The Fastest Indian - Anthony Hopkins

Magnetic Resonance Imaging (MRI)

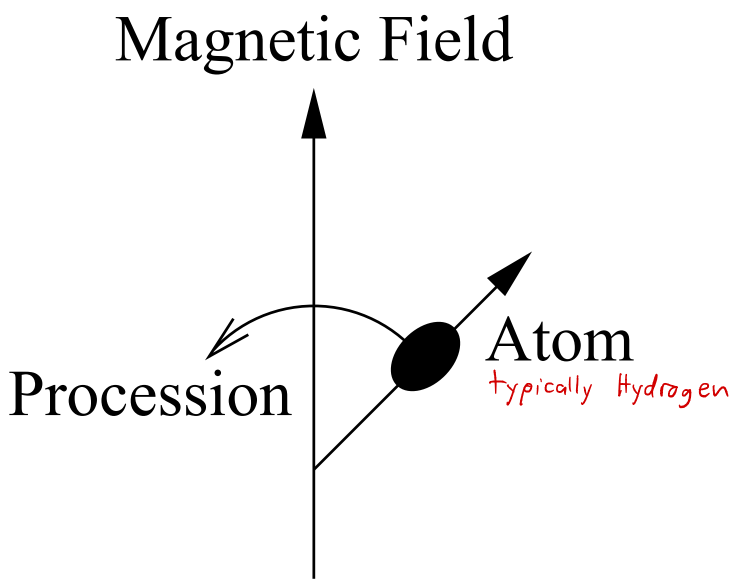
Underlying math is a lot different than in CT

- Can be very high resolution
- No radiation exposure
- Very flexible and programable
- Tends to be expensive, noisy, slow
- Good soft tissue contrast

MRI Attributes

- Based on magnetic resonance effect in atomic species
- Does not require any ionizing radiation
- Numerous modalities
  - Conventional anatomical scans
  - Functional MRI (fMRI) - performed when subject is doing some other task
  - MRI Tagging
- Image formation
  - RF excitation of magnetic resonance modes
  - Magnetic field gradients modulate resonance frequency
  - Reconstruction computed with inverse Fourier transform
  - Fully programmable
  - Requires an enormous (and very expensive) superconducting magnet

Magnetic Resonance



- Atom will precess at the Larmor frequency

$$\overset{[Hz]}{\omega_o} = \overset{[\frac{Hz}{T}]}{L}\overset{[T]}{M}$$

scalar: assuming constant direction

- Quantities of importance

M - magnitude of ambient magnetic field

1.5 T - typical
3.0 T - state of the art

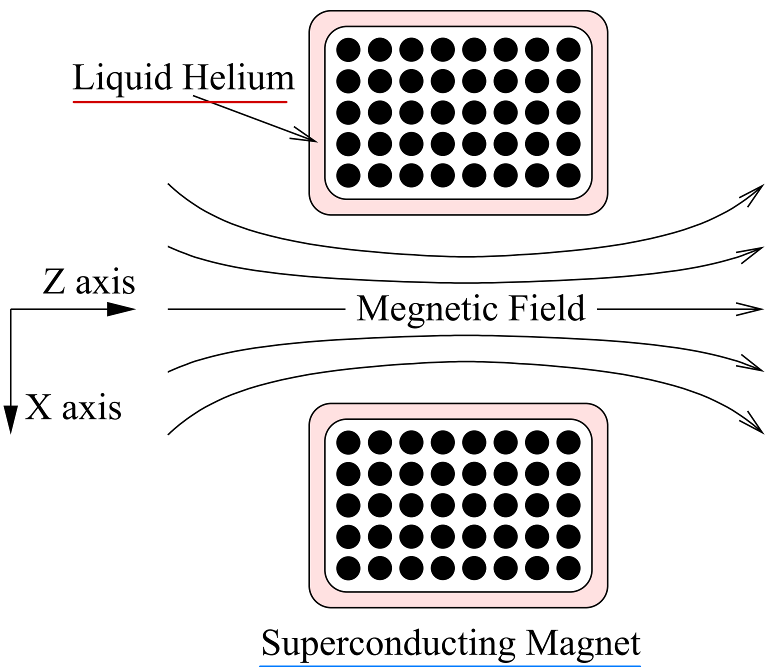
ω_o - frequency of procession (radians per second)

