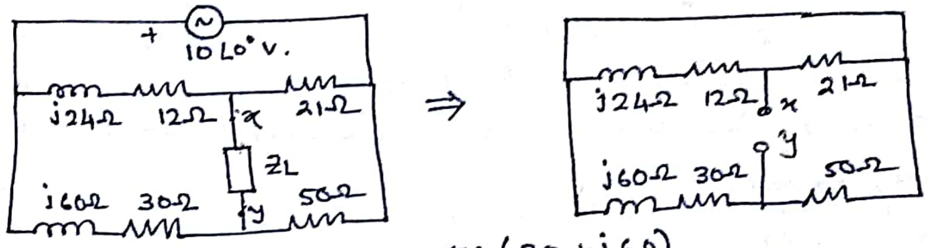
aximum Power Transfer Theorem:

In the network find the value of Z_L so that the power transfer from the source is maximum. Also find P_{max}.



$$Z_{in} = \frac{21(12+j24)}{21+12+j24} + \frac{50(30+j60)}{50+30+j60}$$

$$= \frac{252+j504}{33+j24} + \frac{1500+j3000}{80+j60}$$

$$= \frac{563.44\angle 63.43^\circ}{40.8\angle 36^\circ} + \frac{3354.16\angle 63.43^\circ}{100\angle 36.87^\circ}$$

$$= (42.26+j21.35)\Omega.$$

As per max. power transfer theorem Z_L should be complex conjugate of Z_{in}.

$\therefore Z_L = Z_{in}^* = (42.26 - j21.35)\Omega$.

To Find $V_{o.c}$.

Let at x potential is V_x

at y " is V_y.

$$V_x = \frac{12+j24}{12+j24+21} \times 10\angle 0^\circ. \text{ (voltage division rule).}$$

$$= \frac{268.33\angle 63.43^\circ}{40.8\angle 36^\circ} = 6.577\angle 27.43^\circ \text{ V}.$$

Similarly $V_y = \frac{30+j60}{30+j60+50} \times 10\angle 0^\circ = \frac{670.82\angle 63.43^\circ}{80+j60}$

$= 6.71\angle 26.56^\circ$ V.

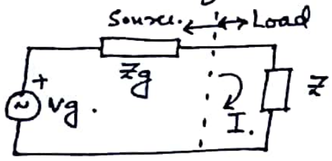
$$V_{o.c} = V_x - V_y$$

$$= 0.1657\angle 170^\circ \text{ V}.$$

$$\therefore P_{max} = \frac{V_{oc}^2}{4R_L} = \frac{0.1657^2}{4\times 42.16} = 0.1624 \text{ mW}.$$

⊛ Maximum Power Transfer Theorem in A C Circuit :

In a linear network having energy sources and imp -ces, maximum amount of power is transferred from source to load impedance if the load impedance is complex conjugate of the total impedance of the netwo if the source impedance is $(R_g \pm jX_g)\,\Omega$, to have maxim power transfer, the load impedance must be $(R_g \mp jX_g$



Obviously, $I = \frac{V_g}{Z_g + Z_L} = \frac{V_g}{(R_g+R_L)+j(X_g+X_L)}$

∴ Power (real power) $P_L = I^2 R_L$.

$$= \frac{V_g^2}{(R_g+R_L)^2+(X_g+X_L)^2} \times R_L.$$

Condition for maximum power flow $\frac{dP_L}{dX_L} = 0$ (

$$\frac{dP_L}{dX_L} = \frac{d}{dX_L}\left[\frac{V^2 R_L}{(R_g+R_L)^2+(X_g+X_L)^2}\right]$$

$$= \frac{-V^2 R_L \ast 2(X_g+X_L)}{\left[(R_g+R_L)^2+(X_g+X_L)^2\right]^2}$$

Setting $\frac{dP_L}{dX_L} = 0$, $X_g = -X_L$.

Substituting $X_g = -X_L$,

$$\ast\ P_L = \frac{V_g^2 R_L}{(R_g+R_L)^2} = \frac{V_g^2}{4R_g}\left[1 - \left(\frac{R_g-R_L}{R_g+R_L}\right)^2\right].$$

It may be seen that P_L would attain maximum provided $R_g = R_L$. Thus maximum power transfer place in a.c n/w provided $R_g = R_L$ and $X_g = -X_L$ in other words $(R_g + jX_g) = (R_L - jX_L)$ i.e. $Z_g = Z_L^{*}$. This means load impedance is the complex conjug of the source impedance.

$$P_{max} = \frac{V_g^2}{4R_L}, \quad \eta = 50\%.$$

$$\ast\ \frac{dP_L}{dR_L} = 0, \quad \frac{dP_L}{dR_L} = \frac{V_g^2(R_g+R_L)^2 - 2V_g^2 R_g(R_g+R_L)}{(R_g+R_L)^4} = 0$$

or, $V_g^2(R_g+R_L) - 2V_g^2 R_R = 0$

or, $R_g = R_L$.

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SUPERPOSITION THEOREM

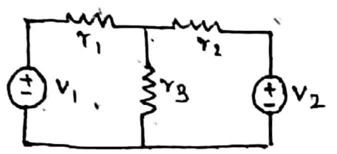
P. Nabanita Chatterjee
(EE Dept.)

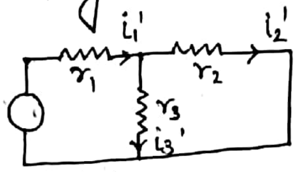
Statement:

If a no. of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebric sum of the currents that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.

Explanation:

In fig. to apply superposition theorem, let us first take the source V_1 alone ~~re~~placing V_2 by short circuit. (a).



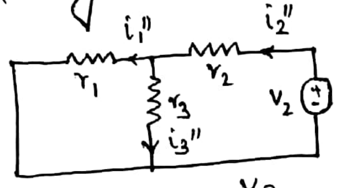


(a)

Here, $$i_1' = \frac{V_1}{\frac{r_2 r_3}{r_2+r_3} + r_1}$$

$$i_2' = i_1' \frac{r_3}{r_2+r_3} \quad \text{and} \quad i_3' = i_1' - i_2'$$

Next, removing V_1 by short circuit, let the circuit be energized by V_2 only.



Here, $$i_2'' = \frac{V_2}{\frac{r_1 r_3}{r_1+r_3} + r_2} \quad \text{and} \quad i_1'' = i_2'' \frac{r_3}{r_1+r_3}$$

Also, $i_3'' = i_2'' - i_1''$

As per superposition Theorem

$$i_3 = i_3' + i_3''$$
$$i_2 = i_2' - i_2''$$
$$i_1 = i_1' - i_1''$$

It may be noted that during application of superposition the directions of currents calculated for each source should be taken care of.

Steps for Solving a Network Using the Principle of Superpositi

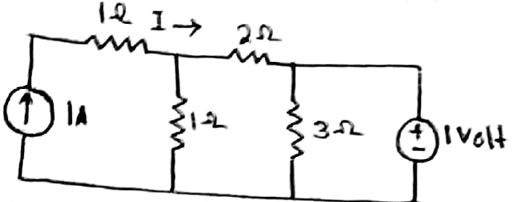
Step 1: Take only one independent source of voltage/current, deactivate the other independent voltage/current sou For voltage sources, remove the source and short ci the respective circuit terminals and for current s just delete the source keeping the respective circuit t open. Obtain branch currents.

Step 2: Repeat the above step for each of the independent sou

Step 3: To determine the net branch current utilising Superposition Theorem, just add the currents obta in step1 and step2 for each branch. If the curr obtained in step1 and step2 are in same directio just add them; on the other hand, if the respectiv currents are directed opposite in each step, assum direction of the clockwise current to be +ve and the current obtained in the next step from the origi current. The net current in each branch is then -ed.

roblems:

Find I in the circuit shown.

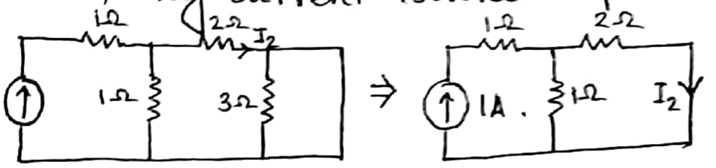


ol. Principle of superposition is applied by taking 1V source only at first.

$$I_S = \frac{1V}{[(1+2)\|3]\Omega} = (1/1.5)A.$$

$$I_1 = I_S \cdot \frac{3}{3+2+1} = \frac{1}{1.5} \times \frac{3}{6} = \frac{1}{3}A.$$ [by current division]

Next, only current source is present.



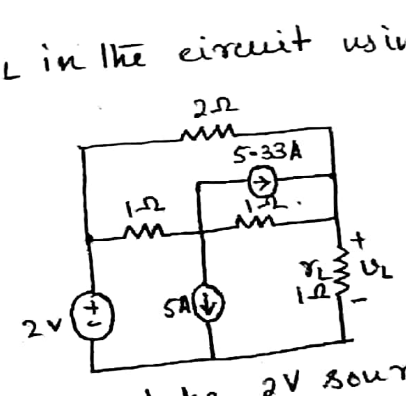
$$I_2 = 1 \times \frac{1}{1+2} = \frac{1}{3} A.$$

It may be observed that utilising the principle of superposition, the net response can be obtained when both the sources (1A and 1V) are present. The current through 2Ω resistor is obtained as.

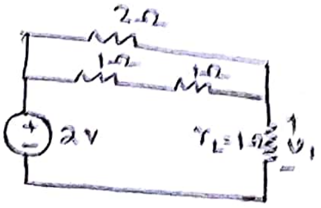
$$I = (I_1 - I_2) = \frac{1}{3} - \frac{1}{3} = 0$$

[I_1 and I_2 being directed reverse]

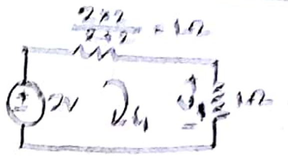
② Find v_L in the circuit using super position theorem.



Let us first take 2V source deactivating the current sources.



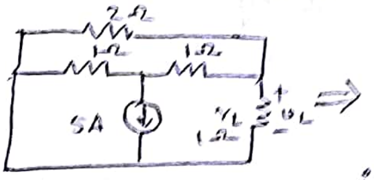
⇒



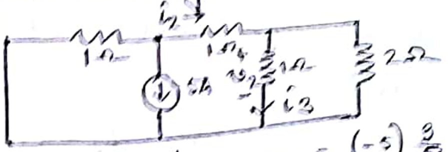
$i_1 = \frac{2}{1+1} =$

$\therefore v_1$ (drop across r_L due to 2V)
$= 1\times1 = 1V.$

Next Lower current source only



⇒

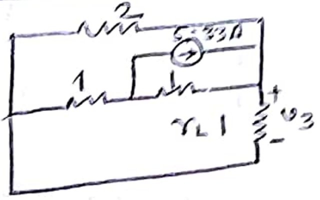


$$i_2 = (-5)\frac{1}{1+1+\frac{2}{3}} = (-5)\frac{3}{8} = -\frac{15}{8}$$

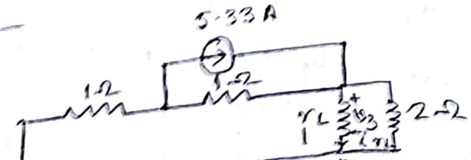
$$\therefore i_3 = -\left(\frac{15}{8}\right)\times\frac{2}{2+1} = -\left(\frac{15}{8}\right)\frac{2}{3} = -\frac{5}{4}\text{ A}.$$

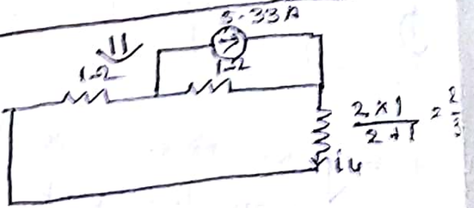
This gives $v_2 = -\frac{5}{4}\times1 = -\frac{5}{4}$ V.

Next 5.33A source only



⇒





$$i_4 = 5.33\cdot\frac{1}{2+\frac{2}{3}} = 3\text{ A}.$$

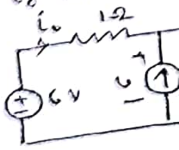
This gives $i_{r_L} = 3\,\frac{2}{2+1} = 2\text{ A}.$

$\therefore v_3 = 2\times1 = 2V.$

By superposition,

$$v_L = v_1+v_2+v_3 = 1+\left(-\frac{5}{4}\right)+2 = \frac{7}{4}\text{ V}$$
$$= 1.75\text{ V}$$

Find i_o and i



1A source a

1 =

But $i_o'' =$

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i.e. i_o''

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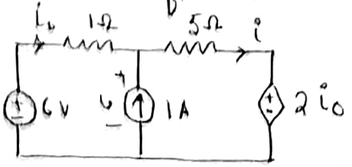
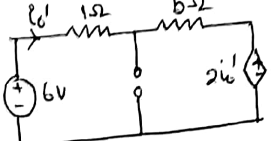
Find i_o and i from the circuit using superposition Th.



Assume only 6V source to be active.

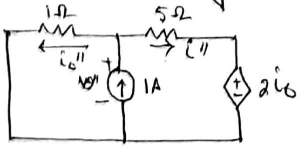


$$-6 + (1+5)i_o' + 2i_o' = 0$$

or, $8i_o' = 6$ ∴ $i_o' = \frac{3}{4}$ A.

∴ $i_o' = i' = \frac{3}{4}$ A.

1A source active only



$$1 = i_o'' + i''$$
$$= \frac{v''}{1} + \frac{v'' - 2io}{5}$$
$$= 1.2v'' - 0.4i_o''$$

But $i_o'' = \frac{v''}{1}$

We finally get, $1 = 1.2i_o'' - 0.4i_o'' = 0.8i_o''$

i.e. $i_o'' = 1.25$ A and $i'' = \frac{v'' - 2io''}{5}$

$$= -i_o''/5 = -0.25\text{A}.$$

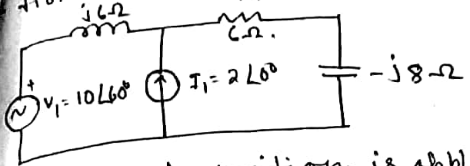
Using the principle of super position,

$$i_o = i_o' - i_o'' = \frac{3}{4} - 1.25 = -0.5\text{A}.$$

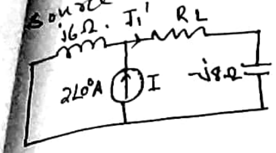
and $i = i_o' + i'' = \frac{3}{4} - 0.25 = 0.5$ A.

Superposition (with A.C):

Find the current in the resistor (R_L) using the principle of superposition.



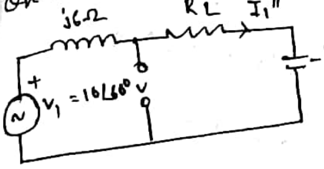
Principle of superposition is applied in the given ckt. taking each source at a time.



$$I_1' = I \frac{j6}{j6+6+(-j8)} = \frac{2\angle 0^\circ \times 6\angle 90^\circ}{6-j2} = \frac{12\angle 90^\circ}{6.32\angle -18.43^\circ}$$

$$= 1.9\angle 108.43^\circ \text{ A}.$$

On the other hand, in next case.

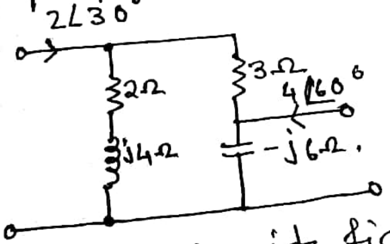


$$I_1'' = \frac{V_1}{j6+6-j8} = \frac{10\angle 60^\circ}{6-j2} = \frac{10\angle 60^\circ}{6.32\angle -18.43^\circ}$$

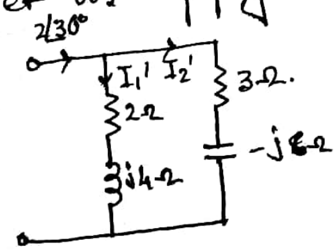
∴ I, the total current through R_L is $I = I_1' + I_1''$ (since both are directed in same dir).

$$I = 1.9\angle 108.43^\circ + 1.58\angle 78.43^\circ$$
$$= -0.6 + j1.8 + 0.32 + j1.55$$
$$= (0.28 + j3.35) = 3.362\angle 94.71^\circ \text{ A}.$$

2) Find the current in the (-j6Ω) capacitive reactance using superposition theorem in the circuit.



Let us apply it first to the fig.

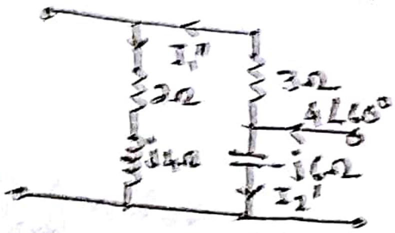


$$I_2' = 2\angle 30^\circ \frac{(2+j4)}{(2+j4)+(3-j6)}$$

$$= \frac{8.944\angle 93.44^\circ}{5-j2} = \frac{8.944\angle 93.44^\circ}{5.385\angle -21.80}.$$

$$= 1.661\angle 115.241.$$

Next it is applied to the next fig.



$$I_2'' = 4\angle 60^\circ \frac{(3+2+j4)}{(3+2+j4)-j6} = \frac{4\angle 60^\circ (5+j4)}{5-j2}$$

$$= \frac{4\angle 60^\circ \times 6.4\angle 38.66^\circ}{5.385\angle -21.80^\circ}$$

$$= 4.754\angle 120.46^\circ \text{ A.}$$

Using the principle of superposition.

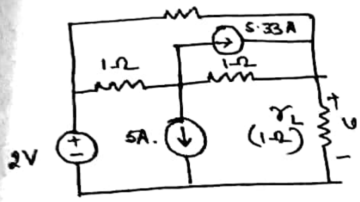
$$I_2 = I_2' + I_2'' = 1.661\angle 115.24 + 4.754\angle 120.46^\circ$$

$$= -0.7083 + j1.5 - 2.41 + j4.098$$

$$= -3.118 + j5.598 = 6.41\angle 119.117^\circ \text{ A.}$$

Thus the current through the capacitor is $6.41\angle 119.117^\circ$ A.

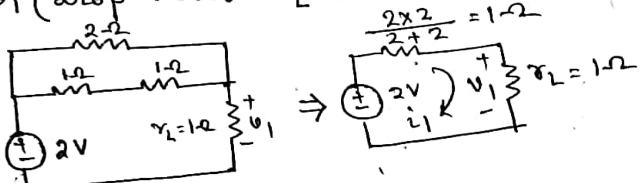
Find V_L in the circuit of Fig. using Superposition theorem.



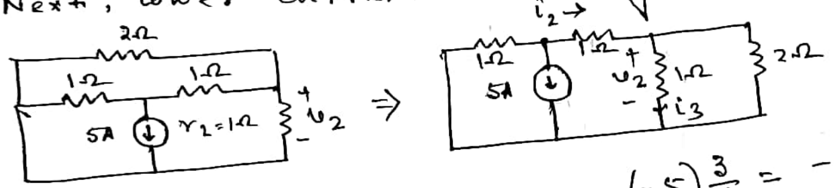
First only 2V source deactivating the ct. sources.

$$i_1 = \frac{2}{\frac{2\times2}{2+2}+1} = 1\ \text{A}.$$

∴ v_1 (drop across r_L due to 2V source) $= 1\times1 = 1$ V.



Next, lower current source only.

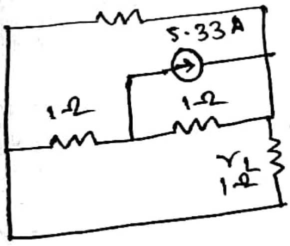


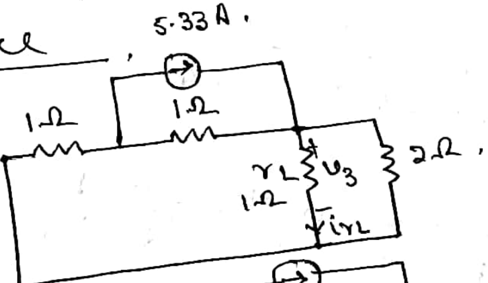
$$i_2 = (-5)\frac{1}{1+1+2/3} = (-5)\frac{3}{8} = -\frac{15}{8}\ \text{A},$$

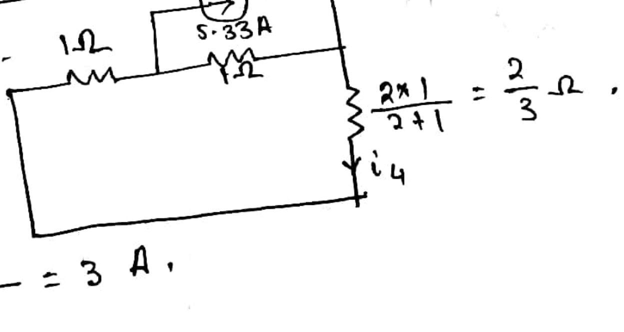
$$i_3 = -\left(\frac{15}{8}\right)\cdot\frac{2}{1+2} = -\frac{5}{4}\ \text{A}.$$

This gives $v_2 = -\frac{5}{4}\times1 = -\frac{5}{4}$ V.

