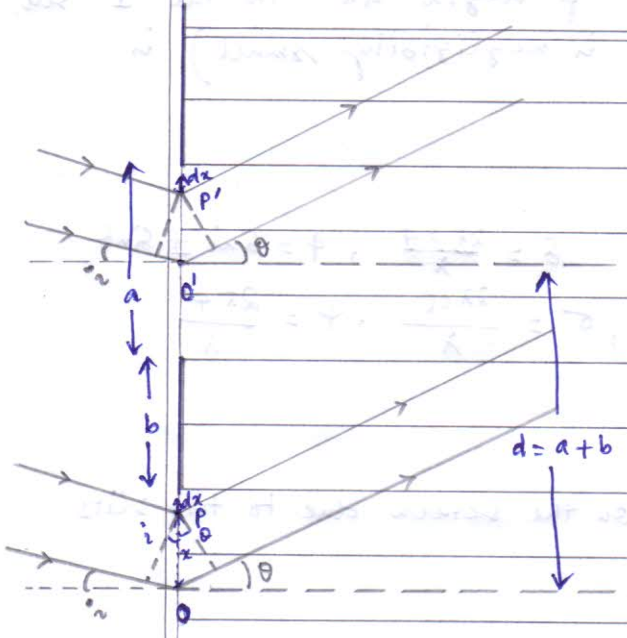
Fraunhofer diffraction through a double slit



Let a parallel beam of light of wavelength λ be made incident at an angle i, with the normal drawn to the surface containing two slits, each of width 'a' and seperated by an opeque space of width 'b'.

The distance between any two pair of corresponding points of the two slits is $d = (a+b)$.

Let us calculate the intensity at a point on the focal plane of a convex lens which receives the rays diffracted from each of the two slits in a direction making and angle 'θ' with the normal to the surface of the slits.

If δ be the phase difference between the rays diffracted from the origin 'O' (mid-pt. of the 1st slit) and from the pt. P (at a ~~position~~ distance x from 'O'), then

$$\delta = \frac{2\pi}{\lambda} x (\sin i \pm \sin\theta)$$

$$= \frac{2\pi x \phi}{\lambda}, \quad \text{where} \quad \phi = \sin i \pm \sin\theta .$$

Let the displacement at a given pt. on the screen due to the rays from the origin 'O' (diffracted at an angle θ) be proportional to $[Re]\ r e^{j\omega t}$, where r be the amplitude of disturbance and $\omega = \frac{2\pi}{\text{Time period}}$.

The displacement at the same pt. on the screen due to waves proceeding from P at a distance x from 'O' is given by

$$y' = [Re]\ r e^{j(\omega t + \delta)}$$

Onward™

Hence, the displacement at the given pt. on the screen due to the secondary waves from an elementary strip of the slit of length 'dx' in the 1st slit (considering the phase change over 'dx' length is negligibily small) is given by:

$$dy \propto y' dx$$

$$\Rightarrow dy = K y' dx \qquad K \to \text{constant}. \qquad \delta = \frac{2\pi x \phi}{\lambda}, \ \phi = \sin i \pm \sin\theta.$$

$$= [Re]\, Kr\, e^{j(\omega t + \delta)} dx \qquad G = Kr, \ \sigma = \frac{2\pi ct}{\lambda}, \ \psi = \frac{2\pi\phi}{\lambda}$$

$$= [Re]\, G\, e^{j\sigma} e^{j\psi x} dx$$

The resultant displacement at the given pt. on the screen due to two slits together would be

$$y = \int_{-a/2}^{+a/2} dy + \int_{d-a/2}^{d+a/2} dy$$

$$= [Re]\, G e^{j\sigma}\left[\int_{-a/2}^{+a/2} e^{j\psi x} dx + \int_{d-a/2}^{d+a/2} e^{j\psi x} dx\right]$$

$$= [Re]\, G e^{j\sigma} \cdot \frac{1}{j\psi}\left[\left(e^{j\psi\frac{a}{2}} - e^{-j\psi\frac{a}{2}}\right) + e^{j\psi d}\left(e^{j\psi\frac{a}{2}} - e^{-j\psi\frac{a}{2}}\right)\right]$$

$$= [Re]\, G e^{j\sigma} \cdot \frac{1}{j\psi} 2j \sin\left(\frac{\psi a}{2}\right)\left[1 + e^{j\psi d}\right]$$

$$= [Re]\, Ga\, e^{j\sigma} \frac{\sin\left(\frac{\psi a}{2}\right)}{\left(\frac{\psi a}{2}\right)}\left[1 + e^{j\psi d}\right] \qquad , \ \alpha = \frac{\psi a}{2}, \ A_0 = Ga$$

$$= [Re]\left\{A_0 \frac{\sin\alpha}{\alpha}\right\} e^{j\sigma}\left[1 + e^{j\psi d}\right] \qquad A = A_0 \frac{\sin\alpha}{\alpha}.$$

$$= [Re]\, A\, (\cos\sigma + j\sin\sigma)\left\{(1 + \cos\psi d) + j\sin\psi d\right\} \qquad C = 1 + \cos\psi d = \xi\cos\gamma$$

$$= [Re]\, A\, (\cos\sigma + j\sin\sigma)(C + jD) \qquad D = \sin\psi d = \xi\sin\gamma$$

