Economics for Engineers

Syllabus for HMTS3101

Module 1 :

a. Market: Meaning of Market, Types of Market, Perfect Competition, Monopoly, Monopolistic and Oligopoly market.

b. The basic concept of economics – Needs, Wants, Utility.

c. National Income-GDP, GNP. Demand & Supply, Law of demand, Role of demand and supply in price determination, Price Elasticity.

d. Inflation: meaning, reasons, etc. (6L)

Module 2 :

a. Business: Types of business, Proprietorship, Partnership, Joint-stock company, and cooperative society – their characteristics.

b. Banking: role of commercial banks; credit and its importance in industrial functioning.

c. Role of central bank: Reserve Bank of India.

d. International Business or Trade Environment. (4L)

Module 3 :

a. Financial Accounting-Journals. Ledgers, Trial Balance, Profit & Loss Account, Balance Sheet.

b. Financial Statement Analysis (Ratio and Cash Flow analysis). (8L)

c. Cost Accounting- Terminology, Fixed, Variable and Semi-variable costs, Break Even Analysis.

d. Cost Sheet, Budgeting and Variance Analysis.

e. Marginal Cost based decisions. (6L)

Module 4 :

a. Time Value of Money: Present and Future Value, Annuity, Perpetuity.

b. Equity and Debt, Cost of Capital.

c. Capital Budgeting: Methods of project appraisal - average rate of return - payback period - discounted cash flow method: net present value, benefit cost ratio, internal rate of return.

d. Depreciation and its types, Replacement Analysis, Sensitivity Analysis.

For HITK students by S.K.Sen

Economics for Engineers

Syllabus for HMTS3101

There will be 2 Internal Tests (One each from Module 1 and Module 2) as per Institute Dairy
Pattern of Qs for Internal tests will be as per HITK template (3 Qs of 10 marks each; no MCQs)
Pattern of Qs for End Sem exam will comprise 5 sections viz. – A, B, C, D and E –

Sec A will have 10 MCQs of 1 mark each (no choice) - 10

Sec B, C, D and E will have 2 Qs each from the Module 1, 2, 3 and 4 respectively [may have a), b), c) or a), b)] - Essay type Qs will be avoided

Students are required to answer 5 out of 8 Qs, selecting at least one from each Module

12x5 = 60

Evaluation : Max marks-100
Internal Test-30
Semester Test-70

Suggested Readings :

1. R. Narayanswami, Financial Accounting- A Managerial Perspective. Prentice-Hall of India Private Limited. New Delhi
2. Horne, James C Van, Fundamentals of Financial Management. Prentice-Hall of India Private Limited, New Delhi
3. H. L. Ahuja., Modern Economic Theory. S. Chand. New Delhi.
4. Newman, Donald G., Eschenbach, Ted G., and Lavelle, Jerome P. Engineering Economic Analysis. New York: Oxford University Press. 2012.

AEIE – Classes will be taken by SKS, AB and KC
ECE-D – Classes will be taken by SKS, AB and JG

DEPRECIATION & ITS TYPES

4B.	1	Equity and debt	• Basic concept • Difference between the two
4B.	1	Cost of capital	• Meaning • Calculation of WACC – problems

4D.	1	Depreciation	• Purpose • Methods of calculation * SLM, WDV, Sum of the digits * Numerical problems
4D.	1	Replacement Analysis Sensitivity analysis	• Basic concept • Basic concept

SLM – Straight Line Method

WDV – Written Down Value

In accountancy, depreciation refers to two aspects of the same concept :

(i) The decrease in value of assets (fair value depreciation)

(ii) The allocation of the cost of assets to periods in which the assets are used (depreciation with the matching principle).

A method of reallocating the cost of a tangible asset over its useful life span of it being in motion. Businesses depreciate long-term assets for both tax and accounting purposes. The former affects the balance sheet of a business or entity, and the latter affects the net income that they report. Generally the cost is allocated, as depreciation expense, among the periods in which the asset is expected to be used. This expense is recognized by businesses for financial reporting and tax purposes. Methods of computing depreciation, and the periods over which assets are depreciated, may vary between asset types within the same business and may vary for tax purposes. These may be specified by law or accounting standards, which may vary by country. There are several standard methods of computing depreciation expense.

Methods of depreciation **:** There are several standard methods of computing depreciation expense, generally based on either the passage of time or the level of activity (or use) of the asset.

We will discuss a few important ones viz. -

1. ***S**traight-line depreciation*
2. ***W**ritten Down Value*
3. ***D**oubling Declining balance method*
4. ***A**nnuity depreciation*
5. ***S**um-of-years-digits method*
6. ***U**nits-of-production depreciation method*
7. ***U**nits of time depreciation*
8. ***G**roup depreciation method*
9. ***C**omposite Depreciation Method*

1. ***Straight-line depreciation*** **:** Straight-line depreciation is the simplest and most often used method. In this method, the company estimates the salvage value (scrap value) of the asset at the end of the period during which it will be used to generate revenues (useful life). (The salvage value is an estimate of the value of the asset at the time it will be sold or disposed of; it may be zero or even negative. Salvage value is also known as scrap value or residual value.) The company will then charge the same amount to depreciation each year over that period, until the value shown for the asset has reduced from the original cost to the salvage value.

For example, a vehicle that depreciates over 5 years is purchased at a cost of \$17,000, and will have a salvage value of \$2000. Then this vehicle will depreciate at \$3,000 per year, i.e. (17-2) / 5 = 3K. This table illustrates the straight-line method of depreciation. Book value at the beginning of the first year of depreciation is the original cost of the asset. At any time book value equals original cost minus accumulated depreciation.

Book value = Original cost − Accumulated Depreciation Book value at the end of year becomes book value at the beginning of next year. The asset is depreciated until the book value equals scrap value.

If the vehicle were to be sold and the sales price exceeded the depreciated value (net book value) then the excess would be considered a gain and subject to depreciation recapture. In addition, this gain above the depreciated value would be recognized as ordinary income by the tax office. If the sales price is ever less than the book value, the resulting capital loss is tax deductible. If the sale price were ever more than the original book value, then the gain above the original book value is recognized as a capital gain.

If a company chooses to depreciate an asset at a different rate from that used by the tax office then this generates a timing difference in the income statement due to the difference (at a point in time) between the taxation department's and company's view of the profit.

2. *Written Down Value Method* :

1. Depreciation is calculated on written down value of the asset.
2. Amount of depreciation keeps on reducing every year.
3. Book value never gets reduced to zero.
4. Applicable for income tax purposes.
5. The total charge remains almost the same as in the initial years repairs are less and depreciation is high while in later years, repairs increase and depreciation decreases.
6. This method is suitable for assets which gives higher utility in the initial years like Machinery etc.

Written Down Value Method (WDV) of Depreciation

It is also known as Reducing Balance or Reducing Installment Method or Diminishing Balance Method. Under this method, the depreciation is calculated at a certain fixed percentage each year on the decreasing book value commonly known as WDV of the asset (book value less depreciation).

The use of book value (the balance brought forward from the previous year) and fixed rate of depreciation result in decreasing depreciation charges over the life span of the asset.

While applying the depreciation rate both salvage or scrap value and removal costs are ignored. It is not possible to reduce the book value to zero; but it can be reduced close to its salvage value at the end of its useful life.

The rate of depreciation may be determined using the following formula **:**

Depreciation Rate = $100[1 - \sqrt[n]{(S/C)}]$
where, n = number of years; S = Salvage value & C = Cost of Asset

For example, if a plant costs Rs. 8,000 with an estimated salvage value of Rs. 1,000 at the end of third year of its useful life, the rate of depreciation will be calculated thus:

Depreciation Rate = $100[1 - \sqrt[n]{(1000/8000)}] = 50\%$

Merits : The following are the advantages of this method

(a) As this method equalizes the total charges of using the asset (i.e., the amount of depreciation plus repair charges) from year to year, it is considered more equitable than straight-line method. This is because depreciation charges decline each year whereas repair charges increase year by year.

(b) It matches the service of the asset with the depreciation charge. When asset is more efficient in the initial years, higher depreciation is charged compared to later years. It is true about fixed assets such as motor vehicles.

(c) It recognizes the risk of obsolescence by charging the major part of depreciation in the early years of the life of the asset.

(d) It results in a better cash flow through tax deferral as under this method, the net income to be taxed is lower in the initial years and higher in subsequent years.

(e) As and when additions are made to the asset, fresh calculations of depreciation are not necessary.

(f) Income-tax authorities recognize this method.

Demerits : The main drawbacks of this method are as follows

(a) In subsequent years the original cost of the asset is completely lost sight of.

(b) The asset can never be reduced to zero.

