Time Value of Money

The concept of time value of money is based on the fact that money has a time value i.e. a rupee today is more important than a rupee tomorrow. It is needless to say that money receivable in future is less valuable than money received today. A rational person has a preference to receive money today than tomorrow as the rupee received today has a higher value than if received in future.

Time preference of money is practically important due to:

1. Inflation: In inflationary situation money received today has more purchasing power than money received in future.
2. Current consumption: A rational person shall defer current consumption only in expectation of increased wealth in the form of investment.
3. Reinvestment purpose: Money can be reinvested to earn interest to beat the inflationary situation and to maintain purchasing power.

Importance of Time Value of Money
The cash inflows and outflows arising at different points of time can be made comparable by using the concept of time value of money:
Investment Decision
Investment decision is concerned with the allocation of capital into long-term investment projects. The cash flow from long-term investment occur at different point in time in the future. They are not comparable to each other and against the cost of the project spent at present. To make them comparable, the future cash flows are discounted back to present value.

Financing Decision
Financing decision is concerned with designing optimum capital structure and raising funds from least cost sources. The concept of time value of money is equally useful in financing decision, especially when we deal with comparing the cost of different sources of financing. The effective rate of interest of each source of financing is calculated based on time value of money concept.

Leasing vs. Buying:
In leasing versus buying decision, we calculate the present value of cost of leasing and cost of buying. The present value of costs of two alternatives are compared against each other to decide on appropriate source of financing.

Valuation of Securities
The concept of time value of money is useful to securities investors. They use valuation models while making investment in securities such as stock and bonds. These security valuation models consider time value of cash flows from securities.
Others: Besides, the concept of time value of money is also used in evaluating proposed credit policies and the firm's efficiency in managing cash collection under current assets management.

Future value of a single flow
Future value of a single cash flow compounded annually
$FV_n = PV\,(1+i)^n$
FV_n = Future value of initial flow after n years
PV = Initial investment
i = Annual rate of investment
n = No. of years of investment
The expression $(1+i)^n$ represents the future value of an initial investment invested today at the end of n years at i rate of interest is referred to as **Future Value Interest Factor [$FVIF_{(i,n)}$]**.

Future value of a lump sum amount when compounding is done more than once a year
$FV_n = PV\left(1+\frac{i}{m}\right)^{m\,x\,n}$
m = No. of times compounding is done during a year

Effective Rate of Interest
$i = \left(1+\frac{i}{m}\right)^{m} - 1$

Present Value of Perpetuity = P/i, where P is the amount payable as perpetuity, and i is the interest rate
Compounding process for multiple flows

We have to add the future compounded values of the sums received over the time period. For example if we are investing Rs. 100, Rs. 200 and Rs. 300 at the beginning of year 1, 2 and 3 respectively at 12% interest the amount that will accumulate at the end of 3 years can be calculated as

Rs. 100 x $FVIF_{(12,3)}$ + Rs. 200 x $FVIF_{(12,2)}$ + Rs. 300 x $FVIF_{(12,1)}$

$FVIF_{(12,3)}$ means Future Value Interest Factor at 12% for 3 years and for quick reference can be looked up in the FVIF table

Future Value of an Annuity

A series of periodic cash flows of equal amounts are known as annuity. These cash flows can be receipts or payments. If the equal amounts of cash flow occur at the end of each period over a specified time horizon, then this stream of cash flow is defined as a **regular annuity** or **deferred annuity**. On the other hand if cash flows occur at the beginning of each period then it is known as **annuity due**.

$$FVA_n = A\left[\frac{(1+i)^n - 1}{i}\right]$$

A = Cash flow at the end of every year for n years
i = rate of interest expressed as decimal e.g. 12% is 0.12
n = time horizon
FVA_n = Accumulated amount at the end of n years

The expression $\left[\frac{(1+i)^n - 1}{i}\right]$ is called **Future Value Interest Factor of an Annuity [$FVIFA_{(i,n)}$]** and it expresses the accumulation of Re. 1 invested or received at the end of every year for a period of n years at i rate of interest.

Sinking Fund Factor

From $FVA_n = A\left[\frac{(1+i)^n - 1}{i}\right]$ we can say

$$A = FVA_n\left[\frac{i}{(1+i)^n - 1}\right]$$

The expression $\left[\frac{i}{(1+i)^n - 1}\right]$ is known as the Sinking Fund Factor. It is the amount that has to be invested at the end of every year for a period of n years at the rate of interest i to accumulate Re. 1 at the end of the period.

Present value of a single flow

With the help of this approach the present value of a future cash flow or a stream of cash flows can be determined.

$$PV = \left[\frac{FV\,n}{(1+i)^n}\right]$$

The expression $\frac{1}{(1+i)^n}$ is defined as **Present Value Interest Factor, $PVIF_{(i,n)}$** and is the reverse of $FVIF_{(i,n)}$

Present value of multiple flows

We have to add the present values of the sums received over the time period. For example if we are to receive Rs. 100, Rs. 200 and Rs. 300 at the end of year 1, 2 and 3 respectively discounted at 12%, the equivalent amount at the present is

Rs. 300 x $PVIF_{(12,3)}$ + Rs. 200 x $FVIF_{(12,2)}$ + Rs. 100 x $PVIF_{(12,1)}$

$PVIF_{(12,3)}$ means Present Value Interest Factor at 12% for 3 years. For quick reference use the PVIF table.

Present Value of an Annuity

The present value of an annuity "A" receivable at the end of every year for a period of "n" years at "i" rate of interest can be expressed as

$$PVA_n = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

The expression $\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ is called **Present Value Interest Factor of an Annuity, [$PVIFA_{(i,n)}$]**

Capital Recovery factor

In order to find out the equated annual instalments to repay a loan taken, we need to find out "A" as PVA_n i.e. the loan amount is known

$$A = PVA_n \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$