- Physics-II, PHYS-2001.

Gr. B → Classical mechanics
→ Quantum mechanics.

Gr. C → Statistical mechanics

Gr. D → Dielectric prop. of matter
→ Magnetic prop of matter

Gr E → Band Theory of solids
→ Superconductivity

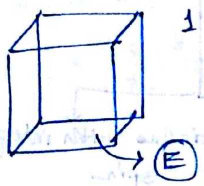
de Broglie's divides the classical & Quantum

- $\lambda < d$ (macroscopic particle)

  $\lambda > d$ (quantum particle

- Gas molec. the mean free path is greater than wavelength associated with gas molec
  → obeys classical stat mechanics.

- Statistically → How particles are distributed over
  system → consists of large no. of particles.
  Large no of gas molecules



at NTP

1 cm^3 of gas contains 2.7×10^{19} no of molecules

how . E is distributed about

Fermi level how e-'s are distributed?

Both classical & quantum particles
(gas molec) (e^-) & photon

how these distribution occurs is stat mech

- As large no of particles ⇒ Energy distribution/ $= \frac{1}{2}kT$.
  ∴ Degrees of freedom

  most prob $= \sqrt{\frac{2kT}{m}}$

  $RMS = \sqrt{\frac{3kT}{m}}$

- Gas molec's are classical particle
  individual particle eq can be solved
  we need to solve $3\times2.7\times10^{19}$ no of Eq
  as each particle has 3 degree of freedom

Gap betwn Mechanical & Thermodynamic prop. of a system
Link provided by Stat Mech

Introduction :-

Statistical Mechanics deals with the mechanical system that are in Thermal Equilibrium. It is statistical because it makes a statistical prediction of a large number of particles (Classical & Quantum), how the particles are distributed among the various energy state or quantum state in the energy level of the system ... statistical mechanics ... formulated as

The main object of statistical mechanics is to establish the laws governing the behavior of macroscopic quantity of a system (pressure (P), V, Energy (E), Entropy (H)) by using the laws governing the behavior of the molecules of that system.

[fermions obey

Statistical mechanics is that branch of physics which studies the macroscopic system from the microscopic or molecular point of view and provides the link between classical mechanics and thermodynamics

Statistical Mechanics

- Classical statistical Mechanics (deals with classical particles)
  - (Obeys Maxwell-Boltzmann stat.) (MB statistics)
  - Ex. gas molecules
  - CE, Equipartition, rms, Avg speed — for derivations (!!)
- Quantum statistical Mechanics (deals with Quantum particle)
  - Particles with $\frac{1}{2}$ (half) integral spin
    - Angular momentum $= \frac{1}{2}\hbar$
    - (obey Fermi-Dirac statistics) — FD stat
    - Eg ~~...~~ Electron, proton, neutron, e, p, n^3
    - ^{3}He atom (antisymmetric wave func)
    - μ-mesons
    - [particles are called Fermions]
  - Particles with integral spin
    - (obey Bose-Einstein stat) (BE stat)
    - Eg photon, Phonon (lattice energy radiated conductivity decreases with ↑ temp)
    - ^{4}He atom,
    - π meson
    - [These are called BOSONS]

Probability :-

$$P(\text{Probability}) = \frac{\text{the no of ways which are favourable to the event}}{\text{total no. of equally likely possible ways}}$$

$P = P_1 \times P_2$ (2 events occur simultaneously) (multiplication)

$P = P_1 + P_2$ (Either or condition) (Addition)

① A bag contains 5 red ball & 3 white ball & all balls identical what is the probability that on 2 independent drawing one ball is red & one is white.

~~$\frac{5}{8} \times \frac{3}{7} \times \frac{15}{56}$~~ $\quad \frac{5}{8} + \frac{3}{7}$

independent drawing.

$$\frac{5}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{5}{8} = \frac{2 \times 15}{64} = \frac{15}{32}$$

$$= \frac{15}{32}$$

② A box is divided into 9 equal size compartment

ⓐ If 9 balls are thrown calculate the probability for a particular compartment to contain a particular ball only $\longrightarrow \frac{8^8}{9^9}$

ⓑ All the nine balls $\rightarrow (\frac{1}{9})^9$

ⓒ None of these balls. $\rightarrow (\frac{8}{9})^9$

~~O~~ O O O O . .

O O O O

$\frac{1}{9} \rightarrow$ first

$P_1 = \frac{1}{9}$

next ball. $P_2 = 8/9$

Third $P_3 = 8/9$

$$\therefore P_a = P_1 \times P_2 \times P_3 \times \cdots$$

$$= \frac{1}{9} \times \left(\frac{8}{9}\right)^8 = \frac{8^8}{9^9}$$

◦ Concept of energy levels, Energy states/quantum states, degeneracy :-

Statistical mechanics applicable for free system & random not bound sys!

