Assignment: Probability & Numerical Methods.

MATH 2202

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1) Bisection Method:-

a) $f(x) = x + \ln x - 2 = 0$. Let x lies in $[1, 2]$

$f(1) = 1 + \ln 1 - 2 = -1$

$f(2) = 2 + \ln 2 - 2 = 0.693$

$\therefore$ The root lies in $[1, 2]$, $\frac{1+2}{2} = 0.5$

$f(0.5) = -0.193$

$\therefore [0.5, 2]$, $\frac{0.5+2}{2} = 1.25$

$f(1.25) = -0.527$.

$\therefore [1.25, 2]$, $\frac{1.25+2}{2} = 1.625$

$f(1.625) = 0.111$

$\therefore [1.25, 1.625]$, $\frac{1.25+1.625}{2} = 1.4375$

$f(1.4375) = -0.1996$.

$\therefore [1.4375, 1.625]$

$f(1.53125) = -0.0426$.

$[1.531, 1.625]$

$f(1.578) = 0.034$

$[1.531, 1.578]$

$f(1.554) = -4.346\times10^{-3}$, $[1.554, 1.578]$

$f(1.566) = 0.0145 \Rightarrow [1.554, 1.566]$

$f(1.56) = 0.00$ $\therefore \boxed{x = 1.56}$

1)(b) $f(x) = 3x - \sqrt{1 - \sin x}$

$f(0) = -1$, $f(1) = 2.008$

$\therefore [0, 1]$.

$f(0.5) = 0.799$.
$(0, 0.5)$

$f(0.25) = -0.248$.
$(0.25, 0.5)$

$f(0.375) = .0128$.
$(0.25, 0.375)$

$f(0.3125) = -0.059$
$(0.3125, 0.375)$

$f(0.343.75) = 0.034$
$(0.3125, 0.34375)$

$f(0.328125) = -0.013$
$(0.328125, 0.34375)$

$f(0.3359375) = 0.010$.

Ans $x = 0.34$

2) Newton Raphson

$f(x) = \log_e x - \cos x = 0$, $f'(x) = \frac{1}{x} + \sin x$

$f(1) = -0.9998$, $f(2) = -0.3062$, $f(3) = 0.09998$

$[2, 3]$

$x_0 = 2.5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ ~~$\Rightarrow 2.5 -$~~ $\Rightarrow 2.73.22$.

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} =$

4) Gauss Elimination :-

a) $2x + y + z = 10$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 16.$

$$AX = B.$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

<u>Augmented Matrix</u>

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

$R_3 \to 2R_3 - R_2$

$R_2 \to 2R_2 - 3R_1$

$$\left[\begin{array}{cccc} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{array}\right]$$

$R_3 \to R_3 - 7R_2$

$$\left[\begin{array}{cccc} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{array}\right]$$ (upper triangular Matrix)

$$\therefore \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ -20 \end{bmatrix}$$

$-4z = -20 \Rightarrow z = 5$

$y + 3z = 6 \Rightarrow y = -9$

$2x + y + z = 10 \Rightarrow x = 7$

4)(b). $2x - y + 3z = 4$

$x + \quad z = 2.$

$2y + z = 3.$

Augmented Matrix

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 1 & 0 & 1 & 2 \\ 0 & 2 & 1 & 3 \end{array}\right]$$

