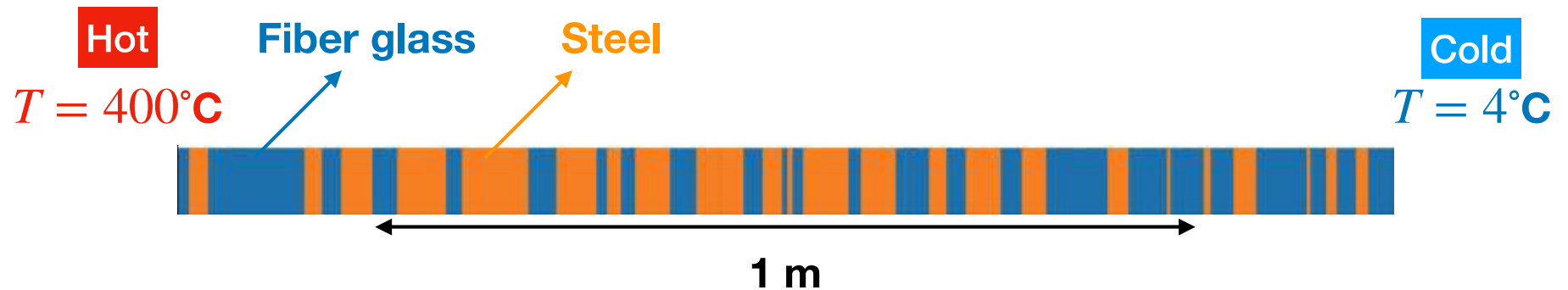


Lecture 10: Quantifying uncertainties in Monte Carlo estimates

Professor Ilias Bilonis

Uncertainty Propagation Through a Boundary Value Problem

The boundary value problem



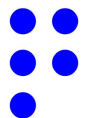
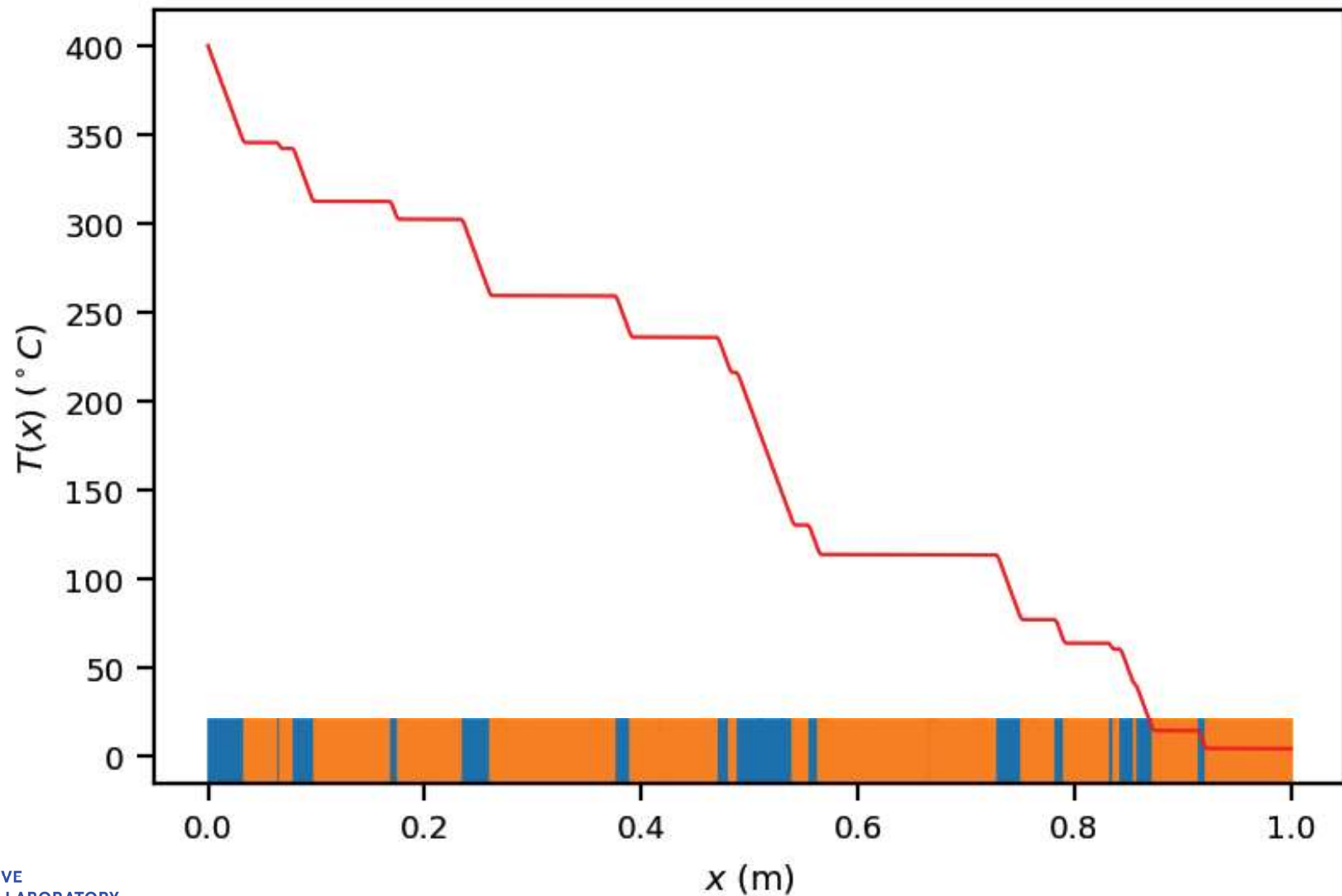
Temperature given by the steady state heat equation:

$$\frac{d}{dx} \left(c(x) \frac{d}{dx} T(x) \right) = 0$$

with boundary conditions:

$$T(0) = 400^{\circ}\text{C} \text{ and } T(1 \text{ m}) = 4^{\circ}\text{C} .$$

Solving the boundary value problem for a specific rod



What do we know about the rod?

- The rod is one meter long.
- The rod is from laminations of two different materials: fiberglass (labeled material 0) and steel (labeled material 1).
- The concentration of the fiberglass is 0.3 and the concentration of steel 0.7.
- The thermal conductivity of the fiberglass $0.045 \text{ Wm}^{-1}\text{K}^{-1}$ and of the steel $38 \text{ Wm}^{-1}\text{K}^{-1}$.
- The rod is made out of D segments and each one of these segments consists of only one of the two types of material. Even though we do not know the exact number of segments, we expect it to be around 100.

Quantifying our uncertainty about the rod

D

- The number of segments is:

$$D \sim \text{Poisson}(100)$$

- The $D + 1$ segment coordinates are:



$$(x_1, \dots, x_{D-1}) = \text{sort}(u_1, \dots, u_{D-1})$$

$$u_i \sim U([0, 1])$$

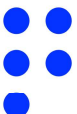
- Finally, the material type on each segment is:

$$M_i \sim \text{Categorical}(0.3, 0.7)$$

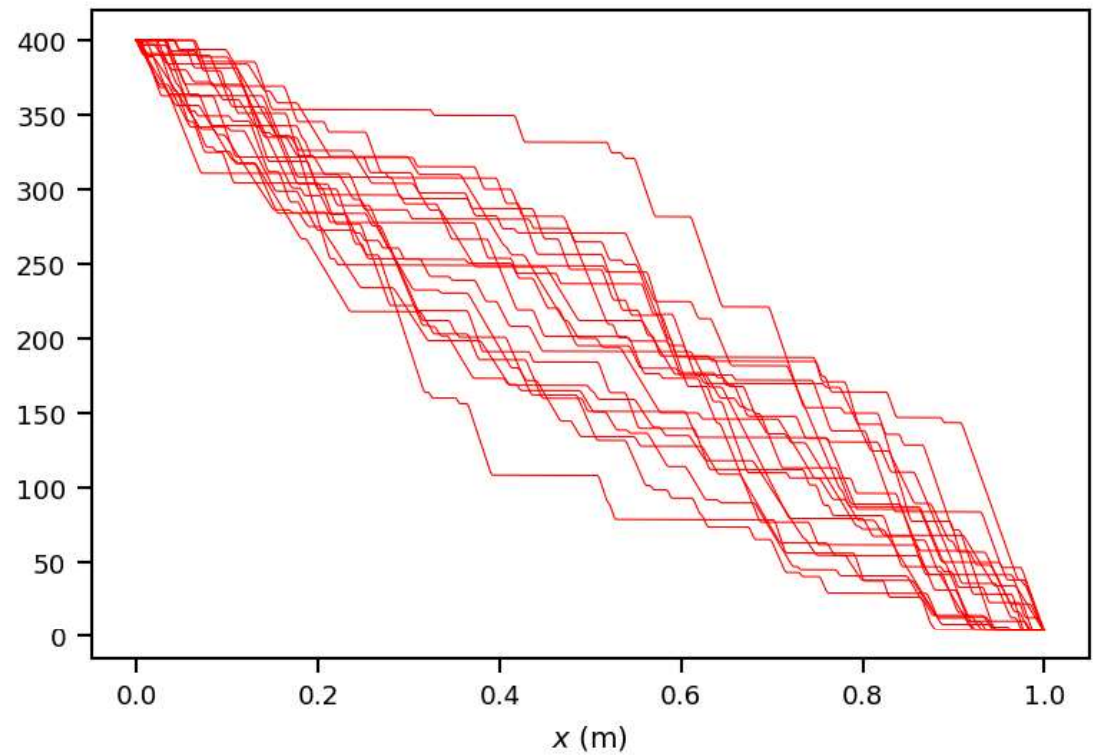
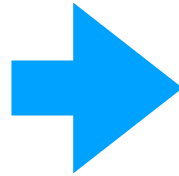
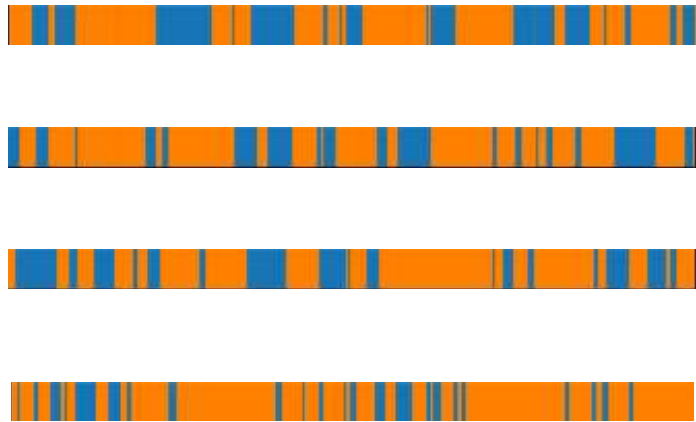
C_0 : the con. of f.g.
 C_1 : the con. of steel

- The thermal conductivity is a random function:

