Digital Halftoning

- Many image rendering technologies only have binary output. For example, printers can either “fire a dot” or not.
- Halftoning is a method for creating the illusion of continuous tone output with a binary device.
- Effective digital halftoning can substantially improve the quality of rendered images at minimal cost.

common application : printing

- similar methods can be applied to signal amplification
- sigma-delta modification

Thresholding

- Assume that the image falls in the range of 0 to 255.
- Apply a space varying threshold, $T(i,j)$.

$$b(i,j) = \begin{cases} 255 & \text{if } X(i,j) > T(i,j) \\ 0 & \text{otherwise} \end{cases} .$$

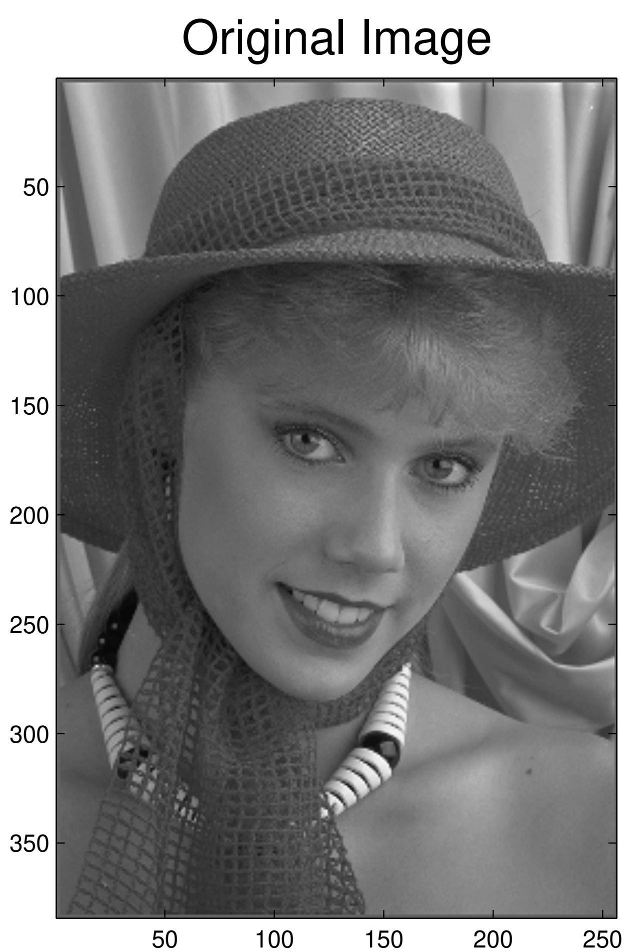
- What is $X(i,j)$?
- Lightness
  - Larger $\Rightarrow$ lighter
  - Used for display
- Absorptance
  - Larger $\Rightarrow$ darker
  - Used for printing
- $X(i,j)$ will generally be in units of absorptance.

Constant Threshold

- Assume that the image falls in the range of 0 to 255.
- $255 \Rightarrow Black$ and $0 \Rightarrow White$
- The minimum squared error quantizer is a simple threshold
$$b(i,j) = \begin{cases} 255 & \text{if } X(i,j) > T \\ 0 & \text{otherwise} \end{cases} .$$
where $T = 127$. - this is just MMSE
- This produces a poor quality rendering of a continuous tone image.

The Minimum Squared Error Solution

- Threshold each pixel
  - Pixel$>$ 127 Fire ink
  - Pixel$\leq$ 127 do nothing



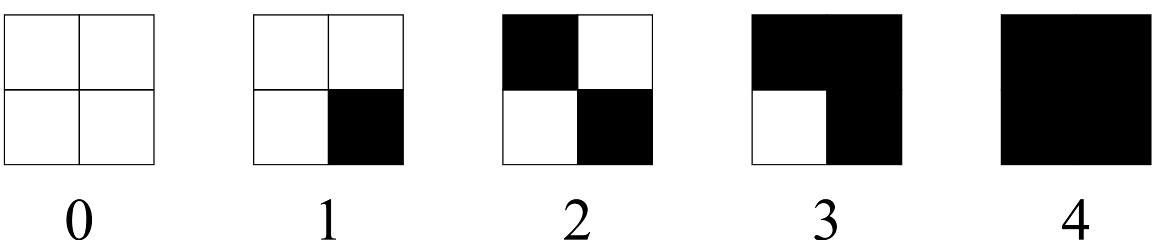
MMSE not a good measure of quality

Ordered Dither

- For a constant gray level patch, turn the pixel "on"in a specified order.
- This creates the perception of continuous variations of gray.
- An $N \times N$ index matrix specifies what order to use.

$$I_2(i,j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

- Pixels are turned on in the following order.



impact printers ~ typewriters, etc

Implementation of Ordered Dither via Thresholding

- The index matrix can be converted to a "threshold matrix" or "screen" using the following operation.

$$T(i,j) = 255\frac{I(i,j)+0.5}{N^2}$$

- The $N \times N$ matrix can then be "tiled" over the image using periodic replication.

$$T(i \bmod N, j \bmod N)$$

- The ordered dither algorithm is then applied via thresholding.

$$b(i,j) = \begin{cases} 255 & \text{if } X(i,j) > T(i \bmod N, j \bmod N) \\ 0 & \text{otherwise} \end{cases} .$$

Clustered Dot Screens

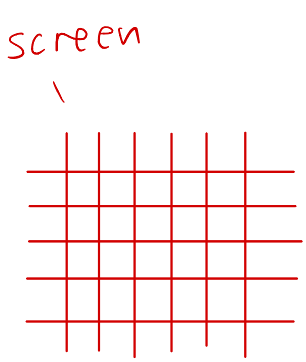
- Definition: If the consecutive thresholds are located in spatial proximity, then this is called a “clustered dot screen.
- Example for 8×8 matrix:

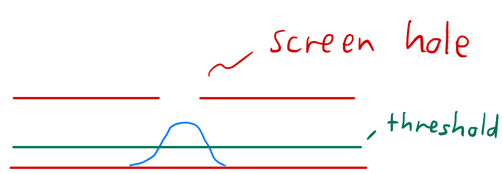
index

N^2+1 gray levels

N^2-1

62	57	48	36	37	49	58	63
56	47	35	21	22	38	50	59
46	34	20	10	11	23	39	51
33	19	9	3	0	4	12	24
32	18	8	2	1	5	13	25
45	31	17	7	6	14	26	40
55	44	30	16	15	27	41	52
61	54	43	29	28	42	53	60