L - Larmor constant. Depends on choice of atom

characteristic to particular atomic species
42.58 MHz/T for proton

Magnetic shielding around entire room

The MRI Magnet



- Large super-conducting magnet
  - Uniform field within bore
  - Very large static magnetic field of M_o

Magnetic Field Gradients

= vector field

- Magnetic field **magnitude** at the location (x, y, z) has the form

static M-field from super-conducting magnet

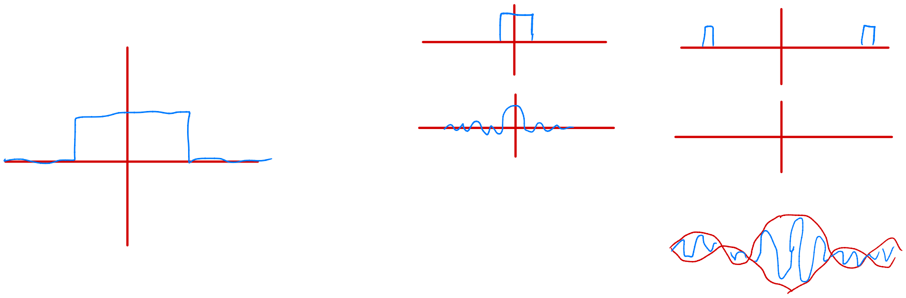
$$M(x, y, z) = M_o + xG_x + yG_y + zG_z$$

three distinct coils: x, y, z

  - G_x, G_y, and G_z control magnetic field gradients
  - Gradients can be changed with time
  - Gradients are small compared to M_o

- For time varying gradients

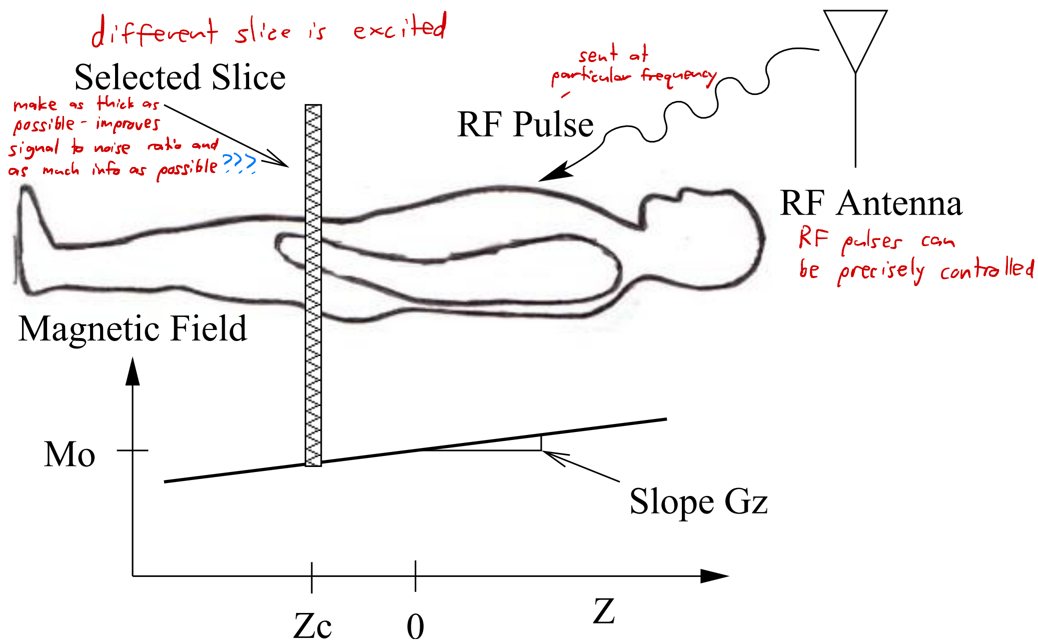
$$M(x, y, z, t) = M_o + xG_x(t) + yG_y(t) + zG_z(t)$$



