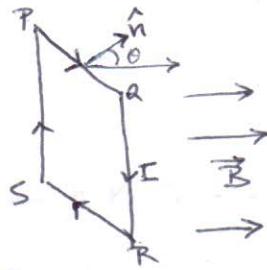


## Magnetic Properties

### 1. The Magnetic Dipole Moment ( $\mu_m$ )



1. **Rectangular**

Current carrying loop PQRS carrying a (steady) current  $I$  as shown in Figure 1, with  $PQ (=RS) = a$  &  $QR (=SP) = b$ . It is placed in a uniform magnetic field  $\vec{B}$  so that the sides QR and SP are perpendicular to  $\vec{B}$ .

Figure 1.

$\hat{n}$  - unit normal to the surface PQRS as shown. Wrap your fingers in the direction of current flow - your extended thumb points towards  $\hat{n}$ . Angle between  $\hat{n}$  and  $\vec{B}$  -  $\theta$ .

Since  $I$  is steady and  $\vec{B}$  is uniform, the net force on the loop is

$$\vec{F} = \int I(d\vec{l} \times \vec{B}) \equiv I(\oint d\vec{l}) \times \vec{B} = 0 \quad (1)$$

But there is a torque acting on the loop.

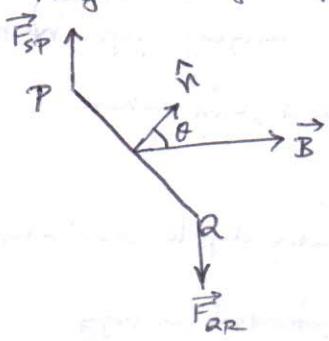
Force on side QR :  $\vec{F}_{QR} = I \vec{QR} \times \vec{B} \rightarrow \perp$  to page coming out

Force on side SP :  $\vec{F}_{SP} = I \vec{SP} \times \vec{B} (= -I \vec{QR} \times \vec{B}) \rightarrow \perp$  to page going in

$\vec{F}_{QR}$  and  $\vec{F}_{SP}$  (with  $\vec{F}_{QR} + \vec{F}_{SP} = 0$ ) form a couple.

(• Students: Verify that  $\vec{F}_{PQ} + \vec{F}_{RS} = 0$  but  $\vec{F}_{PQ}$  and  $\vec{F}_{RS}$  do NOT form a couple.)

The torque on the rectangular loop due to  $\vec{F}_{QR}$  and  $\vec{F}_{SP}$  will try to rotate the loop clockwise (when viewed along the axis of the loop from the top) so as to reduce  $\theta$  and bring  $\hat{n}$  along  $\vec{B}$ . (At  $\theta=0$ , lines of action of  $\vec{F}_{QR}$  and  $\vec{F}_{SP}$  match and they no longer form a couple.)



When viewed along the axis of the loop from the top,  $\vec{F}_{SP}$  and  $\vec{F}_{QR}$  are as shown in Figure 2.

$$|\vec{F}_{SP}| = |\vec{F}_{QR}| = I b B \sin 90^\circ = I b B \quad (3)$$

Torque on the current carrying loop due to  $\vec{B}$  is

$$\vec{\tau}_B = \vec{PQ} \times \vec{F}_{QR} \quad (\text{or } \vec{SP} \times \vec{F}_{SP}) \quad (4)$$

Direction of  $\vec{\tau}_B$  - along  $\vec{QR}$  in Figure 1

- perpendicular to the page and pointing into the page in Figure 2

• Students: Make sure that you understand this.

Then,

$$\vec{\tau}_B = |\vec{\tau}_B| = |PQ||F_{\text{ext}}| \sin \theta = a \cdot IbB \sin \theta$$

$$\Rightarrow \vec{\tau}_B = I(ab)B \sin \theta = IA B \sin \theta \quad (5)$$

where,

$A = ab$  is the area of the loop.

(For  $N$  turns in the loop, each turn carrying a current  $i$ , the equation becomes  $\vec{\tau} = NiA B \sin \theta$   
 $\Rightarrow \vec{\tau} = IAB \sin \theta, I = Ni$ )

We now define a vector  $\vec{\mu}_m$  such that

$$\vec{\mu}_m = IA \hat{n}, \quad (6)$$

then

$$\mu_m = |\vec{\mu}_m| = IA, \quad (7)$$

and Eq.(5) becomes,

$$\vec{\tau}_B = \mu_m B \sin \theta \quad (8)$$

In vector form, Eq.(8) can be written as,

$$\vec{\tau}_B = \vec{\mu}_m \times \vec{B}, \quad (9)$$

as it is clear that the cross product  $\vec{\mu}_m \times \vec{B}$  consistently gives both the magnitude (see Eq.(3)) and the direction of the torque.

Recall that the torque on an electric dipole of dipole moment  $\vec{p}$  in an electric field  $\vec{E}$  is given by

$$\vec{\tau}_E = \vec{p} \times \vec{E} \quad (10)$$

We call  $\vec{\mu}_m$  as the magnetic dipole moment of the current carrying loop. In other words, we refer to the current carrying loop as a magnetic dipole of moment  $\vec{\mu}_m$  where

$$\mu_m = IA.$$

The effect of the torque  $\vec{\tau}_B$  is to rotate the magnetic dipole and align it with  $\vec{B}$ . Consequently the dipole has an orientational potential energy given by

$$U = -\vec{\mu}_m \cdot \vec{B} \quad (11)$$

- Students - Prove Eq.(11). Where is the 'zero' of the potential energy? Does this choice matter? Why or why not?

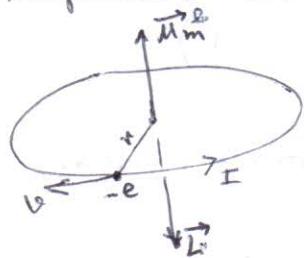
\* In general, it can be shown that

$\vec{\mu}_m = IA \hat{A}$  for a loop carrying current  $I$  and having the "vector area"  $\hat{A}$ . If the loop is flat (as considered in Figure),  $\hat{A}$  is the ordinary area enclosed, with the direction assigned by the usual right-hand rule (fingers in the direction of the current).

## 2. Orbital and Spin Magnetic Dipole Moments ( $\vec{\mu}_m$ and $\vec{\mu}_s$ respectively):

The current loops in an atom are composed of rotating electrons. In that case we can establish a simple relation between the magnetic dipole moment  $\vec{\mu}_m$  that results from a rotating electron and its <sup>orbital</sup> angular momentum  $\vec{L}$ : Our derivation will be based on classical physics. Later on, results from quantum mechanics will be used to further develop the ideas. This procedure is justified by the fact that the final results agree with those of completely quantum mechanical treatments.

Consider an electron of mass 'm' and charge '-e' moving with velocity of magnitude 'v' in a circular Bohr orbit of radius 'r' as shown in Figure 3. The charge circulating in a loop constitutes a current of magnitude 'I', where



$$I = \frac{e}{T}, \quad (1)$$

where  $T$  = orbital period of the electron. Then,

$$vT = 2\pi r, \text{ i.e., } T = \frac{2\pi r}{v} \quad (2)$$

$$\therefore I = \frac{e}{T} = \frac{ev}{2\pi r} \quad (3)$$

Figure 3

The area of the loop is

$$A = \pi r^2 \quad (4)$$

Then the magnitude of the magnetic moment due to orbital motion  $\mu_m^l$  of the equivalent magnetic dipole is [See Eq.(7), Section 1],

$$\mu_m^l = I \cdot A = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2} \quad (5)$$

Because the electron has a negative charge, its orbital magnetic dipole moment  $\vec{\mu}_m^l$  is antiparallel to its orbital angular momentum  $\vec{L}$ , whose magnitude is given by

$$L = mv \times r \quad (6)$$

and whose direction is illustrated by Figure 3. From (5) and (6),

$$\frac{\mu_m^l}{L} = \frac{evr}{2} \cdot \frac{1}{mv \times r} = \frac{e}{2m}, \quad (7)$$

which is a combination of universal constants. In vector form,

$$\vec{\mu}_m^l = -\frac{e}{2m} \vec{L} \quad (8)$$

Now, from (7),

$$M_m^l = \frac{e}{2m} L \quad (9)$$

The quantum mechanical expression for the orbital angular momentum is given by,

$$L = \sqrt{l(l+1)} \ h \quad (10)$$

where  $l$  - orbital angular momentum quantum number of the electron, and

$$h = \frac{h}{2\pi}, h - \text{Planck's constant.}$$

From (9) and (10),

$$M_m^l = \frac{e}{2m} \sqrt{l(l+1)} h, \quad (11)$$

which is exactly the expression of  $M_m^l$  that is obtained from a rigorous quantum mechanical treatment. We write (11) as,

$$M_m^l = \frac{eh}{2m} \sqrt{l(l+1)} \equiv \mu_B \sqrt{l(l+1)} \quad (12)$$

where,

$$\mu_B = \frac{eh}{2m} = 0.927 \times 10^{-23} \text{ ampere-m}^2 \quad (13)$$

The quantity  $\mu_B$  forms a natural unit for the measurement of atomic magnetic dipole moments, and is called the Bohr magneton. Equation (11) has, of course, been backed by experimental evidence.

Experimental results also lead to the conclusion that an electron has an intrinsic (built-in) magnetic dipole moment  $\vec{\mu}_m^s$ , due to the fact that it has an intrinsic angular momentum  $\vec{S}$  called its spin. The magnitude  $S$  of the spin angular momentum is given by the quantization relation:

$$S = \sqrt{s(s+1)} h \quad (14)$$

where  $s = \frac{1}{2}$  is known as the spin quantum number. The spin magnetic dipole moment and the spin angular momentum are related as follows:

$$\vec{\mu}_m^s = -\frac{e}{m} \vec{S} \quad (15)$$

Recall from Eq. (8) that  $\vec{M}_m^l = -\frac{e}{2m} \vec{L}$ . This equation was obtained using

classical measures. Spin, however, has no classical analogue. Its origin is completely quantum mechanical. From (14) and (15), the magnitude  $M_m^s$  is given by,

$$M_m^s = \frac{e}{m} \sqrt{s(s+1)} \ h = 2 \left( \frac{e h}{2m} \right) \sqrt{s(s+1)} \ h = 2\mu_B \sqrt{s(s+1)} \quad (16)$$

Note: The z-components of  $\vec{L}$  and  $\vec{S}$  are also quantized; they are respectively given by.

$$L_z = m_l h \quad \& \quad S_z = m_s h$$

For a given  $l$ ,  $m_l$  goes from  $-l$  to  $+l$  in integral steps;  $m_s = \pm \frac{1}{2}$ . Thus the z-components of the orbital and spin magnetic dipole moments are also quantized - using Eqs. (8) and (15), their magnitudes are respectively given by

$$M_m^{lz} = \frac{e}{2m} L_z = \frac{e h}{2m} m_l = \mu_B m_l \quad \& \quad M_m^{sz} = \frac{e}{m} S_z = 2 \cdot \frac{e h}{2m} \cdot m_s = 2\mu_B m_s \quad (18)$$

For further details and a more exact treatment, see Resnick & Eisberg, Chapter 8.

