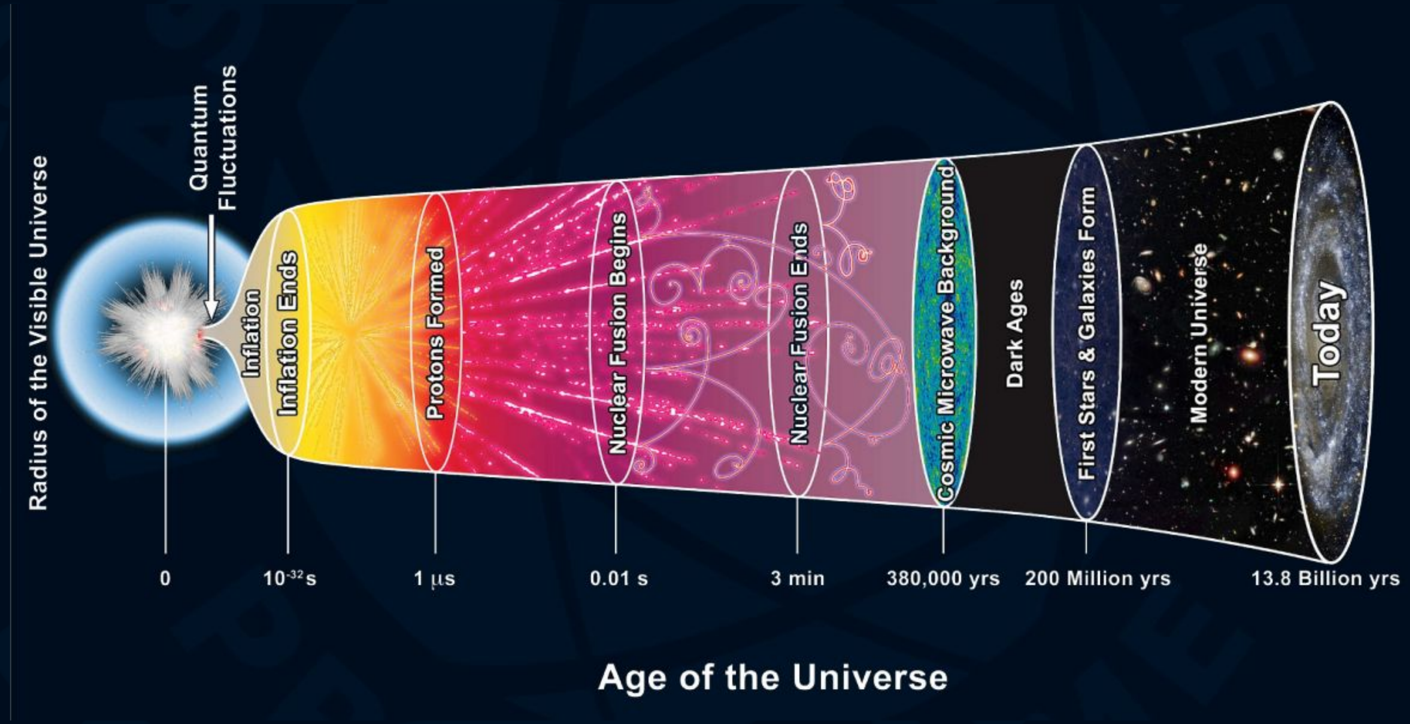
Cosmology Big Data

Rens Verkade

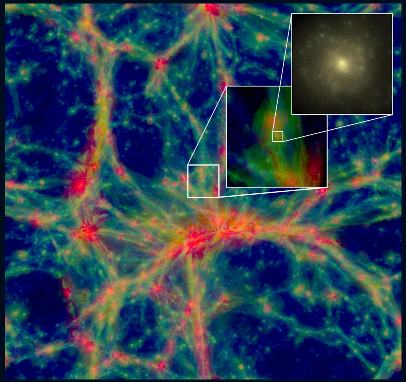
Cosmology in a nutshell



BICEP2/CERN/NASA

Cosmological model

- Current ‘standard’ $\Lambda\mathrm{CDM}$ model
- A universe with Cold Dark Matter and a cosmological constant Λ
- Cold Dark matter allows for bottom-up clustering
- The Cosmological constant plays an important role in the expansion of the universe



EAGLE simulation

Standard candles

- Type I A Supernovae
- White dwarf in a binary system (usually with a red giant)

Nasa

Standard candles

- Type I A Supernovae

- White dwarf in a binary system (usually with a red giant)