ⓐ For a bound electron:

$$E_n = -\frac{13.6 Z^2}{n^2}.eV$$

[specified by n, l, m_l, m_s)

$n =$ Principle

$l =$ Azimuthal Ang mom — $-l$ to l ...

$m_l =$ direction of ...

$m_s =$ spin quantum.

∴ Energy level	n	l	m_l	m_s	Energy State
2	2	0	0	$-\frac{1}{2}, +\frac{1}{2}$	$(2,0,0,+\frac{1}{2}), (2,0,0,-\frac{1}{2})$
		1	−1	$+\frac{1}{2}, -\frac{1}{2}$	$(2,1,-1,+\frac{1}{2}), (2,1,-1,-\frac{1}{2})$
			0	$+\frac{1}{2}, -\frac{1}{2}$	$(2,1,0,+\frac{1}{2}), (2,1,0,-\frac{1}{2})$
			+1	$+\frac{1}{2}, -\frac{1}{2}$	$(2,1,+1,+\frac{1}{2}), (2,1,1,-\frac{1}{2})$

So 8 different energy states in 2nd energy level

For $n=3$, 18 different energy states

Irrespective of energy, all in 2nd energy level → so fixed

• Many or different energy state may correspond to a particular energy level and the no. of energy states to a particular energy level is known as Degeneracy no (g_i) in i^{th} energy level.

And the energy levels are called Degenerate Energy Level

For degenerate energy level $\boxed{g_i > 1}$

For non-degenerate energy level $\boxed{g_i = 1}$ (particles can move from one energy level to other level but not in one to energy state as no energy state)

[All energy levels are discrete

each energy level contains more than one energy state

if $g_i > 1$ more than one energy state particles can move from one energy state to other.

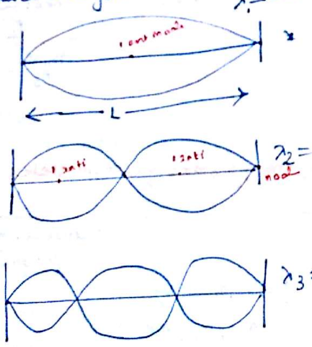
$\boxed{27.1.17}$

b) for a free particle in a cubical box

The propagation of a transverse wave in a stretched string

wave is propagated reflected & re-reflected in the string.

This standing waves are formed.



$\lambda = 2L$

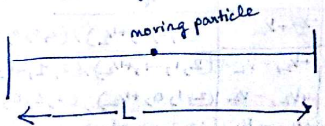
$\lambda_2 = \frac{1}{2} \times 2L$

$\lambda_3 = \frac{1}{3} \times 2L$

$\therefore \boxed{\lambda_j = \frac{1}{n_j} \cdot 2L}$

This is analogous to motion of a particle moving back and forth from 0 to L ... L to 0

KE const.



This is analogous to the motion of a particle moving freely back and forth along a straight line separated by a distance L.

According to de Broglie, particle wave.

$$\lambda = \frac{h}{p}$$

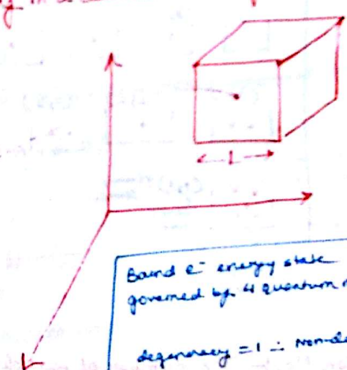
$$p = \frac{h}{\lambda}$$

$$\boxed{p_j = \frac{h}{\lambda_j} = n_j \frac{h}{2L}} \rightarrow \text{momentum is discrete}$$

• The momentum of a particle is permitted to have any of these values $j = 1, 2, \ldots$

• The condition is particle moves from 0 to L as boundary condition bet^n 2 node nodes.

• If a particle is freely moving in a cubical box of length L and size



p_x = momentum along x axis

n_x = integer along x axis

Now $p_x = n_x \frac{h}{2L}$

$p_y = n_y \frac{h}{2L}$

$p_z = n_z \cdot \frac{h}{2L}$

Bound e^- energy state governed by 4 quantum no.

degeneracy = 1 ∴ non-degenerate

momentum is vector

$$\therefore p^2 = p_x^2 + p_y^2 + p_z^2$$

$$p^2 = (n_x^2 + n_y^2 + n_z^2) \frac{h^2}{4L^2}$$

our aim is Energy calculation

Now energy of the particle in cubical box $= \boxed{E = \frac{p^2}{2m} = (n_x^2 + n_y^2 + n_z^2) \frac{h^2}{8mL^2}}$

