Assignment – Math 2202 (Modules II and III)

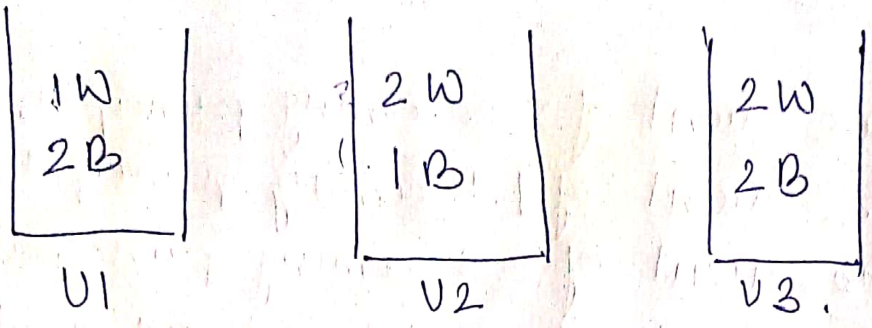
1) Let A be the event of X becoming principal,
B be the event of Y becoming principal,
C be the event of Z becoming principal and
E be the event where "student aid fund" is introduced.

$\therefore$ $P(A) = 0.3$
$P(B) = 0.5$
$P(C) = 0.2$
$P(E/A) = 0.4$
$P(E/B) = 0.6$
$P(E/C) = 0.1$
(given)

$$\therefore P(E) = P(A)P(E/A) + P(B)P(E/B) + P(C)P(E/C)$$
$$= 0.3 \times 0.4 + 0.5 \times 0.6 + 0.2 \times 0.1$$
$$= 0.12 + 0.3 + 0.02$$
$$= 0.44$$

$$\therefore P(B/E) = \frac{P(B)P(E/B)}{P(E)}$$
$$= \frac{0.5 \times 0.6}{0.44} = \frac{0.3}{0.44} = 0.68 \text{ (Ans)}$$

2)



Let A be the event where white ball is transferred from U1 → U2.

B be the event black ball is transferred U1 → U2

M be the event white ball transferred from U2 → U3

N be the event black ball transferred from U2 → U3

E be the event drawn ball is white.

$\therefore P(A) = 1/3.$

$P(B) = 2/3.$

$\therefore P(M/A) = 3/4.$

$P(M/B) = 1/2.$

$\therefore P(M) = P(A)P(M/A) + P(B)P(M/B)$

$= 1/3 \times 3/4 + 2/3 \times 1/2$

$= 1/4 + 1/3$

$= \frac{7}{12}$

$\therefore P(N/A) = 1/4$

$P(N/B) = 1/2$

$\therefore P(N) = P(A)P(N/A) + P(B)P(N/B)$

$= 1/3 \times 1/4 + 2/3 \times 1/2 = 1/12 + 1/3 = 5/12.$

$\therefore P(E/M) = 3/5.$

$\therefore P(E/N) = 2/5.$

$\therefore P(E) = P(M)P(E/M) + P(N)P(E/N)$

$= 7/12 \times 3/5 + 5/12 \times 2/5$

$= \frac{21}{60} + \frac{10}{60}$

$= \frac{31}{60}$ (Ans).

$\therefore P(\bar{E}) = 1 - \frac{31}{60}$

$= \frac{29}{60}.$

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3) Total no. of possible outcomes =

$\{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\} = 8.$

Total no. of favourable outcomes = 6.

$\therefore$ P(game will end after first round) $= \cancel{6}^{3}/\cancel{8}_{4} = 3/4$ (Ans).

Now,

$P(H) = 1/4 \quad P(T) = 3/4.$

Let E be the event where the game will end after the first round.

$\therefore$ Favourable cases for $\bar{E} = \{(HHH), (TTT)\}$.

$\therefore P(HHH) = 1/64.$

$P(TTT) = 27/64.$

$\therefore P(\bar{E}) = 1/64 + 27/64$

$= \cancel{28}^{7}/\cancel{64}_{16} = 7/16$

$\therefore P(E) = 1 - P(\bar{E})$

$= 1 - 7/16$

$= 9/16$ (Ans).

4) $P(A\cap B\cap C)=0$ (given).