More precisely, Eqs. (11) and (16) are respectively written as,

$$M_m^l = g_e \mu_B \sqrt{l(l+1)} \quad (19)$$

$$\& \quad M_m^s = g_s \mu_B \sqrt{s(s+1)} \quad (20)$$

where

$$g_e = 1, \quad (21)$$

is known as the orbital g factor; and

$$g_s = 2, \quad (22)$$

is known as the spin g factor. (Experiments have shown that the actual value of the spin g factor is  $g_s = 2.00232$ , but (22) is adequate for most purposes.)

The orbital angular momentum and spin may be combined vectorially to give the total angular momentum  $\vec{J}$ , i.e.,

$$\vec{J} = \vec{L} + \vec{S} \quad (23)$$

The magnitude of  $\vec{J}$  is also quantized according to the usual condition,

$$J = \sqrt{j(j+1)} \ h, \quad (24)$$

where 'j' is the total angular momentum quantum number of the electron.

If the electron has a certain ' $l$ ', then  $j$  can accept the values  $l \pm \frac{1}{2}$ , i.e.,

$$j = l + \frac{1}{2} \quad \text{or} \quad l - \frac{1}{2} \quad (25)$$

(since  $s = \frac{1}{2}$ ). We next consider multi-electron atoms.

Consider an atom containing a number of electrons — typically such an atom contains a core of completely filled subshells surrounding the nucleus, plus several electrons in a partially filled outer subshell. The orbital angular momentum vectors of all the electrons may be combined to form a resultant  $\vec{L}$ , and the spin angular momentum vectors of all the electrons may be combined to form a resultant  $\vec{S}$  — this is known as LS coupling; it is the only type of coupling that we shall consider. The resultant  $\vec{L}$  and  $\vec{S}$  then combine to form the total angular momentum  $\vec{J}$  of the whole electron system of the atom. Now it follows from Pauli's exclusion principle that when a subshell is completely filled, the only allowed state is one in which the total orbital angular momentum, the total spin angular momentum, and the total angular momentum are all zero for this subshell. So the core of completely filled subshells do not contribute to  $\vec{L}$ ,  $\vec{S}$ , and  $\vec{J}$ ; as a consequence a completely filled subshell has no net magnetic dipole moment. Therefore, only the few electrons in an atom that are not in filled subshells contribute to  $\vec{L}$ ,  $\vec{S}$ , and  $\vec{J}$  and hence to the magnetic moment of the atom. Thus the magnetic moment in atoms must result from incompletely filled shells.

So if  $\vec{S}_1, \vec{S}_2, \vec{S}_3, \dots$  are the individual spin angular momenta of the electrons in the unfilled subshells of the atom, they combine to form a total  $\vec{S}$ , where

$$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \dots \quad (26)$$

Also the individual orbital angular momenta  $\vec{L}_1, \vec{L}_2, \vec{L}_3, \dots$  of the electrons in the unfilled subshells of the atom combine to form a total  $\vec{L}$ , where

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \dots \quad (27)$$

$\vec{L}$  and  $\vec{S}$  subsequently combine to give the total angular momentum  $\vec{J}$ :

$$\vec{J} = \vec{L} + \vec{S} \quad (28)$$

Now according to quantum mechanics,  $\vec{S}$  has a constant magnitude given by:

$$S = \sqrt{S(S+1)} \text{ h}, \quad (29)$$

where  $S$  is the total spin quantum number of the atom as a whole.

Next,  $\vec{L}$  also has a constant magnitude satisfying the quantization condition

$$L = \sqrt{L(L+1)} \ h \quad (30)$$

where  $L$  is the total orbital angular momentum (of all the electrons) of the atom as a whole. <sup>quantum number</sup>

Finally,  $\vec{J}$  also has a constant magnitude and is quantized according to

$$J = \sqrt{J(J+1)} \ h \quad (31)$$

where  $J$  is the total angular momentum quantum number (of all the electrons) of the atom as a whole.

Analogous to Eqs. (19) and (20), the magnitudes of the orbital and spin magnetic dipole moments are given by,

$$\mu_m^L = g_e \mu_B \sqrt{L(L+1)}, \quad [\vec{\mu}_m^L = -g_e \mu_B \frac{\vec{L}}{h}] \quad (32)$$

$$\mu_m^S = g_s \mu_B \sqrt{S(S+1)}, \quad [\vec{\mu}_m^S = -g_s \mu_B \frac{\vec{S}}{h}] \quad (33)$$

where  $g_e = 1$  and  $g_s = 2$ .

The average component of the total magnetic dipole moment along  $\vec{J}$  is given by,

$$\mu_m^T = g \mu_B \sqrt{J(J+1)}, \quad [\vec{\mu}_m^T = -g \mu_B \frac{\vec{J}}{h}] \quad (34)$$

where

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad (35)$$

'g' is called the Landé g factor.

Note:  $\mu_m^T = g \sqrt{J(J+1)} \ \mu_B \equiv \mu_{\text{eff}} \mu_B \quad (34a)$

where  $\mu_{\text{eff}} = g \sqrt{J(J+1)}$  can be calculated from susceptibility measurements.

The values of  $L$ ,  $S$ , and  $J$  for a given atom can be found using a combination of Pauli's exclusion principle and Hund's rules.

The purpose of the above discussion is to convince the students that the origin of magnetic dipole moments <sup>in atoms</sup> is quantum mechanical in nature.

References: i). Solid State Physics by A J Dekker, ii) Quantum Physics by Resnick & Eisberg.

### 3. Diamagnetism and Paramagnetism

Now that we have some understanding of the quantum origins of the magnetic dipole moment of an atom, we will proceed to gain a basic understanding of the magnetic properties of solids. It is convenient to group the magnetic properties of solids under the following headings:

- (i) diamagnetism
- (ii) paramagnetism
- (iii) ferromagnetism, antiferromagnetism, ferrimagnetism

In this and the following sections we shall consider the dia- and paramagnetic behaviour of solids for static applied magnetic fields. The properties corresponding to group (iii) will be discussed in Sections 7 and 8.

Magnetic effects in diamagnetic and paramagnetic materials are typically a thousand to a million times less than the magnetic effects in ferromagnetic materials. Paramagnetic materials are attracted toward magnetic fields while diamagnetic materials are repelled; both attractive and repulsive forces are quite weak. (On the other hand, ferromagnetic materials are strongly attracted by a magnetic field.) Bismuth is one of the strongest diamagnetic materials, but even with it, the effect is still quite weak. Diamagnetism typically is always very weak. Substances like aluminum are called paramagnetic. They are (weakly) attracted by magnetic fields.

We want now to describe briefly the mechanisms of these two effects. First, in many substances the atoms have no permanent magnetic dipole moments, or rather, all the magnets within each atom balance out so that the net magnetic dipole moment of the atom is zero. The electron spins and orbital motions all exactly balance out, so that any particular atom has no average magnetic moment. In these circumstances, when you

turn on a magnetic field little extra currents are generated inside the atom by induction. According to Lenz's law, these currents are in such a direction as to oppose the increasing field. So the induced magnetic moments of the atoms are directed opposite to the magnetic field. This is the mechanism of diamagnetism. The magnetic susceptibility (see Section 4) in this case is therefore negative.

In paramagnetic materials, the atoms do have a permanent magnetic moment - in which the electron spins and orbits have a net circulating current that is not zero. So besides the diamagnetic effect (which is always present) there is also the possibility of lining up the individual permanent atomic magnetic moments with the magnetic field. This is because each permanent atomic magnetic dipole experiences a torque tending to line it up parallel to the field. (The mechanism is similar to the way the permanent dipoles of a dielectric tend to line up with the electric field.) As a consequence, in paramagnetic materials, the induced magnetism tends to enhance the magnetic field. That is, the magnetic susceptibility is positive.

Paramagnetism is generally fairly weak because the lining up forces are relatively small compared with the forces from the thermal motions which try to derange the order. It also follows that paramagnetism is usually sensitive to the temperature. (The paramagnetism arising from the spins of the electrons responsible for conduction in a metal constitutes an exception. We will not be discussing this phenomenon here.) For ordinary paramagnetism, the lower the temperature, the stronger the effect. There is more lining-up at low temperatures when the deranging effects of the collisions are less. We have, in the above, basically given a qualitative description that leads to Curie's law. Diamagnetism, on the other hand, is more or less independent of the temperature. In any substance whose atoms have permanent magnetic dipole moments there is a diamagnetic as well as a paramagnetic effect, but the paramagnetic effect usually dominates.

Now that a qualitative explanation of diamagnetism and paramagnetism has been given, it must be emphasized that magnetic effects are a completely quantum mechanical phenomenon. It is not possible to understand the magnetic effects of materials in any honest way from the point of view of classical physics. However we will mostly follow a classical approach as it does give us some useful insights regarding behavior in magnetic materials.

Reference: The Feynman Lectures on Physics, Volume 2.

#### 4. Magnetization &

The Magnetic Susceptibility Reference: Introduction to Electrodynamics by Griffiths

To review our discussion in Section 3 above, matter becomes magnetized in the presence of a magnetic field. That is, upon microscopic examination, it will be found to contain many tiny dipoles, with a net alignment along some direction. We have discussed two mechanisms that account for this magnetic polarization: (1) paramagnetism (the permanent magnetic dipole associated with the atoms experience a torque tending to line them up parallel to the magnetic field) and (2) diamagnetism (the orbital speed of the electrons is altered in such a way as to change the orbital dipole moment in a direction opposite to the field). Whatever the cause, we describe the state of magnetic polarization by the vector quantity

$\vec{M}$  = magnetic dipole moment per unit volume.

$\vec{M}$  is called the magnetization; it plays a role analogous to the polarization  $\vec{P}$  in electrostatics.

The individual electronic currents of the magnetic dipoles in a magnetized material may be considered to 'add up' resulting in both a bound surface current density,  $\vec{K}_b$ , as well as a bound volume current density,  $\vec{J}_b$ . In other words, the effect of magnetization is to establish  $\vec{J}_b$  within the material and  $\vec{K}_b$  on the surface. It can be shown that  $\vec{J}_b$  and  $\vec{K}_b$  are related to the magnetization  $\vec{M}$  through the equations:

$$\vec{J}_b = \vec{\nabla} \times \vec{M} \quad (1)$$

and

$$\vec{K}_b = \vec{M} \times \hat{n}, \quad (2)$$

$\hat{n}$  - unit normal to the surface, etc. The field due to magnetization of the medium is just the field produced by these bound currents. Equations (1) and (2) can be quite simply obtained from a consideration of the potential due to a single dipole followed by an integration over the entire magnetized material. Interested students can look up Griffiths.

