The ***elasticity of demand*** *is the measure of responsiveness of demand for a commodity to the change in any of its determinants,* viz., price of the commodity, price of the substitutes and complements, consumers' income and consumer expectations regarding prices. Accordingly, there are several kinds of elasticities of demand—***price elasticity, cross elasticity, income elasticity*** and ***elasticity of price expectations***. In the following section, we have discussed ***price elasticity of demand***. The other kinds of elasticities have been discussed in the subsequent sections.

4.1 PRICE ELASTICITY OF DEMAND

The price ***elasticity***[1] ***of demand*** is defined as *the degree of responsiveness or sensitiveness of demand for a commodity to the change in its price.* More precisely, elasticity of demand is the percentage change in the quantity demanded of a commodity as a result of a certain percentage change in its price. A formal definition of price-elasticity of demand (e_p) is given below.

$$e_p = \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}}$$

A more general formula for calculating ***coefficient of price-elasticity*** is given as

$$e_p = -\frac{\Delta Q}{Q_0} \div \frac{\Delta P}{P_0}$$

$$= -\frac{\Delta Q}{Q_0} \cdot \frac{P_0}{\Delta P}$$

$$= -\frac{\Delta Q}{\Delta P} \cdot \frac{P_0}{Q_0} \qquad (4.1)$$

where Q_0 = original quantity demanded, P_0 = original price, ΔQ = change in quantity demanded, and ΔP = change in price.

To measure price elasticity numerically by using the formula given in Eq. (4.1), let us suppose that price of a commodity X decreases from Rs 10 per unit to Rs 8 per unit and quantity demanded of X increases from 50 units to 60 units per time unit. Thus, ΔP = Rs 10 – Rs 8 = Rs 2 and $\Delta Q = 50 - 60 = -10$. By substituting these values in elasticity formula, we get

$$e_p = -\frac{-10}{2} \cdot \frac{10}{50} = 1.0$$

Thus, elasticity co-efficient (e_p) equals 1.

Note that a minus sign (–) is inserted in the formula (Eq. 4.1) with a view to making **elasticity coefficient a** ***non-negative value***. The coefficient of price-elasticity calculated without minus sign in the formula will always be negative, because either ΔP or ΔQ will carry a negative sign depending on whether price increases or decreases. But a negative coefficient of elasticity is rather misleading because elasticity cannot be negative–less than zero. The 'minus' sign is, therefore, inserted in the price-elasticity formula as a matter of 'linguistic convenience' to make the coefficient of elasticity a non-negative value. Sometimes, it is also advised to ignore the negative sign of ΔP or ΔQ. The price-elasticity measure is, however, always reported with a negative sign just to indicate inverse relationship between price change and quantity demanded.

1. Generally, the adjective 'price' is omitted in 'price-elasticity of demand'. In fact, the term 'elasticity of demand' refers to the elasticity with respect to price. The concept of the elasticity used to denote other kinds of elasticities, however, the relevant adjective is generally used, e.g., income-elasticity and cross-elasticity and so on.

4.1.1 Arc and Point Elasticity

When price–elasticity of demand is measured between any two finite points on a demand curve, it is called ***arc elasticity*** and elasticity measured at a point on the demand curve is called ***point elasticity***. As noted above, the elasticity of demand measures the percentage change in quantity demanded due to a certain percentage change in price. The percentage change in price may be considerably high (e.g., 10 per cent, 20 per cent or even higher) or it may be very small—so small that it is not significantly different from zero. When change in price is significantly high, it shows a movement from one point on the demand curve to another point, making an *arc*. Therefore, price elasticity measured for a considerably high change in price, is called ***arc elasticity of demand***. And, when price elasicity is measured for very small changes in price—not significantly different from zero—it is called ***point elasticity***.

4.1.2 Measuring Arc Elasticity

The elasticity co-efficient between any two finite points on a demand curve, i.e., ***arc elasticity***, can be measured by using the formula given in Eq. (4.1). For example, the measure of elasticity between points J and K on the demand curve PM in Fig. 4.1 is the measure of arc elasticity. The movement from point J to K on the straight line demand curve PM shows a fall in price of commodity X from Rs 25 to Rs 15 and the consequent increase in demand from 30 units to 50 units. Here, $\Delta P = 25 - 15 = 10$ and $\Delta Q = 30 - 50 = -20$. The arc elasticity between points J and K (moving from J to K) can be calculated as given below:

$$e_p = -\frac{\Delta Q}{\Delta P} \cdot \frac{P_0}{Q_0}$$

$$e_p = -\frac{-20}{10} \cdot \frac{25}{30} = 1.66 \qquad (4.2)$$

Interpretation: Elasticity coefficient is interpreted as percentage change in demand due to one percent change in price. For example, in Eq. (4.2), ***elasticity coefficient is 1.66***. The *elasticity coefficient (1.66)* will be ***interpreted as a 1 per cent decrease in price of commodity X results in a 1.66 per cent increase in demand for it***.