$\therefore P(X|C)$

$= P(C)\,P(X\cap C)$

$= P(C)\,P((A\cup B)\cap C)$ [$\because x = A\cup B$]

$= P(C)\,P((A\cap C)\cup(B\cap C))$ [~~Demorgan's~~ Distributive Law]

$= P(C)\,P((A\cap C)+(B\cap C)-\cancel{(A\cap B\cap C)}^{0})$ [given].

$= P(C)\,P((A\cap C)+(B\cap C))$.

$= P(C)\,P(A\cap C)+P(C)\,P(B\cap C)$.

$= P(A|C)+P(B|C)$ [Proved].

5) $P(\text{controller functioning}) = a$.

$P(\text{a peripheral fails}) = b$.

~~System will fail not be "up"~~

System will be up if the controller and at least 2 of the peripherals are functioning.

$\therefore P(\text{System is "up"}) = a\{3(1-b)^2b+(1-b)^3\}$.

$= a(1-b)^2(3b+1-b)$.

$= a(1-b)^2(2b+1)$ (Ans).

5) Let $x=i$ ~~dep~~ denote the ~~probability~~ event of i no. of peripherals ~~too~~ functioning.

$P(\text{a peripheral failing}) = b$.

$\therefore P(\text{a peripheral functioning}) = 1-b$.

$\therefore P(x \geq 2) = P(x=2) + P(x=3)$

$= {}^3C_2 (1-b)^2 b + {}^3C_3 (1-b)^3$

$= \frac{3\times\cancel{2}}{\cancel{2}} (1-b)^2 b + (1-b)^3$.

$= 3(1-b)^2 b + (1-b)^3$

$= (1-b)^2 (3+1-b)$.

$= (1-b)^2 (2-b)$

$\therefore P(\text{"up"}) = P(\text{controller functioning}) \cdot P(x \geq 2)$

$= a(1-b)^2 (2-b)$. (Ans).

6) $P(\text{Error}) = 10^{-3}$.

$\therefore$ Let $x=i$ denote the event that error occurs i times.

$\therefore P(x \geq 2) = P(x=2) + P(x=3)$

$= {}^3C_2 (10^{-3})^2 (1-10^{-3}) + \cancel{{}^3C_3}^{1} (10^{-3})^3$

$= 3\times10^{-6}(1-10^{-3}) + 10^{-9}$

$= 10^{-\cancel{3}6}(3 - 3\times10^{-3} + 10^{-3})$

$= 10^{-6}(3 - 2\times10^{-3})$ (Ans).

7) P(defective chip) = $5/100 = 1/20$.

Let $x=i$ be the event that i chips are not defective.

Let the no. of chips bought be 8.

$\therefore P(x \geq 8) = P(x=8)$

$= {}^8C_8 (1-\frac{1}{20})^8 (1/20)^0$

$= \left(\frac{19}{20}\right)^8$

$\approx 0.6634, < 0.9$.

So, more ~~that~~ than 8 chips are needed.

Let the no. of chips bought be 9.

$\therefore P(x \geq 8) = P(x=8) + P(x=9)$

$= {}^9C_8 (1-1/20)^8 (1/20)^1 + {}^9C_9 (1-1/20)^9$

$= 9 \times \left(\frac{19}{20}\right)^8 \cdot \frac{1}{20} + \left(\frac{19}{20}\right)^9$

$= 0.2985 + 0.6302$

$= 0.9287 > 0.9$

Hence, the probability of getting 8 ~~ch~~ good chips will be higher than 0.9 if he buys 9 chips.

8)

x	1	2	3	4	5	6	7
$P(x=x_i)$	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

i) $P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)$
$+ P(x=7) = 1$.

$\Rightarrow K+2K+2K+3K+K^2+2K^2+7K^2+K = 1$.

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$\Rightarrow 9K + 10K^2 = 1.$

$\Rightarrow 10K^2 + 9K - 1 = 0.$

$\Rightarrow \cancel{K = \dfrac{-9 \pm \sqrt{81+40}}{20}}$

$\cancel{= \dfrac{-9 \pm 11}{20}}$

$\Rightarrow 10K^2 + 10K - K - 1 = 0.$

$\Rightarrow 10K(K+1) - 1(K+1) = 0.$

$\Rightarrow (10K-1)(K+1) = 0.$

$\Rightarrow K = 1/10$ or $K = -1.$

$\because$ Probability can't be negative

$\therefore K = 1/10$ (Ans).

ii) $P(x < 6) = P(1) + P(2) + P(3) + P(4) + P(5)$

$= K + 2K + 2K + 3K + K^2.$

$= 8K + K^2$

$= \dfrac{8}{10} + \dfrac{1}{100}$

$= \dfrac{81}{100}$ (Ans).

