

1

## QUESTIONS

D.M

Q1). What do you mean by macrostate and microstate?

Ans  $\Rightarrow$  Whether the Particles are distinguishable or not, a specification of set of occupation number i.e the number of Particles in each energy level of the system defines the macrostate of the system.

If the Particles are indistinguishable, a specification of ( $W$ ) i.e the number of Particles in each energy state of the system defines the microstate of the system.

Q2). Define thermodynamic probability and most probable state of a system.

Ans  $\Rightarrow$  the number of microstates ( $W$ ) corresponding to a given macrostate ( $M$ ) is known as the thermodynamic probability of a given macrostate.

The most probable state is simply the macrostate which has the maximum number of microstates.

Q3). State the fundamental Postulate of Statistical mechanics.

Ans  $\Rightarrow$  the fundamental postulate of statistical mechanics states that, in an isolated and closed system, all the possible microstates are equally probable.

Q4) What is meant by occupation number, degenerate, non-degenerate energy levels.

Ans  $\Rightarrow$  the number of Particles in each energy levels ( $N_i$ ) is called occupation number.

If  $g_i = 1$ , the energy levels are called non-degenerate energy levels.

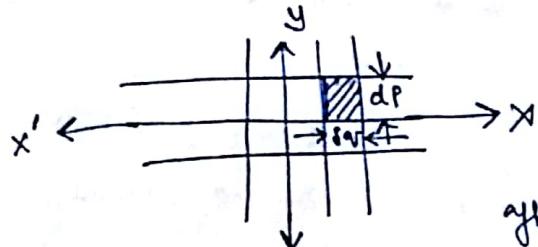
If  $g_i > 1$ , the energy levels are called degenerate energy levels.

Q5) What is phase Space?

Ans  $\Rightarrow$  Combination of Position and momentum space is called phase space.

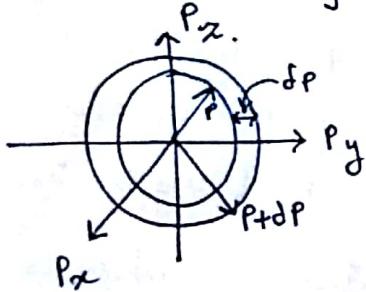
Q6) Define  $\mu$ -phase space and  $\Gamma$ -phase space. Show that the volume of  $\mu$ -phase corresponding to a single quantum state for such particles is  $\hbar^3/V$ , where  $\hbar$  is Planck's constant and  $V$  is the physical volume of the system.

Ans  $\Rightarrow$



The area of each cell/quantum states =  $dx \cdot dy$   
=  $\hbar$ .

For 6-D phase space,



The volume of each cell/quantum state  
=  $dx \cdot dy \cdot dz \cdot dp_x \cdot dp_y \cdot dp_z$   
=  $\hbar \cdot \hbar \cdot \hbar$   
=  $\hbar^3$ .

The total volume of the phase space, =  $\iiint dx \cdot dy \cdot dz \cdot dp_x \cdot dp_y \cdot dp_z$

The volume of momentum space, corresponding to momentum range  $p$  to  $p+dp$ . =  $\sqrt{4\pi p^2 dp}$

$$V = \frac{4}{3} \pi (p+dp)^3 - \frac{4}{3} \pi p^3$$

$$\cancel{\sqrt{4\pi}}$$

The number of energy states within the momentum range  $p$  to  $p+dp$   $\Rightarrow g(p) dp = \frac{V \times 4\pi p^2 dp}{h^3}$

$$\Rightarrow \boxed{h^3/V = \frac{4\pi p^2 dp}{g(p) dp}} \Rightarrow \text{Volume of momentum space corresponding to single quantum state.}$$

Q7). Derive the expression for density of energy state of a system b/w  $E$  to  $E + \delta E$ . (2)

Density of state is defined as the number of energy states per unit volume per unit energy within the energy interval  $E$  to  $E + \delta E$ .

