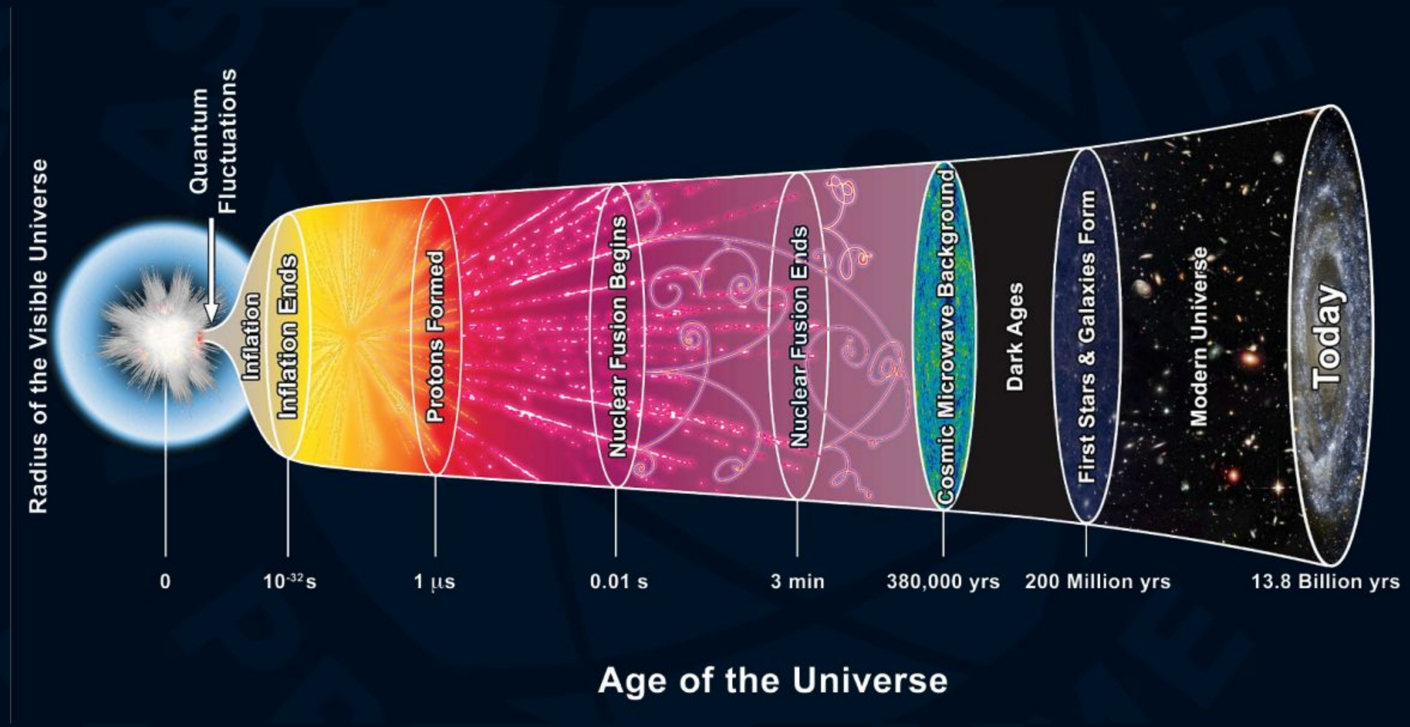
Cosmology Big Data

Rens Verkade

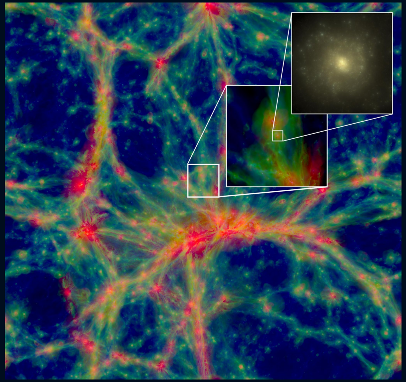
Cosmology in a nutshell



BICEP2/CERN/NASA

Cosmological model

- Current 'standard' $\Lambda\mathrm{CDM}$ model
- A universe with Cold Dark Matter and a cosmological constant Λ
- Cold Dark matter allows for bottom-up clustering
- The Cosmological constant plays an important role in the expansion of the universe



EAGLE simulation

Standard candles

- Type I A Supernovae

- White dwarf in a binary system (usually with a red giant)

Nasa

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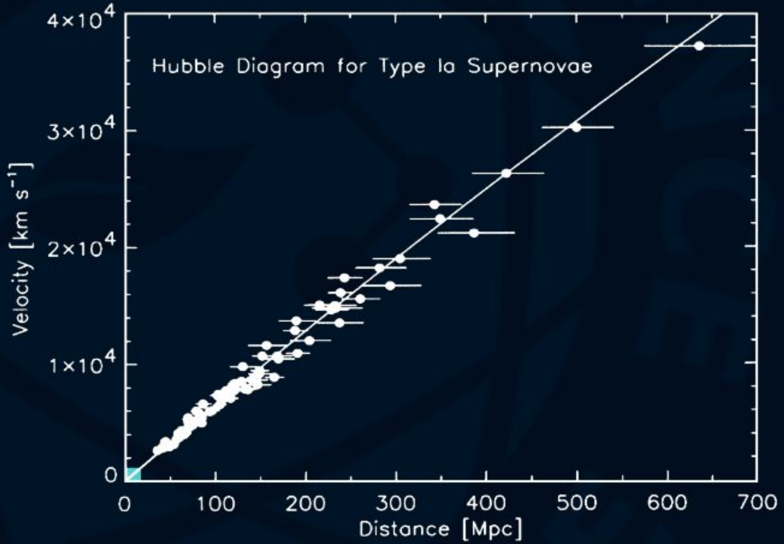
- The Chandrasekhar limit where $R_{WD} = 0$
- White dwarf contracts heavily and a runaway thermo-nuclear reaction produces the supernova