$P(x \geq 6) = P(6) + P(7)$

$= 2K^2 + 7K^2 + K$

$= 9K^2 + K$

$= \dfrac{9}{100} + \dfrac{1}{10} = \dfrac{19}{100}$ (Ans).

$\because$ x can't be greater than 7.

$\therefore P(x > 7) = 0.$

$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4).$

$= K + 2K + 2K + 3K$

$= 8K$

$= \frac{\cancel{8}^{4}}{\cancel{10}_{5}} = 4/5$ (Ans).

iii) $P(x \leq 1) = 1/10$

$P(x \leq 2) = \frac{3}{10}$

$P(x \leq 3) = 5/10 = 1/2$

$\therefore$ for $P(x \leq a) > 1/2$, the minimum value of a ~~sho~~ should be 4.

$P(x \leq 4) = 8/10 > 1/2.$

v) $E(x) = 1.K + 2.(2K) + 3(2K) + 4(3K) + 5(K^2) + 6(2K^2) + 7(7K^2 + K)$

$= K + 4K + 6K + 12K + 5K^2 + 12K^2 + 49K^2 + 7K$

$= 30K + 66K^2.$

$= \frac{30}{10} + \frac{66}{100}$

$= \frac{366}{100}$

$= 3.66$ (Ans).

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$$var(x) = E(x^2) - \{E(x)\}^2$$

$$= 1K + 4(2K) + 9(2K) + 16(3K) + 25(K^2) + 36(2K^2) + 49(7K^2 + K) - (3.66)^2$$

$$= K + 8K + 18K + 48K + 25K^2 + 72K^2 + 343K^2 + 49K - (3.66)^2$$

$$= 124K + 440K^2 - (3.66)^2$$

$$= \frac{124}{10} + \frac{440}{100} - (3.66)^2$$

$$= \frac{168}{10} - (3.66)^2$$

$$= 3.4 \text{ (Ans)}.$$

9) $f(x) = \begin{cases} x & : 0 < x < 1 \\ k - x & : 1 < x < 2 \\ 0 & : \text{otherwise}. \end{cases}$

if $f(x)$ is a probability density function, then

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{0} \cancel{0}\,dx + \int_{0}^{1} x\,dx + \int_{1}^{2} (k-x)\,dx + \int_{2}^{\infty} \cancel{0}\,dx = 1.$$

$$\Rightarrow \frac{1}{2}[x^2]_0^1 + \left[kx - \frac{x^2}{2}\right]_1^2 = 1.$$

$$\Rightarrow \frac{1}{2} + \left\{\left(2k - 2\right) - \left(k - \frac{1}{2}\right)\right\} = 1.$$

$\Rightarrow \cancel{\frac{1}{2}} + 2k - 2 - k + \frac{1}{2} = \cancel{1}$.

$\Rightarrow k = 2$.

$\therefore f(x)$ is a density function for $k=2$. (Ans).

The probability distribution function of x is given by,

$$P(X \le x) = \begin{cases} 0, & \text{for } x \le 0 \\ \int_0^{x} x\,dx, & \text{for } 0 < x \le 1 \\ \frac{1}{2} + \int_1^{x} (2-x)\,dx, & \text{for } 1 < x \le 2 \\ 1, & \text{for } x > 2. \end{cases}$$

$$P\left(\tfrac{1}{2} < x < \tfrac{3}{2}\right) = \int_{1/2}^{3/2} f(x)\,dx.$$

$$= \int_{1/2}^{1} x\,dx + \int_{1}^{3/2} (2-x)\,dx.$$

$$= \frac{1}{2}\left[x^2\right]_{1/2}^{1} + \left[2x - \frac{x^2}{2}\right]_{1}^{3/2}$$

