Lecture 21: Gaussian process regression

Professor Ilias Bilionis

Sampling from a Gaussian process

Sampling from a Gaussian process

$$f(\cdot) \sim GP(m(\cdot), c(\cdot,\cdot))$$

Take a finite # of input $x_{1:n} = (x_1, \dots, x_n)$

Consider the function values $f_{1:n} = (f(x_1), \dots, f(x_n))$ — random vector

By definition of the GP: $f_{1:n} \sim N(m_{1:n}, C_n)$

$m_{1:n} = \begin{pmatrix} m(x_1) \\ \vdots \\ m(x_n) \end{pmatrix}$ $\quad C_n = (c(x_i, x_j))$ — $n \times n$

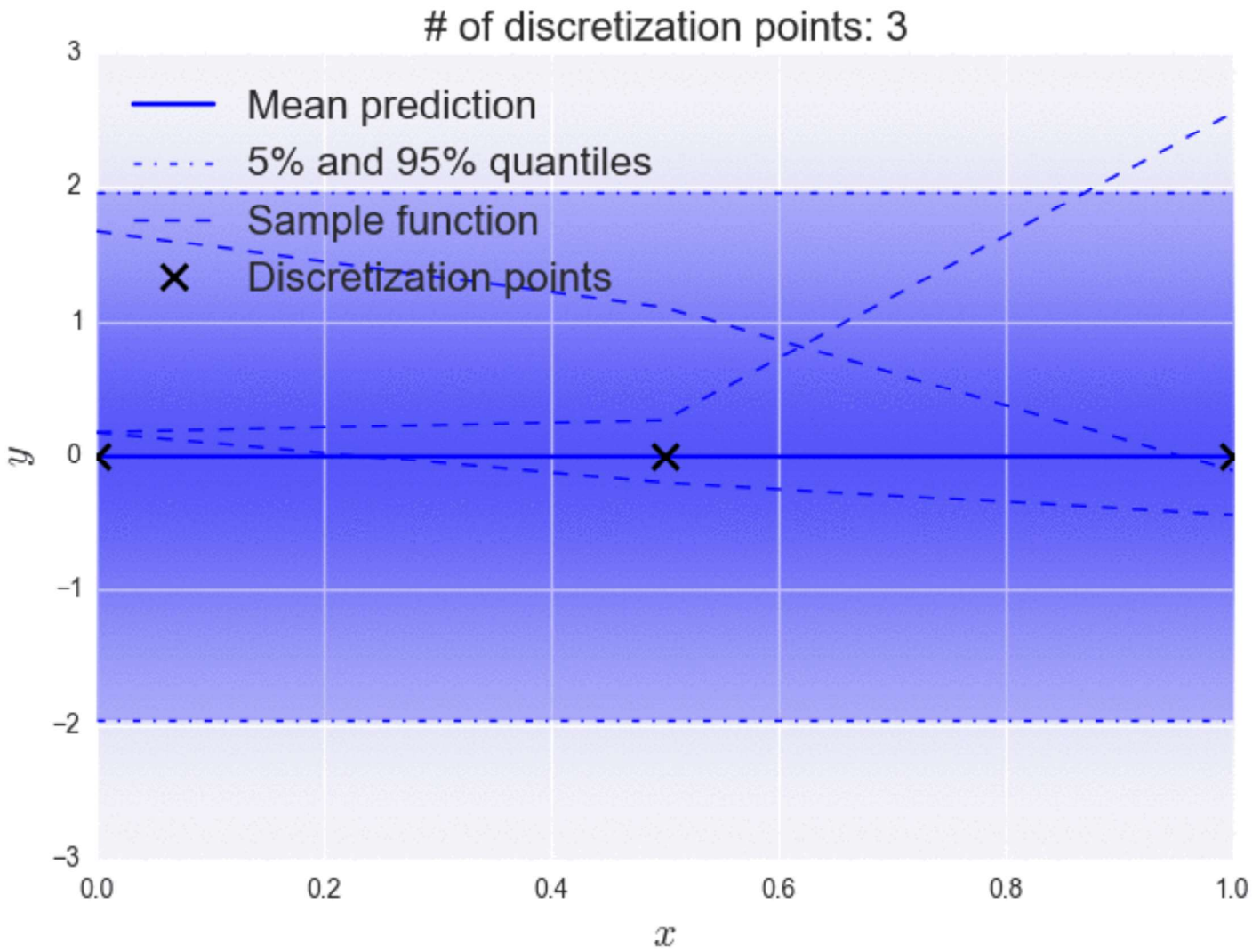
Steps:

- Find a square root of C_n, e.g.

$$C_n = L_n \cdot L_n^T \quad \text{(Cholesky decomp.)}$$

- Sample $z \sim N(0_n, I_n)$ — standard normal in n dimensions
- Evaluate $f_{1:n} = m_{1:n} + L_n z$.

Sampling from a Gaussian process



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