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ECE 637
Final Exam

$\boxed{2}$ Assume $f(i,j)=g$

2.1) Yes, $B(i,j)$ is strict sense stationary because all distributional properties are not a function of space

2.2) $\mu = E[B(i,j)] = 1 \cdot P\{g \geq T(i,j)\} + 0 \cdot P\{g < T(i,j)\}$
$= 1 \cdot P\{T(i,j) \leq g\}$
$= \boxed{g}$

2.3) $\sigma^2 = E[(B(i,j)-\mu)^2] = E[(B(i,j)-g)^2]$
$= E[B(i,j)^2 - 2B(i,j)g + g^2]$
$= E[B(i,j)^2] - 2g^2 + g^2$
$= (1^2 \cdot P\{g \geq T(i,j)\} + 0^2 \cdot P\{g < T(i,j)\}) - g^2$
$= g - g^2 = \boxed{g(1-g)}$

2.4) $R_D(m,n) = E[D(i,j)D(i+m,j+n)]$
$= E[(f(i,j)-B(i,j))((f(i+m,j+n)-B(i+m,j+n))]$
$= E[(g-B(i,j))(g-B(i+m,j+n))]$
$= E[g^2 - B(i,j)g - B(i+m,j+n)g + B(i,j)B(i+m,j+n)]$

if $(m,n)=0$:

$R_D(m,n) = E[g^2 - 2B(i,j)g + B(i,j)^2]$
$= g^2 - 2g^2 + E[B(i,j)^2]$
$= -g^2 + (1 \cdot P\{g \geq T(i,j)) + 0 \cdot P\{g < T(i,j)\})$
$= -g^2 + g = g(1-g)$

if $(m,n) \neq 0$

$R_D(m,n) = E[g^2] - gE[B(i,j)] - gE[B(i+m,j+n)] + E[B(i,j)B(i+m,j+n)]$
$= g^2 - g^2 - g^2 + E[B(i,j)]E[B(i+m,j+n)]$
$= g^2 - g^2 - g^2 + g^2 = 0 \rightarrow \boxed{R_D(m,n) = g(1-g)\delta(m,n)}$