Next upper ct. source,







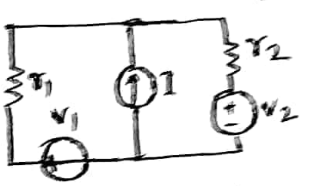
$$i_4 = 5.33\cdot\frac{1}{\frac{2}{3}+2} = 3\ \text{A}.$$

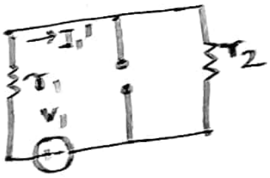
This gives. $i_{r_L} = 3 \cdot \frac{2}{2+1} = 2A$.

$\therefore v_3 = 2\times1 = 2V$.

By superposition, $v_L = v_1 + v_2 + v_3$
$= 1 + (-5/4) + 2 = 7/4\,V$.
$= 1.75V$.

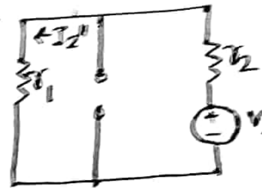
* Find current through r_1, [$V_1 = 12V$, $r_1 = 4\Omega$, $I = 3A$, $r_2 = 2\Omega$, V





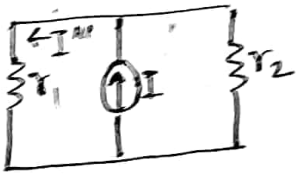
$I' = \frac{V_1}{r_1+r_2} = \frac{12}{4+2}$

Next only V_2 ⇒



$I_2'' = \frac{V_2}{r_1+r_2} = \frac{24}{4+2}$

Next I only ⇒



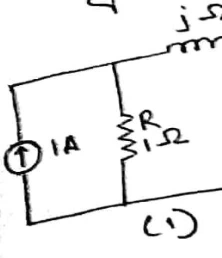
Here $I''' = I\frac{r_2}{r_1+r_2} = 3\frac{2}{2+4} = 1A$.

Taking note that I'' and I''' are in reverse direc in comparison to I', using the principle of Superpositi

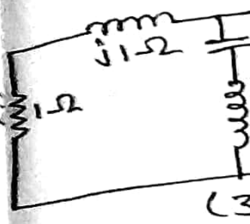
$I = I' - I'' - I'''$
$= 2 - 4 - 1 = -3A$.

∴ The net current through r_1 will flow downward and its magnitude will be 3A.

Calcul
by the

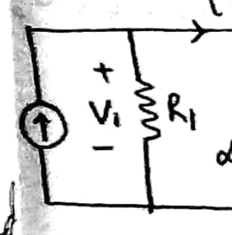


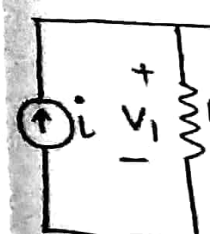
Making V

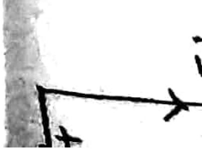


Therefore
voltage a
simultan

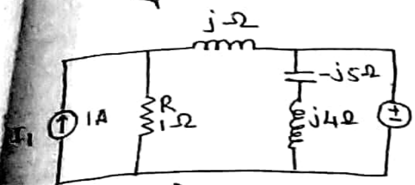
Using the
voltage
C = 2F



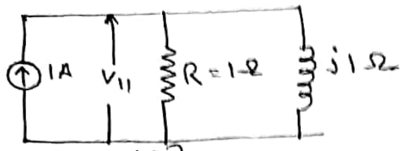




[illegible] Calculate Voltage V across the resistance R in fig. by the principle of superposition.

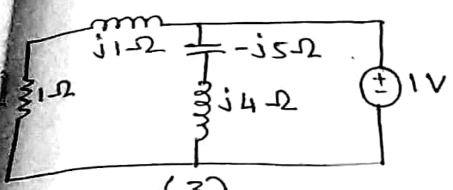


(1)



(2)

Making $V_2 = 0$ (shorted), $V_{11} = \frac{1}{1-j}$



(3)

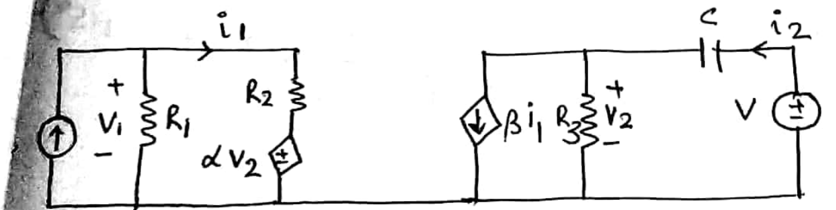
Making $I_1 = 0$ (opened),

$V_{12} = \frac{1}{1+j}$.

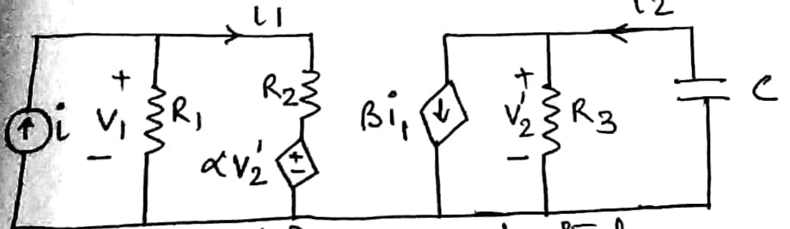
Therefore, by the superposition principle the voltage across R when both I_1 and V2 are applied simultaneously,

$V = V_{11} + V_{12} = 1$ volt.

Using the principle of superposition, calculate the voltage $v_2(t)$ in fig. Assume $R_1 = R_2 = 1\Omega$, $R_3 = 0.5\Omega$, $C = 2$ F, $\beta = 2$, $\alpha = 1$, $i(t) = \sin(t)$, $v(t) = t$.



(a)



(b) V(t) shorted,

(c) i(t) opened.

Set $v(t) = 0$. Let $V_1(s) = \mathcal{L}v_1(t)$, $I(s) = \mathcal{L}i(t)$. KCL and KVL equations are as follows:

$$I(s) = \frac{V_1(s)}{R_1} + I_1(s)$$

$$V_1(s) = \alpha V_2'(s) + R_2 I_1(s)$$

$$V_2'(s) = -\beta I_1(s) \cdot \frac{1}{1/R_3 + sC}$$

Eliminating $I_1(s)$, $V_1(s)$ and substituting parameter values,

$$v_2'(t) = \mathcal{L}^{-1} V_2''(s) = \mathcal{L}^{-1} - \frac{1}{2}\left(-\frac{1}{s+0.5}\right)\left(\frac{1}{s^2+1}\right)$$

$$= -\frac{2}{5}e^{-0.5t} - \frac{1}{\sqrt{5}}\sin(t - 63.5^\circ).$$

Set $i(t) = 0$. We get KCL and KVL equations as

$$V(s) = \frac{I_2(s)}{sC} + V_2''(s)$$

$$\frac{V_2''(s)}{R_3} = I_2(s) - \beta I_1(s)$$

$$I_1(s) = -\frac{\alpha V_2''(s)}{R_1 + R_2}$$

Eliminating $I_1(s)$ and $I_2(s)$ and substituting parameter values,

$$v_2''(t) = \mathcal{L}^{-1} V_2''(s) = \mathcal{L}^{-1}\left(\frac{s}{s+2}\right)\frac{1}{s^3}$$

$$= \left(\frac{1}{4}e^{-2t} + \frac{t}{2} - \frac{1}{4}\right).$$

Then, the total solution,

$$v_2(t) = v_2'(t) + v_2''(t)$$

$$= \left[-\frac{2}{5}e^{-t/2} + \frac{1}{4}e^{-2t} - \frac{1}{4} - \frac{1}{\sqrt{5}}\sin(t - 63\right.$$

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Debasmita Chatterjee (EE Dept).

NORTON'S THEOREM

Norton Theorem is converse of Thevenin's Theorem. It consists of equivalent current source instead of equivalent voltage source as done in Thevenin's Theorem. The determination of internal resistance of the source network is identical in both theorems. However, in final stage, i.e., in the Norton equivalent circuit, the current generator is placed in parallel to the internal resistance unlike to that in Thevenin's Theorem where the equivalent voltage source was placed in series with the internal resistance.

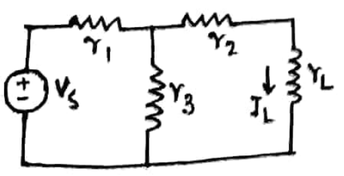
Statement:

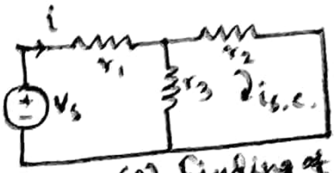
A linear active network consisting of independent and or dependent voltage and current sources and linear bilateral network elements can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance, the current source being the short circuited current across the load terminal and the resistance being the internal resistance of the source network looking through the open circuited load terminals.

Explanation:

In order to find the current through r_L, the load resistance, by Norton's Theorem, let us replace r_L by short circuit (fig (a)).

Obviously, $i = \dfrac{V_s}{r_1 + \dfrac{r_2 r_3}{r_2 + r_3}}$ and $i_{s.c} = i\,\dfrac{r_3}{r_3 + r_2}$.

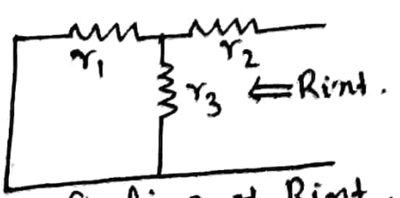




(a) Finding of $i_{s.c.}$

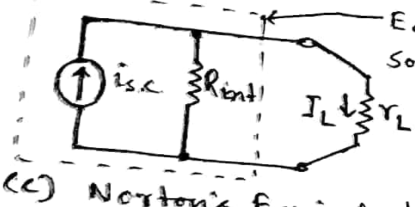
Next, the short circuit is removed and the independent source is deactivated as done in Thevenin's Theorem. [fig (b)].

Here $R_{int} = r_2 + \dfrac{r_1 r_3}{r_1 + r_3}$.



(b). Finding of Rint.

As per Norton's theorem, the equivalent source circuit contain a current source in parallel to the internal r the current source being the short circuited current the shorted terminals of the load resistor [fig.(c)].



(c) Norton's Equivalent Circuit.

Obviously $I_L = i_{sc}\cdot\frac{Rin}{Rint}$

It may be noted here that determination of Rint for source system in Norton Theorem is identical to that for Thevenin's Theorem.

Steps for Solving a N/w using Norton's Theorem:

Step1: Remove the load resistor and find the internal of the source network by deactivating the constant Let this resistance be Rint [same as R_{Th}].

Step2: Next, short the load terminals and find the short current flowing through the shorted load termin using conventional network analysis. Let this cu be is.c.

Step3: Norton's Equivalent circuit is drawn by keepi Rint in parallel to is.c. as shown is fig. (c).

Step4: Reconnect the load resistor (R_L) across the load and the current through it (I_L) is then gi

$$I_L = i_{sc} = \frac{Rint}{Rint + R_L}.$$

Problems:

Find Norto in x-y

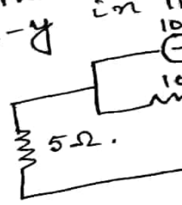


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Fig. (a)

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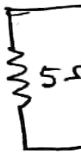


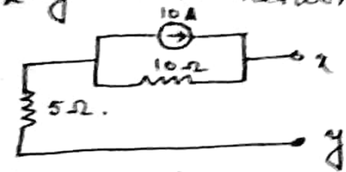
Here

Norton

Here

Problems :

1. Find Norton's Equivalent circuit to the left of terminals x-y in the network shown.



Let us first short the terminals x-y

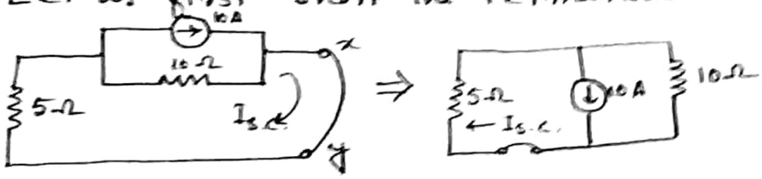
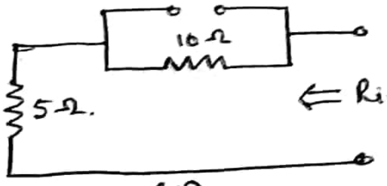


Fig. (a)

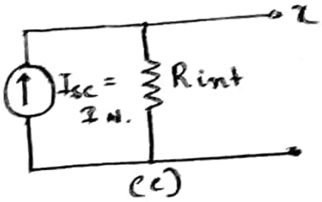
Here, Is.c. is the current through the 5Ω resistor

$\therefore I_{sc} = 10 \times \frac{10}{10+5} = 6.67$ A. [by ct. divider rule].

To determine the equivalent resistance of the circuit, looking through x-y, the constant source is deactivated as shown in fig. (b).



(b)



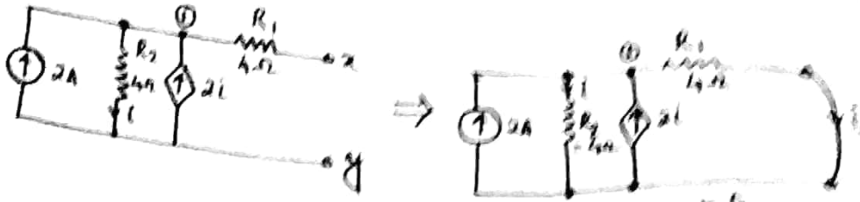
(c)

Here Rint $= 10 + 5 = 15\Omega$.

Norton's Equivalent circuit has been shown in fig. (c).

Here $I_N = 6.67$ A , Rint $= 15\Omega$.

2.6. 2. In the circuit if $R_1 = R_2 = 4\Omega$, find the short circuit curr x-y terminals using Norton's Theorem. What will happ $R_1 = 2R_2$.



short circuited across x-y

Application of KCL at node 1, yields

$$2 + 2i - 1 - i_{s.c} = 0$$

$$\text{i.e. } i + 2 = i_{s.c} \quad \ldots ①$$

Application of KVL in the circuit

$$i_{s.c} R_1 - i R_2 = 0 \quad \ldots ②$$

Provided $R_1 = R_2$, $i_{sc} = i$

Using the value of $i (= i_{s.c})$ in (1), we get

$$i_{s.c} = \infty \text{ A}.$$

This shows that Norton Equivalent Circuit can n drawn for the given fig.

In case $R_1 = 2R_2$, we find from

$$i_{sc} R_1 - i R_2 = 0$$

$$2 i_{sc} R_2 - i R_2 = 0$$

$$\text{or, } R_2 (2 i_{sc} - i) = 0 \quad \ldots ③$$

However, R_2 is not equal to zero. Thus from ③

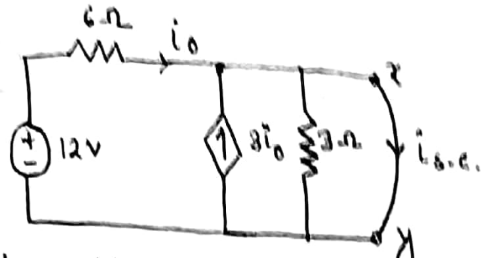
$$2 i_{sc} - i = 0$$

$$\text{or, } 2 i_{sc} = i$$

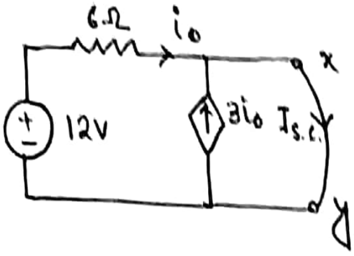
Thus from ① $i_{sc} = -2A$.

Norton Theorem is then applicable in the given p provided $R_1 \neq R_2$.

Find the current through R_L in the circuit of fig using Norton's Theorem.



Let us first remove R_L from x-y terminals and short x-y.



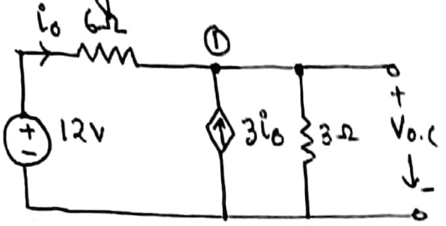
It is evident from fig.

$I_{s.c} = 3i_o + i_o = 4i_o$

But $i_o = \frac{12}{6} = 2$ A.

$\therefore I_{s.c} = 4 \times 2 = 8$ A

Let us now remove the short circuit and it is open circuited at x-y.



Nodal Analysis at node 1 gives,

$$i_o + 3i_o - \frac{V_{o.c}}{3} = 0$$

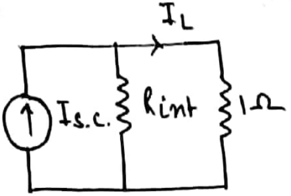
or, $4i_o - \frac{V_{oc}}{3} = 0$

or, $4\left(\frac{12 - V_{o.c}}{6}\right) - \frac{V_{o.c}}{3} = 0$

or, $8 - \frac{2V_{oc}}{3} - \frac{V_{oc}}{3} = 0$

or, $V_{oc} = 8$V.

This gives $R_{int} = \frac{V_{oc}}{I_{sc}} = \frac{8}{8} = 1\Omega$.

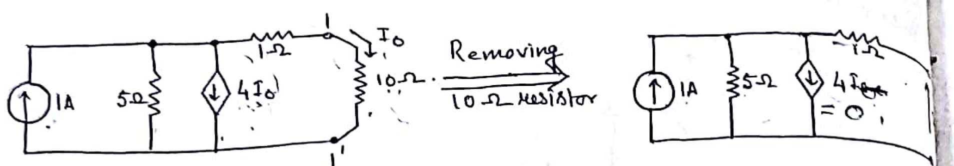


Norton's Equivalent Circuit

I_L (current through 1Ω resistor)

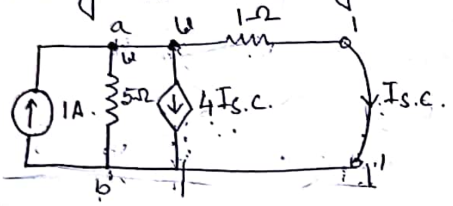
$$= I_{sc} \cdot \frac{R_{int}}{R_{int} + R_L} = 8 \cdot \frac{1}{1+1} = 4 \text{ A.}$$

4. Find the power loss in the 10Ω resistor in the circuit Norton's Theorem.



$V_{o.c} = (V_{1-1'}) = 1 \times 5 = 5\ V.$

Next short circuiting x-y terminals and assuming voltage at 'a' being u, nodal analysis yields.



$1 = \frac{u}{5} + 4I_{sc} + I_{sc}$

or, $5I_{sc} + \frac{u}{5} = 1$. ---- ①

Also $I_{sc} = \frac{u}{1} = u$ A.

From (1) $5I_{sc} + \frac{I_{sc}}{5} = 1$

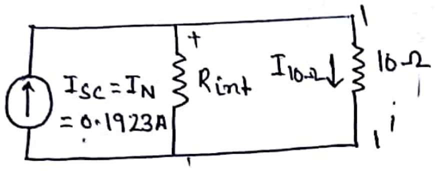
or, $I_{sc} = 0.1923$ A.

This gives $R_{int} = \frac{V_{o.c}}{I_{s.c}} = \frac{5}{0.1923} = 26\Omega$.

Thus Norton Equivalent Circuit is given by

$I_{sc} = 0.1923$ A.

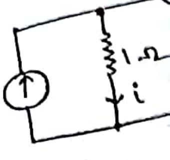
$R_{int} = 26\Omega$.



$I_{10\Omega} = I_{sc} \cdot \frac{R_{int}}{R_{int} + 10} = 0.14$ A

∴ Power Loss in the 10Ω resistor $= (I_{10})^2 \times 10$
$= 0.195\Omega$.