$$\Rightarrow \xi = \sqrt{C^2 + D^2}$$

$$= A\left[C\cos\sigma - D\sin\sigma\right]$$

$$= A\xi\cos(\sigma + \gamma)$$

$$= A\sqrt{C^2 + D^2}\cos(\sigma + \gamma)$$

$\therefore$ Intensity $I = \left\{A\sqrt{C^2+D^2}\right\}^2 = A^2(C^2 + D^2) = A^2\left\{(1 + \cos\psi d)^2 + \sin^2\psi d\right\} = 2(1 + \cos\psi d)A^2$

$$= 4A^2\cos^2\frac{\psi d}{2}.$$

$\therefore$ $\boxed{I = 4A^2\cos^2\beta}$ where $\beta = \frac{\psi d}{2}$

Expt. No.

Intensity due to double slit Fraunhofer diffraction:

$\phi = \sin i \pm \sin\theta$

$$\boxed{I = 4A_0^2 \frac{\sin^2\alpha}{\alpha^2}\cos^2\beta}$$

$\alpha = \frac{\phi a}{2} = \frac{\pi a}{\lambda}(\sin i \pm \sin\theta)$

$\beta = \frac{\phi d}{2} = \frac{\pi d}{\lambda}(\sin i \pm \sin\theta)$

$d = a + b$

The resultant intensity thus depends on two factors:

(i) $\left(4A_0^2 \frac{\sin^2\alpha}{\alpha^2}\right)$ – which gives the diffraction pattern of a single slit and

(ii) $(\cos^2\beta)$ – which gives interference pattern of the diffracted light beams from the two slits.

Condition for minima:- Assume: normal incidence $i = 0°$.

Diffraction minima:- when $\frac{\sin^2\alpha}{\alpha^2} = 0 \Rightarrow \sin\alpha = 0 \Rightarrow \alpha = s\pi$
(also when $\alpha \neq 0$) $s = \pm 1, \pm 2, \dots$

$$\boxed{\alpha = s\pi \Rightarrow a\sin\theta = s\lambda, \quad s = \pm1, \pm2, \dots \text{etc.}} \rightarrow \text{Diff. min.}$$

Interference minima:- when $\cos^2\beta = 0$

$$\Rightarrow \boxed{\beta = (2n+1)\frac{\pi}{2} \Rightarrow d\sin\theta = (2n+1)\frac{\lambda}{2}}, \quad n = 0, \pm1, \pm2, \dots \text{etc.} \rightarrow \text{Int. min.}$$

Condition for maxima:-

Principal maxima:- when $\alpha = 0$ $\quad \lim_{\alpha\to 0}\frac{\sin\alpha}{\alpha} = 1 \Rightarrow$ for $\alpha = 0$ Intensity is maxima.
$\Downarrow$
Princ. maxima.

$\therefore \boxed{\alpha = 0} \rightarrow$ Princ. max.

Secondary maxima:

Interference maxima:- when $\cos^2\beta = 1 \Rightarrow \beta = n\pi$

$\Rightarrow d\sin\theta = n\lambda \quad , n = 0, \pm1, \pm2, \dots$

$$\therefore \boxed{\beta = n\pi \Rightarrow d\sin\theta = n\lambda, \quad n = 0, \pm1, \pm2, \dots \text{etc.}} \Rightarrow \text{Secondary max.}$$

Summary: (for double slit)

1. Principal Max: $\alpha = 0$ → (1)
2. Interference Max: $\beta = n\pi$ or, $d\sin\theta = n\lambda$, $n = 0, \pm1, \pm2, \cdots$ etc. → (2)
3. Interference Min.: $\beta = (2n+1)\frac{\pi}{2}$ or, $d\sin\theta = (2n+1)\frac{\lambda}{2}$, $n = 0, \pm1, \pm2, \cdots$ etc. → (3)
4. Diffraction Min: $\alpha = s\pi$ or, $a\sin\theta = s\lambda$, $s = \pm1, \pm2, \cdots$ etc. → (4)

Missing Order :-

If the condition for maxima of interference pattern [Eqn. (2)] and minima of diffraction pattern [Eqn (4)] are simultaneously satisfied for a given value of θ then the corresponding interference maxima will be missing.

Int. Max. condn. $d\sin\theta = n\lambda$, $n = 0, \pm1, \pm2, \cdots$ etc.

Diff. Min condn. $a\sin\theta = s\lambda$, $s = \pm1, \pm2, \cdots$ etc.