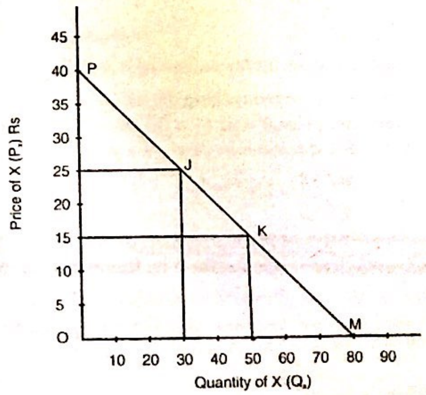

Fig. 4.1 Change in Price and Elasticity Coefficient

Problem in Using Arc Elasticity

The arc elasticity coefficeint should be used carefully because the measure of arc elasticity between any two finite points on a demand curve produces *two different* elasticity coefficients for the same fall and rise in price. In other words, *the arc elasticity coefficient varies between the same two finite points on a demand curve when the direction of change in price is reversed.* It is, therefore, open to misinterpretation. Consider, for example, arc elasticity of the demand curve *PM* between points *J* and *K* (Fig. 4.1). Price elasticity of demand for a fall in price from Rs 25 to Rs 15 is estimated to be 1.66 (see Eq. 4.2). This measure of arc elasticity can be mistaken to be the price elasticity of demand curve *PM* between points *J* and *K*, irrespective of the direction of change in price, whereas this elasticity coefficient is relevant only for the downward movement on the demand curve. It is not relevant for the upward movement from point *K* to *J*. The movement from point *K* to *J* implies a different arc elasticity, as shown below.

In case of the movement from point *K* to *J*, i.e., for rise in price, we have

$$P = 15, \Delta P = 15 - 25 = -10$$

and $$Q = 50, \Delta Q = 50 - 30 = 20$$

Substituting these values into the elasticity formula (Eq. 4.1), we get

$$e_p = -\frac{20}{10}\cdot\frac{15}{50} = 0.60 \qquad (4.3)$$

Note that price elasticity coefficient (0.60) for increase in price by Rs 5 is materially different from price elasticity (1.66) the same for the same decrease in price. Clearly, arc elasticity between any two finite points on a demand curve depends also on the *direction of change in price*.

Suggested Modifications

Economists have suggested some modifications in the elasticity formula to remove this anomaly in the concept of arc elasticity.

One, it is suggested that the problem arising due to the change in the direction of price-change may be avoided by using the lower values of *P* and *Q* in the elasticity formula. The formula is then

$$e_p = -\frac{\Delta Q}{\Delta P}\cdot\frac{P_l}{Q_l} \qquad (4.4)$$

where subscript *l* denotes lower values of *P* and *Q*.

Going by this formula for measuring elasticity between points *J* and *K* in Fig. 4.1, we use $P_l = 15$ (the lower one of the two prices) and $Q_l = 30$ (the lower one of the two quantities). By sustituting these values in Eq. 4.4, for decrease in price, we get

$$e_p = -\frac{20}{10}\cdot\frac{15}{30} = 1.0$$

This method, however, violates the rule of computing percentage change. The choice of the lower values of *P* and *Q* is arbitrary. This method is, therefore, devoid of any logic.

Two, another method suggested* to resolve this problem is to use average of the upper and lower values of *P* and *Q* in the fraction *P/Q*. The suggested formula can be written as

* See, for example, K. Lancaster, *Introduction of Modern Microeconomics*, 2nd Edn., 1974, p. 28.

$$e_p = -\frac{\Delta Q}{\Delta P}\cdot\frac{(P_u + P_l)/2}{(Q_l + Q_u)/2}$$

or

$$e_p = -\frac{Q_l - Q_u}{P_u - P_l}\cdot\frac{(P_u + P_l)/2}{(Q_l + Q_u)/2} \qquad (4.5)$$

(where subscripts *u* and *l* refer to upper and lower values, respectively.)

Substituting the values from our example, we get

$$e_p = -\frac{30 - 50}{25 - 15}\cdot\frac{(25 + 15)/2}{(30 + 50)/2} = 1.0$$

This method measures the elasticity mid-way between points *J* and *K*. The elasticity coefficient (1.0) is not applicable to the whole range of price-quantity combination between points *J* and *K* (see Fig. 4.1). It does not resolve the problem that arises due to the change in the direction of the price. It gives only the mean of the elasticities between the two points.

An ***alternative method*** to avoid this problem is to use point elasticity.

4.1.3 Measuring Point Elasticity

Point elasticity *is the measure of price elasticity at a finite point on a demand curve*. Point elasticity is, in fact, the measure of the proportionate change in quantity demanded in response to a *very* small proportionate change in price. The concept of point elasticity is useful where change in price and the consequent change in quantity demanded are very small. Besides, it offers an alternative to the arc elasticity. Point elasticity may be symbolically expressed as

$$e_p = \frac{\partial Q}{\partial P}\cdot\frac{P}{Q} \qquad (4.6)$$

The method of measuring price elasticity on linear and non-linear demand curves is explained below.

(i) Point elasticity of a linear demand curve. To illustrate the measurement of point elasticity on a linear demand curve, let us suppose that a *linear* demand curve is given by *MN* in Fig. 4.2 and that we need to measure elasticity at point *P*. Let us now substitute the values from Fig. 4.2 in Eq. 4.6. It is obvious from the figure that *P* = *PQ* and *Q* = *OQ*. What we need to find now are the values for ∂Q and ∂P. These can be obtained by assuming a *very small* change in price. But it will be difficult to depict these changes graphically as $\partial P \to 0$ and hence $\partial Q \to 0$. There is, however, an easy way to find the value for $\partial Q/\partial P$. In fact, the ratio $\partial Q/\partial P$ gives the reciprocal of the slope of the demand curve, *MN*. *The reciprocal of the slope of a straight line, MN*, at point P is geometrically given by *QN/PQ*. Therefore,

$$\frac{\partial Q}{\partial P} = \frac{QN}{PQ}$$

Since at point *P*, *P* (price) = *PQ* and *Q* = *OQ*, by substituting these values in (ignoring the minus sign), Eq. (4.6), we get

$$e_p = \frac{QN}{PQ}\cdot\frac{PQ}{OQ} = \frac{QN}{OQ} \qquad (4.7)$$

X

It can be proved geometrically that

$$e_p = \frac{QN}{OQ} = \frac{PN}{PM}$$

P-TO

Proof: To prove that $QN/OQ = PN/PM$, let us draw a horizontal line from P to the vertical axis. We have now three triangles ΔMON, ΔMRP and ΔPQN (Fig. 4.2.). Note that $\angle MON$, $\angle MRP$ and $\angle PQN$ of these triangles are right (90°) angles. Therefore, the other corresponding angles of the three triangles are equal. Given these properties ΔMON, ΔMRP and ΔPQN are **similar triangles.**

