III. Damped Oscillation

In most of the real situations the free oscillation never sustains but dies down due to the presence of a several type of decaying force leading to a steady decrease of the total energy of the system. The presence of such damping force, as we will see, can affect the system in various ways (even destroying the very oscillatory character of the system under certain specific situation) causing the motion to cease eventually. In the following we will consider various case of damped motion characterized by the relations between the natural frequency of vibration (ω_0) and the so-called damping constant.

Constitutive equation of damped motion

Recalling the energy theorem (Theorem-1) we can write the rate of change of total energy in presence of damping as

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega_0^2 x^2\right) < 0$$

The left hand side can be equated with the power dissipate due the presence of damping force. In various physical situations the damping force $F_{damping} = +\Gamma v$, $\Gamma(>0)$ bing a const. Hence, the power dissipated is given by

$$P = F_{damping}\, v = \Gamma v^2 = \Gamma \dot{x}^2$$

and,

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega_0^2 x^2\right) = -\Gamma\dot{x}^2$$

$$\Rightarrow \quad \dot{x}\ddot{x} + \omega_0^2 x\dot{x} = -\frac{\Gamma}{m}\dot{x}^2$$

$$\Rightarrow \quad \boxed{\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0} \quad \cdots\cdots [20]$$

- **Remark**: 1. Equation-[20] can also be constructed by considering the damping force ($\Gamma\dot{x}$) acting along the same direction of the restoring force thus constituting the Newton's law for the system.

  2. Considering $\gamma\dot{x}$ has the dimension of acceleration i.e.

$$[\gamma][\dot{x}] = \cancel{[M]}[L][T]^{-2}$$

$$\Rightarrow [\gamma][L][T]^{-1} = [L][T]^{-2} \Rightarrow [\gamma] = [T]^{-1}$$

So γ has the dimension of frequency (unit = sec^{-1})

3. The effect of the damping constant γ on the free oscillation of frequency ω_0 can be understood in the following theorem.

- <u>Theorem-4</u>: The solution of eqn.-[20] under the initial condition
$$x(t=0)=0$$
$$v(t=0)=v_0 \quad \text{is}$$
(i) Non-oscillatory for $\gamma \geq \omega_0$ *
(ii) Oscillatory for $\gamma < \omega_0$

Proof: (i) Given $\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$ .... [20]

We take a trial solution $x(t) = \exp(-\gamma t) f(t)$

$$\dot{x} = \frac{dx}{dt} = \exp(-\gamma t)\dot{f}(t) - \gamma\exp(-\gamma t) f(t)$$

$$\ddot{x} = \frac{d^2x}{dt^2} = \exp(-\gamma t)\ddot{f}(t) - \gamma\exp(-\gamma t)\dot{f}(t)$$
$$-\gamma\exp(-\gamma t)\dot{f}(t) + \gamma^2\exp(-\gamma t) f(t)$$

Substituting all these results in eqn. [20] we get,

$$\exp(-\gamma t)\left[\ddot{f}(t) - 2\gamma\dot{f}(t) + \gamma^2 f(t)\right]$$
$$+ 2\gamma\exp(-\gamma t)\left[\dot{f}(t) - \gamma f(t)\right] + \omega_0^2 e^{-\gamma t} f(t) = 0$$

$$\Rightarrow \exp(-\gamma t)\left[\ddot{f}(t) + (\omega_0^2 - \gamma^2) f(t)\right] = 0 \quad \cdots [21]$$

As eqn. [21] is valid for all $t \in [0, \infty]$

$$\boxed{\ddot{f}(t) + (\omega_0^2 - \gamma^2) f(t) = 0} \quad \cdots\cdots [22]$$

a. For $\gamma > \omega_0$, $f(t) = A\exp(\alpha t) + B\exp(-\alpha t)$

where, $\alpha = \sqrt{\gamma^2 - \omega_0^2} > 0$

Hence, $x(t) = A\exp(\alpha - \gamma)t + B\exp(-\alpha - \gamma)t$

Now, $x(0) = A + B = 0$ (Given)

$\dot{x}(0) = A(\alpha - \gamma) - B(\alpha + \gamma) = v_0$ (Given)

* A solution is said to be nonoscillatory if it crosses $x=0$ not more than once.

