$S \rightarrow AB|ab|aA$

$A \rightarrow a|\varepsilon$

$B \rightarrow b$

Removal of Null Production:

$S \rightarrow AB|ab|aA|B|a$

$A \rightarrow a$

$B \rightarrow b$

By removing null production, we reduce the number of intermediate states
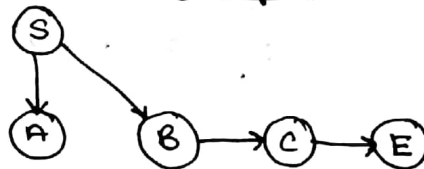
Removal of Unit Production:

$S \rightarrow AB|ab|aA|b|a$

$A \rightarrow a$

$B \rightarrow b$

• **Parsing:** whether the string is possible for the given language or not

Reachibility Graph:

$S \rightarrow AB|ab|aA|B|a$

$A \rightarrow a$

$B \rightarrow b|cC$

$C \rightarrow E$
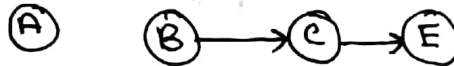
$S \rightarrow AB|ab|aA$

$A \rightarrow a|\varepsilon$

$B \rightarrow b|\varepsilon|cC$

$C \rightarrow E$

$\longrightarrow$

→ S becomes indirectly nullable

Convert To CNF:

1.
$$S \rightarrow Aa \mid bs \mid \varepsilon$$
$$A \rightarrow aA \mid bB \mid \varepsilon$$
$$B \rightarrow aA \mid bc \mid \varepsilon$$
$$c \rightarrow ac \mid bc$$

Remove ε-production —

$$S \rightarrow Aa \mid bS \mid a \mid b \mid \varepsilon$$
$$A \rightarrow aA \mid bB \mid a \mid b$$
$$B \rightarrow aA \mid bc \mid a$$
$$c \rightarrow ac \mid bc$$

Now, there is no unit production and useless states.

Cannot be done
$$\begin{cases} S \rightarrow AS \quad [\because S \rightarrow a \text{ is there}] \\ S \rightarrow SS \\ S \rightarrow a \\ S \rightarrow b \\ S \rightarrow \varepsilon \end{cases}$$

$S \rightarrow AD$	$A \rightarrow DA$	$B \rightarrow DA$	$C \rightarrow Dc$
$S \rightarrow ES$	$A \rightarrow EB$	$B \rightarrow EC$	$C \rightarrow E F$
$D \rightarrow a$	$A \rightarrow a \mid b$	$B \rightarrow a$	$F \rightarrow c$
$E \rightarrow b$			
$S \rightarrow \varepsilon$			
$S \rightarrow a \mid b$			

2. $S \rightarrow aA \mid aBB$

$A \rightarrow aAA \mid \varepsilon$

$B \rightarrow bB \mid bbC$

$C \rightarrow B$

Removing Null Production:

$S \rightarrow aA \mid aBB \mid a$

$A \rightarrow aAA \mid aA \mid a$

$B \rightarrow bB \mid bbC$

$C \rightarrow B$

No Unit Productions.

B and C are useless symbols.

∴ $S \rightarrow aA \mid a$

$A \rightarrow aAA \mid aA \mid a$

$S \rightarrow DA$	$A \rightarrow DE$
$D \rightarrow a$	$E \rightarrow AA$
$S \rightarrow a$	$A \rightarrow DA$
	$A \rightarrow a$

3. $S \rightarrow aAbB$

$A \rightarrow abAB \mid aAA \mid a$

$B \rightarrow bBaA \mid bBB \mid b$

4. $S \rightarrow bA \mid aB$

$A \rightarrow bAA \mid aS \mid a$

$B \rightarrow aBB \mid bS \mid b$

1) $D \rightarrow a \mid b \mid Da \mid Db \mid D0 \mid D1$

$F \rightarrow D \mid (E)$

$T \rightarrow F \mid T * F$

$E \rightarrow T \mid E + T$

3. $\cancel{S \rightarrow aAbB}$

$\checkmark S \rightarrow X \bullet Y$ $\checkmark A \rightarrow EF$

$\cancel{X \rightarrow aAb}$ $E \rightarrow ab \mid \checkmark E \rightarrow \cancel{\;\;} CD$

$X \rightarrow aA \mid \checkmark X \rightarrow CA$ $\checkmark F \rightarrow AB$