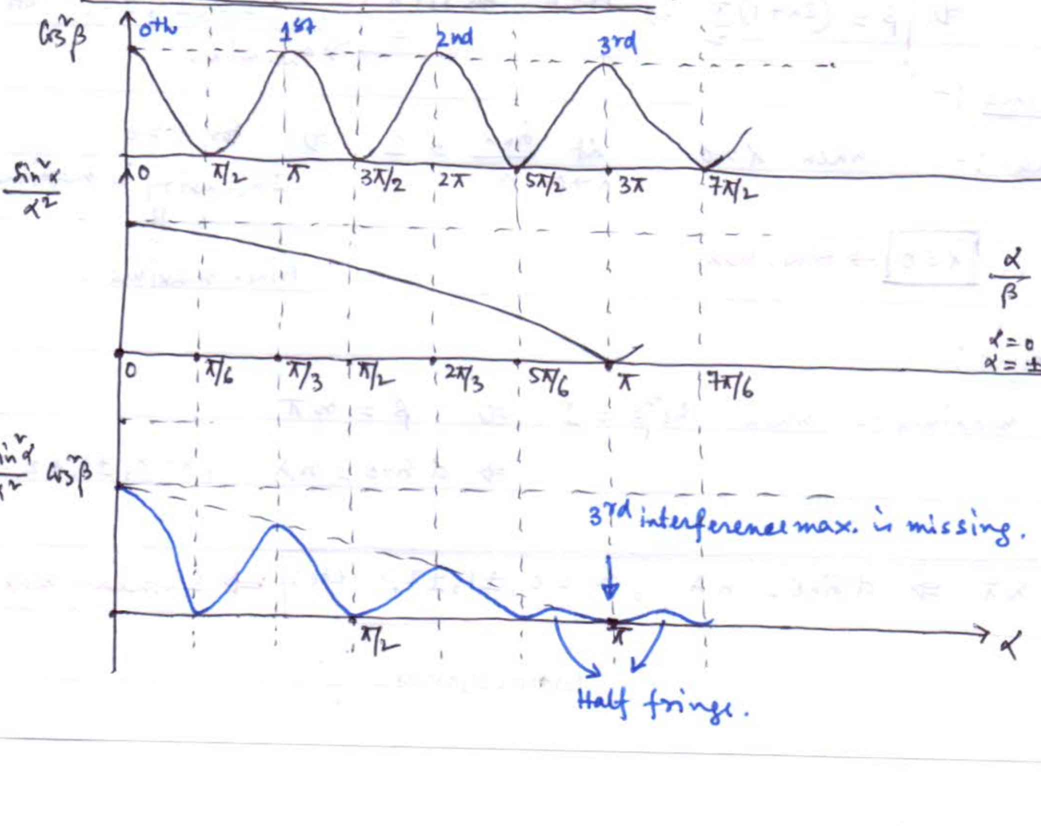
if $\theta \to$ const. $\therefore \frac{d}{a} = \frac{n}{s}$

$\theta \neq 0$.

Again $\frac{d}{a} = \frac{\beta}{\alpha}$.

$\therefore \boxed{\frac{\beta}{\alpha} = \frac{d}{a} = \frac{n}{s}}$ formula to find out the missing orders.

for example lets take $d = 3a$:-



$\frac{\alpha}{\beta} = \frac{a}{d} = \frac{a}{3a} = \frac{1}{3} \Rightarrow \alpha = \frac{\beta}{3}$

$\alpha = 0 \to$ Princ. max

$\alpha = \pm\pi \to$ Diff min.

$\therefore \frac{n}{s} = \frac{d}{a}$

$\Rightarrow n = \frac{3a}{a}\cdot s$

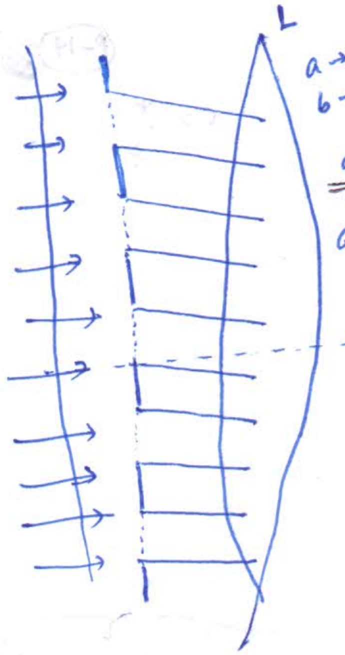
$= 3s$

$\because s = \pm1, \pm2, \cdots$ etc.

$\therefore$ 3rd, 6th, 9th ... orders interference max. will be absent.

GRATING

PLANE ~~TRANSMISS~~ DIFFRACTION



$a \to$ width of clear space

$b \to$ width of the ruled line

$d = a + b$

Grating element

From our previous discussion, we know,

$$dy = \cancel{Re}\, G\, e^{j(\omega t + \delta)}\, dx$$

$$= \cancel{Re}\, G\, e^{j\sigma} e^{j\psi x}\, dx$$

$$\delta = \frac{2\pi x \phi}{\lambda}$$

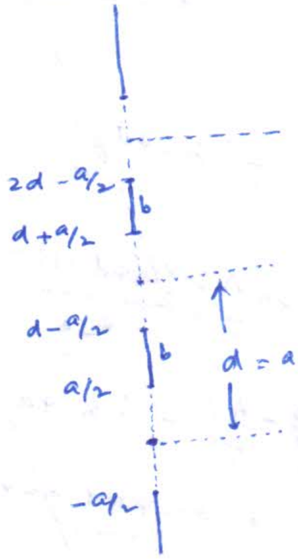
$$\sigma = \frac{2\pi c t}{\lambda}$$

$$\psi = \frac{2\pi \phi}{\lambda} = \frac{2\pi}{\lambda}(\sin i \pm \sin\theta)$$