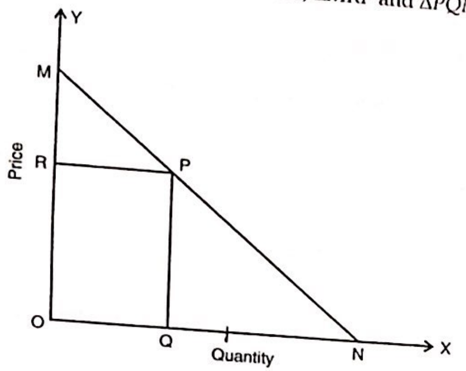

Fig. 4.2 Point Elasticity on a Linear Demand Curve

According to geometrical properties of similar triangles, the ratio of any two sides of a triangle is equal to the ratio of the corresponding sides of the other triangles. Therefore, in ΔPQN and ΔMRP,

$$\frac{QN}{PN} = \frac{RP}{PM}$$

Since $RP = OQ$, by substituting OQ for RP, we get

$$\frac{QN}{PN} = \frac{OQ}{PM}$$

By proportionality rule, $\frac{QN}{OQ} = \frac{PN}{PM}$

It is, thus, proved that $QN/OQ = PN/PM$. (4.8)

Note that PN and PM are two lower and upper segments of the demand curve, MN. It may thus be said that price elasticity at any point on a straight line demand curve is given by

$$e_p = \frac{\text{Lower segment}}{\text{Upper segment}}$$

(ii) Measuring point elasticity on a non-linear demand curve. Point elasticity of a non-linear demand curve is measured by drawing a tangent to the demand curve at the chosen point and measuring the elasticity of the tangent at this point. This gives the elasticity of the demand curve at the chosen point. Suppose we want to measure the elasticity of demand curve DD' at point P in Fig. 4.3. Let us now draw a line tangent to the demand curve DD', at point P as shown by the tangent MN. Since demand curve DD' and the line MN pass through the same point (P), the elasticity of demand curve DD' at point P is equal to the elasticity to the tangent, MN, at point P. By measuring the elasticity at point P on the tangent MN, we get the elasticity at point P on the demand curve DD'. The elasticity of the tangent MN at point P is given by

$$e_p = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q}$$

By substitution,

$$e_p = \frac{QN}{PQ} \cdot \frac{PQ}{OQ} = \frac{QN}{OQ}$$

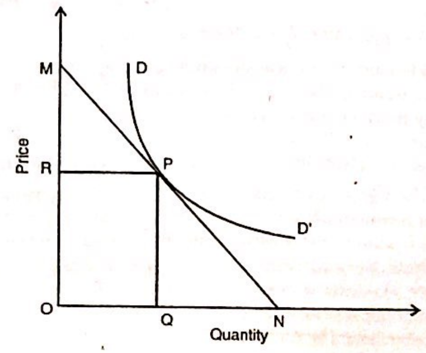

Fig. 4.3 Point Elasticity on a Non Linear Demand Curve

Geometrically, $QN/OQ = PN/PM = e$. (For Proof, see the preceding section).

4.1.4 Price Elasticity Along the Demand Curve

The price elasticity of demand varies all along a demand curve. Consider a linear demand curve MN in Fig. 4.4. At one and only one point, $e_p = 1$. At all other points (except terminal points), $e_p < 1$ or $e_p > 1$. At terminal point N, $e_p = 0$ and at terminal point M, elasticity is ***undefined***. This point is explained below.

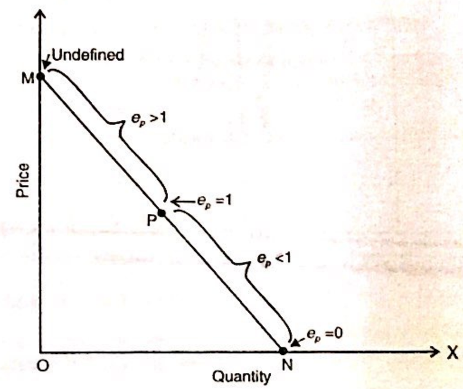

Fig. 4.4 Point Elasticities of Demand

We know that if a point on a demand curve is marked, it divides the demand curve into two parts. For example, if we choose a point mid-way, point P, on demand curve MN in Fig. 4.4, it divides the demand curve into two parts: PM (the upper segment) and PN (the lower segment). Given the above measure of point elasticity ($e_p = PN/PM$), the elasticity at a point on a linear demand curve may be interpreted as the ratio of the lower segment (PN) to the upper segment (PM) of the demand curve. That is,

$$e_p = \frac{\text{Lower segment}}{\text{Upper segment}} = \frac{PN}{PM}$$

Since in Fig. 4.4, $PN = PM$, $e_p = 1$. It follows that:

(a) at mid-point on a linear demand curve, $e_p = 1$.
(b) at any point on the upper (half) segment, $e_p > 1$;
(c) at any point on the lower (half) segment, $e_p < 1$;
(d) at point N, $e_p = 0$; and
(e) at point M, elasticity is undefined reason given below.