Solving $A = -B = \frac{v_0}{2\alpha}$

Hence $x(t) = \exp(-\gamma t)\left[\frac{v_0}{2\alpha}\{\exp(\alpha t) - \exp(-\alpha t)\}\right]$

$$\Rightarrow \boxed{x(t) = \exp(-\gamma t)\frac{v_0}{\alpha}\sinh(\alpha t)} \quad \text{---} [23]$$

b. For $\gamma = \omega_0$ equation -[22] yields, $\ddot{f} = 0$ i.e; $f(t) = At + B$.

This gives, $x(t) = \exp(-\gamma t) f(t) = \exp(-\gamma t)(At + B)$

Now, $\left.\begin{aligned} x(0) &= 0 = B \\ \dot{x}(0) &= v_0 = A \end{aligned}\right\}$ (Given)

Hence, $\boxed{x(t) = v_0 t \exp(-\gamma t)}$ ---- [24]

(ii) For $\gamma < \omega_0$ equation -[22] yields

$\ddot{f} = -\tilde{\omega}^2 f$ i.e.; $f(t) = A\exp(+i\tilde{\omega}t) + B\exp(-i\tilde{\omega}t)$

where $\tilde{\omega} = (\omega_0^2 - \gamma^2)^{1/2} > 0$

and $i = \sqrt{-1}$

This gives $x(t) = A\exp[-(\gamma - i\tilde{\omega})t] + B\exp[-(\gamma + i\tilde{\omega})t]$

Now $x(0) = 0 = A + B$.

$\dot{x}(0) = v_0 = -A(\gamma - i\tilde{\omega}) - B(\gamma + i\tilde{\omega})$

Solving $A = -B = \frac{v_0}{2i\tilde{\omega}}$

Hence $x(t) = \frac{v_0}{2i\tilde{\omega}}\left[\exp(-\gamma t)\{e^{+i\tilde{\omega}t} - e^{-i\tilde{\omega}t}\}\right]$

$$\Rightarrow \boxed{x(t) = \frac{v_0}{\tilde{\omega}}\exp(-\gamma t)\sin(\tilde{\omega}t)} \quad \text{-----} [25]$$

Equation [23] and [24] are non oscillatory solutions while Equation [25] is giving an oscillatory solution.

• Remark: 1. Equation -[23] represents motion in presence of large damping ($\gamma > \omega_0$) when the oscillator starts from the origin ($x=0$) with an initial velocity (v_0). The displacement always takes up positive values, For small values of t the exponential (γt) is close to unity and $x(t)$, therefore, increases as $\sinh(\alpha t)$ is increasing. Finally, the $\exp(-\gamma t)$ dominates and the ~~oscillation~~ motion dies down to zero. The turning point happens when,

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$$\dot{x}(t) = \frac{v_0}{\alpha}\left[e^{-\gamma t}\alpha \cosh \alpha t - \gamma e^{-\gamma t} \sinh \alpha t \right] = 0$$

$$\Rightarrow \quad \tanh \alpha t_0 = \frac{\alpha}{\gamma}$$

$$\Rightarrow \quad \boxed{t_0 = \frac{1}{\alpha}\tanh^{-1}\left(\frac{\alpha}{\gamma}\right)}$$

Such type of nonoscillatory motion is called <u>dead beat</u>. (Represented in the diagram - 4a)

2: Equation -[24] represents what is known as <u>critical damping</u> ($\gamma = \omega_0$). For small values of t, $x(t)$ increases almost linearly as the exponential term is close to 1. At some point of time t_0, $x(t)$ falls exponentially to zero. The turning point happens when

$$\dot{x}(t) = v_0\left[e^{-\gamma t} - \gamma t e^{-\gamma t} \right] = 0$$

$$\Rightarrow \quad \boxed{t_0 = \frac{1}{\gamma} = \frac{1}{\omega_0}}$$

critical damping is often preferred over the dead beat when we desire a high rate of decay without oscillation. (see diagram — 4-b)

3. Equation - [25] represents an oscillatory behavior known as <u>weak damping</u>. The oscillatory behavior is ensured by $\sin(\tilde{\omega} t)$ which crosses the time axis when-ever $t = \frac{n\pi}{\tilde{\omega}}$. <u>The so-called amptitude part now depends upon time exponentially and vanishes for large t, thus ceasing the oscillation.</u> Though the motion is no longer periodic one can define a time interval between two successive zeros of $x(t)$, which is clearly $\frac{\pi}{\tilde{\omega}}$. This is also the time interval between a maximum and the next minimum value of the displacement, but maxima & minima are not exactly half-way between the zeros. For maxima or minima, $\dot{x} = 0$

giving $\cos\tilde{\omega}t - \frac{\gamma}{\tilde{\omega}}\sin\tilde{\omega}t = 0$ or

$$\boxed{\tan\tilde{\omega}t = \frac{\tilde{\omega}}{\gamma}}$$

For $\gamma/\tilde{\omega} \to 0$ $\quad \cancel{\tan}\,\tilde{\omega}t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\tilde{\omega}}$

