CHAPTER

3

Dielectrics

3.1 INTRODUCTION

A dielectric is an insulating material in which all the electrons are tightly bound to the nuclei of the atom and there are no free electrons available for the conduction of current. The difference in the name between dielectric and insulator lies in the application for which these materials are used. When these materials are used to prevent the flow of electricity through them or the application of potential difference, then they are called insulators or passive dielectrics. On the other hand, if they are used for charge storage, they are called dielectrics or active dielectrics. Materials such as glass, rubber, mica, porcelain and polymers are examples of dielectrics.

3.2 POLARIZATION

Polarization is defined as the process of creating or inducing dipoles in a dielectric material by an external electric field.

In an atom, there is a positively charged nucleus at the center surrounded by orbiting electrons which are negatively charged. In the absence of an electric field an isolated atom does not have any dipole moment, since the centroids of positive and negative charge coincide [Fig. 3.1(a)]. Suppose now the atom is placed

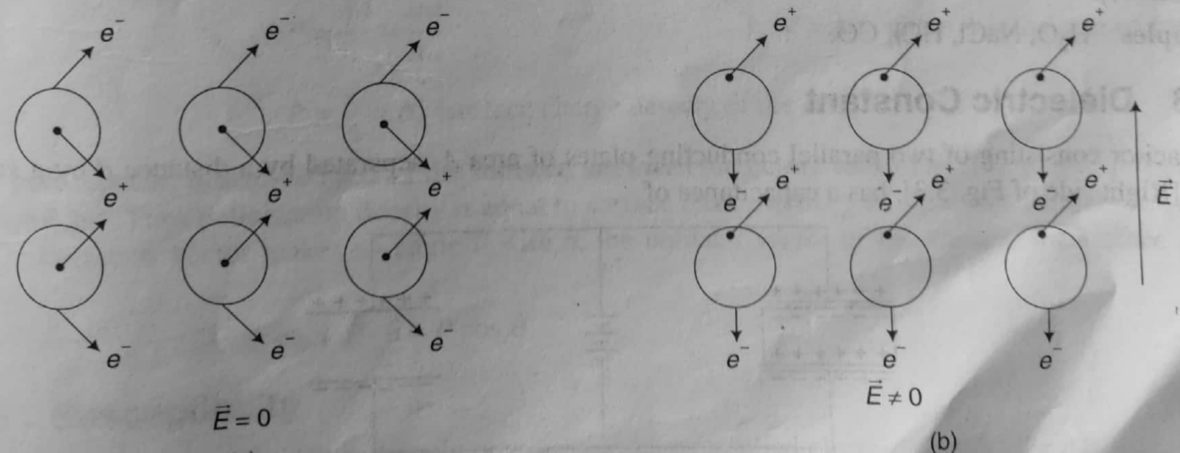

Fig. 3.1 **(a) No polarization. (b) Polarization.**

in an external electric field. The field will push the positively charged nucleus slightly in the direction of the field and the negatively charged electrons in the opposite direction [Fig. 3.1(b)]. The centroids of the positive and negative charges now no longer coincide and as a result an electric dipole is induced in the atom. The amount of dipole moment induced is proportional to the field because a large field displaces charges more than a smaller field. We say that the atoms are polarized under the influence of the external field.

3.3 TYPES OF DIELECTRICS

On the basis of the polarization concept, dielectrics are the materials that have either permanent dipoles or induced dipoles in the presence of an applied electric field. They are classified into two categories, namely, polar and non-polar dielectrics.

3.3.1 Non-polar Dielectrics

A dielectric material in which, there is no permanent dipole existence in the absence of an external field is called 'non-polar' dielectrics.

For non-polar dielectrics, the center of gravity of the positive and negative charges of the molecules coincide. So such molecules do not have any permanent dipole moment [Fig. 3.2(a)].

Examples O_2, H_2, N_2, CO_2, H_2O_2.

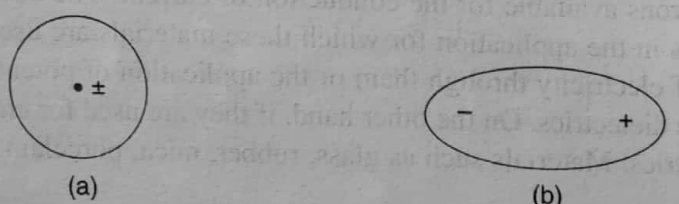

Fig. 3.2 **(a) Non-polar dielectrics. (b) Polar dielectrics.**

3.3.2 Polar Dielectrics

A dielectric material in which there is an existence of permanent dipole even in the absence of an external field is called polar dielectrics.

For non-polar dielectrics, the center of gravity of the positive charges is separated by finite distance from that of the negative charges of the molecules. So such molecules possess permanent electric dipole [Fig. 3.2(b)].

Examples H_2O, NaCl, HCl, CO.

3.3.3 Dielectric Constant

A capacitor consisting of two parallel conducting plates of area A, separated by a distance d by a vacuum space [Right side of Fig. 3.3], has a capacitance of

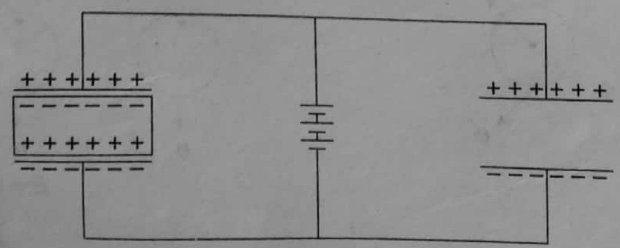

Fig. 3.3 **Two identical capacitors: one evacuated and other filled with dielectric.**

$$C_0 = \frac{\varepsilon_0 A}{d} \qquad ...(3.1)$$

where ε_0 is the permittivity of free space. If C is the capacitance when the space is filled with dielectric material [Left side of Fig. 3.3], then

$$C = \frac{\varepsilon A}{d} \qquad ...(3.2)$$

where ε is the permittivity of the dielectric. Now the dielectric constant of the material

$$K = \frac{C}{C_0} = \frac{\varepsilon}{\varepsilon_0} \qquad ...(3.3)$$

The dielectric constant of a material is the ratio of the capacitance of a given capacitor completely filled with that material to the capacitance of the same capacitor in vacuum. In other words, the ratio of permittivity of medium to that of the vacuum is also known as dielectric constant or relative permittivity (ε_r).

$$\varepsilon_r = K = \frac{\varepsilon}{\varepsilon_0} \qquad ...(3.4)$$

ε_r is a dimensionless quantity and varies widely from material to material. ε_r has a value unity for vacuum and for all other dielectrics ε_r is always greater than 1. For most materials the value of ε_r varies between 1 to 10.