$$G(E) \delta E = \frac{g(E) \delta E}{V \delta E}$$

We know,  $N(E) \delta E = \frac{g(E) \delta E}{e^{\alpha + \beta E}} \rightarrow [M-B \text{ Distribution}]$

$$\text{and, } g(E) \delta E = \frac{4\pi p^2 \delta p}{h^3}$$

$$\text{but, } p^2 = 2mE$$

$$\delta p \delta p = 2m \delta E.$$

$$p \delta p = m \delta E.$$

∴ the number of energy state within the energy interval  $E$  to  $E + \delta E$ ,

$$g(E) \delta E = \frac{4\pi V}{h^3} \sqrt{2mE} \cdot m \cdot \delta E.$$

$$\therefore g(E) \delta E = 2\pi V \sqrt{\left(\frac{2m}{h^2}\right)^{3/2} E^{1/2}} \delta E.$$

$$\therefore G(E) \delta E = \frac{2\pi V \cdot \left(\frac{2m}{h^2}\right)^{3/2} E^{1/2} \delta E}{\cancel{\delta E}}$$

$$G(E) \delta E = 2\pi \left(\frac{2m}{h^2}\right)^{3/2} E^{1/2} \delta E$$

(Hence proved)

Q8) Assuming the expression for the M-B count for isolated system of identical and distinguishable non-particles, derive the M-B distribution law of energy among molecules of an ideal gas.

First, we assume,  $N(E)dE = \frac{2\pi N}{(\pi kT)^{3/2}} e^{-E/kT} \cdot E^{1/2} dE$ .

Assuming M-B distribution,

for continuous case,

$$U = \int_0^{\infty} E N(E) dE.$$

$$U = \int_0^{\infty} \frac{2\pi N}{(\pi kT)^{3/2}} \cdot e^{-E/kT} \cdot E^{1/2} dE.$$

$$= \frac{2\pi N \cdot (kT)^{5/2}}{(\pi kT)^{3/2}} \int_0^{\infty} e^{-x} \cdot x^{3/2} dx.$$

$$U = \frac{2\pi N}{(\pi kT)^{3/2}} \times (kT)^{5/2} \times \frac{3}{2} \times \frac{\sqrt{\pi}}{2}.$$

$$U = \frac{3}{2} N k T$$

(Derived)

Q9) Derive equipartition law of energy, also derive it from M-B distribution law.

Q10) Derive Maxwell's law of distribution of molecular speeds.

We have,  $N(E)dE = \frac{2\pi N}{(\pi kT)^{3/2}} e^{-E/kT} \cdot E^{1/2} dE$ .

For speed distribution,

$$N(v) dv = \frac{2\pi N}{(\pi kT)^{3/2}} \left( \frac{1}{\sqrt{2}} \cdot \sqrt{m} \cdot m \right) e^{-\frac{mv^2}{2kT}} \cdot v^2 dv.$$

$$N(v) dv = \sqrt{2} \pi N \left( \frac{m}{kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} \cdot v^2 dv$$

(Derived)

Q11) Assuming Maxwell's law of distribution of molecular speeds, derive expressions for most probable, average and root mean square speeds.

$$N(E) dE = \left[ \frac{2\pi N}{(\pi kT)^{3/2}} \right] e^{-E/kT} \cdot E^{1/2} dE.$$

$$N(v) dv = \left[ \sqrt{2\pi N} \left( \frac{m}{\pi kT} \right)^{3/2} \right] e^{-\frac{mv^2}{2kT}} \cdot v^2 dv$$

# Most probable Speed.  $\rightarrow A$  (say).

$$N(v) = A \cdot e^{-\frac{mv^2}{2kT}} \cdot v^2.$$

taking  $(\ln)$  on both sides,

$$\ln [N(v)] = \ln A - \frac{mv^2}{2kT} + 2 \ln v.$$

$$\begin{aligned} d[\ln N(v)] &= -\frac{2mv}{2kT} + \frac{2}{v} \\ &= -\frac{mv}{kT} + \frac{2}{v}. \end{aligned}$$

At  $v=v_p$  (most probable speed),

$$\left. \frac{d \{ \ln N(v) \}}{dv} \right|_{v=v_p} = 0.$$

$$\therefore \frac{2}{v_p} = \frac{mv_p}{kT}$$

$$v_p = \sqrt{\frac{2kT}{m}} \rightarrow \text{most probable speed.}$$

Root Mean Square speed of gas molecules is not important for either Semester or 2nd internal as told by D.M SIR.

average speed of gas molecules

Q12) Define Fermi distribution function and Fermi level. Give a physical interpretation of Fermi level at  $T=0K$  and  $T>0K$  in metal.

