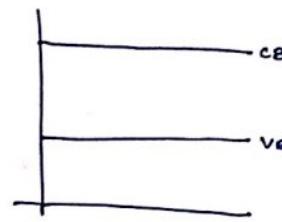
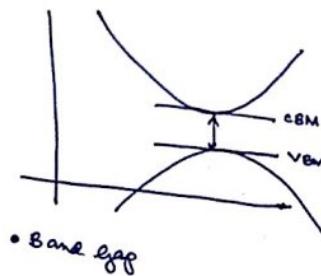


Dielectric Properties of Solids



- The dielectric materials are all insulators.

Electric Dipole: If two equal and opposite charges are separated by a finite distance, then it is said that the two charges form a dipole.

Dipole moment: $\vec{p} = q\vec{d}$



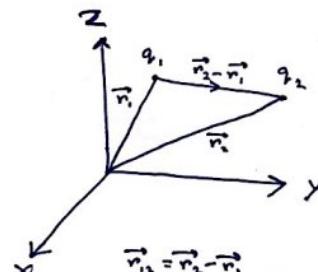
Unit: Coulomb m

$$1 \text{ Debye} = 3.33 \times 10^{-30} \text{ Coulomb m}$$

Coulomb's Law:

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_{12}|^2} \hat{r}_{12}$$

$$\begin{aligned} \epsilon_0 &= 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \\ \frac{1}{4\pi\epsilon_0} &= 9 \times 10^9 \text{ Nm}^2/\text{C}^2 \end{aligned}$$



$\epsilon_0 \rightarrow$ Free Space Permittivity

Electric Field:

$$\vec{E} = \frac{\vec{F}}{q_{12}} = \frac{q_1}{4\pi\epsilon_0 |\vec{r}_{12}|^2} \hat{r}_{12}$$

Electrostatic Potential:

$$V = - \int_{\infty}^r \vec{E} \cdot d\vec{r}$$

$$\vec{E} = - \left(\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)$$

$$= -\vec{\nabla} V$$

$$V = - \int_{\infty}^r \vec{E} \cdot d\vec{r}$$

$$= - \int_{\infty}^r \frac{q_1}{4\pi\epsilon_0 r^2} dr$$

~~if~~

~~then~~ \hat{Q}

$$= - \frac{q_1}{4\pi\epsilon_0} \int_{\infty}^r \frac{1}{r^2} dr$$

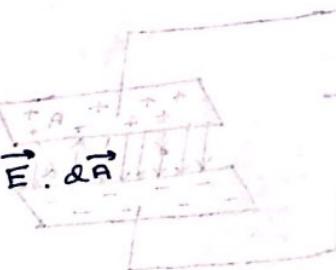
$$= + \frac{q_1}{4\pi\epsilon_0} \left[\frac{1}{r} \right]_{\infty}^r$$

~~neglecting tiny velocity of go~~ \rightarrow ~~neglecting tiny velocity of go~~

$$= + \frac{q_1}{4\pi\epsilon_0 r}$$

Electric Flux

is always ~~at~~ $\phi = \int \vec{E} \cdot d\vec{A}$
 surface flux ~~per~~
 • Flux Density



Gauss's Law:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

Proof:

$$d\phi = \vec{E} \cdot d\vec{A}$$

$$= E dA \cos \theta$$

$$\oint_S \vec{E} \cdot d\vec{A}$$

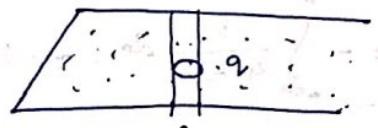
$$= \oint_S E dA \cos \theta$$

$$= \oint_S \frac{q}{4\pi\epsilon_0 r^2} dA \cos \theta$$

$$= \frac{q}{4\pi\epsilon_0} \oint_S \frac{dA \cos \theta}{r^2}$$

$$= \frac{q}{4\pi\epsilon_0} \oint_S d\Omega = \frac{q}{4\pi\epsilon_0} \cdot 4\pi = \frac{q}{\epsilon_0}$$

Electric field due to an infinite two dimensional charged sheet:

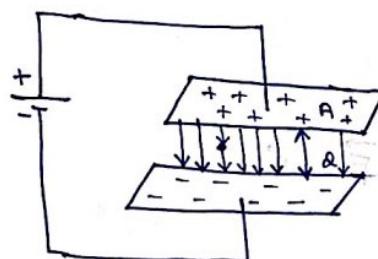


$$\oint \vec{E} \cdot d\vec{r} = \frac{q}{\epsilon_0}$$

$$\Rightarrow E \cdot 2A = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{2\epsilon_0 A} = \frac{\sigma}{2\epsilon_0}$$

Capacitance of a parallel plate capacitor:



The separation of the two fields is very small compared to dimensions of the plates.

$$E_0 = \frac{\sigma}{2\epsilon_0} - \left(-\frac{\sigma}{2\epsilon_0} \right) \quad \begin{matrix} \sigma = \text{Surface charge} \\ \text{density} = \frac{q}{A} \end{matrix}$$

$$= \frac{\sigma}{\epsilon_0}$$

$$= \frac{q}{A\epsilon_0}$$

Potential Difference between the two plates,

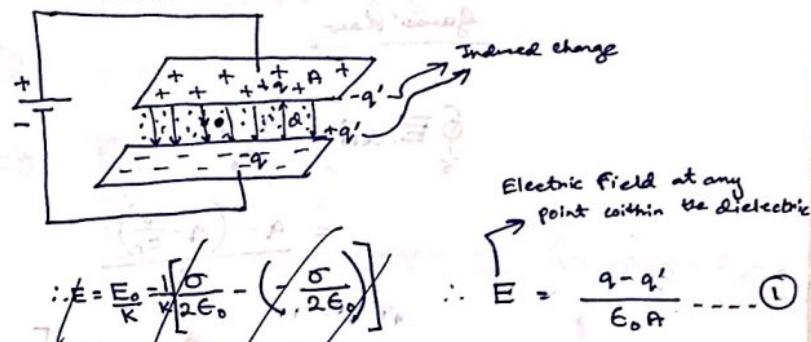
$$V_0 = \frac{qd}{A\epsilon_0}$$

$$\therefore C = \frac{q}{V_0}$$

$$= \frac{q}{\frac{qd}{A\epsilon_0}}$$

$$= \frac{A\epsilon_0}{d}$$

Dielectric field is present in between the capacitor plates:
 (dielectric constant = ϵ_r)



Potential difference between the two plates

$$V = Ed = \frac{q - q'}{\epsilon_0 A} \cdot d \quad \dots \textcircled{2}$$

According to Faraday,

$$\frac{C}{C_0} = \epsilon_r$$

C is capacitance with dielectric
 C_0 is the cap. without dielectric

$$\therefore C = \frac{q}{V}$$

Also, $C = \frac{q}{V}$

$$\therefore \frac{q}{V} = \frac{q}{\frac{q - q'}{\epsilon_0 A} \cdot d}$$

$$= \frac{\epsilon_0 A}{d} \cdot \frac{q}{q - q'}$$

$$= \frac{\epsilon_0 A}{d} \cdot \frac{q}{V_0}$$

$$= \frac{V_0}{V}$$

$$= \epsilon_r \therefore \boxed{\epsilon_r = \frac{\epsilon_0}{V/V_0}}$$

$\frac{q}{V_0} \dots \textcircled{1}$

$$\Rightarrow \frac{E}{E_0} = \frac{V}{V_0} = \frac{V}{V_0} = \frac{1}{\epsilon_r}$$