Now according to Ampere's law (of magnetostatics), if  $\vec{B}(\vec{r})$  is the magnetic field at a point whose position vector is  $\vec{r}$  and  $\vec{J}_t(\vec{r})$  is the total volume current density at that point, then

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{J}_t(\vec{r}) \quad (3)$$

$\mu_0$ -permeability of free space

In magnetized materials, contribution to  $\vec{B}(\vec{r})$  (where  $\vec{r}$  is a position vector of a point within the material) will come from the bound currents as well as from the 'free currents'. The free current might flow through wires embedded in the magnetized substance or, if the latter is a conductor, through the material itself. In any event, the total current (density) can be written as

$$\vec{J}_t(\vec{r}) = \vec{J}_b(\vec{r}) + \vec{J}_f(\vec{r}), \quad (4)$$

where  $\vec{J}_f(\vec{r})$  - the free current density at  $\vec{r}$ . The free current is there because somebody hooked up a wire to a battery - it involves actual transport of charge. The bound current is there because of magnetization. From Eqs. (3) and (4),

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_t = \mu_0 [ \vec{J}_b + \vec{J}_f ]$$

$$\Rightarrow \vec{\nabla} \times \vec{B} = \mu_0 [ \vec{\nabla} \times \vec{M} + \vec{J}_f ] , \text{ from (1)}$$

i.e.,  $\vec{\nabla} \times \left[ \frac{\vec{B}}{\mu_0} - \vec{M} \right] = \vec{J}_f \quad (5)$

We define the quantity  $\vec{H}$  as,

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}. \quad (6)$$

In terms of  $\vec{H}$ , then, Ampere's law reads

$$\vec{\nabla} \times \vec{H} = \vec{J}_f \quad (7)$$

$\vec{H}$  is sometimes called the magnetic field intensity. (Students, you must note that  $\vec{B}$  is indisputably the fundamental quantity. Can you explain why?)  $\vec{H}$  permits us to express Ampere's law in terms of the free current alone - and free current is what we control directly.  $\vec{H}$  plays a role in magnetostatics analogous to  $\vec{D}$  in electrostatics.  $\vec{D}$  originates only with free charges, while  $\vec{H}$  is associated with free currents only. Bound currents, like bound (i.e. polarization) charges, come along for the ride - the material gets magnetized, and this results in bound currents; we cannot turn them on or off independently, as we can free currents.

Now from Eq. (6),

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 (\vec{H} + \vec{M}) \quad (8)$$

In paramagnetic and diamagnetic materials, the magnetization is sustained by the magnetic field  $\vec{B}$ ; when  $\vec{B}$  is removed,  $\vec{M}$  disappears. In fact, for most substances the magnetization is proportional to the field, provided the field is not too strong. Custom dictates that  $\vec{M}$  is written in terms of  $\vec{H}$ , instead of  $\vec{B}$ :

$$\vec{M} = \chi_m \vec{H} \quad (9)$$

The constant of proportionality  $\chi_m$  is called the magnetic susceptibility; it is a dimensionless quantity that varies from one substance to another - positive for paramagnets and negative for diamagnets.

In the last section, it was mentioned that typically both diamagnetism and paramagnetism were weak. This shows up in the generally low values of diamagnetic and paramagnetic susceptibilities - typical values are around  $10^{-5}$ . (see table below)

Diamagnetic	Susceptibility	Paramagnetic	Susceptibility
Bismuth	$-1.7 \times 10^{-4}$	Sodium	$8.5 \times 10^{-6}$
Cold	$-3.4 \times 10^{-5}$	Aluminum	$2.2 \times 10^{-5}$
Copper	$-9.7 \times 10^{-6}$	Platinum	$2.7 \times 10^{-4}$
Water	$-9.0 \times 10^{-6}$	Liquid Oxygen ( $-200^{\circ}\text{C}$ )	$3.9 \times 10^{-3}$

Materials that obey Eq.(9) are called linear media. From (8) and (9),

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \chi_m \vec{H}) = \mu_0 (1 + \chi_m) \vec{H}, \quad (10)$$

for linear media. Thus  $\vec{B}$  is also proportional to  $\vec{H}$ . We write,

$$\vec{B} = \mu \vec{H} \quad (11)$$

where

$$\mu = \mu_0 (1 + \chi_m). \quad (12)$$

$\mu$  is called the permeability of the material. In a vacuum, where there is no matter to magnetize, the susceptibility  $\chi_m$  vanishes (so that  $\vec{M} = 0$ ), and the permeability is  $\mu_0$ . That's why  $\mu_0$  is called the permeability of free space. So in vacuum,  $\mu = \mu_0$ , and,

$$\vec{B} = \mu_0 \vec{H} \quad (\text{vacuum}) \quad (13)$$

Sometimes Eq.(12) is written as,

$$\mu = \mu_0 \mu_r \quad (14)$$

where,

$$\mu_r = 1 + \chi_m, \quad (15)$$

is called the relative permeability. From Eqs.(8) and (9), we can also write,

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 \left( \frac{\vec{M}}{\chi_m} + \vec{M} \right) = \frac{\mu_0}{\chi_m} (1 + \chi_m) \vec{M},$$

$$\vec{M} = \frac{\chi_m}{\mu_0 (1 + \chi_m)} \vec{B} = \frac{\chi_m}{\mu} \vec{B} \quad (16)$$

You can also get this equation using Eqs.(8) and (11).

Now, so far we have mentioned that diamagnetism is very weak, and hence characterized typically by very low values of  $\chi_m$ . There is, however, one important exception. It is an experimental fact that a bulk superconductor in a weak magnetic field will act as a perfect diamagnet, with zero magnetic field in the interior. When a specimen is placed in a magnetic field and is then cooled through the transition temperature for superconductivity, the magnetic flux originally present is ejected from the specimen. This is called the Meissner effect. → See Chapter 10, Solid State Physics by Kittel

Mathematically this means that below the transition temperature,  $\vec{B} = 0$  within the superconducting specimen. Then from Eq. (8),

$$\vec{B} = 0 = \mu_0 (\vec{H} + \vec{M})$$

$$\therefore \vec{M} = -\vec{H} \quad (17)$$

∴ From Eqs (9) & (17),

$$\chi_m = -1, \quad (18)$$

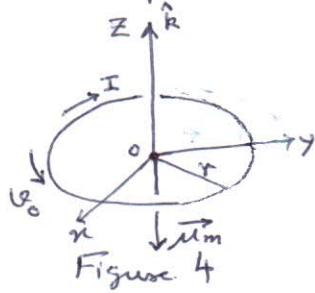
for a superconducting specimen below its transition temperature.

Non superconducting materials only

### 5. Induced Magnetic Moment & Magnetic Susceptibility in Diamagnetism

Diamagnetic materials have atoms or ions with complete shells, so they do not have a net magnetic dipole moment. Their diamagnetic behavior is due to the fact that a magnetic field acts to distort the orbital motion. We will study this using classical physics and then remark on the quantum result.

Let us now establish the fact that the effect of a magnetic field on the orbital motion of an electron is such as to produce a diamagnetic susceptibility. Consider an electron, <sup>(e)</sup> rotating about the nucleus in a circular orbit of radius  $r$  with uniform linear speed  $v_0$  — Figure 4. Assuming the electronic charge to be  $-e$ , the 'loop' current is given by,



where

$$I = \frac{e}{T}, \quad \left. \right\} \quad (1)$$

$$v_0 T = 2\pi r$$

$$I = \frac{ev_0}{2\pi r} \quad (2)$$

Accordingly the orbital dipole moment is,

$$M_m = I \cdot \pi r^2 = \frac{ev_0}{2\pi r} \cdot \pi r^2 = \frac{ev_0 r}{2} \quad (3)$$

If  $\omega_0$  is the angular speed of the electron, then  $v_0 = \omega_0 r$  and

$$M_m = \frac{e\omega_0 r^2}{2} \quad (4)$$

If we consider the orbit to lie on the xy-plane and choose the positive

- References : i) Griffiths  
ii) Dolbear iii) Kittel  
iv) Feynman (Vol. II)

$z$ -direction as shown in Figure 4 ( $\hat{k}$ -unit vector along positive  $z$  axis), then

$$\vec{\mu}_m = \frac{ev_0 r}{2} (-\hat{k}) \quad (5)$$

The centripetal force sustaining the circular orbit of the electron is provided by the Coulomb attraction of the nucleus. Assuming its magnitude to be  $F_0$ ,

$$F_0 = \frac{mv_0^2}{r}, \quad \left( F_0 = \frac{ze \cdot e}{4\pi\epsilon_0 r^2}; z - \text{nuclear charge} \right) \quad (6)$$

where  $m$  - mass of the electron.

Let us now apply a magnetic field  $\vec{B}$  in the positive  $z$ -direction, i.e.,

$$\vec{B} = B \hat{k} \quad (7)$$

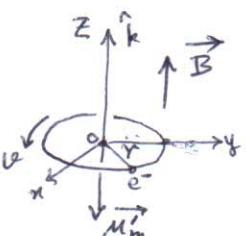


Figure 5

This gives rise to an additional force  $-e\vec{v} \times \vec{B}$  on the electron directed towards the center of the orbit (i.e., radially inwards) at all points on the orbit. Then assuming the radius of the orbit to remain unchanged at  $r$ , the magnitude of the centripetal force

is now increased to  $F_0 + evB$ , so that,

$$F_0 + evB = \frac{mv^2}{r}, \quad (8)$$

where  $v$  is the new speed of the electron. Clearly  $v > v_0$  since the centripetal force has increased due to  $\vec{B}$ , while  $r$  remains the same. The magnitude of the new magnetic dipole moment is,

$$\vec{\mu}'_m = \frac{evr}{2} \quad \left( = \frac{e\omega r^2}{2}, v = \omega r \right) \quad (9)$$

and

$$\vec{\mu}'_m = \frac{evr}{2} (-\hat{k}) \quad (10)$$

Therefore the induced dipole moment due to  $\vec{B}$  is,

$$\vec{\Delta\mu}_m = \vec{\mu}'_m - \vec{\mu}_m = \frac{er}{2}(v - v_0)(-\hat{k}), \quad (11)$$

the applied field

i.e., the induced dipole moment is in a direction opposite to  $\vec{B}$ . This is the essence of diamagnetic behavior. Let us now calculate its magnitude.

