Open questions

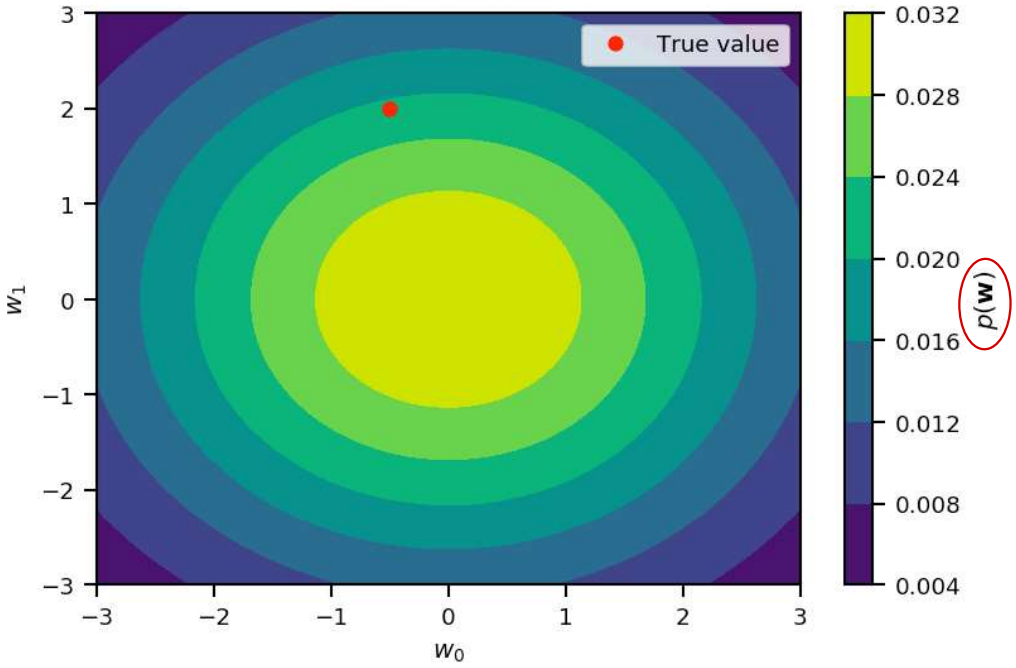
- How do I quantify the measurement noise? ✓
- How do we avoid overfitting? ✓
- How do I quantify epistemic uncertainty induced by limited data?
- How do I choose any remaining parameters?
- How do I choose which basis functions to keep?

Weight prior (linear regression)

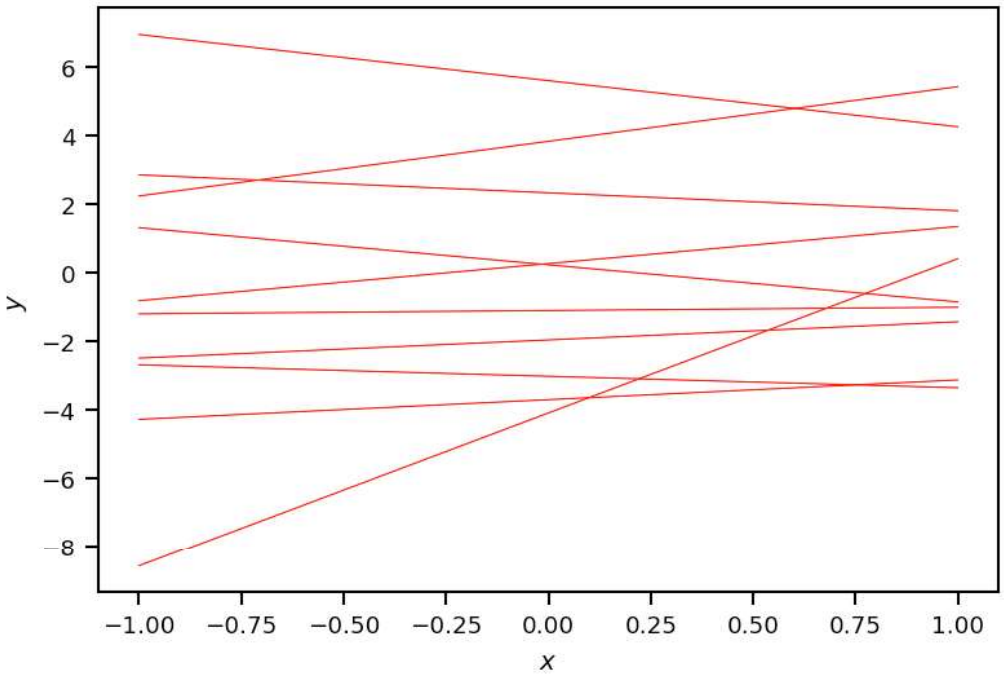
sample & plot

Prior

Weight space



Model space



PREDICTIVE
SCIENCE LABORATORY

Weight posterior

posterior $\propto$ likelihood $\times$ prior

$$p(\underline{w} \mid \underline{x}_{1:N}, \underline{y}_{1:N}, \sigma) \propto p(\underline{y}_{1:N} \mid \underline{x}_{1:N}, \sigma, \underline{w}) \cdot p(\underline{w})$$

