$$\frac{d\langle x\rangle}{dt}=\frac{ih}{2m}\int_{-\infty}^{\infty} x\frac{\partial}{\partial x}\left(\psi^*\cdot\frac{\partial\psi}{\partial x}-\frac{\partial\psi^*}{\partial x}\cdot\psi\right)dx \quad 1.29$$

This expression can be simplified using integration by parts

$$u=x\,;dv=\frac{\partial}{\partial x}\left(\psi^*\cdot\frac{\partial\psi}{\partial x}-\frac{\partial\psi^*}{\partial x}\cdot\psi\right)dx$$

$$du=dx\,;v=\psi^*\cdot\frac{\partial\psi}{\partial x}-\frac{\partial\psi^*}{\partial x}\cdot\psi$$

$$\frac{d\langle x\rangle}{dt}=\frac{ih}{2m}\int_{-\infty}^{\infty} u\cdot dv=\frac{ih}{2m}u\cdot v\Big|_{-\infty}^{\infty}-\frac{ih}{2m}\int_{-\infty}^{\infty} v\cdot du=\frac{ih}{2m}\cdot x\cdot\left(\psi^*\cdot\frac{\partial\psi}{\partial x}-\frac{\partial\psi^*}{\partial x}\cdot\psi\right)\Bigg|_{-\infty}^{\infty}-\frac{ih}{2m}\int_{-\infty}^{\infty}\left(\psi^*\cdot\frac{\partial\psi}{\partial x}-\frac{\partial\psi^*}{\partial x}\cdot\psi\right)dx$$

now, the author indicates that $x\cdot\left(\psi^*\cdot\frac{\partial\psi}{\partial x}-\frac{\partial\psi^*}{\partial x}\cdot\psi\right)\Big|_{-\infty}^{\infty}=0$

but I don’t see how to use L’Hopital’s rule here. There is no clearly defined numerator or denominator.

Looking on Paul’s online math notes, you can use the trick $x=\frac{1}{1/x}$

$$\frac{\left(\psi^*\cdot\frac{\partial\psi}{\partial x}-\frac{\partial\psi^*}{\partial x}\cdot\psi\right)}{1/x}\Bigg|_{-\infty}^{\infty} => \frac{\partial}{\partial x} => \frac{\left(\frac{\partial\psi^*}{\partial x}\cdot\frac{\partial\psi}{\partial x}+\psi^*\cdot\frac{\partial^2\psi}{\partial x^2}\right)-\left(\frac{\partial^2\psi^*}{\partial x^2}\cdot\psi+\frac{\partial\psi^*}{\partial x}\cdot\frac{\partial\psi}{\partial x}\right)}{1/1}\Bigg|_{-\infty}^{\infty}$$

From here, we can use the knowledge that the wave function approaches 0 at $x=\pm\infty$ (and therefore that its derivative does as well) to set this term equal to 0.

$$\frac{d\langle x\rangle}{dt}=-\frac{ih}{2m}\int_{-\infty}^{\infty}\left(\psi^*\cdot\frac{\partial\psi}{\partial x}-\frac{\partial\psi^*}{\partial x}\cdot\psi\right)dx \quad 1.30$$

1.31 continued on next page

$$\frac{d\langle x\rangle}{dt}=-\frac{ih}{2m}\int_{-\infty}^{\infty}\left(\psi*\cdot\frac{\partial\psi}{\partial x}-\frac{\partial\psi*}{\partial x}\cdot\psi\right)dx \quad 1.30$$

$$\frac{d\langle x\rangle}{dt}=-\frac{ih}{2m}\int_{-\infty}^{\infty}\psi*\cdot\frac{\partial\psi}{\partial x}dx+\frac{ih}{2m}\int_{-\infty}^{\infty}\frac{\partial\psi*}{\partial x}\cdot\psi\, dx$$

The two halves of the integral can be combined using integration by parts

we will transform the second half to the form of the first:

$$\int_{-\infty}^{\infty}\frac{\partial\psi*}{\partial x}\cdot\psi\, dx$$

$$u=\psi\, ; dv=\frac{\partial\psi*}{\partial x}dx$$

$$du=\frac{\partial\psi}{\partial x}; v=\psi*$$

$$\int_{-\infty}^{\infty}\frac{\partial\psi*}{\partial x}\cdot\psi\, dx=\int_{-\infty}^{\infty}u\cdot dv=u\cdot v|_{-\infty}^{\infty}-\int_{-\infty}^{\infty}v\cdot du$$

$$\int_{-\infty}^{\infty}\frac{\partial\psi*}{\partial x}\cdot\psi\, dx=\psi\cdot\psi*|_{-\infty}^{\infty}-\int_{-\infty}^{\infty}\psi*\cdot\frac{\partial\psi}{\partial x}$$

once again we will use the fact that the wave function approaches 0 at $x=\pm\infty$

$$\int_{-\infty}^{\infty}\frac{\partial\psi*}{\partial x}\cdot\psi\, dx=-\int_{-\infty}^{\infty}\psi*\cdot\frac{\partial\psi}{\partial x}$$ This itself is an interesting equality that we should remember

$$\frac{d\langle x\rangle}{dt}=-\frac{ih}{2m}\int_{-\infty}^{\infty}\psi*\cdot\frac{\partial\psi}{\partial x}dx-\frac{ih}{2m}\int_{-\infty}^{\infty}\psi*\cdot\frac{\partial\psi}{\partial x}dx$$

$$\frac{d\langle x\rangle}{dt}=-\frac{ih}{m}\int_{-\infty}^{\infty}\psi*\cdot\frac{\partial\psi}{\partial x}dx \quad 1.31$$