Lecture 8: The Monte Carlo method for estimating expectations

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The curse of dimensionality

PREDICTIVE
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The curse of dimensionality

$\underline{X} = (X_1, \dots, X_d)$, $X_i \sim U([0,1])$ independent

$p(\underline{x}) = \prod_{i=1}^{d} p(x_i) = \prod_{i=1}^{d} 1_{[0,1]}(x_i) = 1_{[0,1]^d}(\underline{x})$

sort of like inner product?

- Take the d-dimensional uniform: $X \sim U([0,1]^d)$.

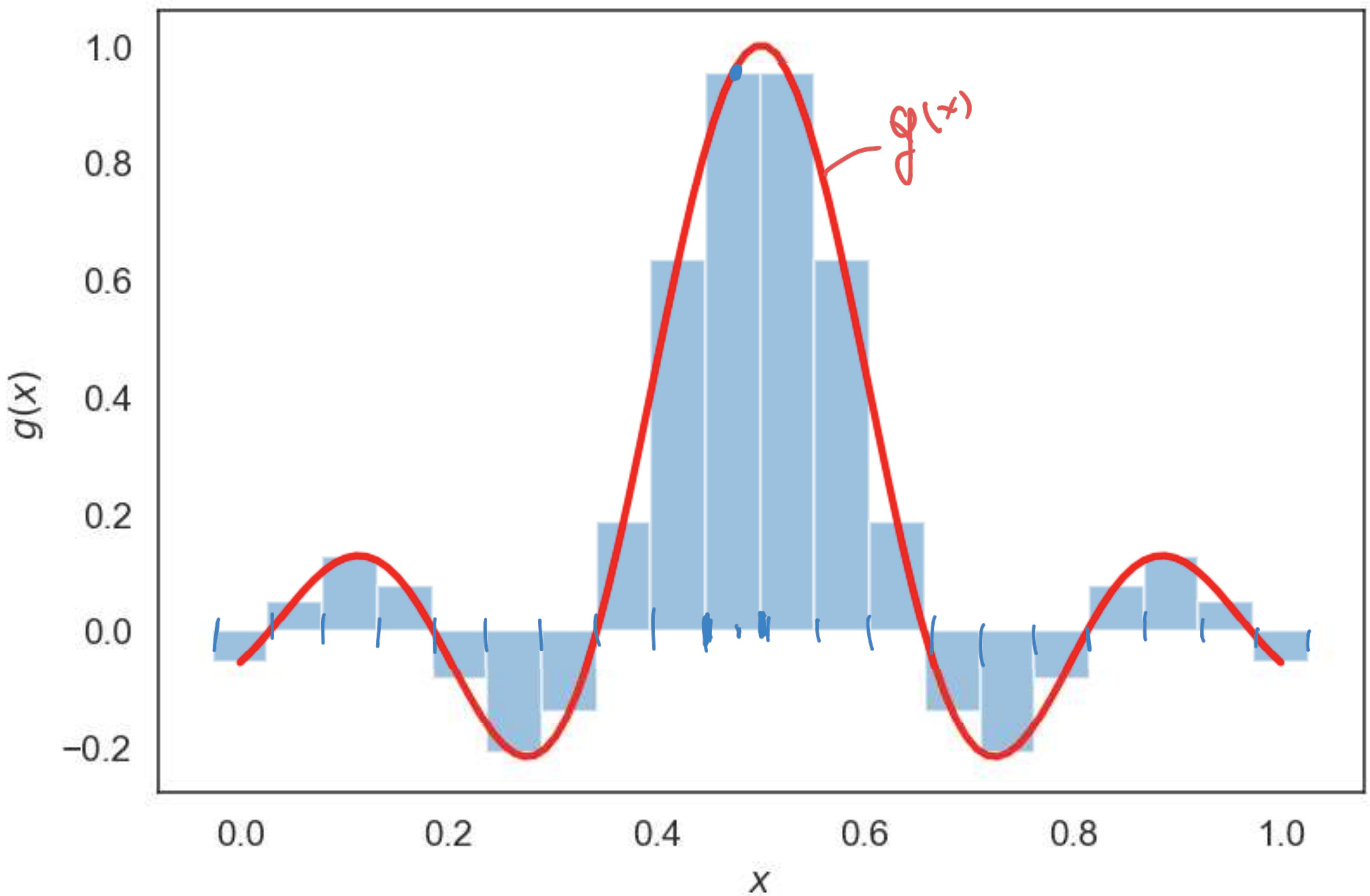
- Take a function $g(x)$.