- Accretion until $M_{WD} \simeq 1.43 M_{\odot}$

Nasa

Standard candles

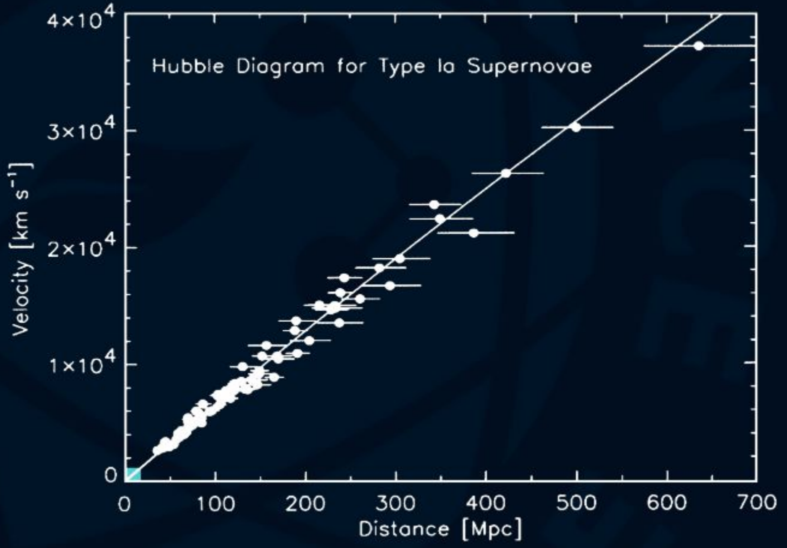
- Type I A Supernovae
- White dwarf in a binary system (usually with a red giant)
- Accretion until $M_{WD} \simeq 1.43 M_{\odot}$
- The Chandrasekhar limit where $R_{WD} = 0$
- White dwarf contracts heavily and a runaway thermo-nuclear reaction produces the supernova

Nasa

An expanding universe

- Type Ia Supernovae have the same peak absolute magnitude
- Difference to observed magnitude gives distance
- Velocity from redshift of peak and spectral lines
- Hubble finds a linear relation!

$$v = H_0 d$$



E. Hubble, PNAS March, 1929, 15 (3) 168-174

Hubble parameter

- The Hubble rate parameterises the growth/shrinkage of the universe

$$H(t) = \frac{\dot{a}}{a}$$ a Scale factor, $a(0) = 1$

Hubble parameter

- The Hubble rate parameterises the growth/shrinkage of the universe

$$H(t) = \frac{\dot{a}}{a} \qquad a \text{ Scale factor,} \qquad a(0) = 1$$

- Little h is an often used unitless version $$h \equiv \frac{H_0}{100\,\mathrm{km/s/Mpc}}$$

Hubble parameter

- The Hubble rate parameterises the growth/shrinkage of the universe

$$H(t) = \frac{\dot{a}}{a} \qquad a \text{ Scale factor,} \qquad a(0) = 1$$

- Little h is an often used unitless version

$$h \equiv \frac{H_0}{100\,\mathrm{km/s/Mpc}}$$

- Depends on the cosmological contents

$$H(z) = H_0\sqrt{\Omega_\Lambda + \Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4}$$

$$\Omega_i = \frac{\rho_i}{\rho_{crit}} \qquad \Omega_k = 1 - \Omega_m - \Omega_r - \Omega_\Lambda$$

Some statistical tools

- Probability of B, given A $P(B|A)$

Some statistical tools

- Probability of B, given A $P(B|A)$

- Our goal is how likely the values of some parameters θ are, given how well they reproduce the data D.

- Bayes' theorem $$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$

Some statistical tools

- Probability of B, given A $$P(B|A)$$
- Our goal is how likely the values of some parameters θ are, given how well they reproduce the data D.
- Bayes' theorem $$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$
- Posterior $$P(\theta|D)$$
- Likelihood $$P(D|\theta)$$

Some statistical tools

- Probability of B, given A $P(B|A)$
- Our goal is how likely the values of some parameters θ are, given how well they reproduce the data D.
- Bayes' theorem $$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$
- Posterior $P(\theta|D)$
- Likelihood $P(D|\theta)$
- Prior $P(\theta)$
- Evidence $P(D)$

Some statistical tools

- The statistic we are interested in depends on what you want to know
- Posterior tells you how likely your theory model is given the data $P(\theta|D)$

Some statistical tools

- The statistic we are interested in depends on what you want to know
- Posterior tells you how likely your theory model is given the data $P(\theta|D)$
- In discoveries we are more interested in the likelihood $P(D|\theta)$
- Given your hypothesised model, probability of finding the data you have measured. Often we first predict and then measure. (Higgs!)