convolution in frequency
modulation in time
sinc convolved with cos in freq

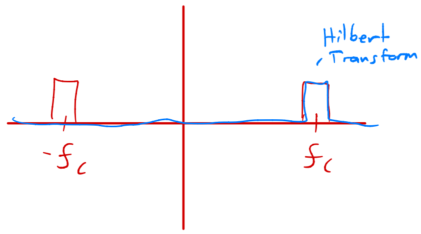
sinc modulated w/cos in space

MRI Slice Select



- Design RF pulse to excite protons in single slice
  - Turn off x and y gradients, i.e. $G_x = G_y = 0$.
  - Set z gradient to fix positive value, $G_z > 0$.
  - Use the fact that resonance frequency is given by

$$\omega = L\left(M_o + zG_z\right) \ .$$



$s(t) = e^{j2\pi f_o} \operatorname{sinc}(t\Delta f)$

$s(t) = A(t)\, e^{j w_o t}$

$= \underbrace{A(t)}_{\text{bandwidth (complex envelope)}}\, e^{j 2\pi \overbrace{f_o}^{\text{carrier frequency}} t}$

$A(t)$ usually small compared to f_c

$\frac{A(t)}{f_c}$ = fractional bandwidth

Slice Select Pulse Design

- Design parameters
  - Slice center = z_c.
  - Slice thickness = Δz.
- Slice centered at $z_c \Rightarrow$ pulse center frequency

$$f_c = \frac{LM_o}{2\pi} + \frac{z_c L G_z}{2\pi} = f_o + \frac{z_c L G_z}{2\pi} .$$

- Slice thickness $\Delta z \Rightarrow$ pulse bandwidth

$$\Delta f = \frac{\Delta z L G_z}{2\pi} .$$

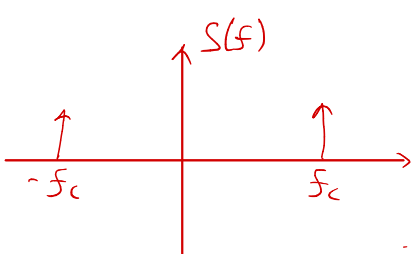
- Using these parameters, the pulse is given by

$$s(t) = e^{j2\pi f_c t} \operatorname{sinc}\left(t \underbrace{\Delta f}_{\text{bandwidth of the pulse}}\right)$$

