CAPITAL BUDGETING (Methods of project appraisal– ARR, PB, DCF, NPV, Benefit Cost Ratio & IRR)

Capital budgeting, or investment appraisal, is the planning process used to determine whether an organization's long term investments such as new machinery, replacement machinery, new plants, new products, and research development projects are worth the funding of cash through the firm's capitalization structure (debt, ...

NPV is used in capital budgeting to analyze the profitability of an investment or project.

The internal rate of return (IRR) is a rate of return used in capital budgeting to measure and compare the profitability of investments.

Payback period in capital budgeting refers to the period of time required to recoup the funds expended in an investment, or to reach the break-even point.

Capital budgeting, or investment appraisal, is the planning process used to determine whether an organization's long term investments such as new machinery, replacement machinery, new plants, new products, and research development projects are worth the funding of cash through the firm's capitalization structure (debt, equity or retained earnings). It is the process of allocating resources for major capital, or investment, expenditures. One of the primary goals of capital budgeting investments is to increase the value of the firm to the shareholders.

Many formal methods are used in capital budgeting, including the techniques such as -

(i) Accounting rate of return (ii) Average accounting return
(iii) Payback period (iv) Net present value
(v) Profitability index (vi) Internal rate of return
(vii) Modified internal rate of return (viii) Equivalent annual cost
(ix) Real options valuation

These methods use the incremental cash flows from each potential investment, or *project*. Techniques based on accounting earnings and accounting rules are sometimes used - though economists consider this to be improper - such as the *accounting rate of return*, and "return on investment." Simplified and hybrid methods are used as well, such as *Payback Period* and *Discounted Payback Period*

Net Present value **:** Capital budgeting projects are classified as either Independent Projects or Mutually Exclusive Projects. An Independent Project is a project whose cash flows are not affected by the accept/reject decision for other projects. Thus, all Independent Projects which meet the Capital Budgeting criterion should be accepted.

Mutually exclusive projects are a set of projects from which at most one will be accepted. For example, a set of projects which are to accomplish the same task. Thus, when choosing between "mutually exclusive projects", more than one project may satisfy the capital budgeting criterion. However, only one, i.e., the best, project can be accepted.

Of these three, only the net present value and internal rate of return decision rules consider all of the project's cash flows and the time value of money. As we shall see, only the net present value decision rule will always lead to the correct decision when choosing among mutually exclusive projects. This is because the net present value and internal rate of return decision rules differ with respect to their reinvestment rate assumptions. The net present value decision rule implicitly assumes that the project's cash flows can be reinvested at the firm's cost of capital, whereas the internal rate of return decision rule implicitly assumes that the cash flows can be reinvested at the project's IRR. Since each project is likely to have a different IRR, the assumption underlying the net present value decision rule is more reasonable.

Internal rate of return **:** The internal rate of return (IRR) is defined as the discount rate that gives a net present value (NPV) of zero. It is a commonly used measure of investment efficiency.

The IRR method will result in the same decision as the NPV method for (non-mutually exclusive) projects in an unconstrained environment, in the usual cases where a negative cash flow occurs at the start of the project, followed by all positive cash flows. In most realistic cases, all independent projects that have an IRR higher than the hurdle rate should be accepted. Nevertheless, for mutually exclusive projects, the decision rule of taking the project with the highest IRR - which is often used - may select a project with a lower NPV.

In some cases, several zero NPV discount rates may exist, so there is no unique IRR. The IRR exists and is unique if one or more years of net investment (negative cash flow) are followed by years of net revenues. But if the signs of the cash flows change more than once, there may be several IRRs. The IRR equation generally cannot be solved analytically but only via iterations.

One shortcoming of the IRR method is that it is commonly misunderstood to convey the actual annual profitability of an investment. However, this is not the case because intermediate cash flows are almost never reinvested at the project's IRR; and, therefore, the actual rate of return is almost certainly going to be lower. Accordingly, a measure called Modified Internal Rate of Return (MIRR) is often used.

Despite a strong academic preference for NPV, surveys indicate that executives prefer IRR over NPV although they should be used in concert. In a budget-constrained environment, efficiency measures should be used to maximize the overall NPV of the firm. Some managers find it intuitively more appealing to evaluate investments in terms of percentage rates of return than dollars of NPV.

Equivalent annuity method **:** The *equivalent annuity* method expresses the NPV as an annualized cash flow by dividing it by the present value of the annuity factor. It is often used when assessing only the costs of specific projects that have the same cash inflows. In this form it is known as the *equivalent annual cost* (EAC) method and is the cost per year of owning and operating an asset over its entire lifespan.

It is often used when comparing investment projects of unequal lifespans. For example, if project A has an expected lifetime of 7 years, and project B has an expected lifetime of 11 years it would be improper to simply compare the net present values (NPVs) of the two projects, unless the projects could not be repeated. The use of the EAC method implies that the project will be replaced by an identical project.

Alternatively the *chain method* can be used with the NPV method under the assumption that the projects will be replaced with the same cash flows each time. To compare projects of unequal length, say 3 years and 4 years, the projects are *chained together*, i.e. four repetitions of the 3-year project are compare to three repetitions of the 4-year project. The chain method and the EAC method give mathematically equivalent answers.

