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ECE 637
Exam #1

$\boxed{2}$ $$y(m,n) = x(m,n) + \lambda\left(x(m,n) - \frac{1}{9}\sum_{k=-1}^{1}\sum_{\ell=-1}^{1} x(m-k, n-\ell)\right)$$

2.1) $y(m,n) = h(m,n) * x(m,n)$

$$\boxed{\begin{aligned} h(m,n) &= \delta(m,n) + \lambda\left(\delta(m,n) - h_1(m,n)h_2(m,n)\right) \\ \text{where} \quad h_1(m,n) &= \frac{1}{3}\left(\delta(m-1,n) + \delta(m,n) + \delta(m+1,n)\right) \\ \text{and} \quad h_2(m,n) &= \frac{1}{3}\left(\delta(m,n-1) + \delta(m,n) + \delta(m,n+1)\right) \end{aligned}}$$

2.2) $$H(e^{j0}, e^{j0}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} h(m,n) = \boxed{1}$$

2.3) No, because it is not the product of any two 1D functions

$$H(e^{j\mu}, e^{j\nu}) \neq H_a(e^{j\mu})H_b(e^{j\nu})$$

2.4) $$H_1(e^{j\mu}, e^{j\nu}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{1}{3}\left(\delta(m-1,n) + \delta(m,n) + \delta(m+1,n)\right)e^{-j(\mu m + \nu n)}$$

$$= \sum_{m=-1}^{1} \frac{1}{3} e^{-j(\mu m + \nu 0)} = \frac{1}{3}\left(e^{j\mu} + 1 + e^{-j\mu}\right)$$

$$= \boxed{\frac{1}{3}\left(1 + 2\cos(\mu)\right)}$$

$$H_2(e^{j\mu}, e^{j\nu}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{1}{3}\left(\delta(m,n-1) + \delta(m,n) + \delta(m,n+1)\right)e^{-j(\mu m + \nu n)}$$

$$= \sum_{n=-1}^{1} \frac{1}{3} e^{-j(0m + \nu n)} = \frac{1}{3}\left(e^{j\nu} + 1 + e^{-j\nu}\right)$$

$$= \boxed{\frac{1}{3}\left(1 + 2\cos(\nu)\right)}$$