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ECE 637
Lab 5: Eigen-decomposition of Images

1. Introduction

No Deliverables

2. Multivariate Gaussian Distributions and Whitening

1. Scatter plots for W, $\tilde{X}$, and X

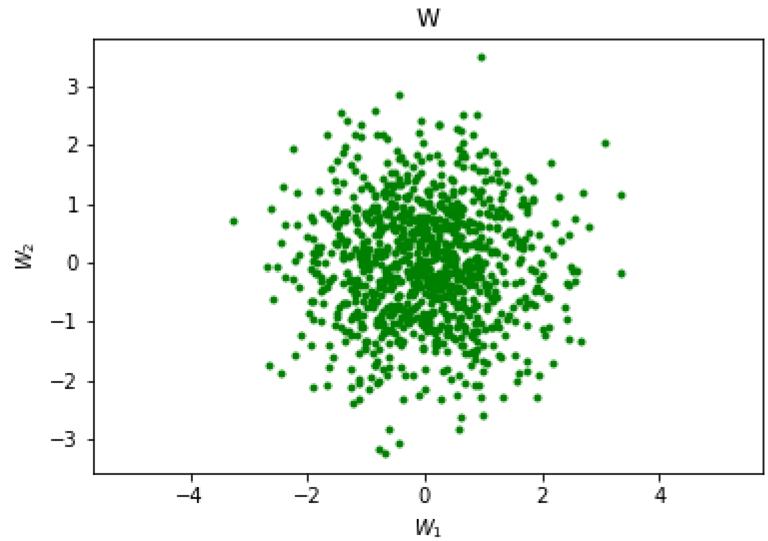

Figure 1: W

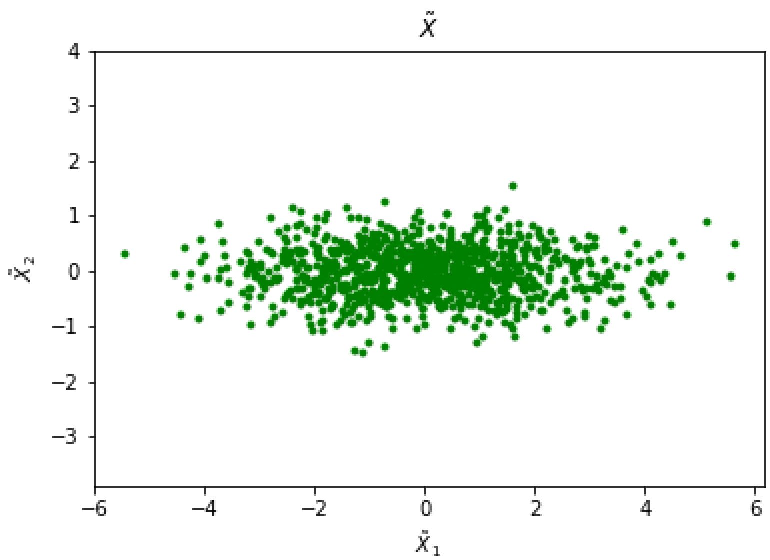

Figure 2: $\tilde{X}$

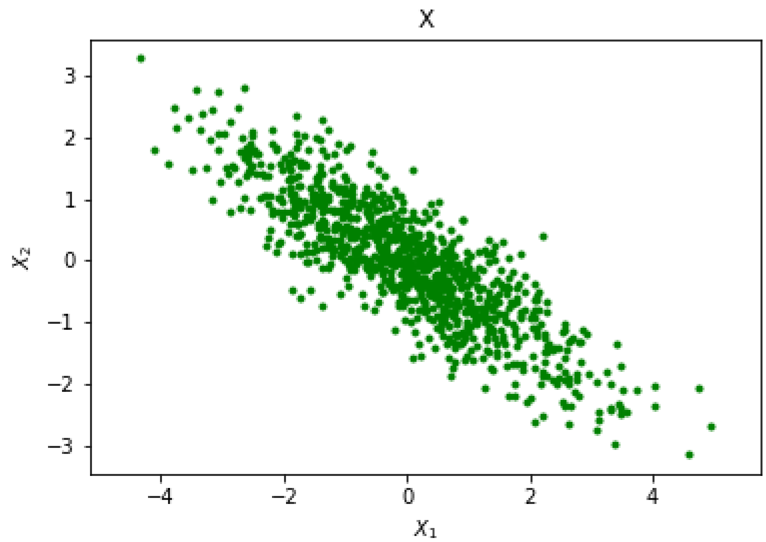

Figure 3: X

2. Print out of the image showing the connected set for s = (67, 45), and T = 3
    a. Theoretical value of the covariance matrix R_X