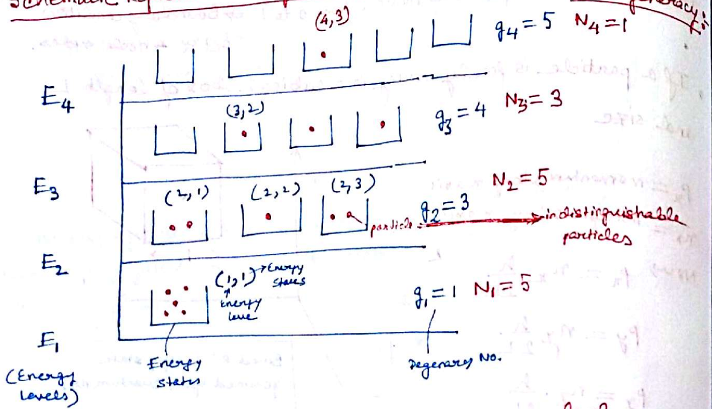
Energy is not continuous but all levels are discrete

Depending on n_x, n_y, n_z E value changes. n_x, n_y, n_z min value is 1.

Energy level	Energy state (n_x, n_y, n_z)	Energy value	degeneracy
1. Lowest or ground states ($n_x = n_y = n_z = 1$)	(1, 1, 1)	$\frac{3h^2}{8mL^2}$	$g_1 = 1$ (No of energy states in this level) (No degeneracy)
2. 1st excited state (2nd energy level)	(2, 1, 1), (1, 2, 1), (1, 1, 2).	$\frac{6h^2}{8mL^2}$	$g_2 = 3$ (degeneracy)
3. 2nd excited ~~level~~ state (3rd energy level)	(2, 2, 1), (2, 1, 2), (1, 2, 2)	$\frac{9h^2}{8mL^2}$	$g_3 = 3$ (degenerate Energy level)
4. 3rd excited state	(1, 1, 3), (1, 3, 1), (3, 1, 1)	$\frac{11h^2}{8mL^2}$	$g_4 = 3$

5. 4^{th} Energy State	$(2,2,2)$	$\frac{12h^2}{8mL^2}$	$g_5 = 1$ (Non degenerate)
6. 5^{th} Energy state	$(1,2,3), (2,3,1), (3,1,2), \ldots$ 6	$\frac{14h^2}{8mL^2}$	$g_6 = 6$ (degenerate)

- **Schematic Representation of Energy levels, Energy states / quantum states & degeneracy:**



- **Occupation No.:–** The number of particles in each energy level is called Occupation No. (N_i) ↳ i^{th} no. of energy level

Eg $\therefore N_1 = 5$

- For a closed system → total no. of particles of the system is constant

ie $N = N_1 + N_2 + N_3 \cdots + N_i = \text{const}$

$$\Rightarrow N = \sum_{i=1} N_i = \text{const.}$$

- For an isolated system; total energy of the system is constant

Each particle in i^{th} energy level has the energy E_i irrespective of the energy state it occupies at that level

$$U = E_1 N_1 + E_2 N_2 + \cdots + E_i N_i = \text{const}$$

$$= \sum_i E_i N_i = \text{const} \text{ (For Isolated System)}$$

~~4~~ 9
= ~~4~~5

- Large no of photons, electrons all particles moves from energy state to other or one energy ~~stat~~ level to other

if one particles move from one level to other → weight

- **Macro State & Micro State:– (Important !!)**

classical particles → distinguishable quantum partize → indistinguishable

- **macrostate:–**

whether the particles are distinguishable or not, a specification of the no. of particles in each energy levels of the system is said to define macrostate of the system.

A macrostate is defined by M

$$M = (N_1, N_2, N_3, \ldots, N_i)$$ ie a set of occupation no.'s

Eg. $M = (5, 5, 3, 1)$.

If dynamic system, macrostate changes as particles move betn energy levels

- **Micro state:–** depends on nature of particle

① If the particles are indistinguishable (Quantum particle) a specification of the no. of particles in each energy state of the system is said to define Microstate of the system.

5 particles in (1,1) energy state at an instant of time.

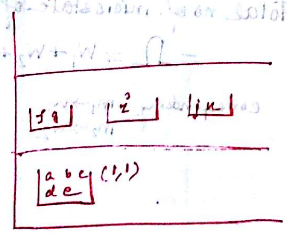
2 . - - - (2,1) - - - - - -

1 . - - . (2,2) - . - -

2 . - - - . (2,3) - - . - -

if one particle moves from (2,1) to (2,3) the ~~micro~~ microstate changes but macrostate is fixed.

