Jack Girard
ECE637
Lab 1 Work

Let $h(m,n)$ be the impulse response of an IIR filter with corresponding difference equation

$$y(m,n) = 0.01x(m,n) + 0.9(y(m-1,n) + y(m,n-1)) - 0.81y(m-1,n-1)$$

where $x(m,n)$ is the input and $y(m,n)$ is the output.

Identity Used:

$$e^{i\phi} = \cos\phi + i\sin\phi$$

Taking Z-Transformation of difference equation:

$$Y(z_1,z_2) = 0.01X(z_1,z_2) + 0.9z_1^{-1}Y(z_1,z_2) + 0.9z_2^{-1}Y(z_1,z_2) - 0.81z_1^{-1}z_2^{-1}Y(z_1,z_2)$$

$$Y(z_1,z_2) - 0.9z_1^{-1}Y(z_1,z_2) - 0.9z_2^{-1}Y(z_1,z_2) + 0.81z_1^{-1}z_2^{-1}Y(z_1,z_2) = 0.01X(z_1,z_2)$$

$$Y(z_1,z_2)(1 - 0.9z_1^{-1} - 0.9z_2^{-1} + 0.81z_1^{-1}z_2^{-1}) = 0.01X(z_1,z_2)$$

$$H(z_1,z_2) = \frac{Y(z_1,z_2)}{X(z_1,z_2)} = \frac{0.01}{1 - 0.9z_1^{-1} - 0.9z_2^{-1} + 0.81z_1^{-1}z_2^{-1}}$$

$$H(e^{j\mu},e^{j\nu}) = \frac{0.01}{1 - 0.9e^{-j\mu} - 0.9e^{-j\nu} + 0.81e^{-j\mu}e^{-j\nu}}$$

$$\boxed{H(e^{j\mu},e^{j\nu}) = \frac{0.01}{1 - 0.9e^{-j\mu} - 0.9e^{-j\nu} + 0.81e^{-j(\mu+\nu)}}}$$

Further Simplifying $H(e^{j\mu},e^{j\nu})$ and calculating $|H(e^{j\mu},e^{j\nu})|$

$$= \frac{0.01}{1 - 0.9(\cos\mu - j\sin\mu) - 0.9(\cos\nu - j\sin\nu) + 0.81(\cos(\mu+\nu) - j\sin(\mu+\nu))}$$

$$= \frac{0.01}{[1 - 0.9\cos\mu - 0.9\cos\nu + 0.81\cos(\mu+\nu)] + [0.9j\sin\mu + 0.9j\sin\nu - 0.81j\sin(\mu+\nu)]}$$

$$= \frac{0.01}{\underbrace{[1 - 0.9(\cos\mu + \cos\nu) + 0.81\cos(\mu+\nu)]}_{=a} + j\underbrace{[0.9(\sin\mu + \sin\nu) - 0.81\sin(\mu+\nu)]}_{=b}}$$

$$= \frac{0.01}{a + jb} = \frac{0.01}{a + jb} \cdot \frac{a - jb}{a - jb} = \frac{0.01(a - jb)}{a^2 + b^2} = k(a - jb) \; ; \; k = \frac{0.01}{a^2 + b^2}$$

$$|k(a - jb)| = \sqrt{(ka)^2 + (kb)^2} = k\sqrt{a^2 + b^2} \; ; \; \begin{aligned} a &= 1 - 0.9(\cos\mu + \cos\nu) + 0.81\cos(\mu+\nu) \\ b &= 0.9(\sin\mu + \sin\nu) - 0.81\sin(\mu+\nu) \end{aligned}$$