Lecture 3: Discrete Random Variables

Professor Ilias Bilionis

Variance of a discrete random variable

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Expectation of a random variable

- The variance of a random variable is:

$$= \sum_x \underbrace{(x-\overbrace{\mathbb{E}[X]}^{\mu})^2}_{g(x)} p(x)$$

$$\mathbb{V}[X] := \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$$

- You can think of the variance as the spread of the random variable around its expectation.

- However, do not take this too literally for discrete random variables.

Properties of the variance

- Take any constant c:

$$\mathbb{V}[X + c] = \mathbb{V}[X]$$

Proof:

$$\mathbb{V}[X+c] = \mathbb{E}\left[\left(X+\cancel{c} - \underbrace{\mathbb{E}[X+c]}_{\mathbb{E}[X]+\cancel{c}}\right)^2\right] \quad \text{c's cancel}$$

$$= \mathbb{E}\left[\left(X - \mathbb{E}[X]\right)^2\right] \quad \blacksquare$$

$$= \mathbb{V}[X]$$

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Properties of the variance

- Take any constant λ:

$$\mathbb{V}[\lambda X] = \lambda^2 \mathbb{V}[X]$$

Proof:

$$\mathbb{V}[\lambda X] = \mathbb{E}\left[\left(\lambda X - \underbrace{\mathbb{E}[\lambda X]}_{\lambda \mathbb{E}[X]}\right)^2\right]$$

$$= \mathbb{E}\left[\left(\lambda X - \lambda \mathbb{E}[X]\right)^2\right]$$

$$= \mathbb{E}\left[\lambda^2\left(X - \mathbb{E}[X]\right)^2\right]$$

$$= \lambda^2 \mathbb{E}\left[\left(X - \mathbb{E}[X]\right)^2\right]$$

$$= \lambda^2 \mathbb{V}[X] \quad \blacksquare$$

from quiz: see utility theory

Properties of the variance

- It holds that:

used later for sampling average approximations

$$\boxed{\mathbb{V}[X] = \mathbb{E}\left[X^2\right] - (\mathbb{E}[X])^2}$$

Proof:

$$\mathbb{V}[X] = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$$

$$= \mathbb{E}\left[\underbrace{X^2 - 2X\,\mathbb{E}[X]}_{\text{r.v.}} + \underbrace{(\mathbb{E}[X])^2}_{\text{const.}}\right]$$

$$= \mathbb{E}\left[X^2 - 2X\,\mathbb{E}[X]\right] + (\mathbb{E}[X])^2$$

$$= \underbrace{\mathbb{E}[X^2] - 2(\mathbb{E}[X])^2}_{\text{linearity}} + (\mathbb{E}[X])^2 =$$

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