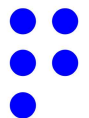
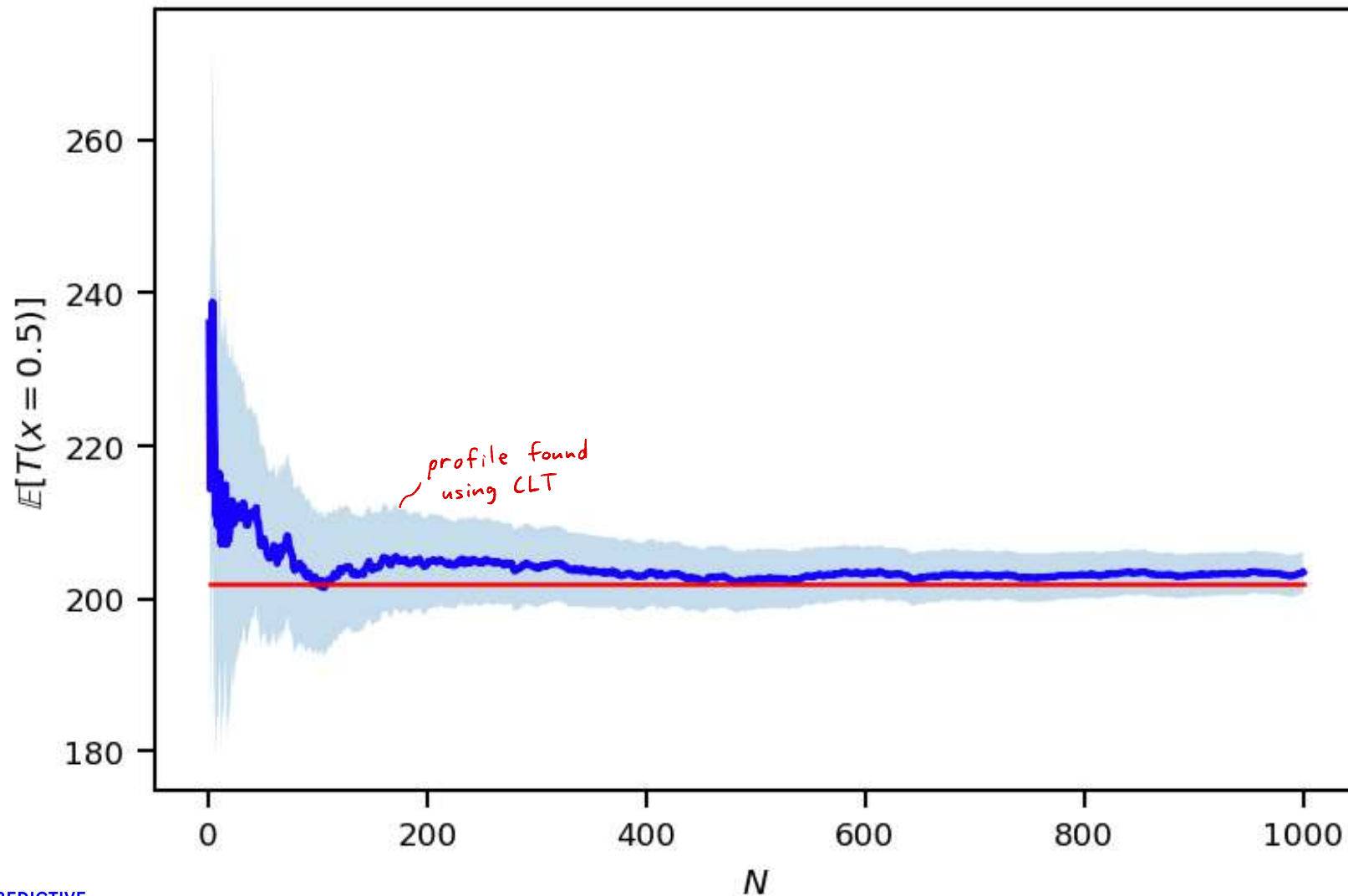
$$c(x) = \sum_{i=1}^D \mathbb{1}_{[x_{i-1}, x_i]}(x) \cdot c_{M_i}$$