② If the particles are distinguishable, a specification of the energy state of each particle of the system is said to define microstate of the system:



microstate is defined as

a,b,c,d,e particles in (1,1) Energy State.

fg . - - - . (2,1) —

i - - - - . (2,2) —

jk - - - - (2,3)

if indistinguishable particle interchange of states in same energy level the microstate remains same

if distinguishable particles interchange states in same energy level the microstate changes

- Each particle can only be in one quantum state at a time. The system taken as a whole can only be in one ~~quantum~~ micro state at a time.
- Changing the quantum state of any one particle of a system will change the microstate of the system as a whole.
- There are actually a large number of ways in which these transfers can be made without changing the ~~microstate~~ macrostate of the system.

Definition :-

- The no. of microstates (no of ways to change) to a particular macrostate is known as Thermodynamic Probability of a given macrostate (W).

10. 2. 17.

Say, In a closed system large no of Macrostates are possible.

Ex. $N = 14$, $U = 28\,eV$.

$E_1 = 1eV$ $\quad E_2 = 2eV$ $\quad E_3 = 3eV$ $\quad E_4 = 4eV$

Calculate no of macrostates

$5\times1 + 5\times2 + 3\times3 + 1\times4$
$= 5 + 10 + 9 + 4 = 28$

$M_1 = (5, 5, 3, 1)$ → Possible macrostate

↳ W_1 no of microstates

$M_2 = (8, 1, 2, 3) \rightarrow W_2$

For a closed isolated system there may be large no of macrostates and under it many microstates

- Total no of microstate of the system

$$= \Omega = W_1 + W_2 + \dots + W_K$$

corresponding $M_1 \rightarrow W_1$, $M_2 \rightarrow W_2$

$$\therefore \boxed{\Omega = \sum W_K}$$

↳ Thermodynamic Probability of ~~Total~~ given system

Fundamental Postulate of Quantum Mechanics :-

- For a closed and isolated system all possible microstates are equally probable.
- The probability of a given microstate is $\boxed{P = \frac{1}{\Omega}}$ → total no of microstate of system
- The probability of a given macrostate → $P_{M_K} = \frac{W_K}{\Omega}$ (W_K → that given no of microstate; Ω → total no of microstates)

- **Most Probable State**: The macrostate which has maximum number of microstate of the system.
(Particle moves from lower to higher probable state)

non-degenerate
(2,1) $g_2 = 1$ — 2 rooms without bench
(1,1) $g_1 = 1$ — 2 compartments

Q.1) 4 particles are distributed over 2 non-degenerate energy level, find the no. of macrostate and microstate of the system. if the particles are — (i) distinguishable (ii) indistinguishable.

Distinguishable (a, b, c, d)

macrostates (N_1, N_2)	Energy states (1,1)	Energy states (2,1)	microstates
$M_1 = (4,0)$	abcd	0	$W_1 = 1$
	they in diff energy level		only one way as 4 to 0
$M_2 = (3,1)$	abc	d	$W_2 = 4$
	abd	c	
	acd	b	
	bcd	a	
$M_3 = (2,2)$	ab	cd	$W_3 = 6$
	~~ac~~	~~abd~~	
	ac	bd	
	bc	ad	
	bd	ae	
	cd	ab	
$M_4 = (1,3)$			$W_4 = 4$
$M_5 = (0,4)$			$W_5 = 1$
			+ 16 microstate

Total macrostate = 5

∴ Each

(2,2) → most probable

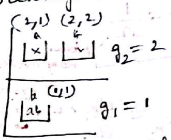
Indistinguishable (·,·,·,·)

Macrostate	Energy state	Microstate
$M_1 = (4,0)$		$W_1 = 1$
$M_2 = (3,1)$		$W_2 = 1$
$M_3 = (2,2)$		$W_3 = 1$
$M_4 = (1,3)$		$W_4 = 1$
$M_5 = (0,4)$		$W_5 = 1$
		5

Scanned by CamScanner

2) 2 particles are distributed over 2 energy level with degeneracy 1 and 2 respectively, find microstate & macrostate of a system

if ① distinguishable ② Non-distinguishable



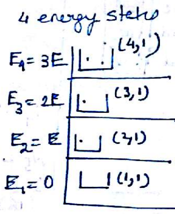
	distinguishable (ab)	microstate		indistinguishable (·, ·)	mi
					$W_1 = 1$
$m_1 = (2,0)$	ab	$W_1 = 1$	$m_1 = (2,0)$	:	$W_2 = 2$
$m_2 = (1,1)$	b x / a ; a x / b ; b / x a ; a x / b	$W_2 = 4$	$M_2 = (1,1)$	· x / x ; x · / x	$W_3 = 3$
$m_3 = (0,2)$	ab x ; x ab ; a b / x ; b a / x	$W_3 = 4$	$M_3 = (0,2)$	· · / x	6
		9 ways			

③ 3 particles each of which can be in one of 0, E, 2E, 3E Energy state here the total energy 6E, find the microstate and macrostate of the system.

