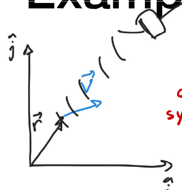
Example: Object tracking



$$m\ddot{\vec{r}} = \vec{F} = u_x\hat{i} + u_y\hat{j} \quad (*)$$

state consists of $\vec{r}$ & $\vec{v}$

ODE system

$$\frac{d\vec{r}}{dt} = \vec{v} \quad (1)$$

$$(*) \Rightarrow \frac{d\vec{v}}{dt} = \frac{u_x}{m}\hat{i} + \frac{u_y}{m}\hat{j} \quad (2)$$

Timestep Δt : discretize

$$\star \begin{cases} (1) \Rightarrow \dfrac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \vec{v} \Rightarrow \vec{r}(t+\Delta t) = \vec{r}(t) + \vec{v}\cdot\Delta t & (3) \\ (2) \Rightarrow \vec{v}(t+\Delta t) = \vec{v}(t) + \underbrace{\dfrac{\vec{F}}{m}}_{\vec{a}}\Delta t = \vec{v}(t) + \dfrac{u_x\Delta t}{m}\hat{i} + \dfrac{u_y\Delta t}{m}\hat{j} & (4) \end{cases}$$

Newton's Laws

$$\vec{r} = r_x\hat{i} + r_y\hat{j}\,; \quad \vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$(3) \Rightarrow \quad r_x(t+\Delta t) = r_x(t) + \Delta t\cdot v_x(t) \quad (5)$$

$$r_y(t+\Delta t) = r_y(t) + \Delta t\cdot v_y(t) \quad (6)$$

$$(4) \Rightarrow \quad v_x(t+\Delta t) = v_x(t) + \frac{\Delta t}{m}u_x(t) \quad (7)$$

$$v_y(t+\Delta t) = v_y(t) + \frac{\Delta t}{m}u_y(t) \quad (8)$$

$$\Vert \quad x_{n+1} = A\cdot x_n + Bu_n$$

Discretized in time

state $x_n = \begin{pmatrix} r_x(n\Delta t) \\ r_y(n\Delta t) \\ v_x(n\Delta t) \\ v_y(n\Delta t) \end{pmatrix} \in \mathbb{R}^4, \; d=4 \;; \; A = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

forces $u_n = \begin{pmatrix} u_x(n\Delta t) \\ u_y(n\Delta t) \end{pmatrix} \in \mathbb{R}^2, \; m=2 \;; \; \underset{4\times 2}{B} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \Delta t/m & 0 \\ 0 & \Delta t/m \end{pmatrix}$

Mapped discret. notion eq. to linear dynamical system formalism

$$x_{n+1} = A x_n + Bu_n + \underbrace{z_n}_{\text{process noise}}$$

? capture all uncertain things including

$$z_n \sim N(0, Q)$$

$$\underset{4\times 4}{Q} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 \\ 0 & 0 & \sigma_q^2 & 0 \\ 0 & 0 & 0 & \sigma_q^2 \end{pmatrix}$$

assume uncorrelated process noise

$$x_{n+1} = A x_n + Bu_n + z_n$$

$$y_n = C x_n + w_n$$

observation $y_n = C x_n + w_n$

measurement noise

GPS measurement of location $\mathbb{R}^2$

$$\underset{2\times 4}{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad w_n \sim N(0, R)$$

$$\text{p.d.} - \underset{2\times 2}{R} = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{pmatrix}$$

assume uncorrelated measurement noise

$\Downarrow$ now ready for

Kalman Filter

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