Fermi distribution function,  $f(E)$  is defined as the number of particles per unit Energy within the energy range  $E$  to  $E+dE$  to the number of energy states per unit energy within the energy range  $E$  to  $E+dE$ .

$$f(E) = \frac{N(E)}{g(E)}$$

Fermi level is the ~~energy level~~ term used to describe the top of the collection of electron energy levels at absolute zero temperature.

Case 1:  $E < E_f$  and  $T=0K$ .

$$f(E) = \frac{1}{e^{\frac{E-E_f}{kT}} + 1} = \frac{1}{e^{-\frac{(E_f-E)}{kT}} + 1} = \frac{1}{e^{-\infty} + 1} \Rightarrow \frac{1}{0+1} = 1.$$

$$\therefore f(E) = 1.$$

So, at  $T=0K$ , all the energy states in an energy levels below fermi levels are fully occupied.

Case 2:

$E > E_f$  and  $T=0K$ .

$$f(E) = \frac{1}{e^{\frac{E-E_f}{kT}} + 1} = \frac{1}{\infty + 1} = 0 \quad \therefore N(E) = 0.$$

At  $T=0K$ ,  $E > E_f$ , all the energy levels above fermi levels are fully unoccupied.

Case 3:

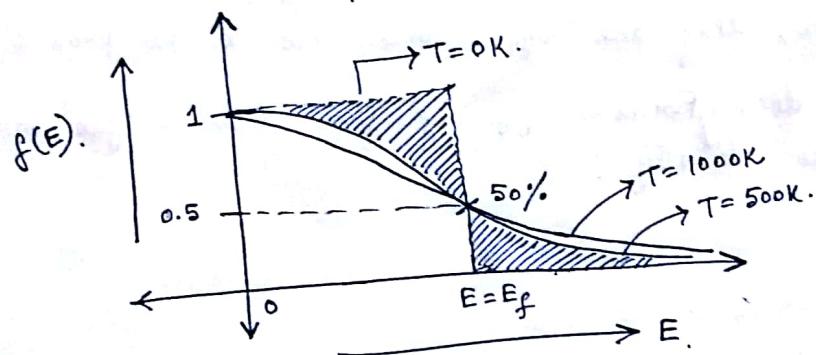
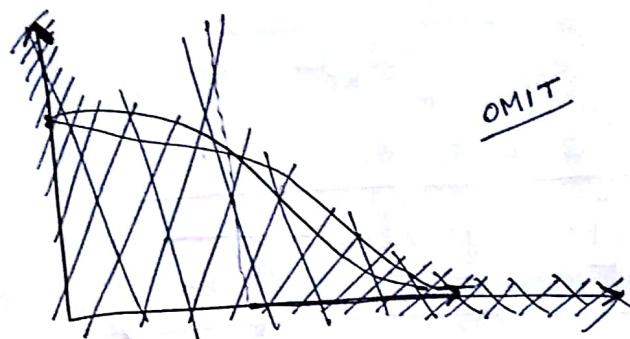
$E = E_f$  &  $T > 0K$ .

$$f(E) = \frac{1}{e^{\frac{(E-E_f)/kT}{kT} + 1}} = \frac{1}{e^0 + 1} = \frac{1}{2}.$$