$\frac{q}{V_0} \dots \textcircled{2}$

$$\Rightarrow E = \frac{E_0}{\epsilon_r}$$

∴ In $\textcircled{1}$,
 and difference between the two plates

$$E = \frac{q - q'}{\epsilon_0 A}$$

$$\Rightarrow \frac{E_0}{\epsilon_r} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A}$$

$$\Rightarrow \frac{q}{\epsilon_0 A \epsilon_r} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \Rightarrow \frac{q}{\epsilon_r} = q - q'$$

$$\therefore q' = q - \frac{q}{\epsilon_r}$$

Amount of induced charge

Gauss' Law

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q - q'}{\epsilon_0}$$

$$= \frac{q - (q - \frac{q}{\epsilon_r})}{\epsilon_0}$$

$$= \frac{q}{\epsilon_r \epsilon_0} = \frac{q}{\epsilon_0} \quad \left[\begin{array}{l} \text{where } \epsilon = \epsilon_r \epsilon_0 \text{ is the} \\ \text{absolute permittivity} \end{array} \right]$$

Now,

$$q' = q - \frac{q}{\epsilon_r}$$

$$\Rightarrow q = \frac{q}{\epsilon_r} + q'$$

$$\Rightarrow \frac{q}{A} = \frac{q}{A \epsilon_r} + \frac{q'}{A}$$

$$\Rightarrow \sigma = \epsilon_0 \left(\frac{q}{\epsilon_r \epsilon_0 A} \right) + \left(\frac{q'}{A} \right) \quad \begin{array}{l} \xrightarrow{\text{polarization (P)}} \text{Induced} \\ \text{charge per unit surface} \\ \text{area} \end{array}$$

$$\Rightarrow \sigma = \epsilon_0 E + P$$

Unit: C/m²

Electric Displacement Vector:

$$\cancel{\vec{D}} = \epsilon_0 \vec{E}$$

$$\boxed{\vec{D} = \epsilon_0 \vec{E} + \vec{P}} \quad \dots \dots (1)$$

- \vec{D} is numerically equal to σ .
- If there is no dielectric material, i.e. in the free space, $\vec{D} = \epsilon_0 \vec{E}$ and $\sigma = \epsilon_0 E$.
L... (2)

Within a dielectric medium of dielectric constant ϵ_r ,

$$\vec{D} = \epsilon_r \epsilon_0 \vec{E} \quad \dots \dots (3)$$

Put equation (3) in (1),

$$\epsilon_r \epsilon_0 \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

$$\Rightarrow \vec{P} = \epsilon_r \epsilon_0 \vec{E} - \epsilon_0 \vec{E}$$

$$\Rightarrow \vec{P} = \epsilon_0 (\epsilon_r - 1) \vec{E}$$

$$\Rightarrow \frac{\vec{P}}{\epsilon_0 \vec{E}} = (\epsilon_r - 1)$$

$\hookrightarrow \text{Dielectric Susceptibility: } X = \frac{\vec{P}}{\epsilon_0 \vec{E}} = \epsilon_r - 1$

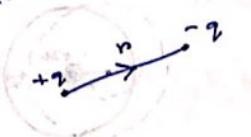
Susceptibility:

It represents how easily and upto what extent a dielectric material can be polarized with the application of external electric field.

Polarization

Dipole moment:

$$\vec{P} = q\vec{r}$$



Polarization: It is defined as the total dipole moment of a material per unit volume.

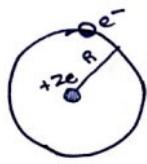
$$\vec{P} = \frac{\sum_{i=1}^N \vec{P}_i}{V}$$

$$= \frac{N \vec{P}_0}{V} = n \vec{p} \quad [n = \frac{N}{V} \rightarrow \text{number density}]$$

Mechanisms of Polarization

1. Electronic Polarization
2. Ionic Polarization
3. Orientation Polarization
4. Space charge Polarization

1. Electronic Polarization



atomic number = Z

Consider the atom of an inert gas.

Let the radius of the atom be R ; and the atomic number be Z .

see below

Considering the spherical shape of atom, the volume charge density of the atom ~~is~~ is given by

$$\text{electronic density} \rho = -\frac{Ze}{\frac{4}{3}\pi R^3}$$

Applying electric field to the atom,



- The nucleus does not shift as it is a lot heavier. Only the electronic cloud shifts.

Taking x (x is the shift) as the radius, draw an imaginary sphere around the negative charge centre of the shifted sphere.

Now, the nucleus is situated on the surface of the imaginary surface.



The charge inside the sphere of radius x ,

$$q = \frac{4}{3}\pi x^3 \rho$$

$$= \frac{4}{3}\pi x^3 \left(-\frac{Ze}{\frac{4}{3}\pi R^3} \right)$$

$$= -\frac{Zex^3}{R^3}$$

$$E \propto \frac{q}{r^2}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$Eq = \frac{q^2}{4\pi\epsilon_0 R^2}$$

The attractive force between the shifted electronic charge distribution and the nucleus is given by

$$\text{Coulomb's Force} \quad F_c = \frac{Ze}{4\pi\epsilon_0 r^2} \left(-\frac{Zex^3}{R^3} \right)$$

$$= -\frac{Zex}{4\pi\epsilon_0 R^3}$$

The force on the electronic charge distribution due to the external electric field is given by

$$\text{Lorentz Force} \quad F_L = -ZeE$$

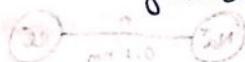
At equilibrium condition, these two forces balance each other.

$$F_L = F_c$$

$$\Rightarrow -ZeE = -\frac{Zex^2}{4\pi\epsilon_0 R^3}$$

$$\Rightarrow x = \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

x is the separation.



When the applied electric field is of the order of 30 kV per metre, then the separation between the nucleus and the electronic charge centre is of the order of 10^{-14} m.

i. the dipole moment of the dipole is given by

$$p = Ze x$$

$$= Ze \frac{4\pi\epsilon_0 R^3}{Ze} E$$

$$= 4\pi\epsilon_0 R^3 E$$

For a particular atom, R is constant.

$$\therefore p \underset{\text{proportional}}{\sim} E$$

$$\Rightarrow p = \alpha_E E$$

where $\alpha_E = 4\pi\epsilon_0 R^3$ is called the electronic polarisability.

of the atom.
against motion of dipole and motion along dipole axis
and against motion of dipole axis.

Polarization:

Let there are N number of atoms per unit volume
within a material.

