Lecture 6: Random Vectors

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Random vectors

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Joint pdf of many random variables

- Take N random variables $X_1, \ldots, X_N$.
- $\mathbf{X} = (X_1, \ldots, X_N)$ is called a **random vector.**
- We will refer to their joint pdf as:

$$p(\mathbf{x}) = p(x_1, \ldots, x_N)$$

usual properties hold

$p(x) \geq 0$

$\int p(x)\, dx_1\, dx_2 \ldots dx_N = 1$

$p(x_i) = \int p(x)\, dx_2 \ldots dx_{i-1}\, dx_{i+1} \ldots dx_N$

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Expectation of a random vector

- The expectation of a random vector is the vector of expectations of each component:

$$\underbrace{\mathbb{E}[\mathbf{X}]}_{\in \mathbb{R}^N} = \begin{pmatrix} \mathbb{E}[X_1] \\ \vdots \\ \mathbb{E}[X_N] \end{pmatrix}$$

Covariance matrix of two random vectors

- Let $\mathbf{X}$ be an N-dimensional random vector.
- Let $\mathbf{Y}$ be an M-dimensional random vector.
- The covariance of $\mathbf{X}$ and $\mathbf{Y}$ is the $N \times M$ matrix consisting of all covariances between the components of $\mathbf{X}$ and $\mathbf{Y}$, i.e.,

$$\mathbb{C}[\mathbf{X}, \mathbf{Y}] = \left(\mathbb{C}[X_i, Y_j]\right)_{ij}$$

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Covariance matrix of two random vectors

- Let $\mathbf{X}$ be an N-dimensional random vector.
- Let $\mathbf{Y}$ be an M-dimensional random vector.
- We can easily show that:

outer product

$$\mathbb{C}[\mathbf{X},\mathbf{Y}] = \mathbb{E}\Big[\underbrace{\underbrace{(X-\mathbb{E}[X])}_{N\times 1}\cdot\underbrace{(Y-\mathbb{E}[Y])^T}_{1\times M}}_{N\times M}\Big]$$

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Self-covariance of a random vector

- Let $\mathbf{X}$ be an N-dimensional random vector.
- The self-covariance of a random vector is the $N \times N$ matrix:

$$\mathbb{C}[\mathbf{X}, \mathbf{X}] = \mathbb{E}\left[(\mathbf{X} - \mathbb{E}[\mathbf{X}])(\mathbf{X} - \mathbb{E}[\mathbf{X}])^T\right]$$