$Y \rightarrow bB \mid \checkmark Y \rightarrow DB$ $\checkmark A \rightarrow XA$

      $\checkmark C \rightarrow a$ $\checkmark A \rightarrow a$

      $\checkmark D \rightarrow b$

$\checkmark B \rightarrow YX$

$\checkmark B \rightarrow YB$

$\checkmark B \rightarrow b$

4. $S \rightarrow CA$ $A \rightarrow EA$ $B \rightarrow FB$

      $S \rightarrow DB$ $E \rightarrow CA$ $F \rightarrow DB$

      $C \rightarrow b$ $\cancel{X \rightarrow \;\;}$ $B \rightarrow CS$

      $D \rightarrow a$ $A \rightarrow DS$ $B \rightarrow b$

                    $A \rightarrow a$

Remove unit production

Solution:

1) $D \rightarrow a \mid b \mid Da \mid Db \mid D0 \mid D1$

$F \rightarrow a \mid b \mid Da \mid Db \mid D0 \mid D1 \mid (E)$

$T \rightarrow a \mid b \mid Da \mid Db \mid D0 \mid D1 \mid (E) \mid T * F$

$E \to a \mid b \mid Da \mid Db \mid D0 \mid D1 \mid (E) \mid T*F \mid E+T$ (Ans)

GNF

$X \to aY \mid b$, $\alpha \in \Sigma$, $\beta \in \Sigma$, $Y \in V^* \cup \Sigma$

Bottom-Up

$S \to BC$
$A \to ab \mid aa$
$B \to Ab \mid bA$
$C \to Ba$
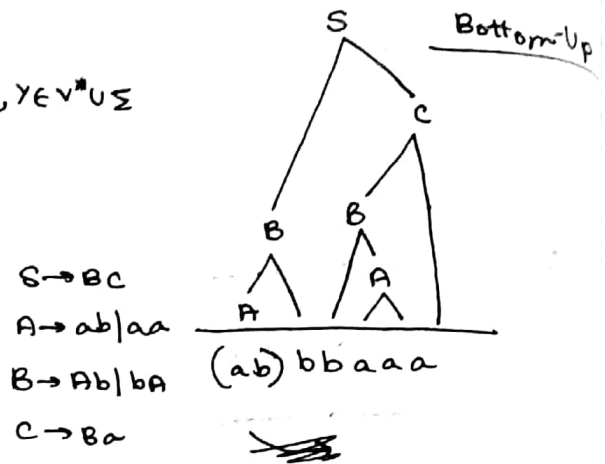
$(ab) \, bb \, a \, a \, a$

$S \to BC$
$B \, Ba$
$B \, bAa$
$Ab \, baaa$

$\longrightarrow$ Top-Down
Parsing

$\boxed{a b \, b \, b \, a \, a \, a \cdot}$

CNF
$X \to AB \mid \alpha$

GNF
$X \to \alpha A \mid \alpha$

Convert to GNF —

Steps:

1) Convert to CNF
2) Rename all the variables using $A_1, A_2, \dots, A_n$ $A \in V^*$
3) $A_i \to A_j A_k$ 4) $A_i \to \beta A_j \mid i < j$
   $A_i \to \alpha$

1.
$S \to XY1 \mid 0$

$X \to 00X \mid Y \mid 1$

$Y \to 1X1$

$S \to XY1 \mid 0$
$X \to 00X \mid 1X1 \mid 1$ } Removing Unit Production
$Y \to 1X1$

$S \to XZ\!\!\!/$ $X \to AX \!\!\!/ \!\!\!0 \!\!\!/$ $\!\!\!X \to W X 0 1\!\!\!/$

$Z \to YW$ $A \to BB$ $X \to WP$ $Y \to WP$

$W \to 1$ $B \to 0$ $P \to XW$

$S \to 0$ $X \to 1$

i) $A_1 \rightarrow A_2 A_3$ v) $A_2 \rightarrow A_6 A_2$ xi) $A_4 \rightarrow A_5 A_8$