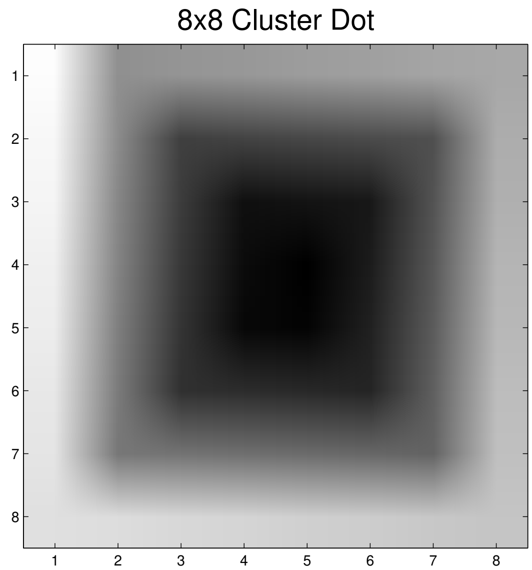


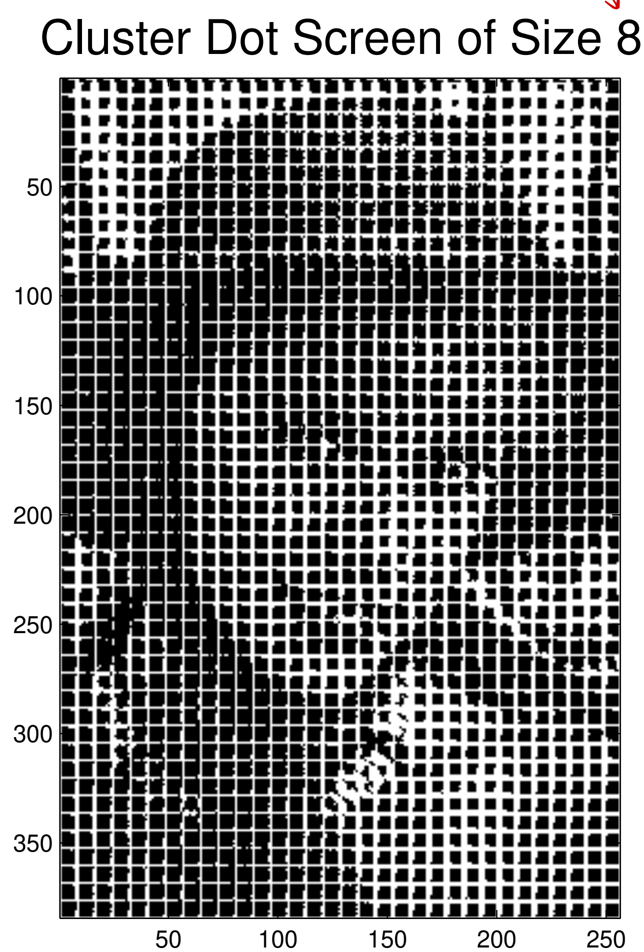


dot size proportional to the screen size & the amount of light coming in

Example: 8×8 Clustered Dot Screening

resolution reduced by a factor of

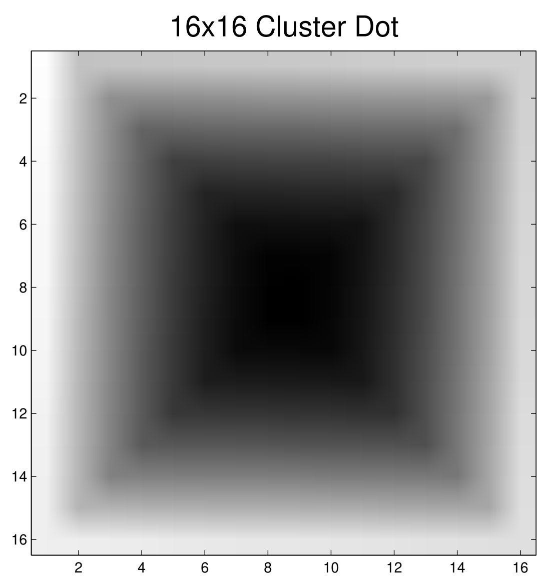


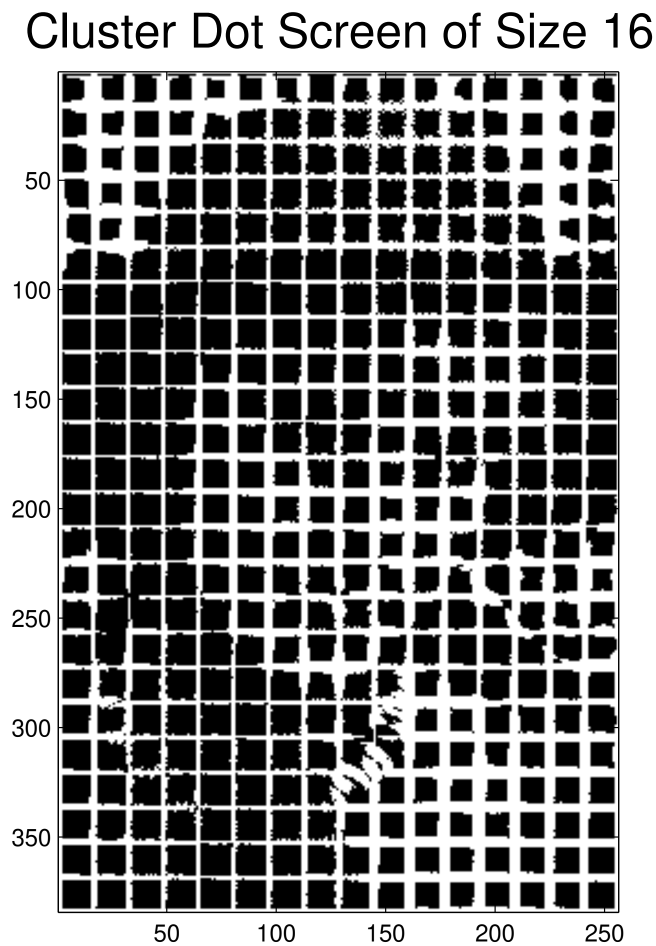


256/8 = 32 Squares

- Only supports 65 gray levels.

Example: 16×16 **Clustered Dot Screening**





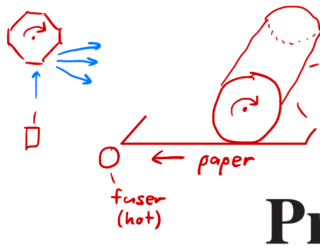
- Support a full 257 gray levels, but has half the resolution.

Printer Types

Electro-photographic

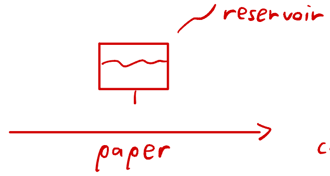
(Xerography)
(Laser Printer)

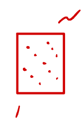
- individual dots are unstable



opc (optical photo conductor) - "drum" - static charged

Inkjet





can print individual dots

conversion to raster usually done on the computer

Properties of Clustered Dot Screens

- Requires a trade-off between number of gray levels and resolution.
- Relatively visible texture
- Relatively poor detail rendition
- Uniform texture across entire gray scale.
- Robust performance with non-ideal output devices
  - Non-additive spot overlap
  - Spot-to-spot variability
  - Noise
  - prints stably for electro-photographic printers

Dispersed Dot Screens

- Bayer's optimum index Matrix (1973) can be defined recursively.

$$I_2(i,j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$I_{2n} = \begin{bmatrix} 4 * I_n + 1 & 4 * I_n + 2 \\ 4 * I_n + 3 & 4 * I_n \end{bmatrix}$$