Subtracting (6) from (8),

$$e\mathbf{v}\cdot\mathbf{B} = \frac{mv^2}{r} - \frac{mv_0^2}{r} = \frac{m}{r} (\mathbf{v} + \mathbf{v}_0) \cdot (\mathbf{v} - \mathbf{v}_0) \equiv \frac{m}{r} (\mathbf{v} + \mathbf{v}_0) \cdot \Delta\mathbf{v} \quad (12)$$

Even for the largest fields that can be obtained in the laboratory ( $B \approx 100 T$ ),  $\Delta\mathbf{v}$  ( $\equiv \mathbf{v} - \mathbf{v}_0$ ) is small compared to  $\mathbf{v}_0$ . Then Eq.(12) can be approximated by,

$$e\mathbf{v}_0\cdot\mathbf{B} = \frac{m}{r} (\mathbf{v}_0 + \mathbf{v}_0) \cdot \Delta\mathbf{v}$$

$$\Rightarrow e\mathbf{v}_0\cdot\mathbf{B} = \frac{m}{r} \cdot 2\mathbf{v}_0 \cdot \Delta\mathbf{v},$$

i.e.,

$$\Delta\vartheta \in (\mathbf{v} - \mathbf{v}_0) = \frac{e\mathbf{B}}{2m} r \quad (13)$$

Note: Since  $\mathbf{v} = \omega\mathbf{r}$  &  $\mathbf{v}_0 = \omega_0\mathbf{r}$ ,  $\mathbf{v} - \mathbf{v}_0 = (\omega - \omega_0)\mathbf{r}$ , i.e.,

$$\Delta\mathbf{v} = \Delta\omega \cdot \mathbf{r}$$

where

$$\Delta\omega = \frac{e\mathbf{B}}{2m} \quad (14)$$

The quantity  $\frac{e\mathbf{B}}{2m}$  is known as the Larmor frequency. For its physical significance, see Section 18-3, Dekker.

From Eqs(11) and (13), the magnitude of the induced magnetic dipole moment is,

$$\Delta M_m = \frac{er}{2} (\mathbf{v} - \mathbf{v}_0) = \frac{er}{2} \cdot \frac{e\mathbf{B}}{2m} \cdot r = \frac{e^2 r^2}{4m} \mathbf{B}. \quad (15)$$

Then

$$\Delta \vec{M}_m = \frac{er}{2} (\mathbf{v} - \mathbf{v}_0) (-\hat{\mathbf{k}}) = \frac{e^2 r^2}{4m} \mathbf{B} (-\hat{\mathbf{k}}) = -\frac{e^2 r^2}{4m} \vec{\mathbf{B}}, \quad (16)$$

since  $\vec{\mathbf{B}} = B\hat{\mathbf{k}}$ .

- Students: Show that  $\vec{M}_m$  will be opposite to applied field  $\vec{\mathbf{B}}$ , and Eq.(16) will hold if we had chosen  $\vec{\mathbf{B}} = -B\hat{\mathbf{k}}$ .

Equation(16) shows explicitly that the induced magnetic dipole moment is opposite in direction to the applied field  $\vec{\mathbf{B}}$ . It is critical to understand this if we wish to understand diamagnetism. Diamagnetism is the result of Lenz's law operating on an atomic scale. Upon the application of a magnetic field, the electronic currents in each atom are modified in such a way that they tend to weaken the effect of this field.

Now suppose the atom has  $Z$  electrons; then Eq.(16), which was

developed for one electron must be suitably modified. However first we would like to write Eq.(16) a bit differently. Figure 5 shows the nucleus to be at the origin of the coordinate system with a single electron rotating about the nucleus in a circular orbit of radius  $r$ . This motion is confined to the  $xy$ -plane with the applied magnetic field  $\vec{B}$  along the  $Z$ -axis. Now clearly this picture of a circular orbit is both simplistic and unrealistic. In an atom, the motion of an electron around the nucleus is in three dimensions, not two as envisaged in Figures 4 and 5. Further the nature of the motion is quite complicated. So to be physically realistic, we need to replace  $r^2$  in Eq.(16) by  $\langle r^2 \rangle$  where  $\langle r^2 \rangle$  is the mean square of the perpendicular distance of the electron from the  $Z$ -axis (remember that the nucleus is at the origin, and  $\vec{B}$  is along the  $Z$ -axis). So Eq.(16) becomes,

$$\vec{\Delta M_m} = -\frac{e^2 \langle r^2 \rangle}{4m} \vec{B} \quad (17)$$

where,

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle \quad (18)$$

Now the mean square distance of the electron from the nucleus is

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle \quad (19)$$

If we consider spherically symmetric atoms (i.e., the electronic charge distribution surrounding the nucleus of the atom is spherically symmetric), we have

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle \quad (20)$$

so that

$$\langle r^2 \rangle = 2 \langle x^2 \rangle \quad & \quad \langle r^2 \rangle = 3 \langle x^2 \rangle \quad (21)$$

$$\therefore \langle r^2 \rangle = \frac{2}{3} \langle r^2 \rangle \quad (22)$$

From (17) and (22),

$$\vec{\Delta M_m} = -\frac{e^2 \langle x^2 \rangle}{6m} \vec{B}, \quad (23)$$

i.e., the magnitude of the induced dipole moment is,

$$\Delta M_m = \frac{e^2 \langle r^2 \rangle}{6m} B \quad (24)$$

We are still left with the problem: What is the mean square radius,  $\langle r^2 \rangle$ ? Classical mechanics cannot supply an answer. We must go back and start over with quantum mechanics. In an atom we cannot really say where an electron is, but only know the probability that it will be at some place. If we interpret  $\langle r^2 \rangle$  to mean the average of the square of the distance from the center for the probability distribution, the magnitude of the diamagnetic moment given by quantum mechanics is just the same as given in Eq. (24). Now Eq. (24), of course, is the moment for one electron. The total induced diamagnetic moment is given by the sum over all the  $Z$  (say) electrons of the atom and hence will have the magnitude.

$$\mu_{\text{ind}}^{\text{dia}} = Z \Delta M_m = \frac{Z e^2 \langle r^2 \rangle}{6m} B \quad (25)$$

&amp;

$$\therefore \vec{\mu}_{\text{ind}}^{\text{dia}} = - \frac{Z e^2 \langle r^2 \rangle}{6m} \vec{B} \quad (26)$$

Then if  $\vec{M}$  is the magnetization,  $N$  is the number of atoms per unit volume, and  $\vec{\mu}_{\text{ind}}^{\text{dia}}$  is the average induced diamagnetic moment per atom, then

$$\vec{M} = N \vec{\mu}_{\text{ind}}^{\text{dia}} = - \frac{N Z e^2 \langle r^2 \rangle}{6m} \vec{B} \quad (27)$$

If  $\chi_m^{\text{dia}}$  is the diamagnetic susceptibility, then

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}), \quad \vec{M} = \chi_m^{\text{dia}} \vec{H}, \quad (28)$$

$$\vec{B} = \mu_0 (\vec{H} + \chi_m^{\text{dia}} \vec{H}) = \mu_0 (1 + \chi_m^{\text{dia}}) \vec{H},$$

i.e.

$$\vec{B} \approx \mu_0 \vec{H}, \quad (29)$$

since  $\chi_m^{\text{dia}} \approx 10^{-5} \ll 1$ . Therefore from Eqs. (27) and (29),

$$\vec{M} = -\frac{N Z e^2 \langle r^2 \rangle}{6m} M_0 \vec{H}, \quad (30)$$

and with  $\vec{M} = \chi_m^{\text{dia}} \vec{H}$  we get the diamagnetic susceptibility as,

$$\chi_m^{\text{dia}} = -\frac{N Z e^2 \langle r^2 \rangle M_0}{6 m} \quad (31)$$

This is the classical Langevin result. With  $\langle r^2 \rangle \approx 10^{-20} \text{ m}^2$ ,  $N \approx 5 \times 10^{23} / \text{m}^3$ , one obtains  $|\chi_m^{\text{dia}}| \approx 10^{-6} Z \approx 10^{-5}$  - compare this with experimental values given in page 12. Diamagnetism associated with free electrons in a metal is not discussed here - not in your syllabus.

## 6. Induced Magnetic Moment & Magnetic Susceptibility in Paramagnetism

Paramagnetism requires the existence of permanent magnetic dipoles, and the paramagnetic behavior is the analogue of the orientational polarization associated with permanent electric dipoles.

The classical theory of paramagnetism is just like the theory of orientational polarization that we discussed during our study of dielectric properties. One assumes that each of the atoms in the paramagnetic specimen has a permanent magnetic dipole moment  $\vec{\mu}_m^0$ , which always has the same magnitude  $\mu_m^0$ , but which can point in any direction. Suppose that the paramagnetic specimen has  $N$  such (identical) atoms per unit volume. Without an external magnetic field, the magnetic moments of these atoms will be oriented at random, and the specimen as a whole will have no resulting magnetization. In the presence of a magnetic field  $\vec{B}$ , a torque  $\vec{\mu}_m^0 \times \vec{B}$  is exerted on each magnetic dipole of moment  $\vec{\mu}_m^0$ , tending to orient it towards  $\vec{B}$ . This ordering tendency is countered by the thermal motion of the dipoles. In the classical theory, the dipoles are assumed to be freely rotating. From the above discussion, the close analogy with orientational polarization is quite apparent. Our calculations will closely follow this analogy.

The potential energy of each magnetic dipole  $\vec{\mu}_m^0$  in the magnetic field  $\vec{B}$  is,

$$U = -\vec{\mu}_m \cdot \vec{B} = -\mu_m^o B \cos \theta, \quad (1) \text{ See Figure 6.}$$

where  $\mu_m^o = |\vec{\mu}_m|$  &  $B = |\vec{B}|$ . According to statistical mechanics, the probability  $P(\theta)d\theta$  for a magnetic dipole to make an angle between  $\theta$  and  $\theta + d\theta$  with the magnetic field  $\vec{B}$  is,

$$P(\theta)d\theta = \frac{\sin \theta d\theta e^{-U/kT}}{\int_0^\pi \sin \theta d\theta e^{-U/kT}} \quad (2)$$

Figure 6

From Eqs (1) and (2),

$$P(\theta)d\theta = \frac{\sin \theta d\theta e^{\mu_m^o B \cos \theta / kT}}{\int_0^\pi \sin \theta d\theta e^{\mu_m^o B \cos \theta / kT}}, \quad (3)$$

where  $k$  - Boltzmann constant &  $T$  - temperature of the paramagnetic specimen. The average component of the dipole moment in the direction of the field  $\vec{B}$  is given by

$$\langle \mu_m \rangle = \langle \mu_m^o \cos \theta \rangle = \mu_m^o \langle \cos \theta \rangle, \quad (4)$$

where,

$$\langle \cos \theta \rangle = \int_0^\pi \cos \theta P(\theta) d\theta = \frac{\int_0^\pi \cos \theta \sin \theta d\theta e^{\mu_m^o B \cos \theta / kT}}{\int_0^\pi \sin \theta d\theta e^{\mu_m^o B \cos \theta / kT}}, \quad (5)$$

i.e.,

$$\langle \cos \theta \rangle = \frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a} = L(a) \quad (6)$$

where  $L(a)$  is the familiar Langevin function, and,

$$a = \frac{\mu_m^o B}{kT} \quad (7)$$

$$\therefore \langle \mu_m \rangle = \mu_m^o \langle \cos \theta \rangle = \mu_m^o L(a) \quad (8)$$

Clearly  $\langle \mu_m \rangle$  can be interpreted as the average induced dipole moment per atom in the direction of  $\vec{B}$ .  $\langle \mu_m \rangle$  &  $\vec{B}$  have the same direction, because the dipole moments tend to orient along  $\vec{B}$ . That is, the susceptibility is positive.