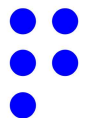
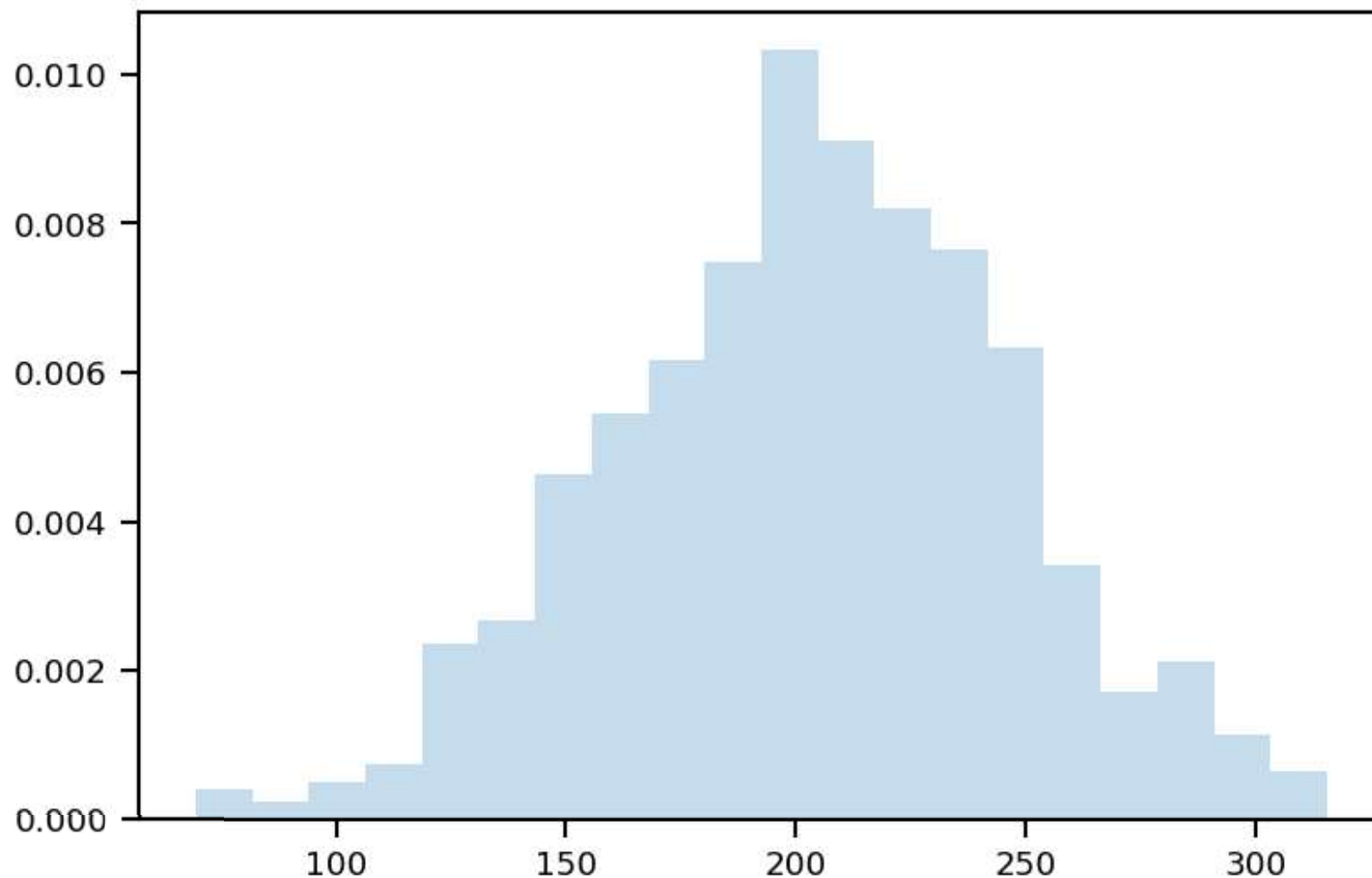
Now we can sample random rods!



Estimating the expected temperature at the center



Estimating the probability density of the temperature at the center



95% Predictive quantiles