(c) This method does not take into consideration the interest on capital invested in the asset.

(d) This method requires elaborate book-keeping. The determination of correct rate of depreciation is a complex task.

Suitability : This method is most suitable to those assets that have more efficiency in the beginning and late on decreases year after year. This method is usually adopted for plant and machinery, fixtures and fittings, motor vehicles, etc.

3. *__Doubling Declining balance method__* **:** Suppose a business has an asset with \$1,000 original cost, \$100 salvage value, and 5 years of useful life.

First, the straight-line depreciation rate would be 1/5, i.e. 20% per year. Under the double-declining-balance method, double that rate, i.e. 40% depreciation rate would be used. The table below illustrates this :

Depreciation rate	Depreciation expense	Accumulated depreciation	Book value at end of the year
		original cost \$1,000	
40%	400	400	600
40%	240	640	360
40%	144	784	216
40%	86.4	870.4	129.6
129.60 - 100.00	29.6	900	scrap value 100.00

For HITK students
by S.K.Sen

When using the double-declining-balance method, the salvage value is not considered in determining the annual depreciation, but the book value of the asset being depreciated is never brought below its salvage value, regardless of the method used. Depreciation ceases when either the salvage value or the end of the asset's useful life is reached.

Since double-declining-balance depreciation does not always depreciate an asset fully by its end of life, some methods also compute a straight-line depreciation each year, and apply the greater of the two. This has the effect of converting from declining-balance depreciation to straight-line depreciation at a midpoint in the asset's life.

With the declining balance method, one can find the depreciation rate that would allow exactly for full depreciation by the end of the period, using the formula:

$$\text{Depreciation rate} = 1 - \sqrt[N]{\{\text{Residual Value / Cost of Fixed Asset}\}}$$

where N is the estimated life of the asset (for example, in years).

For HITK students by S.K.Sen

4. *Annuity depreciation* : Annuity depreciation methods are not based on time, but on a level of Annuity. This could be miles driven for a vehicle, or a cycle count for a machine. When the asset is acquired, its life is estimated in terms of this level of activity. Assume the vehicle above is estimated to go 50,000 miles in its lifetime. The per-mile depreciation rate is calculated as: ($17,000 cost - $2,000 salvage) / 50,000 miles = $0.30 per mile. Each year, the depreciation expense is then calculated by multiplying the number of miles driven by the per-mile depreciation rate.

5. *Sum-of-years-digits method* : Sum-of-years-digits is a depreciation method that results in a more accelerated write-off than the straight line method, and typically also more accelerated than the declining balance method. Under this method the annual depreciation is determined by multiplying the depreciable cost by a schedule of fractions.

Sum of the years' digits method of depreciation is one of the accelerated depreciation techniques which are based on the assumption that assets are generally more productive when they are new and their productivity decreases as they become old. The formula to calculate depreciation under SYD method is **:**

SYD Depreciation = Depreciable Base x (Remaining Useful Life / Sum of the Years' Digits)

Depreciable Base = Cost - Salvage Value

Example : If an asset has original cost of \$1000, a useful life of 5 years and a salvage value of \$100, compute its depreciation schedule.

First, determine years' digits. Since the asset has useful life of 5 years, the years' digits are : 5, 4, 3, 2, and 1.

Next, calculate the sum of the digits : $5+4+3+2+1=15$

The sum of the digits can also be determined by using the formula $(n^2 + n) / 2$ where n is equal to the useful life of the asset in years. The example would be shown as $(5^2 + 5) / 2=15$

Depreciation rates are as follows :
5/15 for the 1st year, 4/15 for the 2nd year, 3/15 for the 3rd year, 2/15 for the 4th year, and 1/15 for the 5th year.

Depreciable base	Depreciation rate	Depreciation expense	Accumulated depreciation	Book value at end of the year
				original cost \$1,000
900	5/15	300 = (900 x 5/15)	300	700
900	4/15	240 = (900 x 4/15)	540	460
900	3/15	180 = (900 x 3/15)	720	280
900	2/15	120 = (900 x 2/15)	840	160
900	1/15	60 = (900 x 1/15)	900	100 (scrap value)

6. ***<u>Units-of-production depreciation method</u>*** **:** Under the units-of-production method, useful life of the asset is expressed in terms of the total number of units expected to be produced:

$$\text{Annual Depreciation Expense} = \frac{\text{(Cost of Fixed Asset – Residual Value)}}{\text{Estimated Total Production}} \times \text{Actual Production}$$

Suppose, an asset has original cost \$70,000, salvage value \$10,000, and is expected to produce 6,000 units.

Depreciation per unit = (\$70,000−10,000) / 6,000 = \$10
10 × actual production will give the depreciation cost of the current year.

The table below illustrates the units-of-production depreciation schedule of the asset.

For HITK students
by S.K.Sen

Units of production	Depreciation cost per unit	Depreciation expense	Accumulated depreciation	Book value at end of the year
				original cost -$70,000
1,000	10	10,000	10,000	60,000
1,100	10	11,000	21,000	49,000
1,200	10	12,000	33,000	37,000
1,300	10	13,000	46,000	24,000
1,400	10	14,000	60,000	10,000 (scrap value)

Depreciation stops when book value is equal to the scrap value of the asset. In the end, the sum of accumulated depreciation and scrap value equals the original cost.

7. ***<u>Units of time depreciation</u>*** **:** Units of time depreciation is similar to units of production, and is used for depreciation equipment used in mining or natural resource exploration, or cases where the amount the asset is used is not linear year to year.

A simple example can be given for construction companies, where some equipment is used only for some specific purpose. Depending on the number of projects, the equipment will be used and depreciation charged accordingly.

8. *<u>Group depreciation method</u>* : Group depreciation method is used for depreciating multiple-asset accounts using straight-line-depreciation method. Assets must be similar in nature and have approximately the same useful lives. ‘

9. *<u>Composite depreciation method</u>* : The composite method is applied to a collection of assets that are not similar, and have different service lives. For example, computers and printers are not similar, but both are part of the office equipment. Depreciation on all assets is determined by using the straight-line-depreciation method.

REPLACEMENT ANALYSIS

For HITK students
by S.K.Sen

The replacement problems are concerned with the situations that arise, when some items such as men, machines, vehicles etc. need replacement due to their decreased efficiency, frequent failure or breakdown. Such decreased efficiency or complete breakdown may either be gradual or all of a sudden.

The replacement problems arise because of the following factors –

1) The old item has become in worse condition and work badly or require expensive maintenance. Thus, the existing assets might be good technically, yet on economic consideration, it may not be worthwhile continuing with them and hence replacement becomes unavoidable. Machinery, equipment, building etc. fall in this category.

2) The old item has failed due to accident or otherwise and does not work at all or the old item is expected to fail shortly. Bulbs, tubes, vehicles, electronic components etc. fall in this category.

3) A better or more efficient design of machine or equipment has become available in the mkt, ie., obsolescence due to new discoveries and better designs of the equipment.

Thus, the problem of replacement is to decide best policy to determine an age at which the replacement is most economical instead of continuing at increased cost. The need for replacement arises in many situations so that different types of decisions may be taken.

For example –

(i) We may decide whether to wait for complete failure of the item (which might cause some loss), or to replace earlier at the expense of the higher cost of the item.

(ii) The expensive items may be considered individually to decide, whether, we should replace now or, if not, when it should be considered for replacement.

(iii) It may be decided, whether, we should replace by the same type of item or by different type / model of item.

Failure Mechanism of Items : The term ‘failure’ has a wider meaning in business than, what in our daily life. The failure mechanism of items or the systems can be divided in two categories as depicted below –

1) Gradual Failure **:** Efficiency of the item deteriorates with usage / time causing –

a) Increased expenditure of operating cost.

b) Decreased productivity of the eqpt.

c) Decrease in the value of the eqpt, ie., the resale value decreases eg. – items like piston rings, bearings, tyres etc.

2) *Sudden Failure* **:** Applicable to those items, that do not deteriorate markedly with service but which ultimately fail after some period of use. The period between installation and failure is not constant for any

particular type of eqpt but will follow some frequency distribution which may be progressive, retrogressive or random in nature.

a) Progressive failure : Probability of failure increases with increase in the life of an item. For example – electric bulbs, tubes, automobile etc.

b) Retrogressive failure : Certain items have more probability of failure in the beginning of their life and as the time passes the chances of failure become less ie., if the eqpt survive in initial period then its expected life increases. For example – Industrial eqpt, aircraft engines, satellite etc.

c) Random failure : Probability of failure is associated with items that fail from random causes but are not directly related to the age. In such a case, virtually all items fail before aging has any effect. For example – vacuum tubes in air-borne eqpt have been seen to fail at a rate independent of the age of the tube.