Important. The last point needs a clarification. It is a general practice of the text book authors to show $e_p = \infty$ at terminal point on the vertical axis, i.e., at point M in Fig. 4.4. This is mathematically incorrect. The reason is, measuring elasticity at point M involves division by zero, and division by zero is undefined. For example, at point M, lower segment equals MN and upper segment equals zero. Therefore, elasticity at point M is undefined. To quote Baumol, "Here [at point M] elasticity is not even defined because an attempt to evaluate the fraction p/x at that point forces us to commit the sin of dividing by zero. The readers who have forgotten why division by zero in immoral may recall that division is the reverse operation of multiplication. Hence, in seeking the quotient $c = a/b$ we look for a number, c, which when multiplied by b gives us the number a, i.e., for which $cb = a$. But if a is not zero, say $a = 5$, and b is zero, there is no such number because there is no c such that $c \times 0 = 5$".[1]

(i) Constant elasticity demand curve. The elasticity of most demand curves is not the same throughout. It varies from zero (0) to close to infinity, i.e., $0 < e_p < \infty$. In case of some demand curves, however, elasticity remains the same throughout their length, as shown in Fig. 4.5. Such demand curves are placed in the following categories.

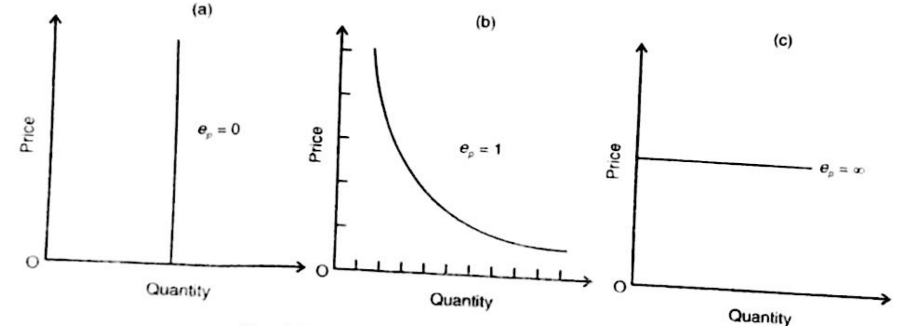

Fig. 4.5 Constant Elasticity Demand Curve

1 Baumol, W.J. *Economic Theory and Operation Analysis*, (Prentice Hall of India Private Limited, New Delhi), 4th Edn., p. 187.

(i) A perfectly inelastic demand curve — it has $e_p = 0$ throughout;
(ii) A unitary elastic demand curve — it has $e_p = 1$ throughout;
(iii) A perfectly elastic demand curve — it has $e_p = \infty$ throughout.

The three kinds of demand curves are shown in Fig. 4.5. (a), (b) and (c), respectively.

4.2 SLOPE AND PRICE ELASTICITY OF DEMAND CURVE

The elasticity of a demand curve is often judged by its appearance: the flatter the demand curve, the greater the elasticity, and *vice versa*. But such conclusions may be incorrect because two demand curves with different slopes may have the same elasticity *at a given price*. In fact, what the appearance of a demand curve reveals is its *slope*, not the elasticity. The *slope* of the demand curve is the relationship between marginal change in price (ΔP) and the resulting change in quantity demanded (ΔQ). The slope of demand curve is expressed as $\Delta P/\Delta Q$.

It is shown below (i) that demand curves having different slopes may have the same elasticity at a given price, and (ii) that demand curves having the same slope may have different elasticities at a given price.

4.2.1 Elasticity of Demand Curves With Different Slopes

Let us first illustrate that two demand curves with different slopes have the same elasticity at a given price. In Fig. 4.6, demand curves AB and AD have different slopes, as shown below.

Slope of demand curve $AB = \dfrac{OA}{OB}$; and

Slope of demand curve $AD = \dfrac{OA}{OD}$.

Note that term OA is common to both the ratios, but $OB < OD$. Therefore,

$$\frac{OA}{OB} > \frac{OA}{OD}.$$

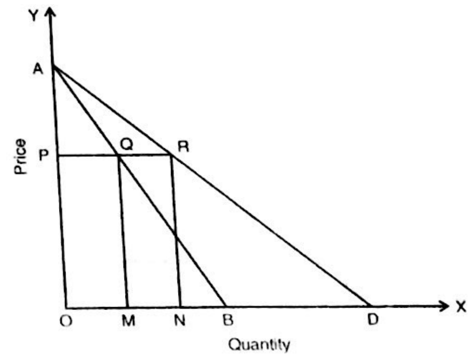

Fig. 4.6 Demand Curves having Different Slopes

Obviously, the slopes of the two demand curves are different. Let us now show that at a given price, both the demand curves have the same elasticity. As shown in Fig. 4.6, at price OP, *the relevant points for measuring the elasticity are* Q *and* R on the demand curves AB and AD, respectively. As we have already shown, price elasticity at a point on a linear demand curve is obtained as follows.

$$e_p = \frac{\text{Lower segment}}{\text{Upper segment}}$$

Thus, at point Q on the demand curve AB, $e_p = QB/QA$, and at point R on the demand curve AD, $e_p = RD/RA$.