$$= \frac{1}{2}\left(1 - \frac{1}{4}\right) + \left\{\left(3 - \frac{9}{8}\right) - \left(2 - \frac{1}{2}\right)\right\}$$

$$= \frac{1}{2}\cdot\frac{3}{4} + 3 - \frac{9}{8} - 2 + \frac{1}{2}$$

$$= \frac{3}{4} \text{ (Ans)}.$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx.$$

$$= \int_0^1 x^2\, dx + \int_1^2 (2x - x^2)\, dx.$$

$$= \frac{1}{3}\left[x^3\right]_0^1 + \left[x^2 - \frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{3} + \left\{\left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)\right\}.$$

$$= \frac{1}{3} + 4 - \frac{8}{3} - 1 + \frac{1}{3}.$$

$$= 3\, \cancel{} - \cancel{\frac{6}{3}}\,2$$

$$= 1.$$

$$Var(x) = E(x^2) - \{E(x)\}^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - 1.$$

$$= \int_0^1 x^3\, dx + \int_1^2 (2x^2 - x^3)\, dx - 1$$

$$= \frac{1}{4}\left[x^4\right]_0^1 + \left[\frac{2}{3}x^3 - \frac{x^4}{4}\right]_1^2 - 1.$$

$$= \frac{1}{4} + \left\{\frac{16}{3} - \frac{16}{4} - \frac{2}{3} + \frac{1}{4}\right\} - 1.$$

$$= \frac{1}{2} + \frac{14}{3} - 4 - 1.$$

$$= \frac{3 + 28 - 24 - 6}{6} = \frac{1}{6} \text{ (Ans)}.$$

10) $f(x) = ce^{-|x|}, \ -\infty < x < \infty.$

$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1.$

$\Rightarrow \int_{-\infty}^{\infty} ce^{-|x|}dx = 1.$

$\Rightarrow \int_{-\infty}^{0} c.e^{x}dx + \int_{0}^{\infty} c.e^{-x}dx = 1.$

$\Rightarrow c\,[e^{x}]_{-\infty}^{0} - c[e^{-x}]_{0}^{\infty} = 1.$

$\Rightarrow c[1-0] - c[0-1] = 1.$

$\Rightarrow 2c = 1.$

$\Rightarrow c = \frac{1}{2}$ (Ans).

10) $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \frac{1}{2}\int_{-\infty}^{0} xe^{x}\,dx + \frac{1}{2}\int_{0}^{\infty} xe^{-x}\,dx$

$= \frac{1}{2}[xe^{x} - e^{x}]_{-\infty}^{0} + \frac{1}{2}[-xe^{-x} - e^{-x}]_{0}^{\infty}$

$= \frac{1}{2}[(-1) - 0] + \frac{1}{2}[0 - (0-1)].$

$= -\frac{1}{2} + \frac{1}{2}$

$= 0$ (An).

$$Var(x) = \frac{1}{2}\int_{-\infty}^{0} x^2 e^x \, dx + \frac{1}{2}\int_{0}^{\infty} x^2 e^{-x} \, dx - 0.$$

$$= \frac{1}{2}\left[e^x(x^2 - 2x + 2)\right]_{-\infty}^{0} + \frac{1}{2}\left[e^{-x}(x^2 + 2x + 2)\right]_{0}^{\infty}$$

$$= \frac{1}{2}\{(2) - 0\} + \frac{1}{2}\{(0) - (2)\}.$$

$$= 1 - 1$$

$$= 0 \text{ (Ans).}$$

iii) P(a microprocessor is active after t sec) $= e^{-\lambda t}$.

$\therefore$ P(a microprocessor is not active after t sec) $= 1 - e^{-\lambda t}$

~~As~~ For the system to fail, all the microprocessors have to stop working after t secs.

$\therefore$ P(~~3 mi~~ all microprocessors not working after t sec)

$$= (1 - e^{-\lambda t})^3.$$

$\therefore$ P(system not operating after t sec) $= (1 - e^{-\lambda t})^3$

$\therefore$ P(system still operating after t secs) $= 1 - (1 - e^{-\lambda t})^3$.

12) 1 min $\rightarrow$ 6000 req.

$\Rightarrow$ 60 secs $\rightarrow$ 6000 req.

$\Rightarrow$ 1 sec $\rightarrow$ 100 req.