& N = 3 , U = 6E

0, E, 2E
4 energy states there
Each energy state values are different, they belong to different energy level

4 energy states



$m_1 = (0,1,1,1)$ ✓
$m_2 = (1,0,0,2)$
$m_3 = (0,0,3,0)$

∴ macrostate formula not there as macrostate

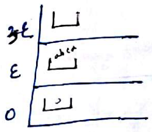
Distinguishable · micro	Indistinguishable
$m_1 = (0,1,1,1) \rightarrow W_1 = 6$	$m_1 = (0,1,1,1) \rightarrow W_1 = 1$
$m_2 = (1,0,0,2) \rightarrow W_2 = 3$	$m_2 = (1,0,0,2) \rightarrow W_2 = 1$
$m_3 = (0,0,3,0) \rightarrow W_3 = 1$	$m_3 = (0,0,3,0) \rightarrow W_3 = 1$
10	3

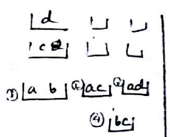
If particles
Here 3 fermions are mere as → non distinguishable
no 2 electrons can't have same energy level.
∴ $W_1 = 1$ 1 macrostate possible

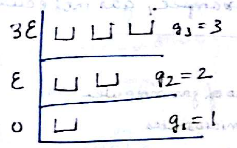
1.2.17

distinguishable
· 4 particles are distributed into 3 energy levels of energies 0, E, 3E so that the total energy 4E, Find macrostate & microstate of the system

∴ 3 energy levels are non-degenerate
degeneracy of the energy level 1, 2, 3 resp say degenerate







N = 4 . U = 4E .

macrostates	microstates
~~$m_1 = (0,2,1)$~~	
~~$m_2 = (0,$~~	
$m_2 = (0,4,0)$	$\rightarrow W_2 = 1$
$m_1 = (2,1,1)$	$\rightarrow W_1 = 12$

macrostate	microstate
$m_1 = (2,1,1) =$	large no. $W_1 =$
$m_2 = (0,4,0)$	$W_2 = 16$

Maxwell Boltzmann Statistics :-

① The particles are ~~iden~~ considered to be ~~an~~ identical and distinguishable

② The total no of particles of the system is const.

②. $N = N_1 + N_2 + \dots + N_i = const$ (closed system)

$$N = \sum N_i = const$$

③ $U = E_1 N_1 + E_2 N_2 + \dots + E_i N_i = const$

$$U = \sum E_i N_i = const.$$

④ (Classical) particles do not obey Pauli's Exclusion Principal

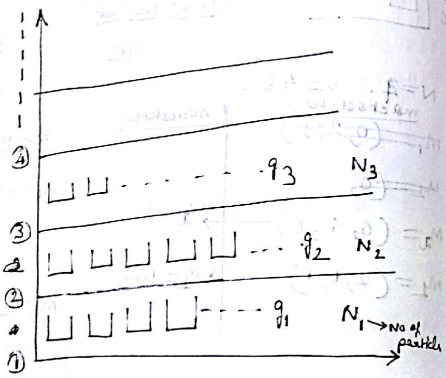
Each energy state may contain any no. of particles

⑤ Particles do not obey ~~Paulis~~ Heisenberg Uncertainty Principal

⑥. Example, gas molecules

N no of gas molecules

All microstates are equally probable to go in each energy level



∴ collect N_1 particles from N particles

N_2 ------ $(N - N_1)$ ---

The arrange

No we will calculate possible microstates under the macrostates

$M(N_1, N_2, \dots, N_i)$

① The no. of ways to collect N_1 particles from N particles in first energy level. ${}^{N}C_{N_1}$ ways

Similarly.

N_2 particles from $(N - N_1)$ particles in 2nd Energy level

${}^{N-N_1}C_{N_2}$ and so on.....

All collections occur simultaneously.

∴ Total no of ways to collect N_1, N_2 upto N_i particles from 1st, 2nd upto i^{th} energy level

Total Probability $= P_1 \times P_2 \times P_3 \times \dots$

$$= {}^{N}C_{N_1} + {}^{N-N_1}C_{N_2} \times \dots$$

$$= \frac{N!}{\cancel{(N-N_1)!}\, N_1!} \times \frac{\cancel{(N-N_1)!}}{(N-N_1-N_2)!\, N_2!} \times \dots$$

$$= \frac{N!}{N_1!\, N_2! \dots N_i!}$$

[Last term den $\frac{1}{N-(N_1+N_2+\dots+N_i)!}$ $= \frac{1}{(N-N)!} = \frac{1}{0!}$

No of energy states g_1

∴ The no of ways to arrange N_1 particles in g_1 energy states in 1st energy level (do not obey pauli's)

∴ ~~I~~ For each energy

N_1 —— g_1 — 1st Energy level

$$= g_1 \times g_1 \times \dots \times g_1 = g_1^{N_1}$$

∴ ~~In~~ Similarly g_i has $g_i^{N_i}$ ways.