Then the electronic polarization is given by

$$P_e = N \alpha_e E$$

---> Total number of molecules
per unit volume

Susceptibility due to electronic polarization

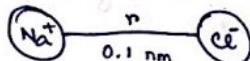
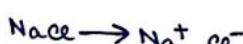
$$\chi_e = \frac{P_e}{\epsilon_0 E}$$

$$= \frac{N \alpha_e E}{\epsilon_0 E}$$

$$\chi_e = \frac{N \alpha_e}{\epsilon_0}$$

2. Ionic Polarization

• Ionic Polarization < Electronic
Polarization



Polarization

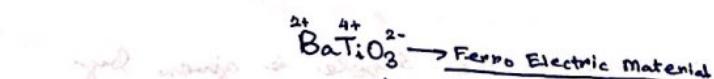
$$P_i = N p_i \rightarrow \text{Dipole}$$

$$P_i = \epsilon r$$

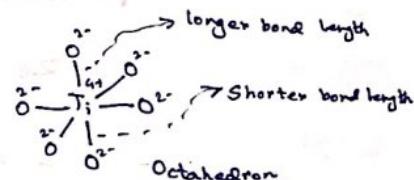
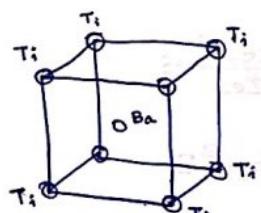
where r is the distance between the ions

$$P_i = \alpha_i E$$

where α_i is the ionic polarisability



Spontaneous electric dipole moment

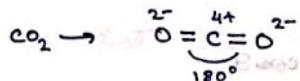
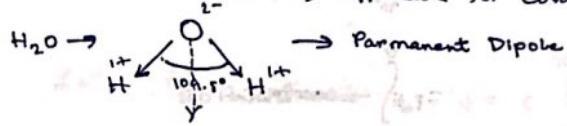


$$P_e + P_i = N \alpha_e E + N \alpha_i E$$

$$= NE(\alpha_e + \alpha_i)$$

Orientation Polarization

Applicable for Covalent compounds



Saturation Polarization:

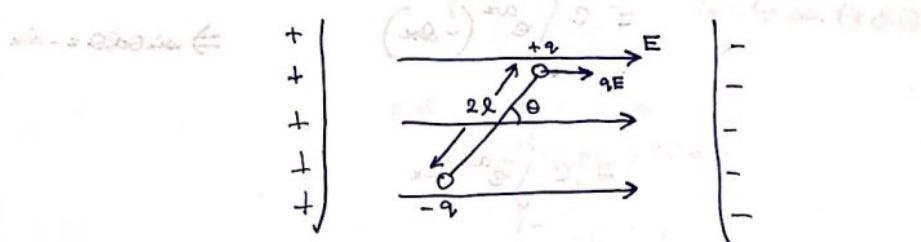
Not Possible at room temperature

N = Number of Molecules per unit Volume

P_m = Molecular Dipole Moment

Due to finite Temperature, the molecules have a thermal energy and due to this thermal energy, the randomness of the molecules increase. If the potential energy of the molecules be W and the temperature is T , the probability of getting a molecule ~~within~~ with this condition is determined by the Boltzmann distribution function $e^{-W/kT}$.

Potential energy of a molecule due to an applied electric field



$$\text{Dipole Moment } P_m = 2qL$$

$$\text{Torque, } \vec{\tau} = \vec{r} \times \vec{F}$$

$$= 2qL E \sin\theta$$

$$= P_m E \sin\theta$$

∴ Potential energy

$$W = \int_{90^\circ}^0 \tau d\theta$$

$$= \int_{90^\circ}^0 P_m E \sin \theta d\theta$$

$$= P_m E \int_{90^\circ}^0 \sin \theta d\theta$$

$$= -P_m E \cos \theta$$

i. the number of molecules per unit volume having orientations between θ and $\theta + d\theta$ is given by

$$dN \propto e^{-\frac{W}{KT}} \sin \theta d\theta$$

$$\Rightarrow dN = C e^{-\frac{W}{KT}} \sin \theta d\theta \quad [C \text{ is a constant}]$$

$$\Rightarrow dN = C e^{-\frac{P_m E \cos \theta}{KT}} \sin \theta d\theta$$

∴ the total number of molecules having orientations between 0 to π is given by

$$N = \int dN = \int_0^\pi C e^{-\frac{P_m E \cos \theta}{KT}} \sin \theta d\theta$$

$$\text{Let } \frac{P_m E}{KT} = a$$

$$\therefore N = C \int_0^\pi e^{a \cos \theta} \sin \theta d\theta$$

$$= C \int_{-1}^1 e^{ax} (-dx)$$

$$= C \int_{-1}^1 e^{ax} dx$$

$$= C \left[\frac{e^{ax}}{a} \right]_{-1}^1$$

$$= \frac{c}{a} (e^a - e^{-a})$$

$$\Rightarrow C = \frac{Na}{e^a - e^{-a}}$$

Orientational Polarization (Langevin's Theory)

Total number of molecular dipole per unit volume within $0 \text{ to } \pi$.

$$N = \int dN = \int_0^\pi c e^{\frac{pE \cos \theta}{kT}} \sin \theta d\theta$$

$$\text{Let } \frac{pE}{kT} = a. \text{ Then, } N = c \int_0^\pi e^{a \cos \theta} \sin \theta d\theta$$

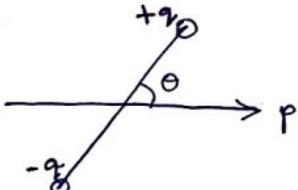
$$\text{Let } \cos \theta = x \Rightarrow -\sin \theta d\theta = -dx$$

(Previous class)

The dipole moment per unit volume for the molecules having orientations between θ and $\theta + d\theta$

$$dP = p \cos \theta dN$$

$$= p \cos \theta c e^{\frac{pE \cos \theta}{kT}} \sin \theta d\theta$$



The polarization for all the molecules oriented from 0 to π ,

$$P = \int dP = \int_0^\pi p c \cos \theta e^{\frac{pE \cos \theta}{kT}} \sin \theta d\theta$$

$$\text{Let } \frac{pE}{kT} = a = (a)$$

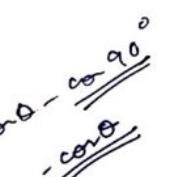
$$P = pc \int_0^\pi e^{ax} \cos \theta \sin \theta d\theta$$

$$\text{Let } \cos \theta = x$$

$$-\sin \theta d\theta = dx$$

$$\therefore P = pc \int_{-1}^1 e^{ax} x (-dx)$$

$$= pc \int_{-1}^1 e^{ax} x dx$$



$$\left[\int_{-1}^1 e^{ax} dx \right]$$

$$\begin{aligned}
 &= p c \left[x \int e^{ax} dx - \int \frac{d}{dx} \int e^{ax} dx \right] \\
 &= p c \left[\frac{x e^{ax}}{a} - \int \frac{e^{ax}}{a} \right] \\
 &= p c \left[\frac{x e^{ax}}{a} - \frac{e^{ax}}{a^2} \right] \\
 &= p c \left[\left(\frac{e^a}{a} - \frac{e^{-a}}{a^2} \right) - \left(-\frac{e^{-a}}{a} - \frac{e^{-a}}{a^2} \right) \right]
 \end{aligned}$$

where at $x = 0$, under zero neg. T from diagram all

$$\begin{aligned}
 P &= p \frac{aN}{e^a - e^{-a}} \left[\frac{e^a + e^{-a}}{a} - \frac{e^a - e^{-a}}{a^2} \right] \\
 &= N p \left[\frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a} \right] \\
 &= N p \left[\coth(a) - \frac{1}{a} \right]
 \end{aligned}$$