- Examples

2×2

1	2
3	0

4×4

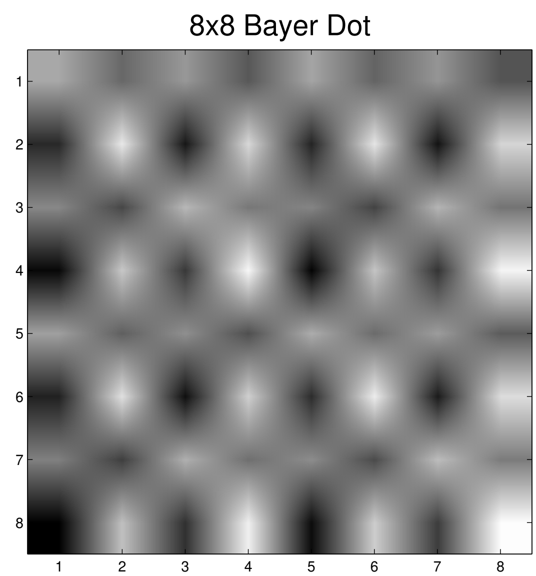
5	9	6	10
13	1	14	2
7	11	4	8
15	3	12	0

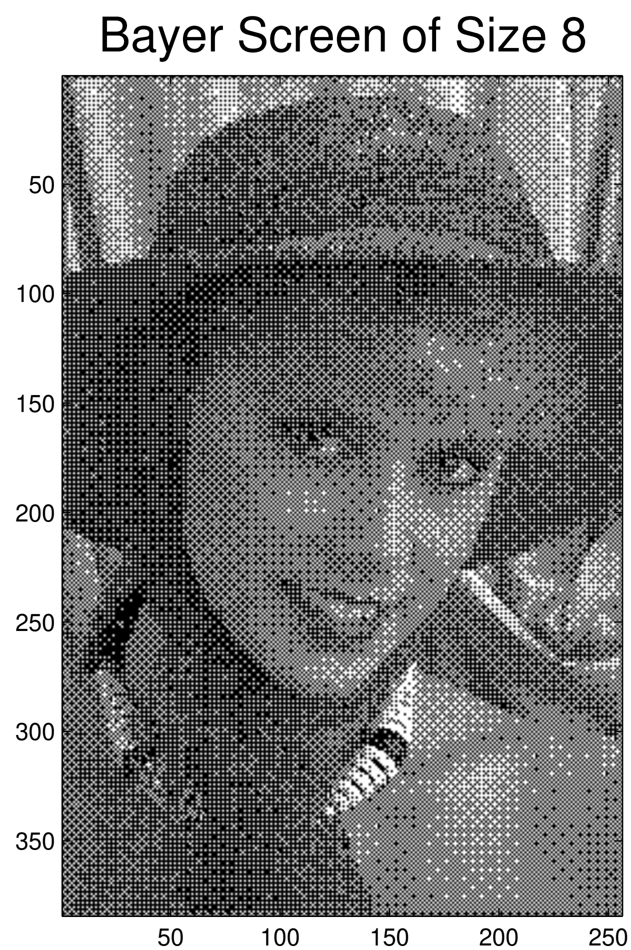
8×8

21	37	25	41	22	38	26	42
53	5	57	9	54	6	58	10
29	45	17	33	30	46	18	34
61	13	49	1	62	14	50	2
23	39	27	43	20	36	24	40
55	7	59	11	52	4	56	8
31	47	19	35	28	44	16	32
63	15	51	3	60	12	48	0

- Yields finer amplitude quantization over larger area.
- Retains good detail rendition within smaller area.

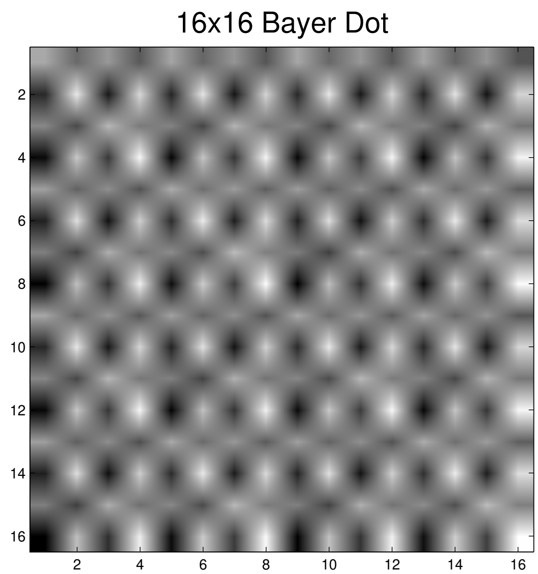
Example: 8×8 Bayer Dot Screening

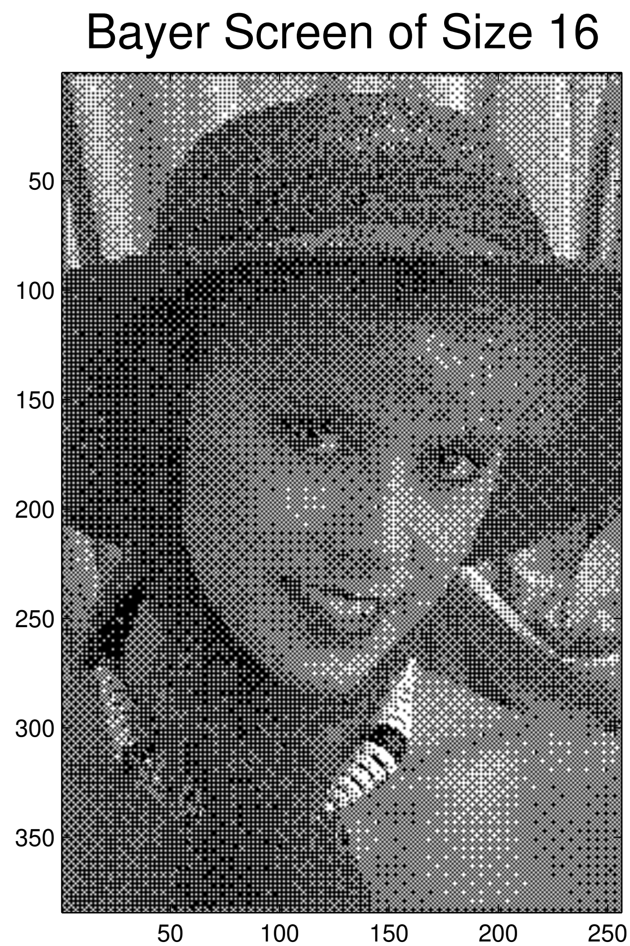




- Again, only 65 gray levels.