3.3.4 Polarization Vector or Polarization Density

There are two kinds of dipoles in materials—those that are induced and those that are permanent and both cause polarization or charge separation. A dipole moment μ is defined as $\mu = qd$, where q is the magnitude of the charge and d is the distance separating the pair of opposite charges. Dipole moment is a vector, pointing from negative towards positive charges. Polarization vector measures the extent of polarization in a unit volume of dielectric matter. It is defined as the induced dipole moment per unit volume of the dielectric. If N is the number of molecules per unit volume, then the polarization vector or polarization density

$$P = N\mu \qquad ...(3.5)$$

The direction of P is along the direction of the applied field.

If a dielectric slab of thickness d and volume V is kept between the two plates of a capacitor [left-hand side of Fig. 3.3], then the dipole moment is $\mu = qd$, where $+q$ and $-q$ are induced charges on the respective faces of the slab.

The polarization is given

$$P = \frac{qd}{V} = \frac{qd}{Ad} \qquad [\because V = Ad, \text{ where } A \text{ is the area of the slab}]$$

$$\therefore \quad P = \frac{q}{A} = \sigma_P \text{ (surface charge density of the slab)} \qquad ...(3.6)$$

So, **polarization is also defined as the induced surface charge per unit area.** The unit of polarization is coulomb/m^2. Thus polarization density is equal to surface charge density on the dielectric slab. In general, if the polarization vector makes an angle θ with $\hat{n}$, the outward vector of the surface, the surface charge density

$$\sigma_P = \vec{P} \cdot \hat{n} = P \cos\theta \qquad ...(3.6a)$$

3.3.5 Susceptibility

The strength of polarization (P) is directly proportional to the applied electric field (E) for dielectrics and is given by

$$P = \varepsilon_0 \chi_e E \qquad ...(3.7)$$

The constant of proportionality is usually written as $\varepsilon_0 \chi_e$, where χ_e is known as electric susceptibility of the medium. χ_e is a dimensionless parameter.

Now

$$\chi_e = \frac{P}{\varepsilon_0 E} \qquad ...(3.8)$$

Thus, **susceptibility is the ratio of polarization to the net electric field $\varepsilon_0 E$ as modified by the induced charges on the surface of the dielectric.**

3.4 RELATION BETWEEN DIELECTRIC CONSTANT AND ELECTRICAL SUSCEPTIBILITY

We consider a parallel-plate capacitor which has vacuum between its plates. When it is charged with a battery, the electric field of strength E_0 is set up between the plates of the capacitor [Fig. 3.4(a)]. If σ and $-\sigma$ are the surface charge densities of the two plates of the capacitor, then the electric field developed between the plates is given by

$$E_0 = \frac{\sigma}{\varepsilon_0} \qquad ...(3.9)$$

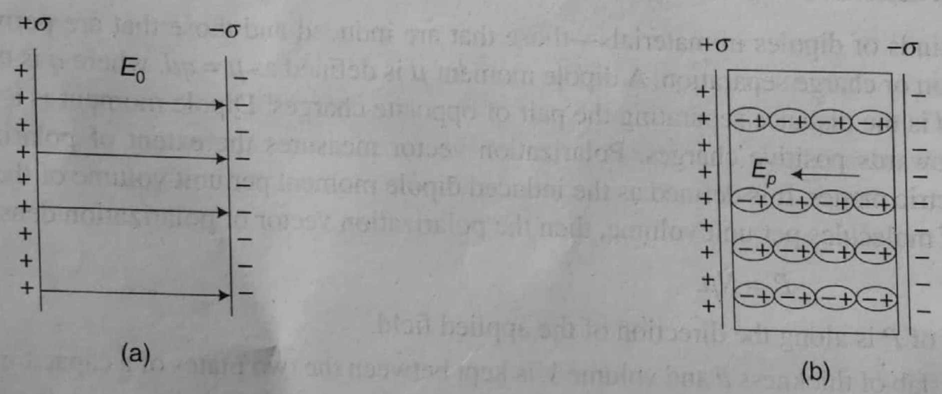

Fig. 3.4 (a) Capacitor with vacuum space. (b) Capacitor filled with dielectric.

If a dielectric slab is placed between the plates of the capacitor; then due to polarization charges, appear on the two faces of the slab and establish another field E_p within the dielectric [Fig. 3.4(b)]. This field will be in a direction opposite to the E_0. Under this situation, the net electric field in the dielectric is given by

$$E = E_0 - E_P \qquad ...(3.10)$$

If σ_s is the surface charge density on the slab, then by following Eq. (3.9), we can write

$$E_p = \frac{\sigma_P}{\varepsilon_0} \qquad ...(3.11)$$

Now, from Eqs. (3.9), (3.10) and (3.11),

$$E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_P}{\varepsilon_0} = \frac{1}{\varepsilon_0}(\sigma - \sigma_P)$$

or, $$\varepsilon_0 E = \sigma - \sigma_P = \sigma - P \qquad [\because P = \sigma_P]$$

or, $$\sigma = \varepsilon_0 E + P \qquad ...(3.12)$$

Again, by Gauss' law, electric flux density or electric displacement vector D is given by

$$D = \sigma \qquad ...(3.13)$$

Now, from Eq. (3.12)

$$D = \varepsilon_0 E + P \qquad ...(3.14)$$

Again, from Eq. (3.7)

$$P = \varepsilon_0 \chi_e E \qquad ...(3.15)$$

and from electrostatics we know

$$D = \varepsilon E = \varepsilon_0 \varepsilon_r E \qquad ...(3.16)$$

Therefore, from Eqs. (3.15), (3.16) and (3.14)

$$\varepsilon_0 \varepsilon_r E = \varepsilon_0 E + \varepsilon_0 \chi_e E$$

or, $$\varepsilon_r = 1 + \chi_e$$

or $$\chi_e = \varepsilon_r - 1 \qquad ...(3.17)$$

3.5 POLARIZABILITY

Let us consider an individual atom in a dielectric material and the material be subjected to an electric field E. The strength of the dipole induced in an atom is proportional to the actual field acting on the particle, and is given by

$$\mu = \alpha E \qquad ...(3.18)$$

where α is the proportionality constant called polarizability. Its unit is Fm^2.

