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ECE637
Lab 1 Work

Let $h(m,n)$ be a low pass filter. For our purposes use

$$h(m,n) = \begin{cases} 1/25 & \text{for } |m| \le 2 \text{ and } |n| \le 2 \\ 0 & \text{otherwise} \end{cases}$$

The unsharp mask filter is then given by

$$g(m,n) = \delta(m,n) + \lambda(\delta(m,n) - h(m,n))$$

where λ is a constant greater than zero.

Identity Used:

$$sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

$$cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

Apply formula for DSFT & Simplify:

$$H(e^{j\mu}, e^{j\nu}) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h(m,n) e^{-j(\mu m + \nu n)}$$

$$= \sum_{n=-2}^{2} \sum_{m=-2}^{2} \frac{1}{25} e^{-j(\mu m + \nu n)}$$

$$= \frac{1}{25} \sum_{n=-2}^{2} e^{-j\mu m} \sum_{m=-2}^{2} e^{-j\nu n}$$

$$= \frac{1}{25} \left[\left(1 + e^{-2\mu} + e^{2\mu} + e^{-\mu} + e^{\mu}\right)\left(1 + e^{-2\nu} + e^{2\nu} + e^{-\nu} + e^{\nu}\right) \right]$$

$$= \frac{1}{25} \left[\left(1 + 2cos(2\mu) + 2cos(\mu)\right)\left(1 + 2cos(2\nu) + 2cos(\nu)\right) \right]$$

$$= \boxed{\frac{1}{25} \left[\left(1 + 2\sum_{k=1}^{2} cos(k\mu)\right)\left(1 + 2\sum_{l=1}^{2} cos(l\nu)\right) \right]}$$

Apply formula for DSFT & Simplify, knowing $H(e^{j\mu}, e^{j\nu})$:

$$G(e^{j\mu}, e^{j\nu}) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} g(m,n) e^{-j(\mu m + \nu n)} = 1 + \lambda(1 - H(e^{j\mu}, e^{j\nu}))$$

$$= \boxed{1 + \lambda \left[1 - \frac{1}{25} \left[\left(1 + 2\sum_{k=1}^{2} cos(k\mu)\right)\left(1 + 2\sum_{l=1}^{2} cos(l\nu)\right) \right] \right]}$$