Find the cu



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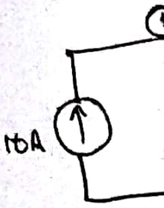

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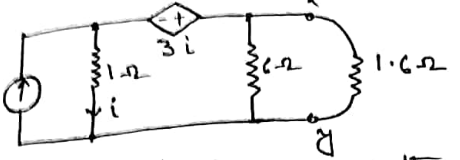
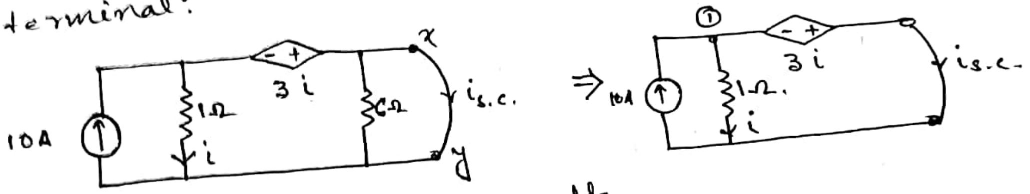
Find the current through 1·6Ω resistor in the ckt.



Let us first remove the 1·6Ω resistor and short x-y terminal.



Let node voltage at (1) be v volts.

$\therefore\ i = \frac{v}{1} = v$ A, ----(1).

KCL at node 1.

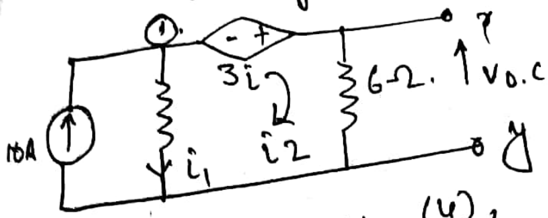
$10 = i + i_{sc}$. ---(2)

Also KVL at right loop yields, $-v - 3i = 0$

or, $v = -3i$ ---(3)

Comparison of (1) and (3) yields that these eqns are only valid if $v = i = 0$.

This gives $i_{sc} = 10$ A.



At node 1, $10 = i_1 + i_2$ ...(4)

$i_2 = \frac{v + 3i_1}{6}$

$i_2 = \frac{i_1 \times 1 + 3i_1}{6} = \frac{2}{3} i_1$

Thus from eqn. (4),

$i_1 + \frac{2}{3} i_1 = 10$, $i_1 = 6$ A.

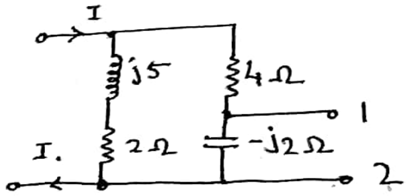
Also, $V_{o.c} = i_2 \times 6 = (10 - i_1)\,6 = 24$ V

Hence $R_{int} = \frac{V_{o.c}}{I_{sc}} = \frac{24}{10} = 2{\cdot}4\,\Omega$.

Current through 1·6Ω resistor ~~is~~.

$I_{(1\cdot6\Omega)} = I_{sc} \cdot \frac{R_{int}}{R_{int} + 1{\cdot}6} = 10\,\frac{2{\cdot}4}{2{\cdot}4 + 1{\cdot}6} = 6$ A.

Find Norton's Equivalent Circuit for the network shown.
Assume $I = 5\angle 0^\circ$ A.



Terminals 1-2 are shorted. No current would flow through (-j2Ω) capacitive reactance due to shorting of terminal 1-2.

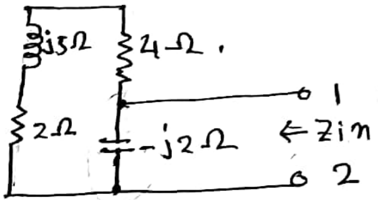
Obviously,

$$I_{s.c} = \frac{(2+j5)}{(2+j5+4)} \times I = \frac{5\angle 0^\circ (2+j5)}{6+j5}$$

$$= \frac{10+j25}{6+j5} = \frac{26.926\angle 68.198^\circ}{7.81\angle 39.81^\circ}$$

$$= 3.45\angle 28.39^\circ \text{ A}.$$

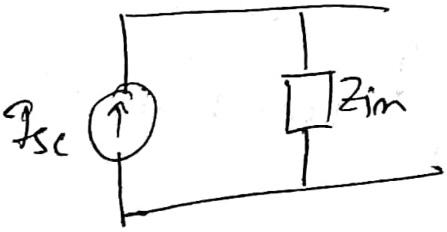
Internal impedance can be obtained by deactivating the current source.



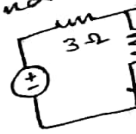
$$Z_{in} = \frac{-j2(4+2+j5)}{-j2+4+j5+2} = \frac{-j8-j4+10}{6+j3}$$

$$= \frac{10-j12}{6+j3} = \frac{15.62\angle -50.19^\circ}{6.71\angle 26.56^\circ}$$

$$= 2.33\angle -76.75^\circ \,\Omega.$$



• Find the



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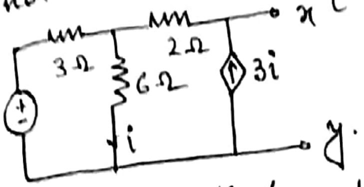
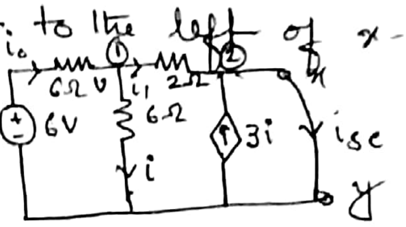
- Find the Norton equivalent to the left of x - y.





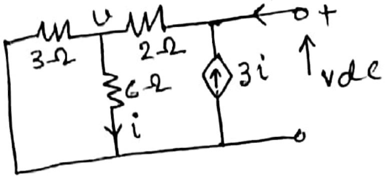
At node 1, $\frac{v-6}{3} + \frac{v}{6} + \frac{v}{2} = 0$,

or, $v = 2V$, this gives $i = v/6 = 1/3$ A.

$i_o = \frac{v-6}{3} = -\frac{2-6}{3} = 4/3$ A., $i_1 = i_0 - i = \frac{4}{3} - 1/3 = 1A$.

At node 2: $i_1 + 3i = Isc$, ∴ $Isc = 1 + 3 \times 1/3 = 2$ A.

To find Rint x-y is open circuited and the const. source is deactivated. A dc. voltage Vd. c is applied such that the i/p ct. is ~~2~~ idc.



Here, $\frac{v_{dc} - v}{2} = i_{dc} + 3i$

also $i = \frac{v}{6}$ and $\frac{v_{dc} - v}{2} = \frac{v}{6} + \frac{v}{3}$

i.e. $v = 0.5\, v_{dc}$.

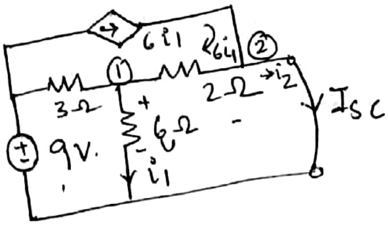
Thus, $i = v/6 = \frac{0.5 v_{dc}}{6}$

Then $\frac{v_{dc} - v}{2} = i_{dc} + 3 \cdot \frac{0.5 \, v_{dc}}{6}$ ∴ $Rint = \frac{v_{dc}}{i_{dc}} = \infty \Omega$

or, $i_{dc} = 0$,

Thus $Rint = \infty \Omega$, $Isc = I_N = 2A$.

- Find Norton's Equivalent to the left of x - y.



Here, $i_1 = v/6$.

At ② $i_2 + 6 i_1 = Isc$,

$v/2 + 6 \cdot v/6 = Isc$. [∵ $i_2 = v/2$].

or, $3/2\, v = Isc$, i.e. $Isc = 1.5$ V.

Again at Node ① $\frac{v-9}{3} + v/6 + v/2 = 0$,

or, $v = 3$ V, Thus $Isc = 1.5 \times 3 = 4.5A = I_N$.

To find Voc. open x-y:

At ① $\frac{v_1 - 9}{3} + i_1 = 6 i_1$ ⇒ ~~i_1 =~~ $\frac{v_1}{3} - 3 - 5 i_1 = 0$.

But $i_1 = v_1/6$ A.

Thus we get, $\frac{v_1}{3} - 3 - \frac{5}{6} v_1 = 0$

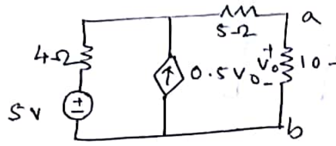
or, $v_1 = -6$ V.

However, $v_{oc} = v_1 + (2\Omega \times 6 i_1 A) = v_1 + 12 i_1$

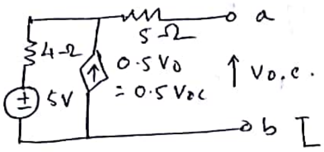
$= v_1 + 12 \cdot \frac{v_1}{6} = 3v_1 = -18V$

Thus $Rint = \frac{Voc}{Isc} = 4\Omega$.

- What is the power loss in the 10Ω resistor of the circu



Let us first remove the 10Ω resistor from a-b terminals. In the left most loop, the circulating current will obviously be $(0.5V_0)$A.



Applying KVL in the leftmost loop,

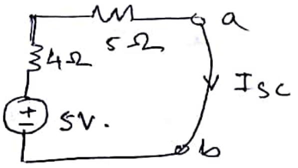
$5 - (-0.5V_0) \times 4 = V_{OC}$.

[∵ the ct. $(0.5V_0)$ A is anticlockwise causing voltage drop additive to the 5V source, also, current thru 5Ω resistor being nil, V_{oc} is the voltage drop across x-y terminal]..

∴ $V_{oc} = 5 + 2V_0$ ---(1)', But with ref. to fig. (2),

$V_0 \equiv V_{oc}$, from (1) $V_{oc} = -5V$ [b is +ve).

From fig. (b). a-b is shorted. it is evident that $0.5V_0 = 0$, as there is no drop across short path.



$$\therefore I_{sc} = \frac{5V}{(4+5)\Omega} = 5/9 \text{ A}.$$

$$\text{Thus } R_{int} = (R_{th}) = \frac{V_{oc}}{I_{sc}} = \frac{5}{5/9} = 9\Omega.$$

[-ve sign of $V_{o.c}$ is neglected].

Thus, following Thevenin's equivalent ckt.

I_{th} = ct. thru. 10Ω resistor,

$$= \frac{V_{oc}}{R_{int}+10} = \frac{5}{9+10} = 5/19 \text{ A}.$$ [∵ V_{oc} is -ve, I_{th} will flow from b to a].

P_{loss} in 10Ω resistor is $(5/19)^2 \times 10$ i.e. 692.5 mW.

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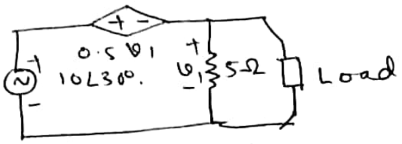
Find the Thevenin's eq. n/w. for the n/w shown across the load



Removing the load, assuming the open ckt. voltage to be Voc, it is seen that Voc = v_1. Applying KVL in the left loop of fig. (a).

$$-10\angle 30^\circ + 0.5\,v_1 + v_1 = 0$$

or, $1.5\,v_1 = 10\angle 30^\circ$.

∴ $v_1 = 6.67\angle 30^\circ$ V (= Voc.),

Applying short circuit across the o/p., it is clear that $v_1 = 0$. This makes the dependent source also to be zero. $I_{sc} = \frac{10\angle 30^\circ}{0}$ [∵ there is no other impedance in the loop]

i.e. $I_{sc} = \infty$, $Z_{int} = \frac{V_{oc}}{I_{sc}} = \frac{V_{oc}}{\infty} = 0\Omega$.

Maximum Power Transfer Theorem

This theorem is used to find the value of load resistance for which there would be maximum amount of power transfer from source to load.

Statement:

A resistance load, being connected to a d.c. network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's Equivalent resistance) of the source network as seen from the load terminals.

Explanation:

A variable resistance R_L is connected to a dc source network as shown in fig. (a) while fig. (b) represents the Thevenin voltage V_0 and Thevenin resistance R_{Th} of the source network. The aim is to determine the value of R_L such that it receives maximum power from the dc. source.

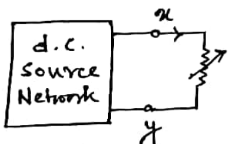


Fig. (a)

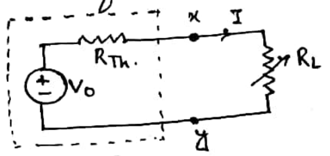


Fig. (b).

With reference to fig (b), $I = \frac{V_0}{R_{Th} + R_L}$

While the power delivered to the resistive load is,

$$P_L = I^2 R_L = \left(\frac{V_0}{R_{Th}+R_L}\right)^2 \times R_L .$$

P_L can be maximised by varying R_L and hence, maximum power can be delivered when $(dP_L/dR_L) = 0$.

However,

$$\frac{dP_L}{dR_L} = \frac{1}{[(R_{Th}+R_L)^2]^2} \left[(R_{Th}+R_L)^2 \frac{d}{dR_L}(V_0^2 R_L) - V_0^2 R_L \frac{d}{dR_L}(R_{Th}+R_L)^2 \right]$$

$$= \frac{1}{(R_{Th}+R_L)^4} \left[(R_{Th}+R_L)^2 V_0^2 - V_0^2 R_L \times 2 (R_{Th}+R_L) \right]$$

$$= \frac{V_0^2 (R_{Th} + R_L - 2R_L)}{(R_{Th}+R_L)^3} = \frac{V_0^2 (R_{Th} - R_L)}{(R_{Th}+R_L)^3}$$

But $\frac{dP_L}{dR_L} = 0$

∴ Finally, $\frac{V_0^2(R_{Th}-R_L)}{(R_{Th}+R_L)^3} = 0$

which gives $(R_{Th}-R_L)=0$ or $R_{Th}=R_L$.

Hence, it has been proved that power transfer from a d network to a resistive network is maximum when the internal -tance of the dc source network is equal to the load resistan

Again, with $R_L = R_{Th}$, the system being perfectly matched and source, the power transfer becomes maximum and amount of power (P_{max}) can be obtained as

$$P_{max} = \frac{V_0^2 R_{Th}}{(R_{Th}+R_{Th})^2}$$

$$= V_0^2/4R_{Th}.$$

(It may be noted that this is the power consumed by the loa

Obviously, the power transfer by the source would be als $V_0^2/4R_{Th}$, the load power and source power being the sam The total power supplied is thus

$$P = 2\frac{V_0^2}{4R_{Th}} = \frac{V_0^2}{2R_{Th}}.$$

During maximum power transfer the efficiency η becomes

$$\eta = \frac{P_{max}}{P} \times 100 = 50\%.$$

The concept of maximum power transfer by mat the source resistance equal to the load resistance ha wide application in communication circuits where th magnitude of power transfer is sufficiently smal To achieve maximum power transfer, then the sour load resistances are matched and with flow of m power from source to load, low efficiency of 50 tolerated. On the other hand, in electric power trans systems, the load resistance being sufficiently greate the source resistance, it is difficult to achieve the for max. power transfer ordinarily. Moreover in powe emphasis is given to keep voltage drops and line to a minimum value and hence operation of the system, operating with bulk power transmission capabil becomes uneconomical if it be operated with only 50% just for the sake of max. power transfer. Hence in po transmission system this criterion of max. power transfer is -dom realised.

Steps for Solution:

step 1: Remove the load resistance and find Thevenin's Resistance (R_{TH}) of the source network looking through the open circuited load terminals.

step 2: As per maximum power transfer theorem, this R_{Th} is the load resistance of the network i.e. $R_L = R_{Th}$ that follows maximum power transfer.

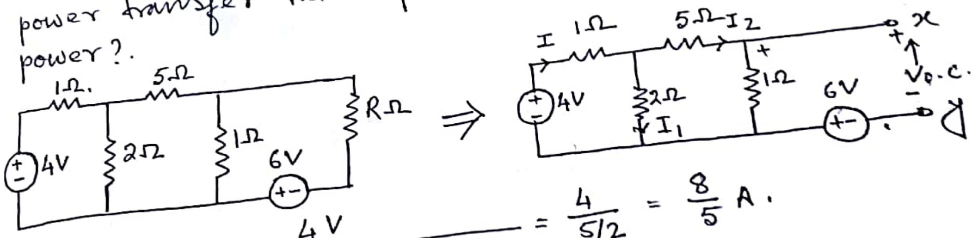
step 3: Find the Thevenin's Voltage (V_0) across the open circuited load terminals.

step 4: Maximum power transfer is given by:

$$\frac{V_0^2}{4R_{Th}}.$$

Example:

. Find the value of R in the circuit of fig. such that maximum power transfer takes place. What is the amount of this power?.



$$\text{Here } I = \frac{4V}{[\{(5+1) \parallel 2\} + 1]\,\Omega} = \frac{4}{5/2} = \frac{8}{5}\,A.$$

$$\therefore \quad I_2 = I\,\frac{2}{2+5+1} = \frac{8}{5} \times \frac{1}{4} = \frac{2}{5}\,A.$$

The drop across a-b branch is then,

$$V_{a-b} = \frac{2}{5} \times 1 = 2/5\ V.$$

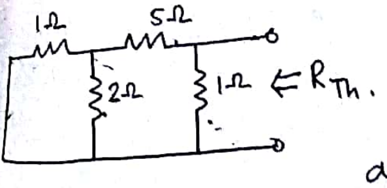
Obviously,

$$V_{o.c} = V_{a\text{-}b} + 6V = \frac{2}{5} + 6 = \frac{32}{5}\,V.$$

or, $V_{o.c} = 6.4\,V.$

$$R_{Th} = (1 \parallel 2 + 5) \parallel 1 = \frac{17}{3} \parallel 1 = \frac{17}{20}\,\Omega$$

$$= 0.85\,\Omega.$$

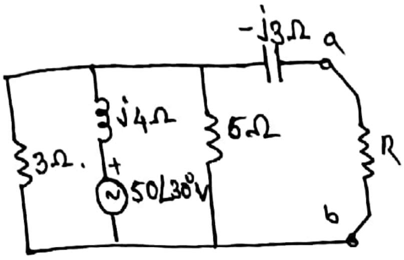


As per max. power transfer $R = R_{Th} = 0.85\,\Omega$.

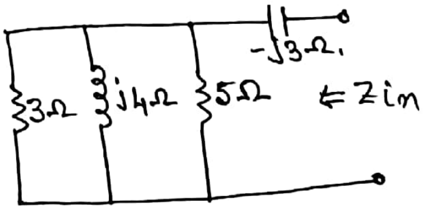
and $$P_{max} = \frac{V_{o.c}^2}{4R} = \frac{(6.4)^2}{4 \times 0.85} = 12\ \text{Watt}.$$

Maximum Power Transfer

4/ What should be the value of R_L so the maximum Power can be transferred from the source to R_L.



Sol. Removing R_L, deactivating the source, the internal impedance can be found out as follows.



$$Z_{int} = (-j3) + \frac{1}{1/5 + 1/3 + 1/j4}.$$

$$= (-j3) + \frac{1}{0.2 + 0.33 - j0.25}.$$

$$= \left[-j3 + \frac{1}{(0.533 - j0.25)}\right]\Omega.$$

$$= 2.75\angle -56^\circ\,\Omega. \quad = (1.54 - j2.282).$$

Current from $50\angle 30^\circ$ V source is,

$$I = \frac{50\angle 30^\circ}{j4 + \frac{5\times3}{5+3}}\text{ A} = 11.32\angle -34.89^\circ\text{ A}.$$

$$\therefore V_{o.c} = I \times \frac{3}{3+5} = 11.32 \times 3/8$$

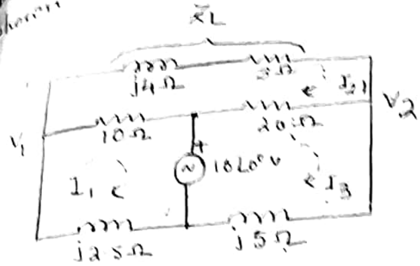
~~Z_{in}~~' R_L = Real Part of $Z_{int} = 1.54\Omega$.

$$P_{max} = \frac{V_{o.c}^2}{4R_L}.$$

$$I = \frac{50\angle 30^\circ}{j4 + 1.875} = \frac{50\angle 30^\circ}{4.417\angle 64.88} = 11.31\angle -34.88.$$

ESH ANALYSIS : (With AC Source) PROBLEMS :

Find the current through Z_L using mesh analysis for the network shown.



