TIME VALUE of MONEY

Present and Future Value, Annuity, Perpetuity

For HITK students
by S.K.Sen

Economics for Engineers

Syllabus for HMTS3101

Module 1 :

a. Market: Meaning of Market, Types of Market, Perfect Competition, Monopoly, Monopolistic and Oligopoly market.
b. The basic concept of economics – needs, wants, utility.
c. National Income-GDP, GNP. Demand & Supply, Law of demand, Role of demand and supply in price determination, Price Elasticity.
d. Inflation: meaning, reasons, etc. (6L)

Module 2 :

a. Business: Types of business, Proprietorship, Partnership, Joint-stock company, and cooperative society – their characteristics.
b. Banking: role of commercial banks; credit and its importance in industrial functioning.
c. Role of central bank: Reserve Bank of India.
d. International Business or Trade Environment. (4L)

Module 3 :

a. Financial Accounting-Journals. Ledgers, Trial Balance, Profit & Loss Account, Balance Sheet.
b. Financial Statement Analysis (Ratio and Cash Flow analysis). (8L)

Module 4 :

a. Cost Accounting- Terminology, Fixed, Variable and Semi-variable costs.
b. Break Even Analysis. Cost Sheet. Budgeting and Variance Analysis.
c. Marginal Cost based decisions. (6L)

Module 5 :

a. Time Value of Money: Present and Future Value, Annuity, Perpetuity.
b. Equity and Debt, Cost of Capital. (4L)

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Module 6 :

a. Capital Budgeting: Methods of project appraisal - average rate of return - payback period - discounted cash flow method: net present value, benefit cost ratio, internal rate of return.

b. Depreciation and its types, Replacement Analysis, Sensitivity Analysis. (8L)

Evaluation: Max marks-100
Internal Test-30
Semester Test-70

Suggested Readings :

1. R. Narayanswami, Financial Accounting- A Managerial Perspective. Prentice-Hall of India Private Limited. New Delhi

2. Horne, James C Van, Fundamentals of Financial Management. Prentice-Hall of India Private Limited, New Delhi

3. H. L. Ahuja., Modern Economic Theory. S. Chand. New Delhi.

4. Newman, Donald G., Eschenbach, Ted G., and Lavelle, Jerome P. Engineering Economic Analysis. New York: Oxford University Press. 2012.

The time value of money (TVM) is the idea that money available at the present time is worth more than the same amount in the future due to its potential earning capacity. This core principle of finance holds that, provided money can earn interest, any amount of money is worth more the sooner it is received.

The present value is always less than or equal to the future value because money has interest-earning potential, a characteristic referred to as the time value of money.

The payback period is considered a method of analysis with serious limitations and qualifications for its use, because it does not account for the time value of money, risk, financing, or other important considerations, such as the opportunity cost.

Discounting is a technique that translates future costs and benefits into present-day values to account for the time value of money.

The time value of money describes the greater benefit of receiving money now rather than later. It is founded on time preference.

The principle of the time value of money explains why interest is paid or earned. Interest, whether it is on a bank deposit or debt, compensates the depositor or lender for the time value of money.

It also underlies investment. An investor is willing to forego spending their money now if they expect a favorable return on their investment.

Time value of money problems involve the net value of cash flows at different points in time.

In a typical case, the variables might be: a balance (the real or nominal value of a debt or a financial asset in terms of monetary units), a periodic rate of interest, the number of periods, and a series of cash flows. (In the case of a debt, cash flows are payments against principal and interest; in the case of a financial asset, these are contributions to or withdrawals from the balance.) More generally, the cash flows may not be periodic but may be specified individually. Any of the variables may be the independent variable (the sought-for answer) in a given problem. For example, one may know that: the interest is 0.5% per period (per month, say); the number of periods is 60 (months); the initial balance (of the debt, in this case) is

25,000 units; and the final balance is 0 units. The unknown variable may be the monthly payment that the borrower must pay.

For example, £100 invested for one year, earning 5% interest, will be worth £105 after one year; therefore, £100 paid now *and* £105 paid exactly one year later *both* have the same value to a recipient who expects 5% interest assuming that inflation would be zero percent. That is, £100 invested for one year at 5% interest has a *future value* of £105 under the assumption that inflation would be zero percent.