<u>Replacement Models</u> : It is evident that the study of replacement is a field of application rather than a method of analysis and mostly concerns with methods of comparing alternative replacement policies.

The replacement situations may be placed in following four categories –

(i) Replacement of eqpt, whose efficiency deteriorates with time.

(ii) Replacement of items that fail completely.

(iii) Problem of mortality & staffing and

(iv) Miscellaneous problems.

For the above mentioned replacement situations, the various types of replacement models will be –

(i) *Replacement of eqpt, whose efficiency deteriorates with time* **:** In such models, the systems can maintain the level of performance by installing a new unit at the beginning of some unit of time (year, month, week etc.) and decide to keep it upto some suitable period so as to minimize the total maintenance and other costs.

The simplest replacement model in such cases is one where the deterioration process is predictable and is represented by an increasing maintenance cost and possibly decreasing scrap value of the eqpt. Here the optimum life of the eqpt is determined on the assumption that increased age reduces the efficiency.

The problem is explained by deterministic models and is similar to inventory models where deterioration corresponds to demand against the desired level of efficiency. The cost of new machine is like a cost of replacement and maintenance costs corresponding to holding costs. These models can be solved by two methods namely –

(a) Calculation of cost per unit of time.
(b) Present value concept, which represents time value of money and enables the comparison on a one number basis.

(ii) *Replacement of items that fail completely & is expensive to be replaced* **:** The models are used in those situations, where replacement of units is done in anticipation of failure. In these models, the units are assumed to have relatively constant efficiency until they fail or die. These models require the knowledge of stochastic process involving probabilities of failures. Here, a replacement policy is formulated to balance the wasted life of items replaced before failure against the costs incurred, when items fails in service.

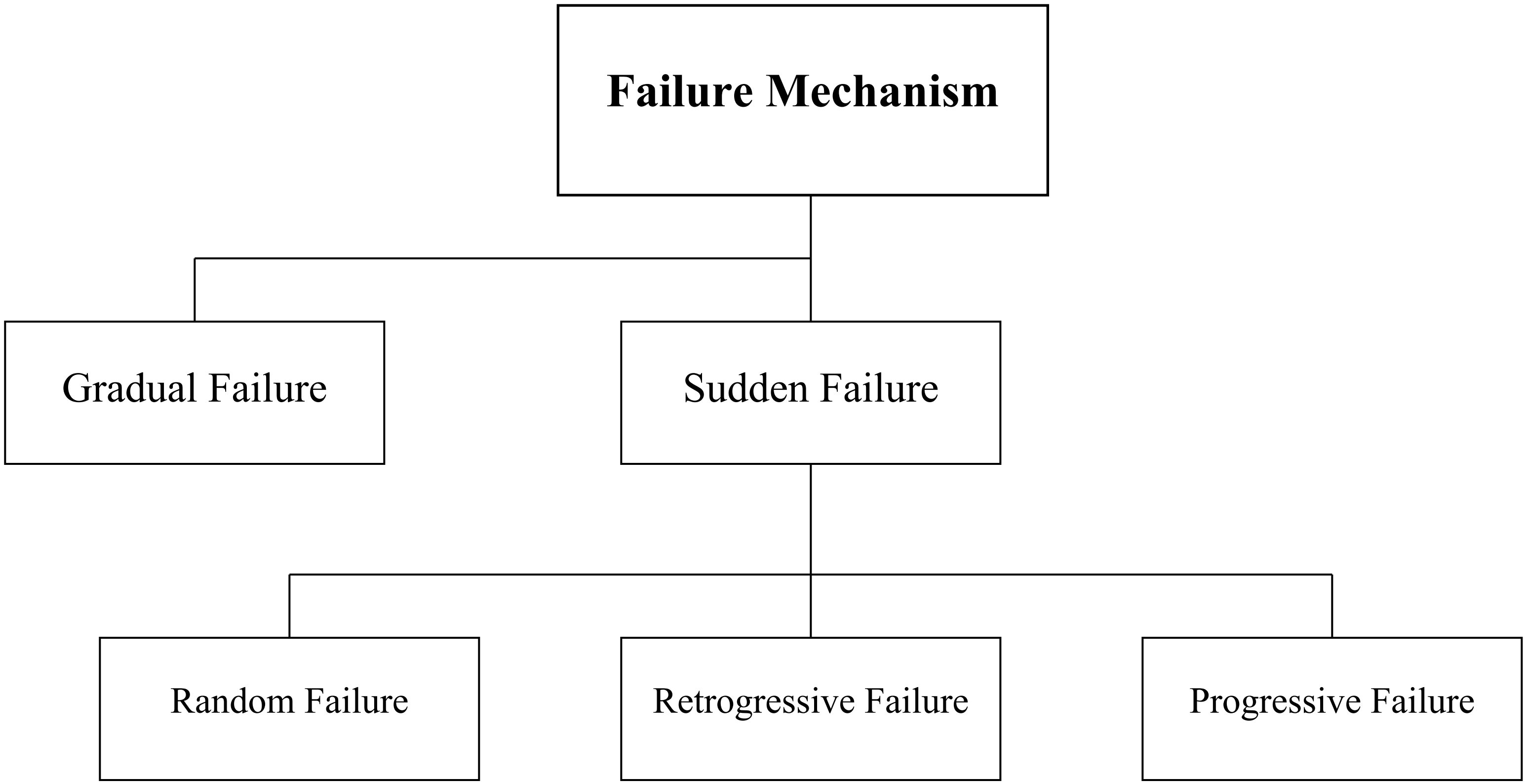
(iii) *Replacement of human beings in the orgn or staffing problem* **:** Requires a separate study and involve the life distribution for the service of staff in a system.

(iv) *Miscellaneous problems* **:** There can be eqpt, where the replacement of items may be necessary due to new research, development, designs etc. otherwise the eqpt may become obsolete. The construction and solution of such models will vary under different situations.

Factors for replacement

- Reduction of efficiency & expensive maintenance
- Complete failure of old items or failure expected shortly
- Obsolescence of existing system / improved m/cs or designs available

FAILURE MECHANISM



SENSITIVITY ANALYSIS

$$y = f(x_1,\ x_2,\ x_3, \dots\ x_n)$$

Dependent Variable (LHS) is a function of Independent Variable (RHS)

For HITK students
by S.K.Sen

What is a 'Sensitivity Analysis' : A sensitivity analysis is a technique used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. This technique is used within specific boundaries that will depend on one or more input variables, such as the effect that changes in interest rates will have on a bond's price.

Sensitivity analysis is a way to predict the outcome of a decision if a situation turns out to be different compared to the key prediction(s).

Sensitivity analysis is very useful when attempting to determine the impact the actual outcome of a particular variable will have if it differs from what was previously assumed. By creating a given set of scenarios, the analyst can determine how changes in one / more variable(s) will impact the target variable.

For example, an analyst might create a financial model that will value a company's equity (the dependent variable) given the amount of earnings per share (an independent variable) the company reports at the end of the year, and the company's price-to-earnings multiple (another independent variable) at that time. The analyst can create a table of predicted price-to-earnings multiples and a corresponding value of the company's equity based on different values for each of the independent variables.

The process of recalculating outcomes under alternative assumptions to determine the impact of variables under sensitivity analysis can be useful for a range of purposes, including -

- Testing the robustness of the results of a model or system in the presence of uncertainty.
- Increased understanding of the relationships between input and output variables in a system or model.

- Uncertainty reduction: identifying model inputs that cause significant uncertainty in the output and should therefore be the focus of attention if the robustness is to be increased (perhaps by further research).
- Searching for errors in the model (by encountering unexpected relationships between inputs and outputs).
- Model simplification – fixing model inputs that have no effect on the output, or identifying and removing redundant parts of the model structure.
- Enhancing communication from modelers to decision makers (e.g. by making recommendations more credible, understandable, compelling or persuasive).
- Finding regions in the space of input factors for which the model output is either maximum or minimum or meets some optimum criterion.
- In case of calibrating models with large number of parameters, a primary sensitivity test can ease the calibration stage by focusing

on the sensitive parameters. Not knowing the sensitivity of parameters can result in time being uselessly spent on non-sensitive ones.

Taking an example from economics, in any budgeting process there are always variables that are uncertain. Future tax rates, interest rates, inflation rates, headcount, operating expenses and other variables may not be known with great precision. Sensitivity analysis answers the question, "if these deviate from expectations, what will the effect be (on the business, model, system, or whatever is being analyzed), and which variables are causing the largest deviations?“

A mathematical model is defined by a series of equations, input variables and parameters aimed at characterizing some process under investigation. Some examples might be a climate model, an economic model, or a finite element model in engineering. Increasingly, such models are highly complex, and as a result their input/output relationships may be poorly understood. In such cases, the model can be viewed as a black box, i.e. the output is an opaque function of its inputs.

