

# Fact Sheet

- Function definitions

$$\text{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1 - |t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- CTFT

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

- CTFT Properties

$$x(-t) \stackrel{CTFT}{\longleftrightarrow} X(-f)$$

$$x(t - t_0) \stackrel{CTFT}{\longleftrightarrow} X(f) e^{-j2\pi f t_0}$$

$$x(at) \stackrel{CTFT}{\longleftrightarrow} \frac{1}{|a|} X(f/a)$$

$$X(t) \stackrel{CTFT}{\longleftrightarrow} x(-f)$$

$$x(t) e^{j2\pi f_0 t} \stackrel{CTFT}{\longleftrightarrow} X(f - f_0)$$

$$x(t)y(t) \stackrel{CTFT}{\longleftrightarrow} X(f) * Y(f)$$

$$x(t) * y(t) \stackrel{CTFT}{\longleftrightarrow} X(f)Y(f)$$

$$\int_{-\infty}^{\infty} x(t)y^*(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f)Y^*(f)df$$

- CTFT pairs

$$\text{sinc}(t) \stackrel{CTFT}{\longleftrightarrow} \text{rect}(f)$$

$$\text{rect}(t) \stackrel{CTFT}{\longleftrightarrow} \text{sinc}(f)$$

For  $a > 0$

$$\frac{1}{(n-1)!} t^{n-1} e^{-at} u(t) \stackrel{CTFT}{\longleftrightarrow} \frac{1}{(j2\pi f + a)^n}$$

- CSFT

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux+vy)} dx dy$$

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$

- DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

- DTFT pairs

$$a^n u(n) \stackrel{DTFT}{\longleftrightarrow} \frac{1}{1 - ae^{-j\omega}}$$

$$(n+1)a^n u(n) \stackrel{DTFT}{\longleftrightarrow} \frac{1}{(1 - ae^{-j\omega})^2}$$

- Rep and Comb relations

$$\text{rep}_T[x(t)] = \sum_{k=-\infty}^{\infty} x(t - kT)$$

$$\text{comb}_T[x(t)] = x(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

$$\text{comb}_T[x(t)] \stackrel{CTFT}{\longleftrightarrow} \frac{1}{T} \text{rep}_{\frac{1}{T}}[X(f)]$$

$$\text{rep}_T[x(t)] \stackrel{CTFT}{\longleftrightarrow} \frac{1}{T} \text{comb}_{\frac{1}{T}}[X(f)]$$

- Sampling and Reconstruction

$$y(n) = x(nT)$$

$$Y(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right)$$

$$s(t) = \sum_{k=-\infty}^{\infty} y(k) \delta(t - kT)$$

$$S(e^{j\omega}) = Y(e^{j\omega T})$$