and its CTFT is given by

$$S(f) = \operatorname{rect}\left(\frac{(f - f_c)}{\Delta f}\right)$$

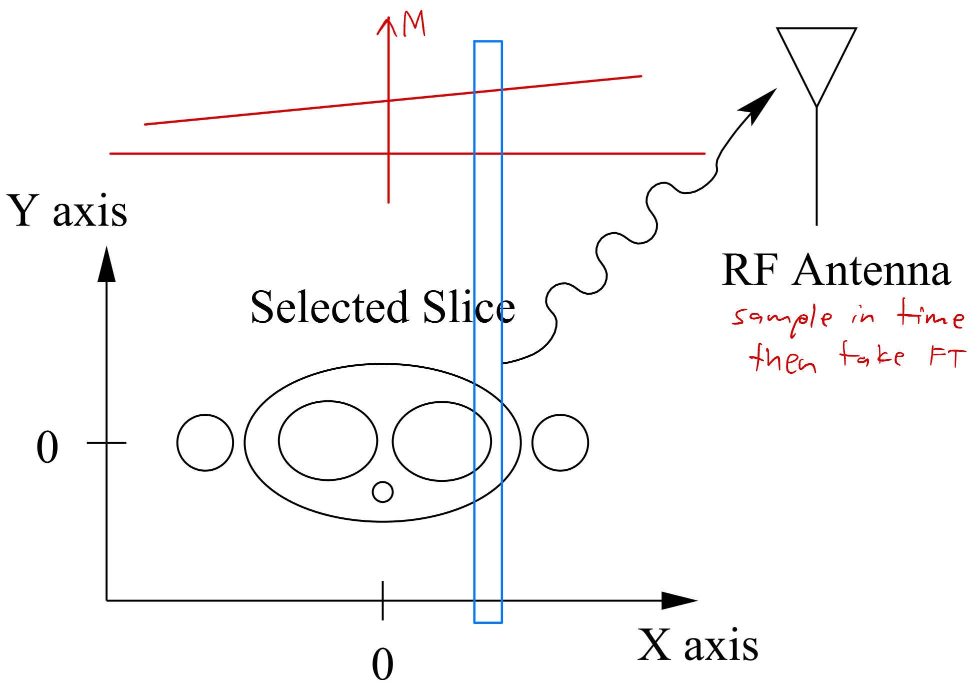
Recall Euler Identity



$s(t) = \cos(2\pi f_c t) = \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}$

Voxel - volumetric element

How Do We Imaging Selected Slice?

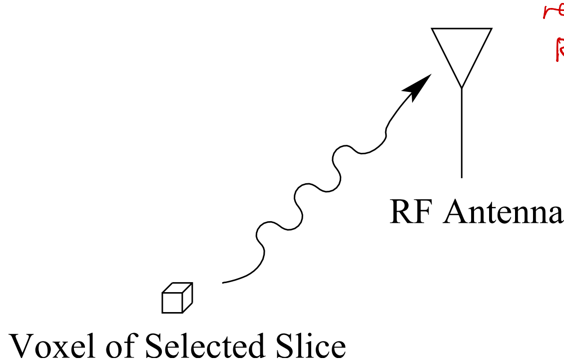


Sagital, coronal, axial

- Precessing atoms radiate electromagnetic energy at RF frequencies
- Strategy
  - Vary magnetic gradients along x and y axies, changes the strength
  - Measure received RF signal
  - Reconstruct image from RF measurements

impose gradient: different voxels oscillate at different frequencies
then, take FT to separate frequencies and voxels can be reconstructed

Signal from a Single Voxel



response to RF signal is linear

- RF signal from a single voxel has the form

phase of returned signal

$$r(x, y, t) = f(x, y)e^{j\phi(t)}$$

$\phi(t) = 2\pi f_c t$

$\frac{d\phi}{dt} = 2\pi f_c$

time derivative of phase is the frequency

$f(x, y)$ voxel dependent weighting

- Depends on properties of material in voxel
- Quantity of interest
- Typically "weighted" by T1, T2, or T2*

$\phi(t)$ phase of received signal

- Can be modulated using G_x and G_y magnetic field gradients
- We assume that $\phi(0) = 0$

Analysis of Phase

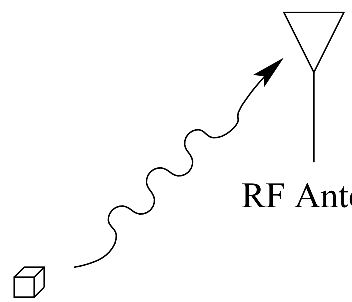
- Frequency = time derivative of phase

$$
\begin{aligned}
\frac{d\phi(t)}{dt} &= L\,M(x,y,t) \\
\phi(t) &= \int_0^t L\,M(x,y,\tau)d\tau \\
&= \int_0^t LM_o + xLG_x(\tau) + yLG_y(\tau)d\tau \\
&= \underbrace{\omega_o}_{\text{carrier frequency}} t + xk_x(t) + yk_y(t)
\end{aligned}
$$

where we define

$$
\begin{aligned}
\omega_o &= L\,M_o \\
k_x(t) &= \int_0^t LG_x(\tau)d\tau \\
k_y(t) &= \int_0^t LG_y(\tau)d\tau
\end{aligned}
$$