Resultant displacement on the screen

$$y = \int_{-a/2}^{+a/2} dy + \int_{d-a/2}^{d+a/2} dy + \int_{2d-\frac{a}{2}}^{2d+\frac{a}{2}} dy + \cdots \int_{(N-1)d-\frac{a}{2}}^{(N-1)d+\frac{a}{2}} dy$$

$$= [Re]\, G e^{j\sigma}\left[\int_{-a/2}^{+a/2} e^{j\psi x} dx + \int_{d-a/2}^{d+a/2} e^{j\psi x} dx + \int_{2d-\frac{a}{2}}^{2d+\frac{a}{2}} e^{j\psi x} dx + \cdots \int_{(N-1)d-\frac{a}{2}}^{(N-1)d+\frac{a}{2}} e^{j\psi x} dx\right]$$



For the 2nd slit

$$\frac{a}{2} + \underline{b} + \frac{a}{2} + \frac{a}{2} + \underline{b} = 2b + a + \frac{a}{2}$$

$$= 2b + \frac{3a}{2} + \frac{a}{2} - \frac{a}{2}$$

$$= 2b + 2a - \frac{a}{2}$$

$$= 2d - \frac{a}{2}$$

$$\therefore y = \cancel{Re}\, \frac{G e^{j\sigma}}{j\psi}\left[\left| e^{j\psi x} \right|_{-a/2}^{a/2} + \left| e^{j\psi x} \right|_{d-\frac{a}{2}}^{d+\frac{a}{2}} + \left| e^{j\psi x} \right|_{2d-\frac{a}{2}}^{2d+\frac{a}{2}} + \cdots + \left| e^{j\psi x} \right|_{(N-1)d-\frac{a}{2}}^{(N-1)d+\frac{a}{2}} \right]$$

$$= \cancel{Re}\, \frac{G e^{j\sigma}}{j\psi}\Big[\left(e^{j\psi\frac{a}{2}} - e^{-j\psi\frac{a}{2}}\right) + \left(e^{j\psi d}\cdot e^{j\psi\frac{a}{2}} - e^{j\psi d}\cdot e^{-j\psi\frac{a}{2}}\right)$$

$$+ \left(e^{j\psi\cdot 2d} e^{j\psi\frac{a}{2}} - e^{j\psi\cdot 2d}\cdot e^{-j\psi\frac{a}{2}}\right) + \cdots$$

$$+ \left(e^{j\psi(N-1)d} e^{j\psi\frac{a}{2}} - e^{j\psi(N-1)d} e^{-j\psi\frac{a}{2}}\right)\Big]$$

$$= \cancel{Re}\, \frac{G e^{j\sigma}}{j\psi}\underbrace{\left(e^{j\psi\frac{a}{2}} - e^{-j\psi\frac{a}{2}}\right)}_{2j\sin\frac{\psi a}{2}}\left[1 + e^{j\psi d} + e^{j\psi\cdot 2d} + \cdots e^{j\psi(N-1)d}\right]$$

$$1 + e^{j\psi d} + e^{j\psi d\cdot 2} + \cdots + e^{j\psi d\cdot(N-1)} \qquad \left[x = e^{j\psi d}\right]$$

$$= 1 + x + x^2 + \cdots + x^{N-1} = \frac{1 - x^N}{1 - x} = \frac{1 - e^{j\psi N d}}{1 - e^{j\psi d}}$$

P-14

$$\alpha = \frac{\psi a}{2}$$

$$y = \frac{aG\, e^{j\sigma}}{j\psi a}\, 2j \sin\frac{\psi a}{2} \cdot \frac{1-e^{j\psi N d}}{1-e^{j\psi d}}$$

$$= Ga\, \frac{\sin\alpha}{\alpha}\left[e^{j\sigma} \cdot \frac{1-e^{j\psi N d}}{1-e^{j\psi d}}\right]$$

$$e^{j\sigma}\, \frac{(1-e^{j\psi N d})(1-e^{-j\psi d})}{(1-e^{j\psi d})(1-e^{-j\psi d})}$$

$$= e^{j\sigma}\, \frac{(1-e^{j\psi N d})(1-e^{-j\psi d})}{1-e^{-j\psi d}-e^{j\psi d}+1}$$

$$A = 2(1-\cos\psi d)$$

$$= e^{j\sigma}\, \frac{1-e^{-j\psi d}-e^{j\psi N d}+e^{j\psi d(N-1)}}{2-(\cos\psi d - j\sin\psi d + \cos\psi d + j\sin\psi d)}$$

$$= e^{j\sigma}\, \frac{1-e^{-j\psi d}+e^{j\psi N d}(e^{-j\psi d}-1)}{2(1-\cos\psi d)}$$

$$= \frac{(\cos\sigma + j\sin\sigma)\{1-\cos\psi d + j\sin\psi d + e^{j\psi N d}(\cos\psi d - 1 - j\sin\psi d)\}}{2(1-\cos\psi d)}$$

$$= \frac{(\cos\sigma + j\sin\sigma)}{A}\left[1-\cos\psi d + j\sin\psi d + (\cos\psi N d + j\sin\psi N d)\{\underbrace{(\cos\psi d - 1)}_{-\frac{A}{2}} - j\sin\psi d\}\right]$$