Now  $\mu_m^o$  is of the order of 1 Bohr magneton, i.e.,  $\mu_m^o \approx 10^{-23} \text{ Amp} \cdot \text{m}^2$ , so that for a magnetic field of 1 Tesla,  $\mu_m^o B \approx 10^{-23} \text{ Joule}$ . At room temperature ( $T \approx 300\text{K}$ ),  $kT \approx 400 \times 10^{-23} \text{ Joule}$ . So if the magnetic field is not too high and the temperature not too low, the condition  $\mu_m^o B / kT \ll 1$  is satisfied.

And when  $\frac{\mu_m^o B}{kT} = a \ll 1$ , then  $L(a) \approx a/3$ , i.e.,

$$L(a) \approx \frac{a}{3} = \frac{\mu_m^o B}{3kT}, \quad a \ll 1 \quad (9)$$

From (8) and (9), for  $\mu_m^o B \ll kT$ ,

$$\langle M_m \rangle = \mu_m^o L(a) = \frac{\mu_m^{o2} B}{3kT} \quad (10)$$

Since  $\langle \vec{M}_m \rangle$  &  $\vec{B}$  have the same direction, we can write,

$$\langle \vec{M}_m \rangle = \frac{\mu_m^{o2}}{3kT} \vec{B} \quad (11)$$

Now with  $N$  atoms per unit volume, the magnetization  $\vec{M}$  is given by,

$$\vec{M} = N \langle \vec{M}_m \rangle = \frac{N \mu_m^{o2} \vec{B}}{3kT}, \quad (12)$$

for  $\mu_m^o B \ll kT$ . Now,  $\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \chi_m^{\text{para}} \vec{H})$ , i.e.,

$$\vec{B} = \mu_0 (1 + \chi_m^{\text{para}}) \vec{H} \quad (13)$$

But the paramagnetic susceptibility  $\chi_m^{\text{para}} \approx 10^{-5} \ll 1$ , i.e.,

$$\vec{B} \approx \mu_0 \vec{H} \quad (14)$$

∴ From (12) and (14),

$$\vec{M} = \frac{N \mu_m^{o2} \mu_0}{3kT} \vec{H} = \chi_m^{\text{para}} \vec{H}, \quad (15)$$

so that the paramagnetic susceptibility is given by,

$$\chi_m^{\text{para}} = \frac{N \mu_m^{o2} \mu_0}{3kT} = \frac{C}{T} \quad (16)$$

where

$$C = \frac{N \mu_m^{o2} \mu_0}{3k} \quad (17)$$

The constant  $C$  is known as the Curie constant, and (16) is known

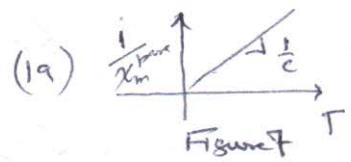
as the Curie law. Remember that the Curie law holds only in the region  $M_m^o B \ll kT$ .

Quantum mechanics essentially leads to the same expression for  $\chi_m^{\text{para}}$  (i.e.,  $\chi_m^{\text{para}} = \frac{N M_m^o^2 \mu_0}{3kT}$ ) if we set,

$$M_m^o = \mu_m^o = g M_B N \sqrt{J(J+1)}, \quad (18)$$

i.e. we use the quantum mechanical expression for the permanent magnetic dipole moment  $\mu_m^o$  using Eq.(34) on page 7. Next, from (6),

$$\frac{1}{\chi_m^{\text{para}}} = \frac{T}{C}$$



So a plot of  $\frac{1}{\chi_m^{\text{para}}}$  vs  $T$  should yield a straight line. See Figure 5, p 305 of Kittel (8th Ed.) for a nice plot for a gadolinium salt,  $\text{Gd}(\text{C}_2\text{H}_3\text{SO}_4)_3 \cdot 9\text{H}_2\text{O}$ .

Now, the discussion above shows that paramagnetism is temperature dependent, unlike diamagnetism. (However there is also a class of paramagnetism called Van Vleck paramagnetism where the susceptibility is independent of temperature - not in your syllabus.)

- References:
1. Solid State Physics by Dekker (Chapter 18)
  2. Solid State Physics by Kittel (Chapter 11)
  3. Feynman Lectures on Physics Vol II (Chapters 34 & 35)

## 7. Ferromagnetism

In this section we will discuss some materials in which the net effect of the magnetic dipole moments in the material is much greater than in the case of paramagnetism or diamagnetism. The phenomenon is called ferromagnetism. Recall that

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) \quad (1)$$

In paramagnetic and diamagnetic materials the induced magnetic moments are usually so weak that we don't have to worry about the additional fields produced by the magnetic moments. In other words, the magnetic susceptibilities are usually so low ( $\approx 10^{-5}$ ) in paramagnetism and diamagnetism that the term

$$\vec{M} = \chi_m \vec{H} \quad (2)$$

can be ignored in (1), so that

$$\vec{B} \approx \mu_0 \vec{H}, \quad (3)$$

i.e.,  $\vec{B}$  in paramagnetic or diamagnetic materials is almost the same as in free space. In fact we have actually used this approximation to find both paramagnetic and diamagnetic susceptibilities. For ferromagnetic materials, however, the magnetic moments induced by applied magnetic fields are quite enormous and have a great effect on the fields themselves. In fact, the induced moments are so strong that they are often the dominant effect in producing the observed fields.

In other words, when a ferromagnetic material is magnetized, we will often have  $|\vec{M}| \gg |\vec{H}|$ , so that

$$\vec{B} \approx \mu_0 \vec{M} \quad (4) \rightarrow \begin{array}{l} \text{See Figure} \\ 19-1, p. 465 \\ \text{Dekker \&} \\ \text{Fig. 6.29} \\ \text{Griffiths} \\ \text{3rd Ed.} \end{array}$$

The question is, why are the magnetic moments so strong - how does it all work? A detailed explanation can only be provided by quantum mechanics. However we will take a classical approach suggested by Weiss who put forward the following hypotheses in 1907 (i.e., well before quantum mechanics was developed):

- (1) A ferromagnetic specimen of macroscopic dimensions contains, in general, a

number of small regions (domains) which are spontaneously magnetized; the magnitude of the spontaneous magnetization of the specimen is determined by the vector sum of the magnetic moments of the individual domains.

- (ii) Within each domain the spontaneous magnetization is due to the existence of a "molecular field" which tends to produce a parallel alignment of the atomic dipoles.

Of the elements, only Fe, Ni, Co, Cd, and Dy are ferromagnetic, although there are a relatively large number of ferromagnetic alloys and oxides. Above a critical temperature  $\theta_f$ , known as the ferromagnetic Curie temperature, the spontaneous magnetization vanishes and the material becomes paramagnetic. Well above  $\theta_f$  the <sup>paramagnetic</sup> susceptibility follows the Curie-Weiss law,

$$\chi_{m,f}^{\text{para}} = \frac{C}{T - \theta} \quad (5)$$

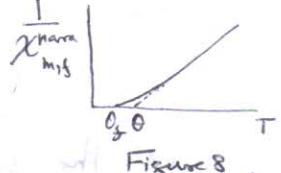


Figure 8

where C is the Curie constant; the temperature  $\theta$  is called the paramagnetic Curie temperature and is usually some degrees higher than  $\theta_f$ . According to the

	Fe	Co	Ni
$\theta_f$ (K)	1043	1393	631
$\theta$ (K)	1093	1428	650

Curie-Weiss law, a plot of  $\frac{1}{\chi}$  vs T should yield a straight line, the intercept along the T-axis being equal to  $\theta$ . Experiments show that this law is indeed satisfied with considerable accuracy except in the region close to the Curie point.

Figure 8 shows a schematic representation of the behavior of the ferromagnetic metals above the Curie point; the slight curvature leads to the distinction between the ferromagnetic and paramagnetic Curie points.

Students, it is important for you to understand that the spontaneous magnetization (below  $\theta_f$ ) refers to a single domain - one of many within a ferromagnetic material below its  $\theta_f$ . Within each domain the spontaneous magnetization has a specific direction but this direction may vary randomly from one domain to the other. For a ferromagnetic specimen that has never been subjected to an external magnetic field; below its  $\theta_f$  the sum of the vectors representing the resultant magnetic dipole moments of the individual domains may therefore vanish. Thus in spite of having spontaneously magnetized domains at room temperature, an "ordinary"

piece of iron is not necessarily magnetized as a whole.

### The Curie-Weiss Law

Spontaneous magnetization implies cooperation between the atomic dipoles within a single domain, i.e., there must be some kind of interaction between the atoms which produces the tendency for parallel alignment of the atomic magnetic dipoles. In order to obtain a phenomenological description of spontaneous magnetization, Weiss assumed that the molecular field  $H_m$  acting on a given dipole may be written in the form

$$H_m = H + H_{int} \quad (6)$$

where  $H$  is the applied field and  $H_{int}$  is the internal field which is proportional to the magnetization, that is,

$$H_{int} = \gamma M, \quad (7)$$

so that

$$H_m = H + \gamma M, \quad (8)$$

where  $M$  is the magnetization and  $\gamma$  is the Weiss constant. It is this internal field which provides the cooperative effect that leads to spontaneous magnetization for  $T < \theta_f$  in a ferromagnetic material. The orienting effect (or cooperative effect) of this internal field is opposed by thermal agitation, and for  $T > \theta_f$  the ordered orientation of the atomic dipoles in the same direction is destroyed. The material becomes paramagnetic. We will now show that a molecular field of the type given in Eq.(8) indeed leads to the Curie-Weiss law.

The paramagnetic susceptibility as given by Curie's law is,

$$\chi_m^{\text{para}} = \frac{M}{H} = \frac{C}{T} \quad (9)$$

where  $M$  - magnetization of the paramagnetic specimen,  $T$  - its temperature,  $C$  - Curie constant for the specimen,  $H$  - applied field.