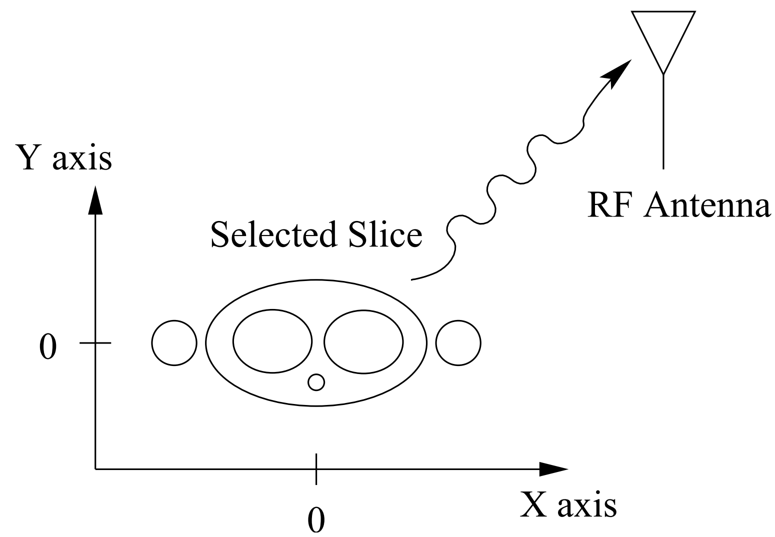
Received Signal from Voxel



- RF signal from a single voxel has the form

$$
\begin{aligned}
r(t) &= f(x,y)e^{j\phi(t)} \\
&= f(x,y)e^{j\left(\omega_o t + xk_x(t) + yk_y(t)\right)} \\
&= f(x,y)\underbrace{e^{j\omega_o t}}_{\text{carrier frequency}} e^{j\left(xk_x(t) + yk_y(t)\right)}
\end{aligned}
$$

Received Signal from Selected Slice



- RF signal from the complete slice is given by

$$
\begin{aligned}
R(t) &= \int_{\mathbb{R}}\int_{\mathbb{R}} r(x,y,t)\,dx\,dy \\
&= \int_{\mathbb{R}}\int_{\mathbb{R}} f(x,y)e^{j\omega_o t}e^{j\left(xk_x(t)+yk_y(t)\right)}dx\,dy \\
&= e^{j\omega_o t}\int_{\mathbb{R}}\int_{\mathbb{R}} f(x,y)e^{j\left(x\overbrace{k_x(t)}^{-u}+y\overbrace{k_y(t)}^{-v}\right)}dx\,dy \\
&= \underset{\text{carrier frequency}}{e^{j\omega_o t}}\,\underset{\text{complex envelope}}{F(-k_x(t),-k_y(t))}
\end{aligned}
$$

were $F(u,v)$ is the CSFT of $f(x,y)$

K-Space Interpretation of Demodulated Signal

- RF signal from the complete slice is given by

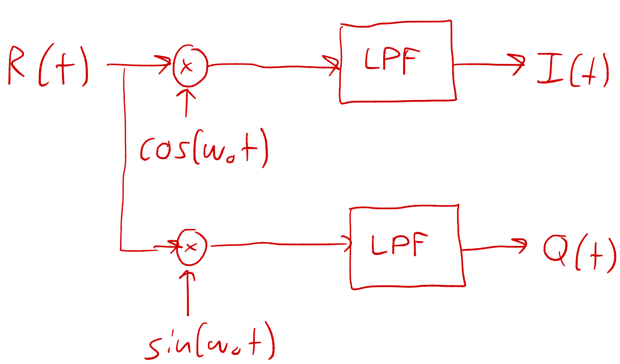
$$F\underbrace{(-k_x(t), -k_y(t))}_{\text{spatial frequencies}} = \underbrace{R(t)}_{\text{received signal}}e^{-j\omega_o t}$$