$$= N(\underline{y}_{1:N} \mid \underline{\underline{\Phi}}\, \underline{w}, \sigma^2 \underline{\underline{I}}_N) \times N(\underline{w} \mid \underline{0}, a^{-1} \underline{\underline{I}}_m)$$

identity; 0-mean; precision

$$\underline{\underline{\Phi}} = \begin{pmatrix} \underline{\varphi}^T(x_1) \\ \vdots \\ \underline{\varphi}^T(x_N) \end{pmatrix} \underline{w}$$

prediction at each observation

$$= N(\underline{w} \mid \underline{m}, \underline{\underline{S}})$$

$$\begin{cases} \underline{\underline{S}} = (\sigma^{-2} \underline{\underline{\Phi}}^T \underline{\underline{\Phi}} + a \underline{\underline{I}}_m)^{-1} \\ \underline{m} = \sigma^{-2} \underline{\underline{S}} \cdot \underline{\underline{\Phi}}^T \underline{y} \end{cases}$$

Gaussian $\times$ Gaussian = Gaussian

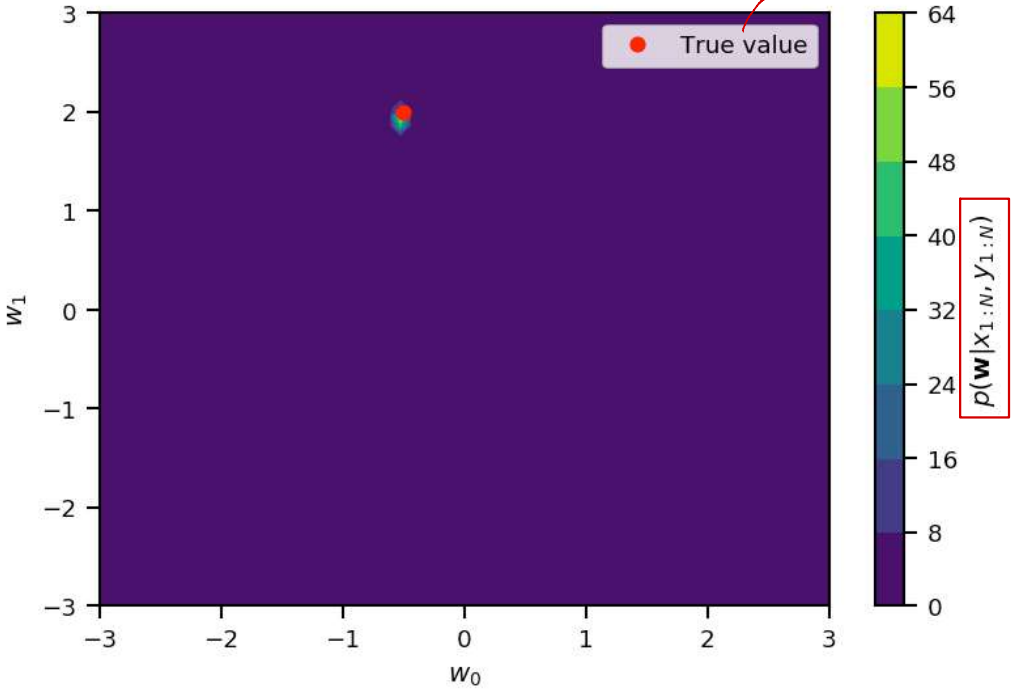
Weight posterior (linear regression)