ADDITIONAL / OPTIONAL

DEPRECIATION

Example of Composite Depreciation Method

Asset	Historical cost	Salvage value	Depreciable cost	Life (yrs)	Depreciation per year
Computers	$5,500	$500	$5,000	5	$1,000
Printers	$1,000	$100	$900	3	$300
Total	$6,500	$600	$5,900	4.5	$1,300

Composite life equals the total depreciable cost divided by the total depreciation per year. $5,900 / $1,300 = 4.5 years. Composite depreciation rate equals depreciation per year divided by total historical cost. $1,300 / $6,500 = 0.20 = 20%

Depreciation expense equals the composite depreciation rate times the balance in the asset account (historical cost). (0.20 * $6,500) $1,300. Debit depreciation expense and credit accumulated depreciation.

When an asset is sold, debit cash for the amount received and credit the asset account for its original cost. Debit the difference between the two to accumulated depreciation. Under the composite method no gain or loss is recognized on the sale of an asset. Theoretically, this makes sense because the gains and losses from assets sold before and after the composite life will average themselves out.

To calculate composite depreciation rate, divide depreciation per year by total historical cost. To calculate depreciation expense, multiply the result by the same total historical cost. The result, not surprisingly, will equal to the total depreciation Per Year again.

Common sense requires depreciation expense to be equal to total depreciation per year, without first dividing and then multiplying total depreciation per year by the same number.

9. ***Tax depreciation*** **:** Most income tax systems allow a tax deduction for recovery of the cost of assets used in a business or for the production of income. Such deductions are allowed for individuals and companies. Where the assets are consumed currently, the cost may be deducted currently as an expense or treated as part of cost of goods sold. The cost of assets not currently consumed generally must be deferred and recovered over time, such as through depreciation. Some systems permit full deduction of the cost, at least in part, in the year the assets are acquired. Other systems allow depreciation expense over some life using some depreciation method or percentage. Rules vary highly by country, and may vary within a country based on type of asset or type of taxpayer. Many systems that specify depreciation lives and methods for financial reporting require the same lives and methods be used for tax purposes. Most tax systems provide different rules for real property (buildings, etc.) and personal property (equipment, etc.).

10. ***Capital allowances*** **:** A common system is to allow a fixed percentage of the cost of depreciable assets to be deducted each year. This is often referred to as a capital allowance, as it is called in the United Kingdom. Deductions are permitted to individuals and businesses based on assets placed in service during or before the assessment year. Canada's Capital Cost Allowance are fixed percentages of assets within a class or type of asset. Fixed percentage rates are specified by type of asset. The fixed percentage is multiplied by the tax basis of assets in service to determine the capital allowance deduction. The tax law or regulations of the country specifies these percentages. Capital allowance calculations may be based on the total set of assets, on sets or pools by year (vintage pools) or pools by classes of assets... Depreciation has got three methods only.

11. ***<u>Tax lives and methods</u>*** **:** Some systems specify lives based on classes of property defined by the tax authority. Canada Revenue Agency specifies numerous classes based on the type of property and how it is used. Under the United States depreciation system, the Internal Revenue Service publishes a detailed guide which includes a table of asset lives and the applicable conventions. The table also incorporates specified lives for certain commonly used assets (e.g., office furniture, computers, automobiles) which override the business use lives. U.S. tax depreciation is computed under the double declining balance method switching to straight line or the straight line method, at the option of the taxpayer. IRS tables specify percentages to apply to the basis of an asset for each year in which it is in service. Depreciation first becomes deductible when an asset is placed in service.

12. *Additional depreciation* **:** Many systems allow an additional deduction for a portion of the cost of depreciable assets acquired in the current tax year. The UK system provides a first year capital allowance of £50,000. In the United States, two such deductions are available. A deduction for the full cost of depreciable tangible personal property is allowed up to $500,000 through 2013. This deduction is fully phased out for businesses acquiring over $2,000,000 of such property during the year. In addition, additional first year depreciation of 50% of the cost of most other depreciable tangible personal property is allowed as a deduction. Some other systems have similar first year or accelerated allowances.

13. ***Real property*** **:** Many tax systems prescribe longer depreciable lives for buildings and land improvements. Such lives may vary by type of use. Many such systems, including the United States and Canada, permit depreciation for real property using only the straight line method, or a small fixed percentage of cost. Generally, no depreciation tax deduction is allowed for bare land. In the United States, residential rental buildings are depreciable over a 27.5 year or 40 year life, other buildings over a 39 or 40 year life, and land improvements over a 15 or 20 year life, all using the straight line method.

13. ***Averaging conventions*** **:** Depreciation calculations require a lot of record-keeping if done for each asset a business owns, especially if assets are added to after they are acquired, or partially disposed of. However, many tax systems permit all assets of a similar type acquired in the same year to be combined in a "pool". Depreciation is then computed for all assets in the pool as a single calculation. These calculations must make assumptions about the date of acquisition. The United States system allows a taxpayer to use a half year convention for personal property or mid-month convention for real property. Under such a convention, all property of a particular type is considered to have been acquired at the midpoint of the acquisition period. One half of a full period's depreciation is allowed in the acquisition period (and also in the final depreciation period if the life of the assets is a whole number of years). United States rules require a mid-quarter convention for per property if more than 40% of the acquisitions for the year are in the final quarter.

REPLACEMENT ANALYSIS

Notes :

1. The general approach to solve replacement problems

2. Although the actual time of failure is unpredictable for any particular item, the likely failure pattern could be established by observations and experiences. The optimum replacement policy can be formulated by generating the probability distribution for the given situation and then using them in conjunction with relevant cost information. The analysis needs following information –

(i) Objective assessment of the probability of the item failing at a particular point in time.

(ii) Assessments of the cost of replacement in terms of

- Actual cost of the item
- Direct costs of labour involved in replacement
- Costs of disruption in terms of lost production, lost orders etc. These approaches are discussed in detail in the remaining sections of this chapter.

Replacement of Items whose Efficiency Deteriorates with Time : Generally the cost of maintenance increases and repair increases with time and a stage comes when these costs become so large that, it is better and economical to replace the item with new one. Sometimes in a system there may be a no. of alternative choices and we may like to make a comparison between various alternatives by considering the cost involved in each choice. In this section, we shall derive various techniques for making such comparisons under different conditions on the basic assumption that suitable expressions for maintenance costs are available.

The various techniques for analyzing such situations can be broadly classified in the following categories –

Replacement of Items whose Maintenance cost increases with Time & Value of Money remains constant for the period **:** The aim here is to determine the optimum replacement age of an item / eqpt whose repair and maintenance costs increases with time and value of money remains constant for the period.

When we say that the value of money remain constant for the period, we mean that there is no time value of money. That is to say, a rupee received at the end of period, say 5 years, is regarded as good as rupee recd now.

We know that the cost of a eqpt over a given time period, say n years, has three elements & the total cost will be given by –

Total Cost = Purchase Price – Value remaining after *n* yrs (scrap value) + Mainte cost for n yrs

Let C = Purchase price of the eqpt
S_n = The scrap value of the eqpt at the end of *n* yrs
R_n = The repair and maintenance cost of the eqpt in *n* yrs.

Case I **:** When time is a continuous variable – In this case , it is not necessary that time should be considered as 1, 2, 3 etc. yrs. The time period can be continuous to take any value. If the eqpt is used for n yrs, the total maintenance cost is given by –

Total repair and maintenance cost

$$(R_n) = \int_0^n R_t.dt$$

The total cost, T(n), of owning and maintaining the eqpt for n yrs will be given by –

$$T(n) = C - S_n + \int_0^n R_t.dt$$

Average annual cost incurred on the eqpt / item will be given by –

$$A(n) = \{ C - S_n + \int_0^n R_t.dt\} / n$$

Rule for replacement, when time is continuous – Average annual cost will be minimized by replacing the machine / items when the average cost to date becomes equal to the current maintenance cost.

<u>*Case II*</u> **:** When time *'t'* is a discrete variable - In this case , the period of time is taken as one year and *t* can take the values 1, 2, 3,. . . etc. Then the total repair and maintenance cost is given by –

Total repair and maintenance cost

$$(R_n) = \sum_{0}^{n} R_t$$

The total cost, T(n), of owning and maintaining the eqpt for n yrs will be given by –

$$T(n) = C - S_n + \sum_{0}^{n} R_t \quad \text{where, } n = 1,\ 2,\ 3,\ \ldots\ldots, n)$$

Correspondingly, Average annual cost incurred on the eqpt / item will be given by –

$$A(n) = \{\, C - S_n + \sum_{0}^{n} R_t \} / n$$

If S_n is assumed to be monotonically decreasing and $\sum R_t$ is assumed to be monotonically increasing, then a method of providing an optimum value of n can be developed. In addition, decision rules for replacement can also be established. Under these assumptions there will exist a value for n that results in the minimum average cost.