The mesh ct. being given by dotted arrowhead lines, mesh analysis gives.

$$(10+j2.5)I_1 - 10I_2 = -10\angle 0^\circ \quad \ldots\ldots (1)$$

$$(20+j5)I_3 - 20I_2 = 10\angle 0^\circ \quad \ldots\ldots (2)$$

$$-10I_1 - 20I_3 + I_2(33+j4) = 0 \quad \ldots\ldots (3).$$

Rearranging in matrix form,

$$\begin{bmatrix}(10+j2.5) & -10 & 0\\ 0 & -20 & (20+j5)\\ -10 & (33+j4) & -20\end{bmatrix}\begin{bmatrix}I_1\\ I_2\\ I_3\end{bmatrix} = \begin{bmatrix}-10\angle 0^\circ\\ 10\angle 0^\circ\\ 0\end{bmatrix}$$

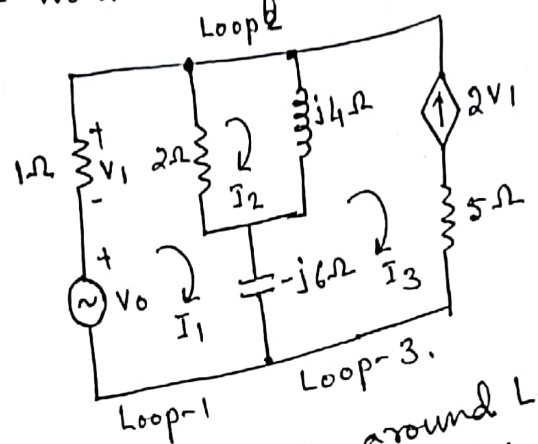
$$\therefore\ I_2 = \frac{\begin{vmatrix}10+j2.5 & -10\angle 0^\circ & 0\\ 0 & 10\angle 0^\circ & 20+j5\\ -10 & 0 & -20\end{vmatrix}}{\begin{vmatrix}10+j2.5 & -10 & 0\\ 0 & -20 & (20+j5)\\ -10 & (33+j4) & -20\end{vmatrix}} = \frac{(10+j2.5)10\angle 0^\circ \times (-20) - (-10\angle 0^\circ)\{10(20+j5)\}}{\Delta Z}$$

where ΔZ = determinant of the matrix in the denominator.

$$= \frac{-2000 - j500 + 2000 + j500}{\Delta Z} = \frac{0}{\Delta Z} = 0.$$

Thus it is seen that no current would flow through Z_L, since I_2 is found to be zero and at the starting of the solution, I_2 had been the current flowing through Z_L.

Find the mesh transformation Maxtrix of the given c



Sol. Application of KVL around Loop-1 yields.

$+V_0 - 1.I_1 - 2(I_1 - I_2) - (-j6)(I_1 - I_3) = 0$

$-j6.I_3 = -V_0.$

or, $-I_1(3-j6) + 2I_2$ $+ j6I_3.$

or, $V_0 = I_1(3-j6) - 2I_2 +$

Application of KVL at loop-2 yields.

$-2(I_2 - I_1) - j4(I_2 - I_3) = 0$

or, $-2I_1 + I_2(2+j4) - (j4)I_3 = 0.$

Presence of current source in the third loop makes the KVL redundant and using KCL.

$I_3 = -2V_1 = -2I_1$

or, $I_3 + 2I_1 = 0.$

Thus.

$$\begin{bmatrix} (3-j6) & -2 & +j6 \\ -2 & (2+j4) & (-j4) \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \\ 0 \end{bmatrix}$$

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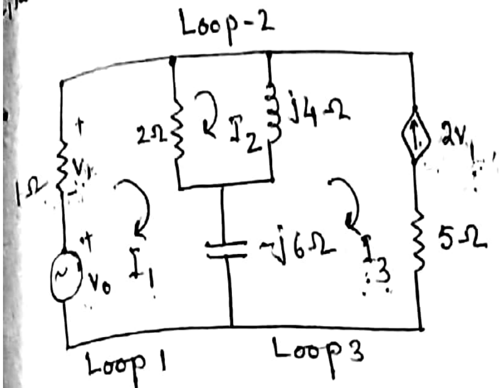
ind the mesh transformation matrix of the given circuit.



Sol. Application of KVL around Loop-1 yields.

$V_0 - 1.I_1 \mp 2(I_1 - I_2) - (-j6)(I_1 - I_3) = 0.$

or, $-I_1(3 - j6) + 2I_2 - j6.I_3 = V_0$ . . . . (1)

KVL at loop-2 yields.

$-2(I_2 - I_1) - j4(I_2 - I_3) = 0$

or, $2I_1 - I_2(2 + j4) + (j4)I_3 = 0$ . . . . (2)

Presence of current source in the third loop makes the use of KVL redundant and using KCL,

$I_3 = -2V_1 = -2I_1$ . . . . (3).

or, $I_3 + 2I_1 = 0$.

In matrix form.

$$\begin{bmatrix} (3-j6) & -2 & +j6 \\ 2 & -(2+j4) & -j4 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \\ 0 \end{bmatrix}.$$

$\Downarrow$

Mesh Transformation Matrix.

$A = (3-j6)[-2-j4) + 2(2+j8) + j6(4+j8)$

$= -6 - j\cancel{12} + \cancel{12}j - 24 + 4 + 16j + 24j - 48$

$= -74 + 40j$

Say $V_0 = 5\angle 0^\circ$.

$$A_1 = \begin{bmatrix} V_0 & -2 & +j6 \\ 0 & -(2+j4) & -j4 \\ 0 & 0 & 1 \end{bmatrix}$$

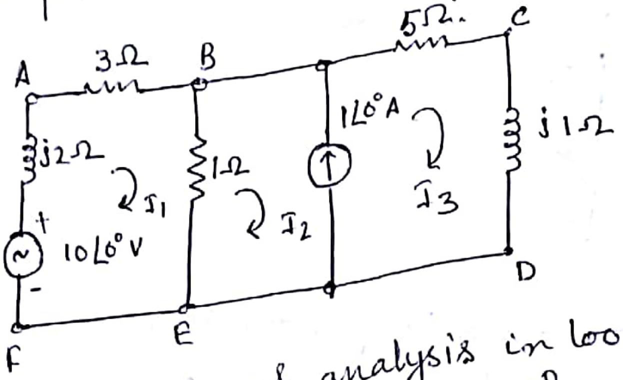
$= V_0[-(2+j4) - 0] \Rightarrow (5 + j0)(-2 - j4) = -10 - 20j + 0$

$= -(10 + 20j).$

$$I_1 = \frac{A_1}{A}, = \frac{-10 - 20j}{-74 + 40j} = \frac{22.36\angle -116.56}{84.11\angle 151.60}$$

$= .265\angle$

Prob. Develop mesh equations for the network shown the power absorbed by the 3 Ω resistor.



Application of mesh analysis in loop ABEF yields.

$I_1(4+j2) - I_2 \times 1 = 10\angle 0^\circ \ldots\ldots (1)$

For loop BCDE.

$(5+j1)I_3 + I_2 - I_1 = 0$

or, $-I_1 + I_2 + (5+j1)I_3 = 0 \ldots\ldots (2)$

But it is evident from fig. $I_3 - I_2 = 1\angle 0^\circ \ldots\ldots (3)$.

Next, subtracting equation (3) from (1),

$(4+j2)I_1 - I_3 = 10\angle 0^\circ - 1\angle 0^\circ$

$= 9. \ldots (4)$

or, $I_3 = [(4+j2)I_1 - 9]. \ldots$

Also adding equation (1) to (2), we get,

$(4+j2)I_1 - I_1 + (5+j1)I_3 = 10\angle 0^\circ.$

$(4+j2)I_1 - I_1 + (5+j1)[(4+j2)I_1 - 9] = 10.$

or, $(3+j2)I_1 + (5+j1)(4+j2)I_1 - 9(5+j1) = 10$

or, $3I_1 + j2I_1 + (5+j1)(4+j2)I_1 - 45 - j9 = 10$

or, $3I_1 + j2I_1 + 20I_1 + j10I_1 + j4I_1 - 2I_1 = 10 + 45 + j9$

or, $21I_1 + j16I_1 = 55 + j9.$

$= 55 + j9.$

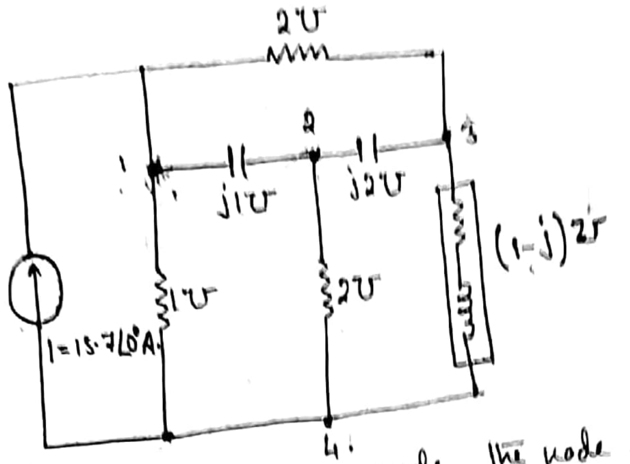
$I_1 = \frac{55+j9}{21+j16} = \frac{55.73\angle 9.29^\circ}{26.4\angle 37.3^\circ}$

$= 2.11\angle -28.01^\circ$ A.

Power absorbed by 3 Ω resistor is.

$(2.11)^2 \times 3 = 13.36$ Watt.

lculate the Power Delivered by the source in the circuit.



Taking Node 4 as the datum node, the node equations are,

Node 1: $V_1(1+2+j1) - jV_2 - 2V_3 = 15.7\angle 0^\circ$.

Node 2: $-jV_1 + V_2(2+j1+j2) - 2jV_3 = 0$

Node 3: $-2V_1 - 2jV_2 + V_3(1-j1+j2+2) = 0$.

In Matrix Form!

$$\begin{bmatrix} 3+j1 & -j & -2 \\ -j & 2+j3 & -2j \\ -2 & -2j & 3+j1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 15.7 \\ 0 \\ 0 \end{bmatrix}.$$

$$V_1 = \frac{\begin{vmatrix} 15.7 & -j & -2 \\ 0 & 2+3j & -2j \\ 0 & -2j & 3+j \end{vmatrix}}{\begin{vmatrix} 3+j & -j & -2 \\ -j & 2+3j & -2j \\ -2 & -2j & 3+j \end{vmatrix}} = \frac{15.7(7+11j)}{13+j29}.$$

Power Delivered by the source $= V_1 I \cos\theta = \text{Real}[V_1 I]$, which on simplification equals 100 watts.

$V_1 I$ = Apparent Power ∴

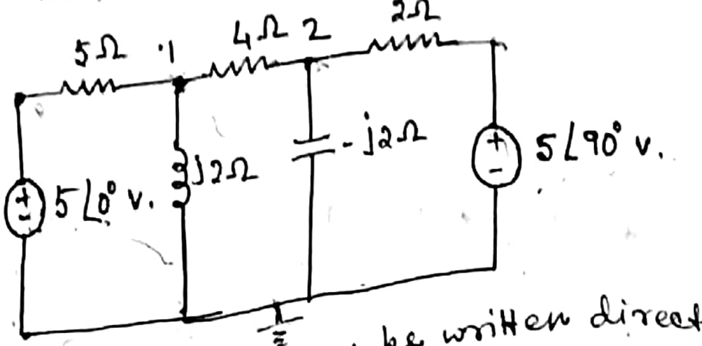
$r\angle\theta^\circ = r\cos\theta + jr\sin\theta$.

$a + jb$.

$r = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1} b/a$.

2. For the network shown, determine the node voltages.



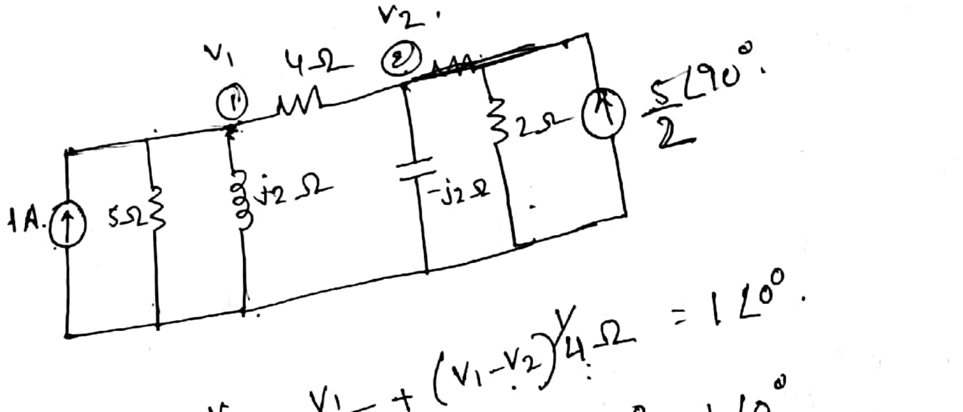
The nodal equations can be written directly in the m… after source transformation.

$$\begin{bmatrix}\frac{1}{5}+\frac{1}{j2}+\frac{1}{4} & -\frac{1}{4}\\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{-j2}+\frac{1}{2}\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix} = \begin{bmatrix}\frac{5}{5}\angle 0^\circ\\ \frac{5\angle 90^\circ}{2}\end{bmatrix} = \begin{bmatrix}1\\ j2.5\end{bmatrix}$$

$$V_1 = \frac{1}{\Delta}\begin{vmatrix}1 & -0.25\\ j2.5 & 0.75+j0.5\end{vmatrix} = 2.48\angle 72.10$$

$$V_2 = \frac{1}{\Delta}\begin{vmatrix}0.45-j0.5 & 1\\ -0.25 & j2.5\end{vmatrix} = 3.44\angle 26.47^\circ$$

$$\text{where } \Delta = \begin{vmatrix}0.45-j0.5 & -0.25\\ -0.25 & 0.75+j0.5\end{vmatrix} = 0.545\angle -15.8^\circ$$



$$(V_1-V_2)/4\,\Omega = 1\angle 0^\circ$$

at Node 1: $\frac{V_1}{5} + \frac{V_1}{j2} + (V_1 - V_2)/4\,\Omega = 1\angle 0^\circ$

$$V_1\left[\frac{1}{5}+\frac{1}{j2}+\frac{1}{4}\right] - V_2\frac{1}{4} ~~1\angle 0^\circ~~ = 1\angle 0^\circ$$

at node 2: $\frac{V_2}{-j2} + \frac{V_2}{2} + (V_2 - V_1)\frac{1}{4} = \frac{5}{2}\angle 90^\circ$

$$-\frac{1}{4}V_1 + V_2\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{-j2}\right] = 2.5\angle 90^\circ = 0 + j2.5$$

Jey,

n the

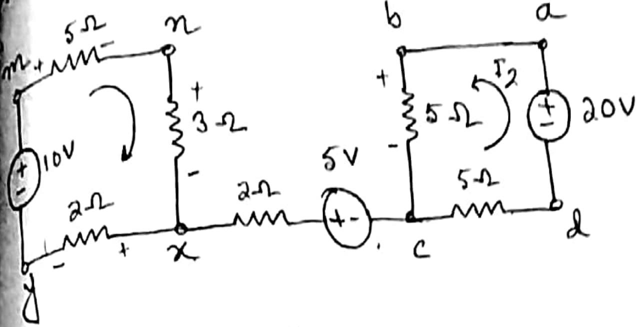
mesh

$= \begin{bmatrix} 1 \\ j2.5 \end{bmatrix}$

15.80

lem (Network Analysis)

nd the drop between the terminals y and d in the network.



Sol. The current in loop m-n-x-y is given by

$$I_1 = \frac{10}{5+2+3} = 1\ A.$$

$\therefore$ Drop in m-n branch $V_{m-n} = 5 \times 1 = 5V.$

Drop in n-x branch $V_{n-x} = 3 \times 1 = 3V.$

Drop in x-y branch $V_{x-y} = 2 \times 1 = 2V.$

Current in loop a-b-c-d, $I_2 = \frac{20}{5+5} = 2A.$

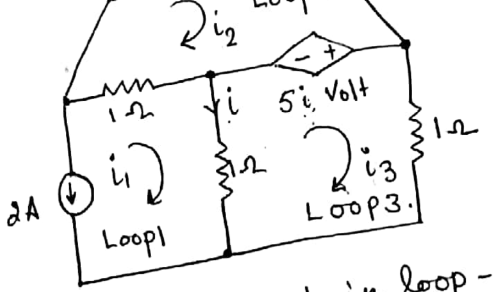
Drop in b-c, $V_{b-c} = 5 \times 2 = 10V,$

Drop in c-d, $V_{c-d} = 5 \times 2 = 10V.$

Inspecting the respective polarities,

$$V_{y-d} = V_{y-x} + V_{x-c} + V_{c-d} = -2+5+10 = 13V.$$

Q. Using Mesh Analysis, find the magnitude of the current dependent source and the current through the 2Ω resistor.



Sol. It is evident that in loop-1,

$i_1 = -2A.$

In loop-3, $i_3 \times 1 + (i_3 - i_1)1 - 5i = 0$

However, $i = i_1 - i_3$

and $i_1 = -2A.$

Substituting these values

$$i_3 + (i_3 + 2) - 5(-2 - i_3) = 0$$

or, $i_3 + i_3 + 2 + 10 + 5i_3 = 0$

or, $i_3 = -\frac{12}{7} = -1.71\,A$.

In Loop-2,

$$i_2 \times 2 + 5i + (i_2 - i_1) = 0$$

or, $2i_2 + 5(i_1 - i_3) + i_2 - i_1 = 0$

or, $3i_2 + 4i_1 - 5(-1.71) = 0$

or, $4i_1 + 3i_2 = -8.55$

or, $4(-2) + 3i_2 = -8.55$

$\therefore\ i_2 = -0.183\,A$.

Also $i = i_1 - i_3 = -2 - (-1.71) = -0.29\,A$.

Value of dependent source $= 5i = -5 \times 0.29 = -1.45$

Thus the magnitude of dependent source = 1.45 V

The magnitude of the actual current i which as fou out, is upwards in the circuit, the actual polarity dependent source is opposite to that shown.

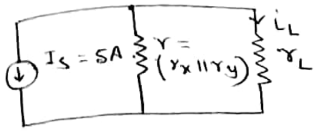
The current through 2Ω resistor is i_2 i.e. 0.183 A flowing anticlockwise in loop-2.

nd Mesh Analysis with DC source:

the circuit find the current through r_L.

①

⇓ Reduced.



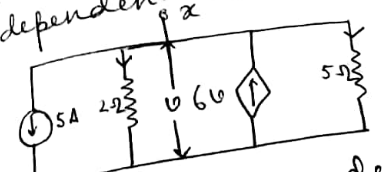
The equivalent current source I_s would be $(10A - 5A) = 5A$ directed downwards, the net resistance of r_x and r_y would be $\frac{5 \times 2}{5+2} = \frac{10}{7}\Omega$.

By current division method

$$i_L = I_s \times \frac{r}{r+r_L} = 5\,\frac{10/7}{\frac{10}{7}+10} := 5 \times \frac{10}{7} \times \frac{7}{80} = \frac{5}{8}A.$$

i.e. $i_L = \frac{5}{8}A$.

Find v in the circuit. Also obtain numerical value of the dependent source.



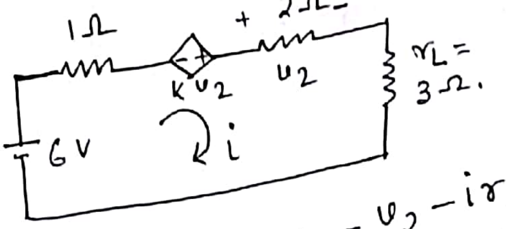
Applying KCL at node 'x',

$$-5 + 6v - \frac{v}{2} - \frac{v}{5} = 0$$

or, $\frac{53}{10}v = 5$, or $v = \frac{50}{53} = 0.943V$.

∴ The numerical value of the dependent source is then $6 \times 0.943 = 5.66A$.

3. In the circuit find the drop across r_L if $k = 2$.



Sol. $6 - i\times 1 + kv_2 - v_2 - ir_L = 0$

or, $6 - i(1+r_L) + v_2(k-1) = 0$.

or, $6 - 4i + v_2 = 0$

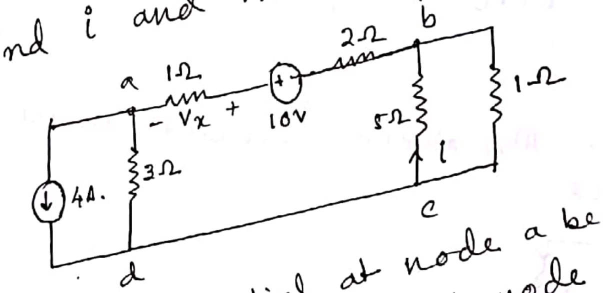
But, $v_2 = i\times 2 = 2i$.