$\sinh x = \frac{e^x - e^{-x}}{2}$
 $\cosh x = \frac{e^x + e^{-x}}{2}$

Diagram cylinder at $x = 0$ mark

$$L(a) = \left[\coth(a) - \frac{1}{a} \right] \Rightarrow \text{this is called Langer's Function}$$

$$\begin{aligned}
 L(a) &= \coth(a) - \frac{1}{a} \\
 &= \frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a}
 \end{aligned}$$

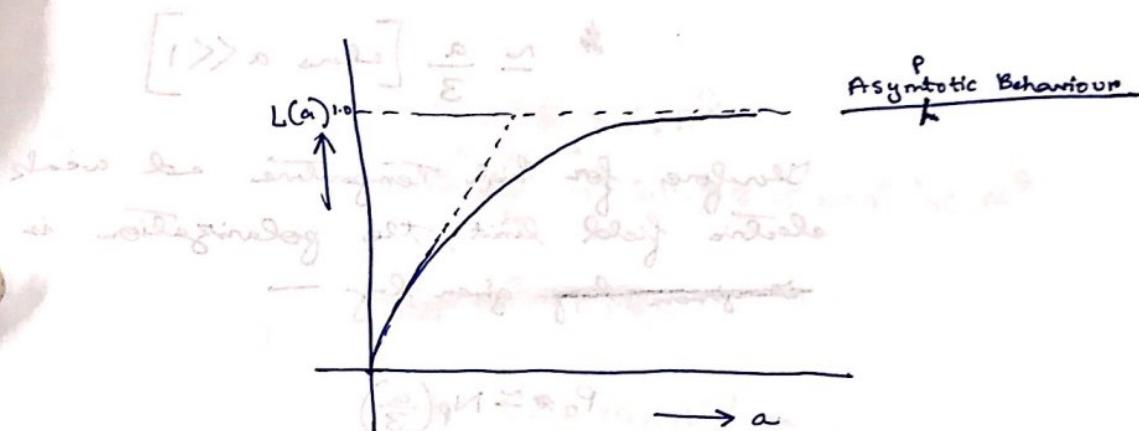
where $a = \frac{pE}{KT}$

When the applied electric field $\rightarrow (E)$ is very high and the temperature is low,

$$a = \frac{PE}{KT} \gg 1$$

$$L(a) = \frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a} \approx 1$$

$$P_s \approx N_p$$



When the applied electric field is low and the temperature is high.

$$a = \frac{PE}{KT} \ll 1$$

$$\therefore L(a) = \frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a}$$

$$\frac{e^x}{e^{-x}} = \frac{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots}{1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\dots}$$

$$L(a) = \frac{\left(1+a+\frac{a^2}{2!}+\frac{a^3}{3!}\right) + \left(1-a+\frac{a^2}{2!}-\frac{a^3}{3!}\right)}{\left(1+a+\frac{a^2}{2!}+\frac{a^3}{3!}\right) - \left(1-a+\frac{a^2}{2!}-\frac{a^3}{3!}\right)}$$

$$= \frac{2\left(1+\frac{a^2}{2}\right)}{2\left(a+\frac{a^3}{6}\right)} - \frac{1}{a}$$

With you in (2) taking into account both terms

$$\begin{aligned} \text{vol. } &= \frac{1 + \frac{a^2}{2} - \left(1 + \frac{a^2}{6}\right)}{a\left(1 + \frac{a^2}{6}\right)} = \frac{1}{9} \\ 1 &\approx \frac{1}{2} - \frac{a^2/2}{3a^2/6} = \frac{a^2}{3a\left(1 + \frac{a^2}{6}\right)} = \frac{a^2}{3a^2 - \frac{a^2}{6}} = \frac{2a^2}{9} \end{aligned}$$

$$\therefore P \approx \frac{a}{3} \quad [\text{where } a \ll 1]$$

Therefore for high Temperature and weak electric field limit, the polarization is given by —

$$P_0 \approx N_p \left(\frac{a}{3}\right)$$

$$= N_p \left(\frac{pE}{3KT}\right)$$

$$= \frac{N_p^2 E}{3KT}$$

Polarizability

$$p = N \alpha E$$

$$\alpha_0 = \frac{P_0}{NE} = \frac{N_p^2 E}{3KT} \cdot \frac{1}{NE} = \frac{p^2}{3KT}$$

$$\boxed{\alpha_0 = \frac{p^2}{3KT}}$$

Susceptibility

$$\begin{aligned} \chi_0 &= \frac{P_0}{\epsilon_0 E} = \frac{N_p^2 E}{3KT} \times \frac{1}{\epsilon_0 E} \\ &= \frac{N_p^2}{3\epsilon_0 K T} \end{aligned}$$

Problems

$\epsilon_0 \times 289.0 = \mu_0 \times \rho_0$ for polarizability $\alpha \propto \frac{1}{\epsilon_r - 1}$

1. Calculate the value of the radius of electron orbit of helium and hence the permittivity for $2.7 \times 10^{25} \text{ atom/m}^3$.
 [Given the electronic polarisability of He = $0.13 \times 10^{-40} \text{ F m}^{-2}$]

(Given) $\rho_0 = 2.7 \times 10^{25} \text{ atom/m}^3$

$$\alpha \propto = 0.13 \times 10^{-40}$$

$$\epsilon = \frac{\rho_0}{2.7 \times 10^{25}}$$

$$N = 2.7 \times 10^{25}$$

$$\alpha \propto = \frac{N e^2}{4 \pi \epsilon_0 R^3}$$

$$\Rightarrow \cancel{\alpha} = 4 \times 3.14 \times 8.85 \times 10^{-12} \times R^3$$

$$\cancel{\alpha} = 1.169$$

$$\Rightarrow R = 5.89 \times 10^{-11} \text{ m}$$

$$\frac{R}{\lambda_0} = 30 \text{ (approx)}$$

$$(\text{Ans}) \text{ Radius of orbit} = \epsilon_r - 1 = X_e$$

$$\Rightarrow \epsilon_r - 1 = \frac{\rho_e}{\epsilon_0 E}$$

$$\Rightarrow \epsilon_r - 1 = \frac{N \rho e}{\epsilon_0 \cancel{E}}$$

$$\Rightarrow \epsilon_r = \frac{N \rho e}{\epsilon_0} + 1$$

$$\text{Ans} \rightarrow \frac{N \rho e}{\epsilon_0} = \frac{2.7 \times 10^{25} \times 0.13 \times 10^{-40}}{8.85 \times 10^{-12}} = 1.000039661$$

$$\frac{N \rho e}{\epsilon_0} = \frac{2.7}{8.85} = X_e \approx 0.302 \text{ (approx)}$$

$$\approx 1.00004 \text{ (Ans)}$$

2. If the susceptibility of CO_2 is 0.995×10^{-3} , calculate the total polarization of CO_2 if its density ρ is 1.977 kg/m^3 .