Thus, $A(n-1) > A(n) < A(n+1)$

From the above expression, the following two inequalities can be established –

$$A(n-1) - A(n) > 0$$

$$A(n+1) - A(n) > 0$$

Thus, we can conclude that, if the loss in resale value for the next period $(S_n - S_{n+1})$ plus running cost, R_{n+1} is greater than the average total cost $A(n)$ of $n^{\underline{th}}$ yr, then it is economic to replace, ie.,

$$A(n) < (S_n - S_{n+1}) + R_{n+1}$$

Rule for replacement, when time is discrete – Average annual cost will be minimized by replacing the machine / items when the next period's maintenance cost becomes greater than the current average cost.

To find A(n) we shall, first of all, determine the maintenance costs accumulated to the n^{th} yr. To this, the cost of eqpt, net of its salvage value, would be added. The two would then be aggregated to get the total cost, T(n), and then the total cost divided by n, the no. of yrs, would give the average cost.

Problem 4.1 **:** The cost of a machine is Rs 6100 and its scrap value is only Rs 100. The maintenance costs are found from experience to be –

When should the machine be replaced?

Solution : Find total cost per yr during the life of the machine as follows –

Total cost in first yr = Purchase cost – scrap value + Mainte cost in 1st yr

= 6100 – 100 + 100 = Rs 6100

Thus, average cost in 1st yr will also be same as Rs 6100.

Total cost upto 2 yrs = Purchase cost – scrap value + Mainte cost in 1st two yrs

= 6100 – 100 + (100 + 250) = Rs 6350

Thus, average cost per yr during 1st two yrs = Rs 6350 / 2 = Rs 3175

In a similar fashion,

Average cost per yr during 1st three yrs = Rs(6350 + 400) / 3 = 6750 / 3 = Rs 2250

Average cost per yr during 1st four yrs = Rs(6750 + 600) / 4 = 7350 / 4 = Rs 1837.5

Average cost per yr during 1st five yrs = Rs(7350 + 900) / 5 = 8250 / 5 = Rs 1650

Average cost per yr during 1st six yrs = Rs(8250 + 1250) / 6 = 9500 / 6 = Rs 1583.33

Average cost per yr during 1st seven yrs = Rs(9500 + 1600) / 7 =11100 / 7 =Rs 1585.71

Average cost per yr during 1st eight yrs = Rs(11100 + 2000) / 8 = 13100 / 8 = Rs 1637.5

These computations may be summarized in the tabular form in next slide –

Replace at the end of yr (n)	Mainte cost (R_n)	Total Mainte cost (R_n)	Difference between price & resale price ($C - S_n$)	Total cost (T_n) 5	Average cost
1	2	3	4	3 + 4	6
1	100	100	6000	6100	6100
2	250	350	6000	6350	3175
3	400	750	6000	6750	2250
4	600	1350	6000	7350	1837
5	900	2250	6000	8250	1650
6	1250	3500	6000	9500	1583
7	1600	5100	6000	11100	1586
8	2000	7100	6000	13100	1638

From the table, we find that A(n) is minimum in the 6th yr being equal to 1583. Also the maintenance cost in the 7th yr is more than 1583. Hence, the machine should be replaced at the end of 6th yr otherwise the average annual cost would increase.

Alternatively, last column of above table shows that the average cost starts increasing in the 7^{th} yr, ie., at the end of 6^{th} yr.

<u>Problem 2</u> **:** A truck owner from his past experience envisaged that the maintenance cost per yr, of a truck whose purchase price is Rs 1,50,000 and the resale value of truck will be as follows –

Determine at which time it is profitable to replace the truck.

Solution : The average cost per yr during the life of the truck is calculated as shown in the table in next slide–

Replace the end of yr (n) 1	Mainte cost (R_n) 2	Cumulative Mainte cost (R_n) 3	Difference between price & resale price ($C - S_n$) 4	Total cost (T_n) 5 = 3 + 4	Average cost per yr Col 6 = Col 5/Col 1
1	10,000	10,000	1,50,000 - 1,30,000 = 20,000	30,000	30,000
2	50,000	60,000	30,000	90,000	45,000
3	20,000	80,000	35,000	1,15,000	38,333
4	25,000	1,05,000	45,000	1,50,000	37,500
5	30,000	1,35,000	60,000	1,95,000	39,000
6	40,000	1,75,000	75,000	2,50,000	41,666
7	45,000	2,20,000	90,000	3,10,000	44,286
8	50,000	2,70,000	1,00,000	3,70,000	46,250

Though the minimum value of the average cost in the last column of the table occurs at the end of $1^{\underline{st}}$ yr, we consider the trend of this cost and find that it becomes minimum at the end of $4^{\underline{th}}$ yr. So, we conclude that, it is profitable to replace the truck at the end of every $4^{\underline{th}}$ yr, as it will be impracticable to replace the truck at the end of every yr.

Simple Interest (SI) = PRT / 100

Compound Interest (CI) = A – P,

where, Amt (Future value) = $P \times (1 + r / 100)^n$

Annuity : Sequence of equal payments made at equal intervals of time.

Present value of Annuity

$$= [A_n / (1 + r)] + [A_n / (1 + r)^2] + [A_n / (1 + r)^3] + \dots\dots + [A_n / (1 + r)^n]$$

where, A_n = Amount of equal instalment or Annuity

n = no. of instalments, r = rate of interest

Problem : The cost pattern for two machines A & B, when time value of money is not considered, is given in the table below -

Year	Cost at the beginning of the yr	
	Machine A	Machine B
1	900	1400
2	600	100
3	700	700
Total outlay	Rs 2200	Rs 2200

Find the cost pattern for each machine, when money is worth 10% per yr, and hence, find which machine is less costly.

Solution **:** The total outlay for three yrs for –
Machine A = Rs (900 + 600 + 700) = Rs 2200
Machine B = Rs (1400 + 100 + 700) = Rs 2200

Here, we observe that the total outlay for both the machines is same for three yrs, when the time value of money is not taken into account. Hence, both the machines will appear to be equally good in this case.

Now consider, when value of money changes with time at the rate of 10% per yr, the discount cost pattern for each machine for 3 yrs is shown in the following table in next slide -

Year	Cost at the beginning of the yr	
	Machine A	Machine B
1	900	1400
2	$600 \text{ x } \{1 / (1 + 0.01)^1\} =$ 545.45	$100 \text{ x } \{1 / (1 + 0.01)^1\} =$ 90.90
3	$700 \text{ x } \{1 / (1 + 0.01)^2\} =$ 578.52	$700 \text{ x } \{1 / (1 + 0.01)^2\} =$ 578.52
Total outlay	Rs 2023.97	Rs 2069.43

The data shows that the total outlay for machine A is less than that for machine B. Hence, machine A will be preferred, when we consider time value of money.

As already mentioned, the money value can be interpreted in two different ways. Accordingly, the optimal replacement policy can be determined by the following two methods –

(i) The maintenance cost increases with time and the money value decreases with constant rate, ie., depreciation value is given.

(ii) The amount to be spent is borrowed at a given rate of interest under the condition of repaying it in pre-decided number of instalments.

To consider the problem of replacement, when time value of money is taken into consideration, we shall assume that -

(i) The eqpt in question has no salvage value and
(ii) The maintenance costs are incurred in the beginning of the different time periods (which we shall take to be yrs).

Working under these assumptions, we first determine the annualized cost or the weighted average of costs for different years of the life of the eqpt. This is done as follows –

<u>Step 1</u> **:** Find the present value of the mainte cost for each of the yrs, by multiplying the cost value by an appropriate present value factor based on the given rate of discount and the time.

<u>Step 2</u> **:** Accumulate present values obtained in step 1 upto each of the yrs 1, 2, 3, ... Add the cost of the eqpt to each of these values.

<u>Step 3</u> **:** Accumulate present value factors upto each of the yrs 1, 2, 3, ...

Step 4 **:** Divide the cost plus cumulative mainte costs for each yr, obtained in step 2, by the corresponding cumulative present value factors obtained in step 3. This gives the annualized costs for the various yrs.

It may be noted that the annualized cost is also the weighted average of costs – the Present Value (PV) factors serve as weights and the average is calculated, as in the usual way, by dividing the summation of the products of the costs and their respective weights by the summation of the weights.