ii) $A_3 \rightarrow A_4 A_5$ vi) $A_6 \rightarrow A_7 A_7$

iii) $A_5 \rightarrow 1$ vii) $A_7 \rightarrow 0$

iv) $A_1 \rightarrow 0$ viii) $A_2 \rightarrow A_5 A_8$

         ix) $A_8 \rightarrow A_2 A_5$

         x) $A_2 \rightarrow 1$

GNF—

i) $A_1 \rightarrow 1 A_3$ iii) $A_5 \rightarrow 1$

vi) $A_6 \rightarrow 0 A_7$ iv) $A_1 \rightarrow 0$

viii) $A_2 \rightarrow 1 A_8$ vii) $A_7 \rightarrow 0$

ix) $A_8 \rightarrow 1 A_5$ x) $A_2 \rightarrow 1$

xi) $A_4 \rightarrow 1 A_8$

• No of production in CNF $\geq$ No. of productions in GNF

ii) $A_3 \rightarrow 1 A_8 A_5$

v) $A_2 \rightarrow 0 A_7 A_2$

2.   Convert To GNF—

        $S \rightarrow AA|a$

        $A \rightarrow SS|b$

$A_i \rightarrow A_j A_k$
$i < j, k$
OR
$i < j$
$i \not< k$

$\boxed{\begin{array}{l} A_1 \rightarrow A_2 A_2 \\ A_1 \rightarrow a \end{array}}$

$A_2 \rightarrow A_1 A_1$ | $A_2 \rightarrow A_2 A_2 A_1$ ⟶ This type of production is known as left recursion.

$\boxed{A_2 \rightarrow b}$ | Removal of left   Left Recursion needs to be removed.

           $\boxed{A_2 \rightarrow b A_3}$ recursion: $X \rightarrow X\beta|\alpha$

           $A_3 \rightarrow A_2 A_1 A_3 | \epsilon$ $X \rightarrow X\beta$

                                $X \rightarrow \alpha A$

           $\boxed{\begin{array}{l} A_3 \rightarrow A_2 A_1 A_3 \\ A_3 \rightarrow A_2 A_1 \end{array}}$ $\begin{cases} A \rightarrow BA|\epsilon \\ A \rightarrow BA|B \end{cases}$

$X \Rightarrow$ W.P

i) $A_1 \to A_2 A_3$ v) $A_2 \to A_6 A_2$ xi) $A_4 \to A_5 A_2$

ii) $A_3 \to A_4 A_5$ vi) $A_6 \to A_7 A_7$

iii) $A_5 \to 1$ vii) $A_7 \to 0$

iv) $A_1 \to 0$ viii) $A_2 \to A_5 A_8$

            ix) $A_8 \to A_2 A_5$

            x) $A_2 \to 1$

GNF —

i) $A_1 \to 1 A_3$ iii) $A_5 \to 1$

vi) $A_6 \to 0 A_7$ iv) $A_1 \to 0$

viii) $A_2 \to 1 A_8$ vii) $A_7 \to 0$

ix) $A_8 \to 1 A_5$ x) $A_2 \to 1$

xi) $A_4 \to 1 A_8$

• No of production in CNF $\gg$ No. of productions in GNF

ii) $A_3 \to 1 A_8 A_5$

v) $A_2 \to 0 A_7 A_2$

2.   Convert To GNF —

         $S \to AA | a$

         $A \to SS | b$

$A_i \to A_j A_x$
$i < j, k$
or
$i < j$
$i \nleq k$

$A_1 \to A_2 A_2$
$A_1 \to a$

$A_2 \to A_1 A_1$ | $A_2 \to A_2 A_2 A_1$
$A_2 \to b$ | Removal of left recursion:
        | $A_2 \to b A_3$
        |
        | $A_3 \to A_2 A_1 A_3 | \epsilon$
        |
        | $A_3 \to A_2 A_1 A_3$
        | $A_3 \to A_2 A_1$

This type of production is known as left recursion.
Left Recursion needs to be removed.

$X \to X\beta | \alpha$

$X \to X\beta$

$X \to \alpha A$

$A \to \beta A | \epsilon$
$A \to \beta A | \beta$

GNF—

$\quad$ $A_1 \to b A_2$
$\quad$ $A_1 \to a$
$\quad$ $A_2 \to b$
$\quad$ $A_2 \to b A_3$

$A_2 \to b A_3 \begin{cases} A_3 \to b A_1 A_3 \\ A_3 \to b A_3 A_1 A_3 \end{cases}$

$A_3 \to A_2 A_1 \begin{cases} A_3 \to b A_1 \\ A_3 \to b A_3 A_1 \end{cases}$

Sequence Detector

Overlapped Sequence Detector $\begin{cases} \text{i/p} - \underline{1 0 1 0}\ 0 0 1\ \underline{0 1 0 1} 0 0 0 1\ 0 0 1 0 \\ \text{o/p} - 0 0 0 1\ 0 0 0 0\ 1 0 1 0 0 0 0 0 0 0 \end{cases}$ $\boxed{1\ 0\ 1\ 0}$

$*$ If $\boxed{011}$, then non-overlapped.