See diagram - 4-c

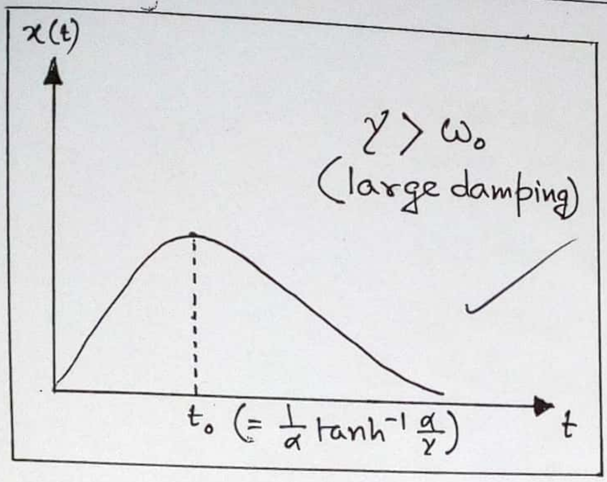

Diagram 4-a

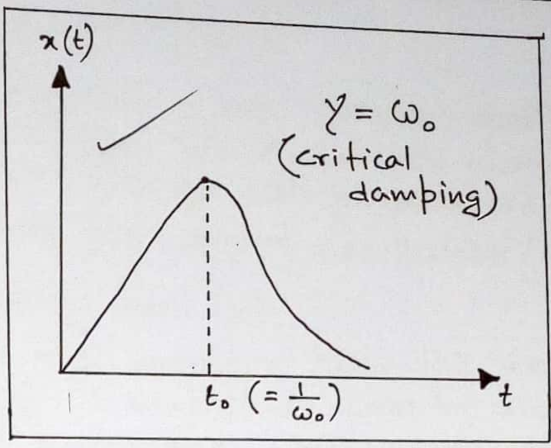

Diagram 4-b.

