Gravitational wave data analysis

1 Matched filter

- Download the data set from [1]. This data set contains a noise time series and an additional gravitational wave signal, where the sampling rate of the time series is 16384Hz. The first column of the data set refers to the time. The second and third columns are the data recorded by LIGO Hanford and Livingston, respectively. For the sake of convenience in data analysis, we downsample the data at a sampling rate of 2048Hz.

- Write down the code to calculate the matched filter between the given data and a template waveform. After that, find out the template waveform for which the signal to noise ratio is maximum. For simplicity, you can assume the template waveform is produced from an equal mass binary system. Produce a plot which shows the SNR versus total mass of the system, where masses of the systems are uniformly distributed in a range $1M_\odot \leq m_{1,2} \leq 3M_\odot$ (use $m_1 = m_2$). Represent the result by fitting the best match template waveform against the data. Also, show the chirp structure of the signal over the time-frequency domain using Q-transformation().

- Here, we would like to investigate the correlation in the parameter space of gravitational waves from compact binary coalescences. Let us consider a gravitational wave $h^*(t; m_1, m_2)$ generated from a binary with masses $m_1 = m_2 = 10M_\odot$. Now, we want to understand how the nearby templates are correlated with that waveform. Calculate the match between $h^*(t; m_1, m_2)$ and the nearby templates that are generated assuming $5M_\odot \leq m_1, m_2 \leq 15M_\odot$. Of course, one should have the maximum match ($= 1$) when $h_i = h^*$. You will see that the match between two nearby waveforms is not zero. So, one can realize the gravitational waveforms are correlated for two nearby points in the parameter space. This fact is used to produce a template bank for detecting gravitational waves.

2 Detector noise

- Load the data of the LIGO Hanford detector from [2]. Plot the noise power spectral density (PSD) of that data. Test whether there is any gravitational wave signal present in the data or not? For this test, you can assume the GW signals can produce only from equal mass binary systems with a range between 5 to 10. We assume a threshold on matched filter SNR of 8 to claim detection of GW.

- Whiten the above data using its noise PSD. You can use PyCBC based function of the Welch method to estimate the PSD. Construct a histogram of the whitened data and show that the whiten data follows a Gaussian distribution with zero mean.

- In gravitational wave data analysis, we consider that the noise is stationary Gaussian. But in reality, the stationary assumption does not hold. Can you think of a test to show that the stationarity does not hold?

  [To devise this test, you can consider the above data (?). Take a arbitrary gravitational waveform $h(f)$ and calculate the optimal SNR ρ_{opt},

$$\rho_{\text{opt}}^2 = 4 \int \frac{|h(f)|^2}{S_n(f)} \, df, \tag{1}$$

  where $S_n(f)$ is noise power spectral density of the data. For the test, you can consider different chunk of data]

3 horizon distance of detectors

- The horizon distance of a GW detector is defined as a distance to the source for which the accumulated signal power (optimal SNR) is equal to a detection threshold of SNR ($\rho_{min} = 8$) when the source is located overhead ($\theta = 0, \pi$). Plot the horizon distance for Advanced LIGO detector (assuming 'H1' noise PSD) assuming a set of equal mass binary with a range of component masses between 5 to 500. In this plot, you can assume that X-axis and Y-axis represent the total mass of the binary and horizon distance, respectively. For this analysis, first, generate the GW waves polarization in frequency domain, $h_+(f), h_\times(f)$ and construct the projection detector frame, $h(f) = F_+ h_+(f) + F_\times h_\times(f)$, where F_+ and $F_\times$ are the antenna pattern functions for the two polarizations. Using the sky location (θ, ϕ) and polarization angle of the incoming GW, one can show that for 'L' shaped detector $F_+, F_\times$ can be written as

$$F_+ = \frac{1}{2}(1 + \cos^2\theta)\cos 2\phi \cos 2\psi - \cos\theta \sin 2\phi \sin 2\psi$$

$$F_\times = \frac{1}{2}(1 + \cos^2\theta)\cos 2\phi \sin 2\psi + \cos\theta \sin 2\phi \cos 2\psi$$

  N.b. The maximum values of $F_+, F_\times$ is 1.

- From the plot of horizon distance, you will see the curve reaches its maximum value for a massive black hole. After that, the horizon distance decreases over mass. Can you explain why do we see such behavior? What can you interpret from the point of view of the detector's capability to detect GWs from the compact binary merger?

- Repeat the same analysis for the future generation gravitational wave detectors (Einstein telescope) and make a plot for horizon distance versus the total mass of the system. Compare this plot with the previous plot for the Advanced LIGO detector and make your conclusion.

[1] link of the data [2] link of the data