Note: In the case of gases, the molecules, for most of the time are far apart, so that local electric field (E_{loc}) is the same as the macroscopic field E.

If N be the number of atoms in a unit volume then polarization vector is

$$P = N\mu$$

$$= N \alpha E = \varepsilon_0 \chi_e E \qquad \text{[From Eq. (3.8)]}$$

Therefore, $$\alpha = \frac{\varepsilon_0 \chi_e}{N} \qquad ...(3.19)$$

Polarizability measures the resistance of the particle to the displacement of its electron cloud.

3.6 TYPES OF POLARIZATION

Three basic types of polarization that contribute to the total magnitude of polarization in a material have been identified.

(i) Electronic polarization

(ii) Ionic polarization

(iii) Orientation polarization

Taking into account the three contributions, the total electrical dipole moment

$$\mu = (\alpha_e + \alpha_i + \alpha_0) E$$

or, polarization

$$P = N\mu = N(\alpha_e + \alpha_i + \alpha_0) E \qquad ...(3.20)$$

$$= N \alpha E$$

where α is total polarizability

α_e is electronic polarizability

α_i is ionic polarizability

α_0 is orientation polarizability

(i) Electronic polarization

Electronic polarization occurs due to the displacement of the positively charged nucleus and negatively-charged electron cloud in opposite directions within a dielectric material upon applying an external electric field E [Fig. 3.5a, b]. The dipole moment (μ_e) induced is proportional to the applied field and the proportionality constant is called electronic polarizability (α_e).

$$\mu_e = \alpha_e E \qquad ...(3.21)$$

If a material has N such atoms per unit volume, subjected to homogeneous field E, then the electronic polarization is

$$P = N\alpha_e E \qquad ...(3.22)$$

The electronic polarizability for a rare gas atom is given by

$$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N} \qquad ...(3.23)$$

The electronic polarization can persist to extremely high field frequencies because electronic standing waves within atoms have very high natural frequencies.

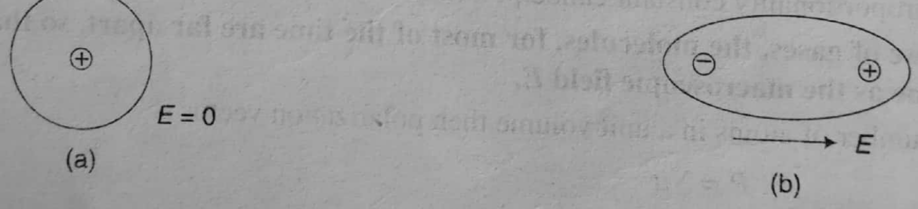

Fig. 3.5 (a) No field applied. (b) Applied electric field.

(ii) Ionic polarization

This type of polarization occurs in ionic materials. In an ionic bond when two different atoms join together, there is transfer of electrons from an atom to another atom, like HCl shows in Fig. 3.6. Even in the absence of an applied field, an HCl molecule has a permanent dipole moment $e \times d$, where d is the distance of separation of ions. In the presence of an applied electric field, the resultant torque lines up the dipoles parallel to the field at absolute zero temperature. The distance between the ions increases from d to $d + x$. The field has induced an additional dipole moment $e \times x$ in the molecule. The induced dipole moment is proportional to the applied electric field and is given by

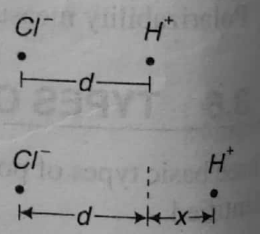

Fig. 3.6 Ionic Polarization.

$$\mu_i = \alpha_i E \qquad ...(3.24)$$

where α_i is the ionic polarizability. Now if N is number of dipole per unit volume, then ionic polarization

$$P_i = N\alpha_i E \qquad ...(3.25)$$

(iii) Orientation polarization

Orientation polarization occurs in dielectric materials which possess molecules with permanent dipole moment, for example, H_2O molecule (polar molecule). In the absence of an external electric field, the permanent dipoles are oriented randomly such that they cancel the effects of each other [Fig. 3.7(a)]. But under the influence of an external applied electric field, each of the dipoles undergo rotation so as to reorient along the direction of the field as shown in Fig. 3.7(b). Thus, the material itself develops electric polarization. This is known as orientation polarization, which depends upon temperature.

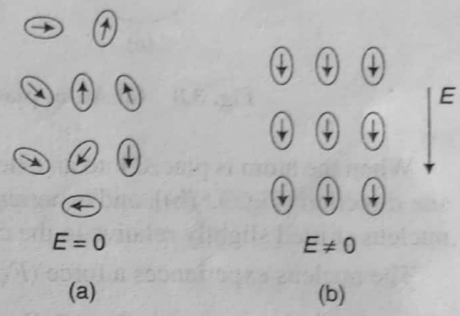

Fig. 3.7 Orientation polarization.

The orientation polarization P_0 is given by Langevin function (1905)

$$P_0 = N\mu L(x) \qquad ...(3.26)$$

where $L(x)$ is known as Langevin function.

Here $x = \dfrac{\mu E}{k_B T}$, k_B is the Boltzmann constant and T is the temperature in Kelvin.