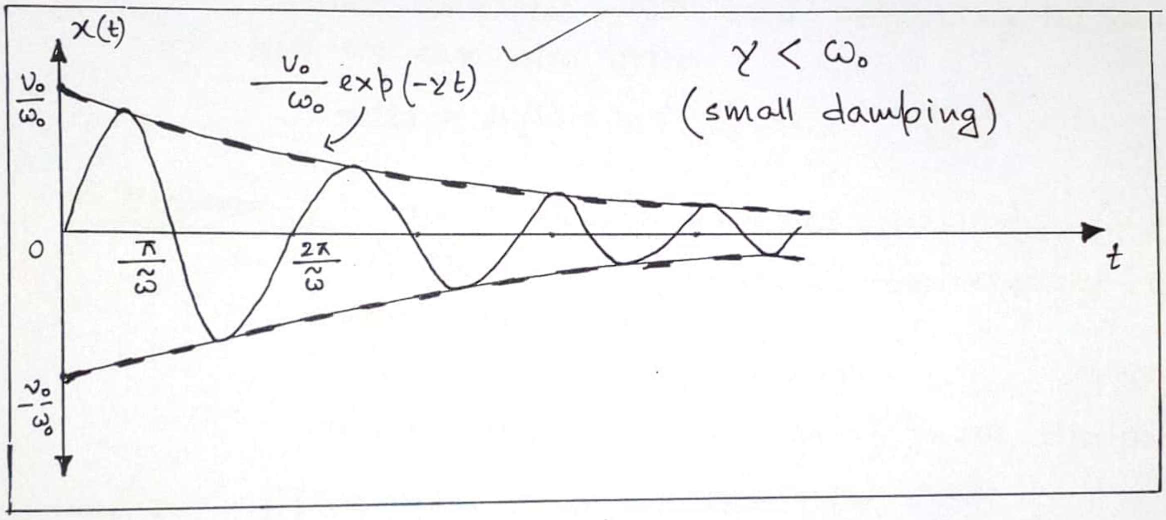

Diagram –4-c

5. The present theorem (Th-4) and the diagrams we obtain are all based on initial condition : at $t=0$ $x(t)=0$ and $\dot{x}(t) = v_0$. One can always start with various other initial conditions (for example at $t=0$ $x(t)=x_0$, $\dot{x}=0$) The decaying part of the above diagrams are almost identical in nature but there can be differences in the initial regimes. In particular, when ~~x≠0~~ at $t=0$ $x=x_0$ and $\dot{x}=v_0$ it is not in general possible to conclude (in case of large damping) that whether the particle will shoot past the origin or whether it will merely fall down to the origin without crossing it (like diagram-4). It all depends upon how γ is related to the initial condition (x_0 & v_0). For some γ the solution remains to be non-oscillatory crossing $x=\hat{0}$ once and for all.

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• Study of weakly damped oscillation

In our preceeding study of weakly damped oscillation we have already observed that the dynamical character of the system is chiefly controlled by two parameters.

(i) The natural frequency of oscillation (ω_0)

(ii) The damping const (γ).

In the following we will consider how the decay in energy and the decay in amplitude can be understood in relation to these two parameters. We will assume the solution –

$$x(t) = \frac{v_0}{\tilde{\omega}} \exp(-\gamma t) \sin(\tilde{\omega} t) \quad \cdots [25]$$

where $\tilde{\omega} = (\omega_0^2 - \gamma^2)^{1/2}$ and replacing the amptitude by $A(t)$ we can also write

$$x(t) = A(t) \sin(\tilde{\omega} t) \quad \cdots [25]'$$

• **Theorem-5**: For a weakly dampled oscillation $(\gamma << \omega_0)$

(i) The time average of total energy $\langle E(t) \rangle$ decays exponentialy.

(ii) $\langle E(t) \rangle = (2\gamma)^{-1} \langle P(t) \rangle$, where $\langle P(t) \rangle$ denotes the time average of power dissipated.

Proof: (i) The total energy $E(t) = T + V$

$$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega_0^2 x^2$$

$$= \frac{1}{2} m \left[\frac{v_0^2}{\tilde{\omega}^2} e^{-2\gamma t} \left\{ \tilde{\omega}^2 \cos^2 \tilde{\omega} t + \gamma^2 \sin^2 \tilde{\omega} t - 2\tilde{\omega}\gamma \cos \tilde{\omega} t \sin \tilde{\omega} t \right\} \right]$$

$$+ \frac{1}{2} m \omega_0^2 \left[\frac{v_0^2}{\tilde{\omega}^2} \exp(-2\gamma t) \sin^2 \tilde{\omega} t \right]$$

Now to find $\langle E(t) \rangle$ we'll take the time averages of the above quantity. For small γ we can take $e^{-2\gamma t}$ outside the integral and use the following results.

$$\left. \begin{aligned} &1.\ \langle \cos^2 \tilde{\omega} t \rangle = \tfrac{1}{2} \\ &2.\ \langle \sin^2 \tilde{\omega} t \rangle = \tfrac{1}{2} \\ &3.\ \langle \sin 2\tilde{\omega} t \rangle = 0 \end{aligned} \right\}.$$

Using all these result we get

$$\langle E(t)\rangle = \frac{1}{\tilde{T}}\int_0^{\tilde{T}} E(t)\,dt \ , \text{ where } \tilde{T} = \frac{2\pi}{\tilde{\omega}}$$

$$= \frac{m v_0^2}{4\tilde{\omega}^2}\, e^{-2\gamma t}\left[\tilde{\omega}^2 + \tilde{\omega}_0 + \gamma^2\right]$$

As $\tilde{\omega}^2 = \omega_0^2 - \gamma^2$

$$\langle E(t)\rangle = \frac{1}{2} m A_0^2 \omega_0^2 e^{-2\gamma t} \ ; \ A_0 = \frac{v_0^2}{\tilde{\omega}^2}.$$

(ii) By definition power consumed by the damping force $2\gamma\dot{x}$ is

$$P(t) = 2\gamma\dot{x}\cdot\dot{x} = 2\gamma\dot{x}^2$$

$$\langle P(t)\rangle = 2\gamma\langle\dot{x}^2\rangle = 2\gamma A_0^2 e^{-2\gamma t}\frac{1}{2}m\left[\frac{\tilde{\omega}^2+\gamma^2}{2}\right]$$

$$= \frac{1}{2} m A_0^2 \omega_0^2 (2\gamma) e^{-2\gamma t}.$$

$$= (2\gamma)\langle E(t)\rangle$$

$$\Rightarrow \quad E(t) = (2\gamma)^{-1}\langle P(t)\rangle$$

• Remark : 1. The time dependence of average energy is given by the following diagram. (Diagram-5)