$R_2 \rightarrow 2R_2 - R_1$

$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}$$

$\therefore \quad 3z = 3 \Rightarrow z = 1$

$\therefore \quad y - z = 0. \Rightarrow y = 1$

$\therefore \quad 2x - y + 3z = 4 \Rightarrow x = 1$

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5) a) Gauss Seidel Method

$10x + y + z = 12$

$2x + 10y + z = 13$ Already diagonally dominant

$2x + 2y + 10z = 14$

$x = \frac{1}{10}(12 - y - z)$

$y = \frac{1}{10}(13 - 2x - z)$

$z = \frac{1}{10}(14 - 2x - 2y)$

Iterations

1st :- $y = 0, \; z = 0.$

$x = \frac{1}{10}(12) = 1.2$

$y = \frac{1}{10}(13 - 2(1.2) - 0) = 1.06$

$z = \frac{1}{10}(14 - 2(1.2) - 2(1.06)) = 0.948$

2nd :-

$x = \frac{1}{10}(12 - 1.06 - 0.948) = 0.999$

$y = \frac{1}{10}(13 - 2(0.999) - 0.948) = 1.0054$

$z = \frac{1}{10}(14 - 2(0.999) - 2(1.0054)) = 0.99912$

3rd

$x = \frac{1}{10}(12 - 1.0054 - 0.99912) = 0.999548$

$y = \frac{1}{10}(13 - 2 \times (0.999548) - 0.99912) = 1.000$

$z = \frac{1}{10}(14 - 2(0.999548) - 2(1.000)) = 1.000$

$\therefore x = 0.99, \; y = 1.00, \; z = 1.00$

9) Forward difference table:-

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
0.10	0.1003	0.0508	0.0008	-1.8×10^{-3}	8.2×10^{-3}
0.15	0.1511	0.0516	-0.001	6.4×10^{-3}	
0.20	0.2027	0.0506	5.4×10^{-3}		
0.25	0.2533	0.056.			
0.30	0.3093				

$$p = \frac{x - x_0}{h} = \frac{x - 0.10}{0.05}$$

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{\lfloor 4}\Delta^4 y_0$$

$$p \text{ for } 0.12 = \frac{0.12 - 0.10}{0.05} = 0.4$$

$$f(x) = 0.1003 + (0.4)(0.0508) + \frac{(0.4)(0.4-1)(0.0008)}{2} + \frac{(0.4)(0.4-1)(0.4-2)(-1.8\times10^{-3})}{3!} + \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)(8.2\times10^{-3})}{4!}$$

$$= 0.1003 + 0.02032 + (-9.6\times10^{-5}) + (-1.152\times10^{-4}) + (-3.412\times10^{-4})$$

$$\boxed{\tan 0.12 = 0.12006.}$$

$\tan 0.26 = 0.2662$

$\tan 0.35 = 0.365300$ (Similarly)

$\tan 0.5 = 0.5543$

(ii)

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$
-1	1			
		0	0	
0	1			-4
		0	-4	
1	1			
		$-4.$		
2	-3			

$$p = \frac{x - x_0}{h} \Rightarrow p = \frac{x+1}{1.}$$

$$f(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0$$

$$= 1 + (x+1)(0) + \frac{(x+1)(x)}{2}(0) + \frac{(x+1)(x)(x-1)}{3\times 2\times 1.}(-4)^2$$

$$= 1 - \frac{2}{3}(x)(x^2-1)$$

$$= 1 - \frac{2}{3}(x^3 - x)$$

$$= \frac{-2}{3}x^3 + \frac{2}{3}x + 1.$$

$$f(x) = -\frac{1}{3}(2x^3 - 2x - 3)$$

$$= -\frac{1}{3}(2\times(0.5)^3 - 2\times 0.5 - 3)$$

$$= 1.25$$

12) Langrangerian Interpolating Polynomial.

x_0	2	y_0	0.69315
x_1	2.5	y_1	0.91629
x_2	3.0	y_2	1.09861.

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_1)(x_2-x_0)} y_2$$

$$y = \frac{(x-2.5)(x-3)}{0.5}(0.69315) + \frac{(x-2)(x-3)}{-(0.5)^2}(0.91629) + \frac{(x-2)(x-2.5)}{0.5}(1.09861)$$

$$f(x) = -0.08164x^2 + 0.81366x - 0.60761$$

for $y = \ln 2.7$.

$x = 2.7$.

$y(2.7) = 0.994164$.

13) Find the value of $\int_0^1 \sqrt{1-x^2}\, dx$ using, $n = 6$.

a) Trapezoidal Rule

b) Simpson's $\frac{1}{3}$ Rule.

$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$.

x_0	0	y_0	1
x_1	1/6	y_1	0.986
x_2	2/6	y_2	0.943
x_3	3/6	y_3	0.866
x_4	4/6	y_4	0.745
x_5	5/6	y_5	0.553
x_6	1	y_6	0

a) $\int_0^1 \sqrt{1-x^2}\,dx = \frac{1}{6\times2}\left[(1+0)+2(0.986+0.943+0.866+0.745+0.553)\right]$

$= 0.76549$

b) $\int_0^1 \sqrt{1-x^2}\,dx = \frac{1}{6\times3}\left[(1+0)+4(0.986+0.866+0.553) + 2(0.943+0.745)\right]$

$= 0.77753$

(4)

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06.