This principle allows for the valuation of a likely stream of income in the future, in such a way that annual incomes are discounted and then added together, thus providing a lump-sum "present value" of the entire income stream; all of the standard calculations for time value of money derive from the most basic algebraic expression for the present value of a future sum, "discounted" to the present by an amount equal to the time value of money. For example, the future value sum FV to be received in one year is discounted at the rate of interest r to give the present value sum PV -

$$PV = FV / (1 + r)$$

Some standard calculations based on the time value of money are :

Present value : The current worth of a future sum of money or stream of cash flows, given a specified rate of return. Future cash flows are "discounted" at the *discount rate;* the higher the discount rate, the lower the present value of the future cash flows. Determining the appropriate discount rate is the key to valuing future cash flows properly, whether they be earnings or obligations.

Present value of an annuity: An annuity is a series of equal payments or receipts that occur at evenly spaced intervals. Leases and rental payments are examples. The payments or receipts occur at the end of each period for an ordinary annuity while they occur at the beginning of each period for an annuity due.

Present value of a perpetuity is an infinite and constant stream of identical cash flows.

Future value: The value of an asset or cash at a specified date in the future, based on the value of that asset in the present.

Future value of an annuity (FVA): The future value of a stream of payments (annuity), assuming the payments are invested at a given rate of interest.

There are several basic equations that represent the equalities listed above. The solutions may be found using (in most cases) the formulas, a financial calculator or a spreadsheet. The formulas are programmed into most financial calculators and several spreadsheet functions (such as PV, FV, RATE, NPER, and PMT).

For any of the equations below, the formula may also be rearranged to determine one of the other unknowns. In the case of the standard annuity formula, however, there is no closed-form algebraic solution for the interest rate (although financial calculators and spreadsheet programs can readily determine solutions through rapid trial and error algorithms).

These equations are frequently combined for particular uses. For example, bonds can be readily priced using these equations. A typical coupon bond is composed of two types of payments: a stream of coupon payments similar to an annuity, and a lump-sum return of capital at the end of the bond's maturity - that is, a future payment. The two formulas can be combined to determine the present value of the bond.

An important note is that the interest rate i is the interest rate for the relevant period. For an annuity that makes one payment per year, i will be the annual interest rate. For an income or payment stream with a different payment schedule, the interest rate must be converted into the relevant periodic interest rate. For example, a monthly rate for a mortgage with monthly payments requires that the interest rate be divided by 12 (see the example below). See compound interest for details on converting between different periodic interest rates.

The rate of return in the calculations can be either the variable solved for, or a predefined variable that measures a discount rate, interest, inflation, rate of return, cost of equity, cost of debt or any number of other analogous concepts. The choice of the appropriate rate is critical to the exercise, and the use of an incorrect discount rate will make the results meaningless.

For calculations involving annuities, you must decide whether the payments are made at the end of each period (known as an ordinary annuity), or at the beginning of each period (known as an annuity due). If you are using a financial calculator or a spreadsheet, you can usually set it for either calculation. The following formulas are for an ordinary annuity. If you want the answer for the Present Value of an annuity due simply multiply the PV of an ordinary annuity by $(1 + i)$.

Formula

The following formula use these common variables:

PV is the value at time = 0 (present value)

FV is the value at time = n (future value)

A is the value of the individual payments in each compounding period

n is the number of periods (not necessarily an integer)

i is the interest rate at which the amount compounds each period

g is the growing rate of payments over each time period

Future value of a present sum

The future value (FV) formula is similar and uses the same variables. $FV = PV(1 + i)^n$

Present value of a future sum

The present value formula is the core formula for the time value of money; each of the other formulae is derived from this formula. For example, the annuity formula is the sum of a series of present value calculations.

The present value (PV) formula has four variables, each of which can be solved for: $PV = FV / (1 + i)^n$

The cumulative present value of future cash flows can be calculated by summing the contributions of FV_t, the value of cash flow at time t

$$PV = \sum_{t=1}^{n} FV_t / (1 + i)^t$$

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Note that this series can be summed for a given value of n, or when n is ∞. This is a very general formula, which leads to several important special cases given below.

Present value of an annuity for n payment periods

In this case the cash flow values remain the same throughout the n periods. The present value of an annuity (PVA) formula has four variables, each of which can be solved for:

$$PV(A) = (A / i)[1 - 1 / (1 + i)^n]$$

To get the PV of an annuity due, multiply the above equation by $(1 + i)$.