∴ Total no. of ways to arrange $= g_1^{N_1} \times g_2^{N_2} \times g_3^{N_3} \times \dots g_i^{N_i}$

∴ The effective no of ways to distribute N particles in different energy levels under different energy states according to M-B statistics.

$$= \underbrace{\left(\frac{N!}{N_1!\, N_2! \dots N_i!}\right)}_{collect} \times \underbrace{\left(g_1^{N_1} \times g_2^{N_2} \times \dots \times g_i^{N_i}\right)}_{arrange}$$

∴ This is max no of ways to distribute in a macrostate which is no of microstate

$$\therefore \boxed{W_{M-B} = N! \prod_i \frac{g_i^{N_i}}{N_i!}}$$ → Ultimate Formula.

For non-generate $g_1 = g_2 = \dots = 1$

$\therefore g_i^{N_i} = 1$

$\therefore W_{M-B} = N! \prod_i \frac{1}{N_i!}$

i = no of energy level

N = no of particle

$N_i = 2!\,1!\,1!$ ($M(2,1,1)$)

$\therefore$ Now,

$M(2,1,1) \to W_1 = \frac{4!}{2!\,1!\,1!} = 12$ (Formula)

If

⊔ ⊔ ⊔
⊔ ⊔
⊔

degenerate

$M(2,1,1) = \frac{4!}{2!\,1!\,1!} \times 1^2 \times 2^1 \times 3^1$

$= 12 \times 6 = 72$ ways

• FERMI-DIRAC STATISTICS

1. The particles are identical and indistinguishable → quantum particle
2. $N = \sum N_i = \text{const}$ (closed system)
3. $U = \sum E_i N_i = \text{const}$ (Isolated system)
4. The quantum particles have half integral spin
5. The particles have antisymmetric wave function

Unlike Boson, symmetric wave function

Because of ⑤

6. Particles obey Pauli's Exclusion Principal
7. Quantum Particle, particles obey Heisenberg's Uncertainty Principal
8. The particles are called fermions.

• (N) no of Fermions

$M(N_1, N_2, \dots N_i)$

As indistinguishable 1 way

Total collect $= 1 \times 1 \times 1 \dots$

$= 1$ ways

4

3

2

1

$g_1\ N_1$

no of ways to arrange N_1 particles in g_1 energy states

g_1

N_1

~~$g_1 \times (g_1 - 2)$~~

Process

$\boxed{N_1}\ g_1 \times (g_1 - 1) \times (g_1 - 2) \times \dots (g_1 - (N_1 - 1))$

1st, 2nd, 3rd, $N_1!$, N_1 particle

(As particles are indistinguishable)

$\therefore \frac{g_1 \times (g_1 - 1) \times (g_1 - 2) \dots (g_1 - (N_1 - 1)) \dots 3 \times 2 \times 1}{N_1!\,(g_1 - N_1)!}$ ×

E_1	E_2	E_3	
•	o	x	✓
x	•	•	✓
•	x	•	✓

2 particles 3 energy state

3 ways.

In M-B

3 energy state

2 particles

⇒ way ⇒ 3^2

E_1	E_2	E_3
ab		
x	ab	x
x	x	ab
a	b	
b	a	
	a	b
	b	a
a		b
b		a

9 ways.

$\therefore = \frac{g_1 (g_1 - 1) \times (g_1 - 2) \dots [g_1 - (N_1 - 1)] \times [g_1 - N_1] (g_1 - N_1 - 1) \dots 3 \times 2 \times 1}{N_1!\,(g_1 - N_1)!}$

collection $= \frac{g_1!}{N_1!\,(g_1 - N_1)!} = {}^{g_1}C_{N_1}$

$\therefore \boxed{W_{F-D} = \prod_i {}^{g_i}C_{N_i}}$ → As collect part is 1

• Q) 3 particles are distributed in 3 energy level with degeneracy 1, 2, 3 respectively. Find no of macrostate & microstate of the system if the particles obey M-B & F-D statistics.

