Lecture 7: Basic Sampling

Professor Ilias Bilionis

Inverse sampling

PREDICTIVE
SCIENCE LABORATORY

Inverse Sampling

- Consider an arbitrary univariate continuous random variable X with CDF $F(x)$, How do you sample from it?

Algorithm:

- Draw a uniform number u. $\sim U([0,1])$
- Set:

$$x = F^{-1}(u)$$

inverse of the CDF

- and you get your sample!

Why does inverse sampling work?

instantiate using rv's

- Let $U \sim U([0,1])$ be a uniform random variable.

- For any CDF $F(x)$ define the random variable:

$$X = F^{-1}(U)$$

- The CDF of X is:

apply F on both sides

$$p(X \leq x) = p\left(F^{-1}(U) \leq x\right) = p\left(F\left(F^{-1}(U)\right) \leq F(x)\right)$$

$$= p\left(U \leq F(x)\right) = F_U\left(F(x)\right) = F(x) \checkmark$$

identity function

X has the right CDF

Example: The exponential distribution

- Take an exponential random variable as an example:

$$X \sim \mathrm{Exp}(r)$$

- The CDF is:

$$F(x) = 1 - e^{-rx}$$

- The inverse of the CDF is:

evaluate

sample

$$F^{-1}(u) = -\frac{\ln(1-u)}{r}$$

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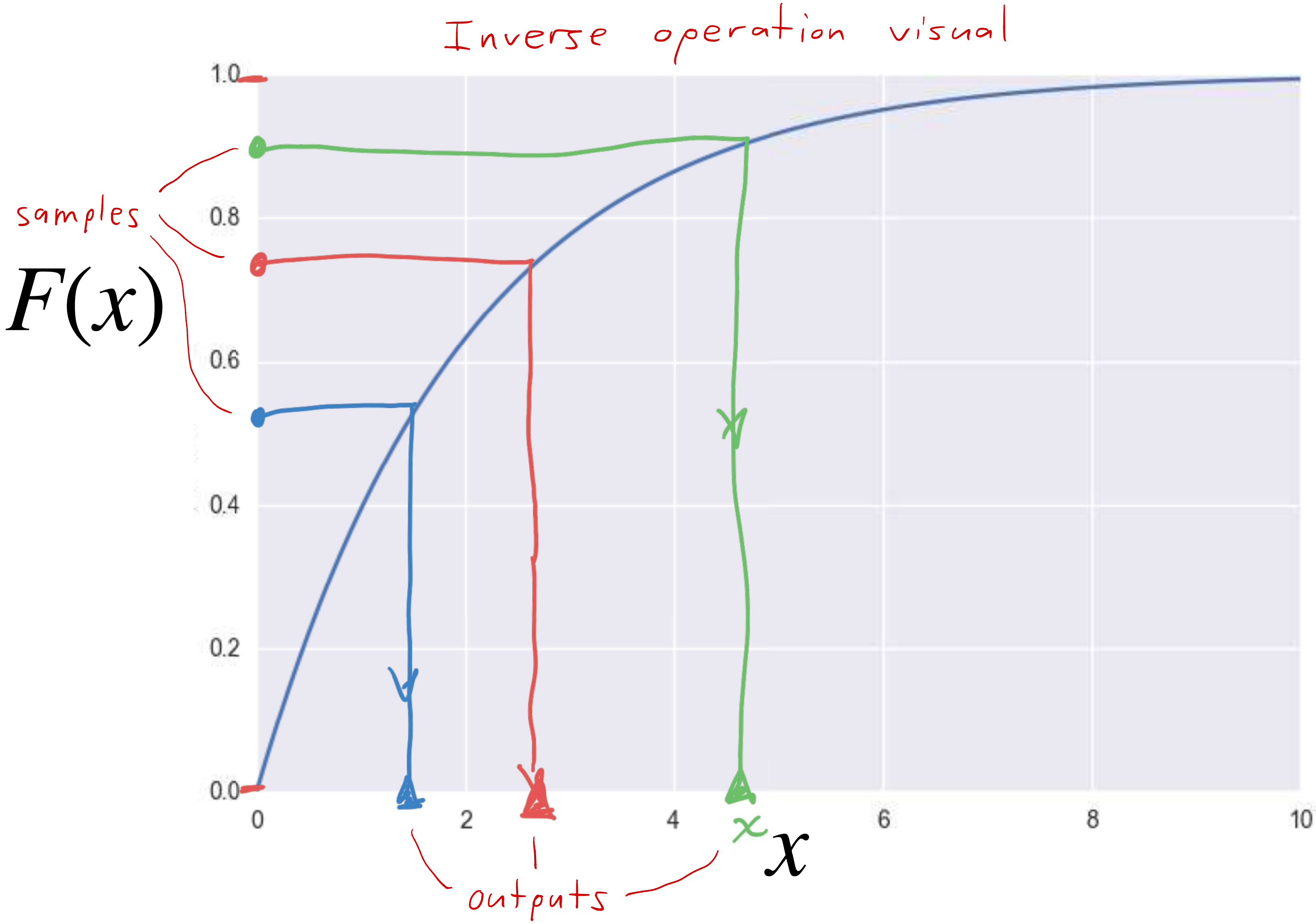
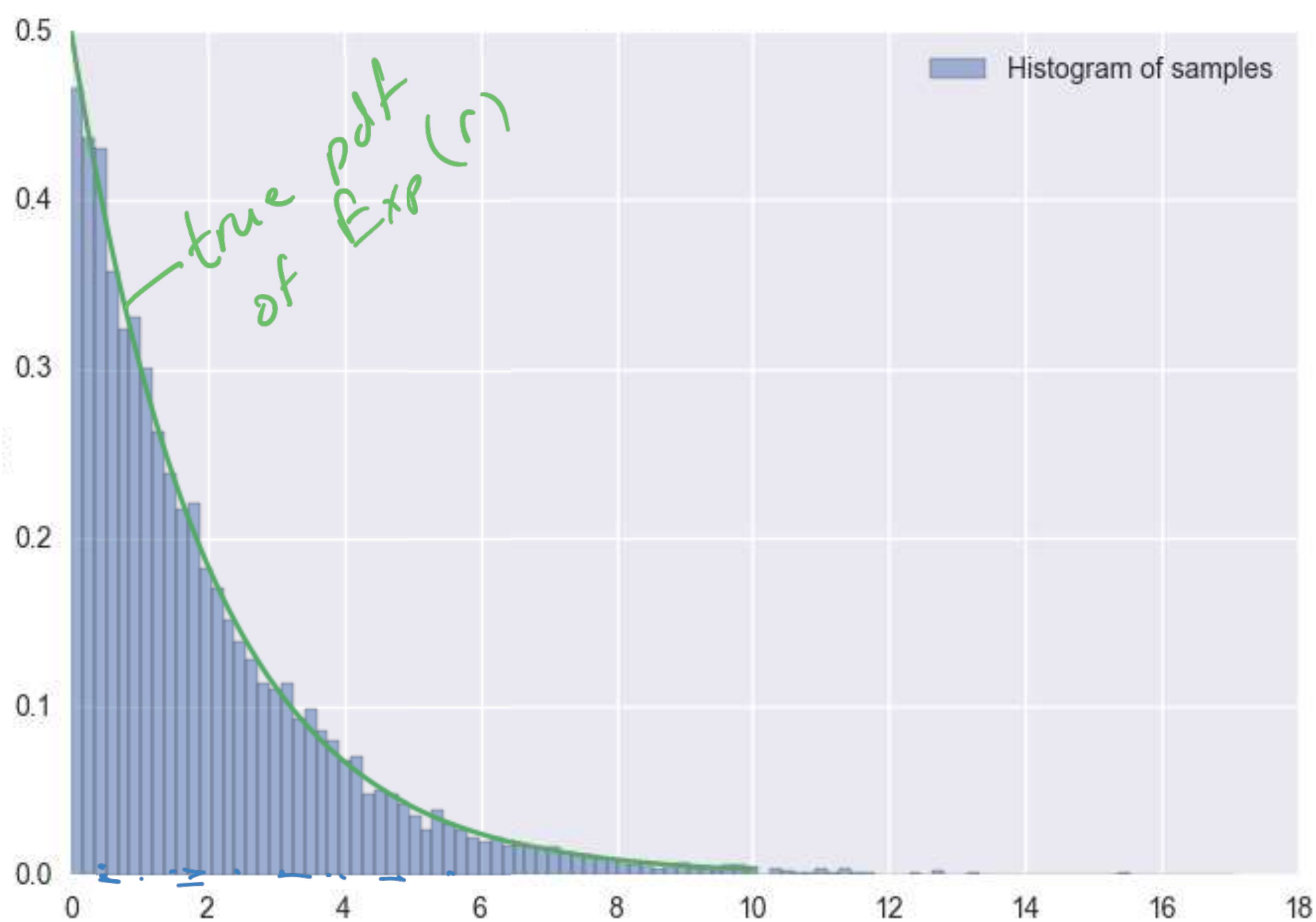
The Exponential Distribution



Inverse Sampling for Exponential

$p(x)$



x