$$= \frac{1}{A}(\cos\sigma + j\sin\sigma)\left[\frac{A}{2} + j\sin\psi d + \cos\psi N d\left(-\frac{A}{2}\right) - j\sin\psi d\cos\psi N d - j\frac{A}{2}\sin\psi N d + \sin\psi N d\sin\psi d\right]$$

$$= \frac{1}{A}(\cos\sigma + j\sin\sigma)\left[\left\{\frac{A}{2} - \frac{A}{2}\cos\psi N d + \sin\psi N d\sin\psi d\right\} - j\left\{\sin\psi d\cos\psi N d + (\sin\psi N d)\frac{A}{2}\right\}\right]$$

$$= \frac{1}{A}\left[\cos\sigma\left\{\frac{A}{2}(1-\cos\psi N d) + \sin\psi d\sin\psi N d\right\} + \sin\sigma\left\{\frac{A}{2}\sin\psi N d + \sin\psi d\cos\psi N d\right\}\right]$$

$$I = y y^* = (Ga)^2\, \frac{\sin^2\alpha}{\alpha^2}\left[e^{j\sigma} \cdot \frac{1-e^{j\psi N d}}{1-e^{j\psi d}}\right]\left[e^{-j\sigma} \cdot \frac{1-e^{-j\psi N d}}{1-e^{-j\psi d}}\right]$$

$$= \left[(Ga)^2\, \frac{\sin^2\alpha}{\alpha^2}\right] \frac{(1-e^{j\psi N d})(1-e^{-j\psi N d})}{(1-e^{j\psi d})(1-e^{-j\psi d})}$$

$$\frac{\left(1-e^{j\psi Nd}\right)\left(1-e^{-j\psi Nd}\right)}{\left(1-e^{j\psi d}\right)\left(1-e^{-j\psi d}\right)}$$

$$= \frac{1-e^{-j\psi Nd}-e^{j\psi Nd}+1}{1-e^{-j\psi d}-e^{j\psi d}+1}$$

$e^{-i\theta}+e^{i\theta}$
$= \cos\theta - i\sin\theta + \cos\theta + i\sin\theta.$

$$= \frac{2-[\cos\psi Nd - j\sin\psi Nd + \cos\psi Nd + j\sin\psi Nd]}{2-2\cos\psi d}$$

$\beta = \frac{\psi d}{2}$.

$$= \frac{1-\cos\psi Nd}{1-\cos\psi d} = \frac{\sin^2\frac{\psi Nd}{2}}{\sin^2\frac{\psi d}{2}} = \frac{\sin^2 N\beta}{\sin^2\beta}.$$

$$\therefore \boxed{I = (Ga)^2\,\frac{\sin^2\alpha}{\alpha^2}\,\frac{\sin^2 N\beta}{\sin^2\beta}}$$

$I_0 = (Ga)^2$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\Rightarrow\ = \cos^2\theta - 1 + \cos^2\theta.$
$= 2\cos^2\theta - 1$

$\boxed{1-\cos 2\theta = 2\sin^2\theta}$

$\alpha = \frac{\psi a}{2} = \frac{\pi a}{\lambda}\sin\theta$, $\beta = \frac{\psi d}{2} = \frac{\pi d}{\lambda}\sin\theta$

$\rightarrow$ Two factors: $\rightarrow$ $I_0\frac{\sin^2\alpha}{\alpha^2} = I_1 \rightarrow$ diff. pattern of a single slit

$\rightarrow$ $\frac{\sin^2 N\beta}{\sin^2\beta} = I_2 \rightarrow$ Interference pattern of the diffracted light beam from N slits.

for $N=2 \Rightarrow I = I_0\frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 2\beta}{\sin^2\beta} = I_0\frac{\sin^2\alpha}{\alpha^2}\frac{4\sin^2\beta\cos^2\beta}{\sin^2\beta}$

$= 4I_0\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta.$

$\rightarrow$ double slit diff. patt.

Principal Maxima.

if a is very small and θ is very small. $\Rightarrow$ variation of $\left(\frac{\sin^2\alpha}{\alpha^2}\right)$ is small.

$\Rightarrow$ Maxima will solely be controlled by $I_2 = \frac{\sin^2 N\beta}{\sin^2\beta}$ factor.

I_2 is max when $\beta = m\pi$, $m = 0, \pm1, \pm2, \ldots$

$\Rightarrow \frac{\pi d}{\lambda}\sin\theta = m\pi \Rightarrow \boxed{d\sin\theta = m\lambda}$ Principle maxima.

$\therefore I_2 = N^2$. $\therefore I_{pm} = I_0\frac{\sin^2\alpha}{\alpha^2}N^2 = I_1 N^2$

$$\lim_{\beta\to m\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to m\pi}\frac{N\cos N\beta}{\cos\beta} = N.$$

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}.$$

$I_1 = I_0 \frac{\sin^2\alpha}{\alpha^2}$

$I_2 = \frac{\sin^2 N\beta}{\sin^2\beta}.$

(P-16)

For α very small and θ small
I_1 non important.

$\frac{\sin^2 N\beta}{\sin^2\beta}$ when max ??

$\lim_{\beta\to m\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to m\pi} N\frac{\cos N\beta}{\cos\beta} = N$

$\therefore \beta = m\pi \Rightarrow \boxed{d\sin\theta = m\lambda} \Rightarrow$ Princ. maxima.