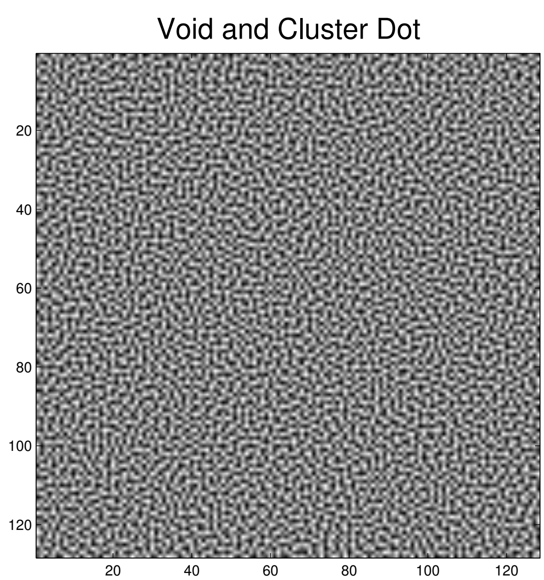
Example: 16×16 Bayer Dot Screening

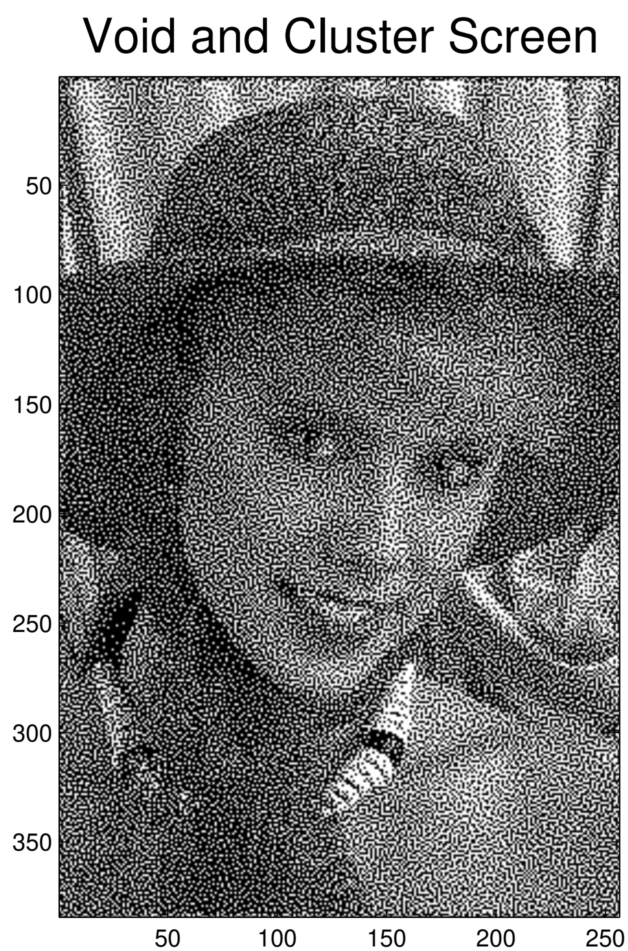




- Doesn't look much different than the 8×8 case.
- No trade-off between resolution and number of gray levels. → can have your cake & eat it too
- Gray level a function of pattern, not ideal or used in printing

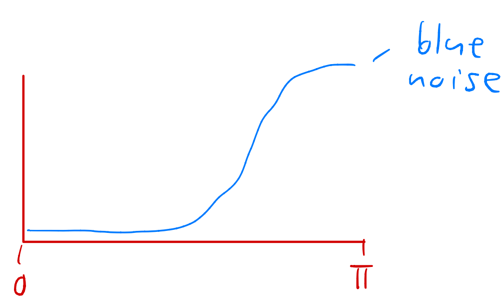
Example: 128×128 Void and Cluster Screen (1989)





screen is big

- Substantially improved quality over Bayer screen.



class D amps
sigma-delta modulation

Properties of Dispersed Dot Screens

- Eliminate the trade-off between number of gray levels and resolution.
- Within any region containing K dots, the K thresholds should be distributed as uniformly as possible.
- Textures used to represent individual gray levels have low visibility.
- Improved detail rendition.
- Transitions between textures corresponding to different gray levels may be more visible.
- Not robust to non-ideal output devices
  - Requires stable formation of isolated single dots.

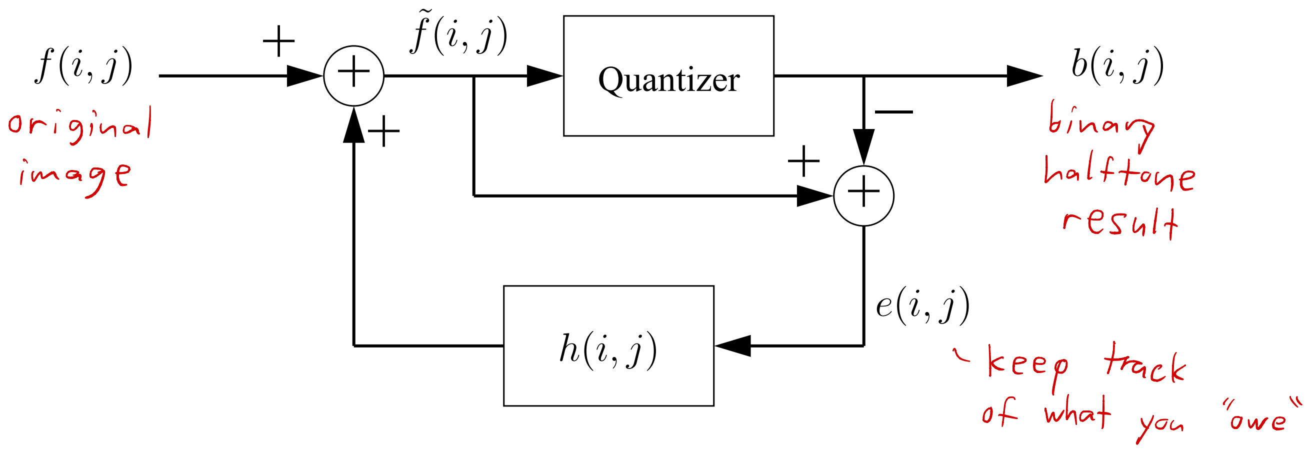
Error Diffusion

- Error Diffusion
  - Quantizes each pixel using a neighborhood operation, rather than a simple pointwise operation.
  - Moves through image in raster order, quantizing the result, and "pushing" the error forward.
  - Can produce better quality images than is possible with screens.

Assumes image is continuous, neighbor pixels related

Uses past knowledge, like an IIR filter

Filter View of Error Diffusion



- Equations are

$$b(i,j) = \begin{cases} 255 & \text{if } \tilde{f}(i,j) > T \\ 0 & \text{otherwise} \end{cases}$$