∴ $2i = 6$

or, $i = 3$ A.

∴ Drop across $r_L = 3\times 3 = 9$V. (with positive polari

4. Find i and V_x in fig.



Sol. Let the potential at node a be V_a and b be V_b.

Applying nodal analysis at node a,

$4 + \frac{V_a}{3} + \frac{V_a - 10 - V_b}{3} = 0$

or, $4 + 0.33V_a + 0.33V_a - 3.33 - 0.33V_b = 0$

or, $0.66V_a - 0.33V_b = -0.67$ .... (1).

At node b,

$\frac{V_b + 10 - V_a}{3} + \frac{V_b}{5} + \frac{V_b}{1} = 0$

or, $0.33V_b + 3.33 - 0.33V_a + 0.2V_b + V_b = 0$

or, $1.53V_b - 0.33V_a = -3.33$.

or, $3.06V_b - 0.66V_a = -6.66$ ..... (2)

From (1) and (2),

$2.73V_b = -7.33$

∴ $V_b = -2.685$V; , $V_a = -2.358$ volts.

Again $i = -\left(\frac{V_b}{5}\right)$. [∵ i shown in is against the dir. of the assumed respective branch ct.].

i.e. $= -\frac{2.685}{5} = 0.537$A.

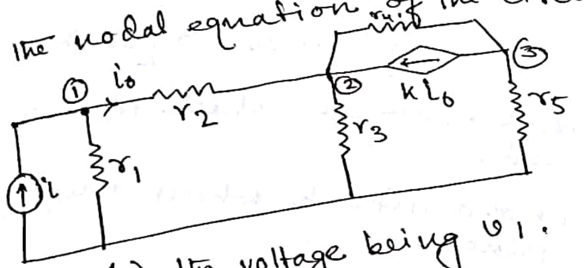
③

…rrent in branch a-b is $\frac{V_a-10-V_b}{3}$ A.

i.e. $\frac{-2.358-10+2.685}{3} = -3.22$ A. (actual dir. will be then b to a)

∴ $V_x = 3.22 \times 1 = 3.22$ V.

Develop the nodal equation of the circuit shown in Matrix form.



Sol. At node (1), the voltage being v_1.

$$i = \frac{v_1}{r_1} + \frac{v_1 - v_2}{r_2}$$

$$\text{or, } v_1\left(\frac{1}{r_1}+\frac{1}{r_2}\right) - v_2\cdot\frac{1}{r_2} = i$$

At node (2),

$$ki_0 = \frac{v_2}{r_3} + \frac{v_2-v_3}{r_4} + \frac{v_2-v_1}{r_2}$$

$$\text{or, } -\frac{v_1}{r_2} + v_2\left(\frac{1}{r_3}+\frac{1}{r_4}+\frac{1}{r_2}\right) - v_3\cdot\frac{1}{r_4} = ki_0.$$

At node (3),

$$ki_0 + \frac{v_3}{r_5} + \frac{v_3-v_2}{r_4} = 0.$$

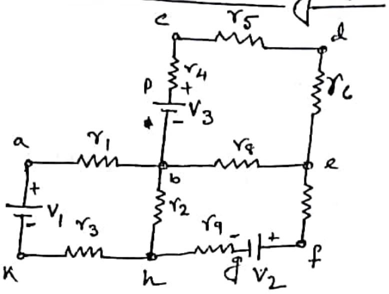
$$\text{or, } v_3\left(\frac{1}{r_4}+\frac{1}{r_5}\right) - v_2\cdot\frac{1}{r_4} + ki_0 = 0.$$

$$\text{or, } -v_2\cdot\frac{1}{r_4} + v_3\left(\frac{1}{r_4}+\frac{1}{r_5}\right) = -ki_0$$

In Matrix form,

$$\begin{bmatrix} (1/r_1+1/r_2) & \left(-\frac{1}{r_2}\right) & 0 \\ -\frac{1}{r_2} & \left(\frac{1}{r_3}+\frac{1}{r_4}+\frac{1}{r_2}\right) & \left(-\frac{1}{r_4}\right) \\ 0 & -\frac{1}{r_4} & \left(\frac{1}{r_4}+\frac{1}{r_5}\right) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i \\ ki_0 \\ -ki_0 \end{bmatrix}.$$

Node and Mesh Analysis :



No. of Circuit elements = 12
[9 resistor + 3 Voltages]

No. of Nodes = 10[a, b, c, d, e, f. g. h,

No. of junction points = 3

No. of branches = 5 [bcde, be, bh,

No. of meshes = 3 [abhk, bcde,

Node : It is an equipotential point at which two or more elements are joined.

Junction : It is that point of a network where three or more elements are joined.

Branch : It is a part of a network which lies between jun points.

Circuit Element	Voltage (volts)	Current (amps)	Power (watts).
R (Ω)	$v = Ri$	$i = \frac{v}{R}$	$p = i^2R$.
L (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + i_0$ [i_0 being the initial ct.].	$p = Li\frac{di}{dt}$.
C (F)	$v = \frac{1}{C}\int i\,dt + v_0$ [v_0 being the initial voltage]	$i = C\frac{dv}{dt}$	$p = Cv\frac{dv}{dt}$

$C = \frac{q}{v}$ i.e. $i = C\frac{dv}{dt}$ [$\because i = \frac{dq}{dt}$].

or, $dv = \frac{1}{C}\,i\,dt$

or, $\int_{v_0}^{v_t} dv \pm \frac{1}{C}\int_0^t i\,dt$. [$v_0$ initial voltage of capacitor, v_t: voltage of capacitor]

or, $v_t - v_0 = \frac{1}{C}\int_0^t i\,dt$.

i.e. $v_t = \frac{1}{C}\int_0^t i\,dt + v_0$.

The power absorbed by the capacitor is given by

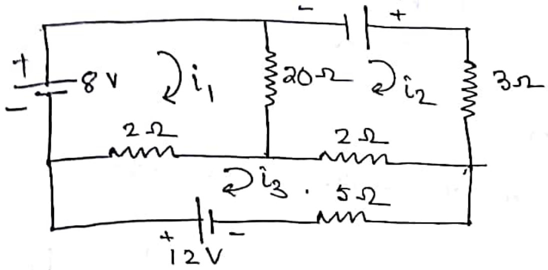
$$p = vi = vC\frac{dv}{dt}.$$

Energy stored by the capacitor is

$$W = \int_0^t p\,dt = \int_0^t vC\frac{dv}{dt}\,dt = \frac{1}{2}Cv^2.$$

Problem

Determine current in 5Ω resistor



For loop 1

$$20(i_1 - i_2) + 2(i_1 - i_3) = 8V$$

or, $22i_1 - 20i_2 - 2i_3 = 8$

For loop 2

$$20(i_2 - i_1) + 3i_2 + 2(i_2 - i_3) = 10$$

or, $-20i_1 + 25i_2 - 2i_3 = 10$

For loop 3

$$2(i_3 - i_1) + 2(i_3 - i_2) + 5i_3 = 12$$

or, $-2i_1 - 2i_2 + 9i_3 = 12$.

$$\therefore \begin{bmatrix} 22 & -20 & -2 \\ -20 & 25 & -2 \\ -2 & -2 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 12 \end{bmatrix}.$$

$$i_3 = \frac{\begin{vmatrix} 22 & -20 & 8 \\ -20 & 25 & -10 \\ -2 & -2 & 12 \end{vmatrix}}{\begin{vmatrix} 22 & -20 & -2 \\ -20 & 25 & -2 \\ -2 & -2 & 9 \end{vmatrix}} = \frac{22(300-20) + 20(-240-20) + 8(40+50)}{22(225-4) + 20(-180-4) - 2(40+50)}$$

$$= \frac{6160 + (-5200) + 720}{4862 + (-3680) - 180}$$

$$= \frac{1680}{1002} \simeq 1.67 \text{ Amp}.$$

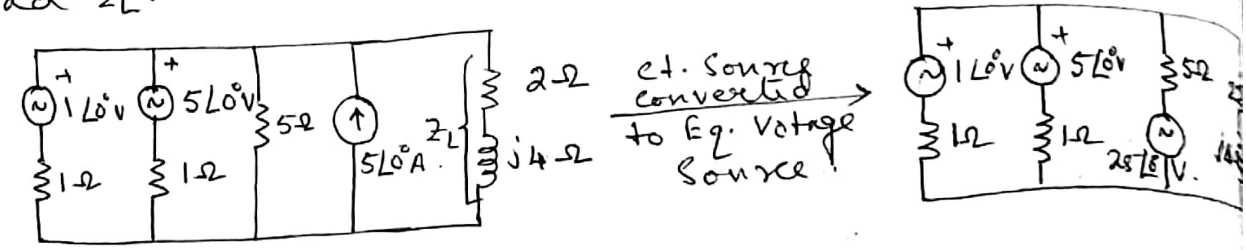
Millman's Theorem for AC Network:

As per Millman's Theorem:

$$V_m = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3 + \cdots + V_n Y_n}{Y_1 + Y_2 + \cdots + Y_n}$$

and $$Z_m = \frac{1}{Y_1 + Y_2 + \cdots Y_n}$$

Using Millman's Theorem find the circuit in the Load Z_L.



Equivalent Valtage Source is given by.

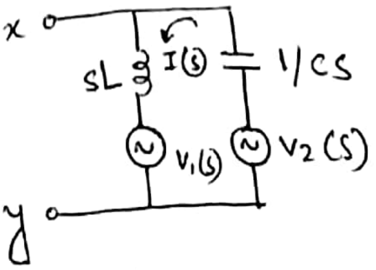
$$E = \frac{E_1 Y_1 + E_2 Y_2 + E_3 Y_3}{Y_1 + Y_2 + Y_3}$$

$$= \frac{1\angle 0° \times 1/1 + 5\angle 0° \times 1/1 + 25\angle 0° \times 1/1}{1/1 + 1/1 + 1/5}$$

$$= \frac{11}{2.2} = 5\angle 0° \text{ V}.$$

$$\therefore \quad I_L = \frac{E}{Z_L} = \frac{5\angle 0°}{2+j4} = 1.12\angle -63.43° \text{ A}.$$

Fig. represent a steady state circuit in s-domain. Find its Thevenin's Equivalent at the right of terminal x-y.



Let I(s) be the circulating loop current as shown.

Obviously. $I(s) = \frac{V_2(s) - V_1(s)}{sL + 1/cs}$.

Assuming the open circuit voltage across x-y terminal to be Vo.c, application of KVL gives.

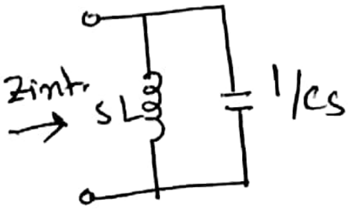
$$V_{o.c} = sLI(s) + V_1(s)$$

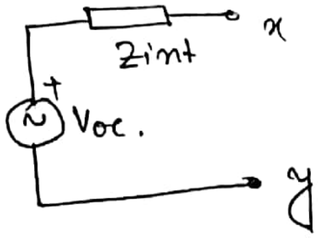
$$= V_1(s) + sL\,\frac{V_2(s) - V_1(s)}{sL + 1/cs}$$

$$= \frac{V_1(s) + s^2LcV_2(s)}{s^2Lc + 1}.$$

With independent sources deactivated, the internal impedance,

$$Z_{int} = sL \parallel \frac{1}{cs} = \frac{sL(1/cs)}{sL + 1/cs} = \frac{sL}{s^2Lc+1}.$$





THEVENIN'S THEOREM

Nabanita Chatterjee
(EE Dept)

It is possibly the most extensively used network theorem. It is applicable where it is desired to determine the current through or voltage across any one element in a network without going through the rigorous method of solving a set of network equations.

It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by a single voltage source with a series resistance. It makes the solution of complicated networks quite quick and easy.

Statement :

Any two terminal bilateral linear d.c. circuits can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

The thevenin voltage e is the open circuit voltage at terminals A and B.

The Thevenin resistance is the resistance seen at AB with all voltage sources replaced by short circuits and all current sources replaced by open circuits.

Explanation

Let us consider a simple d.c. circuit shown in Fig (a). We are to find I_L by Thevenin's Theorem. In order to find the equivalent voltage source, r_L is removed [fig. (b)] and $V_{o.c.}$ is calculated.

(a)

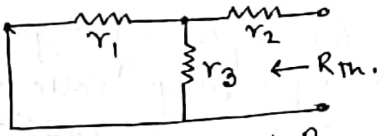
(b) Finding of $V_{o.c.}$

$$V_{o.c.} = I r_3 = \frac{V_s}{r_1 + r_3} \cdot r_3$$

** A more general statement of Thevenin's theorem is that any linear active network consisting of independent and or dependent voltage and current sources and linear bilateral network elements can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance, the voltage source being the open circuited voltage across the open circuited load terminals and the resistance being the internal resistance of the source network looking through the the open circuited load terminals.

Next, to find the internal resistance of the network (Thevenin's -e. or equivalent resistance) in series with Vo.c, the volt source is removed (deactivated) by a short circuit (as the source does not have any internal resistance) as shown fig. (c).

$$R_{Th} = r_2 + \frac{r_1 \cdot r_3}{r_1 + r_3}.$$



(c). Finding of R_{Th}.

As per Thevenin's theorem, the equivalent circuit being Fig. (d)

$$I_L = \frac{V_{o.c}}{R_{Th} + r_L} \text{ Ampere.}$$

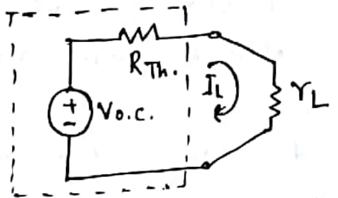


fig.(d) Finding of I_L.

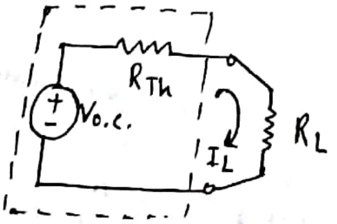
Steps for Solution of a Network Utilizing Thevenin's Theorem:

Step 1: Remove the load resistor (R_L) and find the open circuit valtage (Vo.c.) across the open circuited load terminals.

Step 2: Deactivate the constant sources (for valtage source, remov it by internal resistance and for current source delete the source by open circuit) and find the internal resistance (Thevenin's Resistance) of the source side looking through the open circuited load terminals. Let this resistance be R_{Th}.

Step 3: Obtain Thevenin's equivalent circuit by placing R_{Th} in series with Vo.c.

Step 4: Reconnect R_L across the load terminals as shown.



Equivalent Source.

Thevenin's Equivalent Network

Obviously I (the load current)

$$= \frac{V_{oc}}{R_{Th} + R_L}.$$

Different Methods of Finding R_{Th} :

(Thevenin's Equivalent resistance or Internal impedance)

(a) For Independent Sources:

The most common method of finding R_{Th}, the internal resistance of any linear, bilateral network containing independent current or voltage sources is to deactivate the source by internal resistance i.e. for independent current source, deactivate it by removing the source and for voltage source, deactivate it by shorting it (assuming internal resistance of the voltage source being zero). Then find the internal resistance of the network looking through the load terminals kept open circuited.

(b). For circuit containing dependent sources in addition to or in absence of independent source

1st Method:

Find $V_{o.c.}$ across the open circuited load terminals by conventional network analysis. Next, short the load terminals and find the short circuit current ($I_{s.c.}$) through the shorted terminals.

The internal resistance of the source network is then obtained as

$$R_{Th} = \frac{V_{o.c}}{I_{s.c}}$$

2nd Method:

Remove the load resistance and apply a d.c. driving voltage V_{dc} at the open circuited load terminals. Keep the other independent sources deactivated during this time (i.e. short the voltage source terminals and open the current source terminals). A d.c. driving current i_{dc} will flow in the circuit from the load terminals due to application of V_{dc}.

The internal resistance of the source network is then obtained as

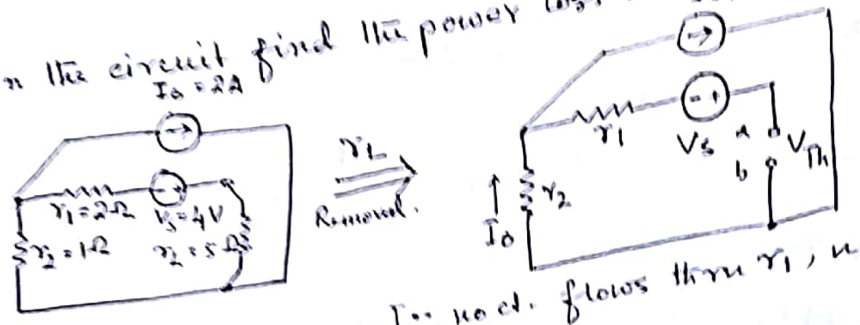
$$R_{Th} = \frac{V_{dc}}{i_{dc}}.$$

$$= \frac{r_1 r_2 r_3 + r_1 r_2 r_4 + r_3 r_4 r_1 + r_3 r_4 r_2}{(r_1+r_4)(r_3+r_2)}$$

∴ In the given bridge circuit,

$V_{o.c} = \frac{E(r_4 r_2 - r_1 r_3)}{(r_2+r_3)(r_1+r_4)}$ and $R_{int} = \frac{r_1 r_2 r_3 + r_1 r_2 r_4 + r_3 r_4 r_1}{(r_1+r_4)(r_3+r_2)}$

③ In the circuit find the power loss in r_L utilising Thevenin's Th

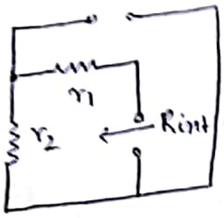


V_{Th} ⇒ open ckt. v across open load termi

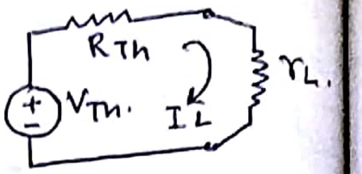
Here $V_{Th} = V_s - I_o r_2$ [∵ no ct. flows thru r_1, no voltage drop acros

or, $V_{Th} = 4 - 2 \times 1 = 2V$.

To find Rint (i.e. internal resistance of the ckt. across a-b), all sour sources are deactivated i.e. $V_s = 0$ and $I_o = 0$.



Obviously $R_{Th} = r_1 + r_2 = 3\Omega$



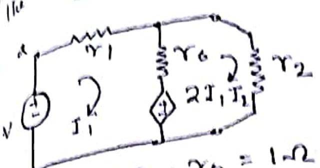
Thus we have obtained the equivalent Thevenin's Circuit whe

$V_{o.c} = V_{Th} = 2V$, $R_{Th} = R_{int} = 3\Omega$.

∴ I_L (load current through r_L) $= \frac{2}{3 + r_L} = \frac{2}{8} = \frac{1}{4} A = 0.25 A$.

∴ Power Loss P_L in $r_L = I_L^2 \times r_L = (0.25)^2 \times 5 = 0.3125$ watt.

Find the Thevenin's equivalent circuit for the network shown at the right of terminal a-b.



$r_1 = r_2 = r_0 = 1\Omega$.

Sol. Let the terminals a-b be open circuited. This leads to $I_1 = 0$ and the depending voltage source $2I_1$ is also zero. Also $I_2 = 0$. Obviously Voc. is zero.

Next, a dc voltage supply V_{dc} be applied across a-b such that the input current be I_1 at terminal a.

Applying KVL at the left loop under this condition,

$$V_{dc} = I_1 r_1 + I_1 r_0 - I_2 r_0 + 2I_1$$

or, $V_{dc} = r_1 I_1 + r_0 (I_1 - I_2) + 2I_1 \ldots (a)$

Applying KVL at the outer loop, $V_{dc} = r_1 I_1 + r_2 I_2 \ldots (b)$.

However $r_1 = r_2 = r_0 = 1\Omega$

$\therefore$ (a) becomes $V_{dc} = I_1 + I_1 - I_2 + 2I_1 \Rightarrow V_{dc} = 4I_1 - I_2 \ldots (c)$

and (b) becomes $V_{dc} = I_1 + I_2 \ldots (d)$.

from (d) $I_2 = V_{dc} - I_1$.