The value of $L(x)$ is $\coth x - \dfrac{1}{x} = \left[\dfrac{e^x + e^{-x}}{e^x - e^{-x}} - \dfrac{1}{x}\right]$

For $\quad x >> 1, P_0 \to 1$

complete alignment, but this does not occur in gases.

For most practical cases (x is small, i.e., at high temperature)

$$x << 1, L(x) = \frac{x}{3}$$

Or,

$$P_0 = N\mu\frac{x}{3} = \frac{N\mu^2 E}{3k_B T} \qquad ...(3.27)$$

The orientation plorizability α_0 is given by

$$\alpha_0 = \frac{\mu^2}{3k_B T} \qquad ...(3.28)$$

3.7 POLARIZATION IN MONOATOMIC GASES

Let us consider one of constituent atom of a dielectric material (rare gases, such as helium and argon) in the absence of an electric field. Let the radius of the atom be a and its atoms number be Z as shown in Fig. 3.8(a). Here positive nucleus $+Ze$ is surrounded by an electronic cloud of charge $-Ze$. Also nucleus is point charge and electron cloud of charge $-Ze$ distributed homogeneously throughout a sphere of radius a. Therefore, the charge density for electron cloud is given by

$$\rho = \frac{-Ze}{\frac{4}{3}\pi a^3} = -\frac{3}{4}\left(\frac{Ze}{\pi a^3}\right) \qquad ...(3.29)$$

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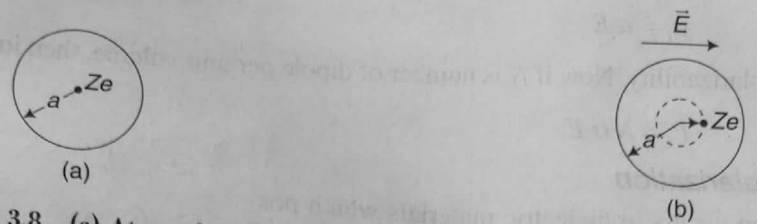

Fig. 3.8 (a) Atom placed in free space. (b) Atom placed into external field.

When the atom is placed into an external electric field E, the nucleus and electron cloud move in the opposite direction [Fig. 3.8(b)], and experience a Lorentz force of magnitude ZeE. Equilibrium is established with nucleus shifted slightly relative to the center of the electron cloud by a distance d.

The nucleus experiences a force (F_N) in the direction of the electric field,

$$F_N = ZeE$$

...(3.30)

and opposing force F_G due to the electric field of the charge located within the sphere of radius d and concentrated at the center of the electron cloud. By Gauss' law, the electric field (E_G) at edge location of the nucleus due to electrons within the sphere of radius d is

$$E_G \times 4\pi d^2 = \text{Total charge } (q) \text{ enclosed in a sphere of radius } d.$$

or,
$$E_G \times 4\pi d^2 = \frac{4}{3}\pi d^3 \rho/\varepsilon_0$$

$$= \frac{4}{3}\pi d^3 \times \left(-\frac{3Ze}{4\pi a^3}\right)\Big/\varepsilon_0$$ [From Eq. (3.29)]

$$= -\frac{Ze\left(\frac{d^3}{a^3}\right)}{\varepsilon_0}$$

Now
$$|F_G| = ZeE_G = \frac{Z^2e^2d}{4\pi\varepsilon_0 a^3}$$

But
$$|F_G| = |F_N|$$

So,
$$d = \frac{4\pi\varepsilon_0 a^3}{Ze}E$$

...(3.31)

Thus, the displacement distance d is proportional to the external electric field E. Due to this displacement, the atom acts as a dipole.

For the single atom, the electronic polarizability of a monoatomic gas can be obtained from induced dipole moment

$$\mu = \alpha_e E = (Ze)\, d = (Ze)\,\frac{4\pi\varepsilon_0 a^3}{Ze}E$$

or
$$\alpha_e = 4\pi\varepsilon_0 a^3$$

...(3.32)

Thus, the electronic polarizability is proportional to the volume of the atom and is independent of temperature.

The polarization vector $P = N\mu$

$$\therefore \quad P = N\alpha_e E \qquad ...(3.33)$$

But we know that $P = \varepsilon_0 E(\varepsilon_r - 1)$ ...(3.34)

Now from Eqs. (3.33) and (3.34)

$$N\alpha_e E = \varepsilon_0 E(\varepsilon_r - 1)$$

or,

$$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N} \qquad ...(3.35)$$

For He, the value of α_e is 0.18×10^{-40} Fm^2, and for Ne, the value of α_e is 0.35×10^{-40} Fm^2. So, bigger the size of the atom, the value of α_e is larger.

3.8 POLARIZATION IN POLYATOMIC GASES

Let us consider a gas containing N molecules per m^3. Assume that each molecule has a permanent electric dipole moment μ.

The polarization is due to the electronic polarization P_e (nucleus shifted slightly relative to the center of the electron cloud), the ionic polarization P_i (ionic nature of bond between atoms) and the orientation polarization P_0 (due to rotation and alignment of the polar molecules in the external electric field). The total polarization of a polyatomic gas is given by

$$P = P_e + P_i + P_0$$

$$= N\alpha_e E + N\alpha_i E + N\frac{\mu^2}{3KT}E$$

$$= N\left(\alpha_e + \alpha_i + \frac{\mu^2}{3KT}\right)E \qquad ...(3.36)$$

Again

$$P = \varepsilon_0 \chi_e E = (\varepsilon_r - 1)\,\varepsilon_0 E \qquad ...(3.37)$$

So,

$$(\varepsilon_r - 1)\,\varepsilon_0 = N\left(\alpha_e + \alpha_i + \frac{\mu^2}{3KT}\right) \qquad ...(3.38)$$

P_e and P_i are essentially independent of the temperature but P_0 is temperature dependent.

At a temperature T and zero external electric field, the molecules will be randomly oriented, so, zero polarization.

When there is an external electric field, the molecules will try to align with the field. Each polar molecule can be considered to be a simple dipole. The force on the dipole provides the torque to rotate the molecule so that they will be in the lowest state where they are parallel to the field. If there was no thermal motion, all dipoles would line up along the external field direction.