$\Rightarrow$ 1000 ~~secs~~ ms $\rightarrow$ 100 req.

$\Rightarrow$ 100 ms $\rightarrow$ 10 req.

$\therefore \lambda = 10$ requests / 100 ms.

a) $P(x=0) = \dfrac{e^{-10} \times 10^0}{0!}$

$= e^{-10}$ (Ans).

b) $P(5 \geq x \geq 10) = P(x=5) + P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$.

$$= \frac{e^{-10} \times 10^5}{5!} + \frac{e^{-10} \times 10^6}{6!} + \frac{e^{-10} \times 10^7}{7!} + \frac{e^{-10} \times 10^8}{8!} + \frac{e^{-10} \times 10^9}{9!} + \frac{e^{-10} \times 10^{10}}{10!}$$

(Ans).

13) a) P(failure of each drive) $= 10^{-3}$

$\therefore$ P(No. of drives ~~to~~ that will fail per day) $= \lambda = 10^{-3} \times 10^{4}$

$= 10.$

$$\therefore P(x=0) = \frac{e^{-10} . 10^{0}}{0!}$$

$$= e^{-10} \text{ (Ans).}$$

b) P(No. of drives that will fail in 2 days) $= \lambda_2 = 2 \times 10 = 20.$

$\therefore P(x<10) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$
$+ P(x=5) + P(x=6) + P(x=7) + P(x=8)$
$+ P(x=9).$

$$= \sum_{k=0}^{9} \frac{e^{-20} . 20^{k}}{k!} \text{ (Ans).}$$

iii) $y = aV + N$

$a = 10^{-2}$

$\mu = 0.$

$\sigma = 2.$

$\therefore P(y < 0) = 10^{-6}$

$\Rightarrow P(aV + N < 0) = 10^{-6}$

$\Rightarrow P(N < -aV) = 10^{-6}$

$\Rightarrow P\left(Z < \frac{-aV - \mu}{\sigma}\right) = 10^{-6}$

$\Rightarrow \phi\left(\frac{-aV - \mu}{\sigma}\right) = 10^{-6}$

$\Rightarrow 1 - \phi\left(\frac{aV + \mu}{\sigma}\right) = 10^{-6}$

$\Rightarrow \phi\left(\frac{aV + \mu}{\sigma}\right) = 1 - 10^{-6}$

$= 1 - 0.000001$

$= 0.999999$

$= \phi(3.62)$

$\Rightarrow aV = 3.62 \times 2 = 7.24$

$\Rightarrow V = 7.24 \times 10^{2}$

$= 724V.$ (Ans).

15) Let x denote the length of the shower.

$\therefore E(x) = \mu = \frac{1}{\lambda} = \frac{1}{2}$

$\therefore \lambda = 2.$

$\therefore f(x) = \lambda e^{-\lambda x} \; ; x \geq 0$

$= 2e^{-2x} \; ; x \geq 0.$

$\therefore P(x \geq 3) = \int_3^\infty 2.e^{-2x} dx.$

$= 1 - \int_0^3 2e^{-2x} dx.$

$= 1 + \frac{\cancel{2}}{\cancel{+2}} \left[e^{-2x} \right]_0^3$

$= 1 + \{ e^{-6} - 1 \}$

$= e^{-6}$ (Ans).

$\therefore P(x \geq 3 | \cancel{P} x \geq 2) = \frac{P(x \geq 3) \cap P(x \geq 2)}{P(x \geq 2)}$

$= \frac{P(x \geq 3)}{P(x \geq 2)}$

$= \frac{e^{-2.3}}{e^{-2.2}}$

$= \frac{e^{-6}}{e^{-4}}$

$= e^{-2}$ (Ans).

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16)

Let the distance from P on AB be x.

$\therefore AP = x$

$\Rightarrow PB = 2a - x$.

$\therefore$ Area of rectangle $= x(2a-x)$

By the Problem,

$x(2a-x) \geq a^2/2$

$\Rightarrow 2ax - x^2 \geq a^2/2$

$\Rightarrow x^2 - 2ax + a^2/2 \leq 0$

$\Rightarrow x^2 - 2ax + a^2 - a^2/2 \leq 0$

$\Rightarrow (x-a)^2 \leq a^2/2$.