It may be geometrically proved that the two elasticities are equal, i.e.,

$$\frac{QB}{QA} = \frac{RD}{RA}$$

Let us first consider ΔAOB. As shown in Fig. 4.6, an ordinate from Q to M at the horizontal axis, forms three triangles – ΔAOB, ΔAPQ and ΔQMB. Note that $\angle AOB$, $\angle APQ$ and $\angle QBM$ are right-angles. Therefore, all the three triangles are **right-angle triangles**. One of the properties of right-angle triangles is that the ratio of their two corresponding sides are always equal. Considering only the relevant triangles, ΔAPQ and ΔQMB, we have

$$\frac{QB}{QM} = \frac{AQ}{AP}$$

Since $QM = OP$, by substituting OP for QM in ratio QB/QM, we get

$$\frac{QB}{OP} = \frac{AQ}{AP}$$

Therefore, $\dfrac{QB}{AQ} = \dfrac{OP}{AP}$ = elasticity of AB at point Q.

It can be similarly proved that

$\dfrac{RD}{RA} = \dfrac{OP}{AP}$ = Elasticity of AD at point R.

It is thus proved that

$$\frac{QB}{QA} = \frac{RD}{RA} = \frac{OP}{AP}$$

It is thus proved that elasticity of demand curves AB and AD at price OP is the same.

4.2.2 Different Elasticity of Parallel Demand Curves at a Price

Let us now show that two demand curves having the same slope have different elasticities at a given price. Consider the demand curves JK and LM in Fig. 4.7. The demand curves JK and LM are parallel and, therefore, have the same slope. Point R on the demand curve JK and point Q on the demand curve LM show the quantities demanded at a given price, OP.

The elasticity at point R on demand curve JK is RK/RJ and elasticity at point Q on demand curve LM is QM/QL. It can be easily proved that

$$\frac{RK}{RJ} \neq \frac{QM}{QL}$$

Following the logic of the preceding section, we can prove that

$$\frac{RK}{RJ} = \frac{PO}{PJ}$$

and

$$\frac{QM}{QL} = \frac{PO}{PL}$$

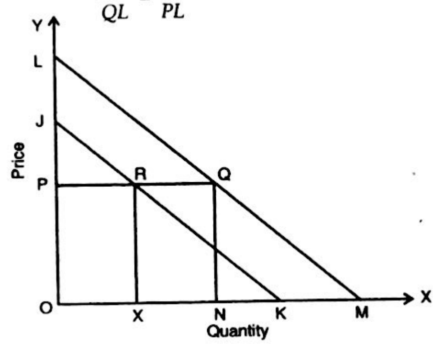

Fig. 4.7 Different Elasticities of Parallel Demand Curves

It can be seen from Fig. 4.7 that $PJ < PL$. Therefore,

$$\frac{PO}{PJ} > \frac{PO}{PL}$$

It is thus proved that

$$\frac{RK}{RJ} > \frac{QM}{QL}$$

It may be concluded from the above conclusions that demand curves having the same slope may have different elasticities, and demand curves having different slopes may have the same elasticities, both at a given price.

4.3 MEASURING PRICE ELASTICITY FROM A DEMAND FUNCTION

Price elasticity of demand can be measured directly from a demand function. In this section, we will describe the method of measuring price elasticities from a given demand function—linear and nonlinear.

(i) Measuring elasticity from a linear demand function. Suppose a linear demand function is given as follows.

$$Q = a - b\,P$$

At a given price, say, P_1, this demand function reads as

$$Q_1 = a - b\,P_1$$

When price changes from P_1 to P_2, then

$$Q_2 = a - b\,P_2$$

To measure the arc elasticity, we need two ratios: $\Delta Q/\Delta P$ and P/Q. Given the two demand functions, ratio $\Delta Q/\Delta P$ can be obtained as follows.

(vii) Direction of change in price. The direction of change in price also determines the elasticity. Between any two finite points on the demand curve, elasticity is higher for the fall in price and *vice versa* (see also pp. 71-72).

4.5 PRICE-ELASTICITY, MARGINAL, AVERAGE AND TOTAL REVENUE

In this section, we look into the relationship between (i) price elasticity of demand and marginal revenue; (ii) marginal revenue and average revenue; and (iii) price elasticity and total revenue. These relationships are of great importance in business analysis.

4.5.1 Price Elasticity and Marginal Revenue

Marginal revenue is the addition to the total revenue (TR) as a result of sale of one additional unit. It is also defined as the first derivative of TR-function, i.e.,

$$MR = \frac{\partial TR}{\partial Q}$$

The relationship between price elasticity and marginal revenue (MR) can be derived as follows. Let us suppose that a given output, Q, is being sold at a price P, so that the total revenue (TR) equals P times Q, i.e.,

$$TR = P.Q$$

The marginal revenue (MR) can be obtained by differentiating $TR = P.Q$ with respect to Q. Thus,

$$\begin{aligned} MR &= \frac{\partial (P.Q)}{\partial Q} \\ &= P\frac{\partial Q}{\partial Q} + Q\frac{\partial P}{\partial Q} \\ &= P + Q\frac{\partial P}{\partial Q} \end{aligned}$$

$$MR = P\left(1 + \frac{Q}{P}\cdot\frac{\partial P}{\partial Q}\right) \tag{4.14}$$

Note that $\frac{Q}{P}\cdot\frac{\partial P}{\partial Q}$ in Eq. (4.14) is the reciprocal of the price elasticity coefficient. It means that

$$\frac{Q}{P}\cdot\frac{\partial P}{\partial Q} = -\frac{1}{e}$$

By substituting $-\frac{1}{e}$ for $\frac{Q}{P}\cdot\frac{\partial P}{\partial Q}$ in Eq. (4.14), we get

$$MR = P\left(1 - \frac{1}{e}\right) \tag{4.15}$$

Eq. (4.15) gives the relationship between price elasticity (e) and MR.

