Lecture 19: State-space models - Filtering

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Basics of Markov models

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The Markov property

Discrete dynamical system

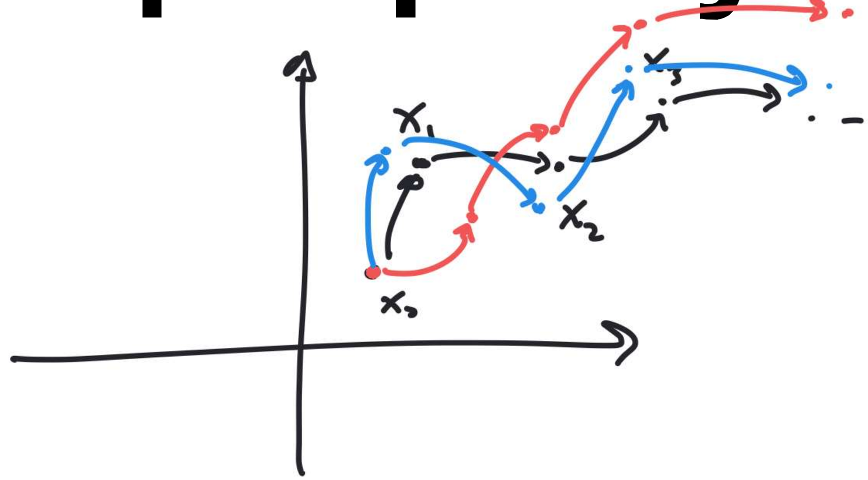
Time: $n = 0, 1, 2, \ldots$

State: $x_n \in \mathbb{R}^d$

Trajectory:

$x_{0:n} = (x_0, x_1, \ldots, x_n)$

$p(x_{0:n}) = ?$



stochastic dynamical systems

$$p(x_{n+1} | x_{0:n}) = p(x_{n+1} | x_n)$$

Markov Property - will hold if x is the complete state of a physical system

$X_0, X_1, X_2, \ldots$ Markov chain

$$X_0 \to X_1 \to X_2 \to X_3 \to \ldots$$

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The joint distribution of a Markov model

$$p(x_{0:n}) = ?$$

$$X_0 \to X_1 \to X_2 \to \cdots$$

$$p(x_0) = \ldots \quad (\text{known})$$

$$p(x_{0:1}) \overset{\text{product rule}}{=} \underbrace{p(x_1 \mid x_0)}_{\text{transition probability}} \underbrace{p(x_0)}_{\text{known}}$$

$$p(x_{0:2}) = p(x_2 \mid x_{0:1}) \underbrace{p(x_{0:1})}_{\text{known}}$$

$\downarrow$ Markov

condition on trajectories up to $n-1$

$$= p(x_2 \mid x_1) \cdot p(x_1 \mid x_0)\, p(x_0)$$

$$p(x_{0:n}) = p(x_0)\, p(x_1 \mid x_0) \cdots p(x_n \mid x_{n-1})$$

$$= p(x_0) \cdot \prod_{t=1}^{n} p(x_t \mid x_{t-1})$$

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joint distribution decomposes into transition probabilities