$\Rightarrow -\frac{a}{\sqrt{2}} \leq x - a \leq \frac{a}{\sqrt{2}}$.

$\Rightarrow a - \frac{a}{\sqrt{2}} \leq x \leq a + \frac{a}{\sqrt{2}}$.

A point on a line follows a uniform distribution over 0 to $2a$.

$$\therefore P\left(a - \frac{a}{\sqrt{2}} \leq x \leq a + \frac{a}{\sqrt{2}}\right) = \frac{1}{2a}\int_{a-a/\sqrt{2}}^{a+a/\sqrt{2}} dx.$$

$$= \frac{1}{2a}\left[\left(a + a/\sqrt{2}\right) - \left(a - a/\sqrt{2}\right)\right],$$

$$= \frac{1}{2\cancel{a}}\left[2 \cdot \frac{\cancel{a}}{\sqrt{2}}\right).$$

$$= \frac{1}{\sqrt{2}} \text{ (Ans)}.$$

19)

Class	Frequency
0-10	14
10-20	22
20-30	27
30-40	x
40-50	23
50-60	20
60-70	15

Mode = 25

$\therefore$ Modal class = 20 - 30

$(l-u) = 10$

$f_0 = 22$

$f_1 = 27$

$f_2 = x$

$l_m = 20$

$$\therefore \text{Mode} = l_m + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times (l-u)$$

$$\Rightarrow 20 + \frac{27-22}{27\times 2 - 22 - x} \times 10 = 25.$$

$$\Rightarrow \frac{50}{32-x} = 5.$$

$\Rightarrow 50 = 5(32-x)$

$= 160 - 5x.$

$\Rightarrow 5x = 110$

$\Rightarrow x = 22.$ (Ans).

20)

Class	cumu. freq.	Freq.
0-10	3	3
10-20	8	5
20-30	17	9
30-40	20	3
40-50	22	2

$\therefore \Sigma f = 22.$

$\therefore \frac{\Sigma f}{2} = 11$

$\therefore$ Median class = 20-30.

$\therefore cf_{i-1} = 5.$ $\quad \therefore l_m = 20$

$f_i = 9.$ $\quad \therefore l-u = 10$

$\therefore \text{Median} = l_m + \frac{\Sigma f/2 - cf_{i-1}}{f_i} \times (l-u).$

$= 20 + \frac{11-5}{9} \times (10).$

$= 20 + \frac{\cancel{60}\,20}{\cancel{9}\,3}$

$= \frac{80}{3} = 26.67$ (Ans).

$\therefore$ Modal class = 20-30

$\therefore \text{Mode} = l_m + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times (l-u)$

$= 20 + \frac{9-5}{18-5-3} \times 10$

$= 20 + \frac{4\cancel{0}}{1\cancel{0}} = 24$ (Ans).

21) $f_x = \begin{cases} 0 ; & x < x_m \\ \alpha \dfrac{x_m^{\alpha}}{\alpha+1} ; & x \geq x_m . \end{cases}$ [Taking $\alpha \cdot \dfrac{x_m^{\alpha}}{x^{\alpha+1}} ; x \geq x_m$]

$\therefore$ ~~Mean~~ $E(x) = \displaystyle\int_{-\infty}^{\infty} x f(x)\, dx.$

$= \displaystyle\int_{x_m}^{\infty} x \cdot \frac{\alpha}{x^{\alpha+1}} \cdot x_m^{\alpha}\, dx.$

$= \alpha \cdot x_m^{\alpha} \displaystyle\int_{x_m}^{\infty} x^{-\alpha}\, dx$

$= \alpha x_m^{\alpha} \dfrac{\left[x^{-\alpha+1}\right]_{x_m}^{\infty}}{1-\alpha}.$

$= \dfrac{\alpha x_m^{\alpha}}{1-\alpha} \cdot \left[0 - x_m^{-\alpha+1}\right]$ ~~for $\alpha>1$~~,

$= -\dfrac{\alpha \cdot x_m}{1-\alpha}$ ~~(Ans)~~ for $\alpha > 1$, or ∞ for $\alpha < 1$ (Ans).