4.5.2 Relation between MR and AR

In Eq. (4.15), P is the same as AR. Eq. (4.15) can therefore be written as

$$MR = AR\left(1 - \frac{1}{e}\right)$$

and $$AR = \frac{MR}{\left(1 - \frac{1}{e}\right)}$$

or $$AR = MR\left(\frac{e}{e-1}\right) \tag{4.16}$$

Eq. (4.16) gives the relationship between AR and price elasticity.

Graphical Proof. Eq. (4.16) gives the relationship between AR and MR and between AR and price elasticity. The relationship between MR and AR can also be derived geometrically. Suppose AR curve is given by the curve AR in Fig. 4.8. Then MR curve is given by the curve AM.

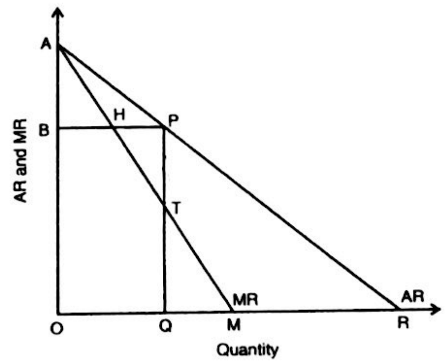

Fig. 4.8 Relationship between *AR* and *MR*

Let us suppose that price is given at PQ ($= BO$). As has been proved above, price elasticity at point P on the AR curve (which is the same as demand curve) can be expressed as

$$e = \frac{QR}{OQ} = \frac{PR}{AP} = \frac{OB}{AB}$$

Considering the last term, i.e., $e = OB/AB$, since $OB = PQ$,

$$\therefore \quad e = \frac{PQ}{AB} \tag{4.17}$$

In Fig. 4.8, $AB = PT$.[1] By substituting PT for AB in Eq. (4.17), we get

1. Proof. At price PQ, total revenue $= PQ \times OQ$, which equals the area $OBPQ$. Considering from MR angle, the total revenue at price PQ is given by the area $OATQ$. Therefore, $OBPQ = OATQ$. It can be observed from Fig. 4.8 that area $OBHTQ$ is common to the areas $OBPQ$ and $OATQ$. Therefore, area of ΔABH = area of ΔTPH. Note that $\angle ABH$ and $\angle TPH$ are right angles. Therefore, $\Delta ABH = \Delta TPH$. The properties of right angle triangles of equal size tell that their corresponding sides are equal. Therefore, $BH = HP$, $AH = HT$, and $AB = PT$.

$$e = \frac{PQ}{PT} \tag{4.18}$$

Since $PT = PQ - TQ$, Eq. (4.18) may be written as

$$e = \frac{PQ}{PQ - TQ} \tag{4.19}$$

It can be seen in Fig. 4.8 that $PQ = AR$ and $TQ = MR$. Therefore, Eq. (4.19) can be expressed as

$$e = \frac{AR}{AR - MR}$$

and

$$MR = AR - \frac{AR}{e} \tag{4.20}$$

or

$$MR = AR\left(1 - \frac{1}{e}\right) \tag{4.21}$$

Then,

$$AR = \frac{MR}{\left(1 - \frac{1}{e}\right)}$$

or

$$AR = MR\left(\frac{e}{e-1}\right) \tag{4.22}$$

Thus, we arrive at the same relationship between MR and AR as given in Eq. (4.16).

4.5.3 Price Elasticity and Total Revenue

Since *total revenue (TR)* and *marginal revenue (MR)* are interrelated, the relationship between TR and *price elasticity* of demand (e_p) can be traced through the relationship between MR and *price elasticity* (e_p). Given the relationship between MR and e_p in Eq. (4.21), the relationship between TR and e_p can be summed up as follows.

(a) where $e_p = 1$, $MR = 0$. Therefore TR does not change with change in price;
(b) where $e_p < 1$, $MR < 0$. Therefore TR decreases with decrease in price and increases with increase in price; and
(c) where $e_p > 1$, $MR > 0$. In this case, TR decreases with increase in price and increases with decrease in price.

This nature of relationships between TR and e_p can be illustrated graphically. We know that $TR = P.Q$. The value for P and Q can be obtained by assuming a demand function. Let us assume a demand function as

$$Q = 100 - 5P$$

Given the demand function, *price function* can be obtained as given below.

$$P = 20 - 0.2Q$$

Now, that we know the value of P, TR can be obtained as follows.

$$TR = P \cdot Q = (20 - 0.2Q)\,Q$$
$$= 20Q - 0.2Q^2$$

From the TR-function, MR-function can be derived as

$$MR = \frac{\partial TR}{\partial Q} = 20 - 0.4Q$$

The TR-function is presented graphically in panel (a) and the demand and MR functions are presented in panel (b) of Fig. 4.9. As the figure shows, at point P on the demand curve, $e_p = 1$ where output, $Q = 50$. Below point P, $e_p < 1$ and above point P, $e_p > 1$. It can be seen in panel (a) of Fig. 4.9 that TR increases over the range of demand curve having $e_p > 1$; TR reaches its maximum level where $e_p = 1$; and it decreases over the range $e_p < 1$.

The relationship between price-elasticity and TR is summed up in Table 4.1. As the table shows, when demand is *perfectly inelastic* (i.e., $e_p = 0$ as in the case of a vertical demand line) there is no decrease in quantity demanded when price is raised and *vice versa*. Therefore, a rise in price increases the total revenue and *vice versa*.

