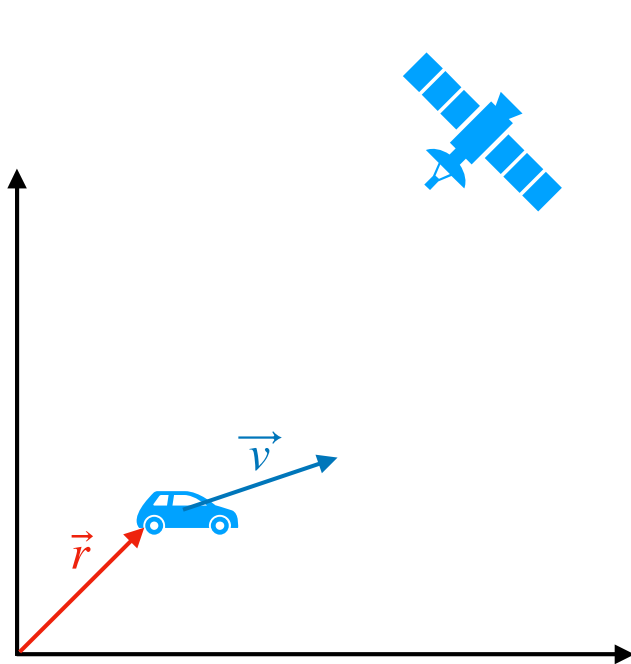# Lecture 20: State-space models - Kalman filters

Professor Ilias Bilionis

Kalman filter - The complete algorithm

Reminder: Object tracking equations in 2D
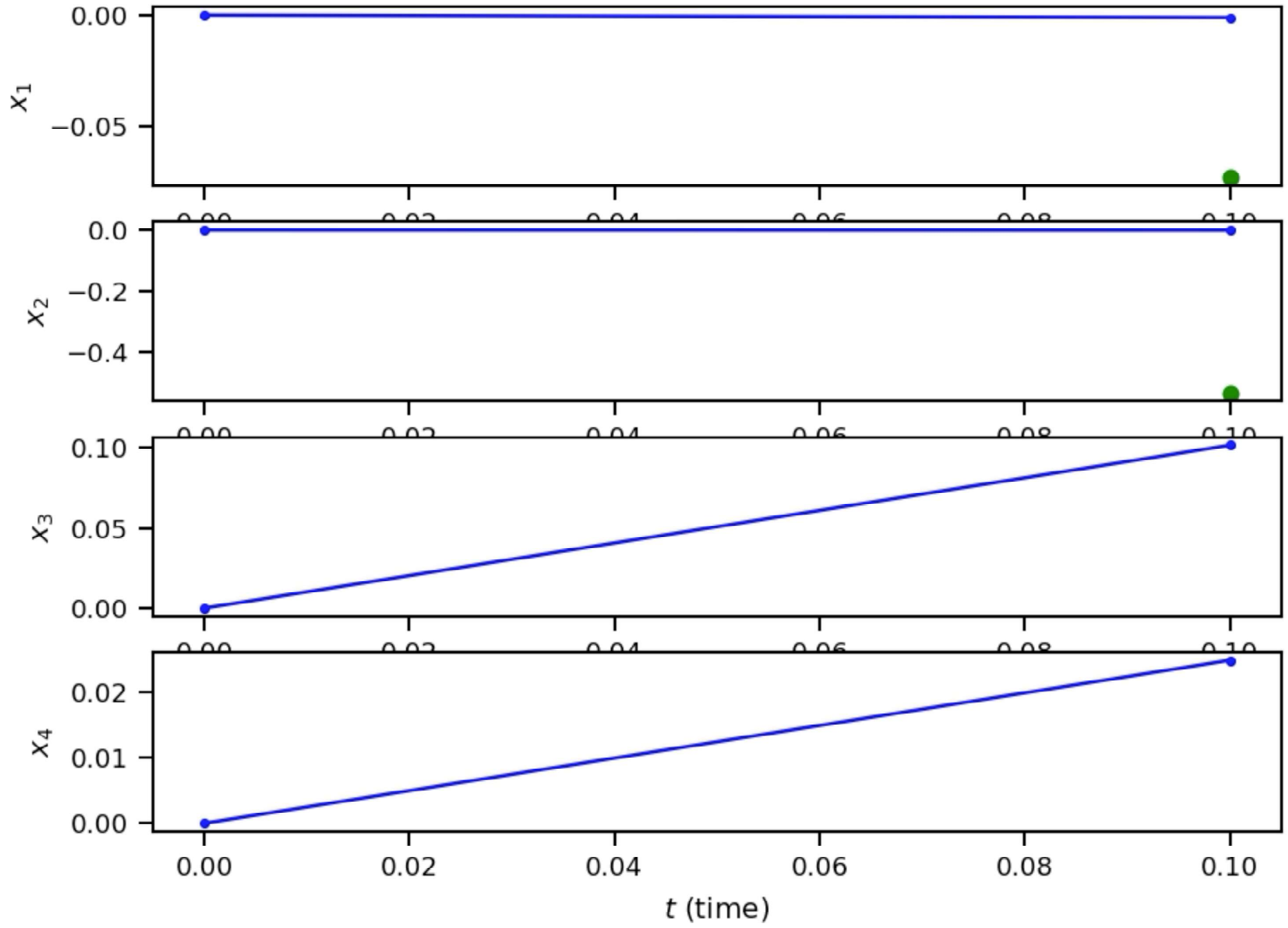
$$m\ddot{\vec{r}} = u_1 \hat{i} + u_2 \hat{j}$$

2nd order ODE to system of first order ODE's

$$\mathbf{x} = \begin{bmatrix} r_1 \\ r_2 \\ v_1 \\ v_2 \end{bmatrix}$$

Discretize in time:

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{B}\mathbf{u}_n + \mathbf{z}_n, \mathbf{z}_n \sim N(0,\mathbf{Q})$$

$$\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \mathbf{w}_n, \mathbf{w}_n \sim N(0,\mathbf{R})$$

$\vec{v}$

$\vec{r}$

**PREDICTIVE
SCIENCE LABORATORY**

Kalman filter algorithm

- Initialize state.

- **Predict** new using transition probabilities.

- Make observation.

- **Update** belief using observation.

- Go to predict.

PREDICTIVE
SCIENCE LABORATORY

observation

mean of posterior state
of knowledge about the state
of the system after update

±2σ

x_1

x_2

x_3

x_4

t (time)