- We would like to estimate:

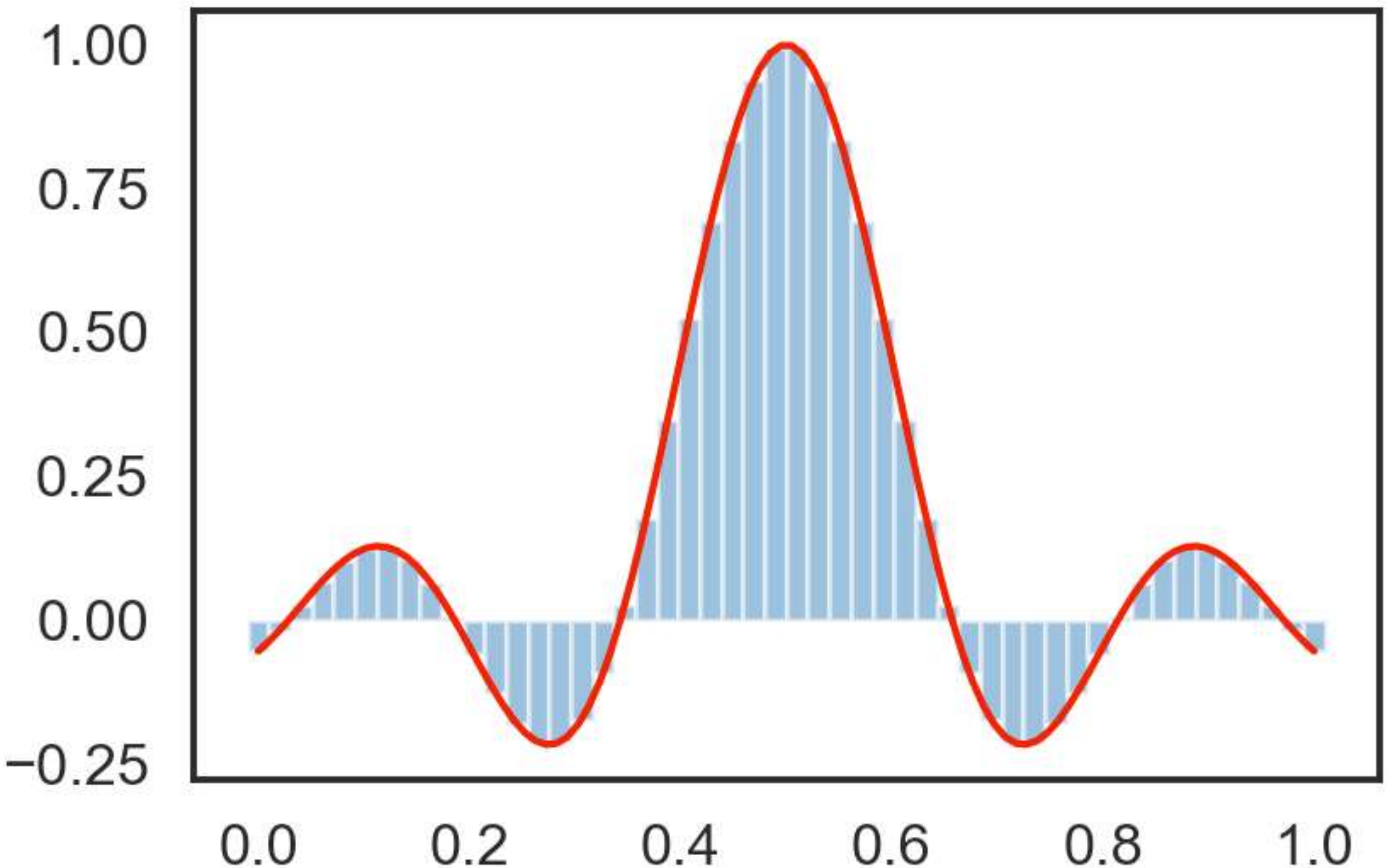
over the d-dimensional euclidean space

$$\boxed{\mathbb{E}[g(X)] = \int g(x)\underline{p(x)}dx}$$

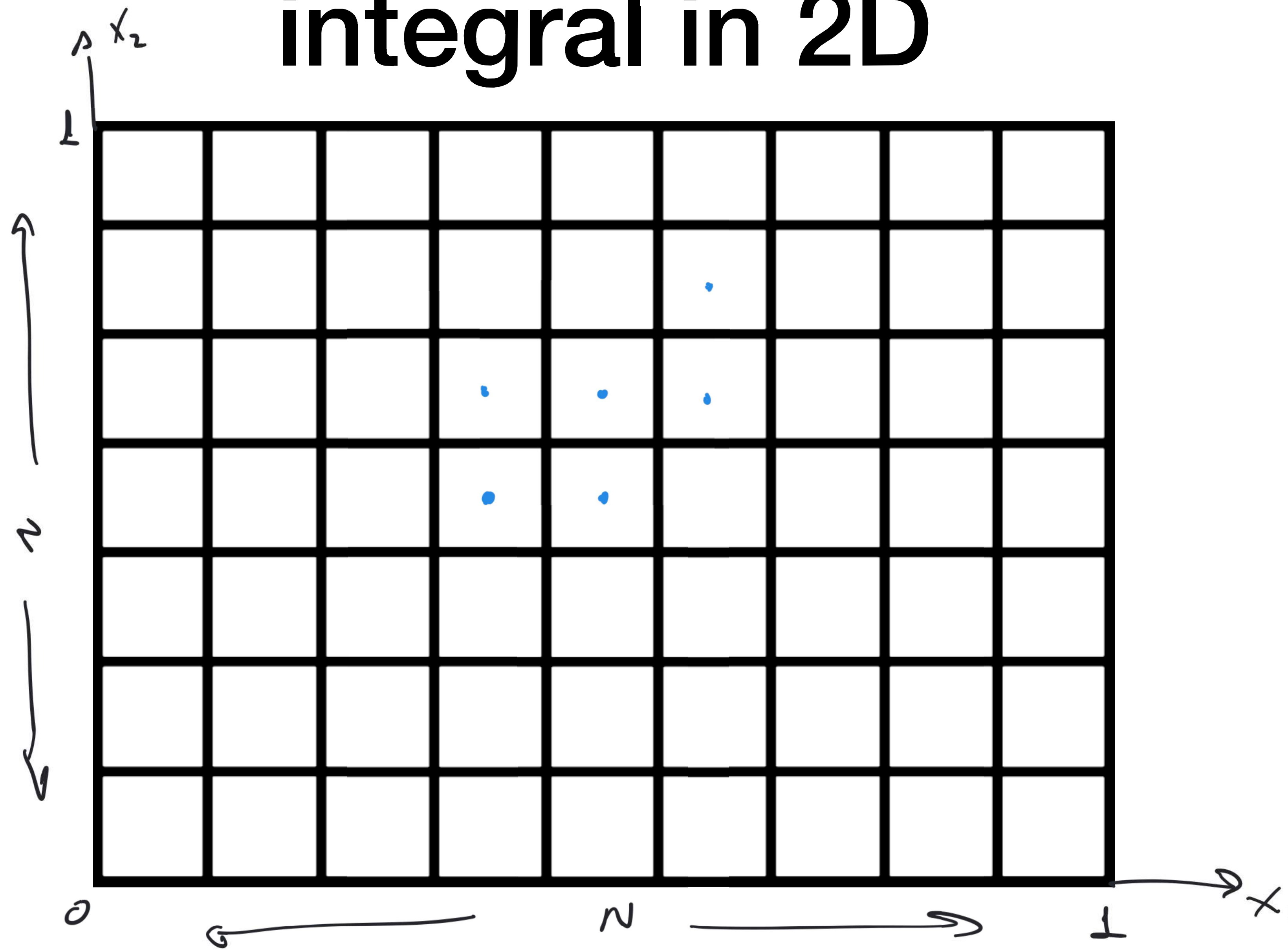
Example: Evaluating integral in 1D



Example: Evaluating integral in 1D



Example: Evaluating integral in 2D



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The curse of dimensionality

- Use n equidistant points per dimension.
- You will have n^d boxes each with volume n^{-d}.
- You can evaluate the integral by:

$$\boxed{\mathbb{E}[g(X)] \approx n^{-d} \sum_{j=1}^{n^d} g(x_{c,j})}$$

The Curse of dimensionality

↳ very hard with high-dimensional integrals

n^d

- Assume it takes a millisecond to evaluate the function. $g(x)$
- Take $n = 10$ points per dimension.
- d=2, needs 0.1 seconds.
- d=3, needs 1 second.
- d=5, needs 100 seconds.
- d=6, needs, 1000 seconds or 16 minutes.
- d=10, needs 115 days…
- d=20, needs 3.17 billion years