Sampling Estimates of Variance

Contents


```
import numpy as np
np.set_printoptions(precision=3)
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
sns.set(rc={"figure.dpi":100, 'savefig.dpi':300})
sns.set_context('notebook')
sns.set_style("ticks")
```

Objectives

- To use the law of large numbers to estimate variances

Estimating the variance

We now want to estimate the variance:

$$V = \mathbb{V}[g(X)],$$

using some sort of sampling average. Notice that:

$$V = \mathbb{V}[g(X)] = \mathbb{E}[g^2(X)] - \left(\mathbb{E}[g(X)]\right)^2.$$

We already know how to estimate the last term, see definition of $\bar{I}_N$ in the previous subsection. To approximate the other term, consider the random variables $g^2(X_1), g^2(X_2), \dots$. These are independent and identically distributed so by the law of large numbers we get that:

$$\frac{g^2(X_1)+\dots+g^2(X_N)}{N} \rightarrow \mathbb{E}[g^2(X)],\ \text{a.s.}$$

Putting everything together, we get that:

$$\bar{V}_N = \frac{1}{N}\sum_{i=1}^N g^2(X_i) - \bar{I}_N^2 \rightarrow V\ \text{a.s.}$$

Example: 1D variance

Let's try it out with a test function in 1D (Example 3.4 of Robert & Casella (2004)). Assume that $X \sim \mathcal{U}([0,1])$ and pick:

$$g(x) = \left(\cos(50x) + \sin(20x)\right)^2.$$

The correct value for the variance is:

$$\mathbb{V}[g(X)] \approx 1.093.$$

Let's find a sampling average estimate of the variance:

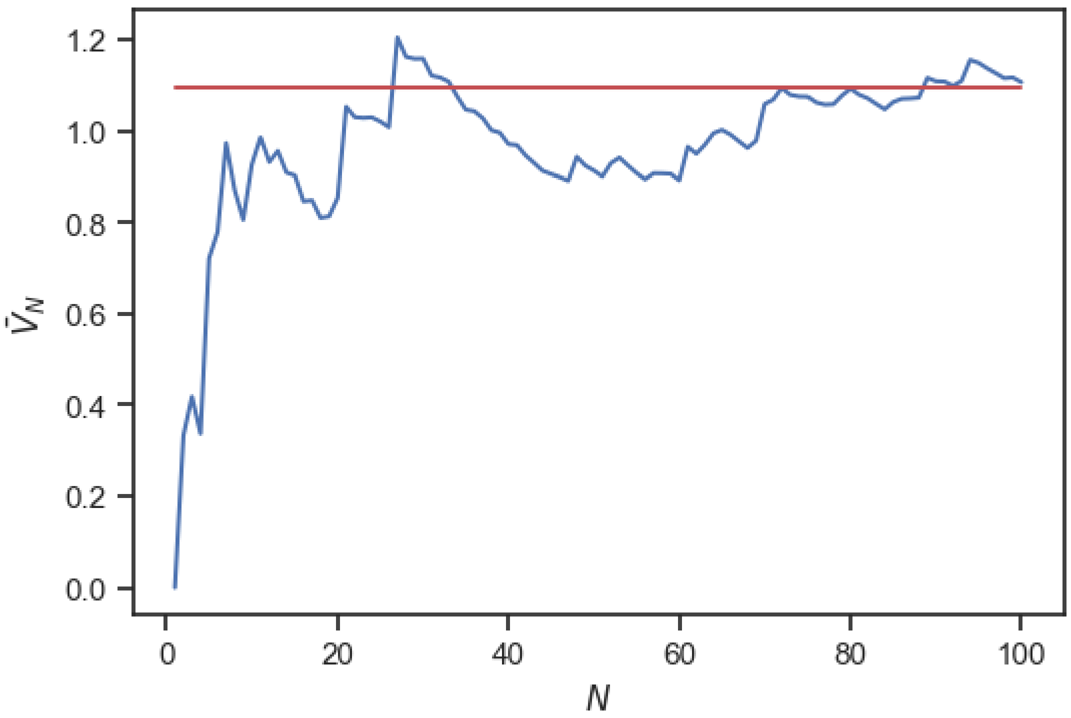
```
# Define the function
g = lambda x: (np.cos(50 * x) + np.sin(20 * x)) ** 2

# Number of samples to take
N = 100
# Generate samples from X
x_samples = np.random.rand(N)
# Get the corresponding Y's
y_samples = g(x_samples)
# Evaluate the sample average E[g(X)] for all sample sizes
I_running = np.cumsum(y_samples) / np.arange(1, N + 1)
# Evaluate the sample average for E[g^2(X)] for all sample sizes
I2_running = np.cumsum(y_samples ** 2) / np.arange(1, N + 1)
# Build the sample average for V[g(X)]
V_running = I2_running - I_running ** 2
```

generate cumulative sums (particular number of points summed divided by the number or points summed at each index location)

Plot a running estimate of the variance:

```
fig, ax = plt.subplots()
ax.plot(np.arange(1, N+1), V_running)
ax.plot(np.arange(1, N+1), [1.093] * N, color='r')
ax.set_xlabel(r"$N$")
ax.set_ylabel(r"$\bar{V}_N$");
```



Questions

- Increase `N` until you get an answer that is close enough to the correct answer (the red line).
- Reduce `N` back to a small number, say 1,000. Run the code 2-3 times to observe that every time you get a slightly different answer...

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