Present value of a growing annuity

In this case each cash flow grows by a factor of $(1 + g)$. Similar to the formula for an annuity, the present value of a growing annuity (PVGA) uses the same variables with the addition of *g* as the rate of growth of the annuity (A is the annuity payment in the first period). This is a calculation that is rarely provided for on financial calculators.

where $i \neq g$: $PV = A / (i - g)[1 - \{(1 + g) / (1 + i)\}^n]$

where $i = g$: $PV = (A \times n) / (1 + i)$

To get the PV of a growing annuity due, multiply the above equation by $(1 + i)$.

Present value of a perpetuity

A perpetuity is payments of a set amount of money that occur on a routine basis and continues forever.

When $n \rightarrow \infty$, the *PV* of a perpetuity (a perpetual annuity) formula becomes simple division

$$PV(P) = A / i$$

Present Value of Int Factor Annuity

$$A = P(1 + r/n)^{nt}$$

Example : Investment P = \$1000, Interest i = 6.90% Compounded Qtrly (4 Times in Year), Tenure Years $n = 5$

$$= 1000 \text{ x } (1 + 0.069 / 4)^{5\text{yrs x 4 qrtrs in a yr}}$$

$$= 1000 \text{ x } (1 + 0.069 / 4)^{20} \approx 1407.84$$

Present value of a growing perpetuity

When the perpetual annuity payment grows at a fixed rate (g) the value is theoretically determined according to the following formula. In practice, there are few securities with precise characteristics, and the application of this valuation approach is subject to various qualifications and modifications. Most importantly, it is rare to find a growing perpetual annuity with fixed rates of growth and true perpetual cash flow generation. Despite these qualifications, the general approach may be used in valuations of real estate, equities, and other assets.

This is the well known Gordon Growth model used for stock valuation.

Future value of an annuity

The future value of an annuity (FVA) formula has four variables, each of which can be solved for:

$$FV(A) = A[\{(1 + i)^n - 1\} / i]$$

To get the FV of an annuity due, multiply the above equation by $(1 + i)$.

Future value of a growing annuity

The future value of a growing annuity (FVA) formula has five variables, each of which can be solved for:

where $i \neq g$:

$$FV(A) = A[\{(1 + i)^n - (1 + g)^n\} / (i - g)]$$

where $i = g$: $FV(A) = A.n(1 + i)^{n-1}$

Financial Evaluation of a Project : Financial Evaluation of a Project helps to determine the most efficient financial strategy for achieving the desired objectives of the organization. The different methods used in financial evaluation of a project are as follows –

Rate or Return (RR) Method : This is probably the most popular of the traditional methods of financial evaluation of a project. In this method, the expected profits or earnings are expressed as a percentage of the initial investment, as given by the following equation –

$$RR = E \times 100 / (C \times N) \text{ percent, where}$$

RR = Percentage rate of return per year,
E = Expected earnings or profit over the project lifecycle,
C = Initial capital investment,
N = Useful life of the project, in years.

Yearwise expected earnings for products A & B in the project

Year	Expected Earnings (Rs.)	
	Product A	Product B
1	3,00,000	4,00,000
2	3,00,000	4,00,000
3	4,00,000	2,00,000
4	2,00,000	3,00,000
5	2,00,000	1,00,000
Total earning	14,00,000	14,00,000

Note **:** Both products are assumed to have lifecycle of 5 years
Initial capital investment = Rs. 10,00,000

RR = 14,00,000 x 100 / (10,00,000 x 5) = 28 percent per year

__Payback (PB) Method__ **:** PB method is another simple method that is traditionally used for investment appraisal in a project. Here, we determine the number of years taken to pay back the initial capital investment from the surplus earnings in a project.

Let us consider the same project given in earlier table. As before, the initial capital investments for products A and B are Rs. 10,00,000 and expected earnings (after depreciation but before tax are as given in the table. Both the products have a payback period of three years, since that is the period of time reqd to earn back the initial capital investment. The RR method has earlier indicated identical rates of return of 28% for products A and B. However, the project involving the two products cannot be financially appraised as being identical; it is clear from the table that product B has a definite advantage in terms of the timing of the earnings – it is expected to earn Rs. 8,00,000 in the first two years, as against Rs. 6,00,000 from product A.

Net Present Value (NPV) Method : This is one of the two discounting methods of investment appraisal, the other one being Discounted Cash Flow (DCF) method. Both these methods are based on two simple concepts, which help to overcome the disadvantages of the traditional methods (RR & PB). These concepts are -

- Money has a time value that refers to the fact that money in hand at present is more valuable than the money in hand in future.
- Investment in a project is more concerned with cash than with entries in the conventional accounts books.