Nasa

An expanding universe

- Type Ia Supernovae have the same peak absolute magnitude
- Difference to observed magnitude gives distance
- Velocity from redshift of peak and spectral lines
- Hubble finds a linear relation!

$$v = H_0 d$$



E. Hubble, PNAS March, 1929, 15 (3) 168-174

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- Depends on the cosmological contents

$$H(z) = H_0\sqrt{\Omega_\Lambda + \Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4}$$

$$\Omega_i = \frac{\rho_i}{\rho_{crit}} \qquad \Omega_k = 1 - \Omega_m - \Omega_r - \Omega_\Lambda$$

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- Prior $P(\theta)$
- Evidence $P(D)$

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- In discoveries we are more interested in the likelihood $P(D|\theta)$
- Given your hypothesised model, probability of finding the data you have measured. Often we first predict and then measure. (Higgs!)

Bayes' Theorem in practice

- Let's say you get tested for some disease
- It's rare so 1 in 100 people with the symptoms actually have the disease
- The test is positive, what is the probability you have the disease
- Not 1/100, we have additional info, the test

Example from Megan Bell https://tomrocksmaths.com/2021/08/31/bayes-theorem-and-disease-testing/

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	Have the disease	Don't have the disease
Test positive	99%	1%
Test negative	6%	94%

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$$P(\text{have disease}|\text{positive test}) = \frac{0.99 \times 0.01}{0.99 \times 0.01 + 0.06 \times 0.99} = \frac{1}{7}$$

Monte Carlo Markov Chain (MCMC)

- MCMC is one way to sample the probability distribution $P(\theta|D)$

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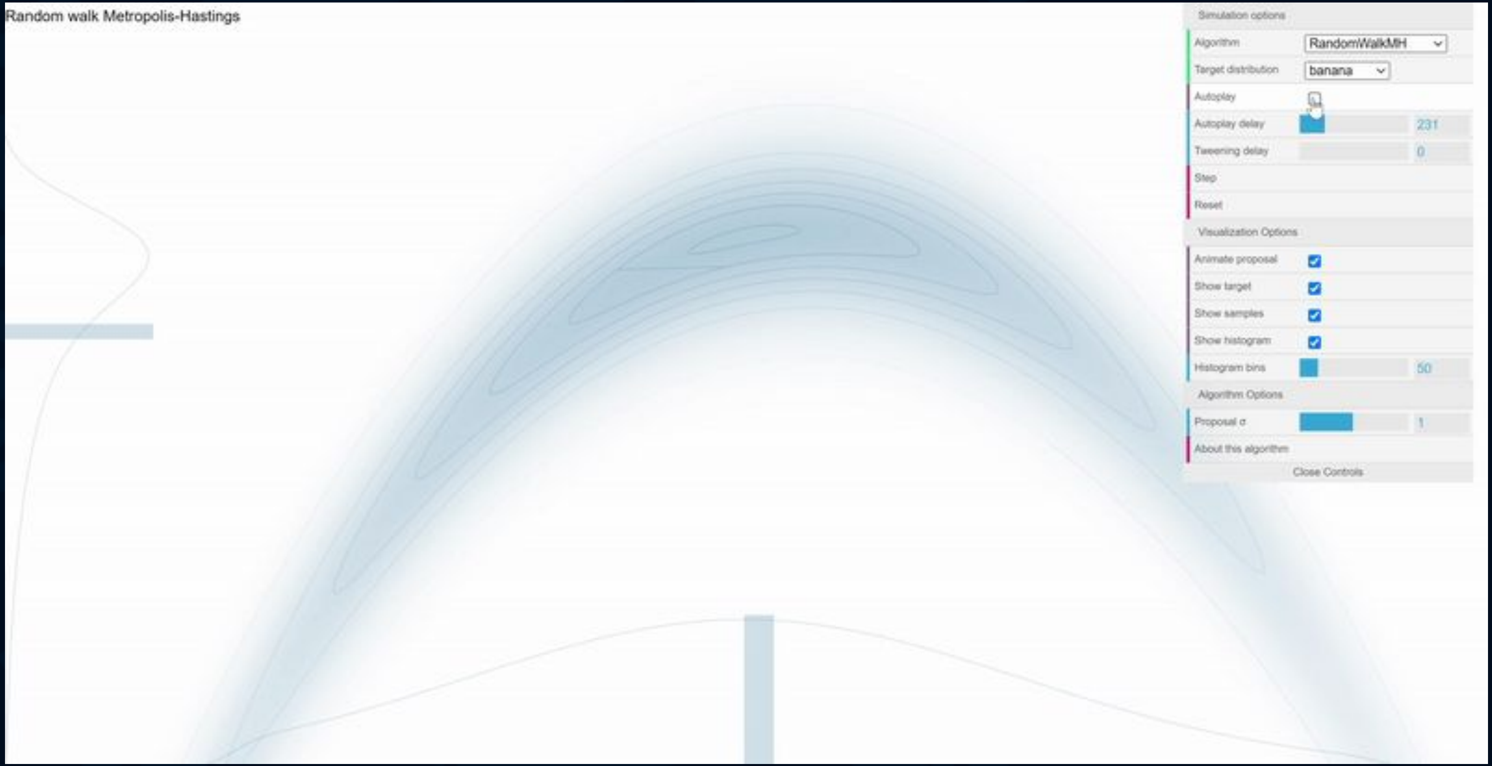

https://chi-feng.github.io/mcmc-demo/

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- Stop after n steps, or preferably after some convergence test is met