where

$$k_x(t) = \int_0^t LG_x(\tau)d\tau$$

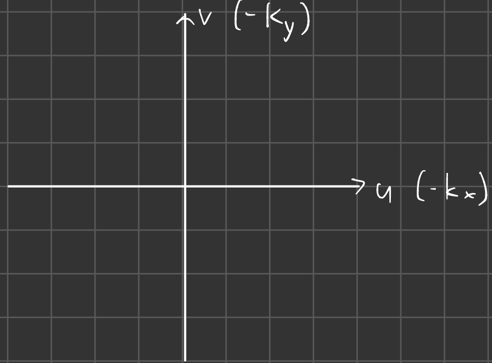
$$k_y(t) = \int_0^t LG_y(\tau)d\tau$$

- Strategy
  - Scan spatial frequencies by varying $k_x(t)$ and $k_y(t)$
  - Reconstruct image by performing (inverse) CSFT
  - $G_x(t)$ and $G_y(t)$ control velocity through K-space



$R(t) = A(t)e^{j\omega t}$

$A(t) = I(t) + jQ(t)$



- move around
- sample
- get complex signal value at position
- k values are double integrals of applied voltage

Controlling K-Space Trajectory

This is how we change $k_x(t)$, $k_y(t)$

- Relationship between gradient coil voltage and K-space

$$L_x \frac{di(t)}{dt} = v_x(t) \quad G_x(t) = M_x i(t)$$

$$L_y \frac{di(t)}{dt} = v_y(t) \quad G_y(t) = M_y i(t)$$

using this results in

$$k_x(t) = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s) ds d\tau$$

position in frequency space

"acceleration" in frequency space

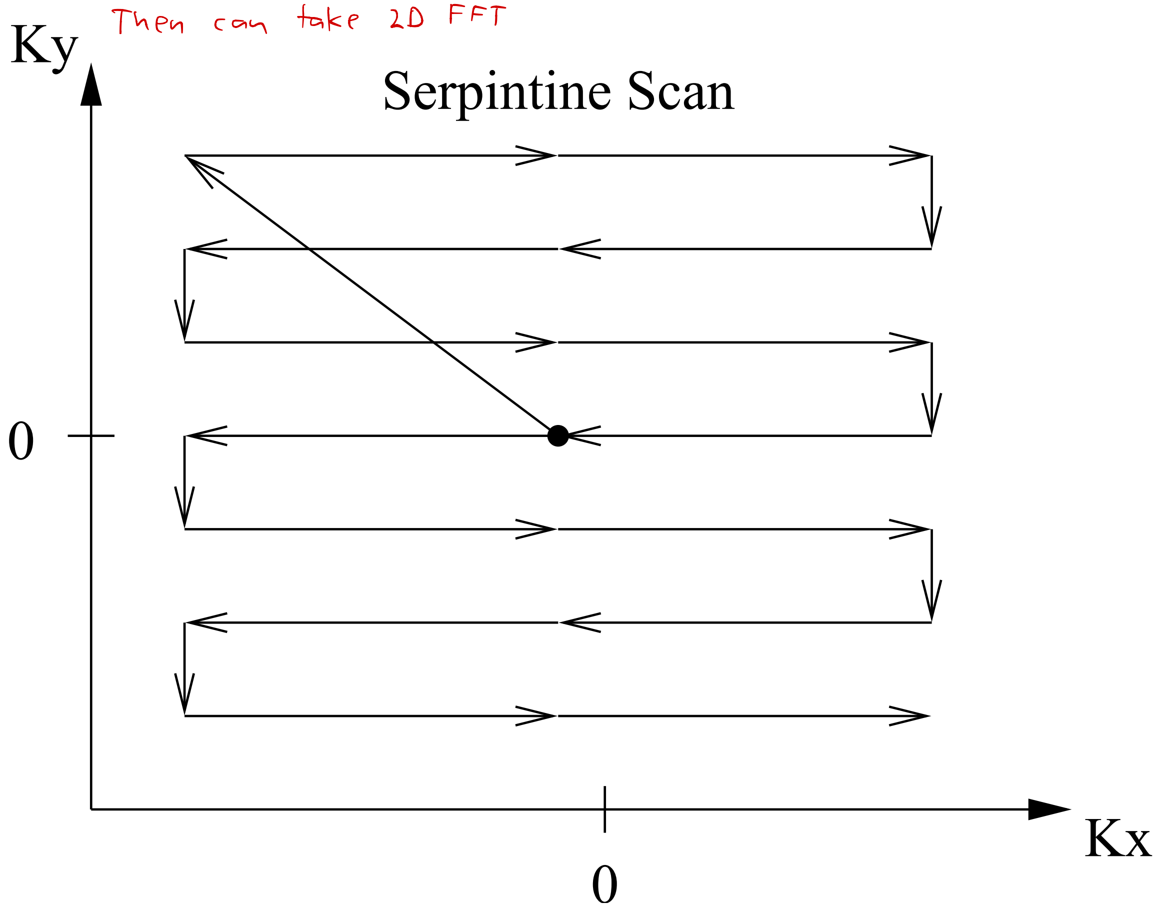
$$k_y(t) = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s) ds d\tau$$