$$e(i,j) = \tilde{f}(i,j) - b(i,j)$$

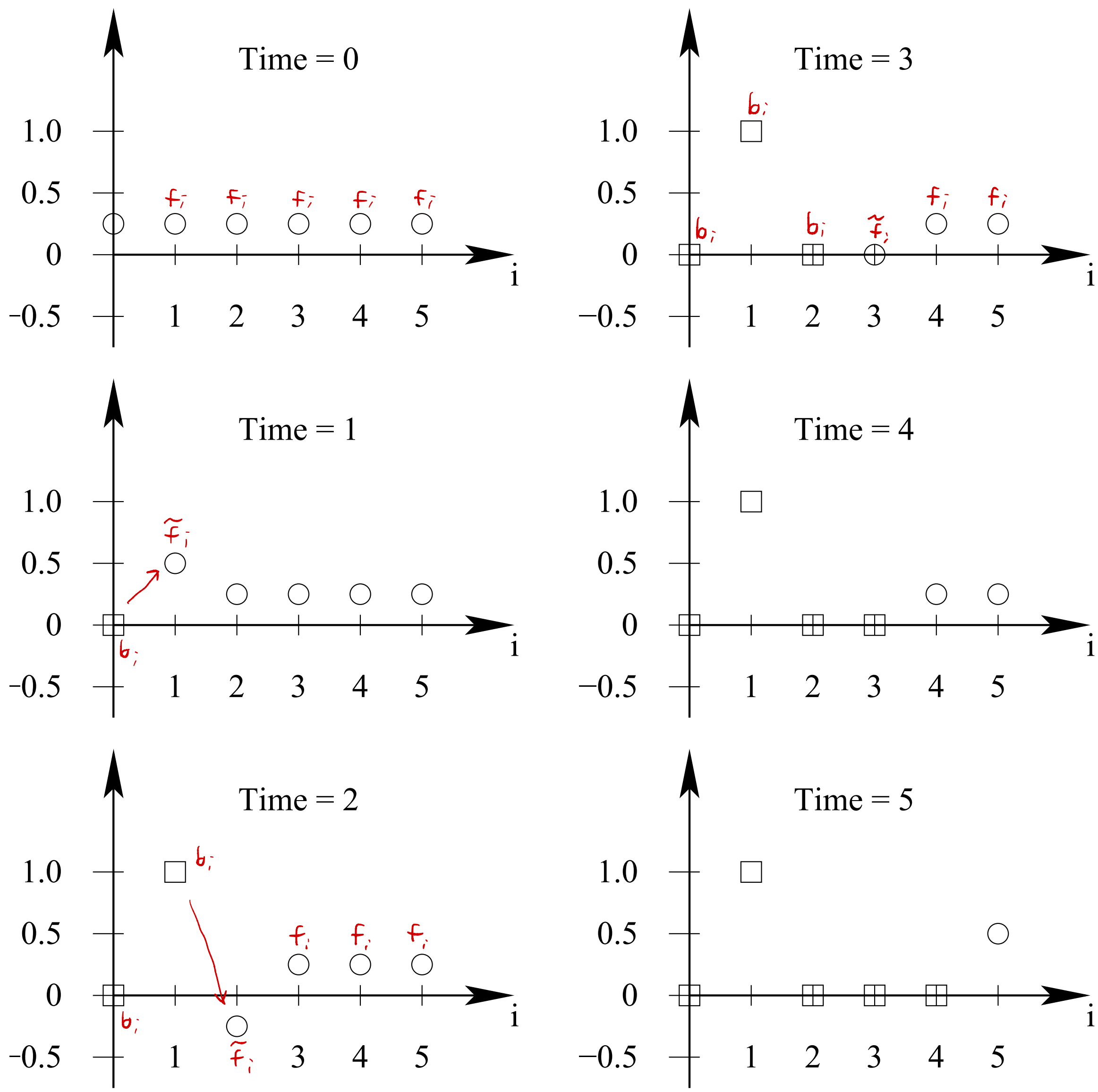
$$\tilde{f}(i,j) = f(i,j) + \sum_{k,l \in S} h(k,l) e(i-k, j-l)$$

- Parameters
  - Threshold is typically $T = 127$.
  - $h(k,l)$ are typically chosen to be positive and sum to 1

1-D Error Diffusion Example

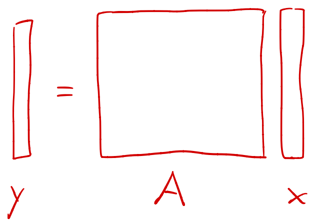
- $\tilde{f}(i) \Rightarrow$ circles
- $b(i) \Rightarrow$ boxes

T=0.5



Important to understand the pulling & pushing idea

$y = Ax$



Two Views of Error Diffusion

- Two mathematically equivalent views of error diffusion
  - Pulling errors forward
  - Pushing errors ahead
- Pulling errors forward
  - More similar to common view of IIR filter
  - Has advantages for analysis
- Pushing errors ahead
  - Original view of error diffusion
  - Can be more easily extended to important cases when weights area time/space varying

ED: Pulling Errors Forward

1. For each pixel in the image (in raster order)

   (a) Pull error forward

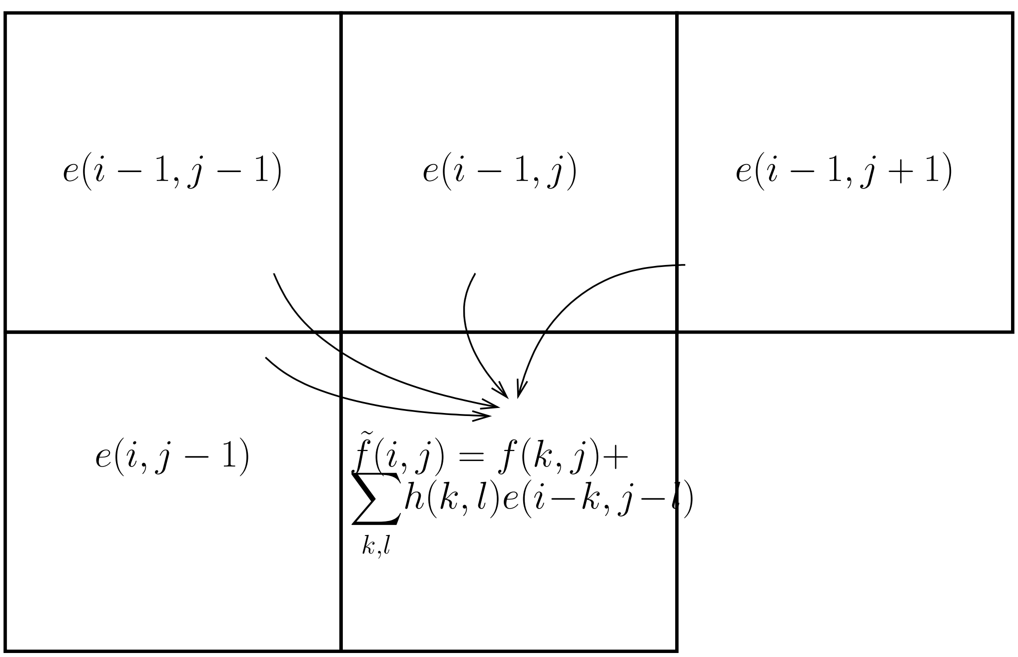
$$\tilde{f}(i,j) = f(i,j) + \sum_{k,l \in S} h(k,l) e(i-k, j-l)$$

   (b) Compute binary output

$$b(i,j) \; = \; \begin{cases} 255 & \text{if } \tilde{f}(i,j) > T \\ 0 & \text{otherwise} \end{cases}$$

   (c) Compute pixel's error

$$e(i,j) = \tilde{f}(i,j) - b(i,j)$$



2. Display binary image $b(i,j)$

ED: Pushing Errors Ahead

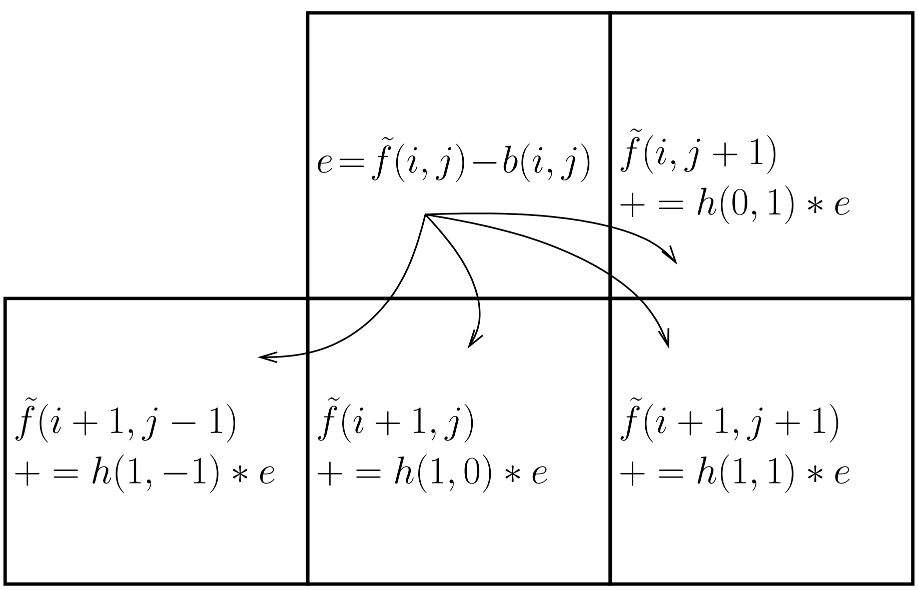
Preferable

1. Initialize $\tilde{f}(i,j) \leftarrow f(i,j)$
2. For each pixel in the image (in raster order)

   (a) Compute

$$b(i,j) = \begin{cases} 255 & \text{if } \tilde{f}(i,j) > T \\ 0 & \text{otherwise} \end{cases}$$