$$\chi = 0.995 \times 10^{-3}$$

1 molecule $\rightarrow 6.023 \times 10^{23}$

$$\rho = 1.9777 \text{ kg/m}^3$$

$$\chi_{\text{eff}} = \frac{\rho}{\epsilon_0 E}$$

1 mol of CO_2 contains N_A molecules

44 g of CO_2 contains 6.023×10^{23} molecules.

$$= \frac{N_A E}{\epsilon_0 E} \quad 1977 \text{ g CO}_2 \text{ contains } \frac{6.023 \times 10^{23}}{44} \times 1977$$

$$= \frac{N_A}{\epsilon_0}$$

$= 2.7 \times 10^{25}$ molecules

$$\therefore N = 2.7 \times 10^{25} / \text{m}^3$$

$$\Rightarrow \alpha = \frac{\epsilon_0 \chi}{N}$$

$$\chi = 3.22 \times 10^{-140} \text{ F m}^2 \text{ (nm)}$$

$$\frac{\sigma}{\epsilon_0} = 1 - \alpha$$

3. In a dielectric material $E_x = 5 \text{ V/m}$ and

$$\vec{P} = \frac{1}{10\pi} (3\hat{i} - \hat{j} + 4\hat{k}) \text{ nC/m}^2$$

Calculate i) χ_e , ii) \vec{E} , and iii) \vec{D} .

$$\text{i)} \chi_{\text{eff}} = 5 \text{ V/m}$$

$$P_x = \frac{3}{10\pi} \text{ nC/m}^2 = \frac{3}{10\pi} \times 10^{-9} \text{ C/m}^2$$

$$\therefore \chi_e = \frac{P_x}{\epsilon_0 E_x} = \frac{3 \times 10^{-9} \times \frac{1}{5} \times 10^{-9}}{10\pi} \\ = 2.158 \text{ (Nm/V)}$$

$$\text{ii) } \vec{P} = \epsilon_0 X_e \vec{E}$$

$$\Rightarrow \vec{E} = \frac{\vec{P}}{X_e \epsilon_0} = \frac{1}{10\pi} (3\hat{i} - \hat{j} + 4\hat{k}) \times 10^{-9} \times \frac{1}{2.158} \times \frac{1}{8.85 \times 10^{-12}}$$

$$= 1.66 (3\hat{i} - \hat{j} + 4\hat{k}) \text{ V/m}$$

$$\text{iii) } \vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$= 8.85 \times 10^{-12} \left[1.66 (3\hat{i} - \hat{j} + 4\hat{k}) \right] \cancel{+} + \frac{1}{10\pi} (3\hat{i} - \hat{j} + 4\hat{k}) 10^{-9}$$

$$= 0.0318 (3\hat{i} - \hat{j} + 4\hat{k})$$

$$= 0.0464 \times 10^{-9} (3\hat{i} - \hat{j} + 4\hat{k}) \text{ C/m}^2$$

(Ans)

4. Dielectric constant of a gas at NTP is 1.00074.
 Calculate the dipole moment of each atom
 of ~~the~~ the gas atom when it is held
 at an external field of $3 \times 10^5 \text{ V/m}$.

Dipole moment

$$p = \frac{P}{N} \xrightarrow{\text{Polarization}}$$

\rightarrow
molar

$$P = \epsilon_0 X E$$

$$= \epsilon_0 (\epsilon_r - 1) E$$

$$= 8.85 \times 10^{-12} \times (1.00074 - 1) \times 3 \times 10^5$$

$$= 1.9647 \times 10^{-10}$$

$$22.4 \text{ litre} \rightarrow N_A$$

$$1 \text{ litre} \rightarrow \frac{N_A}{22.4}$$

$$1 \text{ m}^3 \rightarrow \frac{N_A}{22.4 \times 10^{-3}}$$

$$\therefore N = \frac{6.023 \times 10^{23}}{22.4 \times 10^{-3}}$$

$$= 2.69 \times 10^{25} / \text{m}^3$$

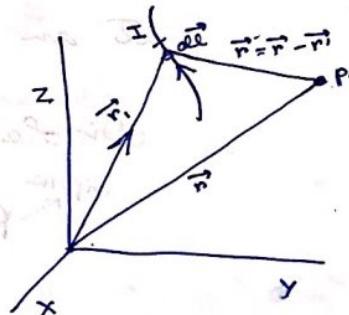
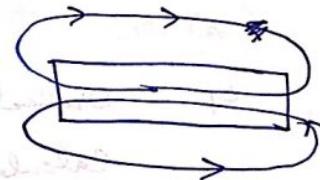
$$\therefore P = \frac{P}{N}$$

$$= 7.3 \times 10^{-36} \text{ cm} (\text{Ans})$$

Magnetic Properties of Solids

Properties of magnetism/magnetic materials:

1. Like poles of magnets repel each other and unlike poles attract each other.
2. Magnetic lines of force do not intersect each other and they are closed curves. The number of lines of force passing through unit cross sectional area normally is defined as magnetic flux density.
3. Magnetic monopoles do not exist.



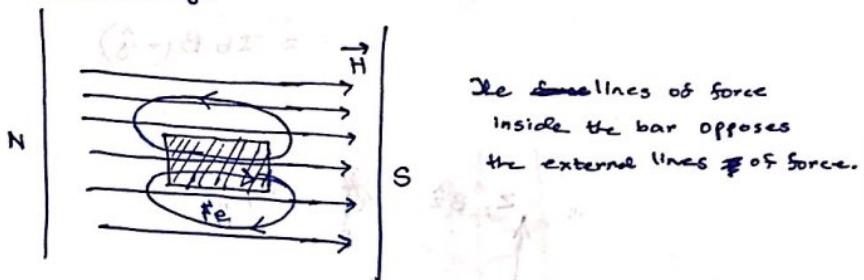
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I d\vec{r}_i (\vec{r}_i) \times \vec{r}}{r^3}$$

$$\nabla \cdot \vec{B} = 0$$

$$\int (\nabla \vec{B}) d\vec{s} = 0$$

$$\Rightarrow \oint \vec{B} \cdot d\vec{s} = 0$$

Magnetic Permeability:



Let an iron bar is kept in a uniform magnetic field.

