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$$5.1) M = \begin{bmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{bmatrix} = \underbrace{\begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix}}_A \underbrace{\begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}}_B \rightarrow A = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix}$$

$$5.2) \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} x_w/y_w \\ 1 \\ z_w/y_w \end{bmatrix}$$

$$\hookrightarrow [\alpha_1 \ \alpha_2 \ \alpha_3]^T = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix}^{-1} \begin{bmatrix} x_w/y_w \\ 1 \\ z_w/y_w \end{bmatrix} = A^{-1} \begin{bmatrix} x_w/y_w \\ 1 \\ z_w/y_w \end{bmatrix}$$

$$5.3) M = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix} = \begin{bmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{bmatrix}$$

5.4) No, the entries of matrix M are not always positive because, as seen previously, M can be written in terms of chromaticity values, which are always ≥ 0

$$M = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix} = \underbrace{\begin{bmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{bmatrix}}_{\text{strictly } \geq 0}$$

\therefore M must have all positive or 0 (non-negative) entries

5.5) No, the entries of M^{-1} are not always positive.

By the relationship:

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = M^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

\uparrow
elements
strictly
 ≥ 0

\hookrightarrow negative values in M^{-1} imply that the XYZ primaries are negative, which is true

$\therefore M^{-1}$ does not always have positive entries