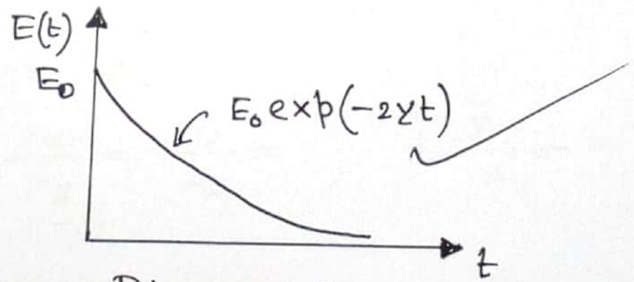

Diagram-5.

• Logarithmic Decrement : Let's consider the following diagram representing a weakly damped motion

$$x(t) = A_0 \exp(-\gamma t)\sin\tilde{\omega}t$$

$$\Rightarrow x(t) = A_0 \exp(-\gamma t)\sin\frac{2\pi}{\tilde{T}}t \quad \cdots [26]$$

where $\tilde{T} = \frac{2\pi}{\tilde{\omega}}$

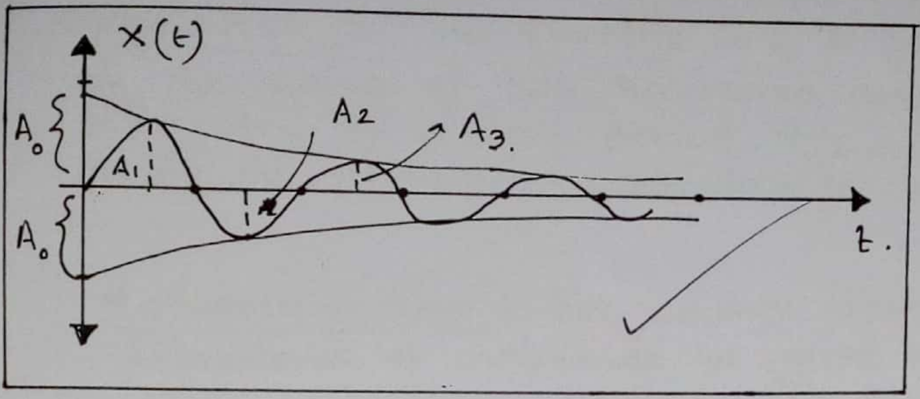

Diagram - 6

Let us call maximum displacement from the origin between two successive zeros of the solution as the 'amplitude'. From diagram - 6 we can construct a decreasing sequence of amplitudes values (irrespective of sign) as $\{A_n \mid n=1,2,3\ldots\}$ occuring at $\{(2n-1)\frac{\tilde{T}}{4} \mid n=1,2,3\cdots\}$ respectively. Then from equation -26 we can write,

1. $$\left|x\left(t=\frac{\tilde{T}}{4}\right)\right| = \left|A_0 \exp\left(-\frac{\gamma\tilde{T}}{4}\right)\sin\left(\frac{2\pi}{\tilde{T}}\frac{\tilde{T}}{4}\right)\right|$$
$$= \left|A_0 \exp\left(-\frac{\gamma\tilde{T}}{4}\right)\right| = A_1.$$

2. $$\left|x\left(t=\frac{3\tilde{T}}{4}\right)\right| = \left|A_0 \exp\left(-\frac{3\gamma\tilde{T}}{4}\right)\sin\left(\frac{2\pi}{\tilde{T}}\frac{3\tilde{T}}{4}\right)\right|$$
$$= \left|-A_0 \exp\left(-\frac{3\gamma\tilde{T}}{4}\right)\right| = A_2$$

3. $$\left|x\left(t=\frac{5\tilde{T}}{4}\right)\right| = \left|A_0 \exp\left(-\frac{5\gamma\tilde{T}}{4}\right)\right| = A_3.$$

and so on.