$SI = PRT/100$, $CI = A - P$, where, $Amt = P \times (1 + r/100)^n$

Annuity **:** Sequence of equal payments made at equal intervals of time.

$$\text{Present value of Annuity} = [A / (1 + r)] + [A / (1 + r)^2] + [A / (1 + r)^3] + [A / (1 + r)^4] + \ldots\ldots + [A / (1 + r)^n],$$

where, A = Amount of equal instalment,

n = no. of instalments, r = rate of interest

Net present values of cash flows for the product start-up project

Year	Product A		Product B	
	Cash flow (Rs.)	NPV (Rs.)	Cash flow (Rs.)	NPV (Rs.)
0	-1000000	-1000000	-1000000	-1000000
1	+3,00,000	+2,72,730	+4,00,000	+3,63,640
2	+3,00,000	+2,47,920	+4,00,000	+3,30,5600
3	+4,00,000	+3,00,520	+2,00,000	+1,50,260
4	+2,00,000	+1,36,600	+3,00,000	+2,04,900
5	+2,00,000	+1,24,180	+1,00,000	+62,090

Note **:** Cash flows have been discounted by 10% per anum

The capital investments (outflows) have been shown with a negative (-) sign, whereas the expected earnings (inflows) are shown with a positive (+) sign. A simple way of looking at the NPV method is to consider the project as a bank account with a nil opening balance. The initial capital investment is an outflow that creates an overdraft, which is reduced by receipts from the project earnings. At the end of the project lifecycle, the account is either in credit (positive balance), or still overdrawn (negative balance) by a certain amount. The NPV is that amount, adjusted to take account of the time value of money.

The NPV analysis of the project reveals that products A and B have positive net present values of Rs. 81,950 and Rs. 1,11,450 respectively, after repayments of the initial capital investments of Rs. 10,00,000 in either case. Hence, product B is to be preferred on financial grounds since it has a higher NPV than product A. The conclusion from NPV method is different from the results of RR and PB analysis. . . . . NPV is qualitatively better appraisal of profitability.

Discounted Cash Flow (DCF) Method : In the NPV analysis, we have assumed rate of interest of 10%, and then calculated the net present value of the project by discounting all cash flows at the above rate. The converse of this approach is to determine the rate of interest (or return) for which the NPV of the project over its lifecycle is zero. This rate of return is known as the Internal Rate of Return (IRR) or the Discounted Cash Flow (DCF) rate of return.

In practice, DCF is frequently preferred to NPV because it gives a single, comprehensive rate of profitability for each project. This helps in comparison of rate of profitability not only against the DCF for other projects, but also against the project organisation's expected rate of return. It also provides an indication of the maxm rate of interest that an orgn undertaking the project can afford to pay.

Table below shows the results of NPV calculations for different rates of return; namely, 10, 12, 13 and 14% per anum for product A of the start-up project. Upto 13% rate NPV is positive; at 14% rate of return, the project NPV is negative. Thus, the DCF rate of return for zero NPV is about 13.4% (interpolated value).

In the similar manner, we can work out DCF rate of return for the Product B. As shown in the table below, the Rate of return works out to be approximately 15.2% for zero NPV.

Comparing DCF rates of return for products A and B of the project, we can infer that product B is a financially better proposition, since it offers a higher internal rate of return.

<u>Internal Rate of Return (IRR)</u> : IRR is the discount rate at which project's Net Present Value (NPV) is zero. It indicates the net benefits expected from a project's entire life. The internal rate of return is expressed in percentages. In other words, it is the discount rate at which, present value of future earnings is equal to initial investment. It is the value of r in the following equation –

$$\text{Initial Investment (I)} = \frac{\sum C_t}{(1+r)^n} \quad [\text{Sum from } t = 1 \text{ to } t = n]$$

C_t = Cash flow at the end of year ‘t’

r = Internal rate of return

n = Life of the project

In NPV calculation we assume that discount rate is known and hence determine NPV. However, in the IRR calculation we assume NPV equals zero and then determine the discount that satisfies the equation, for example –