$\therefore \text{Var}(x) = \displaystyle\int_{-\infty}^{\infty} x^2 f(x)\, dx. - \frac{\alpha x_m}{1-\alpha}$

$= \dfrac{\alpha}{\cancel{\alpha+1}} \cdot x_m^{\alpha} \displaystyle\int_{x_m}^{\infty} x^{1-\alpha}\, dx. - \frac{\alpha x_m}{1-\alpha}$

$= \alpha x_m^{\alpha} \dfrac{\left[x^{2-\alpha}\right]_{x_m}^{\infty}}{2-\alpha} - \dfrac{\alpha x_m}{1-\alpha}$

$= \dfrac{\alpha}{2-\alpha} x_m^{\alpha} \left\{0 - x_m^{2-\alpha}\right\} - \dfrac{\alpha x_m}{1-\alpha},$ for $\alpha > 2$

or, ∞, for $\alpha \leq 2$

$$= -\frac{\alpha x_m^2}{2-\alpha} - \frac{\alpha x_m}{1-\alpha} (\text{Ans}).$$

$$= \frac{-\alpha x_m^2(1-\alpha) - \alpha(x_m)(2-\alpha)}{(2-\alpha)(1-\alpha)}$$

$$= \frac{-\alpha x_m^2 + \alpha^2 x_m^2 - 2\alpha x_m + \alpha^2 x_m}{(2-\alpha)(1-\alpha)}.$$

22) In exponential function, $P(x \geq t) = e^{-\lambda t}$.

$$\therefore P(x \geq a+b \mid x \geq a) = \frac{P(x \geq a+b) \cap P(x \geq a)}{P(x \geq a)}$$

$$= \frac{P(x \geq a+b)}{P(x \geq a)} \quad [\because a \,\&\, b \text{ are both true int}].$$

$$= \frac{e^{-\lambda(a+b)}}{e^{-\lambda a}}$$

$$= e^{-\lambda b}$$

$$= P(x \geq b) \text{ [Hence shown]}.$$

23) $N = 400$

i) $P(x=200) = {}^{400}C_{200} (\frac{1}{2})^{200} (\frac{1}{2})^{200}$

≈ 0.5. (Ans).

ii) $P(190 \leq x \leq 210) = \sum_{i=190}^{210} {}^{400}C_i (\frac{1}{2})^{400}$. (Ans).

24) $P(\text{head}) = p$

$\therefore \cancel{E(\text{head}) = p + (1-p)p + (1-p)^2 p + (1-p)^3 p + \cdots}$

$\cancel{= \frac{p}{1-(1-p)}}$

No. of turns.	Probability of head.
1	p
2	$p(1-p)$
3	$p(1-p)^2$
4	$p(1-p)^3$
⋮	⋮

$\therefore E[\text{no. of turns}] = 1 \cdot p + 2p(1-p) + 3p(1-p)^2 + 4p(1-p)^3 + \cdots$

$\therefore E(1-p) = p(1-p) + 2p(1-p)^2 + 3p(1-p)^3 + 4p(1-p)^4 + \cdots$

$\therefore E - E \cdot (1-p) = p + p(1-p) + p(1-p)^2 + p(1-p)^3 + \cdots$

$= \frac{p}{1-(1-p)} = \frac{p}{p} = 1.$

$\Rightarrow E.(1-1+p) = 1.$

$\Rightarrow E = 1/p$ (Ans).

25) The possibility of order of opening of doors are as follows $\rightarrow D_1$

$D_2 \rightarrow D_1$

$D_3 \rightarrow D_1$

$D_2 \rightarrow D_3 \rightarrow D_1$

$D_3 \rightarrow D_2 \rightarrow D_1$.

$\therefore$ ~~Expected~~ time =

25) ~~E(D1)~~ = Let X be the event that the miner reaches safety.

$\therefore E(X|D1) = 2$

$E(X|D2) = 5+E(X)$

$E(X|D3) = 7+E(X)$.

$P(D_1) = P(D_2) = P(D_3) = 1/3$.

$\therefore E(X) = P(D_1)E(X|D1) + P(D_2)E(X|D_2) + P(D_3)E(X|D_3)$

$= \frac{1}{3}[2+5+E(x)+7+E(x)]$

$= \frac{1}{3}[14+2E(x)]$

$\Rightarrow E(x) = 14$ hours (Ans).