# Lecture 12: Analytical examples of Bayesian inference

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## Predictive checking

# Replicating the experiment using the model

- Assume that we have built model using data $x_{1:n}$.

$p(\theta | x_{1:n})$

- What would get it we ran the experiment again?

- The *replicated data* $x^{rep}_{1:n}$ are given by the following process:

sample from the posterior.

$$\theta_s | x_{1:n} \sim p(\theta | x_{1:n})$$

$$x^{rep}_{1:n,s} | \theta = \theta_s \sim p(x^{rep}_{1:n} | \theta = \theta_s)$$

sample a dataset from this sampled $\theta_s$ from the likelihood

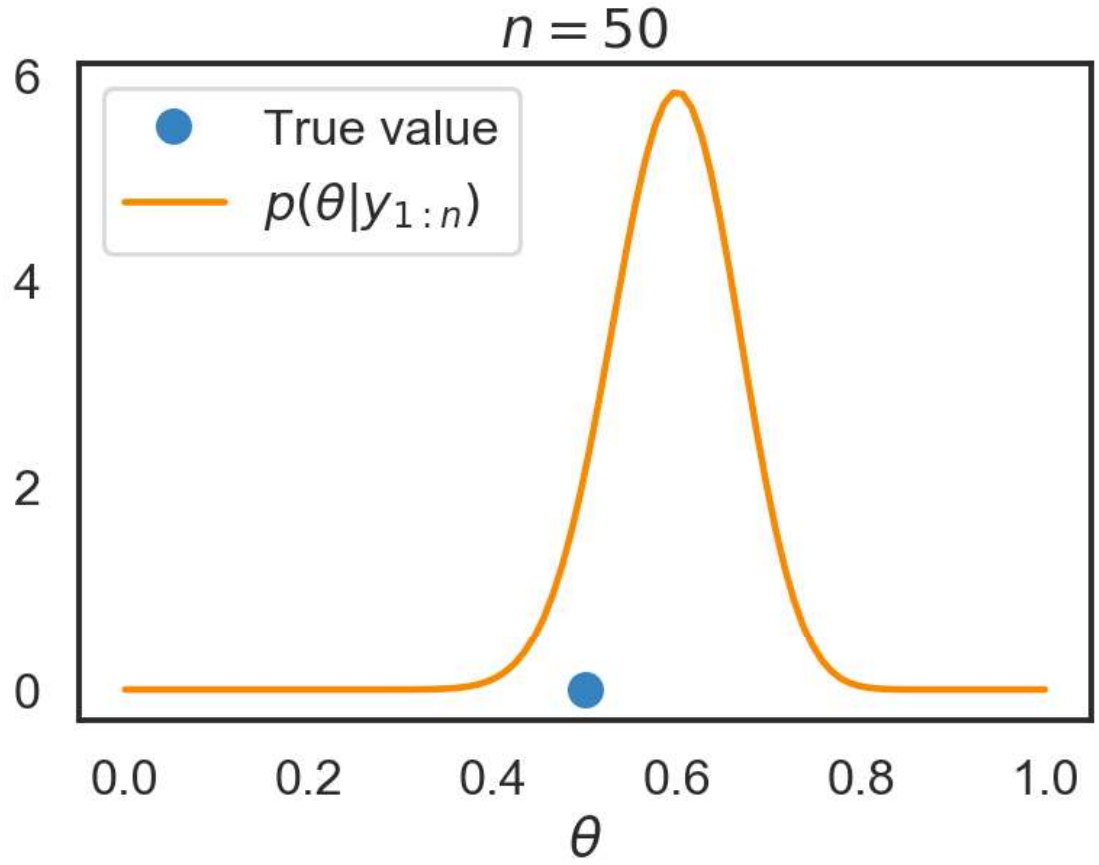
# Posterior Predictive Checking

The idea is to sample $x^{\text{rep}}_{1:n}$ and compare their characteristics to the observed data $x_{1:n}$.

# Example: Coin toss case studies

- Case study 1: I simply generate 50 coin tosses from a fair coin using Numpy.
- Case study 2: I just picked 50 coin tosses by hand trying to be as fair as possible.

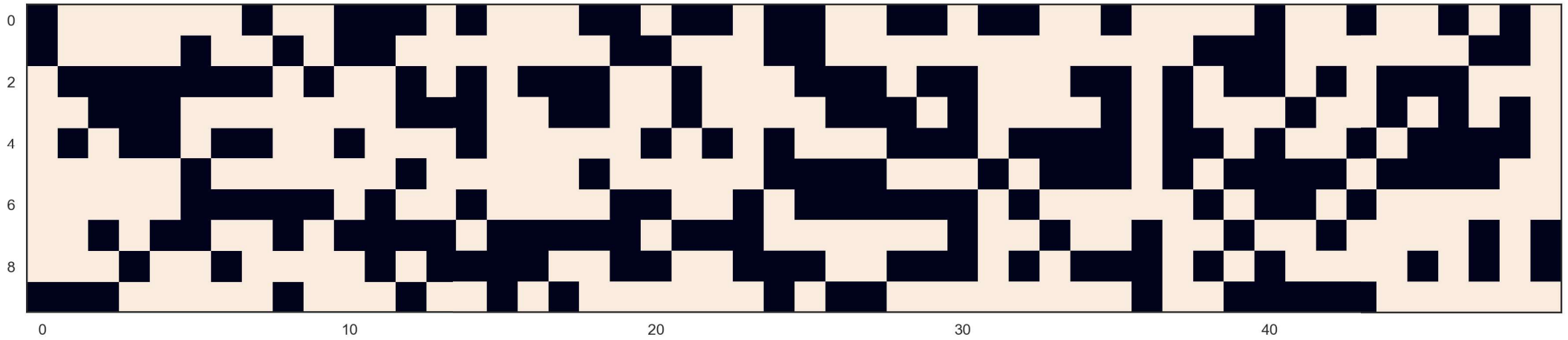
# Example: Inferring the probability of a coin toss