So,

$$M = \frac{C}{T} H \quad (10)$$

for a paramagnetic material. Now consider the paramagnetic phase of a ferromagnetic material (i.e., temperature greater than  $\theta$ ): an applied field  $H$  will cause a finite magnetization  $M$  and this in turn will cause a finite internal field  $H_{int}$  ( $= \gamma M$ ).

If  $\chi_{m,f}^{\text{para}}$  is the paramagnetic susceptibility of the ferromagnetic specimen, then

$$\chi_{m,f}^{\text{para}} = \frac{M}{H} \quad (11)$$

where,

$$M = \frac{c}{T} (H + H_{\text{int}}) = \frac{c}{T} (H + \lambda M) \quad (12)$$

$$\Rightarrow MT - c\lambda M = CH \Rightarrow M(T - c\lambda) = CH$$

$$\therefore M = \frac{c}{T - c\lambda} H \quad (13)$$

$$\chi_{\text{mf}}^{\text{para}} = \frac{M}{H} = \frac{c}{T - c\lambda} = \frac{c}{T - \theta}, \quad (5)$$

where,

$$\theta = c\lambda \quad (15)$$

Not in syllabus but relevant information

- It can be shown through simple physical arguments that  $\lambda \sim 10^4$ . This large value plays a crucial role in spontaneous magnetization. See Dekker p. 472.
- For  $T < \theta_f$ , the spontaneous magnetization itself is a function of temperature. - Dekker p. 469
- Experiments have shown that the magnetization is essentially associated with electron spins rather than with their orbital motions. - Dekker Section 1a-3 p. 468.

### 6.4.2 Ferromagnetism

In a linear medium the alignment of atomic dipoles is maintained by a magnetic field imposed from the outside. Ferromagnets—which are emphatically *not* linear<sup>7</sup>—require no external fields to sustain the magnetization; the alignment is “frozen in.” Like paramagnetism, ferromagnetism involves the magnetic dipoles associated with the spins of unpaired electrons. The new feature, which makes ferromagnetism so different from paramagnetism, is the interaction between nearby dipoles: In a ferromagnet, *each dipole “likes” to point in the same direction as its neighbors*. The reason for this preference is essentially quantum mechanical, and I shall not endeavor to explain it here; it is enough to know that the correlation is so strong as to align virtually 100% of the unpaired electron spins. If you could somehow magnify a piece of iron and “see” the individual dipoles as tiny arrows, it would look something like Fig. 6.25, with all the spins pointing the same way.

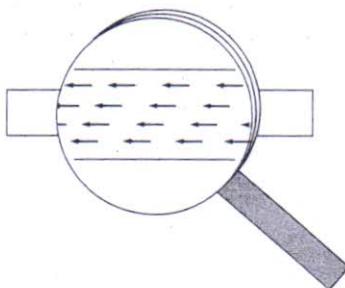
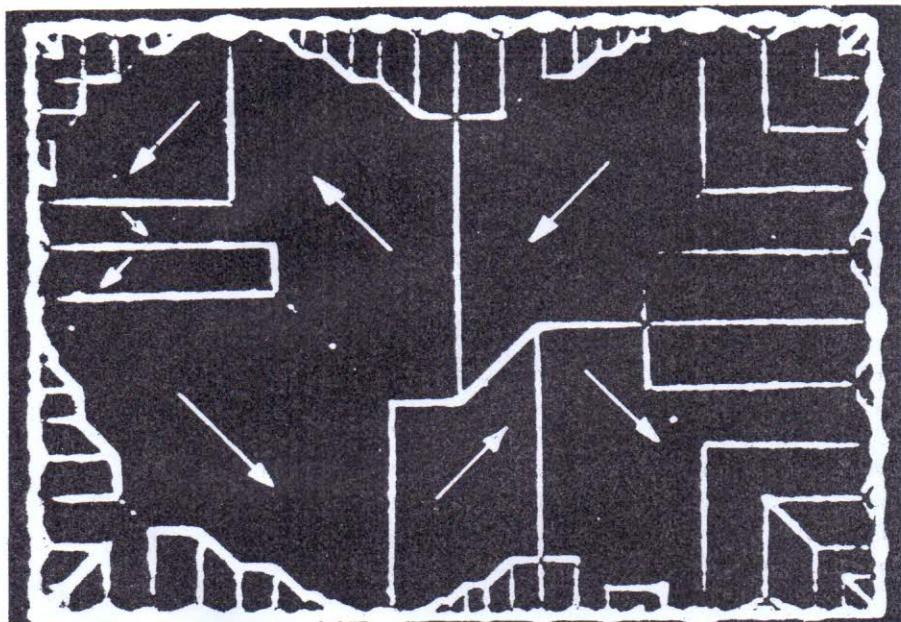


Figure 6.25

But if that is true, why isn’t every wrench and nail a powerful magnet? The answer is that the alignment occurs in relatively small patches, called **domains**. Each domain contains billions of dipoles, all lined up (these domains are actually *visible* under a microscope, using suitable etching techniques—see Fig. 6.26), but the domains *themselves* are randomly oriented. The household wrench contains an enormous number of domains, and their magnetic fields cancel, so the wrench as a whole is not magnetized. (Actually, the orientation of domains is not *completely* random; within a given crystal there may be some preferential alignment along the crystal axes. But there will be just as many domains pointing one way as the other, so there is still no large-scale magnetization. Moreover, the crystals themselves are randomly oriented within any sizable chunk of metal.)

How, then, would you produce a **permanent magnet**, such as they sell in toy stores? If you put a piece of iron into a strong magnetic field, the torque  $\mathbf{N} = \mathbf{m} \times \mathbf{B}$  tends to align the dipoles parallel to the field. Since they like to stay parallel to their neighbors, most of the dipoles will resist this torque. However, at the *boundary* between two domains, there

<sup>7</sup>In this sense it is misleading to speak of the susceptibility or permeability of a ferromagnet. The terms *are* used for such materials, but they refer to the proportionality factor between a *differential* increase in  $\mathbf{H}$  and the resulting *differential* change in  $\mathbf{M}$  (or  $\mathbf{B}$ ); moreover, they are not *constants*, but functions of  $\mathbf{H}$ .



Ferromagnetic domains. (Photo courtesy of R. W. DeBlois)

Figure 6.26

are *competing neighbors*, and the torque will throw its weight on the side of the domain most nearly parallel to the field; this domain will win over some converts, at the expense of the less favorably oriented one. The net effect of the magnetic field, then, is to *move the domain boundaries*. Domains parallel to the field grow, and the others shrink. If the field is strong enough, one domain takes over entirely, and the iron is said to be "saturated."

It turns out that this process (the shifting of domain boundaries in response to an external field) is not entirely reversible: When the field is switched off, there will be *some* return to randomly oriented domains, but it is far from complete—there remains a preponderance of domains in the original direction. The object is now a permanent magnet.

A simple way to accomplish this, in practice, is to wrap a coil of wire around the object to be magnetized (Fig. 6.27). Run a current  $I$  through the coil; this provides the external magnetic field (pointing to the left in the diagram). As you increase the current, the field increases, the domain boundaries move, and the magnetization grows. Eventually, you reach the saturation point, with all the dipoles aligned, and a further increase in current has no effect on  $\mathbf{M}$  (Fig. 6.28, point *b*).

Now suppose you *reduce* the current. Instead of retracing the path back to  $M = 0$ , there is only a *partial* return to randomly oriented domains.  $M$  decreases, but even with the current off there is some residual magnetization (point *c*). The wrench is now a permanent

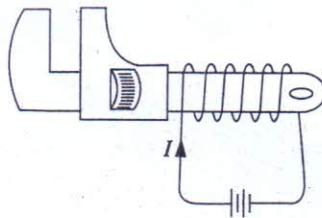


Figure 6.27

magnet. If you want to eliminate the remaining magnetization, you'll have to run a current backwards through the coil (a negative  $I$ ). Now the external field points to the right, and as you increase  $I$  (negatively),  $M$  drops down to zero (point  $d$ ). If you turn  $I$  still higher, you soon reach saturation in the other direction—all the dipoles now pointing to the right ( $e$ ). At this stage switching off the current will leave the wrench with a permanent magnetization to the right (point  $f$ ). To complete the story, turn  $I$  on again in the positive sense:  $M$  returns to zero (point  $g$ ), and eventually to the forward saturation point ( $b$ ).

The path we have traced out is called a **hysteresis loop**. Notice that the magnetization of the wrench depends not only on the applied field (that is, on  $I$ ), but also on its previous magnetic "history."<sup>8</sup> For instance, at three different times in our experiment the current was zero ( $a$ ,  $c$ , and  $f$ ), yet the magnetization was different for each of them. Actually, it is customary to draw hysteresis loops as plots of  $B$  against  $H$ , rather than  $M$  against  $I$ . (If our coil is approximated by a long solenoid, with  $n$  turns per unit length, then  $H = nI$ , so  $H$  and  $I$  are proportional. Meanwhile,  $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ , but in practice  $M$  is huge compared to  $H$ , so to all intents and purposes  $\mathbf{B}$  is proportional to  $\mathbf{M}$ .)

→ See Figure 6.29

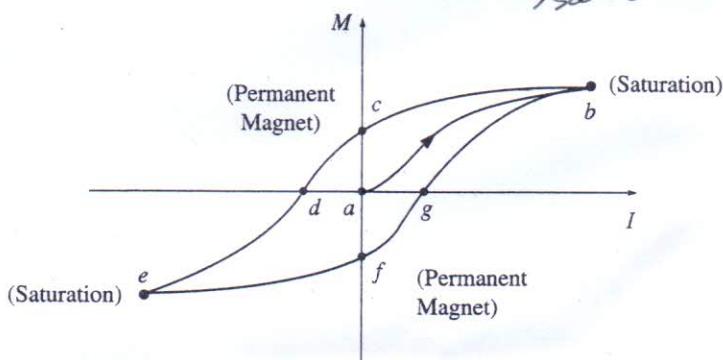


Figure 6.28

<sup>8</sup>Etymologically, the word *hysteresis* has nothing to do with the word *history*—nor with the word *hysteria*. It derives from a Greek verb meaning "to lag behind."

To make the units consistent (teslas), I have plotted  $(\mu_0 H)$  horizontally (Fig. 6.29); notice, however, that the vertical scale is  $10^4$  times greater than the horizontal one. Roughly speaking,  $\mu_0 \mathbf{H}$  is the field our coil *would* have produced in the absence of any iron;  $\mathbf{B}$  is what we *actually* got, and compared to  $\mu_0 \mathbf{H}$  it is gigantic. A little current goes a long way when you have ferromagnetic materials around. That's why anyone who wants to make a powerful electromagnet will wrap the coil around an iron core. It doesn't take much of an external field to move the domain boundaries, and as soon as you've done that, you have all the dipoles in the iron working with you.

