vertices of degree 4. How many pendant vertices the tree should have?

26. If a tree has exactly 2 pendant vertices then prove that degree of every other vertex is exactly 2.
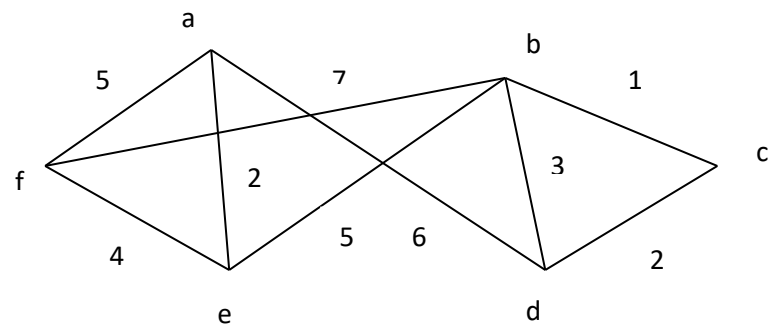
27. Find the minimal spanning tree for the following graph by Kruskal's algorithm.
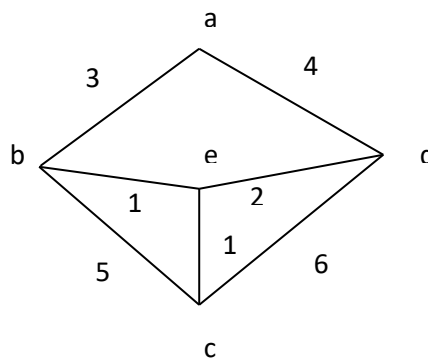
28. Construct a spanning tree by DFS algorithm form the following graph.
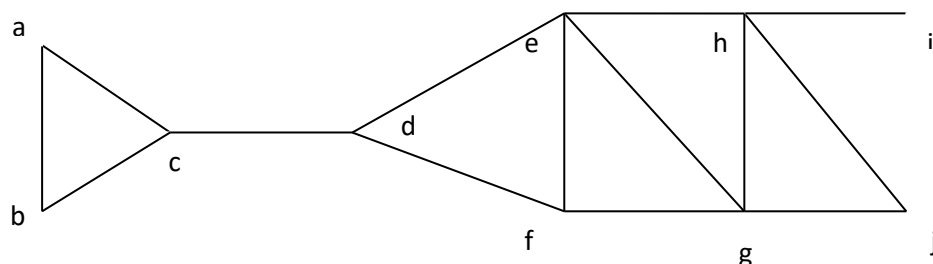
29. Find the minimal spanning tree for the following graph by Prim's algorithm.

30. Construct a spanning tree by BFS algorithm form the following graph.