**With data from fair coin.**

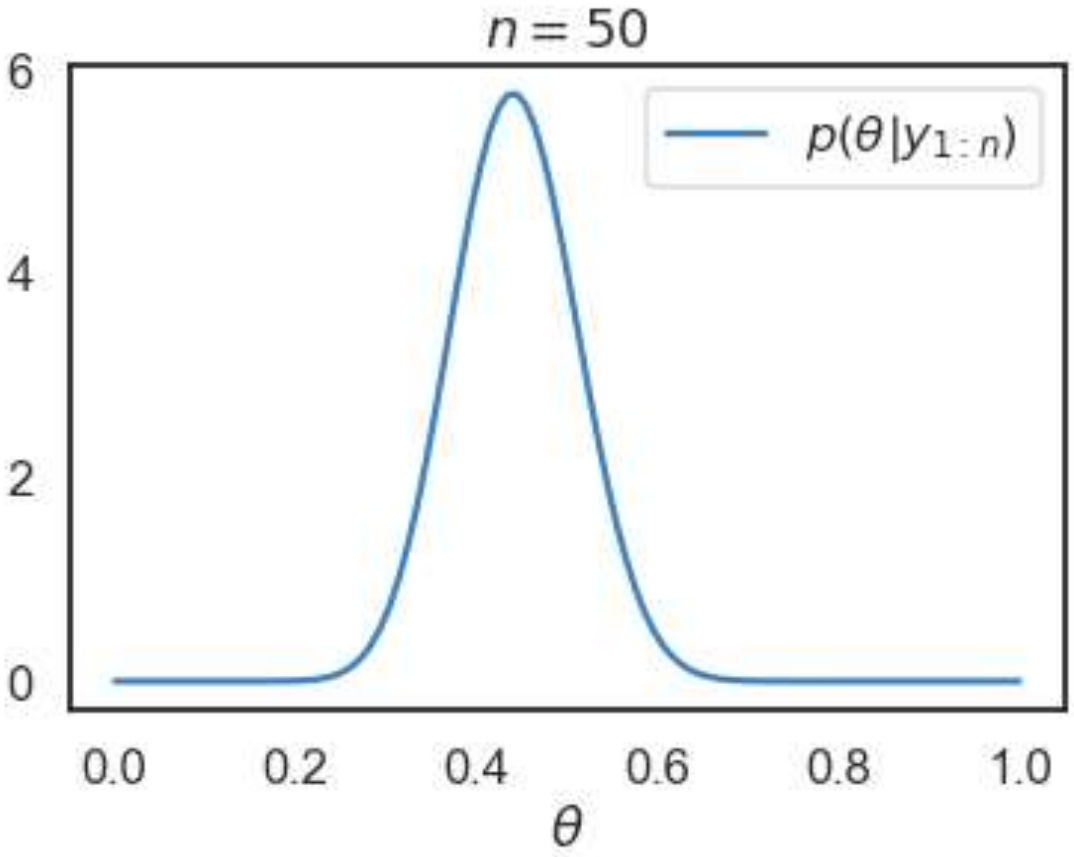
# Posterior Predictive Checking

First row is the original data set
Black - heads
White - tails




**With data from fair coin.**

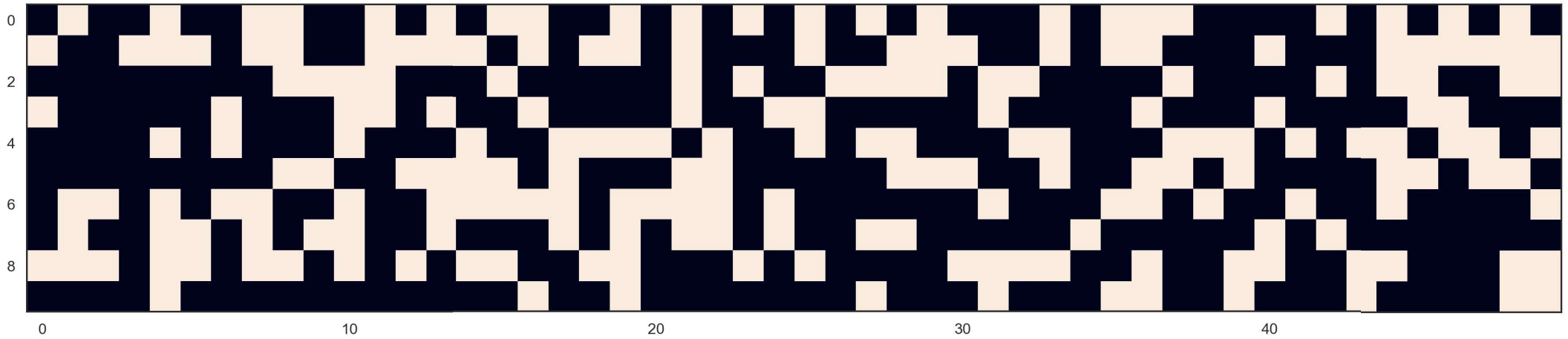
# Example: Inferring the probability of a coin toss




**With made-up data.**

# Posterior Predictive Checking

First row was generated by hand
↳ has more switches between heads & tails
- replicated data has longer sequences




**With made-up data.**