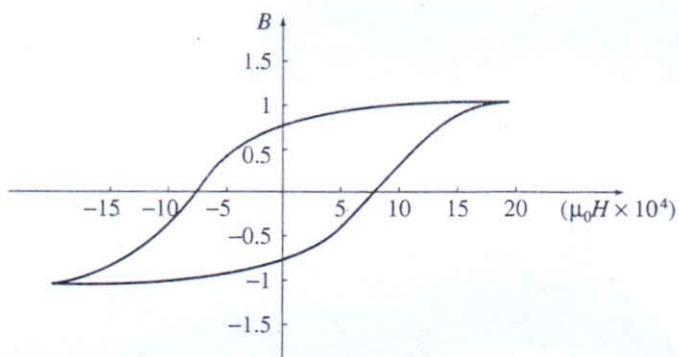


Figure 6.29

One final point concerning ferromagnetism: It all follows, remember, from the fact that the dipoles within a given domain line up parallel to one another. Random thermal motions compete with this ordering, but as long as the temperature doesn't get too high, they cannot budge the dipoles out of line. It's not surprising, though, that *very* high temperatures do destroy the alignment. What *is* surprising is that this occurs at a precise temperature (770° C, for iron). Below this temperature (called the **Curie point**), iron is ferromagnetic; above, it is paramagnetic. The Curie point is rather like the boiling point or the freezing point in that there is no *gradual* transition from ferro- to para-magnetic behavior, any more than there is between water and ice. These abrupt changes in the properties of a substance, occurring at sharply defined temperatures, are known in statistical mechanics as **phase transitions**.

**Problem 6.20** How would you go about *demagnetizing* a permanent magnet (such as the wrench we have been discussing, at point *c* in the hysteresis loop)? That is, how could you restore it to its original state, with  $M = 0$  at  $I = 0$ ?

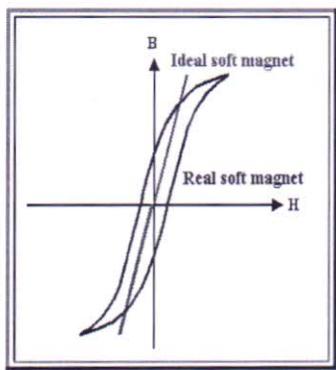
**Problem 6.21**

- (a) Show that the energy of a magnetic dipole in a magnetic field  $\mathbf{B}$  is given by

$$U = -\mathbf{m} \cdot \mathbf{B}. \quad (6.34)$$

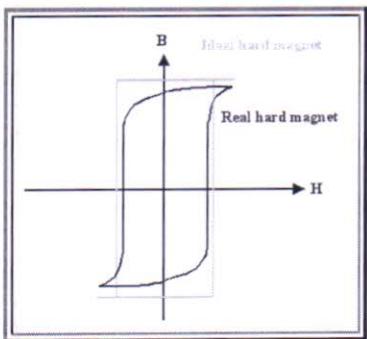
### Definitions

- Let's quickly go over the three questions from the preceding sub-chapter
  - 1. What kind of hystereses curve do I need for the application I have in mind?
- Lets look at two "paradigmatic" applications: A **transformer core** and a **magnetic memory**.
  - The transformer core is ferromagnetic in order to "transport" a large magnetic flux  $B$  produced by the primary coil to the secondary coil. What I want is that the induced flux  $B$  follows the primary field  $H$  as closely as possible.
  - In other words: There should be *no* hystereses loop - just a straight line, as shown below



- The ideal curve, without any hystereses, does not exist. What you get is something like the curve shown for a real **soft magnet** - because that is what we call a material with a kind of slender hystereses curve and thus small values of coercivity and remanence.
- If we switch on a positive field  $H$  and then go back to zero again, a little bit of magnetization is left. For a rather small reverse field, the magnetic flux reverses, too - the flux  $B$  follows  $H$  rather closely, if not exactly.
- Hystereses losses are small, because the area enclosed in the hystereses loop is small.
- But *some* losses remain, and the "transformer core" industry will be very happy if you can come up with a material that is just 1 % or 2 % "softer" than what they have now.
- Beside losses, you have another problem: If you vary  $H$  sinusoidally, the output will be a somewhat distorted sinus, because  $B$  does not follow  $H$  linearly. This may be a problem when transforming *signals*.

- A soft magnetic material will obviously not make a good **permanent magnet**, because its remaining magnetization (its remanence) after switching off the magnetic field  $H$  is small.
  - But a permanent magnet is what we want for a **magnetic storage material**. Here we want to induce a large permanent magnetization by some external field (produced by the "writing head" of our storage device) that stays intact for many years if needs be. Some more information about magnetic storage can be found in an extra module.
  - It should be *strong enough* - even so it is contained in a tiny area of the magnetic material on the tape or the storage disc - to produce a measurable effect if the reading head moves over it. It should not be *too strong*, however, because that would make it too difficult to erase it if we want to overwrite it with something else. In short, it should look like this



- We can define what we want in terms of coercivity and remanence. Ideally, the hystereses curve is very "square".
- At some minimum field, the magnetization is rather large and does not change much anymore.
- If we reverse the field direction, not much happens for a while, but as soon as we move above slightly above the coercivity value, the magnetization switches direction completely.
- Ferromagnetic losses are unavoidable, we simply must live with them

- Pretty much all possible applications - consult the list in the next section - either calls for *soft* or for *hard* magnets; there isn't much in between.

- So we now must turn to the second and third question:

### Tailoring Hystereses Curves

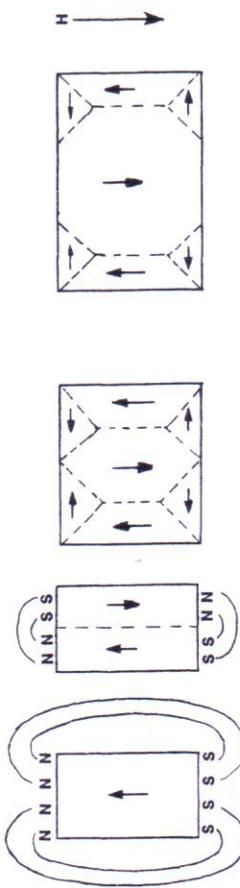
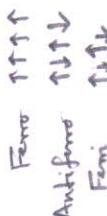


Figure 15.9. (a) Spontaneous alignment of all spins in a single direction. (b) Division into two magnetic domains having opposite spin directions. (c) Closure domains in a cubic crystal. (d) Growth of a domain whose spins are parallel to an external magnetic field. (The domain walls are *not* identical with the grain boundaries.)

heard when a permanent magnet approaches the iron rod. The wall motions may be impeded by imperfections in the crystal, such as by particles of a second phase, oxides, holes, or cracks. A second type of impediment to free domain wall motion stems from dislocations, i.e., from residual stresses in the crystal caused by grinding, polishing, or plastic deformation.

**Cold work enlarges the coercivity** and the area within the hysteresis loop. Further, cold work decreases the permeability and causes a clockwise rotation of the hysteresis curve. In short, mechanical hardness and magnetic hardness parallel each other in many cases. (There exist exceptions, however, such as in the case of silicon additions to iron, which makes the material magnetically softer and mechanically harder, see Section 17.2.3.) Recrystallization and grain growth by annealing at suitable temperatures relieve the stresses and restore the soft-magnetic properties.

We shall return to ferromagnetism in Section 15.4 and Chapter 16.



### ✓ 15.1.4. Antiferromagnetism

Antiferromagnetic materials exhibit, just as ferromagnetics, a spontaneous alignment of moments below a critical temperature. However, the responsible neighboring atoms in antiferromagnetics are aligned in an antiparallel fashion (Fig. 15.10). Actually, one may consider an antiferromagnetic crystal to be divided into two interpenetrating sublattices, A and B, each of which has a spontaneous parallel alignment of spins. Figure 15.10 depicts the spin alignments for two manganese compounds. (Only the spins of the manganese ions contribute to the antiferromagnetic behavior.) Figure 15.10(a) im-

parallel to each other. It is this interplay between exchange energy, which demands parallel spin alignment, and magnetostatic energy, which supports antiparallel spins, that leads eventually to an energetically most favorable domain size (which is about 1–100  $\mu\text{m}$ ).

A further reduction in magnetostatic energy can be obtained if the magnetic flux follows a completely closed path within a crystal so that no exterior poles are formed. Indeed, “closure” **domain structures**, as shown in Fig. 15.9(c), are observed in cubic crystals.

Another question which needs to be answered pertains to whether the flip from one spin direction into the other occurs in one step, i.e., between two adjacent atoms, or instead over an extended range of atoms. Again, the above-mentioned exchange energy, which supports a parallel spin alignment, hinders a spontaneous flip-over. Instead, a gradual rotation over several hundred atomic distances is energetically more favorable. The region between individual domains in which the spins rotate from one direction into the next is called a domain wall or a **Bloch wall**.

Finally, we may ask the question whether and how those domain walls can be made visible. The most common method, devised by Bitter in 1931, utilizes an aqueous suspension of very finely dispersed  $\text{Fe}_3\text{O}_4$  particles which is applied to the polished surface of a test material. These particles are attracted to the domain wall endings and can then be observed as fine lines under an optical microscope. Another method exploits the rotation of the plane of polarization of reflected light from differently magnetized areas (**Kerr effect**).

We mentioned above that an external magnetic field causes a movement of the domain walls. The movement is, as a rule, not continuous, but occurs most of the time in distinct jumps. This is known as the **Barkhausen effect**, which utilizes an induction coil wound around a ferromagnetic rod. The former is connected to an amplifier and a loudspeaker. Audible clicks are

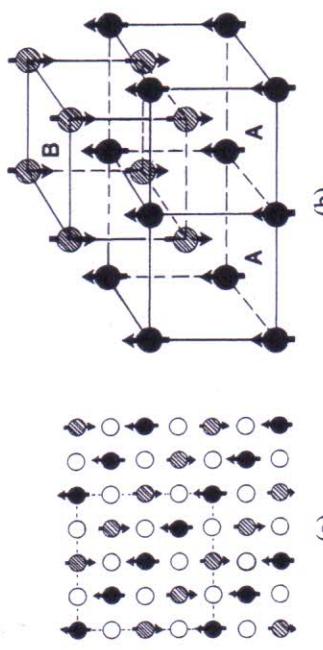


Figure 15.10. Schematic representation of spin alignments for antiferromagnetics at 0 K. (a) Display of a (100) plane of  $\text{MnO}$ . The gray (spin down) and black (spin up) circles represent the  $\text{Mn}$  ions. The oxygen ions (open circles) do not contribute to the antiferromagnetic behavior.  $\text{MnO}$  has a  $\text{NaCl}$  structure. (b) Three-dimensional representation of the spin alignment of manganese ions in  $\text{MnF}_2$ . (The fluorine ions are not shown.) This figure demonstrates the interpenetration of two manganese sublattices, A and B, having antiparallel aligned moments.