$h = 0.01.$

Area $= \frac{0.01}{2}\left[(1.93+2.06)+2(1.95+2.01+1.98+2.03)\right]$

$= 0.09965.$

15) $\int_1^5 \log_{10} x\,dx$, $n=8$. , $h = \frac{5-1}{8} = \frac{1}{2}$.

x_0	1	y_0	0
x_1	1.5	y_1	0.17609
x_2	2.0	y_2	0.30103
x_3	2.5	y_3	0.39794
x_4	3.0	y_4	0.47712
x_5	3.5	y_5	0.544068
x_6	4.0	y_6	0.602059
x_7	4.5	y_7	0.653225
x_8	5.0	y_8	0.6989700.

$$\text{Area} = \frac{1}{4}\left[(y_0+y_8) + 2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]$$

$$= 1.75050225$$

17) Euler's Method

$$\frac{dy}{dx} = x^2+4y \quad , \text{ find } y \text{ at } x = 0.2$$
$$h = 0.05$$

$$y_{n+1} = y_n + h(f(x_n, y_n)) \quad -(i)$$

	x		y
x_0	0	y_0	1
x_1	0.05	y_1	1.2
x_2	0.10	y_2	1.440125
x_3	0.15	y_3	1.72865
x_4	0.20	y_5	2.0785

using (i) at every step.

18) Modified Euler's Method

$$\frac{dy}{dx} = \log_{10}(x+y).$$

$y_0 = 1, \quad x_0 = 0, \quad h = 0.2.$

$n=0, \quad x_1 = 0.2.$

$y_1 = y_0 + h f(x_0, y_0).$
$= 1 + 0.2(0) = 1.$

$\Rightarrow y_1 = 1.$

$n=0, m=1.$

$$y_1' = y_0 + \frac{0.2}{2}(0 + \log_{10}(0.2+1)) = 1.00792.$$

$n=0, m=2$

$$y_1'' = y_0 + \frac{0.2}{2}(\log(0.2+1.00792)) = 1.0082$$

$n=1,\ x_2=0.4$

$y_2 = y_1 + h(f(x_1, y_1))$

$y_2 = 1.02463.$

$n_1 = 1,\ m=1,\ x_2 = 0.4$

$y_2^1 = y_1 + \frac{h}{2}\left(f(x_1, y_1) + f(x_2, y_2^0)\right)$

$= 1.0052 + \frac{0.2}{2}\left(\log_{10}(0.2 + 1.0082) + \log_{10}(0.4 + 1.02463)\right)$

$= 1.03178$

$y_2^2 = y_1 + \frac{h}{2}\left(f(x_1, y_1) + f(x_2 + y_2^1)\right)$

$= 1.03200.$

$n=2,\ x_3 = 0.5,\ h = 0.1$

$y_3 = y_2 + h f(x_2, y_2)$

$= 1.032 + (0.1 \times \log((0.4 + 1.032))$

$= 1.04759$

$y_3^1 = y_2 + \frac{h}{2}\left(f(x_2, y_2) + f(x_3, y_3^0)\right)$

$= 1.04927$

$y_3^{11} = y_2 + \frac{h}{2} f(x_2, y_2) + f(x_3, y_3^1)$

$\boxed{y_3(0.5) = 1.04930}$

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19) Runge Kutta's 4 order method

$$\frac{dy}{dx} = \frac{1}{x+y}, \quad y_0 = 1, \quad x_0 = 0, \quad x_1 = 0.5, \quad h = 0.5$$

n = 0

$$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = h f(x_0, y_0) = 0.5 \times \frac{1}{1+0} = 0.5$$

$$k_2 = h f(x_0 + 0.25, 1 + \frac{0.5}{2}) = 0.5 f(0.25, 1.25) = 0.33$$

$$k_3 = 0.5 f(0.+0.25, 1 + \frac{0.33}{2}) = \frac{0.5}{0.25+1.165} = 0.35$$

$$k_4 = 0.5 f(0+.25, (1+0.35)). = 0.2702$$

$$y_1 = 1 + \frac{1}{6}[0.5 + 2 \times 0.33 + 2 \times 0.35 + 0.2702]$$

$$= 1.355$$

Similarly

$$y_2 = 1.584$$