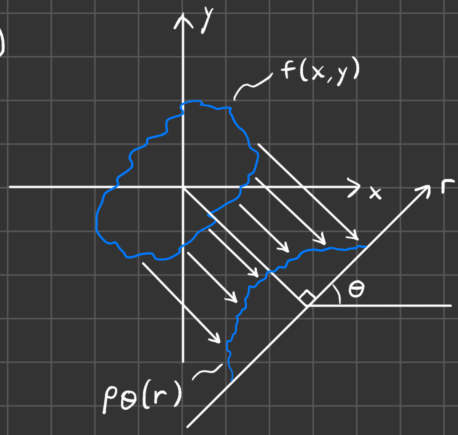
Jack Girard
ECE 637
Final Exam

$\boxed{3}$

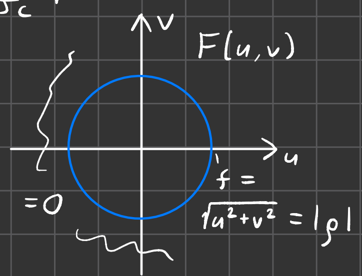
3.1)



3.2) $p_\theta(r) \overset{CTFT}{\longleftrightarrow} P_\theta(\rho)$

$$P_\theta(\rho) = F(\rho\cos\theta, \rho\sin\theta) \rightarrow \boxed{F(\rho\cos\theta, \rho\sin\theta) = \int_{-\infty}^{\infty} p_\theta(r) e^{-j2\pi\rho r} dr}$$

3.3) $F(u,v) = 0$ for $\sqrt{u^2+v^2} \leq f_c$, therefore it is band-limited to $\sqrt{u^2+v^2} \leq f_c$:



$P_\theta(\rho) = F(\rho\cos\theta, \rho\sin\theta) = 0$ for $|\rho| > f_c$, therefore $p_\theta(r)$ is band-limited with cutoff frequency f_c

3.4) $f(x,y) = \begin{cases} 1, & x^2+y^2 \leq 1 \\ 0, & \text{otherwise} \end{cases} \rightarrow \sqrt{x^2+y^2} \leq 1$

$\sqrt{r^2+z^2} = 1 \rightarrow z = \sqrt{1-r^2}\ ;\ |r|<1$

$p_\theta(r) = \int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} 1\,dz = 2\sqrt{1-r^2} \rightarrow p_\theta(r) = \begin{cases} 2\sqrt{1-r^2}, & |r|<1 \\ 0, & |r|>1 \end{cases}$

$\hookrightarrow \boxed{p_\theta(r) = 2\sqrt{1-r^2}\,\text{rect}\left(\frac{r}{2}\right)}$

3.5)