JK FF—

J	K	Present State	Next State
0	0	0 / 1	0 / 1
0	1	0 / 1	0 / 0
1	0	0 / 1	1 / 1
1	1	0 / 1	1 / 0

• Design a sequence detector to detect the sequence
$\quad$ 11011 using JK Flip Flop. (Overlapping is accepted)

$\qquad\qquad\qquad\quad$ 0 1 1 0 1 1 0 1 1 0 1 1 0 1 1
$\qquad$ Overlapping $\to$ $\quad$ 1 $\quad$ 1 $\quad$ 1 $\quad$ 1
$\qquad$ Non Overlapping $\to$ 1 $\qquad\qquad$ 1

<u>Step 1</u>: Calculate the number of states.

1 0 | 1 0
1 0 1 0 1 0
1 0 1 0 1 0 1 0

A —1/—→ B —1/—→ C

∴ N = 5,

⇒ States = 5 in ← 1 1 0 1 1
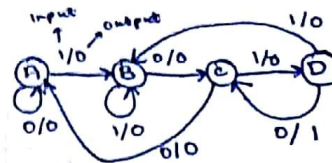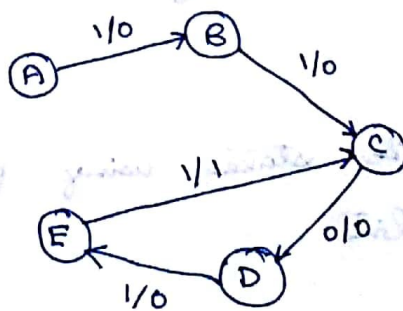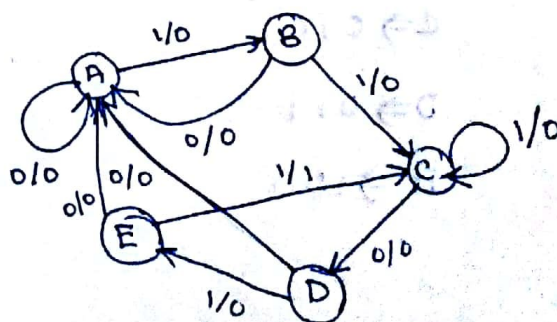

∴ No of bits = No. of States

<u>Step 2</u>: Name the states and describe what for the states are waiting and which sequence they have already met.

	already read	waiting for
A	—	1 1 0 1 1
B	1	1 0 1 1
C	1 1	0 1 1
D	1 1 0	1 1
E	1 1 0 1	1

<u>Step 3</u>: Draw the Transition diagram with the acceptable sequence only.

<u>Step 4</u>: Include all the other Transitions/breaking sequence

Scanned by CamScanner

Step 5: Draw the state Table

	0		1	
A	A	0	B	0
B	A	0	C	0
C	D	0	C	0
D	A	0	E	0
E	A	0	C	1

• 3 bits are required generate 5 states.

∴ 3 Flip Flops are req

Step 6: Derive the Number of Flip Flops.

$$2^{P-1} \leq N \leq 2^P \quad \left[P = no.\ of\ FF\ required \right.$$

$$N = 5$$

$$P = 3 \longrightarrow No.\ of\ flip\ flops.$$

Step 7: Rename the states using P number of binary bits.

$$A \rightarrow 000$$
$$B \rightarrow 001$$
$$C \rightarrow 010$$
$$D \rightarrow 011$$
$$E \rightarrow 100$$

Notice that in state table, for 0 input, output states are A or D. And for input 1, output states are B, C or E.

∴ we divide the states in an appropriate manner.

$$A, D \Rightarrow \text{even number}$$
$$B, C, E \Rightarrow \text{odd number}$$

$$A \Rightarrow 000$$
$$B \Rightarrow 001$$
$$C \Rightarrow 011$$
$$D \Rightarrow 100$$
$$E \Rightarrow 101$$

- Rewrite the state table using the new names.