## 15.1.5. Ferrimagnetism

Ferrimagnetic materials are of great technical importance. They exhibit a spontaneous magnetic moment (Fig. 15.9) and hysteresis (Fig. 15.6) below a Curie temperature, just as iron, cobalt, or nickel. In other words, ferrimagnetic materials possess, similarly as ferromagnetics, small domains in which the electron spins are spontaneously aligned in parallel. The main difference from ferromagnetics is, however, that ferrimagnetics are ceramic materials (oxides) and that they are poor electrical conductors. A large resistivity is often desired for high-frequency applications (e.g., to prevent eddy currents in cores of coils, see Chapter 17).

To explain the spontaneous magnetization in ferrimagnetics, Néel proposed that two sublattices should exist in these materials (just as in antiferromagnetics) each of which contains ions whose spins are aligned parallel to each other. The crucial point is that each of the sublattices contain *different* numbers of magnetic ions. This causes some of the magnetic moments to remain uncanceled. As a consequence, a net magnetic moment results. Ferrimagnetic materials can thus be described as *imperfect antiferromagnetics*. The crystallography of ferrites is rather complex. We defer its discussion until later. For the time being, it suffices to know that there are two types of lattice sites which are available to be occupied by the metal ions. They are called A sites and B sites. (As before, oxygen ions do not contribute to the magnetic moments).

We will now discuss as an example nickel ferrite,  $\text{NiO} \cdot \text{Fe}_2\text{O}_3$ . The  $\text{Fe}^{3+}$  ions are equally distributed between A and B sites (Fig. 15.12), and since ions on A and B sites exhibit spontaneous magnetization in opposite directions, we expect overall cancellation of spins for these ions. Specifically, atomic iron possesses six  $3d$ -electrons and two  $4s$ -electrons ( $3d^6 4s^2$ , see Appendix 3). The  $\text{Fe}^{3+}$  ions are deprived of three electrons, so that five  $d$ -electrons, or five spin moments per atom, remain in its outermost shell. This is indicated in Fig. 15.12.

$$\chi = \frac{C}{T - (-\theta)} = \frac{C}{T + \theta}. \quad (15.4)$$

The Néel temperature is often below room temperature (Table 15.2). Most antiferromagnetics are found among ionic compounds. They are insulators or semiconductors. Essentially no practical application for antiferromagnetism is known at this time. (See, however, the use of “canted” antiferromagnetics, described in Section 17.5, which are materials in which the magnetic moments of the two sublattices are not completely antiparallel. This results in a small net magnetization.)

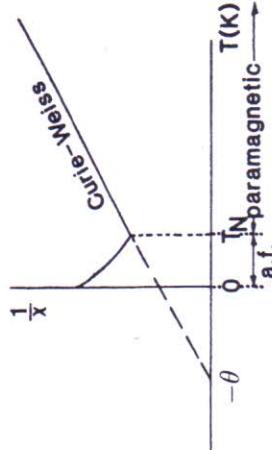


Figure 15.11. Schematic representation of the temperature dependence of a polycrystalline antiferromagnetic (a.f.) material.

plies that the ions in a given  $\{110\}$  plane possess parallel spin alignment, whereas ions in the adjacent plane have antiparallel spins with respect to the first plane. Thus, the magnetic moments of the solid cancel each other and the material as a whole has no net magnetic moment.

Antiferromagnetic materials are paramagnetic above the Néel temperature  $T_N$ , i.e., they obey there a linear  $T = f(1/\chi)$  law (see Fig. 15.11). Below  $T_N$ , however, the inverse susceptibility may rise with decreasing temperature. The extrapolation of the paramagnetic line to  $1/\chi = 0$  yields a negative  $\theta$ . Thus, the Curie–Weiss law (15.2) needs to be modified for antiferromagnetics to read

$$\chi = \frac{C}{T - (-\theta)} = \frac{C}{T + \theta}. \quad (15.4)$$

The Néel temperature is often below room temperature (Table 15.2). Most antiferromagnetics are found among ionic compounds. They are insulators or semiconductors. Essentially no practical application for antiferromagnetism is known at this time. (See, however, the use of “canted” antiferromagnetics, described in Section 17.5, which are materials in which the magnetic moments of the two sublattices are not completely antiparallel. This results in a small net magnetization.)

Table 15.2. Characteristic Data for Some Antiferromagnetic Materials.

Substance	$T_N$ (K)	$-\theta$ (K)
MnO	116	610
$\text{MnF}_2$	67	82
$\alpha$ -Mn	100	?
FeO	198	570
NiO	523	~2000
CoO	293	330
Cr	310	?

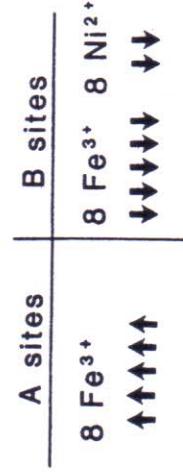


Figure 15.12. Distribution of spins upon A and B sites for the inverse spinel  $\text{NiO} \cdot \text{Fe}_2\text{O}_3$ . The spins within one site are arranged considering Hund's rule (Fig. 15.4). The iron ions are equally distributed among the A and B sites. The nickel ions are only situated on B sites. The relevance of the number of ions per unit cell is explained later on in the text.

Table 15.3. Calculated and Measured Number of Bohr Magnets for Some Ferrites.

Ferrite	Mn	Fe	Co	Ni	Cu
Calculated $\mu_B$	5	4	3	2	1
Measured $\mu_B$	4.6	4.1	3.7	2.3	1.3

The electron configuration of nickel in its atomic state is  $3d^84s^2$ . Two electrons are stripped in the  $\text{Ni}^{2+}$  ion so that eight  $d$ -electrons per atom remain. They are arranged, according to Hund's rule (Fig. 15.4), to yield two net magnetic moments (Fig. 15.12). All nickel ions are accommodated on the B sites. Nickel ferrite is thus expected to have two uncanceled spins, i.e., two Bohr magnetons (per formula unit), which is essentially observed (see Table 15.3).

The small discrepancy between experiment and calculation is believed to be caused by some contributions of orbital effects to the overall magnetic moment, and by a slight deviation of the distribution of metal ions on the A and B sites from that shown in Fig. 15.12.

The unit cell of cubic ferrites contains a total of 56 ions. Some of the metal ions are situated inside a *tetrahedron formed by the oxygen ions*. These are the above-mentioned A sites (Fig. 15.13(a)). Other metal ions are arranged in the center of an *octahedron* and are said to be on the B sites (Fig. 15.13(b)). The A and B sites are nestled inside a unit cell (Fig. 15.13(c)).

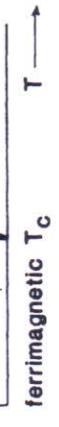


Figure 15.14. Schematic representation of the temperature dependence of the saturation magnetization,  $M_s$ , and the reciprocal susceptibility for ferrites.

Now, only 8 tetrahedral sites and 16 octahedral sites are occupied by metal ions. In  $\text{NiO} \cdot \text{Fe}_2\text{O}_3$  twice as many iron ions as nickel ions are present. Eight of the  $\text{Fe}^{3+}$  ions per unit cell occupy the A sites, eight of them occupy some of the B sites and the eight  $\text{Ni}^{2+}$  ions fill the remaining B sites (Fig. 15.12). This distribution is called an *inverse spinel structure* (in contrast to a *normal spinel*, such as for  $\text{ZnO} \cdot \text{Fe}_2\text{O}_3$ , in which all  $\text{Fe}^{3+}$  ions occupy the B sites).

The temperature dependence of most ferrimagnetics is very similar to ferrromagnetics (Fig. 15.14). The saturation magnetization decreases with increasing temperature until it vanishes at a Curie temperature,  $T_C$ . Above  $T_C$ , ferrimagnetics behave paramagnetically, having a nonlinear  $1/\chi$  versus  $T$  relationship.

In conclusion, this section described, in a mostly qualitative way, the difference between dia-, para-, ferro-, antiferro-, and ferrimagnetism. In the sections to come, we shall again pick up the different forms of magnetism and deepen our understanding of these phenomena by following essentially the train of thought brought forward by Langevin, Weiss, and Néel.

## 15.2. Langevin Theory of Diamagnetism

We shall now develop the classical theory of diamagnetism in a quantitative way as put forward by Langevin at the turn of the 20th century.

We stated before that the orbital motion of an electron about its nucleus induces a magnetic moment,  $\mu_m$ . We compared the latter with a magnetic moment which is created by a current passing through a loop-shaped wire. This magnetic moment is, naturally, larger, the larger the current,  $I$ , and the larger the area,  $A$ , of the orbit or loop:

$$\mu_m = I \cdot A = \frac{e}{t} A = \frac{e}{s/v} A = \frac{ev\pi r^2}{2\pi r} = \frac{evr^2}{2} \quad (15.5)$$

( $e$  is the electron charge,  $r$  is the radius of the orbit,  $s = 2\pi r$  = length of the orbit,  $v$  = velocity of the orbiting electrons, and  $t$  = orbiting time).

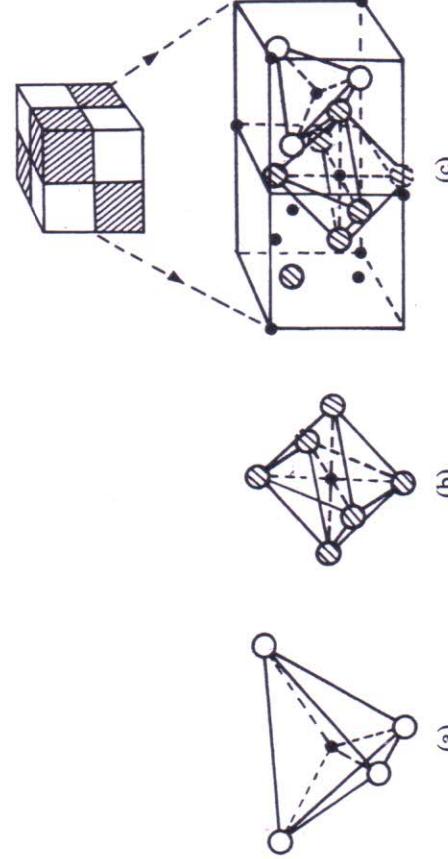


Figure 15.13. Crystal structure of cubic ferrites. The small filled circles represent metal ions, the large open or shaded circles represent oxygen ions: (a) tetrahedral or A sites; (b) octahedral or B sites; and (c) one-fourth of the unit cell of a cubic ferrite. A tetrahedron and an octahedron are marked. Adapted from J. Smit, and H.P.J. Wijn, *Ferrites*, Wiley, New York (1959).