Thus we see that

$$\frac{A_1}{A_2} = \frac{A_2}{A_3} = \frac{A_3}{A_4} = \cdots = \frac{A_{n-1}}{A_n} = \exp\left(\frac{\gamma\tilde{T}}{2}\right) = \Delta$$
(const)

$$\Rightarrow \ln\Delta = \gamma\tilde{T}/2.$$

One can also express

$$\frac{A_1}{A_n} = \frac{A_1}{A_2}\cdot\frac{A_2}{A_3}\cdot\frac{A_3}{A_4}\cdots\frac{A_{n-2}}{A_{n-1}}\;\frac{A_{n-1}}{A_n}$$

$$\Rightarrow \frac{A_1}{A_n} = \cancel{(n-1)}\,\Delta^{n-1} = \cancel{(n-1)\exp\left(\frac{\gamma\tilde{T}}{2}\right)}$$

$$\Rightarrow \ln\frac{A_1}{A_n} = (n-1)\ln\Delta$$

$$\Rightarrow \boxed{\ln\Delta = \frac{1}{n-1}\ln\frac{A_1}{A_n}} \cdots [27]$$

This leads to the following definition—

• <u>Definition-8</u>: The quantity $\ln \Delta$ is the natural logarithm of the ratio of two successive amplitudes that are seperated by a half period $\tilde{T}/2$, the larger amplitude being the numerator. We denote $\lambda = \ln\Delta \doteq \gamma\tilde{T}/2$

• <u>Relaxation time</u>: For weakly damped motion the time dependence of amplitude is given by

$$A(t) = A_0 \exp(-\gamma t)$$

Putting $t = \frac{1}{\gamma}$ we find $A\left(t=\frac{1}{\gamma}\right) = A_0 e^{-1}$. We denote the time $1/\gamma$ by τ defined as below.

• <u>Definition-9</u>: Let γ be the damping factor of a weakly damped oscillation. The time $\tau = \frac{1}{\gamma}$ is required for the amplitude to reduce to $1/e$ th of its initial value. τ is called the relaxation time.

• <u>Remark</u>: 1. We know that $\lambda = \frac{\gamma\tilde{T}}{2}$, and $\tau = \frac{1}{\gamma}$. This gives $\boxed{\lambda\tau = \tilde{T}/2}$

2. Hence λ is the ratio of half the period of oscillation and the relaxation time.

• <u>Quality factor or Q-value</u>: We have a relation between the average energy stored $\langle E(t)\rangle$ and that dissipated is given by the formula,

$$\langle E(t)\rangle = (2\gamma)^{-1}\langle P(t)\rangle$$

Considering one period interval this equation becomes

$$\tilde{T}\langle E(t)\rangle = \tilde{T}(2\gamma)^{-1}\langle P(t)\rangle$$

$$\Rightarrow \frac{2\pi}{\tilde{\omega}}\langle E(t)\rangle = \frac{2\pi}{\tilde{\omega}}(2\gamma)^{-1}\langle P(t)\rangle$$

$$\Rightarrow 2\pi\left(\frac{2\gamma}{\tilde{\omega}}\right)\langle E(t)\rangle = \frac{2\pi}{\tilde{\omega}}\langle P(t)\rangle$$

$$\Rightarrow \frac{2\pi}{Q}\langle E(t)\rangle = \tilde{T}\langle P(t)\rangle$$

$$\Rightarrow Q = 2\pi\frac{\langle E(t)\rangle}{\tilde{T}\langle P(t)\rangle} \quad \text{---} \; [28]$$

In view of equation –[28] we get the following definition

• *Definition–10*: The quality factor Q of a damped oscillation with frequency $\tilde{\omega}$ and damping const γ is defined as

$$Q = \frac{\tilde{\omega}}{2\gamma} = 2\pi \times \frac{\text{average energy stored in one period}}{\text{average energy lost in one period.}} \quad \text{----[29]}$$

• *Remark*: 1. For $\omega_0 >> \gamma$, $\tilde{\omega} \simeq \omega_0$ and $Q = \frac{\omega_0}{2\gamma}$

i.e.; lower the damping higher the Q value.

