Lecture 12: Analytical examples of Bayesian inference

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Decision making

The Decision-making problem

- What if someone asks you to report a single value for θ in the coin toss example?

- What is the correct way of doing this?

- To answer it, you have to quantify the cost of making a mistake and then make a decision that minimizes this cost.

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The Decision-making problem

- The **loss** when we guess θ' and the true value is θ = $\ell(\theta', \theta)$

$$\mathbb{E}[\ell(\theta',\theta) \mid x_{1:N}] = \int \overset{\text{loss}}{\ell(\theta',\theta)} \, \overset{\text{posterior}}{p(\theta \mid x_{1:N})} \, d\theta$$

integrate out the true parameters

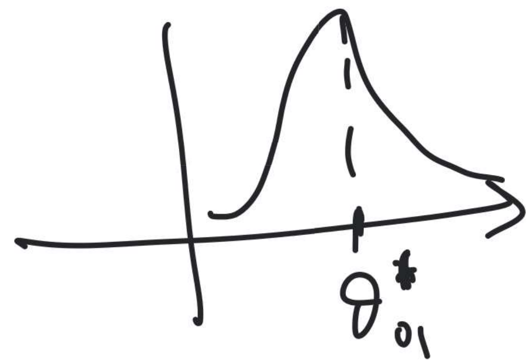
expected loss cond. on the data

$$\min_{\theta'} \mathbb{E}[\ell(\theta',\theta) \mid x_{1:N}]$$

expectation over posterior state of knowledge

The 0-1 Loss

$$\ell_{01}(\theta', \theta) = \begin{cases} 0, & \theta' = \theta \\ 1, & \theta' \neq \theta \end{cases}$$



$$\theta^*_{01} = \arg\max_{\theta} p(\theta \mid x_{1:n})$$

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The Square Loss

$$\ell_2(\theta', \theta) = (\theta' - \theta)^2$$

$$\theta_2^* = \mathbb{E}[\theta | x_{1:N}]$$
$$= \int \theta p(\theta | x_{1:N}) d\theta$$

can estimate this using Monte-Carlo
if you can sample from the posterior

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The Absolute Loss

$$\ell_1(\theta', \theta) = |\theta' - \theta|$$

$\theta_1^* =$ median of the post.

$$p(\theta \leq \theta_1^* | x_{1:N}) = 0.5$$

Example: Coin toss - Picking a value

Using the square loss, we get:

$$\theta_N^* = \frac{1 + \sum_{i=1}^{N} x_i}{N+2}$$

stems from the expectation of a Beta distribution