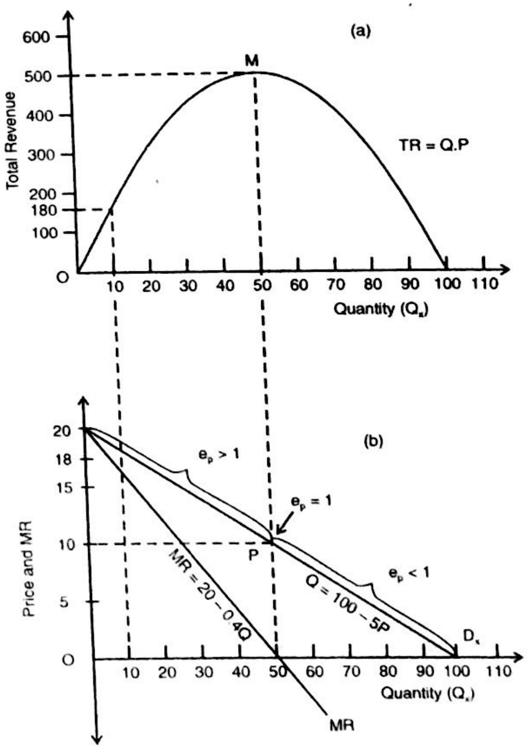

Fig. 4.9 Price Elasticity and Total Revenue

In case of an *inelastic demand* (i.e., $e_p < 1$), quantity demanded increases by less than the proportionate decrease in price and hence the total revenue falls when price falls. The total revenue increases when price increases because quantity demanded decreases by less than the proportionate increase in price.

Table 4.1. Elasticity, Price-change and Change in *TR*

Elasticity Co-efficient	Nature of demand	Change in Price	Change in *TR*
$e_p = 0$	Perfectly inelastic	Increase Decrease	Increase Decreases
$e_p < 1$	Inelastic	Increase Decrease	Increase Decrease
$e_p = 1$	Unitary elastic	Increase Decrease	No change No change
$e_p > 1$	Elastic	Increase Decrease	Decrease Increase

If demand for a product is *unit elastic* ($e_p = 1$) quantity demanded increases (or decreases) in the proportion to decrease (or increase) in the price. Therefore, total revenue remains unaffected.

If demand for a commodity has $e_p > 1$, change in quantity demanded is greater than the proportionate change in price. Therefore, the total revenue increases when price falls and *vice versa*.

4.6 PRICE-ELASTICITY AND CONSUMPTION EXPENDITURE

Another important relationship which is often referred to in economic analysis is one between price elasticity and consumption expenditure. From the law of demand, we know that quantity demanded of a commodity increases when its price falls. But, ***what happens to the total expenditure on that commodity***—does it fall or increase?

The relationship between price elasticity and total consumption expenditure may be derived as follows. The total consumption expenditure (TE_x) on commodity *X*, at a given price P_x, all other prices remaining the same, is given by

$$TE_x = Q_x \cdot P_x \qquad (4.23)$$

By differentiating Eq. (4.23) with respect to P_x, we get marginal expenditure (ME_x) as

$$ME_x = \frac{\partial Q_x \cdot P_x}{\partial P_x} = Q_x + P_x \frac{\partial Q_x}{\partial P_x}$$

$$= Q_x \left[1 + \frac{P_x}{Q_x} \cdot \frac{\partial Q_x}{\partial P_x}\right]$$

In Eq. (4.24), $\frac{P_x}{Q_x} \cdot \frac{\partial Q_x}{\partial P_x} = -e_p$ (4.24)

By substitution, Eq. (4.24) can be written as

$$ME_x = \frac{Q_x \cdot P_x}{\partial P_x} = Q_x(1 - e_p) \qquad (4.25)$$

It may be inferred from Eq. (4.25) that whether the total expenditure increases, decreases or remains constant as a result of change in price depends on whether

$$Q_x(1 - e_p) \gtreqless Q_x$$

Whether $Q_x(1 - e_p)$ is greater than, equal to or less than Q_x depends on whether $e_p \gtreqless 1$.

The relationship between, total consumer expenditure and price elasticity of demand has been summarised up in Table 4.2.

Table 4.2 Elasticity and Consumption Expenditure

Elasticity (e_p)	Price change	Marginal expenditure	Total expenditure
$e_p > 1$	Rise Fall	$ME < 0$ $ME > 0$	Decreases Increases
$e_p = 1$	Rise Fall	$ME = 0$ $ME = 0$	Constant Constant
$e_p < 1$	Rise Fall	$ME > 0$ $ME < 0$	Increases Decreases

As shown in the above table, when $e_p > 1$, i.e., demand is *elastic*, an increase in price causes more than proportionate decrease in quantity demanded. Hence, total expenditure decreases. And, if price decreases, quantity demanded increases more than proportionately. As a result, total expenditure increases.

When $e_p = 1$, a rise (or fall) in price causes a proportionate decrease (or increase) in quantity demanded leaving total expenditure unchanged.

When $e_p < 1$, i.e., when demand is *inelastic*, a rise in price causes increase in the total expenditure because demand decreases less than proportionately, and a fall in price reduces it as quantity demanded increases less than proportionately.

4.7 OTHER ELASTICITIES OF DEMAND

In this section, we will discuss elasticities of demand with respect to some of its other determinants often used in economic analysis.