$I_2 = N^2$ for $\beta \to m\pi$

$m = 0, \pm1, \pm2, \ldots$

At $\beta = m\pi \Rightarrow I_{pm.} = I_0 \frac{\sin^2\alpha}{\alpha^2} N^2 = I_1 N^2.$

as N increases: $\Rightarrow$ Intensity of princ. max. $\uparrow$

But $\left(\frac{\sin^2\alpha}{\alpha^2}\right) \downarrow$ when $\theta\uparrow$

$\Rightarrow$ The intensity of princ. max. decreases with the increase in the order no. of bands.

Secondary minima, maxima.

$$I_2 = \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\frac{dI_2}{d\beta} = 0 \Rightarrow \frac{2N\sin N\beta\cos N\beta}{\sin^2\beta} - \frac{2\sin^2 N\beta}{\sin^3\beta}\cos\beta = 0$$

$$\Rightarrow \frac{2\sin^2 N\beta}{\sin^2\beta}\left[\frac{\cancel{\sin^2\beta}}{2\cancel{\sin^2 N\beta}}\cdot\frac{2N\cancel{\sin N\beta}\cos N\beta}{\cancel{\sin^2\beta}} - \frac{\sin^2\beta}{2\cancel{\sin^2 N\beta}}\cdot\frac{2\cancel{\sin^2 N\beta}}{\sin^3\beta}\cos\beta\right] = 0$$

$$\Rightarrow \frac{2\sin^2 N\beta}{\sin^2\beta}\left[N\frac{\cos N\beta}{\sin N\beta} - \frac{\cos\beta}{\sin\beta}\right] = 0$$

$\therefore$ Either. $\frac{\sin^2 N\beta}{\sin^2\beta} = 0$

OR

$N\cot N\beta = \cot\beta.$

MINIMA when $\sin N\beta = 0$ but $\sin\beta \neq 0$. $\Rightarrow$ Intensity $= 0$.

Thus for minima $N\beta = \pm s\pi \Rightarrow \boxed{d\sin\theta = \pm\frac{s}{N}\lambda}$ $s = 1, 2, 3, \ldots$
$s \neq 0, N, 2N, 3N, \ldots$ etc.

$\beta = \frac{\phi d}{2} = \frac{2\pi\phi}{\lambda}\frac{d}{2} = \frac{\pi d\phi}{\lambda}$

$\therefore \begin{cases} d\sin\theta = m\lambda, & m = 0, \pm1, \pm2, \cdots \text{etc.} \longrightarrow \text{Princ. Max.} \\ d\sin\theta = \frac{s}{N}\lambda, & s = 1, 2, 3, \cdots \text{etc.}; s \neq 0, N, 2N, 3N, \ldots \text{etc.} \to \text{Secondary minima.} \end{cases}$

$\Rightarrow$ Thus, between two consecutive princ. maxima there are $(N-1)$ minima.

$\Rightarrow$ There will be $(N-2)$ other secondary maxima between any two adjacent principal maxima.

Secondary maxima: If $N \cot N\beta = \cot\beta$ then $\frac{dI_2}{d\beta} = 0$.

also it can be shown that

$$\left[\frac{d^2 I_2}{d\beta^2}\right]_{N\cot N\beta = \cot\beta} < 0$$

$\therefore$ $N\cot N\beta = \cot\beta$.

$\Rightarrow$ Maximization condn. [secondary maxima].

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}, \quad I_1 = I_0\frac{\sin^2\alpha}{\alpha^2}$$

$$I_2 = \frac{\sin^2 N\beta}{\sin^2\beta}.$$

Intensity of secondary maxima:

$$N\cot N\beta = \cot\beta$$

$$\Rightarrow \frac{N\cos N\beta}{\sin N\beta} = \frac{\cos\beta}{\sin\beta}$$

$$\Rightarrow N^2\frac{\cos^2 N\beta}{\cos^2\beta} = \frac{\sin^2 N\beta}{\sin^2\beta} \qquad \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}.$$

$$\Rightarrow \frac{N^2(1-\sin^2 N\beta)}{1-\sin^2\beta} = \frac{N^2\sin^2 N\beta}{N^2\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