- $v_x(t)$ and $v_y(t)$ are like the accelerator peddles for $k_x(t)$ and $k_y(t)$

Echo Planer Imaging (EPI) Scan Pattern

- A commonly used raster scan pattern through K-space

Then can take 2D FFT



$$k_x(t) = L\int_0^t G_x(\tau)d\tau = \frac{LM_x}{L_x}\int_0^t\int_0^\tau v_x(s)dsd\tau$$

$$k_y(t) = L\int_0^t G_y(\tau)d\tau = \frac{LM_y}{L_y}\int_0^t\int_0^\tau v_y(s)dsd\tau$$

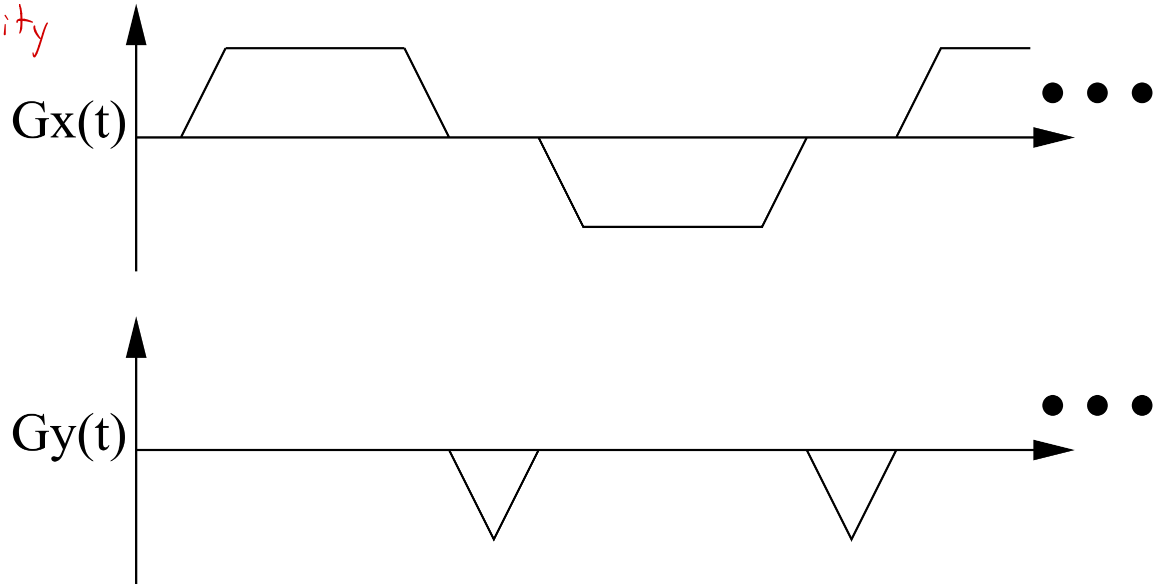
k position adjusted with voltage, or "acceleration"

Gradient Waveforms for EPI

- Gradient waveforms in x and y look like

Velocity

$\frac{dG_x(t)}{dt}$:



- Voltage waveforms in x and y look like

Acceleration