2. We have seen that the loss of energy is given by $\langle E(t) \rangle = E_0 \exp(-2\gamma t)$. Using quality factor $Q = \frac{\omega_0}{2\gamma}$

$$\langle E(t) \rangle = E_0 \exp\left(-\frac{\omega_0 t}{Q}\right)$$

Let, Γ be the time by which the energy falls $\frac{1}{e}$th of its initial value,

$$\frac{1}{e} E_0 = E_0 e^{\left(-\frac{\omega_0 \Gamma}{Q}\right)}$$

$$\Rightarrow \frac{\omega_0 \Gamma}{Q} = 1$$

$$\Rightarrow \boxed{\Gamma = \frac{Q}{\omega_0} = \frac{1}{2\gamma} = \frac{\tau}{2}} \quad \text{----[29a]}$$

Γ is called the mean decay time.

II. Damped Oscillation.

1. A massless spring suspended from a rigid rod carries a mass of 200 g at its lower end. It is observed that the system oscillates with a time period of 0.2 s and the amplitude of oscillations reduces to half of its initial value in 30 seconds. Calculate,

   (i) Relaxation time

   (ii) Quality factor

   (iii) spring constant. and lograrilhmic decrement

[Hint: (i) Considering the equation of motion

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$$

where $\omega_0^2 = \frac{K}{m}$, K = spring constant.

Given $T = \frac{2\pi}{\omega_0} = 0.2\ s.$, $m = 200\ g$

Relaxation time $\tau = \frac{1}{\gamma}$

Now $A(t) = A_0 \exp(-\gamma t)$

$$\Rightarrow -\gamma t = \ln\frac{A(t)}{A_0}, \quad t = 30 \quad A(t) = \frac{1}{2}A_0$$

$$\Rightarrow \gamma = \frac{\ln 2}{30} = 0.023\ s^{-1}$$

$$\Rightarrow \tau = \frac{1}{\gamma} = \frac{30}{\ln 2} = \boxed{43.28\ s}$$

(ii) Quality factor $Q = \frac{\tilde{\omega}}{2\gamma} = \frac{\sqrt{\omega_0^2-\gamma^2}}{2\gamma}$

$$= \frac{1}{2}\left(\left(\frac{\omega_0}{\gamma}\right)^2 - 1\right)^{1/2}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi\ s^{-1} = \overset{31.4}{\cancel{98.6}}\ s^{-1}$$

$$\gamma = \frac{\ln 2}{30} = 0.023\ s^{-1}$$

Hence, $Q = \frac{1}{2}\left[\left(\frac{\overset{31.4}{\cancel{98.6}}}{0.023}\right)^2 - 1\right]^{1/2} = \boxed{683}$

(iii) Spring constant $K = m\omega_0^2 = 200 \times (31.4)^2$

$$= \boxed{197192\ dyne/cm.}$$

Now logarithmic decrement

$$\lambda = \frac{\gamma \tilde{T}}{2} = \frac{\gamma\, 2\pi}{2\tilde{\omega}} = \frac{\gamma \pi}{\tilde{\omega}}$$

$$= \frac{0.023 \times 3.14}{\sqrt{(31.4)^2 - (0.023)^2}} = \boxed{0.0023}$$

2. Calculate the frequency, relaxation time and the quality factor of an LCR circuit with

$L = 1\ mH$

$C = 5\,\mu F$

$R = 0.5\ \Omega.$

[Hint: $L = 1\ mH = 10^{-3}\ H$

$C = 5\,\mu F = 5 \times 10^{-6}\ F$

$R = 0.5\ \Omega.$

Considering the equation: $\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = 0$

and comparing it with $\ddot{q} + 2\gamma\dot{q} + \omega_0^2 q = 0$, the

frequency $\tilde{\omega} = \sqrt{\omega_0^2 - \gamma^2} = \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)^{1/2}$

$= \boxed{1.414 \times 10^4\ rad/s.}$

frequency $\tilde{\nu} = \frac{\tilde{\omega}}{2\pi} = \boxed{0.225 \times 10^4\ Hz.}$

Relaxation time $\tau = \frac{1}{\gamma} = \frac{2L}{R} = \boxed{4 \times 10^{-3}\ s.}$

Quality factor $Q = \frac{\tilde{\omega}}{2\gamma} = 28.3.$]

3. Show that the amplitude of a weakly damped oscillator reduces to half of its initial value in time $t = \tau \ln 2$, τ being the relaxation time.

[Hint: $A(t) = A_0 \exp(-\gamma t)$ and $\tau = \frac{1}{\gamma}$

Hence $A(t) = A_0 \exp(-t/\tau)$

$\Rightarrow A(t = \tau \ln 2) = A_0 \exp(-\ln 2)$

$\Rightarrow A = A_0/2$]