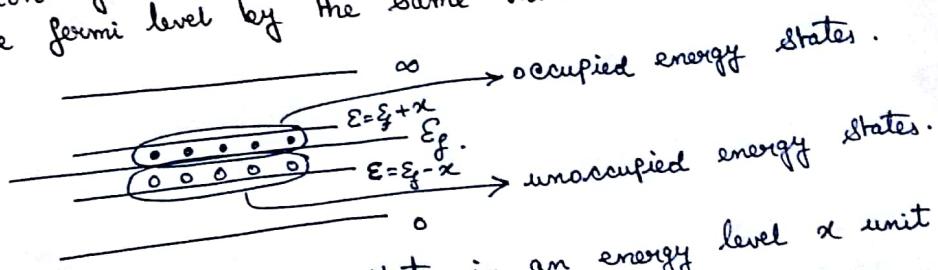
At  $T > 0K$ , fermi level is the energy level at which the probability of finding the particle is half.

and Fermi level. Give a sketch for  $T=0K$  and  $T>0K$  in metal.

- Q13) Sketch the Fermi-distribution function for  $T=0K$  and  $T>0K$  in metal.



- Q14) Verify that the fraction of unoccupied energy states in the energy level below the Fermi level is equal to the fraction of occupied energy states in the energy level above the Fermi level by the same value.



- Fraction of occupied state in an energy level  $x$  unit above Fermi level,

$$f(E) = \frac{1}{e^{(E-E_f)/kT} + 1}$$

$$f(E_f+x) = \frac{1}{e^{x/kT} + 1}$$

- Fraction of unoccupied state in an energy level,  $x$  unit below Fermi level,

$$E = E_f - x$$

$$(1 - f(E)) = 1 - \frac{1}{e^{\frac{(E-E_f)/kT}{}} + 1}$$
$$= \frac{1}{1 + e^{-\frac{(E-E_f)}{kT}}}$$

Now, at  $E = E_f - \alpha$ ,

$$\boxed{[1 - f(E_f - \alpha)] = \frac{1}{e^{\alpha/kT} + 1}}$$

Hence, they are equal which had to be proved.

Q15) How does Fermi energy of free electron gas varies with electron density?

Q. Can an electron have zero energy at 0K?

Ans  $\Rightarrow$  Avg. K.E of an electron at 0K is,

$\bar{E} = \frac{3}{5} E_f$  ; a non-zero quantity.

As, Value

of  $E_f$  in general is about 5 eV.

$$\text{Thus, } \bar{E} = \frac{3}{5} \times 5 \text{ eV} = 3 \text{ eV.}$$

So, an electron can never have zero energy at 0K.

Q17). What do you mean by fermi temperature and fermi velocity of an electron in fermi level?

Ans  $\Rightarrow$  the temperature of an electron corresponding to the Energy value  $E_f$  is called fermi-temperature. ( $T_f$ )

$$k T_f = E_f.$$

but,  $T_f = \frac{E_f}{k}$   $\Rightarrow$  Valid for only for classical particle.

~~T<sub>f</sub>~~

the velocity of an electron corresponding to the Energy value  $E_f$  is called fermi velocity. ( $v_f$ ).

$$\frac{1}{2} m v_f^2 = E_f.$$

$$v_f^2 = \frac{2 E_f}{m} = \frac{2}{m} \times \frac{h^2}{2m} \times \left( \frac{3N}{8\pi V} \right)^{2/3}.$$

$$v_f = \frac{h}{m} \left( \frac{3N}{8\pi V} \right)^{1/3}.$$

Q18) Derive Bose-Einstein Energy distribution law and hence obtain Planck's formula for black body radiation. B)

We know,

$$\begin{aligned} W &= \pi_i g_i + N_i - 1 \\ &\quad \times \frac{c}{N_i} \\ &= \pi_i \frac{(g_i + N_i - 1)!}{N_i! (g_i - 1)!} \end{aligned} \quad \text{--- (1)}$$

taking  $\ln$  on both sides,

$$\ln W = \sum_i \left[ \ln(g_i + N_i - 1)! - \ln N_i! - \ln(g_i - 1)! \right]$$

$g_i + N_i$  is much greater so  $(-1)$  is omitted

using Stirling's approximation,

$$\ln W = \sum_i (g_i + N_i) \ln(g_i + N_i) - (g_i + N_i) - N_i \ln N_i + N_i - (g_i - 1) \ln(g_i - 1) + (g_i - 1).$$

$$d[\ln W] = \sum_i [dN_i \ln(g_i + N_i) + dN_i - dN_i - dN_i \ln N_i - dN_i + dN_i]$$

$$d[\ln W] = \sum_i \ln \left( \frac{g_i + N_i}{N_i} \right) dN_i. \quad \text{--- (3)}$$

For most improbable state,

$W$  is maximum.