Suppose the cash flow of the project undertaken by an orgn are as follows –

Discounted cash flow rate of return for product A of the project

Year	Cash flow (Rs.)	NPV at discounted rate of return			
		10%	12%	13%	14%
0	-10,00,000	-10,00,000	-10,00,000	-10,00,000	- 10,00,000
1	+3,00,000	+2,72,730	+2,67,870	+2,65,500	+2,63,160
2	+3,00,000	+2,47,920	+2,39,160	+2,34,930	+2,30,850
3	+4,00,000	+3,00,520	+2,84,720	+2,77,240	+2,70,000
4	+2,00,000	+1,36,600	+1,27,100	+1,22,660	+1,18,420
5	+2,00,000	+1,24,180	+1,13,480	+108,560	+1,03,880
Project NPV		+81,950	+32330	+8890	-13690

Discounted cash flow rate of return for product B of the project

Year	Cash flow (Rs.)	NPV at discounted rate of return			
		10%	14%	15%	16%
0	-1000000	-1000000	-1000000	-1000000	-1000000
1	+4,00,000	+3,63,640	+3,50,880	+3,47,840	+3,44,840
2	+4,00,000	+3,30,560	+3,07,800	+3,02,440	+2,97,280
3	+2,00,000	+1,50,260	+1,35,000	+1,31,500	+1,28,140
4	+3,00,000	+2,04,900	+1,77,630	+1,71,540	+1,65,690
5	+1,00,000	62,090	51,940	49,720	47,610
Project NPV		+1,11,450	23250	3040	-16,440

Yearwise cash flow of a project

Year	0	1	2	3	4
Cash flow	-1,00,000	+30,000	+30,000	+40,000	+45,000

IRR is the value of ‘r’ that satisfies the following equation –

$$1,00,000 = 30.000 / (1 + r) + 30,000 / (1 + r)^2 + 40,000 / (1 + r)^3 + 45,000 / (1 + r)^4$$

A perpetuity is an annuity that has no end, or a stream of cash payments that continues forever. There are few actual perpetuities in existence (the United Kingdom (UK) government has issued them in the past; these are known and still trade as consols). Real estate and preferred stock are among some types of investments that effect the results of a perpetuity, and prices can be established using techniques for valuing a perpetuity. Perpetuities are but one of the time value of money methods for valuing financial assets. Perpetuities are a form of ordinary annuities.

The concept is closely linked to terminal value and terminal growth rate in valuation.

A perpetuity is an annuity in which the periodic payments begin on a fixed date and continue indefinitely. It is sometimes referred to as a perpetual annuity. Fixed coupon payments on permanently invested (irredeemable) sums of money are prime examples of perpetuities. Scholarships paid perpetually from an endowment fit the definition of perpetuity.

The value of the perpetuity is finite because receipts that are anticipated far in the future have extremely low present value (present value of the future cash flows). Unlike a typical bond, because the principal is never repaid, there is no present value for the principal. Assuming that payments begin at the end of the current period, the price of a perpetuity is simply the coupon amount over the appropriate discount rate or yield, that is:

Where PV = Present Value of the Perpetuity, A = the Amount of the periodic payment, and r = yield, discount rate or interest rate.

To give a numerical example, a 3% UK government War Loan will trade at 50 pence per pound in a yield environment of 6%, while at 3% yield it is trading at par. That is, if the face value of the Loan is £100 and the annual payment £3, the value of the Loan is £50 when market interest rates are 6%, and £100 when they are 3%.

Real-life examples
For example, UK government bonds, called consols, that are undated and irredeemable (e.g. war loan) pay fixed coupons (interest payments) and trade actively in the bond market. Very long dated bonds have financial characteristics that can appeal to some investors and in some circumstances, e.g. long-dated bonds have prices that change rapidly (either up or down) when yields change (fall or rise) in the financial markets.

A more current example is the convention used in real estate finance for valuing real estate with a capitalization rate (cap rate). Using a cap rate, the value of a particular real estate asset is either the net income or the net cash flow of the property, divided by the cap rate. Effectively, the use of a cap rate to value a piece of real estate assumes that the current income from the property continues in perpetuity. Underlying this valuation is the assumption that rents will rise at the same rate as inflation. Although the property may be sold in future (or even the very near future), the assumption is that other investors will apply the same valuation approach to the property.

Another example is the constant growth Dividend Discount Model for the valuation of the common stock of a corporation, which assumes that the market price per share is equal to the discounted stream of all future dividends, which is assumed to be perpetual. If the discount rate for stocks (shares) with this level of systematic risk is 12.50%, then a constant perpetuity of per dollar of dividend income is eight dollars. However if the future dividends represent a perpetuity increasing at 5.00% per year, then the dividend discount model, in effect, subtracts 5.00% off the discount rate of 12.50% for 7.50% implying that the price per dollar of income is $13.33.