Bayes' Theorem in practice

- Let's say you get tested for some disease
- It's rare so 1 in 100 people with the symptoms actually have the disease
- The test is positive, what is the probability you have the disease
- Not 1/100, we have additional info, the test

Example from Megan Bell https://tomrocksmaths.com/2021/08/31/bayes-theorem-and-disease-testing/

Bayes’ Theorem in practice

- Let’s say you get tested for some disease

- It’s rare so 1 in 100 people with the symptoms actually have the disease

Example from Megan Bell https://tomrocksmaths.com/2021/08/31/bayes-theorem-and-disease-testing/

Bayes’ Theorem in practice

- Let’s say you get tested for some disease
- It’s rare so 1 in 100 people with the symptoms actually have the disease
- The test is positive, what is the probability you have the disease

Example from Megan Bell https://tomrocksmaths.com/2021/08/31/bayes-theorem-and-disease-testing/

Bayes’ Theorem in practice

- We need to first know the test’s performance

Bayes’ Theorem in practice

- We need to first know the test’s performance
- Test is 99% sensitive, if you have the disease and get tested, 99% of the tests are positive (True positive)

Bayes’ Theorem in practice

- We need to first know the test’s performance

- Test is 99% sensitive, if you have the disease and get tested, 99% of the tests are positive (True positive)

- Test is 94% specific, if you don’t have the disease and get tested, 94% of tests are negative (True Negative)

Bayes’ Theorem in practice

- We need to first know the test’s performance
- Test is 99% sensitive, if you have the disease and get tested, 99% of the tests are positive (True positive)
- Test is 94% specific, if you don’t have the disease and get tested, 94% of tests are negative (True Negative)

	Have the disease	Don’t have the disease
Test positive	99%	6%
Test negative	1%	94%

Bayes’ Theorem in practice

- If 100 people get tested, we will have 1 true positive and 6 false positive. So probability to have the disease with a positive test 1/7

Bayes' Theorem in practice

- If 100 people get tested, we will have 1 true positive and 6 false positive. So probability to have the disease with a positive test 1/7
- Check using Bayes' theorem

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$

Bayes' Theorem in practice

- If 100 people get tested, we will have 1 true positive and 6 false positive. So probability to have the disease with a positive test 1/7

- Check using Bayes' theorem

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$

$$P(\text{have disease}|\text{positive test}) = \frac{P(\text{positive test}|\text{have disease})P(\text{have disease})}{P(\text{positive test})}$$

Bayes' Theorem in practice

- If 100 people get tested, we will have 1 true positive and 6 false positive. So probability to have the disease with a positive test 1/7

- Check using Bayes' theorem

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$

$$P(\text{have disease}|\text{positive test}) = \frac{P(\text{positive test}|\text{have disease})P(\text{have disease})}{P(\text{positive test})}$$

$$P(\text{have disease}|\text{positive test}) = \frac{P(\text{positive test}|\text{have disease})P(\text{have disease})}{P(\text{positive test}|\text{have disease})P(\text{have disease}) + P(\text{positive test}|\text{no disease})P(\text{no disease}))}$$

Bayes' Theorem in practice

$$P(\text{have disease}|\text{positive test}) = \frac{P(\text{positive test}|\text{have disease})P(\text{have disease})}{P(\text{positive test})}$$

$$P(\text{have disease}|\text{positive test}) = \frac{P(\text{positive test}|\text{have disease})P(\text{have disease})}{P(\text{positive test}|\text{have disease})P(\text{have disease}) + P(\text{positive test}|\text{no disease})P(\text{no disease}))}$$

$$P(\text{have disease}|\text{positive test}) = \frac{0.99 \times 0.01}{0.99 \times 0.01 + 0.06 \times 0.99} = \frac{1}{7}$$