$\ln W$  is also maximum.

using Lagrange multiplier method,

$$d[\ln W_{\max}] - \alpha \sum_i N_i - \beta \sum_i E_i N_i = 0.$$

where  $\alpha$  and  $\beta$  are Lagrange multipliers.

$$\therefore d[\ln W_{\max}] - \alpha \sum_i dN_i - \beta \sum_i E_i dN_i = 0.$$

From (3) R.H.S

$$\sum_i \ln \left( \frac{g_i + N_i}{N_i} \right) dN_i - \alpha \sum_i dN_i - \beta \sum_i E_i dN_i = 0.$$

$$\Rightarrow \sum_i \left[ \ln \left( \frac{g_i + N_i}{N_i} \right) - \alpha - \beta E_i \right] dN_i = 0.$$

but,  $dN_i \neq 0$  as No. of Particles  $\neq 0$ .

$$\therefore \ln \left( \frac{g_i + N_i}{N_i} \right) - \alpha - \beta E_i = 0.$$

$$\frac{N_i}{g_i} = \frac{1}{e^{\alpha + \beta E_i - 1}}$$

## Derivation of Planck's law from B-E Statistics

the number of photons within the frequency interval  $\nu$  to  $\nu + d\nu$  is given by,

$$N(\nu) d\nu = \frac{g(\nu) d\nu}{e^{\alpha + \beta E} - 1}$$

So, for instance two photons can be emitted simultaneously with absorption of a single photon of energy  $2h\nu$ .

$$\therefore \sum N_i \neq \text{constant.}$$

So,  $\alpha = 0$ . [for a particular black body].

$$\text{Now, } N(\nu) d\nu = \frac{g(\nu) d\nu}{e^{h\nu/kt} - 1}$$

$$\text{we know, } g(p) dp = \frac{V \times 4\pi p^2 dp}{h^3}$$

Now, photons have two states of polarization i.e left circular & Right circular Polarised).

making state of Polarisation of photons into consideration,

$$g(p) dp = \frac{2 \times 4\pi p^2 dp \times V}{h^3}$$

$$g(\nu) d\nu = \frac{8\pi p^2 dp \times V}{h^3}$$

$$p = \frac{h\nu}{c}$$

$$dp = \frac{h}{c} d\nu$$

$$g(\nu) d\nu = \frac{8\pi \frac{h^3 \nu^2}{c^3} \times d\nu \times V}{h^3}$$

$$g(\nu) d\nu = \frac{8\pi V \nu^2 d\nu}{c^3}$$

the energy density of black body radiation within the frequency interval  $\nu$  to  $\nu + d\nu$ ,

$$\begin{aligned}
 U_\gamma d\gamma &= \frac{h\gamma \times N(\gamma) d\gamma}{\nu} \\
 &= \frac{h\gamma}{\nu} \times \frac{g(\gamma) d\gamma}{e^{h\gamma/kT} - 1} \\
 &= \frac{h\gamma}{\nu} \times \frac{8\pi\nu \gamma^2}{c^3} d\gamma \cdot \frac{1}{e^{h\gamma/kT} - 1} \\
 &= \frac{8\pi h \gamma^3}{c^3 [e^{h\gamma/kT} - 1]} d\gamma
 \end{aligned}$$

$$\boxed{\therefore U_\gamma d\gamma = \frac{8\pi h \gamma^3}{c^3 [e^{h\gamma/kT} - 1]} d\gamma}$$

Q19) Show that Wien's formula and Rayleigh Jeans formula can be deduced as a particular case of Planck's law.