⊔ ⊔ ⊔
⊔ ⊔
⊔

$g_1 = 1$

$g_2 = 2$

$g_3 = 3$

$N = 3$

M-B Statistics

$M_1 = (3, 0, 0) \to W_1 =$

$M_2 = (0, 3, 0) \to W_2 =$

✓ $M_3 = (0, 0, 3) \to W_3 =$

$M_4 = (2, 1, 0) \to W_4 =$

$M_5 = (2, 0, 1) \to W_5 =$

✓ $M_6 = (1, 0, 2) \to W_6 =$

✓ $M_7 = (1, 2, 0) \to W_7 =$

F-D Statistics

$m_1 = (0, 0, 3)$

$m_2 = (1, 2, 0)$

$W(1,2,0) = {}^{g_1}C_{N_1} \times {}^{g_2}C_{N_2} \times {}^{g_3}C_{N_3}$

$= {}^1C_1 \times {}^2C_2 \times {}^3C_0 =$

$m_8 = (0,1,2)$

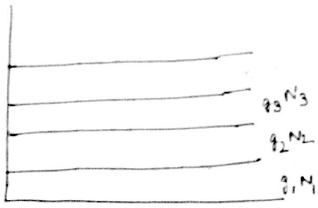
$m_9 = (0,2,1)$

$m_{10} = (1,1,1)$

• No of ways to distribute the particles in macrostate $M(N_1, N_2, N_3, N_4 \ldots N_i)$

$$W_{M-B} = \frac{N!}{N_1!\, N_2! \ldots N_i!} g_1^{N_1} \times g_2^{N_2} \times \ldots \times g_i^{N_i}$$

$$W_{F-D} = 1 \times 1 \times \ldots \times 1 \times {}^{g_1}C_{N_1} \times {}^{g_2}C_{N_2} \times \ldots \times {}^{g_i}C_{N_i}$$



Bose – Einstein statistics :–

The particles are considered to be identical and indistinguishable

1. The particles are considered to be identical and indistinguishable
2. Total no of particles of the system is const $N = \sum N_i = \text{const}$ (closed)
3. Total energy $U = \sum E_i N_i = \text{const}$ (isolated)
4. Particles have integral spin
5. Quantum particles have symmetric wave function (As they don't obey Pauli's Exc. Pr each energy state may occupy any no of particles)
6. The particles do not obey Pauli's Exclusion Principle
7. The particles obey Heisenberg Uncertainty Principle
8. Eg Boson Particles :- He^4 atom, photon, phonon, π – meson

THERMODYNAMIC PROBABILITY :- (The no. of microstate)

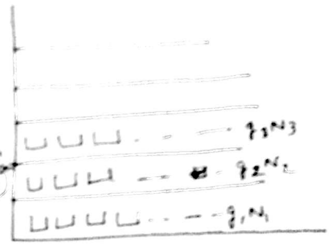
i.e. the thermodynamic probability of the macrostate of the macrostate $M(N_1, N_2, \ldots N_i)$ for the particles (Boson) according to BE statistics

(N) particles

Collect :-

From N_1 particles from N particles – 1 (As particles indistinguishable)

N_2 particles from $(N - N_1)$ particles → 1 way



a b c d e ∴ So total 1 ways.

• NO of ways to arrange N_1 particles in g_1 energy state :-

M-B → BE ← F-D

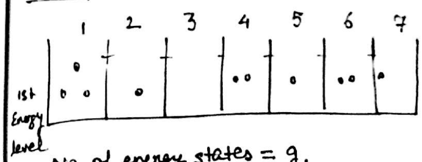
Ex. 3 Energy States & 2 Particles

3 states

1	2	3	
••	×	×	✓
×	••	×	✓
×	×	••	✓
•	•	×	✓
×	•	•	✓
•	×	•	✓

6 ways

$W_{m-B} =$



No. of energy states $= g_1$

No of particles $= N_1$ (say here 10).

No. of partions $= (g_1 - 1) = 6$ (here)

No. of objects $= (N_1 + g_1 - 1)$ (As objects and partitions mixed up and drawn)

The permutation of object among themselves $= (N_1 + g_1 - 1)!$

(As both particles & partitions are indistinguishable)

No. of ways to arrange $= \frac{(N_1+g_1-1)!}{N_1!\,(g_1-1)!}$ [Since the particles and partitions are both are indistinguishable]

$\therefore$ This is $^{(N_1+g_1-1)}C_{N_1}$

$\therefore$ Say in the example we apply this.

$\begin{cases} g_1 = 3 \\ N_1 = 2 \end{cases}$ $^{2+3-1}C_2 = {}^4C_2 = 6$ ways].

For second energy level $\Rightarrow$ $^{N_2+g_2-1}C_{N_2}$

$\therefore$ Total no of ways $^{N_i+g_i-1}C_{N_i}$

$$\boxed{W_{B-E} = 1\times1\times1\times1\times {}^{N_1+g_1-1}C_{N_1} \times \dots \times {}^{N_i+g_i-1}C_{N_i}}$$

Q) A system has 7 particles arranged in 2 compartment. The first compartment is divided into 8 cell and the second compartment into 10 cell. Find the no of micro state to a macrostate (3,4) if the particles obey M-B, F-D, B-E

$N = 7 \quad g_1 = 8 \quad g_2 = 10$

$\therefore W_{M-B} = W_{(3,4)}\Big|_{M-B} = \frac{7!}{3!\,4!} \times 8^3 \times 10^4$