Using this value of I_2 in (c), $V_{dc} = 4I_1 - (V_{dc} - I_1)$

or, $2V_{dc} = 5I_1$

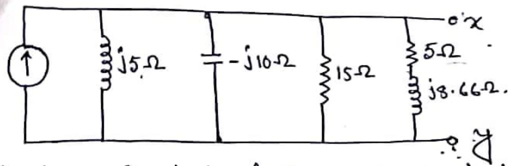
$\therefore \frac{V_{dc}}{I_1} = \frac{5}{2} = 2.5\Omega$.

Thus the internal resistance of the circuit across a-b is 2.5Ω.

The given circuit is thus Thevenised to give

$V_{o.c} = 0V$, $R_{int} = 2.5\Omega$.

5. If $I = 33\angle -13^\circ$ A, find the Thevenin's Equivalent circuit to the terminals x-y in the network.



Sol. Let us first find the equivalent impedance across the current However, assuming the equivalent admittance to be Yeq, we f that,

$$Y_{eq} = Y_1 + Y_2 + Y_3 + Y_4$$

$$Y_1 = \frac{1}{j5} = -j0.2 \text{ mho}, \quad Y_2 = \frac{1}{-j10} = j0.1 \text{ mho}$$

$$Y_3 = \frac{1}{15} = 0.067 \text{ mho}, \quad Y_4 = \frac{1}{5+j8.66} = \frac{1}{10\angle 60^\circ}$$

$$= (0.05 - j0.0866) \text{ mho}$$

$$\therefore Y_{eq} = (0.117 - j0.1866) \text{ mho}.$$

Then, $V_{x-y} = (V_{oc}) = I / Y_{eq}.$

$$= 33\angle -13 / (0.117 - j0.1866) = 150\angle 45^\circ \text{ V}.$$

To find $Z_{in} (= Z_{Th})$, the current source is deactivated and by inspection it is observed that

$$Z_{in} = \frac{1}{Y_{eq}} = \frac{1}{0.117 - j0.1866} = \frac{1}{0.22\angle -58^\circ} = 4.545\angle 58^\circ$$

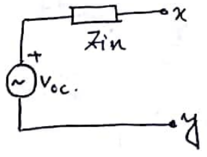


Fig is Thevenin's Equivalent Circuit

$Z_{in} = 4.545\angle 58^\circ \Omega$ and

$V_{oc} = 150\angle 45^\circ$ V.

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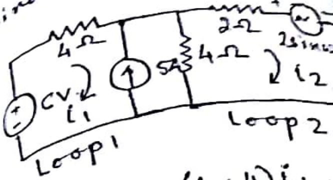
find Thevenin's Equiv



In loop 1. $(4+4)i_1$

$8i_1 -$

In loop 2, $i_2(4+4+$

or, $-4i_1$

or, $-2i_1$

from (1), $8i_1 = 4$

or, i_1

Using the value

$-2(0.5i_2 - 1$

or, $-i_2 + 3.5$

or, $4i_2 = 6.5$

$\therefore i_2 = 1.62$

$\therefore V_{oc} = \cancel{6.5}$

$V_{oc} = 6.$

To find the T

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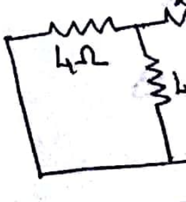
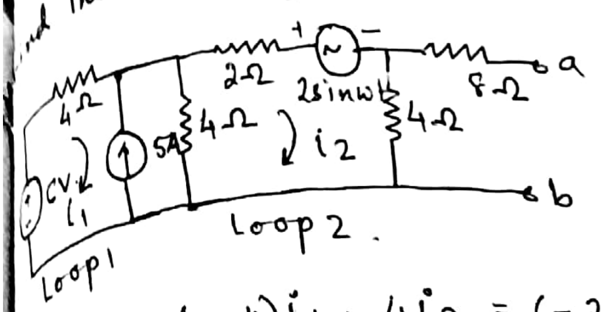


$R_{in} =$

nd Thevenin's Equivalent Circuit of the network shown :



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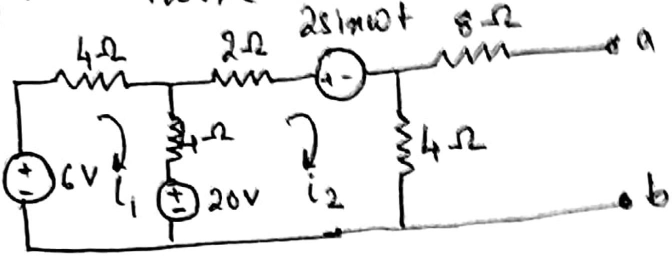


In loop 1. $(4+4)i_1 - 4i_2 = 6-20$

$8i_1 - 4i_2 = -14$ ....(1)

In loop 2, $i_2(4+4+2) - 4i_1 = 20 - 2\sin\omega t$

or, $-4i_1 + 10i_2 = 20 - 2\sin\omega t$

or, $-2i_1 + 5i_2 = 10 - \sin\omega t$ .....(2).

from (1), $8i_1 = 4i_2 - 14$

or, $i_1 = 0.5i_2 - 1.75$ ... (3).

Using the value of i_1 from (3) in (2)

$-2(0.5i_2 - 1.75) + 5i_2 = 10 - \sin\omega t$

or, $-i_2 + 3.5 + 5i_2 = 10 - \sin\omega t$.

or, $4i_2 = 6.5 - \sin\omega t$

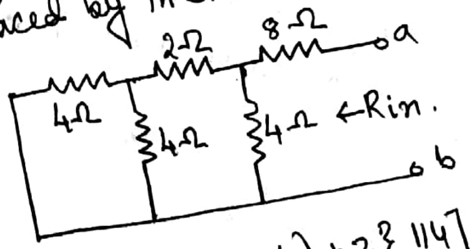
∴ $i_2 = 1.625 - 0.25\sin\omega t$.

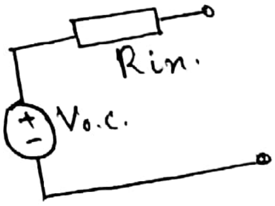
∴ V_{oc} = ~~6.5 − sinωt~~.

$i_2 \times 4 = V_{a-b}$
$= (1.625 - 0.25\sin\omega t)4$.

$V_{oc} = 6.5 - \sin\omega t$.

To find the Thevenin's internal resistance, the sources are replaced by their internal resistances.



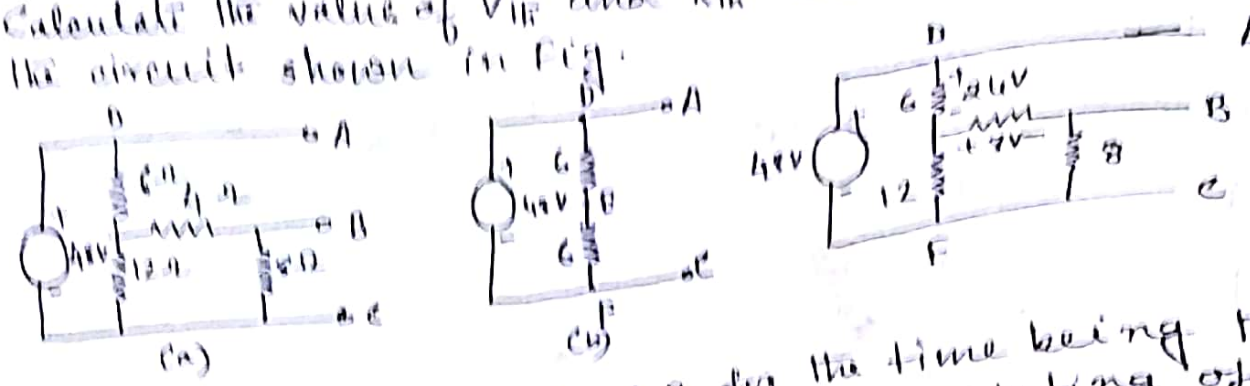


$R_{in} = 10\Omega$

$V_{oc} = (6.5 - \sin\omega t)$ V

$R_{in} = [\{(4 \| 4) + 2\} \| 4] + 8 = 10\Omega$.

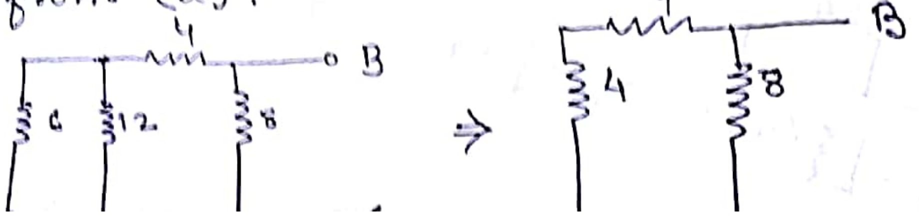
7. Calculate the value of V_{th} and R_{th} between terminals A and the circuit shown in Fig.



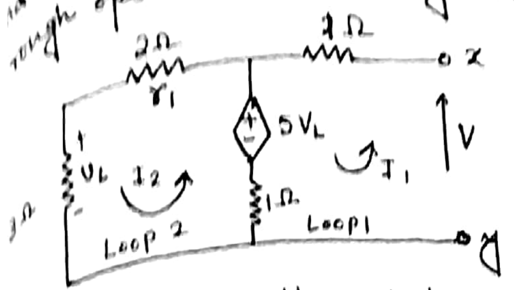
Forgetting about the terminal B for the time being there a one consisting of 12V a parallel paths between E and F: R_{EF} = 12||12 = 6Ω. The the other of (4+8) = 12Ω. Hence resistance source voltage of 48 V drops across two 6Ω resistances connected in series. Hence V_{EF} = 24V. The same 24V a connected directly between E and F across 12Ω resistor connected directly between E and F across two series-connected resistance of 4Ω and 6Ω connected across E and F. Drop across 4Ω resistance = 24 × 4/(4+8) = 8V (fig.c).

Now as we go from B to A via point E, there is rise in voltage of 8V followed by another rise in voltage of 24V thereby giving a total voltage dr of 32V. Hence V_{th} = 32 V with point A positive.

For finding R_{th}, we short-circuit the 48 V source. This short circuiting, in effect, combines the points A, D and F electrically as shown in (a) as seen from (AB).



d the internal resistance of the network given in fig. looking rough open circuited x-y terminal.



Let a d.c. voltage V be applied at x-y [x terminal being +ve]. Obviously, V_L is $(I_2 \times 3)$ or $3I_2$ V.

∴ The magnitude of the dependent source is then $5V_L$ or $(15 \times I_2)$ V.

Applying KVL in loop-1,

$$V = (1+1)I_1 - 5V_L - 1 \times I_2$$

$$V = 2I_1 + 15I_2 - I_2$$

$$= 2I_1 + 14I_2 \quad \text{.....} (1)$$

Applying KVL in loop-2,

$$0 = (1+3+2)I_2 - 5V_L - 1 \times I_1$$

$$0 = 6I_2 - 15I_2 - I_1$$

$$= -9I_2 - I_1 \quad \text{.......} (2)$$

Thus, from (2), $I_2 = -\frac{1}{9} I_1$ ..... (3)

$$\therefore V = 2I_1 - 1.56I_1$$

obviously, from (3),

$$\frac{V}{I_1} = R_{int} = 0.44\Omega.$$

$$2I_1 - 16I_2 = V$$

$$-I_1 + 21I_2 = 0$$

$$I_1 = 21I_2$$

$$\text{or, } I_2 = \cancel{21/I_1}\ I_1/21.$$

$$\therefore V = 2I_1 - \frac{16 \times I_1}{21}$$

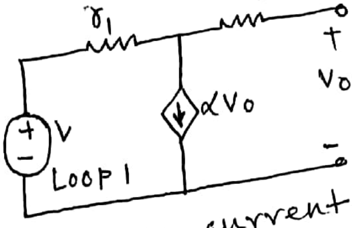
$$= I_1\left(2 - \frac{16}{21}\right).$$

$$= 1.2381 I_1$$

$$\text{or, } \frac{V}{I_1} = 1.2381\Omega.$$

$$= \underline{\underline{R_{int}}}$$

Obtain Thevenin's Equivalent to the left of terminal a-b in the circuit.



In fig., the loop current being αV_o, it could create a drop of $(r_1 \alpha V_o)$ across r_1.

Assuming $V_O \equiv V_{o.c}$

Application of KVL in loop-1 gives.

$$-V_1 + r_1 \alpha V_{o.c} + V_{o.c} = 0$$

$$\text{or, } V_{o.c} = V_1 - r_1 \alpha V_{o.c}$$

$$\text{i.e. } V_{o.c}(1 + r_1 \alpha) = V$$

$$\therefore V_{o.c} = \frac{V}{1 + \alpha r_1}$$

To determine Rint, a-b terminals are shorted, V_o becomes zero. Since $V_o = 0$, the voltage control current source is also equal to zero. Thus r_1, r_2 are in series with V. The circulating current being $I_{s.c}$.

$I_{sc} = \frac{V}{r_1 + r_2}$ and hence, the internal resistance the given network is given by

$$R_{int} = \frac{V_{o.c}}{I_{s.c}} = \frac{V}{1+\alpha r_1} \cdot \frac{r_1 + r_2}{V} = \frac{r_1 + r_2}{1+\alpha r_1}$$

This gives the Thevenin's equivalent circuit as

$$V_{o.c} = \frac{V}{1+\alpha r_1} \; ; \; R_{int} = \frac{r_1 + r_2}{1+\alpha r_1} .$$

ciprocity Theorem Pr

Show the application

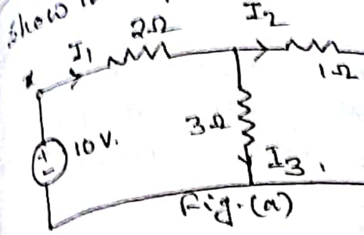


Fig. (a)

$\therefore I_1 = \frac{10}{3.5} =$

$I_3 = 2.86 -$

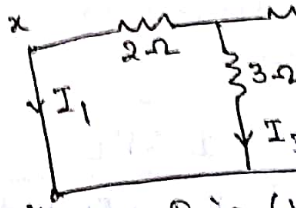


Fig. (

This gives,

Hence we a

x-y, the a

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ciprocity Theorem Problem:

how the application of reciprocity Theorem in the network.

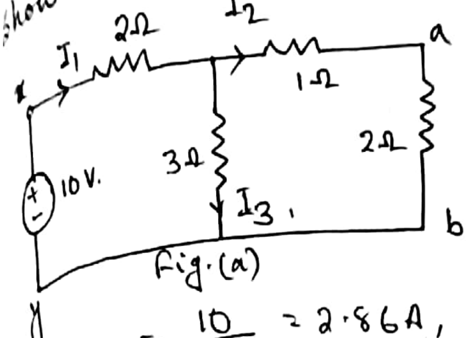


Fig.(a)

The equivalent resistance across x-y is given by

$$R_{eq} = [(2+1)\|3]+2 = 3.5\Omega.$$

$$\therefore I_1 = \frac{10}{3.5} = 2.86A, \quad I_2 = 2.86\times\frac{3}{3+3} = 1.43\,A.$$

$$I_3 = 2.86 - 1.43 = 1.43\,A.$$

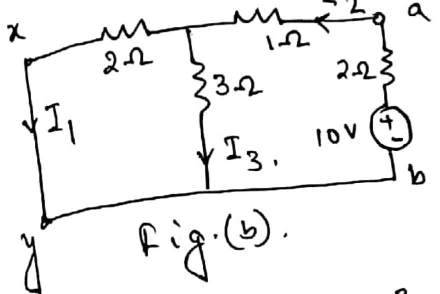


Fig.(b).

With ref to fig.(b).

Req across xy terminal

$$R_{eq} = (2\|3)+1+2 = 4.2\Omega.$$

$$I_2 = \frac{10V}{4.2\Omega} = 2.381\,A.$$

This gives, $I_1 = I_2\frac{3}{3+2} = 2.381\times\frac{3}{5} = 1.43\,A,$

Hence we observe that when the source was in branch x-y, the a-b branch current is 1.43A, again when the source is in branch a-b, the x-y branch current becomes 1.43A.

- In any linear bilateral network if a source of emf in any branch produces a current I in any other branch, then the same emf E acting in the second branch would produce the same current I in the first branch.

In other words, it simply means that E and I are mutually transferrable. The ratio E/I known as the the transfer resistance (or impedance in a.c. system). Another way of stating the above is that the receiving point and the sending point in a n/w are interchangeable. It also means that interchange of an ideal voltage source and an ideal ammeter in any n/w will not change the ammeter reading.

Compensation Theorem

In a linear time-invariant n/w when the resistance (R) an uncoupled branch carrying a current (I) is changed by (ΔR), the currents in all the branches would change and can be obtained by assuming that an ideal voltage source of has been connected in series with (R+ΔR) when all other sources in the n/w are replaced by their internal resistances.

$$I = \frac{V_0}{R_{Th}+R_L} , \quad I' = \frac{V_0}{R_{Th}+R_L+\Delta R_L}$$

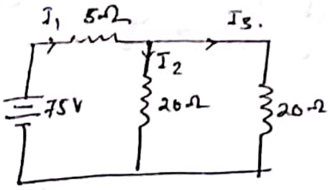
$$\Delta I = I' - I$$

$$= \frac{V_0}{R_{Th}+R_L+\Delta R_L} - \frac{V_0}{R_{Th}+R_L}$$

$$= \frac{V_0[V_{Th}+R_L-R_{Th}-R_L-\Delta R_L]}{(R_{Th}+R_L+\Delta R_L)(R_{Th}+R_L)}$$

$$= -\left(\frac{V_0}{R_{Th}+R_L}\right)\frac{\Delta R_L}{R_{Th}+R_L+\Delta R_L} = -\frac{I\,\Delta R_L}{R_{Th}+R_L+\Delta R_L}$$

$$= \frac{-V_c}{R_{Th}+R_L+\Delta R_L}$$



~~$R_{eq} = \frac{20 \times 20}{40} = \frac{1}{20}+\frac{1}{20}+\frac{1}{5} = \frac{1+1+4}{20} = \frac{6}{20} =$~~

~~$I_1 = \frac{75 \times 10}{3} = 250$~~

en electric current flows thro any resistor, there would be a voltage drop across the resistor acc to Ohm's Law. This dropped voltage opposes the source voltage. Hence voltage drop across an electric resistance in any network can be assumed as a voltage source

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ple:
ing Millman's theor
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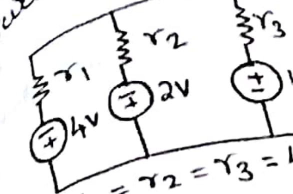


$[r_1 = r_2 = r_3 =$

Let V be the eq
resistance to be
of the Millman's

Here, $V =$

and $R = \frac{1}{G}$

Substituti

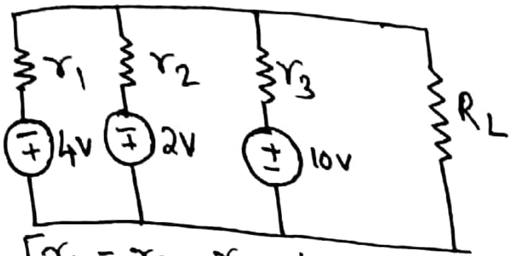
$V =$

and $R =$

The c
the dro

mple:

Using Millman's theorem, find the current through R_L in the circuit and find the voltage drop.



$[r_1 = r_2 = r_3 = 4\Omega,\ R_L = 10\Omega]$

Let V be the equivalent voltage source while R the equivalent resistance to be inserted in series with the voltage source of the Millman's equivalent network.

Here, $V = \dfrac{-V_1G_1 - V_2G_2 + V_3G_3}{G_1 + G_2 + G_3}$

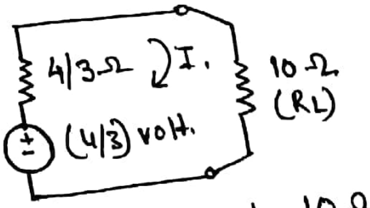
and $R = \dfrac{1}{G} = \dfrac{1}{G_1+G_2+G_3} = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$.

Substituting the numerical values,

$$V = \frac{-4\times\frac{1}{4} - 2\times\frac{1}{4} + 10\times\frac{1}{4}}{1/4 + 1/4 + 1/4} = \frac{-1 - 1/2 + 5/2}{3/4} = 4/3\,V.$$

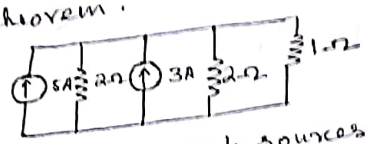
and $R = \dfrac{1}{1/4 + 1/4 + 1/4} = 4/3\ \Omega$.

$$I = \frac{4/3}{4/3 + 10} = \frac{4}{3}\times\frac{3}{34} = 0.12\,A.$$



The current through 10Ω resistor is thus 0.12 A while the drop across it is 10 × 0.12 = 1.2V.