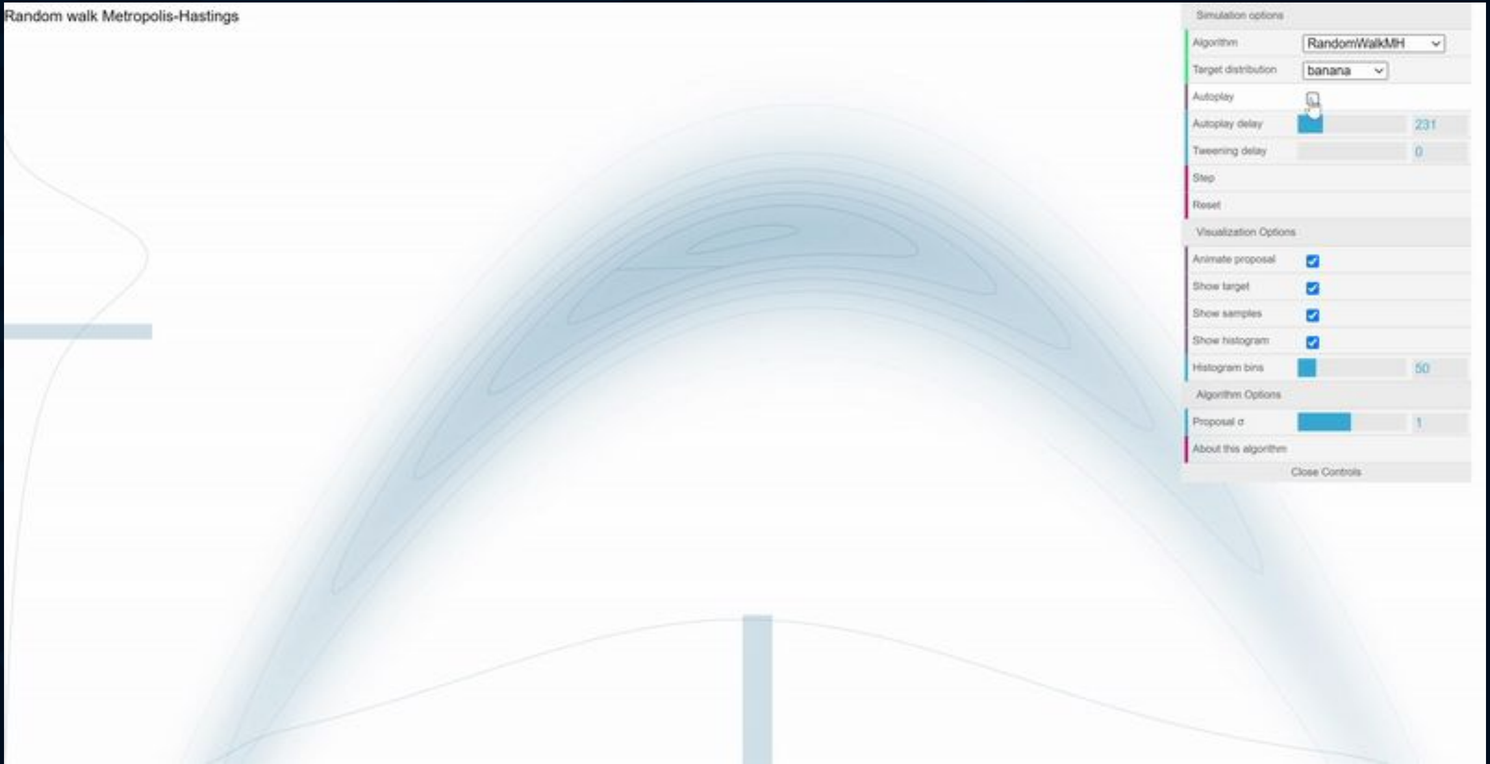
Monte Carlo Markov Chain (MCMC)

- MCMC is one way to sample the probability distribution $P(\theta|D)$

- The algorithm steps randomly and checks if the new Θ is better (χ^2 - test)

- If the step is better we keep the new location, if not try again

Monte Carlo Markov Chain (MCMC)



https://chi-feng.github.io/mcmc-demo/

Monte Carlo Markov Chain (MCMC)

- MCMC is one way to sample the probability distribution $P(\theta|D)$
- The algorithm steps randomly and checks if the new Θ is better (χ^2 - test)
- If the step is better we keep the new location, if not try again
- Stop after n steps, or preferably after some convergence test is met

Hand-in

- Work on the cluster https://jupyterhubroute-pra3024-2025.apps.dsri2.unimaas.nl/
- Notebook is on Canvas and due Tuesday 11 Feb 00:00
  - needs to be compiled (so output visible)
  - needs to be runnable (without extra installations)
  - Have comments AND Markdown (graded)
  - Use of sources is fine but give proper references otherwise it's plagiarism and you fail the exercise
  - Use of AI is allowed but understand the code so explain properly in markdown cells
  - Check rubric on canvas to see how you are graded
- If questions arise email ON TIME
  - rens.verkade@maastrichtuniversity.nl
  - xenofon.chiotopoulos@maastrichtuniversity.nl