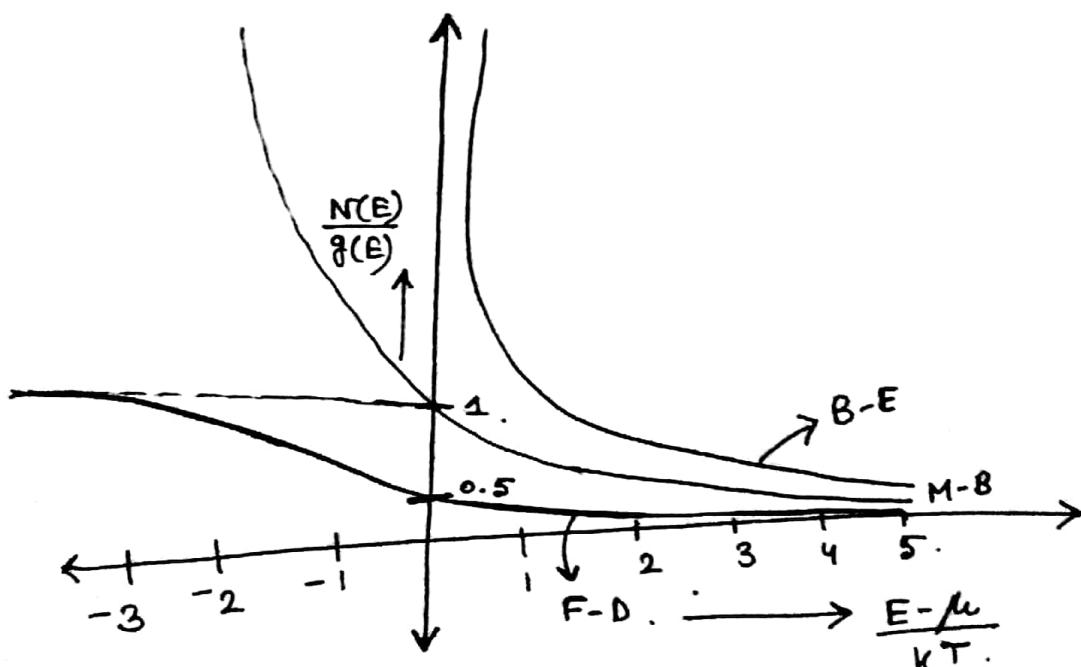
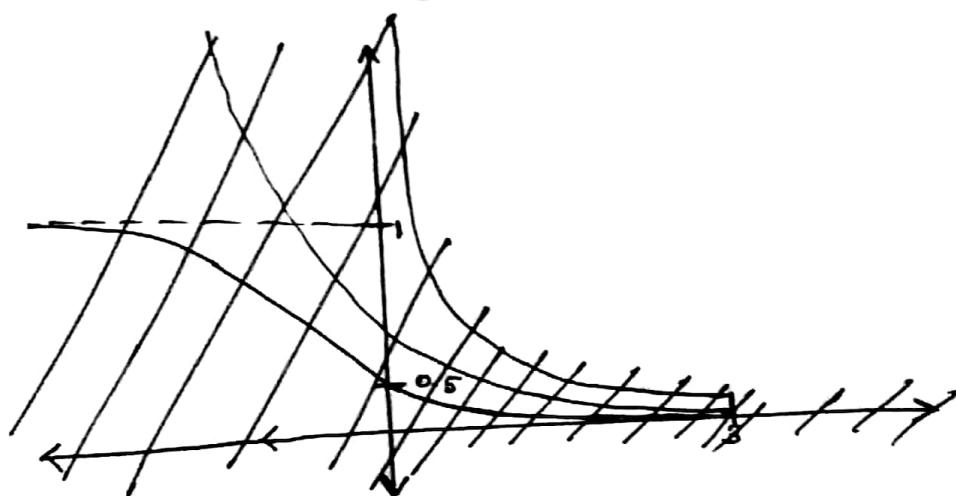
Ans  $\Rightarrow$

Q20). Under what conditions do B-E and F-D statistics yield classical statistics, show it graphically.

