Statistical Inference - Project Part 1

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Overview

This project uses simulation to explore the exponential distribution in R. There are two segments of the project. First, simulate randow draws from the exponential distribution. Second, perform inferential data analysis on the simulation data. This report will look at the sample mean and sample variance versus the theoretical mean and theoretical variance for the distribution.

The probability distribution function (PDF) for the exponential distribution is: $\mathrm{P(x)} = \lambda\ \mathrm{e}^{-\lambda \mathrm{t}}$. The first moment, mean, is $\mu = 1/\lambda$. The second moment, variance, is also $\sigma^2 = 1/\lambda$.

The required libraries are loaded.

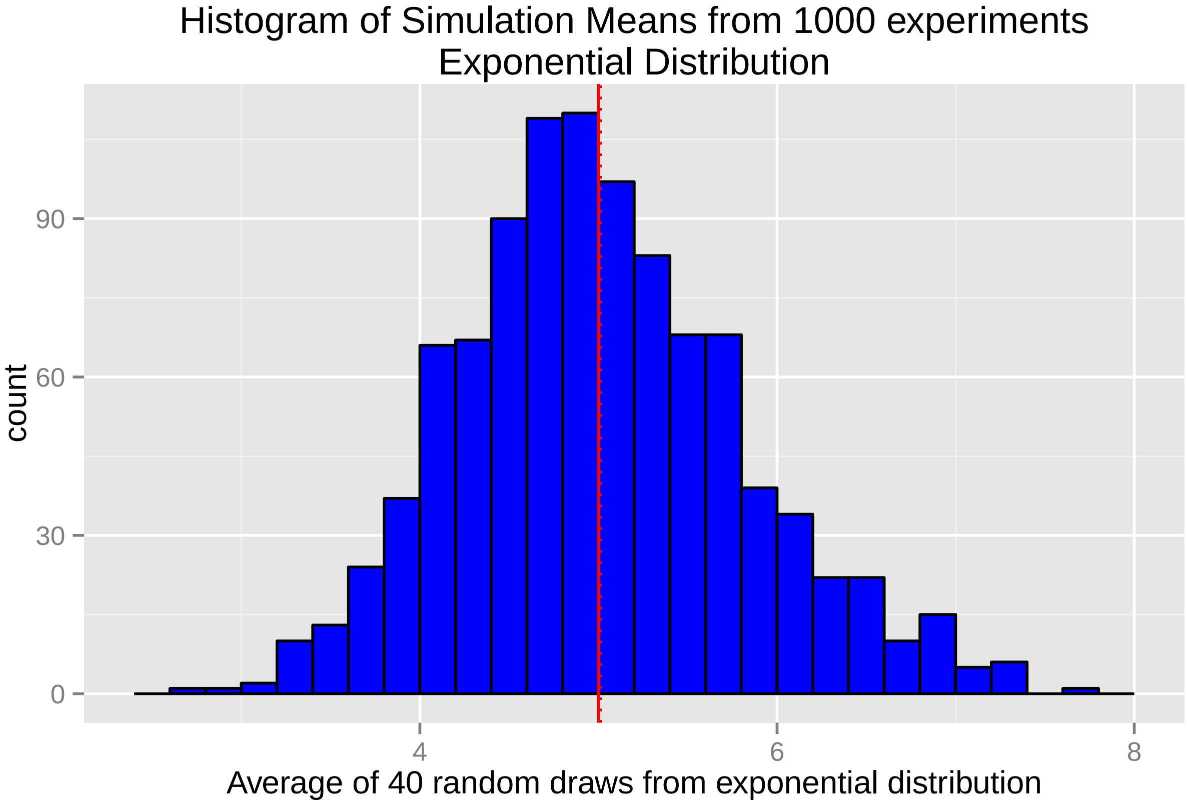
```
library(plyr)
library(ggplot2)
```

Simulations

```
simulations <- 1000
draws <- 40
lambda <- 0.2
```

For the experiment, we run 1000 simulations of 40 draws each from the exponential distribution using a lambda of 0.2 for the distribution. The experiment is run. From each simulation, the mean of 40 draws is calculated. This is our sample population. A data frame is created with the means of each simulation. A histogram of the means is plotted to examine the distribution of the sample means.

```
simdata <- apply(matrix(rexp(simulations * draws, lambda), simulations), 1, mean)
simdata <- data.frame(simdata)
plotfn(simdata, lambda)
```



Sample Mean versus Theoretical Mean

```
# Calculate mean of all experiments (x bar)
simdata.mean <- mean(simdata$simdata)

# Calculate the theoretical mean (mu)
expdist.mean <- 1/lambda
```

In the diagram above, the red solid line shows the theoretical mean of the exponential distribution and the red dotted line shows the mean of the sample population. The theoretical mean of the exponential distributions, μ, is $\mu = 1/\lambda = 5$. The mean of the sample population, $\bar{x} = 5.0104009$. The difference between the two values is very small: μ - $\bar{x}$ = -0.0104009. The percentage error in this amount is -0.2080189%, which indicates this is a good representation.