$$\Rightarrow \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

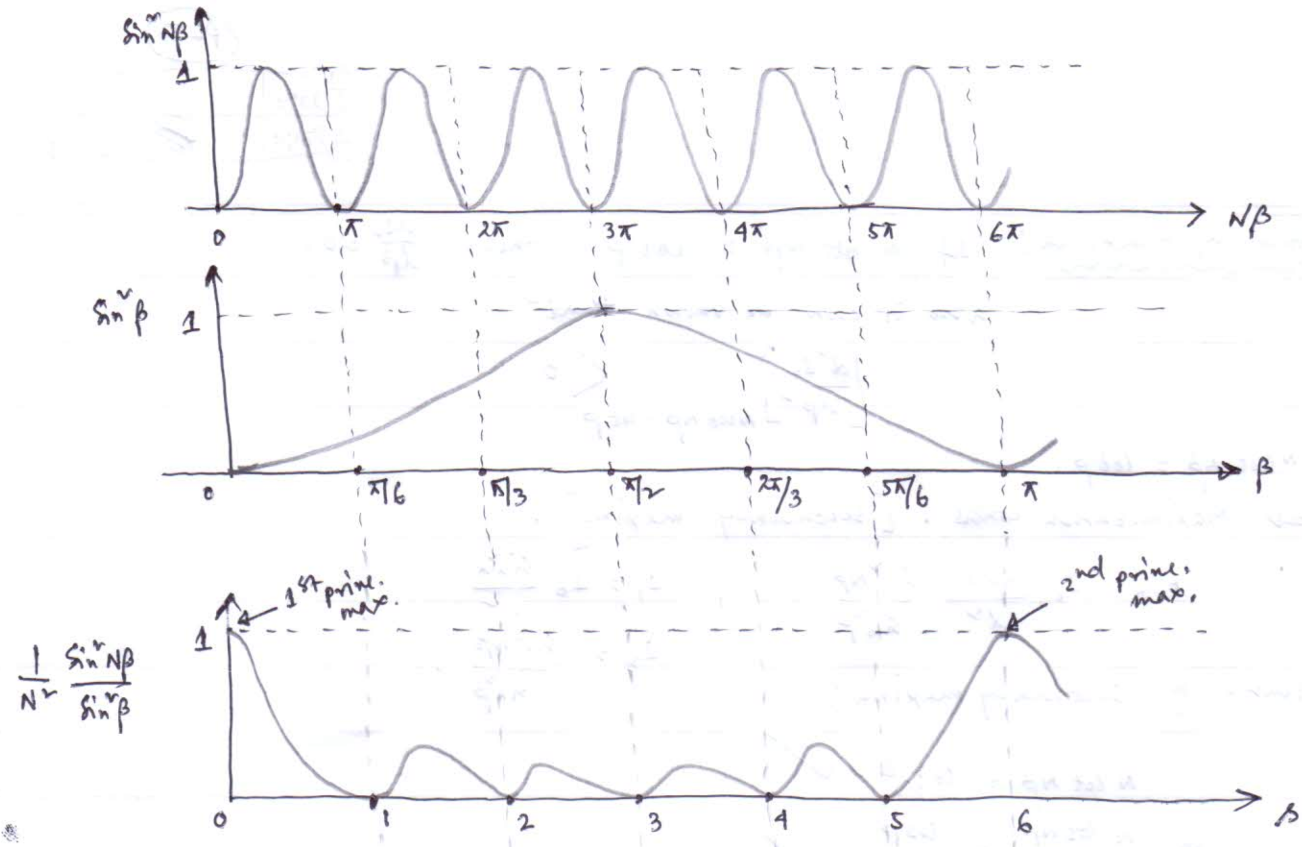
$$\Rightarrow (I_2)_{sm.} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

$\therefore$ Intensity of the secondary maxima is given by:

$$I_{sm} = I_1(I_2)_{sm} = \frac{I_1 N^2}{\{1+(N^2-1)\sin^2\beta\}} = \frac{I_{pm}}{1+(N^2-1)\sin^2\beta}$$

$$\Rightarrow \frac{I_{sm}}{I_{pm}} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

$\Rightarrow$ as $N\uparrow$ $(I_{sm}/I_{pm})\downarrow$



For $N=6$

- $d\sin\theta = m\lambda$, $m = 0, \pm 1, \pm 2, \cdots$ etc. $\longrightarrow$ Princ. Max.
- $d\sin\theta = \frac{s}{N}\lambda$, $s = 1, 2, 3, \cdots$ etc. but $s \neq 0, N, 2N, 3N, \cdots$ etc. $\Rightarrow$ Secondary Minima.
- Between 2 consecutive princ. max. there are $(N-1)$ secondary min. and $(N-2)$ secondary max.

Absent Spectra :-

m^{th} order princ. max. $\quad d\sin\theta = m\lambda$

Suppose the value of 'a' is such that s^{th} order diff. min. occurs for same θ ;

Then $\quad a\sin\theta = s\lambda$

If these two condn's are satisfied simultaneously then m^{th} order princ. max. will be absent from the resulting spectra.

$$\therefore \quad \frac{d}{a} = \frac{m}{s}$$

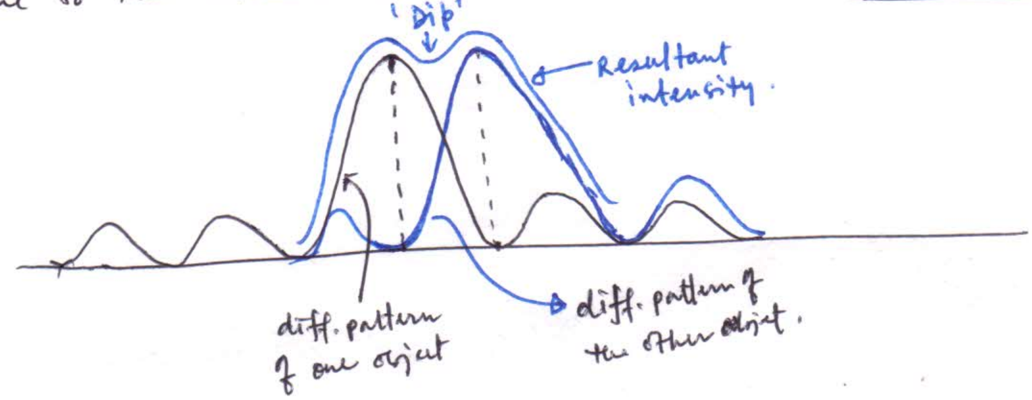
If $d = 2a$ then $m = 2s$ $\therefore$ 2, 4, 6 ....etc. order of princ. maxima will be absent corresponding to the diff. minima $s = 1, 2, 3, \cdots$ etc.