$$M-B = \frac{N(E)}{g(E)} = \frac{1}{e^{\alpha + \beta E}} = \frac{1}{e^{E-\mu/kT}}$$

$$F-D = \frac{N(E)}{g(E)} = \frac{1}{e^{E-\mu/kT} + 1}. \quad \mu \rightarrow \text{chemical potential.}$$

$$B-E = \frac{N(E)}{g(E)} = \frac{1}{e^{E-\mu/kT} - 1}.$$



At low occupation index [ $N(E) \ll g(E)$ ].

and, high energy value, both F-D & B-E statistics approach to M-B statistics.

Using F-D distribution function, derive the concentration conc. of holes in valence band in intrinsic semi-conductors.

(Ans) a) Electron conc. in C.B

$$n_e = \frac{1}{V} \int_{E_c}^{+\infty} N(E) dE = \frac{1}{V} \int_{E_c}^{\infty} f(E) g(E) dE.$$

in metal,  $f(E) = \frac{1}{e^{\frac{E-E_f}{kT}} + 1}$ ,  $E > E_f$ .

$$g(p) dp = \frac{2\pi v \times 4\pi p^2 dp}{h^3} \quad (\text{for metal}).$$

In metal,  $E = E_k = \frac{p^2}{2m}$ .

$$g(E) dE = 4\pi v \left(\frac{2m}{h^2}\right)^{3/2} E^{1/2} dE.$$

In conduction band, electron energy,

$$E - E_c = E_k = \frac{p^2}{2m_e^*}$$

$$g(E) dE = 4\pi v \left(\frac{2m_e^*}{h^2}\right)^{3/2} (E - E_c)^{1/2} dE.$$

$m_e^*$  → effective mass of electron.

$$dE = d(E - E_c) = dE \cdot [as E_c is constant].$$

$$n_e = \frac{1}{V} \int_{E_c}^{\infty} e^{-(E-E_f)/kT} \cdot 4\pi v \left(\frac{2m_e^*}{h^2}\right)^{3/2} (E - E_c)^{1/2} dE.$$

$$= 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} \int_{E_c}^{+\infty} e^{-(E-E_f)/kT} \cdot (E - E_c)^{1/2} dE.$$

$$\Rightarrow 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} e^{(E_f - E_c)/kT} \int_{E_c}^{\infty} e^{-\frac{(E-E_c)}{kT}} \cdot (E - E_c)^{1/2} dE.$$

let  $\frac{E - E_c}{kT} = x$ .  $dE = kT dx$  as  $E_c$  is constant.

$$n_e = 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} e^{-(E_c - E_f)/kT} \int_{-\infty}^{\infty} e^{-x} \cdot x^{1/2} \cdot (kT)^{3/2} dx.$$

$$= 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} e^{-(E_c - E_f)/kT} \int_{-\infty}^{\infty} e^{-x} \cdot x^{1/2} (kT)^{3/2} dx$$

$$\begin{aligned}
 n_h &= 4\pi \left( \frac{2m_e^* kT}{\hbar^2} \right)^{3/2} e^{-(E_c - E_f)/kT} \cdot \frac{1}{2} \sqrt{\pi} \\
 n_e &= 2 \left( \frac{2\pi m_e^* kT}{\hbar^2} \right)^{3/2} e^{-(E_c - E_f)/kT} \cdot \frac{1}{2} \sqrt{\pi}
 \end{aligned}$$

b) Holes conc. in Valence band.