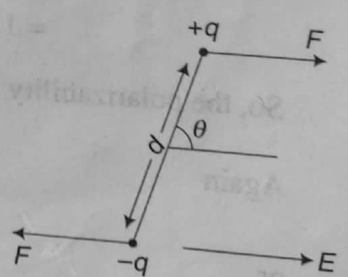

Fig. 3.9 Electric dipole placed in external electric field.

The electric force on the dipole produces a couple [Fig. 3.9] and the torque acting to rotate the dipole is

$$\tau = qEd\sin\theta = \mu E\sin\theta$$

or,

$$\vec{\tau} = \vec{\mu} \times \vec{E} \quad ...(3.39)$$

The potential energy of the dipole for an arbitrary angle θ is given by

$$U(\theta) = -\mu E \cos\theta = -\vec{\mu}\cdot\vec{E} \quad ...(3.40)$$

The dipole has the lowest potential energy when the dipole is parallel to the electric field and the highest potential energy when antiparallel to the field. For no thermal motion, all dipoles would line along the direction of the external electric field. But at a greater temperature, the thermal motion will be greater and there will be small alignment of the dipoles with the field.

Worked Out Problems

Example 3.1 Two parallel plates have equal and opposite charges. When the space between them is evacuated, the electric field intensity is 3×10^5 V/m and when the space is filled with dielectric, the electric intensity is 1.0×10^5 V/m. What is the induced charge density on the surface of the dielectric?

Sol. Given $E_0 = 3 \times 10^5$ V/m and $E = 1.0 \times 10^5$ V/m

We know that $E = E_0 - \dfrac{P}{\varepsilon_0}$

or,
$$P = \varepsilon_0(E_0 - E)$$
$$= 8.85 \times 10^{-12}\,(3-1) \times 10^5$$
$$= 1.77 \times 10^{-6}\ \text{C/m}^2$$

Again $P = \sigma_P$

So, $\sigma_P = 1.77 \times 10^{-6}\ \text{C/m}^2$

Example 3.2 Calculate the polarizability and relative permittivity in hydrogen gas with a density of 9.8×10^{26} atoms/m^3. [Given the radius of the hydrogen atom to be 0.50 Å].

Sol. Given $N = 9.8 \times 10^{26}$ atoms/m^3

$$a = 0.50 \times 10^{-10}\ \text{m}$$

We know that $\alpha_e = 4\pi\varepsilon_0 a^3$

$$= 4 \times 3.14 \times 8.85 \times 10^{-12} \times (0.50 \times 10^{-3})^3$$
$$= 1.38 \times 10^{-41}\ \text{Fm}^2$$

So, the polarizability $\alpha_e = 1.38 \times 10^{-41}\ \text{Fm}^2$

Again
$$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}$$

or,
$$\varepsilon_r = \frac{N\alpha_e}{\varepsilon_0} + 1$$

$$= \frac{9.8 \times 10^{26} \times 1.38 \times 10^{-41}}{8.85 \times 10^{-12}} + 1$$

$$= 1.001$$

So relative permittivity $\varepsilon_r = 1.001$

Example 3.3 In a dielectric material, $E_x = 5$ V/m and $\vec{P} = \frac{1}{10\pi}(3\hat{i} - \hat{j} + 4\hat{k})$ nC/m^2. Calculate (i) χ_e (ii) $\vec{E}$ (iii) $\vec{D}$

Sol. The polarization is given by

$$\vec{P} = \chi_e \varepsilon_0 \vec{E}$$

or,

$$\chi_e = \frac{\vec{P}}{\varepsilon_0 \vec{E}}$$

Here we consider only the x component

So, (i)

$$\chi_e = \frac{P}{\varepsilon_0 E_x} = \frac{3 \times 10^{-9}}{10\pi} \times \frac{36\pi}{10^{-9} \times 5} = 2.16$$

(ii)

$$\vec{E} = \frac{\vec{P}}{\varepsilon_0 \chi_e} = \frac{1}{10\pi}(3\hat{i} - \hat{j} + 4\hat{k}) \times \frac{36\,\pi}{10^{-9} \times 2.16}$$

$$= 5\hat{i} - \frac{5}{3}\hat{j} + \frac{20}{3}\hat{k} \text{ V/m}$$

(iii)

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \frac{10^{-9}}{36\pi}\left(5\hat{i} - \frac{5}{3}\hat{j} + \frac{20}{3}\hat{k}\right) + \frac{10^{-9}}{10\pi}(3\hat{i} - \hat{j} + 4\hat{k})$$

$$= (139.7\,\hat{i} - 46.6\,\hat{j} + 186.3\,\hat{k}) \text{ pC/m}^2$$

Example 3.4 Calculate the dipole moment μ of a molecule of carbon tetrachloride (CCl_4) in a field 10^7 Vm^{-1}. [Given: Density = 1.60 gm/cm^3, Molecular weight = 156, Relative permittivity $\varepsilon_r = 2.24$].

Sol. Molecular density

$$N = \frac{\text{Avogadro's number}}{\text{Molecular weight}} \times \text{Density}$$

$$= \frac{6.02 \times 10^{23}}{156} \times 1.60$$

$$= 6.17 \times 10^{21} \text{ molecules/cm}^3$$

The dipole moment of a single molecule μ is

$$\mu = \frac{\varepsilon_0(\varepsilon_r - 1)}{N} E$$

$$= \frac{8.85 \times 10^{-12} \times 1.24 \times 10^7}{6.17 \times 10^{27}}$$

$$= 1.77 \times 10^{-32} \text{ C/m}$$

There are 74 electrons in each CCl_4 molecule

So, $$\mu = 74\,ed$$

or, $$d = \frac{\mu}{74e} = \frac{1.77 \times 10^{-32}}{74 \times 1.6 \times 10^{-19}}$$

$$= 1.5 \times 10^{-15}\ \text{m}$$

Example 3.5 Dielectric constant of a gas at N.T.P is 1.00074. Calculate the dipole moment of each atom of the gas when it is held in an external field of 3×10^4 V/m.