Once the weighted average of costs for different yrs is obtd, the following rules are followed to decide on the replacement –

a) Do not replace if the next period's cost is less than the weighted average of the previous costs.

b) Replace if the next period's cost is greater than the weighted average of the previous costs.

Therefore, the optimum replacement age "*n*" must satisfy the following relationship –

$$R_{n+1} > \{C + R_1 + R_2 V + R_3 V^2 + \dots + R_n V^{n-1}\} / \{1 + V + V^2 + V^3 + \dots + V^{n-1}\} > R_n$$

or, $R_{n+1} > \{F(n)\} / \sum V^{n-1} > R_n$ where, $V = 1 / (1 + r)$

In operational terms, the criterion for optimal replacement is – replace every *n* yrs, where A(n), the annualized cost of replacing every *n* yrs (that is, the weighted average of costs for *n* yrs) is the minimum.

Selecting Best Eqpt : In the problem of choosing the best eqpt / item, the cost that are constant over time for each given eqpt / item will still have to be taken into.

account, although these costs may differ for each eqpt. Only those costs that are same for the eqpt under comparison can be excluded.

Suppose you have to select among two machines M_1 and M_2. Best selection can be done by adopting the following procedure.

Step 1 **:** Find the best replacement age for machines M_1 and M_2 by using the relationship -

$$R_{n+1} > \{C + R_1 + R_2V + R_3V^2 + \ldots\ldots + R_nV^{n-1}\} / \{1 + V + V^2 + V^3 + \ldots\ldots + V^{n-1}\} > R_n$$

or, $R_{n+1} > \{F(n)\} / \sum V^{n-1} > R_n$ where, $V = 1 / (1 + r)$

Suppose the optimum replacement age for machines M_1 and M_2 comes out to be n_1 and n_2 respectively.

Step 2 : Compute the fixed annual payment, ie., weighted average cost for each machine by using the formula -

$$x = \{C + R_1 + R_2V + R_3V^2 + \ldots\ldots + R_nV^{n-1}\} / \{1 + V + V^2 + V^3 + \ldots\ldots + V^{n-1}\} = \{F(n)\} / \sum V^{n-1}$$

and substitute in this formula $n = n_1$ for machine M_1 and $n = n_2$ for machine M_2. Let it be x_1 and x_2 for machines M_1 and M_2 respectively.

Step 3 : The following possibilities can occur –

If $x_1 < x_2$ then choose machine M_1

If $x_1 > x_2$ then choose machine M_2

If $x_1 = x_2$ then both machines are equally good.

Remark : This method can be extended to any number of eqpts / items. In words the procedural step can be stated as – First find the best replacement ages & corresponding weighted average costs x_1, x_2, x_3 . . . . . for all the n eqpts. Then select the eqpt / item corresponding to the least weighted average cost.

<u>Problem</u> **:** A machine X costs Rs 5000. Its mainte cost is Rs 1000 each of the first 4 yrs & then it increases by Rs 200 every yr. Assuming that the machine has no salvage value and the maintenance cost is incurred in the beginning of each yr, determine the optimal replacement time for the machine assuming time value of money is 10%.

Solution : Presenting the data in the table below –

Year	Mainte cost (R_n)	PV factor (V^{n-1})	PV of Rn = (R_nV^{n-1})	Cost + Cum Rn = $C + \sum R_nV^{n-1}$ = F(n)	Cum PV factor ($\sum V^{n-1}$)	Annualised cost = F(n) / ($\sum V^{n-1}$)
Col 1	Col 2	Col 3	Col 4	Col 5	Col 6	Col 7
1	1000	1	1000	6000	1	6000
2	1000	0.9091	909.1	6909.1	1.9091	3619
3	1000	0.8264	826.4	7735.5	2.7355	2828
4	1000	0.7513	751.3	8486.8	3.4868	2434
5	1200	0.683	819.6	9306.4	4.1698	2232
6	1400	0.6209	869.2	10175.6	4.7907	2124
7	1600	0.5645	903.2	11078.8	5.3552	2069
8	1800	0.5132	923.8	12002.6	5.8684	2045
9	2000	0.4665	933	12935.6	6.3349	2042
10	2200	0.4241	933	13868.6	6.759	2052
11	2400	0.3855	925.2	14793.8	7.1445	2071
12	2600	0.3505	911.3	15705.1	7.495	2095

In the above table, we compare the 2nd column, where values are constantly increasing, with the last column, where values 1st decrease and then increase. Looking to this, we should decide to replace the machine in the latest year that the 7th column value exceeds the corresponding 2nd column value. Here 9th yr is the last because in the yr next to it, the mainte cost of Rs 2200 exceeds the value of Rs 2052 in the 7th column. It is the yr in which the annualized cost is the least. Thus, optimal replacement interval for the machine is the 9th yr.

Problem : A mfrr is offered two machines A and B. A is priced at Rs 5000 and running costs are estimated at Rs 800 for each of the 1st five yrs, increasing by Rs 200 per yr in the 6th and subsequent yrs. Machine

B, which has the same capacity as A, costs Rs 2500 but the running costs of Rs 1200 per yr for six yrs, increasing by Rs 200 per yr thereafter.

If money is worth 10% per yr, which machine should be purchased? Assuming that machines will eventually be sold for scrap at a negligible price.

<u>Solution</u> **:** Since money is worth 10% per anum, discount rate is given by

$$V = 1 / (1 + 0.1) = 0.9091$$

Therefore, the optimum replacement age '*n*' must satisfy the relationship -

$$R_{n+1} > \{C + R_1 + R_2 V + R_3 V^2 + \ldots\ldots + R_n V^{n-1}\} / \{1 + V + V^2 + V^3 + \ldots\ldots + V^{n-1}\} > Rn$$

or, $R_{n+1} > \{F(n)\} / \sum V^{n-1} > R_n$ where, $V = 1 / (1 + r)$

For the best replacement age n, tabulate the required calculations of machine A -

Year	Mainte cost (R_n)	PV factor (V^{n-1})	PV of Rn $= (R_n V^{n-1})$	Cost + Cum Rn $= C + \sum R_n V^{n-1}$ $= F(n)$	Cum PV factor ($\sum V^{n-1}$)	Annualised cost $= F(n) / (\sum V^{n-1})$
1	2	3	4	5	6	7
1	800	1	800	5800	1	5800
2	800	0.9091	737	6527	1.9091	3419
3	800	0.8264	661	7188	2.7355	2628
4	800	0.7513	601	7889	3.4868	2234
5	800	0.683	546	8335	4.1698	1999
6	1000	0.6209	621	8956	4.7907	1896
7	1200	0.5645	677	9633	5.3552	1799
8	1400	0.5132	718	10351	5.8684	1764
9	1600	0.4665	746	11097	6.3349	1752
10	1800	0.4241	763	11860	6.759	1800

It is observed that $R_{10} = 1800$ becomes greater than the weighted average cost Rs 1752 for 9th yr ie., R_9. Hence, it would be best to replace the machine A after 9yrs.

Calculation for machine B

Year	Mainte cost (R_n)	PV factor (V^{n-1})	PV of Rn = ($R_n V^{n-1}$)	Cost + Cum Rn = C + $\sum R_n V^{n-1}$ = F(n)	Cum PV factor ($\sum V^{n-1}$)	Annualised cost = F(n) / ($\sum V^{n-1}$)
1	2	3	4	5	6	7
1	1200	1	1200	3700	1	3700
2	1200	0.9091	1091	4791	1.9091	2510
3	1200	0.8264	991	5783	2.7355	2114
4	1200	0.7513	902	6684	3.4868	1917
5	1200	0.683	820	7504	4.1698	1800
6	1200	0.6209	745	8249	4.7907	1722
7	1400	0.5645	790	9039	5.3552	1688
8	1600	0.5132	821	9860	5.8684	1680
9	1800	0.4665	840	10700	6.3349	1689

For HITK students
by S.K.Sen

Also, from the table above, equivalent fixed annual payment for machine A is read from the last column as x_1 = weighted average for 9 yrs = Rs 1752.

In asimilar fashion, best replacement age for machine B can be calculated. Calculations for machine B are given in the following table –
Since $R_9 > R_8$ or, in other words, weighted average for 9 yrs is greater than that for 8 yrs (1689 > 1680), it would be best to replace machine B after 8 yrs.

The equivalent fixed annual payment for machine B as in the table is x_2 = weighted average for 8 yrs = Rs 1680

Since, $x_1 > x_2$, it would be better to purchase machine B instead of A, although the average of actual payments (without considering time value) is Rs 1578 for A and Rs 1588 for B. [Avg actual payment for A = (5000 + 9200) / 9 = 1578 and for B = (2500 + 10200) / 8 = 1588].