2. Find the current through the 1Ω resistor using Millma
Theorem.



First the current sources are combined such that the total current is obtained as.

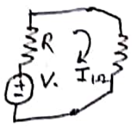
$I = I_1 + I_2 = 5 + 3 = 8A.$

$G = G_1 + G_2 = 1/2 + 1/2 = 1 \text{ mho}.$

Converting current source to an equivalent voltage sou

$V = \frac{I}{G} = \frac{8}{1} = 8V.$

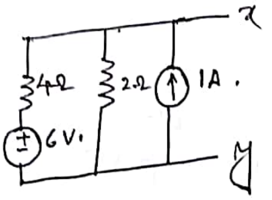
and $R = 1/G = 1\Omega.$



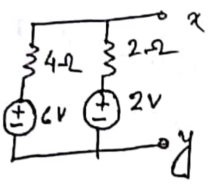
The current thru. 1Ω resistor is

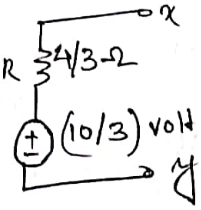
$I_{1\Omega} = \frac{V}{R+1} = \frac{8}{1+1} = 4A.$

3. Find the Millman's Equivalent Circuit for the left of the terminal $x-y$.



Let us first convert the current source of 1A to voltage source.





Millman's Equivalent voltage is then given by

$$V = \frac{V_1 G_1 + V_2 G_2}{G_1 + G_2} = \frac{6 \times 1/4 + 2 \times 1/2}{1/4 + 1/2}$$

$$= 10/3 \text{ V}.$$

$$R = \frac{1}{1/4 + 1/2} = 4/3 \Omega.$$

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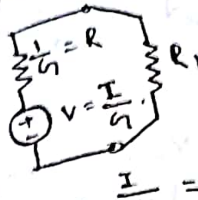
the resulting cu
source.



Thus $V = \frac{I}{G} =$

[+ and − sign
sources may

Also $R = \frac{1}{G}$

and $V =$

where R is th
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Thus finally

Steps:

Step1 Obtai
$(V_1, V_2$
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Step2 Apply
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Step3 Dete
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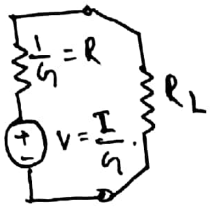
t the resulting current sources is converted to an equivalent
tage source.



Thus $V = \frac{I}{G} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm \cdots \pm I_n}{G_1 + G_2 + G_3 + \cdots G_n}$.

[+ and − signs appeared to include the cases where the sources may not be supplying current in the same direction.]

Also $R = \frac{1}{G} = \frac{1}{G_1 + G_2 + G_3 + \cdots G_n}$.

and $V = \frac{\pm \frac{V_1}{R_1} \pm \frac{V_2}{R_2} \pm \cdots \pm \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$.

where R is the equivalent resistance connected with the equivalent voltage sources in series.

Thus finally, $V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \cdots \pm V_n G_n}{G_1 + G_2 + \cdots + G_n}$.

$$= \frac{\sum_{k=1}^{n} V_k G_k}{\sum_{k=1}^{n} G_k} \quad \text{and } G_k = \frac{1}{R_k}.$$

Steps:

Step1 Obtain the conductance of each valtage source ($V_1, V_2 \cdots$) and find G, the equivalent conductance removing the load.

Step2 Apply Millman's theorem to find V, the equivalent valtage source,
$$V = \frac{V_1 G_1 + V_2 G_2 + \cdots + V_n G_n}{G_1 + G_2 + \cdots + G_n}.$$

Step3 Determine (R), the equivalent series resistance with the equivalent valtage source (V)
$$R = 1/G.$$

Step4 The current through the load is then given by.
$$I_L = \frac{V}{R + R_L}.$$

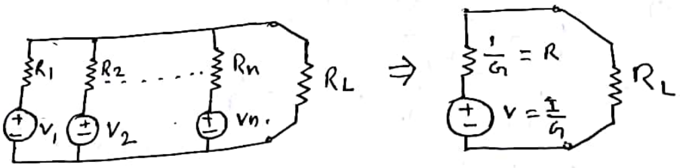
R_L being the load resistance.

Millman's Theorem:

The utility of this theorem is that, any no. of parallel vol sources can be reduced to one equivalent one.

Statement:

When a no. of Voltage sources $(V_1, V_2, \cdots V_n)$ are in parallel having internal resistances $(R_1; R_2, \cdots R_n)$ respective the arrangement can be replaced by a single equivalen voltage source V in series with an equivalent series resistance R as given below.



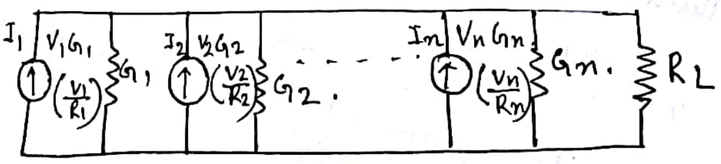
As per Millman's Theorem,

$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \cdots \pm V_n G_n}{G_1 + G_2 + \cdots + G_n}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots + G_n}$$

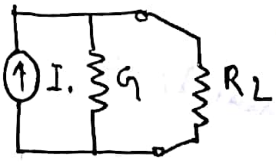
Explanation:

Assuming a dc network of numerous parallel voltage source with internal resistances supplying power to a load resistance R_L, all voltage sources are converted to current sources.



Let I represent the resultant current of the parallel curre sources while G the equivalent conductance such that,

$$I = I_1 + I_2 + I_3 + \cdots ; \quad G = G_1 + G_2 + G_3 + \cdots$$



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pensation Th

In a linear

impedance / res

carrying a ct-

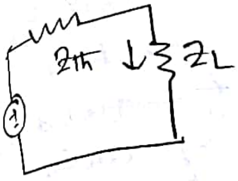
all the branc

obtained by

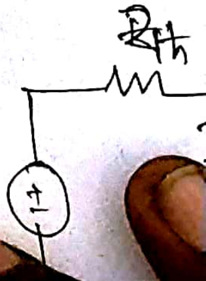
source of Vc

with $(Z + \Delta Z$

are ϕ replac



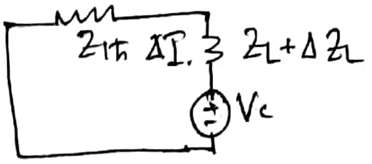
$\Delta I = I'$



pensation Th.

n a linear time invariant n/w, ~~th~~ when the pedance / resistance of a uncoupled branch rrying a ct. I is changed by ΔZ, the cts. in ll the branches would change and can be btained by assuming that an ideal voltage ource of Vc (say) has been connected in series with (Z + ΔZ) when all other sources in the n/w. rce ϕ replaced by the internal resistance.

$$I = \frac{V_0}{Z_{th}+Z_L}.$$

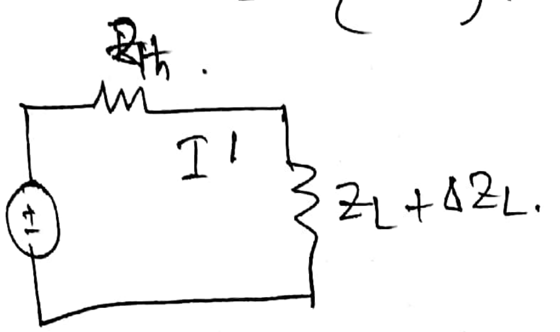


$$I' = \frac{V_0}{Z_{th}+(Z_L+\Delta Z_L)}$$

$$\Delta I = I' - I = \frac{V_0(\cancel{Z_{th}}+\cancel{Z_L}-\cancel{Z_{th}}-\cancel{Z_L}-\Delta Z_L)}{(Z_{th}+Z_L)(Z_{th}+Z_L+\Delta Z_L)}.$$

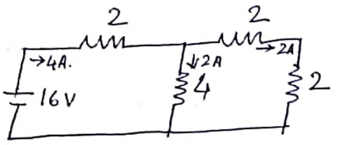
$$= \frac{-V_0\,\Delta Z_L}{(Z_{th}+Z_L)(Z_{th}+Z_L+\Delta Z_L)} = \frac{-I\,\Delta Z_L}{(Z_{th}+Z_L+\Delta Z_L)}.$$

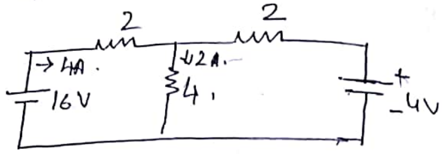
$$= \frac{-V_c}{(\quad)}. \rightarrow \text{compensating voltage.}$$



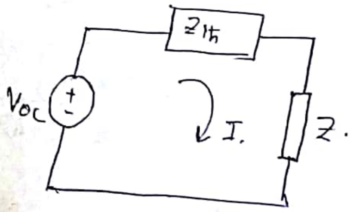
$$I' = I_1 + \Delta I.$$

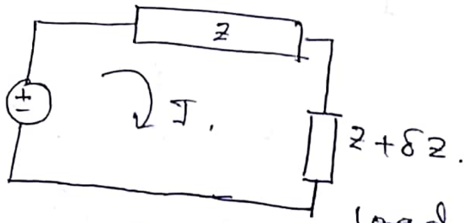
According to this theorem any resistance in ~~any branch~~ a network may be replaced by a voltage source that has zero internal resistance and a voltage equal to the voltage drop across the replaced resistance due to the ct. which was flowing thru it.





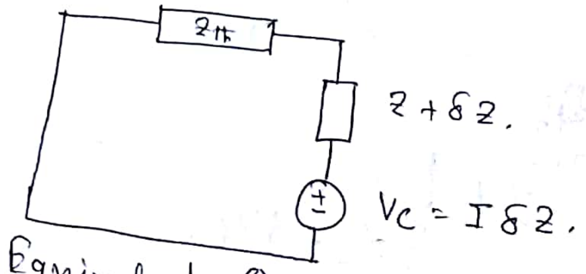
In a linear network N, if the current in a branch is I and the impedance Z of the branch is increased by ΔZ then the increment of voltage and ct. in each branch of the n/w is that voltage or current that would be produced by an opposing voltage source of value $V_C = (I\Delta Z)$ introduced into the altered branch after the modification.





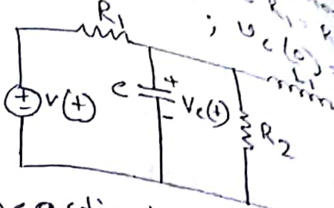
load is changed.

⇓



Equivalent Circuit by the compensation theorem.

Using the superpo $v_c(t)$. Assume R₁ $i(t) = e^{-4t}$; $v_c(0)$



Deactivating i(t)

$V_c'(S) = \frac{\frac{1}{1/R_2}}{\frac{1}{1/R_2+}}$

$v_c'(t) = \mathcal{L}^{-1} v_c'(s$

Setting v(t) = 0.

$v_c''(s) = ($

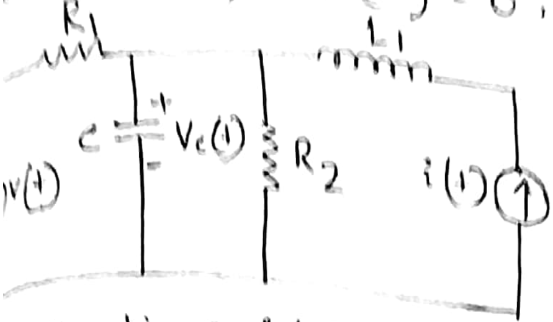
$= ($

$v_c''(t) = L^{-1}$

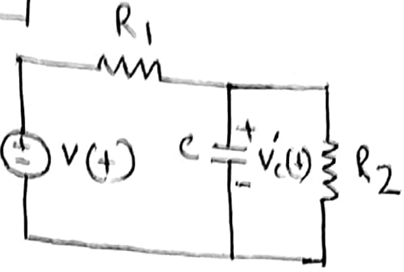
Hence the can be w

$v_c(t) =$

sing the superposition theorem calculate the voltage (t). Assume $R_1 = R_2 = 2\Omega$, $C = 0.5F$, $L = 1H$, $v(t) = t$, t) = e^{-4t}; $v_c(0) = 0$, $i_L(0) = 0$.



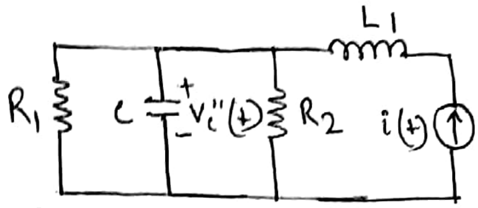
activating i(t) ⇒



$$V_c'(S) = \frac{\frac{1}{1/R_2 + SC}}{\frac{1}{1/R_2 + SC} + R_1} \times V(S) = \frac{V(S)}{(S+2)} = \frac{1}{S^2(S+2)}$$

$$v_c'(t) = \mathcal{L}^{-1} V_c'(S) = \left(\frac{t}{2} + \frac{1}{4}e^{-2t} - \frac{1}{4}\right)u(t).$$

etting v(t) = 0. ⇒



$$V_c''(S) = \left(\frac{1}{1/R_1 + 1/R_2 + 1/CS}\right) I(S)$$

$$= \left(\frac{2}{S+2}\right)\left(\frac{1}{S+4}\right) = \frac{1}{S+2} - \frac{1}{S+4}$$

$$v_c''(t) = \mathcal{L}^{-1} V_c''(S) = \left(e^{-2t} - e^{-4t}\right)u(t).$$

Hence the total response due to both the sources can be written as,

$$v_c(t) = v_c'(t) + v_c''(t) = \left(\frac{5}{4}e^{-2t} - e^{-4t} + \frac{t}{2} - \frac{1}{4}\right)u(t).$$

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Problem on Superposition Theorem :

①

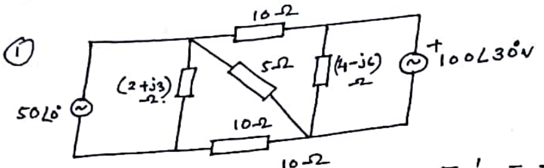
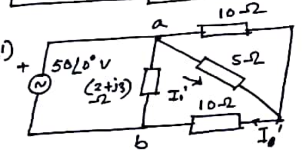


Fig (1)

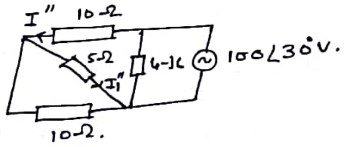


$$I_0' = \frac{50\angle 0^\circ}{(10 \| 5)+10} = 3.75\angle 0^\circ \text{ A}.$$

$$I_1' = I' \frac{10}{10+5} = 2.5 \text{ A} \angle 0^\circ$$

$$I'' = \frac{100\angle 30^\circ}{10 \| 5+10} = 7.5\angle 30^\circ \text{ A}.$$

Fig (2)



This gives

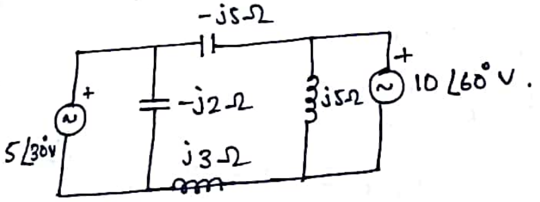
$$I_1'' = I'' \frac{10}{10+5} = 5\angle 30^\circ \text{ A}.$$

Thus the ct. thru 5Ω resistor, using the principle of superposition is

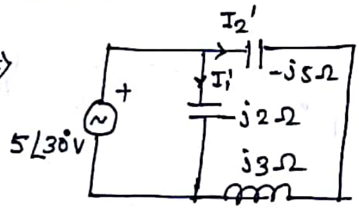
$$I_{5\Omega} = I_1' + I_1'' = 2.5\angle 0^\circ + 5\angle 30^\circ$$

$$= 7.273\angle 20.1^\circ \text{ A} = (6.83 + j2.5)$$

② Find the current through j3Ω inductive reactance using the principle of superposition.

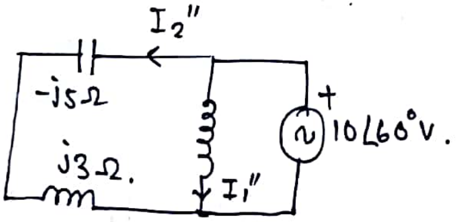


ⓐ ⇒



$$I_2' = \frac{5\angle 30^\circ}{-j5+j3} = 2.5\angle 30^\circ + 90^\circ$$

$$= 2.5\angle 120^\circ \text{ A}.$$



$$I_2'' = \frac{10\angle 60^\circ}{-j5+j3} = 5\angle 150^\circ \text{ A}.$$

$$I_2' - I_2'' = 2.5\angle 120^\circ - 5\angle 150^\circ$$

$$= -1.25 + j2.165 + 4.3$$

$$-j2.5$$

$$= 3.1\angle 6.2^\circ \text{ A}.$$

Mesh Analysis :

Applicable to separate

For each closed path

flow either in cloc

- Next the polarities
be find out by the
- Next we can a
the loop equatio
cts. thru diffe
- No. of mesh e

$m =$

Node Analysis

- To compute
- To write c
respective
other vo
- No. of i

$n =$

Node :

junction :

branch :

cos ϕ

esh Analysis :

Applicable to separate closed paths of a complete ckt / n/w.

For each closed path one particular current is assumed to flow either in clockwise or in anticlockwise direction.

Next the polarities of drops across each element has to be find out by the assumed direction of current.

. Next we can apply KVL for each closed loop and the loop equations has to be solved to find out different cts. thru different elements.

. No. of mesh equation required to form is

$$m = b - j + 1, \quad b \Rightarrow \text{branch..}$$
$$j \Rightarrow \text{junction}$$

Node Analysis :

- To compute all the branch currents.
- To write ct. expression it is assumed that the respective node potential is always higher than the other voltages appearing in the equation.
- No. of independent node-pair equation needed is

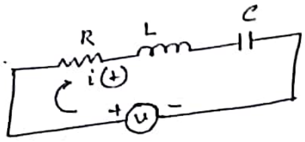
$$n = j - 1, \quad j = \text{no. of junctions.}$$

Node :

junction :

branch :

$$\cos\phi = \frac{V_R}{V_Z} = \frac{IR}{IZ} =$$



Assuming initial condition of the voltage in inductor to be assisting the input current and that in capacitor opposes the input current.

Using KVL.

$$v(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt.$$

$$RI(s) + sLI(s) - Li(0^+) + \frac{1}{sC} I(s) + \frac{V_0}{s} = V(s).$$

Laplace.

$$RI(s) + sLI(s) - Li(0^+) + \frac{I(s)}{Cs} + \frac{Q_0}{Cs} = V(s).$$

or, $RI(s) + sLI(s) = V(s) - \frac{Q_0}{Cs} + Li(0^+).$

$\therefore\ I(s)\left[R + sL + \frac{1}{Cs}\right] = V(s) - \frac{Q_0}{Cs} + Li(0^+).$

$\Delta f = f_2 - f_1$

The power i/p at f_1 and f_2.

$$P = I^2R = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R = \frac{I_{max}^2 R}{2} = \frac{1}{2} P_{max}.$$

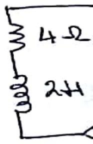
$f_1, f_2 \Rightarrow$ half power points, corresponding bandwidth is termed as half-power bandwidth or -3db bandwidth.

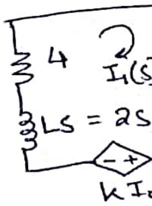
1. The ct. is $I_{max}/\sqrt{2}$.
2. Impedance is $\sqrt{2}R$ or $\sqrt{2}Z_{min}$.
3. $P_1 = P_2 = P_{max}/2$.
4. The ckt. phase angle $\phi = \pm 45^\circ$ or $\pi/4$ rad.
5. Tangent of the ckt. phase angle at the off-resonance freq. f_1 and f_2, $Q = \tan\theta = \tan 45^\circ = 1$.

The impedance is $\sqrt{2}$ times its impedance at resonance so ct is $I_{max}/\sqrt{2}$. But the impedance at resonance $Z = R$ so at half-power points the impedance is $\sqrt{2}R$. Since $Z = \sqrt{R^2 + X^2}$, therefore, $\sqrt{2}R = \sqrt{R^2 + X^2}$ or $X = R$.

④ ... Analysis:

... transform the ... apply mesh ... independent ...