Magnetic flux density = \vec{B}

Magnetic field intensity = \vec{H}

$$\vec{B} \propto \vec{H}$$

$$\Rightarrow \vec{B} = \mu_0 \vec{H}$$

$$\Rightarrow \vec{B} = \mu \vec{H} \quad [\mu = \text{magnetic property}]$$

... ①

In vacuum,

$$\vec{B} = \mu_0 \vec{H}$$

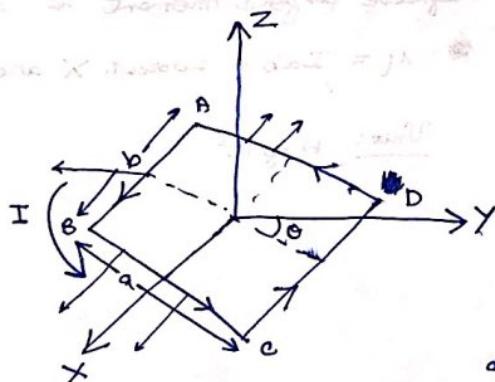
$\mu_0 = \frac{\text{space}}{\text{permeability}}$

$$\frac{\vec{B}_{\text{inside}}}{\vec{B}_0} = \frac{\mu \vec{H}}{\mu_0 \vec{H}} = \frac{\mu}{\mu_0} = \mu_r$$

$\mu_r \rightarrow \text{Relative Permeability}$

Magnetic Dipole Moment and Torque for a rectangular current loop:

in a uniform magnetic field B_0 along \hat{x} direction and I is the current.



First the loop was placed in the xy plane.

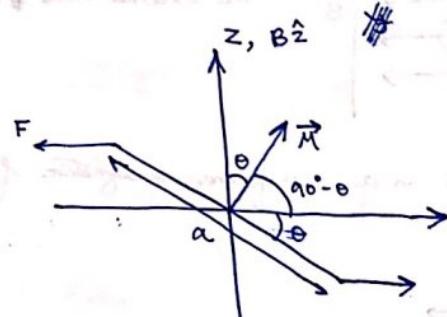
Then it was tilted in the zy direction, so that it makes an angle theta with the plane.

Force on AB,

$$F = I \vec{A} \times \vec{B}$$

$$\vec{F} = I b \hat{x} \times B \hat{z}$$

$$= I b B (-\hat{y})$$



$$\text{Torque } \vec{\tau} = a F \sin \theta \hat{x}$$

$$= a (I b B) \sin \theta \hat{x}$$

$$= (\mu_{ab}) B \sin \theta \hat{x}$$

$$= \mu B \sin \theta \hat{x}$$

where $\mu = I ab$.

$$\therefore \vec{\tau} = \mu \times \vec{B}$$

$$\left[\begin{array}{l} \text{For electric field,} \\ \vec{\tau} = \vec{P} \times \vec{E} \end{array} \right]$$

∴ Magnetic dipole moment is defined as

$$\mu = I ab = \text{current} \times \text{area}$$

Unit: Amp m²

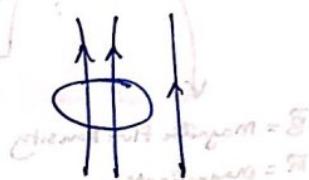
Relation among magnetic field intensity (\vec{H}), magnetic flux density and magnetization of a material.

Magnetization (\vec{M}): It represents total magnetic dipole moment of a material per unit volume.

(If current is flowing) $\vec{B} = (\mu_0 \vec{M}) + \vec{H}$

From Ampere's Law, $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$



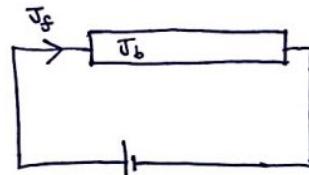
Current in loop $I = 8$
residual current $= 5I$

$$\Rightarrow \oint_s (\nabla \times \vec{B}) \cdot d\vec{s} = \mu_0 \int_{\text{cont}} \vec{J} \cdot d\vec{s} \quad [I = \int \vec{J} \cdot d\vec{s}]$$

$$\Rightarrow \nabla \times \vec{B} = \mu_0 \vec{J}_{enc}$$

When a current is flowing through a metal, we may write $\vec{J}_{enc} = \vec{J}_f + \vec{J}_b$,

where \vec{J}_f is called the free current density,



\vec{J}_b is called the bound current density.

The bound current density (\vec{J}_b) has the origin of atomic magnetic dipole moment.

$$\vec{J}_b = \nabla \times \vec{M}$$

where \vec{M} is the intensity of magnetization of the material.

$$\therefore \nabla \times \vec{B} = \mu_0 (\vec{J}_f + \vec{J}_b)$$

$$\Rightarrow \nabla \times \vec{B} = \mu_0 (\vec{J}_f + \nabla \times \vec{M})$$

$$\Rightarrow \frac{1}{\mu_0} (\nabla \times \vec{B}) = \vec{J}_f + \nabla \times \vec{M}$$

$$\Rightarrow \nabla \times \frac{\vec{B}}{\mu_0} - \nabla \times \vec{M} = \vec{J}_f$$

$$\Rightarrow \nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_s$$

Diagram below shows a rectangular loop of area A.

Let $\left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{H}$ (Intensity of magnetic field) ... ①

$\therefore \nabla \times \vec{H} = \vec{J}_s$ → Differential form of Ampere's Law

\vec{B} = magnetic flux density
 \vec{M} = magnetization

From ①,

$$\frac{\vec{B}}{\mu_0} = \vec{H} + \vec{M}$$

$$\Rightarrow \vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\Rightarrow \vec{B} = \mu_0 \vec{H} \left(1 + \frac{\vec{M}}{\vec{H}} \right)$$

$$\Rightarrow \frac{\vec{B}}{\vec{H}} = \mu_0 \left(1 + \frac{\vec{M}}{\vec{H}} \right) \dots \dots \textcircled{2}$$

We know $\frac{\vec{B}}{\vec{H}} = \mu \dots \dots \textcircled{3}$

Magnetic susceptibility

$$X = \frac{\vec{M}}{\vec{H}}$$

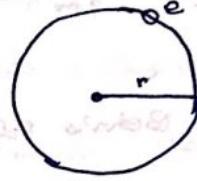
Magnetisation per unit magnetic field

From ②,

$$\therefore \mu = \mu_0 \left(1 + X \right)$$

$$\Rightarrow \frac{\mu}{\mu_0} = \left(1 + X \right)$$

$$\Rightarrow \boxed{\mu_r = (1 + X)} \quad [\mu_r = \text{relative permeability}]$$



Orbital Current

$$I = -\frac{e}{T}$$

Let the velocity of the electron be v .

$$\therefore T = \frac{2\pi r}{v}$$

$$I = -\frac{ev}{2\pi r}$$

The associated (orbital) magnetic moment

$$M_{ee} = I(\pi r^2)$$

$$= -\frac{ev}{2\pi r} \pi r^2 = -\frac{evr}{2} \quad \text{--- ①}$$

Orbital angular momentum of the electron

$$L = mvr \quad [m \text{ is the mass of the electron}]$$

By eq(1) / eq(2),

$$\frac{M_{ee}}{L} = -\frac{evr}{2} \times \frac{1}{mvr} = -\frac{e}{2m}$$

$$\therefore \frac{M_{ee}}{L} = -\frac{e}{2m}$$

↓
Orbital gyromagnetic ratio

$$\gamma = \frac{M_{ee}}{L} = -\frac{e}{2m}$$

Angular Momentum and its Quantization

$$\Rightarrow M_{el} = -\frac{e}{2m} \times L \quad \dots \dots \textcircled{3}$$

According to Bohr's Atomic Orbital Theory,

$$L = \frac{nh}{2\pi} \quad \left[n=1, 2, 3, 4, \dots \right]$$

... $\textcircled{4}$

Cutting $\textcircled{4}$ in $\textcircled{3}$,

$$M_{el} = -\frac{e}{2m} \times \frac{nh}{2\pi}$$

$$= -n \left(\frac{eh}{4\pi m} \right)$$

Quantization of Magnetic moment
Unit of M_B : Amp m²

$$\Rightarrow M_{el} = -nM_B$$

where $M_B = \frac{eh}{4\pi m}$ is called Bohr magneton.