The assumption of the same cash flows for each link in the chain is essentially an assumption of zero inflation, so a real interest rate rather than a nominal interest rate is commonly used in the calculations.

Real options analysis **:** Real options analysis has become important since the 1970s as option pricing models have gotten more sophisticated. The discounted cash flow methods essentially value projects as if they were risky bonds, with the promised cash flows known. But managers will have many choices of how to increase future cash inflows, or to decrease future cash outflows. In other words, managers get to manage the projects - not simply accept or reject them. Real options analysis tries to value the choices - the option value - that the managers will have in the future and adds these values to the NPV.

Ranked projects **:** The real value of capital budgeting is to rank projects. Most organizations have many projects that could potentially be financially rewarding. Once it has been determined that a particular project has exceeded its hurdle, then it should be ranked against peer projects (e.g. – highest Profitability index to lowest Profitability index). The highest ranking projects should be implemented until the budgeted capital has been expended.

Funding sources **:** Capital budgeting investments and projects must be funded through excess cash provided through the raising of debt capital, equity capital, or the use of retained earnings. Debt capital is borrowed cash, usually in the form of bank loans, or bonds issued to creditors. Equity capital are investments made by shareholders, who purchase shares in the company's stock. Retained earnings are excess cash surplus from the company's present and past earnings.

Need for capital budgeting **:** As large sum of money is involved which influences the profitability of the firm making capital budgeting an important task.

Long term investment once made can not be reversed without significant loss of invested capital. The investment becomes sunk, and mistakes, rather than being readily rectified, must often be borne until the firm can be withdrawn through depreciation charges or liquidation. It influences the whole conduct of the business for the years to come.
Investment decision are the base on which the profit will be earned and probably measured through the return on the capital. A proper mix of capital investment is quite important to ensure adequate rate of return on investment, calling for the need of capital budgeting.

The implication of long term investment decisions are more extensive than those of short run decisions because of time factor involved, capital budgeting decisions are subject to the higher degree of risk and uncertainty than short run decision.

5 Methods of Project Appraisal **:** Some of the methods of project appraisal are as follows -

1. *Economic Analysis* :** Under economic analysis, the project aspects highlighted include requirements for raw material, level of capacity utilization, anticipated sales, anticipated expenses and the probable profits. It is said that a business should have always a volume of profit clearly in view which will govern other economic variables like sales, purchases, expenses and alike.

It will have to be calculated how much sales would be necessary to earn the targeted profit. Undoubtedly, demand for the product will be estimated for anticipating sales volume. Therefore, demand for the product needs to be carefully spelled out as it is, to a great extent, deciding factor of feasibility of the project concern.

In addition to above, the location of the enterprise decided after considering a gamut of points also needs to be mentioned in the project. The Government policies in this regard should be taken into consideration. The Government offers specific incentives and concessions for setting up industries in notified backward areas. Therefore, it has to be ascertained whether the proposed enterprise comes under this category or not and whether the Government has already decided any specific location for this kind of enterprise.

2. *Financial Analysis* :** Finance is one of the most important pre-requisites to establish an enterprise. It is finance only that facilitates an entrepreneur to bring together the labour of one, machine of another and raw material of yet another to combine them to produce goods.

In order to adjudge the financial viability of the project, the following aspects need to be carefully analysed -
a). Assessment of the financial requirements both – fixed capital and working capital need to be properly made. You might be knowing that fixed capital normally called 'fixed assets' are those tangible and material facilities which purchased once are used again and again. Land and buildings, plants and machinery, and equipment's are the familiar examples of fixed assets/fixed capital. The requirement for fixed assets/capital will vary from enterprise to enterprise depending upon the type of operation, scale of operation and time when the investment is made. But, while assessing the fixed capital requirements, all items relating to the asset like the cost of the asset, architect and engineer's fees, electrification and installation charges (which normally come to 10 per cent of the value of machinery), depreciation, pre-operation expenses of trial runs, etc., should be duly taken into consideration. Similarly, if any expense is to be incurred in remodeling, repair and additions of buildings should also be highlighted in the project report.

b). In accounting, working capital means excess of current assets over current liabilities. Generally, 2: 1 is considered as the optimum current ratio. Current assets refer to those assets which can be converted into cash within a period of one week. Current liabilities refer to those obligations which can be payable within a period of one week. In short, working capital is that amount of funds which is needed in day today's business operations. In other words, it is like circulating money changing from cash to inventories and from inventories to receivables and again converted into cash.

This circle goes on and on. Thus, working capital serves as a lubricant for any enterprise, be it large or small. Therefore, the requirements of working capital should be clearly provided for. Inadequacy of working capital may not only adversely affect the operation of the enterprise but also bring the enterprise to a grinding halt.

The activity level of an enterprise expressed as capacity utilization, needs to be well spelt out in the business plan or project report. However, the enterprise sometimes fails to achieve the targeted level of capacity due to various business vicissitudes like unforeseen shortage of raw material, unexpected disruption in power supply, inability to penetrate the market mechanism, etc.
Then, a question arises to what extent and enterprise should continue its production to meet all its obligations/liabilities. 'Break-even analysis' (BEP) gives an answer to it. In brief, break-even analysis indicates the level of production at which there is neither profit nor loss in the enterprise. This level of production is, accordingly, called 'break-even level'.

3. *Market Analysis* :** Before the production actually starts, the entrepreneur needs to anticipate the possible market for the product. He/ she has to anticipate who will be the possible customers for his product and where and when his product will be sold. There is a trite saying in this regard: "The manufacturer of an iron nails must know who will buy his iron nails."

This is because production has no value for the producer unless it is sold. It is said that if the proof of pudding lies in eating, the proof of all production lies in marketing /consumption. In fact, the potential of the market constitutes the determinant of probable rewards from entrepreneurial career.

Thus, knowing the anticipated market for the product to be produced becomes an important element in every business plan. The various methods used to anticipate the potential market, what is named in 'Managerial Economics' as 'demand forecasting', range from the naive to sophisticated ones.

The commonly used methods to estimate the demand for a product are as follows:

i). Opinion Polling Method : In this method, the opinions of the ultimate users, i.e. customers of the product are estimated. This may be attempted with the help of either a complete survey of all customers (called, complete enumeration) or by selecting a few consuming units out of the relevant population (called, sample survey).
Let us discuss these in some details -

(a) Complete Enumeration Survey : In this survey, all the probable customers of the product are approached and their probable demands for the product are estimated and then summed. Estimating sales under this method is very simple. It is obtained by simply adding the probable demands of all customers. An example should make it clear.
Suppose, there are total N customers of X product and everybody will demand for D numbers of it. Then, the total anticipated demand will be : $N \sum_{i=1} D_i N$

Though the principle merit of this method is that it obtains the first-hand and unbiased information, yet it is beset with some disadvantages also. For example, to approach a large number of customers scattered all over market becomes tedious, costly and cumbersome. Added to this, the consumers themselves may not divulge their purchase plans due to the reasons like their personal as well commercial/business privacies.

(b) Sample Survey **:** Under this method, only some number of consumers out of their total population is approached and data on their probable demands for the product during the forecast period are collected and summed. The total demand of sample customers is finally blown up to generate the total demand for the product. Let this also be explained with an example.

Imagine, there are 1000 customers of a product spread over the Faridabad market. Out of these, 50 are selected for survey using stratified method. Now, if the estimated demand of these sample customers is D_i, i.e., it refers to 1, 2, 3,…., 50 the total demand for the entire group of customers will be
$50 \sum n_i D_i = n_1 D_1 + n_2 D_2 + n_3 D_3 \ldots\ldots\ldots n_{50} D_{50}$

Where n, is the number of customers in group i, and $n_1 + n_2 + n_3 \ldots + n_{50} = 1000$.
But, if all the 1000 customers of the group are alike, then the selection may be done on a random basis and total demand for the group will be:
$(D_1 + D_2 + D_3 + D_4 \ldots + D_{50})\ 1000 /50$

No doubt, survey method is less costly and tedious than the complete enumeration method.

(c) Sales Experience Method **:** Under this method, a sample market is surveyed before the new product is offered for sale. The results of the market surveyed are then projected to the universe in order to anticipate the total demand for the product.

In principle, the survey market should be the true representative of the national market which is not always true. Suppose, if Delhi is selected as a sample market, it may not be a true representative of a small place, say Silchar in Assam simply because the characteristic features of Delhi are altogether different from those of a small town like Silchar.

Again, if we select Agra as a sample market, sales in Agra would be influenced by the size of the floating tourist's population throughout the year. But this feature is not experienced by many other places again like Silchar in Assam.

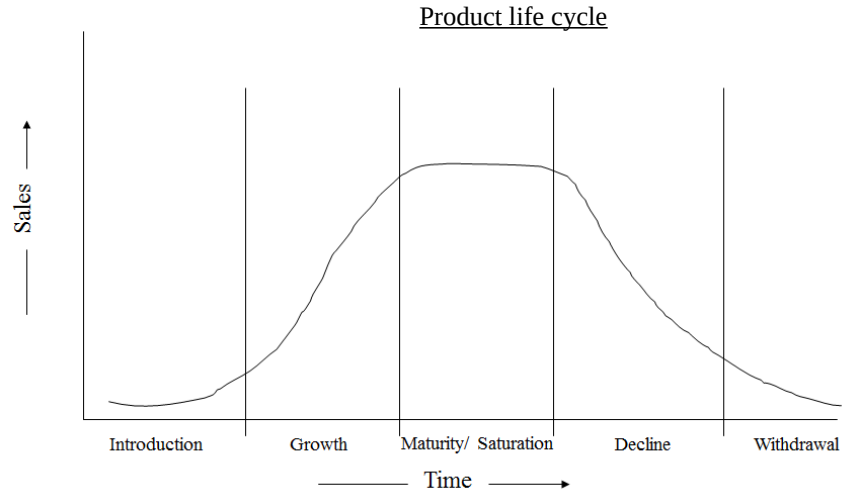
(d) Vicarious Method **:** Under the vicarious method, the consumers of the product are not approached directly but indirectly through some dealers who have a feel of their customers. The dealers' opinions about the customers' opinion are elicited. Being based on dealers' opinions, the method is bound to suffer from the bias on the part of the dealers. Then, the results derived are likely to be unrealistic. However, these hang-ups are not avoidable also.

2. Life Cycle Segmentation Analysis **:** It is well established that like a man, every product has its own life span. In practice, a product sells slowly in the beginning. Backed by sales promotion strategies over period, its sales pick up. In the due course of time, the peak sale is reached. After that point, the sales begin to decline. After, some time, the product loses its demand and dies. This is natural death of a product. Thus, every product passes through its 'life cycle'.

This is precisely the reason why firms go for new products one after another to keep themselves alive.
Based on above, the product life cycle has been divided into the following five stages:
i). Introduction
ii). Growth
iii). Maturity
iv). Saturation

v). Decline / Withdrawl

The sales of the product vary from stage to stage and follows S-shaped curve as shown in Figure 16.1:



Considering the above five stages of a product life cycle, the sales at different stages can be anticipated.

4. ***T****echnical Feasibility* **:** While making project appraisal, the technical feasibility of the project also needs to be taken into consideration. In the simplest sense, technical feasibility implies to mean the adequacy of the proposed plant and equipment to produce the product within the prescribed norms. As regards know-how, it denotes the availability or otherwise of a fund of knowledge to run the proposed plants and machinery.

It should be ensured whether that know-how is available with the entrepreneur or is to be procured from elsewhere. In the latter case, arrangement made to procure it should be clearly checked up. If project requires any collaboration, then, the terms and conditions of the collaboration should also be spelt out comprehensively and carefully.

In case of foreign technical collaboration, one needs to be aware of the legal provisions in force from time to time specifying the list of products for which only such collaboration is allowed under specific terms and conditions. The entrepreneur, therefore, contemplating for foreign collaboration should check these legal provisions with reference to their projects.

While assessing the technical feasibility of the project, the following inputs covered in the project should also be taken into consideration:
(i) Availability of land and site. (ii) Availability of other inputs like water, power, transport, communication facilities. (iii) Availability of servicing facilities like machine shops, electric repair shop, etc. (iv) Coping-with anti-pollution law. (v) Availability of work force as per required skill and arrangements proposed for training-in-plant and outside. (vi) Availability of required raw material as per quantity and quality.

5. ***M****anagement Competence* **:** Management ability or competence plays an important role in making an enterprise a success or otherwise. Strictly speaking, in the absence of managerial competence, the projects which are otherwise feasible may fail.

On the contrary, even a poor project may become a successful one with good managerial ability. Hence, while doing project appraisal, the managerial competence or talent of the promoter should be taken into consideration.
Research studies report that most of the enterprises fall sick because of lack of managerial competence or mismanagement. This is more so in case of small-scale enterprises where the proprietor is all in all, i.e., owner as well as manager. Due to his one-man show, he may be jack of all but master of none.

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A benefit-cost ratio (BCR) is an indicator, used in the formal discipline of cost-benefit analysis, that attempts to summarize the overall value for money of a project or proposal. A BCR is the ratio of the benefits of a project or proposal, expressed in monetary terms, relative to its costs, also expressed in monetary terms. All benefits and costs should be expressed in discounted present values.

Benefit cost ratio (BCR) takes into account the amount of monetary gain realized by performing a project versus the amount it costs to execute the project. The higher the BCR the better the investment. General rule of thumb is that if the benefit is higher than the cost the project is a good investment.

In the absence of funding constraints, the best value for money projects are those with the highest net present value. Where there is a budget constraint, the ratio of NPV to the expenditure falling within the constraint should be used. In practice, the ratio of PV of future net benefits to expenditure is expressed as a BCR. (NPV-to-investment is net BCR.) BCRs have been used most extensively in the field of transport cost-benefit appraisals. The NPV should be evaluated over the service life of the project.

Long-term BCRs, such as those involved in climate change, are very sensitive to the discount rate used in the calculation of net present value, and there is often no consensus on the appropriate rate to use.
The handling of non-monetary impacts also present problems. They are usually incorporated by estimating them in monetary terms, using measures such as WTP (willingness to pay), though these are often difficult to assess. Alternative approaches include the UK's New Approach to Appraisal framework.

A further complication with BCRs concerns the precise definitions of benefits and costs. These can vary depending on the funding agency.

Calculation **:** BCR = Discounted value of incremental benefits ÷ Discounted value of incremental costs
Accept all projects with a BCR greater than 1, when costs and benefits are discounted at the opportunity cost of capital.
In finance, discounted cash flow (DCF) analysis is a method of valuing a project, company, or asset using the concepts of the time value of money. All future cash flows are estimated and discounted by using cost of capital to give their present values (PVs).

Discounted cash flow **:** In finance, discounted cash flow (DCF) analysis is a method of valuing a project, company, or asset using the concepts of the time value of money. All future cash flows are estimated and discounted by using cost of capital to give their present values (PVs). The sum of all future cash flows, both incoming and outgoing, is the net present value (NPV), which is taken as the value or price of the cash flows in question.

Using DCF analysis to compute the NPV takes as input cash flows and a discount rate and gives as output a present value; the opposite process—takes cash flows and a price (present value) as inputs, and provides as output the discount rate—this is used in bond markets to obtain the yield.

Discounted cash flow analysis is widely used in investment finance, real estate development, corporate financial management and patent valuation.

Discount rate **:** The most widely used method of discounting is exponential discounting, which values future cash flows as "how much money would have to be invested currently, at a given rate of return, to yield the cash flow in future." Other methods of discounting, such as hyperbolic discounting, are studied in academia and said to reflect intuitive decision-making, but are not generally used in industry.

The discount rate used is generally the appropriate weighted average cost of capital (WACC), that reflects the risk of the cashflows. The discount rate reflects two things:
(i) Time value of money (risk-free rate) – according to the theory of time preference, investors would rather have cash immediately than having to wait and must therefore be compensated by paying for the delay.
(ii) Risk premium – reflects the extra return investors demand because they want to be compensated for the risk that the cash flow might not materialize after all.
Mathematics **-** Discounted cash flows : The discounted cash flow formula is derived from the future value formula for calculating the time value of money and compounding returns.

$$\mathrm{DCF} = \{\mathrm{CF}_1 / (1 + \mathrm{r})^1\} + \{\mathrm{CF}_2 / (1 + \mathrm{r})^2\} + \ldots + \{\mathrm{CF}_\mathrm{n} / (1 + \mathrm{r})^\mathrm{n}\}$$

$$\mathrm{FV} = \mathrm{DVF}(1 + \mathrm{r})^\mathrm{n}$$

Thus the discounted present value (for one cash flow in one future period) is expressed as **:**

$$\mathrm{DPV} = \mathrm{FV} / (1 + \mathrm{r})^\mathrm{n}$$

where
DPV is the discounted present value of the future cash flow (*FV*), or *FV* adjusted for the delay in receipt;

FV is the nominal value of a cash flow amount in a future period;
r is the interest rate or discount rate, which reflects the cost of tying up capital and may also allow for the risk that the payment may not be received in full;
n is the time in years before the future cash flow occurs.
Where multiple cash flows in multiple time periods are discounted, it is necessary to sum them as follows:

$$\mathrm{DPV} = \sum_{i=0}^{N} \{\mathrm{FV}_t / (1 + \mathbf{r})^t\}$$

for each future cash flow (*FV*) at any time period (*t*) in years from the present time, summed over all time periods.

The sum can then be used as a net present value figure. If the amount to be paid at time 0 (now) for all the future cash flows is known, then that amount can be substituted for *DPV* and the equation can be solved for *r*, that is the internal rate of return.

All the above assumes that the interest rate remains constant throughout the whole period.
If the cash flow stream is assumed to continue indefinitely, the finite forecast is usually combined with the assumption of constant cash flow growth beyond the discrete projection period. The total value of such cash flow stream is the sum of the finite discounted cash flow forecast and the Terminal value (finance).

For continuous cash flows, the summation in the above formula is replaced by an integration.
Example of DCF **:** To show how discounted cash flow analysis is performed, consider the following simplified example.
John Doe buys a house for $100,000. Three years later, he expects to be able to sell this house for $150,000.
Simple subtraction suggests that the value of his profit on such a transaction would be $150,000 − $100,000 = $50,000, or 50%. If that $50,000 is amortized over the three years, his implied annual return (known as the internal rate of return) would be about 14.5%. Looking at those figures, he might be justified in thinking that the purchase looked like a good idea.

1.145^3 x 100000 = 150000 approximately.
However, since three years have passed between the purchase and the sale, any cash flow from the sale must be discounted accordingly. At the time John Doe buys the house, the 3-year US Treasury Note rate is 5% per annum. Treasury Notes are generally considered to be inherently less risky than real estate, since the value of the Note is guaranteed by the US Government and there is a liquid market for the purchase and sale of T-Notes. If he hadn't put his money into buying the house, he could have invested it in the relatively safe T-Notes instead. This 5% per annum can therefore be regarded as the risk-free interest rate for the relevant period (3 years).

Using the DPV formula above (FV = $150,000, i = 0.05,
n = 3), that means that the value of $150,000 received in three years actually has a present value of $129,576 (rounded off). In other words, we would need to invest $129,576 in a T-Bond now to get $150,000 in 3 years almost risk free. This is a quantitative way of showing that money in the future is not as valuable as money in the present ($150,000 in 3 years isn't worth the same as $150,000 now; it is worth $129,576 now).

Subtracting the purchase price of the house ($100,000) from the present value results in the net present value of the whole transaction, which would be $29,576 or a little more than 29% of the purchase price.

Another way of looking at the deal as the excess return achieved (over the risk-free rate) is (114.5 - 105)/(100 + 5) or approximately 9.0% (still very respectable).

But what about risk?
We assume that the $150,000 is John's best estimate of the sale price that he will be able to achieve in 3 years time
(after deducting all expenses, of course). There is of course a lot of uncertainty about house prices, and the outcome may end up higher or lower than this estimate.

(The house John is buying is in a "good neighborhood," but market values have been rising quite a lot lately and the real estate market analysts in the media are talking about a slow-down and higher interest rates. There is a probability that John might not be able to get the full $150,000 he is expecting in three years due to a slowing of price appreciation, or that loss of liquidity in the real estate market might make it very hard for him to sell at all.)

Under normal circumstances, people entering into such transactions are risk-averse, that is to say that they are prepared to accept a lower expected return for the sake of avoiding risk. See Capital asset pricing model for a further discussion of this. For the sake of the

example (and this is a gross simplification), let's assume that he values this particular risk at 5% per annum (we could perform a more precise probabilistic analysis of the risk, but that is beyond the scope of this article). Therefore, allowing for this risk, his expected return is now 9.0% per annum (the arithmetic is the same as above).

And the excess return over the risk-free rate is now (109 - 105)/(100 + 5) which comes to approximately 3.8% per annum.

That return rate may seem low, but it is still positive after all of our discounting, suggesting that the investment decision is probably a good one: it produces enough profit to compensate for tying up capital and incurring risk with a little extra left over. When investors and managers perform DCF analysis, the important thing is that the net present value of the decision after discounting all future cash flows at least be positive (more than zero). If it is negative, that means that the investment decision would actually lose money even if it appears to generate a nominal profit. For instance, if the expected sale price of John Doe's house in the example above was not $150,000 in three years, but $130,000 in three years or $150,000 in five years, then on the above assumptions buying the house would actually cause John to lose money in present-value terms (about $3,000 in the first case, and about $8,000 in the second). Similarly, if the house was located in an undesirable neighborhood and the Federal Reserve Bank was about to raise interest rates by five percentage points, then the risk factor would be a lot higher than 5%: it might not be possible for him to predict a profit in discounted terms even if he thinks he could sell the house for $200,000 in three years.

In this example, only one future cash flow was considered. For a decision which generates multiple cash flows in multiple time periods, all the cash flows must be discounted and then summed into a single net present value.

Methods of appraisal of a company or project
This is necessarily a simple treatment of a complex subject: more detail is beyond the scope of this article.
For these valuation purposes, a number of different DCF methods are distinguished today, some of which are outlined below. The details are likely to vary depending on the capital structure of the company. However the assumptions used in the appraisal (especially the equity discount rate and the projection of the cash flows to be achieved) are likely to be at least as important as the precise model used.

Both the income stream selected and the associated cost of capital model determine the valuation result obtained with each method. This is one reason these valuation methods are formally referred to as the Discounted Future Economic Income methods.

Equity-Approach
Flows to equity approach (FTE) : Discount the cash flows available to the holders of equity capital, after allowing for cost of servicing debt capital

Advantages **:** Makes explicit allowance for the cost of debt capital

Disadvantages **:** Requires judgement on choice of discount rate

Entity-Approach
(i) Adjusted present value approach (APV) **:** Discount the cash flows before allowing for the debt capital (but allowing for the tax relief obtained on the debt capital)

Advantages **:** Simpler to apply if a specific project is being valued which does not have earmarked debt capital finance

Disadvantages **:** Requires judgement on choice of discount rate; no explicit allowance for cost of debt capital, which may be much higher than a risk-free rate
(ii) Weighted average cost of capital approach (WACC) **:** Derive a weighted cost of the capital obtained from the various sources and use that discount rate to discount the cash flows from the project

Advantages **:** Overcomes the requirement for debt capital finance to be earmarked to particular projects

Disadvantages **:** Care must be exercised in the selection of the appropriate income stream. The net cash flow to total invested capital is the generally accepted choice.

Total cash flow approach (TCF) **:** This distinction illustrates that the Discounted Cash Flow method can be used to determine the value of various business ownership interests. These can include equity or debt holders.

Alternatively, the method can be used to value the company based on the value of total invested capital. In each case, the differences lie in the choice of the income stream and discount rate. For example, the net cash flow to total invested capital and WACC are appropriate when valuing a company based on the market value of all invested capital.

Shortcomings **:** Commercial banks have widely used discounted cash flow as a method of valuing commercial real estate construction projects. This practice has two substantial shortcomings. 1) The discount rate assumption relies on the market for competing investments at the time of the analysis, which would likely change, perhaps dramatically, over time, and 2) straight line assumptions about income increasing over ten years are generally based upon historic increases in market rent but never factors in the cyclical nature of many real estate markets. Most loans are made during boom real estate markets and these markets usually last fewer than ten years. Using DCF to analyze commercial real estate during any but the early years of a boom market will lead to overvaluation of the asset.
Discounted cash flow models are powerful, but they do have shortcomings. DCF is merely a mechanical valuation tool, which makes it subject to the principle "garbage in, garbage out". Small changes in inputs can result in large changes in the value of a company. Instead of trying to project the cash flows to infinity, terminal value techniques are often used. A simple perpetuity is used to estimate the terminal value past 10 years, for example. This is done because it is harder to come to a realistic estimate of the cash flows as time goes on involves calculating the period of time likely to recoup the initial outlay.

Another shortcoming is the fact that the Discounted Cash Flow Valuation should only be used as a method of intrinsic valuation for companies with predictable, though not necessarily stable, cash flows. The Discounted Cash Flow valuation method is widely used in valuing mature companies in stable industry sectors such as Utilities. At the same time, this method is often applied to valuation of high growth technology companies. In valuing young companies without much cash flow track record, the Discounted Cash Flow method may be applied a number of times to assess a number of possible future outcomes, such as the best, worst and mostly likely case scenarios.

Internal rate of return The internal rate of return (IRR) or economic rate of return (ERR) is a method of calculating rate of return. The term *internal* refers to the fact that its calculation does not incorporate environmental factors (e.g., the interest rate or inflation).

It is also called the discounted cash flow rate of return (DCFROR).
In the context of savings and loans, the IRR is also called the effective interest rate.

The internal rate of return on an investment or project is the "annualized effective compounded return rate" or rate of return that makes the net present value of all cash flows (both positive and negative) from a particular investment equal to zero. It can also be defined as the discount rate at which the present value of all future cash flow is equal to the initial investment or, in other words, the rate at which an investment breaks even.

Equivalently, the IRR of an investment is the discount rate at which the net present value of costs (negative cash flows) of the investment equals the net present value of the benefits (positive cash flows) of the investment.

Uses of IRR **:** IRR is used in capital budgeting to measure and compare the profitability of investments.
IRR calculations are commonly used to evaluate the desirability of investments or projects. The higher a project's IRR, the more desirable it is to undertake the project. Assuming all projects require the same amount of up-front investment, the project with the highest IRR would be considered the best and undertaken first.
A firm (or individual) should, in theory, undertake all projects or investments available with IRRs that exceed the cost of capital. Investment may be limited by availability of funds to the firm and/or by the firm's capacity or ability to manage numerous projects.

Because the internal rate of return is a rate quantity, it is an indicator of the efficiency, quality, or yield of an investment. This is in contrast with the net present value, which is an indicator of the value or magnitude of an investment.

An investment is considered acceptable if its internal rate of return is greater than an established minimum acceptable rate of return or cost of capital. In a scenario where an investment is considered by a firm that has shareholders, this minimum rate is the cost of capital of the investment (which may be determined by the risk-adjusted cost of capital of alternative investments). This ensures that the investment is supported by equity holders since, in general, an investment whose IRR exceeds its cost of capital adds value for the company (i.e., it is economically profitable).
One of the uses of IRR is by corporations that wish to compare capital projects. For example, a corporation will evaluate an investment in a new plant versus an extension of an existing plant based on the IRR of each project. In such a case, each new capital project must produce an IRR that is higher than the company's cost of capital. Once this hurdle is surpassed, the project with the highest IRR would be the wiser investment, all other things being equal (including risk).

IRR is also useful for corporations in evaluating stock buyback programs. Clearly, if a company allocates a substantial amount to a stock buyback, the analysis must show that the company's own stock is a better investment (has a higher IRR) than any other use of the funds for other capital projects, or than any acquisition candidate at current market prices.

IRR is also suitable for private equity, from the limited partners' perspective, as a measure of the general partner's performance as investment manager.[2] This is because it is the general partner who controls the cash flows, including the limited partners' draw-downs of committed capital.

Calculation **:** Given a collection of pairs (time, cash flow) involved in a project, the internal rate of return follows from the net present value as a function of the rate of return. A rate of return for which this function is zero is an internal rate of return.

Given the (period, cash flow) pairs (n, C_n) where n is a positive integer, total number of perods N and NPV (net present value); IRR is given by r in –

$$\mathrm{NPV} = \sum_{n=0}^{N} \{C_n / (1+r)^n\} = 0$$

The period n is usually given in years, but the calculation may be made simpler if r is calculated using the period in which the majority of the problem is defined (e.g., using months if most of the cash flows occur at monthly intervals) and converted to a yearly period thereafter.
Any fixed time can be used in place of the present (e.g., the end of one interval of an annuity); the value obtained is zero if and only if the NPV is zero.
In the case that the cash flows are random variables, such as in the case of a life annuity, the expected values are put into the above formula.

Often, the value of r cannot be found analytically. In this case, numerical methods or graphical methods must be used.
Example **:** If an investment may be given by the sequence of cash flows

Year (n)	Cash Flow (C_n)
0	-123400
1	36200
2	54800
3	48100

then the IRR r is given by

In this case, the answer is 5.96% (in the calculation, that is, r = .0596).
Numerical solution Since the above is a manifestation of the general problem of finding the roots of the equation NPV(r) = 0, there are many numerical methods that can be used to estimate r. For example using secant method r is given by

$$r_{n+1} = r_n - \mathrm{NPV}_n[(r_n - r_{n-1}) / (\mathrm{NPV}_n - \mathrm{NPV}_{n-1})]$$

where r_n is considered $n^{\underline{th}}$ approximation of IRR.

Problems with using internal rate of return **:** As an investment decision tool, the calculated IRR should *not* be used to rate mutually exclusive projects, but only to decide whether a single project is worth investing in.
NPV vs discount rate comparison for two mutually exclusive projects. Project 'A' has a higher NPV (for certain discount rates), even though its IRR is lower than for project 'B‘.

In cases where one project has a higher initial investment than a second mutually exclusive project, the first project may have a lower IRR (expected return), but a higher NPV (increase in shareholders' wealth) and should thus be accepted over the second project (assuming no capital constraints).

IRR should not be used to compare projects of different duration. For example, the net present value added by a project with longer duration but lower IRR could be greater than that of a project of similar size, in terms of total net cash flows, but with shorter duration and higher IRR.

Modified Internal Rate of Return (MIRR) considers cost of capital, and is intended to provide a better indication of a project's probable return.

In the case of positive cash flows followed by negative ones and then by positive ones (for example, + + − − − +) the IRR may have multiple values. In this case a discount rate may be used for the borrowing cash flow and the IRR calculated for the investment cash flow. This applies for example when a customer makes a deposit before a specific machine is built.

In a series of cash flows like (−10, 21, −11), one initially invests money, so a high rate of return is best, but then receives more than one possesses, so then one owes money, so now a low rate of return is best. In this case it is not even clear whether a high or a low IRR is better. There may even be multiple IRRs for a single project, like in the example 0% as well as 10%. Examples of this type of project are strip mines and nuclear power plants, where there is usually a large cash outflow at the end of the project.

In general, the IRR can be calculated by solving a polynomial equation. Sturm's theorem can be used to determine if that equation has a unique real solution. In general the IRR equation cannot be solved analytically but only iteratively.
When a project has multiple IRRs it may be more convenient to compute the IRR of the project with the benefits reinvested.[4] Accordingly, MIRR is used, which has an assumed reinvestment rate, usually equal to the project's cost of capital.

It has been shown that with multiple internal rates of return, the IRR approach can still be interpreted in a way that is consistent with the present value approach provided that the underlying investment stream is correctly identified as net investment or net borrowing.

Despite a strong academic preference for NPV, surveys indicate that executives prefer IRR over NPV.[7] Apparently, managers find it easier to compare investments of different sizes in terms of percentage rates of return than by dollars of NPV. However, NPV remains the "more accurate" reflection of value to the business. IRR, as a measure of investment efficiency may give better insights in capital constrained situations. However, when comparing mutually exclusive projects, NPV is the appropriate measure.
Mathematics **:** Mathematically, the value of the investment is assumed to undergo exponential growth or decay according to some rate of return (any value greater than −100%), with discontinuities for cash flows, and the IRR of a series of cash flows is defined as any rate of return that results in a net present value of zero (or equivalently, a rate of return that results in the correct value of zero after the last cash flow).
Thus, internal rate(s) of return follow from the net present value as a function of the rate of return. This function is continuous. Towards a rate of return of −100% the net present value approaches infinity with the sign of the last cash flow, and towards a rate of return of positive infinity the net present value approaches the first cash flow (the one at the present). Therefore, if the first and last cash flow have a different sign there exists an internal rate of return. Examples of time series without an IRR:

i). Only negative cash flows - the NPV is negative for every rate of return.
ii). (−1, 1, −1), rather small positive cash flow between two negative cash flows; the NPV is a quadratic function of $1/(1 + r)$, where r is the rate of return, or put differently, a quadratic function of the discount rate $r/(1 + r)$; the highest NPV is −0.75, for $r = 100\%$.

In the case of a series of exclusively negative cash flows followed by a series of exclusively positive ones, the resulting function of the rate of return is continuous and monotonically decreasing from positive infinity (when the rate of return approaches -100%) to the value of the first cash flow (when the rate of return approaches infinity), so there is a unique rate of return for which it is zero. Hence, the IRR is also unique (and equal). Although the NPV-function itself is not necessarily monotonically decreasing on its whole domain, it *is* at the IRR.

Similarly, in the case of a series of exclusively positive cash flows followed by a series of exclusively negative ones the IRR is also unique.

Finally, by Descartes' rule of signs, the number of internal rates of return can never be more than the number of changes in sign of cash flow.

The reinvestment misconception **:** It is often stated that IRR assumes reinvestment of all cash flows until the very end of the project. This is a misconception. There is no hidden reinvestment assumption associated with the calculation of IRR. IRR is simply the solution to the equation in the example shown above. The cash flows are static. The NPV is set at zero. There is only one unknown variable in the equation, namely r.
This misconception likely stems from the modified internal rate of return (MIRR) concept, which allows for inclusion of a second, subsequent investment. If the reinvestment rate is set at IRR, the MIRR equals the IRR. This is hardly a surprise - compounding cash flows (with the IRR) and then discounting them using the same discount factor (the IRR) is obviously a zero-sum game.

Sources stating that there is such a hidden assumption include those cited below. Sources arguing that there is no IRR reinvestment assumption include.

The internal rate of return in personal finance **:** The IRR can be used to measure the money-weighted performance of financial investments such as an individual investor's brokerage account. For this scenario, an equivalent, more intuitive definition of the IRR is, "The IRR is the annual interest rate of the fixed rate account (like a somewhat idealized savings account) which, when subjected to the same deposits and withdrawals as the actual investment, has the same ending balance as the actual investment." This fixed rate account is also called the *replicating fixed rate account* for the investment. There are examples where the replicating fixed rate account encounters negative balances despite the fact that the actual investment did not. In those cases, the IRR calculation assumes that the same interest rate that is paid on positive balances is charged on negative balances. It has been shown that this way of charging interest is the root cause of the IRR's multiple solutions problem. If the model is modified so that, as is the case in real life, an externally supplied cost of borrowing (possibly varying over time) is charged on negative balances, the multiple solutions issue disappears. The resulting rate is called the *fixed rate equivalent* (*FREQ*).

Unannualized internal rate of return **:** In the context of investment performance measurement, there is sometimes ambiguity in terminology between the periodic rate of return, such as the internal rate of return as defined above, and a holding period return. The term *internal rate of return* or *IRR* or *Since Inception Internal Rate of Return (SI-IRR)* is in some contexts used to refer to the unannualized return over the period, particularly for periods of less than a year.

The payback period is expressed in years and fractions of years. For example, if a company invests $300,000 in a new production line, and the production line then produces cash flow of $100,000 per year, then the payback period is 3.0 years ($300,000 initial investment / $100,000 annual payback).

Net Present Value (NPV) is the difference between the present value of cash inflows and the present value of cash outflows. NPV is used in capital budgeting to analyze the profitability of a projected investment or project.