$$\begin{aligned}
 n_h &= \frac{1}{V} \int_{-\infty}^{E_V} N(E) dE \\
 &= \frac{1}{V} \int_{-\infty}^{E_V} [1 - f(E)] g(E) dE. \\
 [1 - f(E)] &= \frac{e^{\frac{E-E_f}{kT}}}{e^{\frac{E-E_f}{kT}} + 1} \rightarrow \text{for metal.}
 \end{aligned}$$

here,  $E < E_f \rightarrow V.B.$

$$E - E_f \leq -3kT.$$

Now,  $[1 - f(E)] \text{ in valence band} = e^{\frac{E-E_f}{kT}}$

in metal,  $E = E_K = \frac{P^2}{2m} \rightarrow \text{for metal.}$

in C.B.,  $E - E_C = E_K = \frac{P^2}{2m_e^*}$

in V.B.,  $E_V - E = E_K = \frac{P^2}{2m_h^*}$

for V.B. density of state,

$$\begin{aligned}
 g(E) dE &= 4\pi \sqrt{\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2} (E_K - E)^2} dE. \\
 n_h &= \frac{1}{V} \times 4\pi \sqrt{\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2}} \int_{-\infty}^{E_V} \frac{E_V - E}{\hbar^2} (E_K - E)^{1/2} dE. \\
 n_h &= 4\pi \left(\frac{2m_h^*}{\hbar^2}\right)^{3/2} \int_{-\infty}^{E_V - E} \frac{E_V - E}{\hbar^2} (E_V - E)^{1/2} dE.
 \end{aligned}$$

$$\text{Let } \frac{E_V - E}{kT} = x.$$

$$-dE = (kT) dx.$$

$$n_h = 4\pi \left(\frac{2m_h^*}{h^2}\right)^{3/2} e^{-\frac{(E_f - E_V)}{kT}} (kT)^{3/2} \int_0^\infty e^{-x} \cdot x^{1/2} dx.$$

$$n_h = 4\pi \left(\frac{2m_h^*}{h^2}\right)^{3/2} e^{-\frac{(E_f - E_V)}{kT}} (kT)^{3/2} \times \frac{\sqrt{\pi}}{2}.$$

$$\therefore n_h = 2 \left( \frac{2\pi m_h^* kT}{h^2} \right)^{3/2} e^{-\frac{(E_f + E_V)}{kT}}$$

Q22) Derive the expression of Fermi-level in intrinsic semi-conductors and also show that at  $T=0K$ , Fermi level is exactly midway b/w C.B and V.B.

For intrinsic semi-conductors,

$$n_e = n_h$$

$$(m_e^*)^{3/2} e^{-\frac{(E_c - E_f)}{kT}} = (m_h^*)^{3/2} e^{-\frac{(E_f - E_V)}{kT}}$$

$$\Rightarrow e^{-\frac{-E_c + E_f + E_f - E_V}{kT}} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

$$\Rightarrow 2E_f - (E_c + E_V) = kT \times \frac{3}{2} \ln \left(\frac{m_h^*}{m_e^*}\right)$$

$$E_f = \frac{E_c + E_V}{2} + \frac{3}{4} kT \ln \left(\frac{m_h^*}{m_e^*}\right)$$

At  $T=0K$ ,

$$E_f = \frac{E_c + E_V}{2}$$

$\rightarrow$  (midway b/w C.B and V.B)

Q23) Derive the expression of carrier concentration in terms of temperature of intrinsic semi-conductor.

$$n_i^2 = n_e \times n_h$$

$$n_i^2 = 4 \left( \frac{2\pi kT}{h^2} \right)^3 (m_e m_h)^{3/2} e^{-\frac{(E_c - E_v)}{kT}}$$

$$n_i = 2 \left( \frac{2\pi kT}{h^2} \right)^{3/2} (m_e m_h)^{3/4} e^{-E_g/2kT}$$

If  $T \downarrow$ ,  $e^{-E_g/2kT} \uparrow$ , and  $n_i \downarrow$

Q24). State and establish the law of mass action in semi-conductors  
What is its significance.

25). What are P-type and N-type Semi-conductors?

Ans  $\Rightarrow$  P-type

After addition of trivalent impurities such as Boron, aluminium or gallium to an intrinsic semi-conductor creates deficiencies of valence electrons, called "holes".  
~~It is opposite to  $B_3$~~

N-type

After addition of pentavalent impurities such as antimony, arsenic contributes free electrons greatly increasing the conductivity of the intrinsic semi-conductor.

Q26) Discuss the variation of fermi level in n-type and p-type semiconductors with temperature.