Origin of magnetic moment (quantum theory)

1. Principal quantum number (n):

$$n = 1, 2, 3, 4, \dots$$

Role: It determines the energy of the electron

2. Orbital Quantum Number (l):

$$l = 0, 1, 2, 3, 4, \dots n-1$$

Role: It determines the magnitude of orbital angular momentum.

$$L = |\vec{l}| = \sqrt{l(l+1)} \hbar \quad \left[\hbar = \frac{h}{2\pi} \right]$$

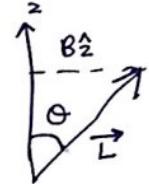
3. Magnetic Quantum Number (m_l):

$$m_l = -l, -l+1, \dots, -1, 0, 1, \dots, l-1, l$$

$$\therefore \text{Total} = 2l + 1$$

Role: Determines the orientations (directions) of orbital angular momentum of electron.

$L_z = m_l \hbar$ \vec{L} component of the orbital angular momentum



Space Quantization of angular momentum:

$$L_z = L \cos \theta$$

$$\Rightarrow \cos \theta = \frac{L_z}{L} = \frac{m_l \hbar}{\sqrt{l(l+1)} \hbar}$$

$$\Rightarrow \cos \theta = \frac{m_l}{\sqrt{l(l+1)}}$$

For p orbital, $l=1$, $m_l = -1, 0, +1$

When $m_l = -1$

$$\cos \theta = \frac{-1}{\sqrt{1(1+1)}} = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = 135^\circ$$

When $m_l = 0$

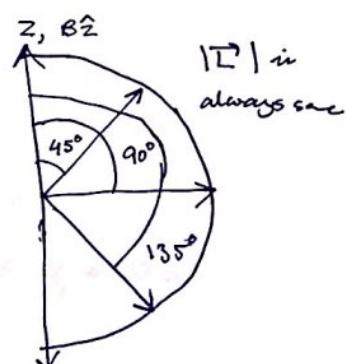
$$\cos \theta = 0$$

$$\Rightarrow \theta = 90^\circ$$

When $m_l = +1$

$$\cos \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = 45^\circ$$



4. Spin Quantum Number (s):

$$S = \frac{1}{2} \text{ for an electron}$$

Role: It determines the magnitude of spin angular momentum of an electron.

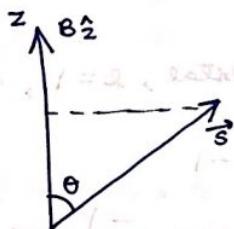
$$S = |\vec{S}| = \sqrt{s(s+1)}\hbar$$

5. Spin Magnetic Quantum Numbers (m_s):

$$m_s = +\frac{1}{2}, -\frac{1}{2}$$

Role: Determines the direction of the spin angular momentum.

$$\vec{S}_z = m_s \hbar$$



$$S_z = S \cos \theta$$

$$\Rightarrow \cos \theta = \frac{S_z}{S}$$

$$= \frac{m_s \hbar}{\sqrt{s(s+1)} \hbar}$$

$$= \frac{m_s}{\sqrt{s(s+1)}}$$

When $m_s = +\frac{1}{2}$,

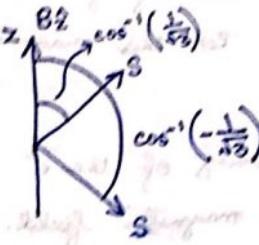
$$\cos \theta = \frac{\frac{1}{2}}{\sqrt{\frac{1}{2}(\frac{1}{2}+1)}}$$

$$= \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\cos \theta = \frac{-\frac{1}{2}}{\sqrt{\frac{1}{2}(\frac{1}{2}+1)}} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1} \left(-\frac{1}{\sqrt{3}} \right)$$



Magnetic Materials

Diamagnetic
(experience repulsive force)

Paramagnetic
(experience quick attractive force when kept in magnetic field)

Antiferromagnetic

Ferromagnetic

Ferrimagnetic

Ferromagnetic:

$$\text{Spin } \omega_{\text{spin}} = \frac{eB}{m_e^2} = \frac{eB}{m_e^2 \cdot 4\pi \rho_0 M_p} = \omega$$

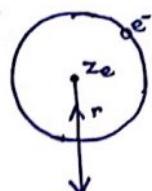
$$\Omega = \frac{eB}{m_e^2 \cdot 4\pi \rho_0 M_p} = \omega_{\text{spin}} + \omega_{\text{ext}}$$

$$\Omega = \omega_{\text{ext}} + \omega_{\text{spin}}$$

Antiferromagnetic:

spin magnetic moment cancel each other.

Diamagnetic Material:



$$\frac{mv_e^2}{r} = \frac{Ze \cdot e}{4\pi \epsilon_0 r^2} \quad [v_e = \text{velocity of electron}]$$

$$\Rightarrow \frac{mv_e^2}{r} = \frac{Ze^2}{4\pi \epsilon_0 r^2}$$

$$V_e = r \omega_0$$

$$\therefore \frac{mv_e^2 \omega_0^2}{r} = \frac{Ze^2}{4\pi \epsilon_0 r^2}$$

$$\Rightarrow \omega_0^2 = \frac{Ze^2}{4\pi\epsilon_0 n^3}$$

$\left[\omega_0 = \text{angular velocity when magnetic field is not applied} \right]$

Let electron's orbit lies in xy plane and a magnetic field B is applied along Z direction.

$F_L = -evB$ (the velocity of the electron will change due to this magnetic force or Lorentz Force)

$$= -e\omega r B$$

$\omega \rightarrow$ angular velocity of the e^- when the atom is kept in a magnetic field.

Electron's motion

So, under magnetic field,

$$m\omega^2 r = \frac{Ze^2}{4\pi\epsilon_0 n^3} - e\omega r B$$

$$\therefore \omega^2 = \frac{Ze^2}{4\pi\epsilon_0 m r^3} - \frac{eB}{m} \omega$$

$$\Rightarrow \omega^2 + \frac{eB}{m} \omega - \frac{Ze^2}{4\pi\epsilon_0 m r^3} = 0$$

$$\Rightarrow \omega^2 + \frac{eB}{m} \omega - \omega_0^2 = 0$$

$\left[\omega_0 = \frac{Ze^2}{4\pi\epsilon_0 m r^3} \right]$

$$\Rightarrow \omega = \frac{-\frac{eB}{m} \pm \sqrt{\frac{e^2 B^2}{m^2} + 4\omega_0^2}}{2}$$

$$\Rightarrow \omega = \frac{-\frac{eB}{2m} \pm \sqrt{\frac{e^2 B^2}{4m^2} + \omega_0^2}}{2}$$

At normal magnetic field,

$$\left(\frac{eB}{2m} \right)^2 \ll \omega_0^2$$



$$\therefore \omega_0 \approx -\frac{eB}{2m} \pm \omega_0$$

Taking +ve value of ω_0 , $\omega < \omega_0$

and B is in same direction as ω_0, S_0 ,
 $\omega < \omega_0$.