sample & plot

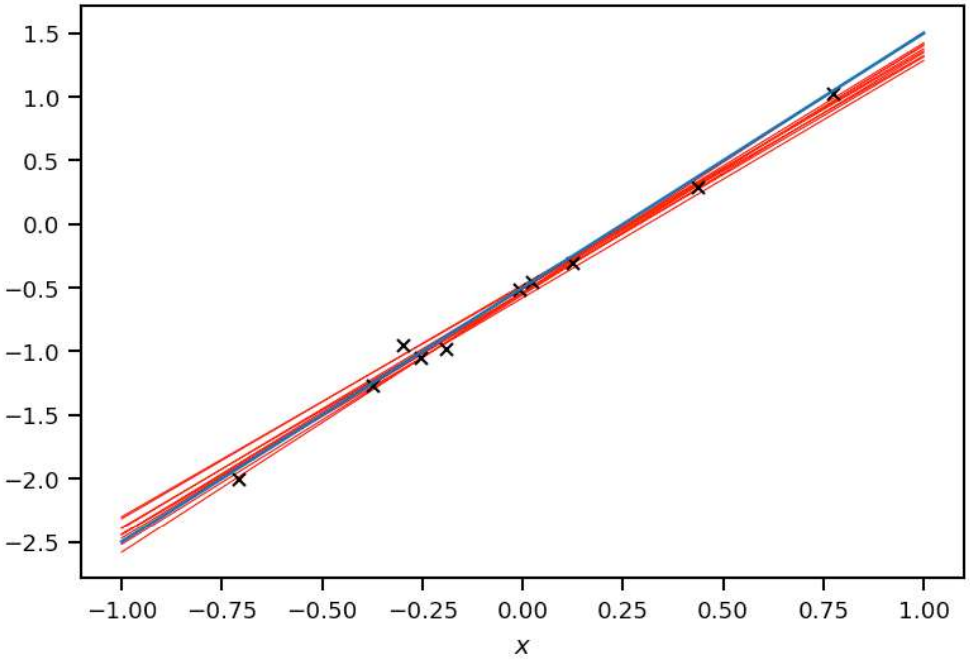
Posterior

Weight space

we know since the data is synthetic

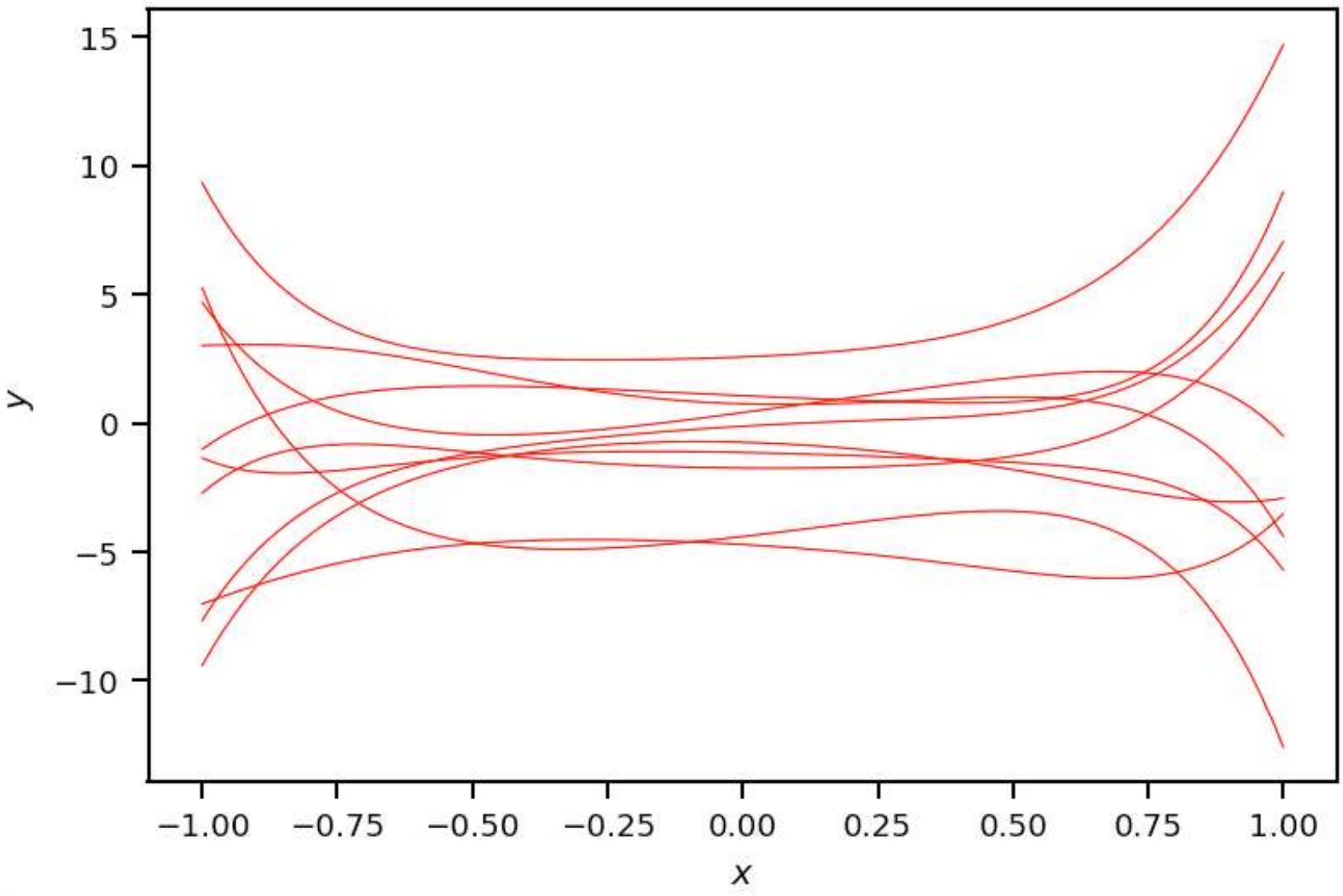


Model space



Example: 7th degree polynomial (prior)

sample 8 weights and plot



Example: 7th degree polynomial (posterior)