Notes :

1) In the above question, we have assumed that the eqpt has no salvage value. We can drop the assumption to consider the salvage value at the end of a particular yr is translated into PV terms by applying an appropriate discount factor (keeping in mind that for a given yr, the mainte cost is assumed to be incurred in the beginning of the yr, while salvage value represents the asset value at the end of the yr, so that same discount rate should not be applied to both of them). This value is then subtracted from the

cost of the eqpt and the difference added to the cumulative mainte costs up to the yr in question.

2) A 2nd assumption made in the above problem is that the costs are incurred in the beginning of each yr. If the costs are incurred during the yr instead, they may be discounted to the start of the yr. If, we could assume that the costs are incurred continuously over the yr, then discounting can be done using integral calculus. A more reasonable assumption for most problems of the type considered, sufficient accuracy may be achieved, if we assume that the expenditure takes place half way through the yr. If j be the rate of interest per half yr, and r is the rate per anum, then one rupee invested for six months would become $(1 + j)$ rupee and if the amount is invested for a 2nd

six-month period, the total at the end of the yr would be $(1 + j)^2$ rupees [principal of compound interest; Amount = P $(1 + j)^n$ where j is converted to fraction ie., j = j% / 100]

3) The implicit assumption in our analysis till this point is that the eqpt / item concerned is to be used for ever. If the eqpt is reqd for a certain fixed time period, then the formulation needs to be modified.

<u>Replacement of Items when it Fails Suddenly</u> : Now we shall consider the determination of optimal policy for replacing items used in large numbers, which do not deteriorate gradually but fails all of a sudden. They may not involve any mainte cost, but fail without warning. Light bulbs and fluorescent tubes are examples of this type. It may, however, be obsd that although the failure of a single unit, a bulb for instance, is random, the behaviour of the group as a whole is expected to be fairly stable.

When immediate replacement of the item may not be available, the break down implies loss in production, idle inventory, idle labour and many other losses, so that the failure of the item puts the orgn to heavy loss. Naturally in such circumstances it is advisable to formulate a suitable replacement policy. Sometimes, the items may be replaced even, when they are in working order, to decrease the probability of breakdown. Any of the following two courses can be followed in such situations.

1) Individual replacement policy. Whenever any item fails, it should be immediately replaced.

2) Grp replacement policy. Here, we decide that all the items of the system should be replaced after a certain period irrespective of the fact that items have failed or not with a provision that, if any item fails before this time it can be replaced individually.

1) *Individual Replacement Policy* **:** Under this policy, an item is immediately replaced as soon as it fails. This policy is adopted, when the tendency of an item to fail increases with the time it has been in service. For this, a record is to be maintained of how long the item has been in use and it is replaced, when either it fails or it reaches some service life, say T, but not failed. This policy can also be adopted, where a no. of items are used simultaneously. The aim here is to determine optimum value of service life, T.

To discuss this type of replacement policy, we consider the problem of human population. No grp of population ever existed under the conditions that –

a) All deaths are immediately replaced by births and
b) There are no other entries or exits.

But the reason for stating this problem under these two assumptions is the analysis of individual replacement, when item fails completely become easy by keeping the virtual human population in mind. Such problem can be translated into industrial items, where death is equivalent to a part failure and birth is equivalent to new replacement. Thus, industries also face a fairly common situation.

The formulation of replacement policy in such situations depends upon the probability of failure. The probability distribution of failure (or life span of an item) can be determined by its mortality table which is explained below –

Mortality Tables **:** Before we know about mortality table, let us understand Mortality Theorem. It states, "*A large population is subject to a given mortality law for a very long period of time. All deaths are immediately replaced by death and there are no other entries or exits. Then the age distribution ultimately become stable and that the number of deaths per unit time becomes constant (which is equal to the size of the total population divided by the mean age of death).*"

We all know that it is difficult to predict, when any item in a system will fail. In order to obtain a probability distribution of failure of items in the system, we make an assumption that failures occur only at the end of each given period. The objective is then to find optimum period, in which items can be replaced, so that sum of the costs involved is minimum.

Mortality table may be used to obtain a probability distribution of life span of item in the system.

Let $S(t)$ = No. of items surviving at time '*t*'.

$S(t-1)$ = No. of items surviving at time '*(t – 1)*'.

N = Total no. of items in the system

Then probability of failure of items during the interval t and (t – 1) is given by –

$$\{S(t-1) - S(t)\} / N$$

The conditional probability that an item survived upto age $(t - 1)$ and will fail in the next period is given by –
$\{S(t - 1) - S(t)\} / S(t - 1)$

This has been explained with the help of a problem.

2) *Group Replacement Policy* **:** Grp replacement policy consists of two steps. Firstly, it consists of individual replacements at the time of failure of any unit in the system and then there is grp replacement of existing live units at some suitable time. Here, the individual replacement at the time of failure ensures running of the system, whereas, grp replacement after some time interval will reduce the probability of failure. Such policy requires two fold consideration namely –
(i) The rate of individual replacement during the period
(ii) The total cost incurred due to individual and grp replacement during the period.

Thus, for formulation of grp replacement policy, one should know –

a) The probability of failure
b) Loss incurred due to these failures
c) Cost of individual replacement and
d) The cost of grp replacement.

Grp replacement policy is defined in the following theorem and later it is explained by numerical problem.

Theorem (Group Replacement Policy) **:**

(a) One should grp replace at the end of t^{th} period, if the cost of individual replacements for the t^{th} period is greater than the average cost per period by the end of t^{th} period.

(b) One should not adopt grp replacement policy at the end of t^{th} period if the cost of individual replacements at the end of $(t-1)^{\underline{th}}$ period is less than the average cost per period through the end of $t^{\underline{th}}$ period.

<u>Problem</u> **:** A computer contains 10,000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Re 1 only. If, all the resistors are replaced at the same time, the cost per resistor would be reduced to 35 paise. The percent surviving say S(t) at the end of month t and P(t) the probability of failure during the month t, are –

What is the optimum replacement plan?

Solution : The whole problem can be divided into two parts –

i) There is a policy of individual replacement.

ii) There is a policy of grp replacement.

It should be noted that no resistor survives for more than 6 months. Thus, a resistor which has survived for 5 months is sure to fail during 6^{th} month. We assume that the resistor failing during a month are replaced just at the end of the month.

Let N_i denote the no. of resistors replaced at the end of i^{th} month. The different values of N_i can be calculated in the following way -

N_0 = No. of resistors at the beginning = 10,000

N_1 = No. of resistors being replaced by the end of 1^{st} month.

= No. of resistors in the beginning x probability that a resistor fails during 1^{st} month of installation

$= N_0 \times P_1 = 10{,}000 \times 0.03 = 300$

For HITK students
by S.K.Sen

N_2 = No. of resistors being replaced by the end of 2nd month.
= No. of resistors in the beginning x Probability that these resistors fail during 2nd month + No. of resistors replaced in 1st month x Probability that these fail during 2nd month

$$= N_0 \times P_1 + N_1 \times P_1 = 10{,}000 \times 0.07 + 300 \times 0.03 = 709$$

Similarly,

$$N_3 = N_0 \times P_3 + N_1 \times P_2 + N_2 \times P_1$$
$$= 10{,}000 \times 0.2 + 300 \times 0.07 + 709 \times 0.03 = 2042$$

$$N_4 = N_0 \times P_4 + N_1 \times P_3 + N_2 \times P_2 + N_3 \times P_1$$
$$= 10{,}000 \times 0.4 + 300 \times 0.2 + 709 \times 0.07 + 2042 \times .03 = 4171$$

$$N_5 = N_0 \times P_5 + N_1 \times P_4 + N_2 \times P_3 + N_3 \times P_2 + N_4 \times P_1$$
$$= 10{,}000 \times 0.15 + 300 \times 0.4 + 709 \times 0.2 + 2042 \times .07 + 4171 \times 0.03 = 2030$$

$$N_6 = N_0 \times P_6 + N_1 \times P_5 + N_2 \times P_4 + N_3 \times P_3 + N_4 \times P_2 + N_5 \times P_1$$
$$= 10{,}000 \times 0.15 + 300 \times 0.15 + 709 \times 0.4 + 2042 \times 0.2 + 4171 \times 0.07 + 2030 \times 0.03 = 2590$$

It can be seen from the above calculations that N_i increases upto 4th month and then decreases. It can also be seen that N_i will later tend to increase & the value of N_i will oscillate till the system acquires a steady state.

