

3) $x(n) = f(nT)$

$$3.1) \left. \frac{df}{dt} \right|_{t=n-\frac{1}{2}} = \frac{x(n) - x(n-1)}{n - (n-1)} = x(n) - x(n-1) = x(n) * h(n)$$

$$\rightarrow h(n) = \delta(n) - \delta(n-1)$$

$$3.2) \left. \frac{df}{dt} \right|_{t=n+\frac{1}{2}} = \frac{x(n+1) - x(n)}{n+1 - n}$$

$$\left. \frac{d^2f}{dt^2} \right|_{t=n} = \frac{x(n+1) - x(n) - (x(n) - x(n-1))}{(n+1 - n) - (n - (n-1))}$$

$$= x(n+1) - 2x(n) + x(n-1) = g(n) * x(n)$$

$$\rightarrow g(n) = \delta(n+1) - 2\delta(n) + \delta(n-1)$$

3.3) First, check to see if the first derivative is larger than some threshold T and that the second derivative changes sign

$$\text{Let } d_1(n) = x(n+1) - x(n) \\ d_2(n) = x(n+1) - 2x(n) + x(n-1)$$

$$B = \{|d_1(n)| \geq T\} \text{ and } \{d_2(n)d_2(n-1) \leq 0\} \text{ or } \{d_2(n+1)d_2(n) \leq 0\}$$

where boolean $B=1$ indicates an edge