In loop 1 :

In loop 2

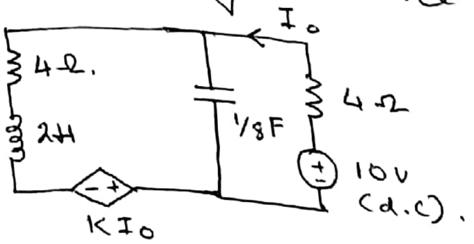
In Ma...

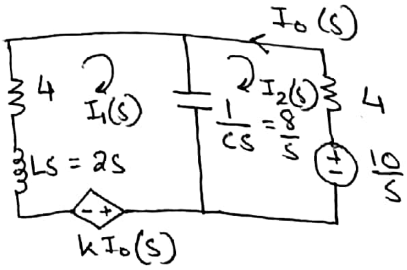
$\therefore$ $I_1(s)$

given

in Analysis:

nsform the circuit given in fig. to s-domain and ly mesh analysis to find the current through the endent voltage source (in s-domain).





In loop 1: $\left(4+2s+\frac{8}{s}\right)I_1(s) - \frac{8}{s}I_2(s) + kI_0(s) = 0.$

or, $\left(4+2s+\frac{8}{s}\right)I_1(s) - \frac{8}{s}I_2(s) - kI_2(s) = 0$

$[\because I_0(s) = -I_2(s)]$

or, $\left(4+2s+\frac{8}{s}\right)I_1(s) - I_2(s)\left(\frac{8}{s}+k\right) = 0.$

In loop 2: $4I_2(s) + \frac{10}{s} + I_2(s)\cdot\frac{8}{s} - I_1(s)\frac{8}{s} = 0$

or, $-\frac{8}{s}I_1(s) + I_2(s)\left(4+\frac{8}{s}\right) = -\frac{10}{s}$

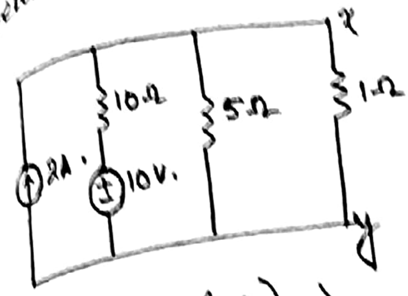
In Matrix Form:

$$\left[\begin{array}{c|c} \left(4+2s+\frac{8}{s}\right) & -\left(\frac{8}{s}+k\right) \\ \hline -\frac{8}{s} & \left(4+\frac{8}{s}\right) \end{array}\right]\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{10}{s} \end{bmatrix}$$

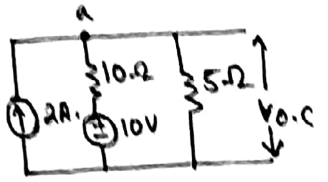
∴ $I_1(s)$, the current through the dependent source is given by,

$$\frac{\begin{vmatrix} 0 & -\frac{8}{s}-k \\ -\frac{10}{s} & 4+\frac{8}{s} \end{vmatrix}}{\Delta} = \frac{-10-s\left(\frac{10}{8}k\right)}{s(s^2+4s+8-k)}$$

the circuit find the power loss in the 1Ω resistor by
venin's Theorem.



Removing R_L (1Ω) ⇒

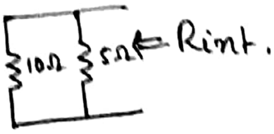


Application of KCL at Node 'a' results.

$$\frac{V_{oc}}{5} + \frac{V_{oc}-10}{10} = 2$$

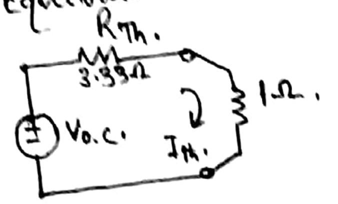
or, $V_{o.c} = 10V.$

To find Rint ⇒



Rint or $R_{Th} = \frac{5\times10}{5+10} = 3.33\Omega.$

Thevenin's Equivalent Circuit.

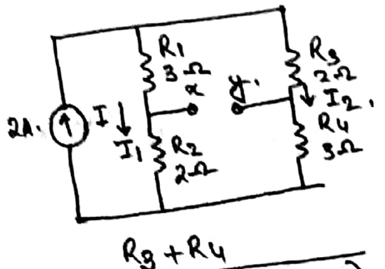


Current through 1Ω resistor is

$$I_{Th} = \frac{V_{oc}}{R_{Th}+1} = \frac{10}{3.33+1} = 2.31A.$$

∴ Power Loss in 1Ω resistor $= (2.31)^2 \times 1 = 5.33\ W.$

In the network, find V_{x-y} and Rint (across x-y) using Thevenin's Theorem.



$$I_1 = I \cdot \frac{R_3+R_4}{(R_1+R_2)+(R_3+R_4)}$$ [by current division Rule].

$= 1A.$

$$I_2 = I \times \frac{R_1+R_2}{(R_1+R_2)+(R_3+R_4)} = 1A.$$

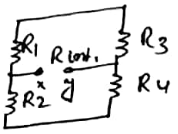
Thus voltage drop across $R_2 = 1\times2 = 2V.$
voltage drop across $R_4 = 1\times3 = 3V.$

∴ $V_{x-y} = V_x - V_y = 2V - 3V = -1V$.

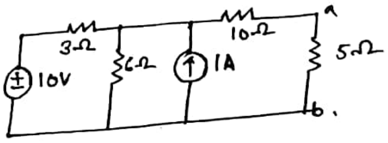
i.e. y is at higher potential.

To find Rint, across x–y, current source is removed and (R_1+R_2) is in parallel to (R_3+R_4).

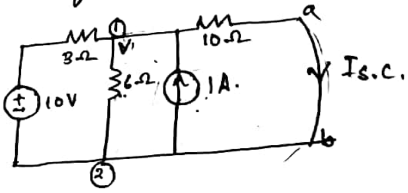
∴ $R_{int} = (R_1+R_2) \parallel (R_3+R_4)$. $= 5 \parallel 5 = 2.5\,\Omega$.



3. Find the current in the 5Ω resistor for the circuit shown.



Sol. Let us first remove the 5Ω resistor and short the a-b termi



Assuming the voltage to be +ve at node 1, nodal analysis gives.

$$\frac{V-10}{3} + \frac{V}{6} - 1 + \frac{V}{10} = 0.$$

or, $0.33V + 0.17V + 0.1V = 4.33$,

or, $V = 7.22V$.

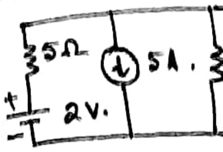
Applying KVL at the right most loop (1–a–b–2),

$V_1 = 10 \times I_{s.c.}$

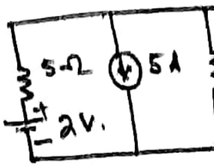
or, $I_{s.c} = 0.722\text{ A } (= I_N)$.

To find Norton's
resistance is
deactivated.

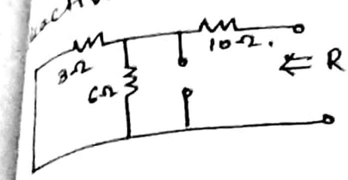
. Find the Norton'



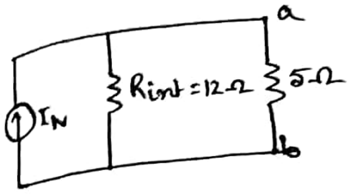
⇓ sh



find Norton's Equivalent resistance through a-b, 5Ω resistance is removed and all the constant sources are deactivated.



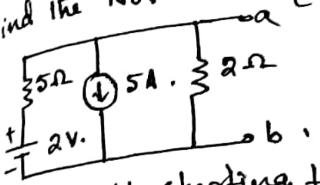
⇐ Rint. Here $R_{int} = \frac{3\times6}{3+6} + 10 = 12\Omega$.



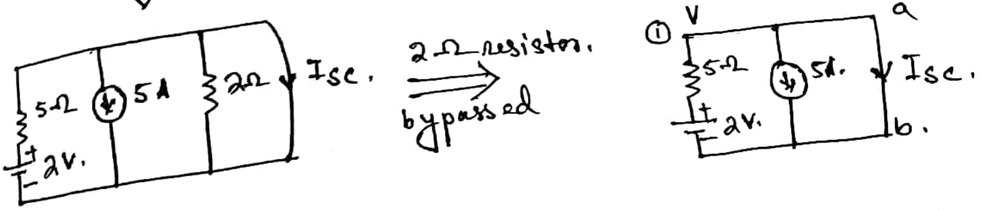
Norton's Equivalent Circuit being shown in fig, current through 5Ω resistor is

$$I_{5\Omega} = I_N \cdot \frac{R_{int}}{R_{int}+5} = 0.722 \times \frac{12}{12+5}$$
$$= 0.5096A \simeq 509.6mA.$$

find the Norton's equivalent circuit across a-b for the n/w shown.

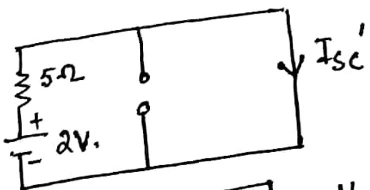


⇓ shorting terminal a-b.

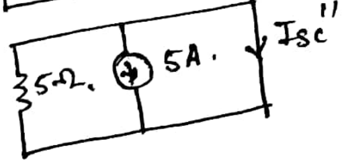


Taking first 2V source;

$2 = 5 \times I_{sc}' \Rightarrow I_{sc}' = 0.4A$.

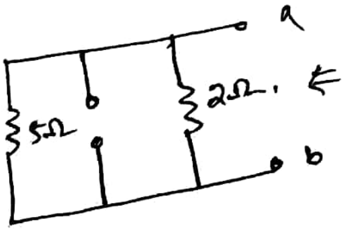


Taking 5A source only, $I_{sc}'' = -5A$.



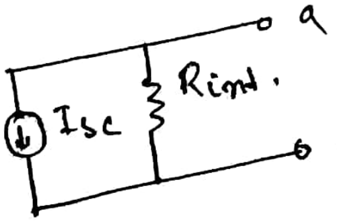
Using principle of Superposition.

$I_{sc} = I_{sc}' + I_{sc}'' = -4.6A$.



⇐ Rint $R_{int} = \frac{5\times2}{5+2} = 1.43\Omega$.

Norton's Equivalent Circuit.



Thus we observe that the voltage across the capacitor being constant, current through it is zero. This means that the capacitor, on application of d.c voltage and with no initial charge first acts as short circuit but as soon as the full charge it retains, the capacitor behaves an open circuit. It can store finite amount of energy, even if the ct. thru it is zero.

Inductance :

Inductance is the property of a material by virtue of which it opposes any change of magnitude or direction of electric current passing thru the conductor.

Any change of field induces a voltage (v_L) across the coil, given by (Lenz's law)

$$v_L = -L\frac{di}{dt}.$$

i is the current through the inductor in ampere.

Thus, in a pure inductive circuit with applied voltage v, we can write. $v + v_L = 0$

or, $v = -v_L = L\frac{di}{dt}$.

The voltage across the inductor would be zero if the current through it remains const. This means that an inductor behaves as a short circuited coil in steady state, when direct steady ct. flows thru. it. However for any small change in current strength or change in direction, inductance will appear. It may also be observed that for a minute change in current within zero time it gives an infinite voltage across the inductor which is physically not at all feasible. Thus in an inductor current cannot change abruptly. Thus it behaves as open circuit just after switching across d.c. voltage but as short circuit at steady state.

Power absorbed $P = v \times i = Li\frac{di}{dt}$ watt.

Energy stored $W = \int_0^t P\,dt = \int_0^t Li\frac{di}{dt}\,dt = \frac{1}{2}Li^2$.

So it is evident that inductor can store finite amount of energy, even the voltage across it may be nil. A pure inductor does not dissipate energy, but only stores it.

Time Domain

R: $v = Ri(t)$

L: L

Initial ct. is $i(0^+)$, clockwise circuit current being $i(t)$;

$$v(t) = L\frac{di(t)}{dt}$$

Current in the inductor is given by

$$i(t) = \frac{1}{L}\int_0^t v(t)\,dt + i(0^+)$$

C: C

Initial voltage is V_0, with +ve polarity at the side of the capacitor that opposes charging ct. $i(t)$ then - $i(t)$

$$v(t) = V_0 + \frac{1}{C}\int_0^t i(t)\,dt.$$

when assist charging ct. $i(t)$

$$v(t) = -V_0 + \frac{1}{C}\int_0^t i(t)\,dt.$$

Current in the capacitor is given by

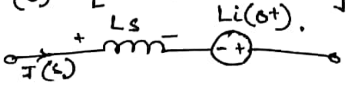
$$i(t) = C\frac{dv(t)}{dt}.$$

S-Domain

$V(S) = RI(S)$.

Ls

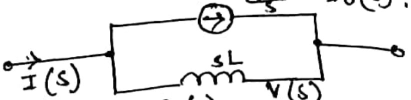
$$V(s) = [sLI(S) - Li(0^+)].$$



$$I(S) = \frac{1}{L}\cdot\frac{V(S)}{S} + \frac{i(0^+)}{S}$$

$$= I_L(S) + I_0(S).$$

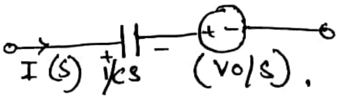
$$\frac{i(0^+)}{S} = I_0(S).$$



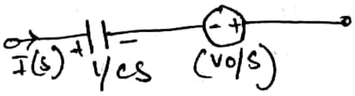
$$I_L(S) = \frac{V(S)}{SL}$$

1/cs.

$$V(S) = \frac{V_0}{S} + \frac{1}{C}\frac{I(S)}{S} = \frac{V_0}{S} + \frac{I(S}{C}$$

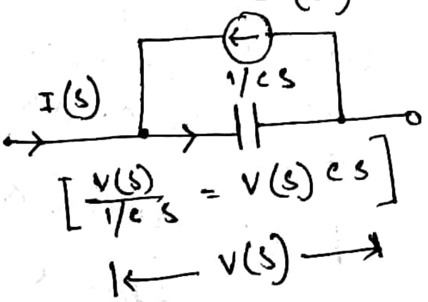


$$V(s) = -\frac{V_0}{S} + \frac{I(S)}{CS}.$$

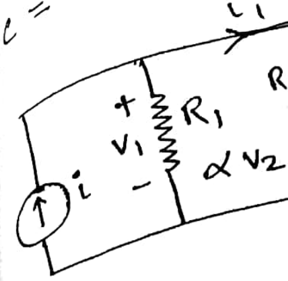


$$I(S) = C[V(S)S - V(0^+)].$$

CV(0+)



Using the princi
voltage V2(t)
C = 2F, β =



Set v(t)

Sol.

Let $V_1(S)$
equations ar
$I(S)$

$V_1(S$

V_2'

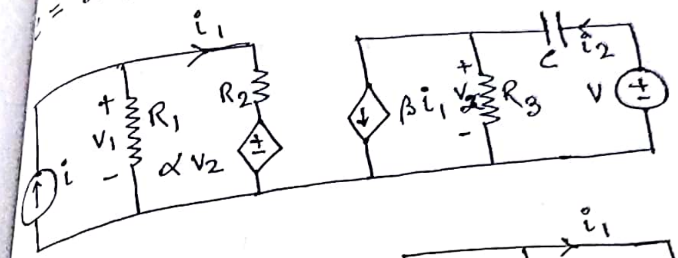
Elimination
values,

Set i(

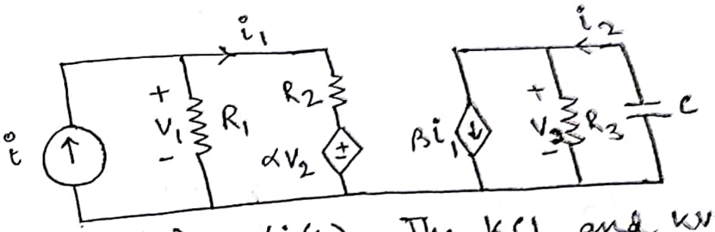
The

Nabanita Chatterjee. (EE Faculty).

ing the principle of superposition, calculate the ltage $v_2(t)$ in fig. Assume $R_1 = R_2 = 1\Omega$, $R_3 = 0.5\Omega$, $= 2F$, $\beta = 2$, $\alpha = 1$, $i(t) = \sin t$, $v(t) = t$.



Set $v(t) = 0$.



Let $V_1(s) = \mathcal{L} V_1(t)$, $I(s) = \mathcal{L} i(t)$. The KCL and KVL quations are,

$$I(s) = \frac{V_1(s)}{R_1} + I_1(s)$$

$$V_1(s) = \alpha V_2'(s) + R_2 I_1(s).$$

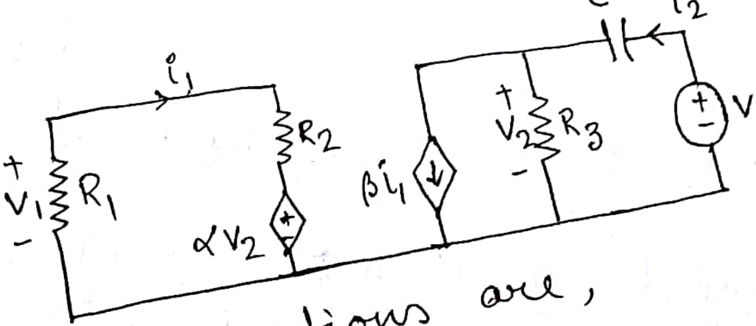
$$V_2'(s) = -\beta I_1(s) \frac{1}{1/R_3 + sC}$$

Eliminating $I_1(s), V_1(s)$ and substituting parameter values,

$$v_2'(t) = \mathcal{L}^{-1} V_2'(s) = \mathcal{L}^{-1} - \frac{1}{2}\left(-\frac{1}{s+0.5}\right)\left(\frac{1}{s^2+1}\right).$$

$$= -\frac{2}{5} e^{-0.5t} - \frac{1}{\sqrt{5}} \sin(t - 63.5°)$$

Set $i(t) = 0$



The KCL and KVL equations are,

$$V(s) = \frac{I_2(s)}{sC} + V_2''(s).$$

$$\frac{V_2''(s)}{R_3} = I_2(s) - \beta I_1(s)$$

$$I_1(s) = \frac{-\alpha V_2''(s)}{R_1 + R_2}.$$

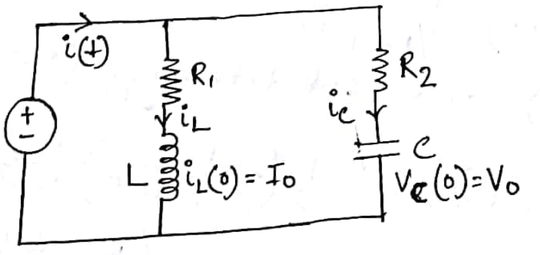
Eliminating $I_1(s)$ and $I_2(s)$ and substituting parameter values,

$$v_2''(t) = \mathcal{L}^{-1} V_2''(s) = \mathcal{L}^{-1}\left(\frac{s}{s+2}\right)\frac{1}{s^3} = \left(\frac{1}{4}e^{-2t} + \frac{t}{2} - \frac{1}{4}\right.$$

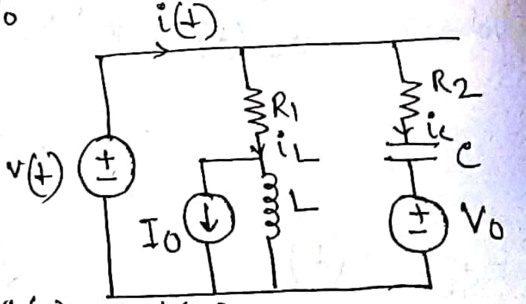
Then the total Solution.

$$v_2(t) = v_2'(t) + v_2''(t) = \left[-\frac{2}{5}e^{-t/2} + \frac{1}{4}e^{-2t} - \frac{1}{4} - \frac{1}{\sqrt{5}}\sin(t - 63.5^\circ) + t/2\right]u(t)$$

2. Calculate the driving point current for the Network by converting all initial conditions into equivalent sources and applying the superposition theorem. Assume $R_1 = 3\Omega$, $R_2 = 1\Omega$, $L = 2H$, $C = 0.5F$, $v_c(0) = V_0 = -3$ volt, ~~Ib~~ $I_L(0) = I_0 = 1A$, $v(t) = u(t)$, a unit step.



Circuit with initial conditions.



i) Set $I_0 = 0$, $V_0 = 0$ and let $i(t) = i'(t)$.

Equivalent circuit with current source eliminated and $V_0 = 0$