Sample Variance versus Theoretical Variance

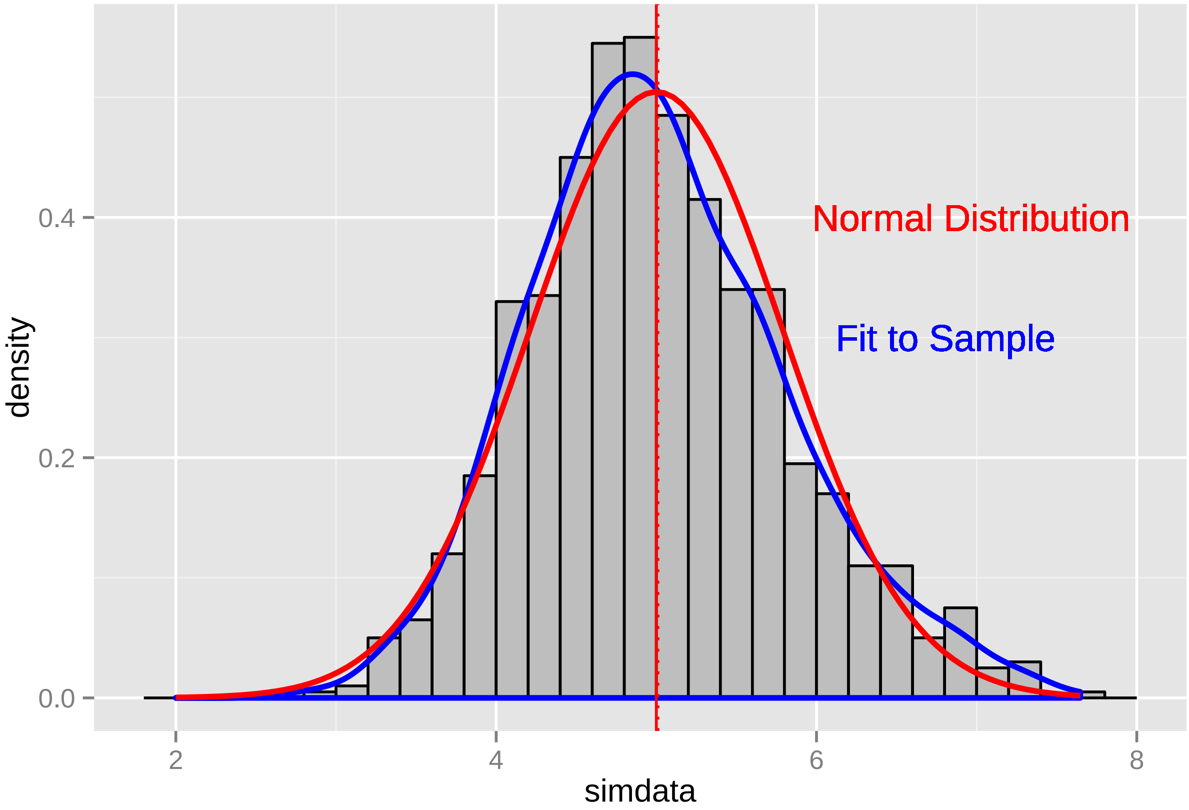
```
# Calculate mean of all experiments (s^2)
simdata.var <- var(simdata$simdata)

# Calculate the theoretical variance (E[x bar] = sigma^2/n)
expdist.var <- expdist.mean^2/draws
```

The theoretical variance of the means of draws from the exponential distribution, σ^2, is $\sigma^2 = (1/\mathrm{n})\ \lambda^{-2} =$ 0.625. The variance of the sample population, s^2 = 0.6391882. The difference between the two values is very small: σ^2 - s^2 = -0.0141882. The percentage error in this amount is -2.2701078%, which indicates this is a good representation.

Distribution

A histogram of the means is again plotted to examine the distribution of the sample means. In the plot below, the histogram was adjusted for y to be between 0 and 1. This enabled an overlay of the Normal Distribution N(5,0.625) in red as defined by the Central Limit Theorem. A curve is fit to the sample distribution is overlayed in blue. This graph shows that the distribution is approximately normal as stated by the CLT.

```
plotdist(simdata, lambda, expdist.mean, expdist.var)
```



Appendix

The report was limited to 3 pages plus up to 3 pages of appendix material if needed (code, figures, etcetera). The code for the two plotting functions was hidden in the document above to keep the length to three pages (minus appendix). The code for the two functions is presented here.

Plot a histogram of the samples with the theoretical exponential distribution mean and sample mean.

```
plotfn <- function(vals, lmd) {
    plot1 <- ggplot(simdata, aes(x = simdata)) +
        geom_histogram(binwidth = 0.2, color="black", fill="blue") +
        labs(title=paste("Histogram of Simulation Means from 1000 experiments\n",
            "Exponential Distribution",sep="")) +
        labs(x="Average of 40 random draws from exponential distribution") +
    geom_vline(xintercept=mean(vals$simdata), linetype="dotted", color="red") +
    geom_vline(xintercept=1/lmd, color="red")
    print(plot1)
}
```

Plot a histogram of the samples with an overlayed curve fit to the histogram and Normal Distribution.

```
plotdist <- function(dst, lmd, nmean, nvar) {
    plot2 <- ggplot(dst) +
        geom_histogram(aes(x = simdata, y=..density..), binwidth = 0.2,
                       color="black", fill="grey") +
        geom_density(aes(x = simdata), color="blue", size=1) +
        stat_function( fun=dnorm ,arg=list(mean=nmean, sd=sqrt(nvar)),
                                            color = "red", size=1) +
        geom_vline(xintercept=mean(dst$simdata), linetype="dotted",
                   color="red") +
        geom_vline(xintercept=1/lmd, color="red") +
        geom_text(aes(2,0.4,label = "Normal Distribution", hjust = -2), color="red") +
        geom_text(aes(2,0.3,label = "Fit to Sample", hjust = -3),
                  color="blue")


    print(plot2)
}
```