   (b) Diffuse error forward using the following scheme



3. Display binary image $b(i,j)$

Commonly Used Error Diffusion Weights

divide by 16 corresponds to a shift operation in memory

- Floyd and Steinberg (1976)

	■	7/16
3/16	5/16	1/16

- Jarvis, Judice, and Ninke (1976)

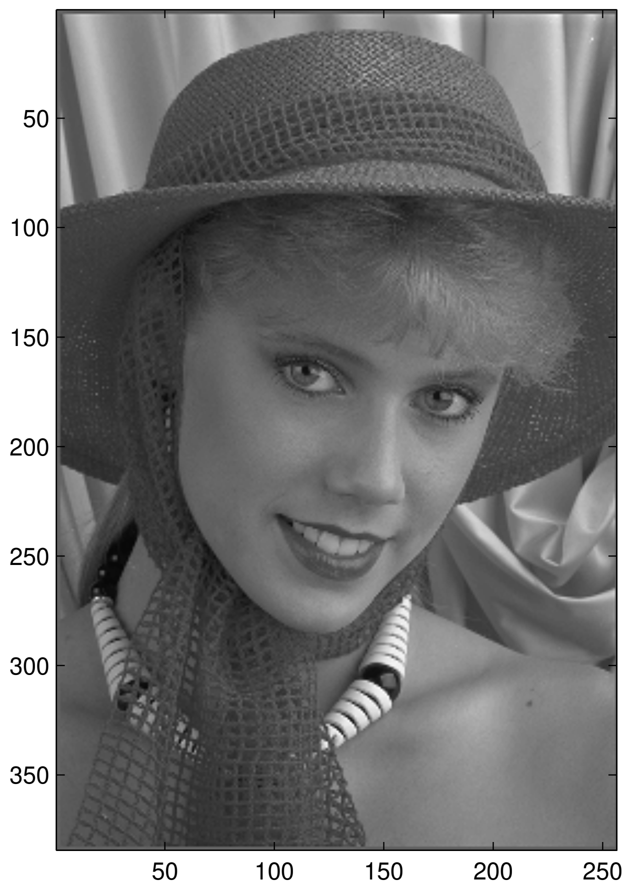
		■	7/48	5/48
3/48	5/48	7/48	5/48	3/48
1/48	3/48	5/48	3/48	1/48

Weights must sum to 1

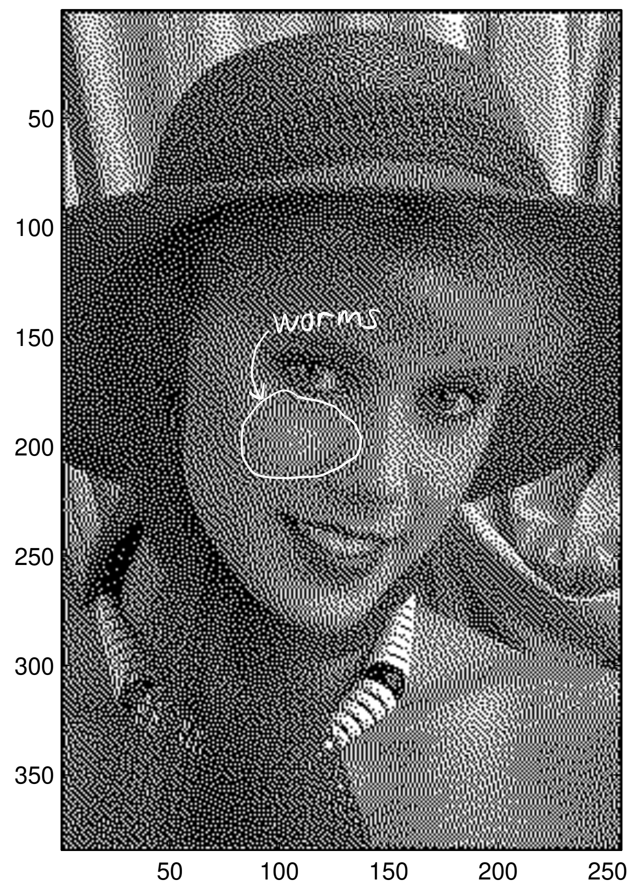
Floyd Steinberg Error Diffusion (1976)

- Process pixels in neighborhoods by "diffusing error" and quantizing.

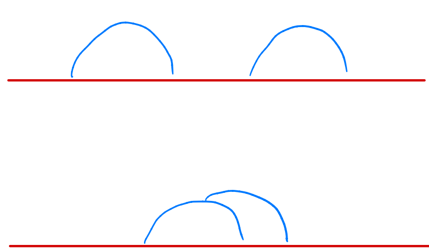
Original Image f



Floyd and Steinberg Error Diffusion B



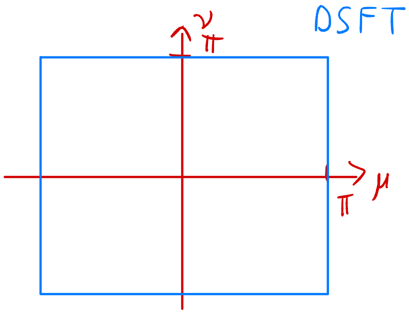
ink dots:



– #1 will be perceived as darker

visual perception is not linear

push error energy to higher frequencies, so they are harder to see with the human eye



$D = B - f$

display error

binary image

image

$f_n = \begin{cases} B_n = 1, & \text{prob. } f_n \\ B_n = 0, & \text{prob. } 1 - f_n \end{cases}$ i.i.d.

where $0 \le f_n \le 1$

"white noise dither"

"white noise screen"