Applying B in opposite direction, $\omega > \omega_0$.

Change in angular frequency of the orbital electron due to the applied magnetic field

$$\Delta \omega = \frac{eB}{2m}$$

$$\omega = 2\pi \nu$$

$$2\pi \Delta \nu = \frac{eB}{2m}$$

$$\Rightarrow \Delta \nu = \frac{eB}{4\pi m} \quad [\Delta \nu = \text{Change in frequency}]$$

This change in frequency of orbital electron induces the magnetic moment —

$$\Delta M = IA$$

$$= -\frac{e}{t} \pi r^2 = -\frac{e}{\Delta t} \pi r^2$$

$$= -e \Delta \nu \pi r^2$$

$$= -e \frac{eB}{4\pi m} \pi r^2$$

$$\Rightarrow \Delta M = -\frac{e^2 B r^2}{4m}$$

The induced magnetic moment due to all the atomic electrons

$$\hat{M} = -\frac{e^2 B}{4m} \sum_{i=1}^n r_i^2$$

$$= -\frac{e^2 B Z}{4m} \langle r^2 \rangle \quad [\text{where } \sum_{i=1}^n r_i^2 = Z \langle r^2 \rangle]$$

$\sqrt{\langle r^2 \rangle}$ = root mean square radius of electronic orbit

$$M_L = -\frac{e^2 B Z \langle r^2 \rangle}{4m}$$

Since the electronic orbits are taken to be circular lying in x-y plane.

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle.$$

Since we have assumed the spherical model of atom therefore the radius of the atom is given by —

$$r_0^2 = x^2 + y^2 + z^2$$

$$\langle r_0^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$$

Since the atom has spherical symmetry,

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$$

$$\therefore \langle r_0^2 \rangle = 3 \langle x^2 \rangle$$

$$\therefore \langle r^2 \rangle = \frac{2}{3} \langle r_0^2 \rangle$$

$$\therefore \frac{\langle r^2 \rangle}{\langle r_0^2 \rangle} = \frac{2}{3}$$

$$\Rightarrow \langle r^2 \rangle = \frac{2}{3} \langle r_0^2 \rangle$$

$$M_L = -e^2 B_z \frac{2}{3} \langle r_0^2 \rangle$$

$$= -e^2 B_z \frac{\langle r^2 \rangle}{6m}$$

If a solid has N number of atoms per unit volume then the magnetization of the solid,

$$M = NM_L = -\frac{Ne^2 B_z \langle r_0^2 \rangle}{6m}$$

Diamagnetism susceptibility —

$$\chi = \frac{M}{H} = \frac{1}{H} \left(-\frac{Ne^2 M_0 H z \langle r_0^2 \rangle}{6m} \right)$$

$$\Rightarrow X = - \frac{N e^2 \mu_0 Z \langle r_0^2 \rangle}{6m}$$

-ve sign indicates that the diamagnetic materials experience a weak repulsive force when kept in a weak magnetic field.

Diamagnetic susceptibility is temperature independent.

Assumptions:

1. The individual spin magnetic moments of the electrons are considered.
2. The spin magnetic moment can take only two components (up and down). When an external magnetic field is applied.
3. The individual spin magnetic moments do not interact with each other.

The number of ~~electrons~~ atoms having energy E at temperature T is proportional to the Boltzmann distribution factor ($e^{-E/kT}$)

The number of ~~electron~~ atoms per unit volume having inclination (spin magnetic moment) between θ and $\theta + d\theta$ is given by

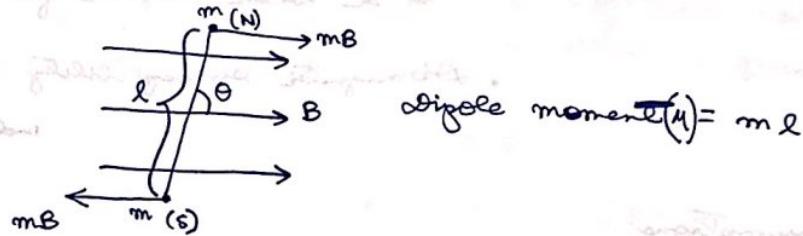
$$dN d\Omega e^{-E/kT} \sin \theta d\theta$$

$$\text{Total dipole moment} = N \int_{-\pi/2}^{\pi/2} m \sin \theta d\theta$$

at T = 0 $\Rightarrow dN = c e^{-E/kT} \sin \theta d\theta$
 -> magnetoo rotation disappears

Equipotential & magnetic plane

-> half dipole moment in a magnetic field of B



$$\text{dipole moment } (\mu) = m l$$

Equipotential

at $\theta = \pi/2$ current dipole angle is $\pi/2$ \Rightarrow Torque experienced by the magnetic dipole —

$$\tau = \frac{l m B \sin \theta}{4\pi} \quad \text{also called as the magnetic dipole moment of the dipole}$$

$$= M B \sin \theta \quad \text{where } M = \frac{l m}{4\pi}$$

-> torque on half dipole moment

The potential energy stored in the dipole —

-> when dipole is in equilibrium \Rightarrow $\tau = 0$

$$E = \int_{90^\circ}^{0^\circ} \tau d\theta$$

$$= \int_{90^\circ}^{0^\circ} M B \sin \theta d\theta$$

It is ignored given number of atoms all unaligned \Rightarrow dipole going to T environment

$$(M^2/3) - M B \cos \theta \quad \text{minimum}$$

$$\therefore dN = c e^{-\frac{MB \cos \theta}{kT}} \sin \theta d\theta$$

number of atoms with dipole angle θ between 0 to π —

$$N = \int_0^\pi dN$$

$$= \int_0^\pi c e^{-\frac{MB \cos \theta}{kT}} \sin \theta d\theta$$

$$\text{Let } \frac{MB}{kT} = a$$

$$N = \int_0^\pi ce^{a\cos\theta} \sin\theta d\theta$$

$$\text{Let } \cos\theta = x$$

$$-\sin\theta d\theta = dx$$

$$\therefore N = c \int_{-1}^1 e^{ax} (-dx)$$

$$= c \int_{-1}^1 e^{ax} dx$$

$$= c \left[\frac{e^{ax}}{a} \right]_{-1}^1$$

$$= \frac{c}{a} (e^a - e^{-a})$$

$$\therefore c = \frac{aN}{e^a - e^{-a}}$$

\therefore Magnetization for all the atomic magnetic dipoles is given by

$$M = \int_0^\pi M \cos\theta dN$$