Sol. Given $E = 3 \times 10^4$ V/m $= 3 \times 10^4$ N/C

and $$K = \varepsilon_r = 1.00074$$

We know $$\varepsilon_r = 1 + \chi_e$$

or, $$\chi_e = \varepsilon_r - 1 = 1.00074 - 1 = 0.00074$$

Polarization density $P = \varepsilon_0 \chi_e E$

$$= 8.85 \times 10^{-12} \times 0.74 \times 10^{-3} \times 3 \times 10^4$$
$$= 1.96 \times 10^{-10}\ \text{C/m}$$

No. of atoms of gas per cubic meter

$$N = \frac{6.06 \times 10^{23}}{22.4 \times 10^{-3}} = 2.7 \times 10^{25}$$

So induced dipole moment of each atom

$$\mu = \frac{P}{N} = \frac{1.96 \times 10^{-10}}{2.7 \times 10^{25}}$$

$$= 7.27 \times 10^{-36}\ \text{C/m}$$

Example 3.6 A dielectric cube of side L and center at the origin has a polarization vector given as $\vec{P} = \hat{i}x + \hat{j}y + \hat{k}z$. Find the volume and surface bound charge densities and show that the total bound charge vanishes in this case.

Sol. The bound surface charge density is $\sigma_b = \vec{P} \cdot \hat{n}$. For each of the six sides of the cube, there exists a surface charge density. For the side located at $x = L/2$, the surface charge density

$$\sigma_b^1 = \vec{P} \cdot \hat{i}|_{L/2} = (\hat{i}x + \hat{j}y + \hat{k}z) \cdot \hat{i}|_{L/2} = x|_{L/2} = L/2$$

∴ the total bound surface charge

$$q_{bs} = \int_s \sigma_b\, ds = 6 \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \sigma_b\, dy\, dz = 3L^3$$

The bound volume charge density is

$$\rho_b = -\vec{\nabla} \cdot \vec{P} = -(1 + 1 + 1) = -3$$

∴ the total bound volume charge is

$$q_{bV} = \int \rho_b\, dV = -3 \int dV = -3L^3$$

Hence, the total bound charge within the cube,

$$q = q_{bs} + q_{bv} = 3L^3 - 3L^3 = 0$$

So, total bound charge vanishes.

Example 3.7 The two plates of a parallel-plate capacitor are identical and carry equal amount of opposite charges. The separation between the plates is 5 mm and the space between the plates is filled with a dielectric of dielectric constant 3. The electric field within the dielectric is 10^6 V/m. Calculate (i) polarization vector $\vec{P}$, and (ii) displacement vector $\vec{D}$.

Sol. (i) The magnitude of the polarization vector is

$$P = \varepsilon_0 (k - 1) E$$
$$= 8.85 \times 10^{-12} (3 - 1) \times 10^6$$
$$= 17.7 \times 10^{-6} \text{ C/m}^2$$
$$= 17.7 \, \mu \text{ C/m}^2$$

(ii) The magnitude of the displacement vector is

$$D = k\varepsilon_0 E$$
$$= 3 \times 8.85 \times 10^{-12} \times 10^6$$
$$= 2.65 \times 10^{-7} \text{ C/m}^2$$
$$= 26.5 \, \mu \text{ C/m}^2$$

Example 3.8 A dielectric cube of side L, centered at the origin, carries a "frozen-in" polarization $\vec{P} = k\vec{r}$, where k is a constant. Find all the bound charges and check that they add up to zero.

Sol. The bound volume charge density ρ_s is equal to

$$\rho_b = -\vec{\nabla} \cdot \vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr) = -3k$$

Since the bound volume charge density is constant, the total bound volume charge in the cube is equal to product of the charge density and the volume

$$q_{bv} = -3k L^3$$

The surface charge density σ_s is equal to

$$\sigma_s = \vec{P} \cdot \hat{n} = k\vec{r} \cdot \hat{n}$$

The scalar product between $\vec{r}$ and $\hat{n}$ can be evaluated easily (see Fig. 3.1W) and is equal to

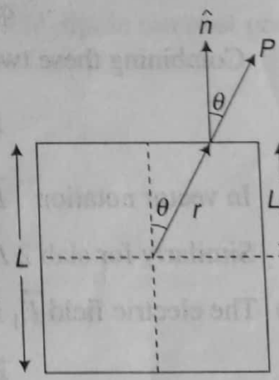

Fig. 3.1W Dielectric cube of side *L*.

$$\vec{r} \,.\, \hat{n} = r \cos \theta = \frac{1}{2} L$$

Therefore the surface charge density is equal to

$$\sigma_s = k\vec{r} \,.\, \hat{n} = \frac{1}{2} k \cdot L$$

The total surface charge density is equal to the product of the surface charge density and the total surface area of the cube

$$q_{bs} = \frac{1}{2} kL \times 6L^2 = 3\ kL^3$$

$\therefore$ the total bound charge on the cube is equal to

$$q = q_{bv} + q_{bs} = -3\ kL^3 + 3\ kL^3$$
$$= 0$$

Example 3.9 The space between the plates of a parallel-plate capacitor [Fig. 3.2W] is filled with two slabs of linear dielectric material. Each slab has thickness S, so that the total distance between the plates is $2S$. Slab 1 has a dielectric constant of 2, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate is $-\sigma$.

(i) Find the electric displacement $\vec{D}$ in each slab.

(ii) Find the electric field $\vec{E}$ in each slab.

(iii) Find the polarization $\vec{P}$ in each slab.

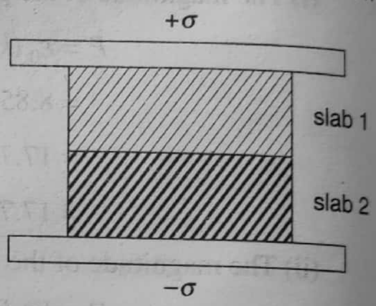

Fig. 3.2W Parallel-plate capacitor filled with two slabs of linear dielectric material.