The image of ~~an optical~~ a point object by an optical system with limited aperture consists of circular diff. pattern.

Resolving power and Rayleigh Criterion of resolution!

The resolving power of an analysing instrument is its ability to resolve two close point objects or to just seperate two close spectral lines in their diffraction patterns.

According to Lord Rayleigh two equally bright pt. sources could be just resolved by any optical system if the distance between them is such that the central maximum in the diff. pattern due to one source coincides exactly with the 1st minimum in the diff. pattern due to the other. This is known as the Rayleigh Criterion of resolution.



Intensity at 'dip' is about $\left(\frac{8}{\pi^2}\right)$ times the intensity of either peak

≈ 81%

Resolving power of a grating: [Ability of a grating to distinguish two close spectral lines]

measured by. $\to \left(\frac{\lambda}{d\lambda}\right)$ $d\lambda \to$ smallest wavelength diff for which spectral lines can be just resolved at wavelength λ.

$$\boxed{\frac{\lambda}{d\lambda} = Nm}$$

N = total no. of rulings.

m = order no. of the spectrum.

P1 Convex lens of focal length 40 cm is employed to focus the Fraunhofer diff pattern of a single slit of 0.3 mm width. . . . .

Calculate the linear distance of the 1st order dark band from the central band.

$\lambda = 589$ nm.

$a \sin\theta = m\lambda$.

$$W = \frac{\lambda f}{a} = \frac{589\,\text{nm} \times 40\,\text{cm}}{0.3\,\text{mm}}$$

$$= \frac{589 \times 10^{-9}\,\text{m} \times 40 \times 10^{-2}\,\text{m}}{0.3 \times 10^{-3}\,\text{m}}$$

$$= \frac{589 \times 40}{0.3} \times 10^{-8}\,\text{m}$$

$$= 78533 \times 10^{-8}\,\text{m}$$

$$= 0.785\,\text{mm}$$

P-2. A ll$^{\text{l}}$ beam of light of $\lambda = 500$ nm is incident normally on a narrow single slit of width $a = 0.2$ mm. For a Fraunhofer diff. pattern find the angular position of the 1st and 2nd maxima.

$\alpha_1 = 1.43\pi \rightarrow$ 1st maxima

$\alpha_2 = 2.46\pi \rightarrow$ 2nd maxima

$$\boxed{\begin{aligned}\alpha &= \frac{\pi a \sin\theta}{\lambda} \\ \theta &= \sin^{-1}\left(\frac{\lambda\alpha}{\pi a}\right)\end{aligned}}$$

$\theta_1 = \sin^{-1}\dfrac{1.43\lambda}{a} = 0.20^\circ$

$\theta_2 = \sin^{-1}\dfrac{2.46\lambda}{a} = 0.35^\circ$

P-3 Show that the intensity of the 1st secondary maximum formed by a single slit for diffraction process is nearly 4.5% of that of the principal maximum.

$I = I_0 \dfrac{\sin^2\alpha}{\alpha^2}$

$I_{second} = I(1.43\pi) = I_0 \dfrac{\sin^2(1.43\pi)}{(1.43\pi)^2} = 0.047\, I_0$

$I_{prime} = \lim_{\alpha \to 0} I = I_0$

$\therefore \dfrac{I_{secn}}{I_{prime}} = 0.047 \rightarrow 4.7\%$.

P-4 Find the missing orders for a double slit Fraunhofer pattern if the width of each slit is 0.15 mm and they are separated by a distance 0.60 mm.

Note here, $a = 0.15$ mm and $d = 0.60$ mm

Diff. min $\Rightarrow \alpha = \dfrac{\pi a}{\lambda}\sin\theta = s\pi$, $s = \pm1, \pm2, \cdots$

Int. max. $\Rightarrow \beta = \dfrac{\pi d}{\lambda}\sin\theta = n\pi$, $n = 0, \pm1, \pm2, \cdots$

$\Rightarrow$ for missing orders $\dfrac{d}{a} = \dfrac{n}{s} \Rightarrow n = \left(\dfrac{d}{a}\right)s$

Here, $\dfrac{d}{a} = \dfrac{0.60}{0.15} = \dfrac{0.45}{0.15} = 4$

$\therefore n = 4s \Rightarrow 4, 8, 12$, etc. int. maxima will be absent.

also $\dfrac{\pi d \sin\theta}{\lambda} = n\pi \Rightarrow \sin\theta = \dfrac{n\lambda}{d} = \dfrac{n\lambda}{4} \Rightarrow$ for $n = 4, 8, 12, \cdots$ etc.

$\sin\theta = (\text{integral}) * (\lambda)$

Diff. min condn.

P-5 Width of each slit of double slit is 0.15 mm and they are separated by a distance of 0.45 mm. If the double slit produces Fraunhofer diffraction, find the angular position of the 1st minimum and missing orders.