$$R_X = \begin{bmatrix} 2 & -1.2 \\ -1.2 & 1 \end{bmatrix}$$

    b. Numerical listing of covariance estimate $\hat{R}_X$

$$\hat{R}_X = \begin{bmatrix} 2.002 & -1.211 \\ -1.211 & 1.021 \end{bmatrix}$$

    c. Scatter plots for $\tilde{X}$ and W

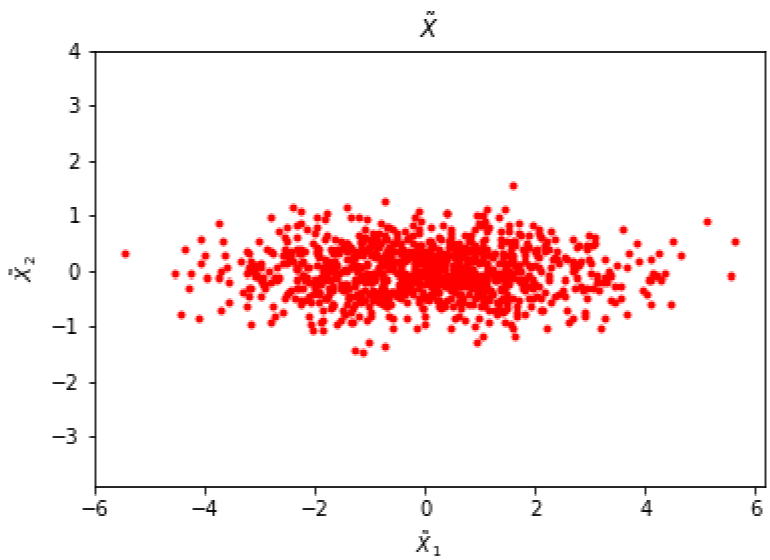

Figure 4: $\tilde{X}$

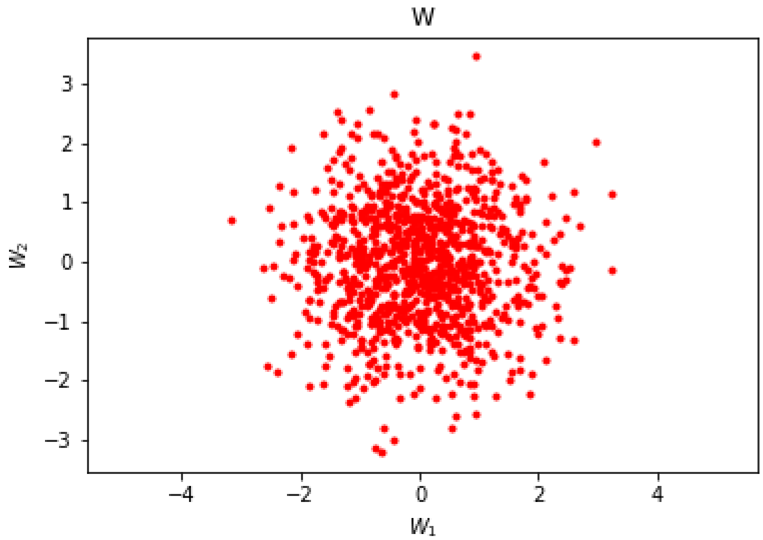

Figure 5: W

d. Numerical listing of covariance estimate $\hat{R}_W$

$$\hat{R}_W = \begin{bmatrix} 1.006 & -0.018 \\ -0.018 & 1.026 \end{bmatrix}$$

3. Estimation of Eigenvectors and Eigenvalues Using the Singular Value Decomposition

No Deliverables

4. Eigenimages, PCA, and Data Reduction

1. Figure of first 12 eigenimages

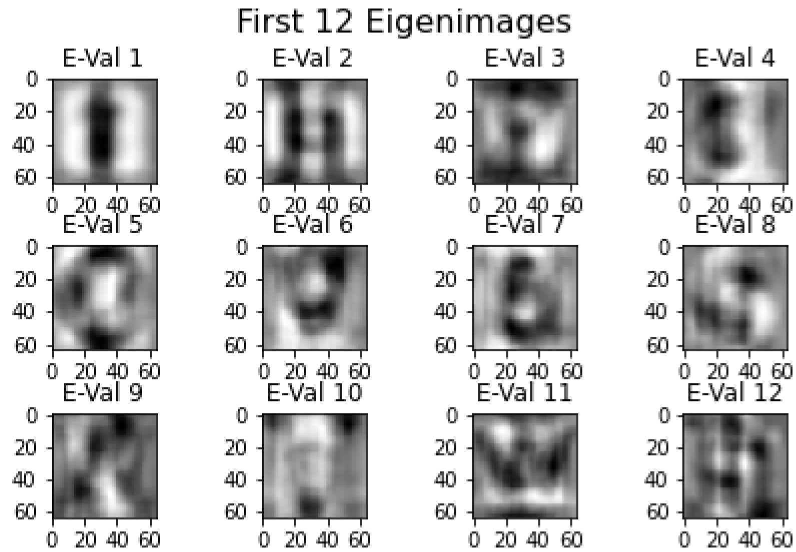

Figure 6: First 12 Eigenimages

2. Plot of projection coefficients vs. eigenvector number

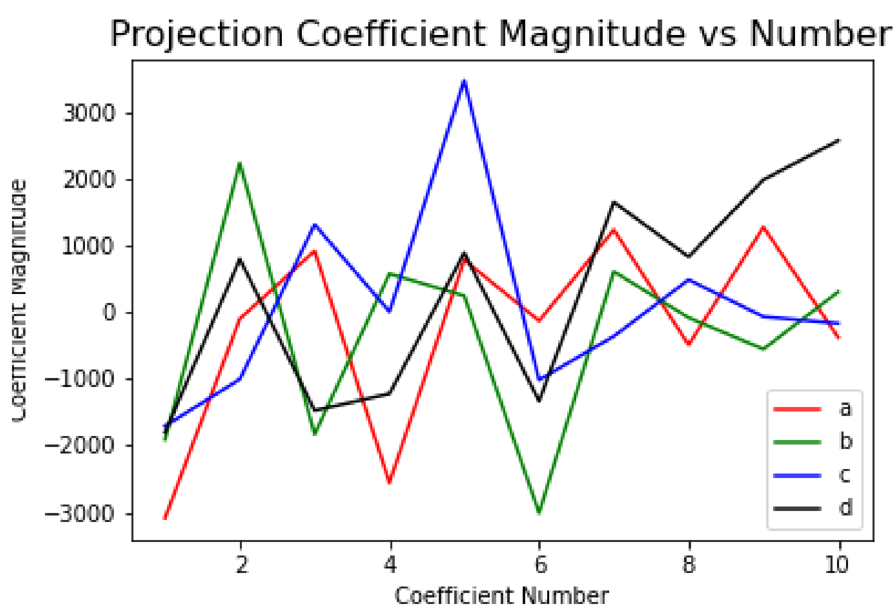

Figure 7: Projection Coefficients vs Eigenvector Number

3. Original image and the 6 resynthesized versions

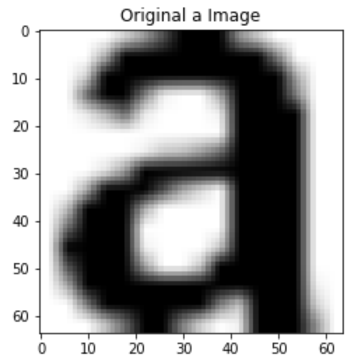

Figure 8: Original 'a' Image

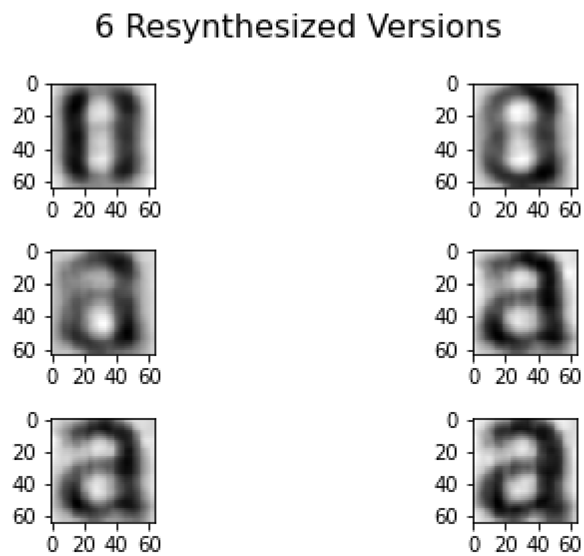

Figure 9: 6 Resynthesized versions of 'a'

5. Image Classification

1. Mis-classified image list using R_k

Actual Letter	**Mis-Classified As**
d	a
j	y
l	i
n	v
p	e
q	a
u	a
y	v

2. Mis-classified image lists using various B_k
    a. Using $B_k = \Lambda_k$

Actual Letter	Mis-Classified As
i	l
y	v

    b. Using $B_k = R_{wc}$

Actual Letter	Mis-Classified As
g	q
y	v

    c. Using $B_k = \Lambda$

Actual Letter	Mis-Classified As
f	t
y	v

    d. Using $B_k = I$

Actual Letter	Mis-Classified As
f	t
g	q
y	v

3. Further Questions:
    a. **1. Which of the above classifiers worked the best in this experiment?**
    Classification using $B_k = \Lambda_k$, $B_k = R_{wc}$, and $B_k = \Lambda$ worked the best in this experiment. This is because these selections led to the least number of mis-classified letters
    b. **In constraining the covariance, what is the trade off between the accuracy of the data model and the accuracy of the estimates?**
    When the covariance is constrained, the accuracy of the estimates is increased as seen in the previous results. However, the accuracy of the data model is decreased because the ground-truth of the training data is not captured. When the covariance matrix is modified, certain statistical relationships among the training data are not maintained.