Sol. (i) The electric displacement $\vec{D}_1$ in slab 1 can be calculated using Gauss' law. Consider a cylinder with cross-sectional area A and axis parallel to the z axis, being used as a gaussian surface. The top of the cylinder is located inside the top metal plate (where electric displacement is zero) and the bottom of the cylinder is located inside the dielectric of slab 1. The electric displacement is directed parallel to the z axis and pointed downwards. So, the displacement flux through this surface is equal to

$$\phi_D = D_1 A$$

The free charge enclosed by this surface is equal to

$$q_{\text{free}} = \sigma A$$

Combining these two we obtain

$$D_1 = \frac{\phi_D}{A} = \frac{q_{\text{free}}}{A} = \sigma$$

In vector notation $\vec{D}_1 = -\sigma\hat{k}$

Similarly for slab 2 $D_2 = -\sigma\hat{k}$

(ii) The electric field $\vec{E}_1$ in slab 1 is equal to

$$\vec{E}_1 = \frac{1}{k_1\varepsilon_0}\vec{D} = -\frac{\sigma}{k_1\varepsilon_0}\hat{k} = -\frac{\sigma}{2\varepsilon_0}\hat{k}$$

The electric field $\vec{E}_2$ in slab 2 is equal to

$$\vec{E}_2 = \frac{1}{k_2\varepsilon_0}\vec{D}_2 = -\frac{\sigma}{k_2\varepsilon_0}\hat{k} = -\frac{2\sigma}{3\varepsilon_0}\hat{k}$$

(iii) The polarization $\vec{P}_1$ of slab 1 is equal to

$$\vec{P}_1 = \vec{D}_1 - \varepsilon_0 \vec{E}_1$$

$$= -\sigma\hat{k} + \frac{\sigma}{2}\hat{k}$$

$$= -\frac{\sigma}{2}\hat{k}$$

The polarization $\vec{P}_2$ of slab 2 is equal to

$$\vec{P}_2 = \vec{D}_2 - \varepsilon_0 \vec{E}_2$$

$$= -\sigma\hat{k} + \frac{2\sigma}{3}\hat{k} = -\frac{\sigma}{3}\hat{k}$$

Example 3.10 The polarizability of a gas is 0.35×10^{-40} Fm^2. If the gas contains 2.7×10^{25} atoms/m^3 at 0°C and one atmospheric pressure, calculate its relative permittivity.

[Given $\alpha = 0.35 \times 10^{-40}$ Fm^2, $N = 2.7 \times 10^{25}$].

Sol. We know

$$\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}$$

$$= \frac{1 + 2.7 \times 10^{25} \times 0.35 \times 10^{-40}}{8.854 \times 10^{-12}}$$

$$= 1 + 0.1067 \times 10^{-3}$$

$$= 1.000107$$

So, the relative permittivity is 1.000107.

Example 3.11 A capacitor uses a dielectric material of relative permittivity $\varepsilon_r = 8$. It has an effective surface area of 0.036 m^2 with a capacitance of 6 μF. Calculate the field strength and dipole moment per unit volume if a potential difference of 15 V exists across the capacitor.

Sol. Field strength $E = \dfrac{V}{d}$ where $d = \dfrac{\varepsilon_0 \varepsilon_r A}{C}$

$$d = \frac{8.85 \times 10^{-12} \times 8 \times 0.036}{6 \times 10^{-6}}$$

$$= 0.42 \times 10^{-6} \text{ m}$$

or, field strength $$E = \frac{V}{d} = \frac{15}{0.42 \times 10^{-6}} = 35.3 \times 10^6 \text{ V/m.}$$

Dipole moment/unit volume $= \varepsilon_0 (\varepsilon_r - 1) E$

$$= 8.85 \times 10^{-12} (8 - 1) \times 3.5 \times 10^6$$

$$= 2.1 \times 10^{-5} \text{ C/m}^2$$

Review Exercises

Part 1: Multiple Choice Questions

1. In vacuum, electric susceptibility is
   (a) greater than 1 (b) zero (c) less than 1 (d) None of these
2. The relation between three electric vectors E, D and P is
   (a) $\vec{D} = \varepsilon_0 (\vec{E} + \vec{P})$ (b) $\vec{D} = \vec{E} + \varepsilon\vec{P}$ (c) $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ (d) $\vec{D} = \frac{1}{\varepsilon_0} (\vec{E} + \vec{P})$
3. The relation between electrical susceptibility and dielectric constant is
   (a) $\chi_e = \varepsilon_0 k$ (b) $\chi_e \doteq k - 1$ (c) $\chi_e = k + 1$ (d) $\chi_e = \frac{k}{\varepsilon_0} - 1$
4. The dimension of polarizability in SI unit is
   (a) Fm^2 (b) Fm (c) Fm^{-1} (d) Fm^{-2}
5. Dielectrics are the substances which are
   (a) semiconductor (b) conductors (c) insulators (d) None of these
6. The electronic polarizability for a rare gas atom is
   (a) $\alpha_e = \frac{(\varepsilon_r - 1)}{\varepsilon_0 N}$ (b) $\alpha_e = N(\varepsilon_r - 1)$ (c) $\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}$ (d) $\alpha_e = \frac{N}{\varepsilon_r - 1}$
7. A medium behaves like dielectric when the
   (a) displacement current is much greater than the conduction current
   (b) displacement current is zero
   (c) conduction current is almost zero
   (d) displacement current is equal to the conduction current
8. The total polarization of a polyatomic gas is
   (a) $P = N(\alpha_e + \alpha_i)$ (b) $P = N\left(\alpha_e + \alpha_i + \frac{\mu}{KT}\right) E$
   (c) $P = N\left(\alpha_e + \alpha_i + \frac{\mu^2}{3KT}\right) E$ (d) $P = \frac{N\mu E}{KT}$
9. The potential energy of the dipole for an arbitrary angle θ
   (a) $U(\theta) = -\vec{\mu} \times \vec{E}$ (b) $U(\theta) = -\vec{\mu} \cdot \vec{E}$ (c) $U(\theta) = \vec{E} \times \vec{\mu}$ (d) None of these
10. The relation between electronic polarizability and atomic radius for monatomic gases is
   (a) $\alpha_e = a^3$ (b) $\alpha_e = 4\pi\varepsilon_0 a^3$ (c) $\alpha_e = 4\pi\varepsilon_0 a^2$ (d) $\alpha_e = 4\pi\varepsilon_0 a$
11. For polar dielectrics, the orientation polarizability α_0 is given by
   (a) $\alpha_0 = \frac{3KT}{\mu^2}$ (b) $\alpha_0 = \frac{\mu^2}{3KT}$ (c) $\alpha_0 = \mu KT$ (d) None of these
12. Electrical susceptibility χ_e is
   (a) $\chi_e = \frac{P}{\varepsilon_0 E}$ (b) $\chi_e = \frac{P}{3\varepsilon_0 E}$ (c) $\chi_e = \varepsilon_0 EP$ (d) $\chi_e = \frac{3\varepsilon_0 E}{P}$