The expected life of each resistor is given by –
Expected life = $x_i P_i$, where x_i is the month and P_i is the corresponding probability of failure
$= (1 \times .03) + (2 \times .07) + (3 \times .2) + (4 \times .4) + (5 \times .15) + (6 \times .15) = 4.02$ months

Therefore, average no. of replacements every month = N / Mean Age = 10000 / 4.02 = 2487.5 resistors or 2488 resistors.

Considering individual replacements average cost of replacements per month = Rs 2488 x 1 Individual replacement cost of resistor is Re 1 per resistor).

For HITK students by S.K.Sen

Considering the policy of grp replacement we get following cost –

End of month	Total cost of grp replacement	Cost per month in Rs
1	300 x 1 + 10000 x .35 = 3800	3800 / 1 = 3800
2	(300 + 709) x 1 + 10000 x .35 = 4509	4509 / 2 = 2254.5
3	(300 + 709 + 2042) x 1 + 10000 x .35 = 6551	6551 / 3 = 2183.66
4	(300 + 709 + 2042 + 4171) x 1 + 10000 x .35 = 10722	10722 / 4 = 2681
5	(300 + 709 + 2042 + 4171 + 2030) x 1 + 10000 x .35 = 12752	12752 / 5 = 2550.4
6	(300 + 709 + 2042 + 4171 + 2030 + 2590) x 1 + 10000 x .35 = 15342	15342 / 6 = 2557

Hence, the minimum cost per month is obtained for grp replacement of all resistors after three months with an average cost of Rs 2183.66 per month. This cost is also less than the average cost of monthly individual replacement policy, ie., Rs 2488. Thus, optimal replacement policy is grp replacement of all resistors after every 3 months.

Recruitment & Promotion Problems : Like industrial replacement problems, principles of replacement are also applicable to the problems of recruitment and staff promotion. The staff of an organization calls for replacements because people leave the ogn for several reasons. In staffing problems with fixed total staff and fixed size of staff grps the proportion of staff in each grp determines the promotion age and conversely.

Here also, the stay of individual employee may be random variable but the characteristics of the grp of employees are likely to be fairly stable. Following problems will make the principles clear.

<u>Problem</u> **:** A research team is planned to raise a strength of 50 chemists and then to remain at that level. The wastage of recruits depends on their length of service which is as follows –

What is the recruitment per year necessary to maintain the reqd strength? There are 8 senior posts for which the length of service is the main criterion. What is the average length of service after which new entrant expects promotion to one of the posts?

<u>Solution</u> **:** Since the percentage of chemists who have left upto the end of the yr is given, the probability P_t of a person leaving during the $t^{\underline{th}}$ year can be determined.

The reqd probability of a person being in service at the end of the yr is calculated in the following table -

Probability of persons in service at the end of year

Year (n)	No. of persons who left at the end of yr	No. of persons in service at the end of yr	Prob of leaving at the end of yr	Prob of persons in service at the end of yr (P_n)
0	0	100	0	1
1	5	95	.05	.95
2	36	64	.36	.64
3	56	44	.56	.44
4	63	37	.63	.37
5	68	32	.68	.32
6	73	27	.73	.27
7	79	21	.79	.21
8	87	13	.87	.13
9	97	3	.97	.03
10	100	0	1	0
	Total	436		

Above table shows that if 100 chemists are appointed each year then the total no. of chemists serving in the orgn would have been 436.

Thus, in order to maintain a strength of 50 chemists, we must recruit, 100 x 50 / 436 = 12 chemists every yr

If P_n denotes the probability of a person to be in service at the end of n^{th} yr, then out of 12 recruits the total no. of survivals at the end of yr '*n*' will be 12 x P_n. Thus, we can construct the following table showing the no. of chemists in service at the end of each yr.

There are 8 senior posts for which the length of service is the main criterion.

From the above table, we observe that there are 3 persons in service during the $6^{\underline{th}}$ yr, 2 persons in service during $7^{\underline{th}}$ yr and 2 persons in $8^{\underline{th}}$ yr, ie., in all 7 persons are there in service from $6^{\underline{th}}$ to $8^{\underline{th}}$ yr which is less than the total no. of senior posts.

Hence, the promotions of new entrants will start by the end of $5^{\underline{th}}$ yr.

SENSITIVITY ANALYSIS

For HITK students
by S.K.Sen

Quite often, some or all of the model inputs are subject to sources of uncertainty, including errors of measurement, absence of information and poor or partial understanding of the driving forces and mechanisms. This uncertainty imposes a limit on our confidence in the response or output of the model. Further, models may have to cope with the natural intrinsic variability of the system (aleatory), such as the occurrence of stochastic events.

Good modeling practice requires that the modeler provides an evaluation of the confidence in the model. This requires, first, a quantification of the uncertainty in any model results (uncertainty analysis); and second, an evaluation of how much each input is contributing to the output uncertainty. Sensitivity analysis addresses the second of these issues (although uncertainty analysis is usually a necessary precursor), performing the role of ordering by importance the strength and relevance of the inputs in determining the variation in the output.

In models involving many input variables, sensitivity analysis is an essential ingredient of model building and quality assurance. National and international agencies involved in impact assessment studies have included sections devoted to sensitivity analysis in their guidelines. Examples are the European Commission (see e.g. the guidelines for impact assessment), the White House Office of Management and Budget, the Intergovernmental Panel on Climate Change and US Environmental Protection Agency's modelling guidelines.

Settings and Constraints **:** The choice of method of sensitivity analysis is typically dictated by a number of problem constraints or settings. Some of the most common are

- Computational expense: Sensitivity analysis is almost always performed by running the model a (possibly large) number of times, i.e. a sampling-based approach. This can be a significant problem when,

- A single run of the model takes a significant amount of time (minutes, hours or longer). This is not unusual with very complex models.
- The model has a large number of uncertain inputs. Sensitivity analysis is essentially the exploration of the multidimensional input space, which grows exponentially in size with the number of inputs.

Computational expense is a problem in many practical sensitivity analyses. Some methods of reducing computational expense include the use of emulators (for large models), and screening methods (for reducing the dimensionality of the problem). Another method is to use an event-based sensitivity analysis method for variable selection for time-constrained applications. This is an input variable selection (IVS) method that assembles together information about the trace of the changes in system inputs and outputs using sensitivity analysis to produce an input/output trigger/event matrix that is designed to map the relationships between input data as causes that trigger events and the output data that describes the actual events. The cause-effect relationship between the causes of state change i.e. input variables and the effect system output parameters determines which set of inputs have a genuine impact on a given output. The method has a clear advantage over analytical and computational IVS method since it tries to understand and interpret system state change in the shortest possible time with minimum computational overhead.

- Correlated inputs: Most common sensitivity analysis methods assume independence between model inputs, but sometimes inputs can be strongly correlated. This is still an immature field of research and definitive methods have yet to be established.

- Nonlinearity: Some sensitivity analysis approaches, such as those based on linear regression, can inaccurately measure sensitivity when the model response is nonlinear with respect to its inputs. In such cases, variance-based measures are more appropriate.
- Model interactions: Interactions occur when the perturbation of two or more inputs simultaneously causes variation in the output greater than that of varying each of the inputs alone. Such interactions are present in any model that is non-additive, but will be neglected by methods such as scatterplots and one-at-a-time perturbations. The effect of interactions can be measured by the total-order sensitivity index.
- Multiple outputs: Virtually all sensitivity analysis methods consider a single univariate model output, yet many models output a large number of possibly spatially or time-dependent data. Note that this does not preclude the possibility of performing different sensitivity analyses for each output of

interest. However, for models in which the outputs are correlated, the sensitivity measures can be hard to interpret.

- Given data: While in many cases the practitioner has access to the model, in some instances a sensitivity analysis must be performed with "given data", i.e. where the sample points (the values of the model inputs for each run) cannot be chosen by the analyst. This may occur when a sensitivity analysis has to be performed retrospectively, perhaps using data from an optimisation or uncertainty analysis, or when data comes from a discrete source.

Core methodology

Ideal scheme of a possibly sampling-based sensitivity analysis. Uncertainty arising from different sources - errors in the data, parameter estimation procedure, alternative model structures - are propagated through the model for uncertainty analysis and their relative importance is quantified via sensitivity analysis.

Sampling-based sensitivity analysis by scatterplots. Y (vertical axis) is a function of four factors. The points in the four scatterplots are always the same though sorted differently, i.e. by Z_1, Z_2, Z_3, Z_4 in turn. Note that the abscissa is different for each plot: (−5, +5) for Z_1, (−8, +8) for Z_2, (−10, +10) for Z_3 and Z_4. Z_4 is most important in influencing Y as it imparts more 'shape' on Y.