4.7.1 Cross-Elasticity of Demand

Cross-elasticity is the measure of responsiveness of demand for a commodity to the changes in the price of its substitutes and complementary goods. For instance, cross-elasticity of demand for tea (*T*) is the percentage change in its quantity demanded due to a change in the price of its substitute, coffee (*C*). Formula for measuring cross-elasticity of demand for tea ($e_{t,c}$) with respect to price of coffee (P_c) is

$$e_{t,c} = \frac{\text{Proportionate change in demand for tea } (Q_t)}{\text{Proportionate change in price of coffee } (P_c)}$$

$$= \frac{P_c}{Q_t} \cdot \frac{\Delta Q_t}{\Delta P_c} \qquad (4.26)$$

The cross-elasticity of demand for coffee (Q_c) with respect to price of tea (P_t) is

$$e_{c,t} = \frac{P_t}{Q_c} \cdot \frac{\Delta Q_c}{\Delta P_t}$$

For a numerical example, suppose that price of coffee (P_c) increases from Rs 10 to Rs 15 per 10 grams, and as a result, demand for tea increases from 20 tons to 30 tons per week, price of tea remaining constant. By substituting these values in Eq. (4.26), we get cross-elasticity of demand for tea with respect to price of coffee, as

$$e_{t,c} = \frac{10}{20} \cdot \frac{20-30}{10-15}$$
$$= \frac{10}{20} \cdot \frac{-10}{-5}$$
$$= 1.0$$

Note that cross–elasticity with respect to substitutes is always positive.

The same formula is used to measure the cross-elasticity of demand for a good in response to change in the price of its ***complementary goods***. Electricity to electrical gadgets, petrol to automobile, butter to bread, sugar and milk to tea and coffee, are the examples of complementary goods.

When two goods are substitutes for each other, their demand has a *positive corss-elasticity* because increase in the price of one increases the demand for the other. But, the demand for complementary goods has *negative cross-elasticity*, for increase in the price of a good decreases the demand for its complementary goods.

An ***important*** aspect of cross-elasticity is that if cross-elasticities between any two goods are positive, the two goods may be considered as substitutes for each other. Also the greater the cross-elasticity, the closer the substitute. Similarly, if cross-elasticity of demand for any two related goods is negative, the two may be considered as complementary for each other: the higher the negative cross-elasticity, the higher the degree of complementarity.

4.7.2 Income-Elasticity of Demand

Apart from price of a product and its substitutes, another important determinant of demand for a product is consumer's income. As noted earlier, the relationship between demand for normal and luxury goods and consumer's income is of positive nature, unlike the negative price-demand relationship. In simple words, the demand for normal goods and services increases with increase in consumer's income and *vice versa*. The responsiveness of demand to the change in consumer's income is known as ***income-elasticity*** of demand.

Income-elasticity e_y of demand for a product, say X with respect to change in money income (M) can be defined as:

$$e_y = \frac{\Delta Q_x / Q_x}{\Delta M / M} = \frac{M}{Q_x} \cdot \frac{\Delta Q_x}{\Delta M} \qquad (4.27)$$

where Q_x = quantity of X demanded; M = disposable money income; ΔQ_x = change in quantity demanded of X; and ΔM = change in income.

Unlike price-elasticity of demand (which is negative except in case of Giffen goods), income-elasticity of demand is positive because of a positive relationship between income and quantity demanded of a product. There is an exception to this rule. Income-elasticity of demand for an *inferior good* is negative, because of negative income-effect. The demand for inferior goods decreases with increase in consumer's income and *vice versa*. When income increases, consumers switch over to the consumption of superior commodities. That is, they substitute superior goods for inferior ones. For instance, when income rises, people prefer to buy more of rice and wheat and less of inferior foodgrains like *bajra*, ragi, etc. and use more of taxi and less of bus service and so on.

Nature of commodity and income-elasticity. For all normal goods, income-elasticity is positive though the degree of elasticity varies in accordance with the nature of commodities. As noted above, consumer goods are generally grouped under three broad categories, viz., necessities (essential consumer goods), comforts, and luxuries. The general pattern of income-elasticities for goods of different categories for increase in income and their impact on sales are given in Table 4.3.

Table 4.3. Nature of Commodities, Income Elasticity and Expenditure

Commodities	Coefficient of income Elasticity	Impact on Expenditure
1. Necessities	Less than unity ($e_y < 1$)	Less than proportionate change in expenditure
2. Comforts	Almost equal to unity ($e_y \cong 1$)	Almost proportionate change in expenditure
3. Luxuries	Greater than unity ($e_y > 1$)	More than proportionate increase in expenditure

Income-elasticity of demand for different categories of goods may however vary from household to household and from time to time, depending on choice, taste and preference of the consumers, levels of their consumption and income, and their susceptibility to 'demonstration effect'. The other factor which may cause deviation from the general pattern of income-elasticities is the frequency of increase in income. If income increases regularly and frequently, income-elasticities will conform to the general pattern, otherwise not.

Uses of Income-Elasticity. Some ***important uses*** of income-elasticity are as follows.

First, the concept of income-elasticity can be used to estimate the future demand for a product provided the rate of increase in income and income-elasticity of demand for the product are known. The knowledge of income-elasticity can be used for forecasting demand, when a changes in personal income is expected, other things remaining the same.

Secondly, the concept of income-elasticity can also be used to define the ***'normal'*** and ***'inferior'*** goods. The goods whose income-elasticity is positive for all levels of income are termed as 'normal goods'. On the other hand, the goods for which income elasticities are negative, beyond a certain level of income, are termed as ***'inferior goods'***.

4.7.3 Elasticity of Price Expectations

Sometimes, consumer's price expectations play a much more important role in determining demand for a commodity than any other factor. The concept of price-expectation-elasticity refers to the expected change in future price (P_t) as a result of change in price of a product in the past (P_p). The elasticity of price-expectation is defined and measured by the following formula: