Module-2
CSEN 3104
Lecture 18
20/08/2019

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Interconnection Networks

Shuffle-Exchange and Omega Networks

- Based on two routing functions --- Shuffle (S) and Exchange (E)
- Let $A = a_{n-1} \dots a_1 a_0$ be the address of a Processing Element (PE)
- The Shuffle function is given by

$$S(a_{n-1} \dots a_1 a_0) = a_{n-2} \dots a_1 a_0 a_{n-1} \text{ where } 0 \le A \le (N-1) \text{ and } n = \log_2 N$$

- Corresponds to cyclic shifting of the bits in A to the left for 1 bit position
- Show figure for perfect shuffle
- This action corresponds to perfect shuffling a deck of N cards
- The inverse perfect shuffle does the opposite to restore ordering (Show figure)
- Corresponds to cyclic shifting of the bits in A to the right for 1 bit position
- The Exchange function is given by

$$E(a_{n-1} \dots a_1 a_0) = a_{n-1} \dots a_1 a_0'$$

- The Exchange function exchanges the data between two PEs with adjacent addresses
- It is to be noted that $E(A) = C_0(A)$, where C_0 was the cube routing function

Shuffle-Exchange and Omega Networks

- The Shuffle-Exchange function can be implemented as
  - Single stage network
  - Multistage network
- Single Stage recirculating shuffle-exchange network (Show figure)
- Dashed lines -> Shuffle　　　　Solid lines -> Exchange
- A number of parallel algorithms can be effectively implemented by using Shuffle-Exchange function. Examples:
  - Fast Fourier Transform (FFT)
  - Polynomial Evaluation
  - Sorting
  - Matrix Transposition etc…

Multistage Omega Networks

- To implement Shuffle-Exchange functions (Show figure)
- An N X N Omega network consists of n (= $\log_2 N$) identical stages
- Perfect shuffle interconnection between two adjacent stages
- Each stage has N/2 numbers of 4-function (straight, exchange, upper broadcast and lower broadcast) switch boxes under independent box control
- The switch boxes can be repositioned without violating the perfect shuffle interconnection between stages (Show figure)
- The n-cube network has the same interconnection topology as the repositioned Omega
- However, they are different in the following two points:
  - Cube NW uses 2-function switch boxes, whereas Omega NW uses 4-function ones
  - The dataflow directions in the two NWs are opposite to each other i.e. the roles of the input-output lines are exchanged in the two networks

Routing Algorithm for Omega Network

- A source S (with address $s_{n-1}\ s_{n-2}\ .\ .\ .\ s_0$) has to be connected to a certain destination D (with address $d_{n-1}\ d_{n-2}\ .\ .\ .\ d_0$)
- Starting at input S, connect the input of the first switch [in the $(n\text{-}1)^{th}$ stage] that is connected to S to
  - the upper output of the switch when $d_{n-1} = 0$
  - otherwise, to the lower output
- In the same way, bit d_{n-2} determines the output of the switch located on the next stage
- This process continues until a path is established between S and D
- In general, the input of the switch on the i^{th} stage is connected to the upper output when $d_i = 0$; Otherwise, the switch is connected to the lower output
- Example: Source 2 (i.e., S = 010) and destination 6 (i.e., D =110) (Show Figure)
- In addition to one-to-one connections, the omega network also supports broadcasting
- Show Figure to explain the paths between source 2 and destinations 4,5,6 and 7

Omega Network (Blocking)

- Omega network is a blocking network
- Because some permutations cannot be established by the network
- For example, a permutation that requires
  - source 3 to be connected to destination 1, and
  - source 7 to be connected to destination 0
- This cannot be established (Show figure)
- However, such permutations can be established in several passes through the network
- For example, when node 3 is connected to node 1, node 7 can be connected to node 0 through node 4
- That is, node 7 sends its packet to node 4, and then node 4 sends the packet to node 0
- Therefore, we can connect node 3 to node 1 in one pass and node 7 to node 0 in two passes

Delta Network

- Recapitulation of Floor Function and Ceiling Function
- Floor(x) = Greatest integer $\leq x$ Floor(2.4) = 2
- Ceil(x) = Least integer $\geq x$ Ceil(2.4) = 3
- Mathematical definition of q-shuffle of qc objects (denoted by S_{q*c}):
- $S_{q*c}(i) = (qi + \text{Floor}(i/c)) \bmod qc$ for $0 \leq i \leq qc-1$
- Alternatively, $S_{q*c}(i)$ $= qi \bmod (qc-1)$ for $0 \leq i < qc-1$
  $= i$ for $i = qc-1$
- Show diagram of a 4-shuffle of 12 indices viz. S_{4*3}
- 2-shuffle is basically the well known perfect shuffle, discussed earlier

Construction of a^n X b^n Delta Network

- An a^n X b^n delta network has a^n sources and b^n destinations
- There are n stages consisting of a X b crossbar modules
- a-shuffle is used as the link pattern between every two consecutive stages
- Numbering of the stages is done as 1, 2, …., n starting at the source side
- a^{n-1} crossbar modules are required in the first stage
- The first stage has $a^{n-1}b$ output terminals and so the second stage must have $a^{n-1}b$ input terminals
- So stage 2 requires $a^{n-2}b$ crossbar modules
- The i^{th} stage has $a^{n-i}b^{i-1}$ crossbar modules of size a X b
- Thus the total number of a X b crossbar modules required in an a^n X b^n delta network can be found as:

$$(a^n - b^n)/(a - b) \qquad \text{for } a \neq b$$

and, $$nb^{n-1} = na^{n-1} \qquad \text{for } a = b$$

Construction of a^n X b^n Delta Network

- The stages are interconnected in such a fashion that there exists a unique path of constant length from any source to any destination
- The path is digit controlled such that a crossbar module connects an input to one of its b outputs depending on a single base-b digit taken from the destination address
- If the destination D is expressed in a base-b system as $(d_{n-1}d_{n-2} \ldots d_1d_0)_b$, where $D = d_0b^0 + d_1b^1 + \ldots\ldots + d_{n-1}b^{n-1}$ and $0 \leq d_i < b$, then the base-b digit d_i controls the crossbar modules of stage (n-i)
- No input or output terminal of any crossbar module is left unconnected
- Show the diagram of 4^2 X 3^2, 2^3 X 2^3 and a^n X b^n delta network

Thank you