Lecture 20: State-space models - Kalman filters

Professor Ilias Bilionis

Derivation of Kalman filter - Overview

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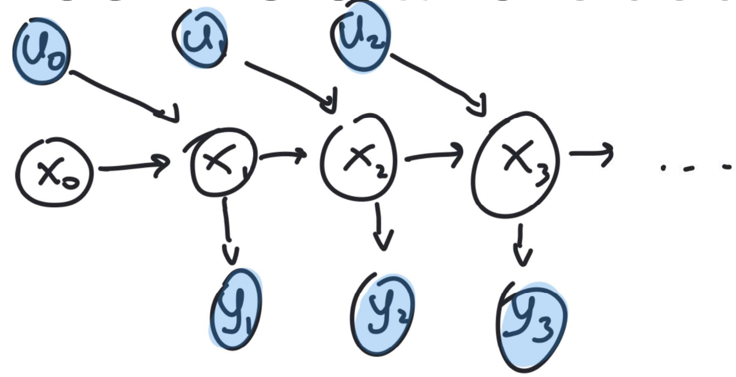
Reminder - Linear dynamical system with controls and observations



control affects transition probabilities

Initial Prob. : $x_0 = \mu_0 + z_0$, $z_0 \sim N(0, V_0)$; $p(x_0) = N(x_0 | \mu_0, V_0)$

Transition Prob. : $x_{n+1} = Ax_n + Bu_n + z_n$, $z_n \sim N(0, Q)$; process covariance

$p(x_{n+1} | x_n, u_n) = N(x_{n+1} | Ax_n + Bu_n, Q)$

Emission Prob. : $y_n = Cx_n + w_n$, $w_n \sim N(0, R)$ measurement covariance

$p(y_n | x_n) = N(y_n | Cx_n, R)$

constants.

Filtering Problem : $p(x_{0:n} | y_{1:n}, u_{1:n}) \propto p(x_{0:n}, y_{1:n} | u_{1:n})$

posterior // joint

$$= p(x_0) \prod_{t=1}^{n} p(x_t | x_{t-1}, u_{t-1}) \, p(y_t | x_t)$$

decompose with transition probabilities

known & Gaussian (see colors)

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Derivation of Kalman filter - Overview

$$p(x_{0:n} \mid y_{1:n}, u_{1:n}) \propto p(x_0) \prod_{t=1}^{n} p(x_t \mid x_{t-1}, u_{t-1})\, p(y_t \mid x_t)$$

KF : $p(x_n \mid y_{1:n}, u_{1:n})$ - posterior of state at last time step

Forward

$p(x_1 \mid y_1, u_1)$

$\downarrow$

$p(x_2 \mid y_{1:2}, u_{1:2})$

$\downarrow$

$\vdots$

$p(x_n \mid y_{1:n}, u_{1:n})$

Backwards

reconstruct entire state, required for certain applications