$D_n = B_n - f_n$

$R_D(m-n) = E[D_n D_m]$

$= \sigma^2 \delta(m-n)$

$= g(1-g)\delta(m-n)$

$E[B_n] - E[f_n]$

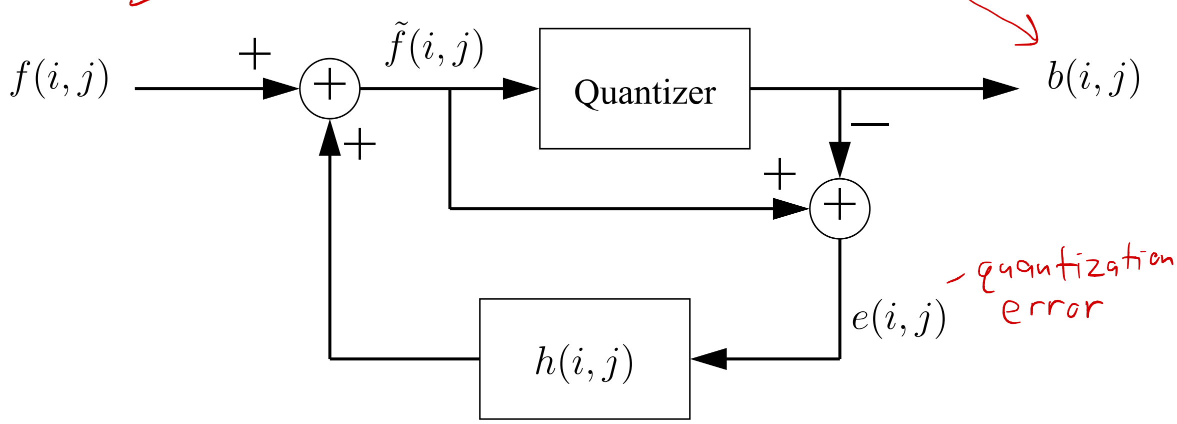
$= 1(f_n) + 0(1-f_n) - f_n = 0$

$R = E[D_n]E[D_m]$ $m \neq n$

$E[D_n^2]$

Quantization Error Modeling for Error Diffusion

display error



- Quantization error is commonly assumed to be:
  - Uniformly distributed on $[-0.5, 0.5]$
  - Uncorrelated in space
  - Independent of signal $\tilde{f}(i,j)$
  - $E[e(i,j)] = 0$ – mean
  - $E[e(i,j)e(i+k,j+l)] = \frac{\delta(k,l)}{12}$ – variance (recall: uniform distribution)

$p(x) = \begin{cases} 1, & -0.5 \le x \le 0.5 \\ 0, & 0 \end{cases}$

Calculating energy of the error – could be an exam problem

$f_n = g \qquad 0 \leq g \leq 1$

Good Exam Question

$$B_n = \begin{cases} 1, & \text{prob. } g \\ 0, & \text{prob. } 1-g \end{cases} \qquad \text{i.i.d.}$$

$$D_n = B_n - f_n = B_n - g$$

$$E[D_n] = E[B_n - g] = E[B_n] - E[g] = g - g = 0$$

$$E[D_n^2] =$$

if $B_n = 1$ $\qquad$ if $B_n = 0$

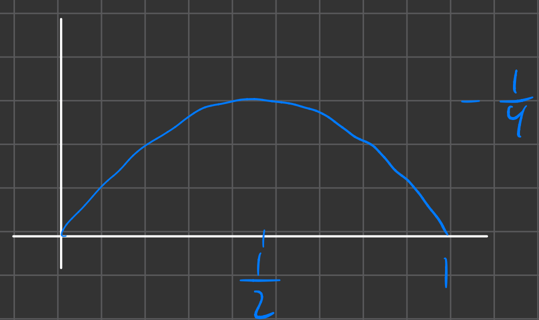
Prob $= g$ $\qquad$ Prob $= 1-g$

$$D^2 = (1-g)^2 g + (-g)^2(1-g)$$

$$= (1-g)^2 g + g^2(1-g) = g(1-g)[(1-g)+g] = g(1-g) \checkmark$$

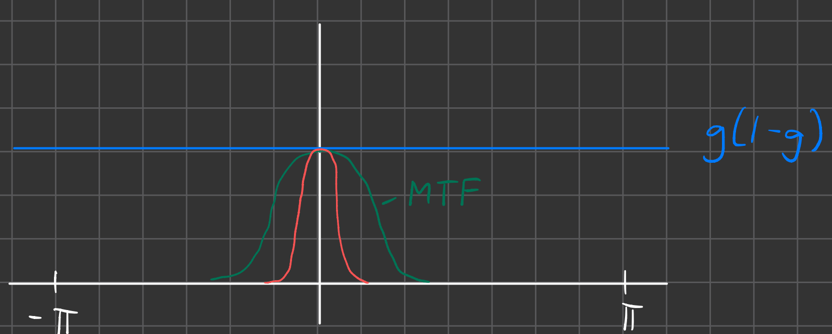
$$= (1-g)g$$

$$g(1-g) = E[D_n^2]$$



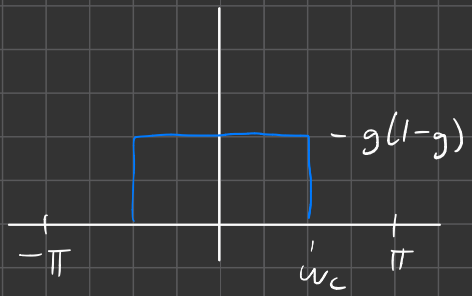
$$R_D(k) = E[D_n D_{n+k}]$$
$$= g(1-g)\delta_k$$

$$S_D(e^{j\omega}) = g(1-g)$$



error energy $= \int_{-\pi}^{\pi} |H(\omega)|^2 g(1-g)\, d\omega$

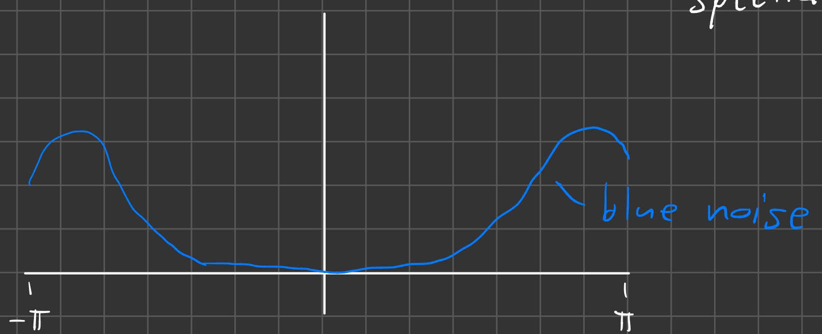
if $H(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$



error energy $= 2\omega_c \cdot g(1-g)$

Better Solution:

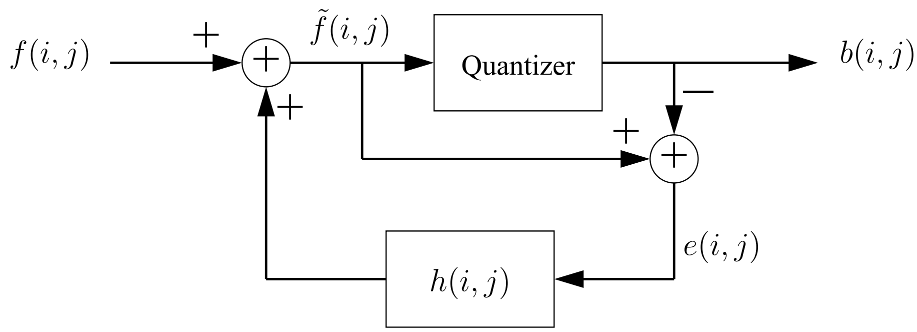
spectral shaping

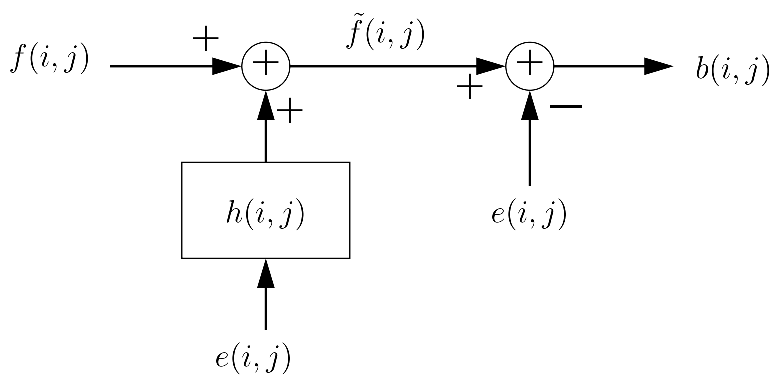


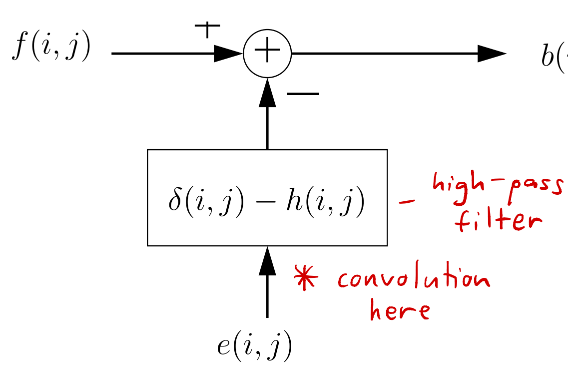
Goal: drive display error to higher spatial frequencies so their perception is less

Modified Error Diffusion Block Diagram

- The error diffusion block diagram can be rearranged to facilitate error analysis







Exam Worthy

Error Diffusion Spectral Analysis

- So we see that

$$b(i,j) = f(i,j) - (\delta(i,j) - h(i,j)) * e(i,j)$$

rewriting ...

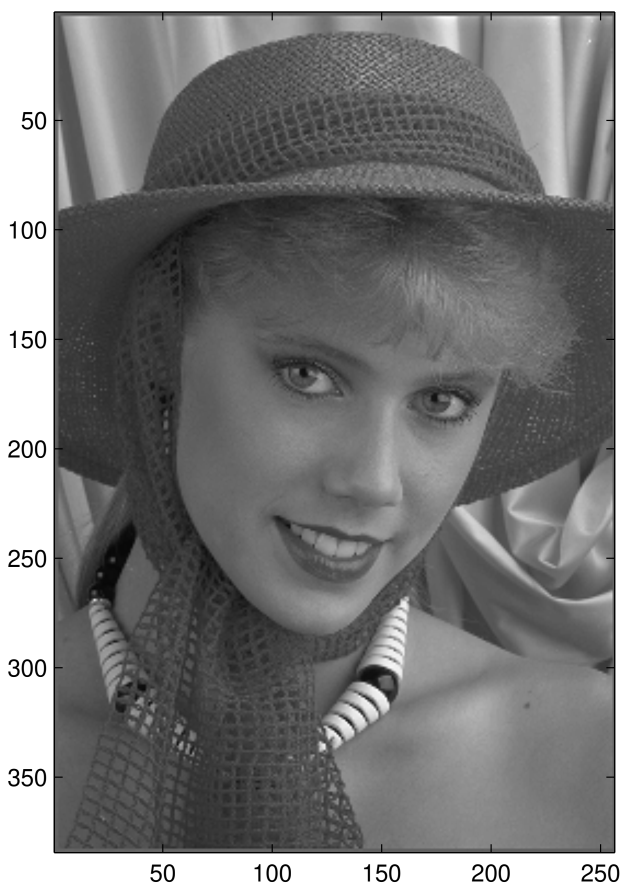
$$f(i,j) - b(i,j) = \underbrace{(\delta(i,j) - h(i,j))}_{\text{high pass filter}} * \underbrace{e(i,j)}_{\text{quantization error}}$$

  - Display error is $f(i,j) - b(i,j)$
  - Quantization error is $e(i,j)$
  - Display error is a high pass version of quantization error
  - Human visual system is less sensitive to high spatial frequencies

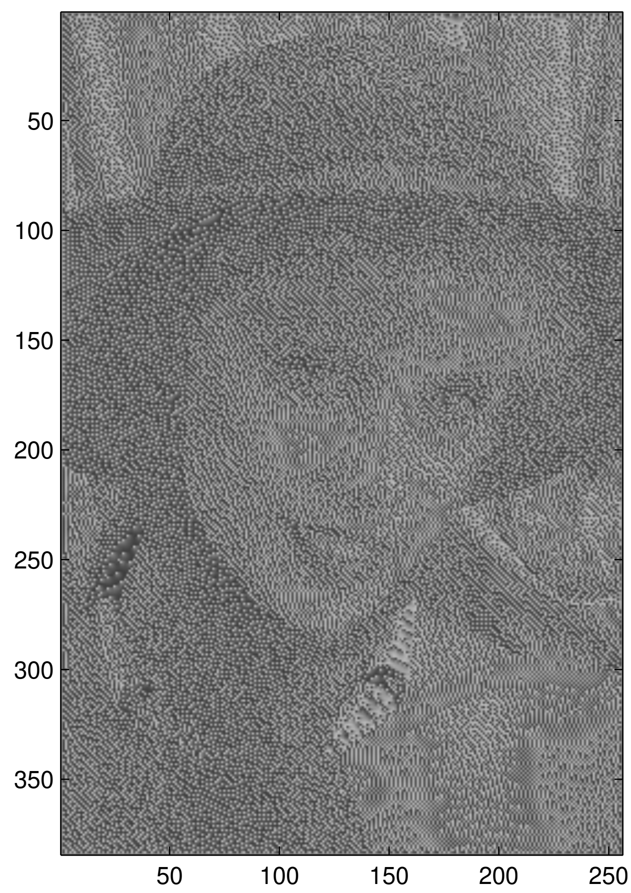
Error Image in Floyd Steinberg Error Diffusion

- Process pixels in neighborhoods by “diffusing error” and quantizing.

Original Image



Quantizer Error Image



Correlation of Quantization Error and Image

- Quantizer error spectrum is unknown
- Quantizer error model

$$
\boxed{\begin{aligned} E(\mu,\nu) &= \rho F(\mu,\nu) + R(\mu,\nu) \\ &= \rho(\text{Image}) + (\text{Residual}) \end{aligned}}
$$

  – ρ represents correlation between quantizer error and image

Review for Exam

Weight	ρ
1-D	0.0
Floyd and Steinberg	-0.55
Jarvis, Judice, and Ninke	- 0.8

high pass filter

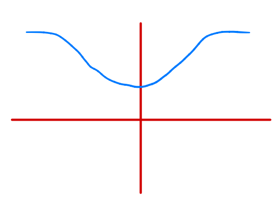
- Using this model, we have

$$
\boxed{\begin{aligned} B(\mu,\nu) &= F(\mu,\nu) - (1 - H(\mu,\nu))\, E(\mu,\nu) \\ &= [1 - \rho\,(1 - H(\mu,\nu))]\, F(\mu,\nu) + \text{noise} \end{aligned}}
$$

- This is unsharp masking

↳ result is a sharper image

looks like



DC ≠ 0

Additional Topics

- Pattern Printing
- Dot Profiles
- Halftone quality metrics
  - Radially averaged power spectrum (RAPS)
  - Weighted least squares with HVS constrast sensitivity function
  - Blue noise dot patterns
- Error diffusion
  - Unsharp masking effects
  - Serpentine scan patterns
  - Threshold dithering
  - TDED
- Least squared halftoning
- Printing and display technologies
  - Electrophotographic
  - Inkjet