$W_{(3,4)_{F-D}} = {}^8C_3 \times {}^{10}C_4$

$W_{(3,4)_{B-E}} = {}^{3+8-1}C_3 \times {}^{4+10-1}C_4$

• Distribution law :-

M-B	F-D	B-E
① $N = \sum N_i =$ const $\Rightarrow \sum dN_i = 0$ --- ①	1. $\sum dN_i = 0$ — ①	1. $\sum dN_i = 0$ - ①
in isolated ② $U = \sum E_i N_i =$ const; $\sum d(E_i N_i) = 0$. $E_1 \to$ energy of 1st level say Bohr's orbit. $E_2 \to$ these are fixed. $\sum N_i dE_i + \sum E_i dN_i = 0$. $\therefore \sum E_i dN_i = 0$ — ② [E_i = const]	2. $\sum E_i dN_i = 0$ — ② ① U + fixed, N↑ ∴ W↑ ② Also, N fixed U↑ ∴ W value also increases	$N_i \neq$ const, $dN_i \neq 0$ } As particles in each level are not const but ② $\sum E_i dN_i = 0$ — ②

from ① + ② using Lagrange's multiplier method.

$\alpha \sum dN_i + \beta \sum E_i dN_i = 0$ --- ③

from energy

α, β are multiplier consts explained later

M-B

$W = N! \prod_i \frac{g_i^{N_i}}{N_i!}$

$W_{M-B} = N! \prod_i \frac{g_i^{N_i}}{N_i!}$

$W_{F-D} = \prod_i {}^{g_i}C_{N_i}$

W_{B-E} $^{N_i+g_i-1}C_{N_i}$

Take log both sm

$\ln W = \ln N! + \sum [N_i \ln g_i - \ln N_i!]$

[Starling approximation $\ln x! = x\ln x - x$ when x is very large]

$\therefore \ln W = N\ln N - N + \sum (N_i \ln g_i - N_i \ln N_i + N_i)$

∴ Derivative

$\ln W = N\ln N - N + \sum[N_i \ln g_i - N_i \ln N_i + N_i]$

As $[N = const]$

$[$ As $N = const$, $g_i = const$, $dN = 0$, $dg_i = 0$ $]$

$\Rightarrow d[\ln W] = \sum(dN_i \ln g_i) - dN_i \ln N_i - dN_i + dN_i]$

$\Rightarrow d[\ln W] = \sum \ln(g_i/N_i) dN_i$ — 3(i)

$\Omega = W_1 + W_2 + \dots + W_n$

For most probable state macrostate with most no. of microstate in W is max. [particle move from lower prob state to more prob state]

W is max

~~ln W is max~~. ln W is max

∴ $d(\ln W) = 0$.

∴ LHS = 0

$\Rightarrow \sum \ln(g_i/N_i) dN_i = 0$ — ④ — only for M-B. (Valid when Eq ③ is valid)

we had,

$\alpha \sum dN_i + \beta \sum N_i dN_i = 0$ — ③ was same for M-B, F-D + BE

(closed system & isolated system)

From eq ③ and ④

$\sum \ln(g_i/N_i) dN_i = \alpha \sum dN_i + \beta \sum E_i dN_i$

$\Rightarrow \sum\left[\ln(g_i/N_i) - \alpha - \beta E_i\right] dN_i = 0.$

∴ We can write (As summation all terms are independent of each other so they are independently 0

but $dN_i \neq 0$.

$\left[\ln(g_i/N_i) - \alpha - \beta E_i\right] dN_i = 0$

∴ $\ln(g_i/N_i) - \alpha - \beta E_i = 0.$

$\Rightarrow g_i/N_i = e^{\alpha + \beta E_i}$

$\Rightarrow \boxed{\dfrac{N_i}{g_i} = \dfrac{1}{e^{\alpha + \beta E_i}}}$

In F-D statistics

$\boxed{\dfrac{N_i}{g_i} = \dfrac{1}{e^{\alpha + \beta E_i} + 1}}$

B-E statistics

$\boxed{\dfrac{N_i}{g_i} = \dfrac{1}{e^{\alpha + \beta E_i} - 1}}$

$\ln W = N\ln N - N + \sum[N_i \ln g_i - N_i \ln N_i + N_i]$

$\Rightarrow d[\ln W] = 0 + 0 + \sum(dN_i \ln g_i + N_i \cdot \frac{1}{g_i} dg_i - dN_i \ln N_i - N_i \cdot \frac{1}{N_i} dN_i)$

$= \sum(dN_i \ln g_i - dN_i \ln N_i - dN_i + dN_i)$

$= \sum(\ln g_i - \ln N_i) dN_i$

$= \sum \ln(g_i/N_i) dN_i$