13. Generally, the dielectrics are
   (a) metallic materials of low specific resistance and have negative temperature coefficient of resistance
   (b) metallic materials of high specific resistance and have negative temperature coefficient of resistance.
   (c) metallic materials of high specific resistance and have positive temperature coefficient of resistance.
   (d) None of these
14. The ionic polarizability is
   (a) independent of temperature
   (b) depends on temperature
   (c) depends on square of the temperature
   (d) None of these

[**Ans.** 1 (b), 2 (c), 3 (b), 4 (a), 5 (c), 6 (c), 7 (a), 8 (c), 9 (b), 10 (b), 11 (b), 12 (a), 13 (b), 14 (a)]

Short Questions with Answers

1. Define polarization.

Ans. Polarization is defined as the process of creating or inducing dipoles in a dielectric material by an external electric field.

2. Define electrical susceptibility.

Ans. The electrical susceptibility is the ratio of polarization (P) to the net electric field ($\varepsilon_0 E$) as modified by the induced charges on the surface of the dielectric.

3. What are non-polar and polar dielectrics?

Ans. A dielectric, in the atoms and molecules of which, the center of gravity of positive and negative charges coincides, is called non-polar dielectric.

A dielectric, in the atoms and molecules of which, the center of gravity of positive and negative charges does not coincide, is called polar dielectric.

4. Define dielectric strength.

Ans. The dielectric strength of a dielectric is defined as the maximum value of the electric field that can be applied to the dielectric without its electric breakdown.

5. What do we mean by 'dielectric constant of glass is 8.5'?

Ans. Dielectric constant of a glass is 8.5 means that the ratio of the capacitance of a capacitor with glass as dielectric to the capacitance of the capacitor with air as dielectric.

6. Define polarizability.

Ans. Polarizability is the ability of an atom or a molecule to become polarized in the presence of an electric field.

7. What is electronic polarization?

Ans. Under the action of an external field, the electron clouds of atoms are displaced with respect to heavy fixed nuclei to a distance less than the dimensions of the atom. This is known as electronic polarization.

Part 2: Descriptive Questions

1. What are polar and non-polar dielectrics? What is meant by polarization of dielectric?
2. Show that $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$.
3. Derive a relation between electric susceptibility and atomic polarizability on the basis of microscopic description of matters at atomic level.
4. Define the following terms: (i) dipole moment, (ii) electrical susceptibility, (iii) relative dielectric constant, and (iv) polarization.
5. Explain the phenomenon of polarization of dielectric medium and show that $K = 1 + \chi_e$, where the symbols have their usual meanings.
6. Show that electronic polarizability α_e is

$$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N},$$ where the symbols have their usual meanings.

7. Derive an expression for electronic polarization of a dielectric medium.
8. What are non-polar and polar dielectrics? Find out the relation between dielectric constant and electrical susceptibility.
9. Define polarization. Show that electronic polarizability is proportional to the volume of the atom and is independent of temperature.
10. Explain polarization in polyatomic gases.

Part 3: Numerical Problems

1. Copper is a FCC crystal with a lattice constant 3.6 Å and atomic number 29. If the average displacement of the electrons relative to the nucleus is 1×10^{-18} m. Applying an electric field, calculate the electronic polarization. **[Ans.** $P = 3.94 \times 10^{-7}$ C/m^2**]**
2. A sphere of radius R carries a polarization $\vec{P}(\vec{r}) = k\vec{r}$ where k is constant and $\vec{r}$ is the vector from the center.
   (a) Calculate the bound charges σ_b and ρ_s.
   (b) Find the field inside and outside the sphere. **[Ans.** $\sigma_b = KR$, $\rho_b = -3k$, 0**]**
3. Two parallel plates of a capacitor having equal and opposite charges are separated by 6.0 mm thick dielectric materials of dielectric constant 2.8. If the electric field strength inside be 10^5 V/m, determine polarization vector and displacement vector. **[Ans.** $P = 1.6 \times 10^{-6}$ C/m^2, $D = 2.5 \times 10^{-6}$ C/m^2**]**
4. Determine the electric susceptibility at 0°C for a gas whose dielectric constant at 0°C is 1.000041. **[Ans.** $\chi_e = 4.1 \times 10^{-5}$**]**
5. The electronic polarizability of argon atom is 1.75×10^{-40} Fm^2. What is the static dielectric constant of solid argon, if its density is 1.8×10^3 kg/m^3 (Given atomic weight of $A_r = 39.95$ and $N = 6.025 \times 10^{26}$/K mole). **[Ans.** $\varepsilon_r = 1.5367$**]**
6. The dielectric constant of helium at 0°C is 1.0000684. If the gas contains 2.7×10^{25} atoms/m^3, find the radius of the electron cloud. **[Ans.** 0.6×10^{-10} m**]**
7. A gas containing 2.7×10^{25} atoms per m^3 has polarizability of 0.2×10^{-40} Fm^2. Calculate the relative permittivity of the gas. **[Ans.** 1.000061**]**