Lecture 3: Discrete Random Variables

Professor Ilias Bilionis

The probability mass function

Probability mass function

Let X be a discrete random variable. The *probability mass function (pmf) of* X *is:*

$p(X = x) =$ Probability that the random variable X takes the value x

$X = \begin{cases} H, & 0.5 \\ T, & 0.5 \end{cases} \rightarrow \begin{array}{l} p(X = H) = 0.5 \\ p(X = T) = 0.5 \end{array}$

PREDICTIVE
SCIENCE LABORATORY

Probability mass function

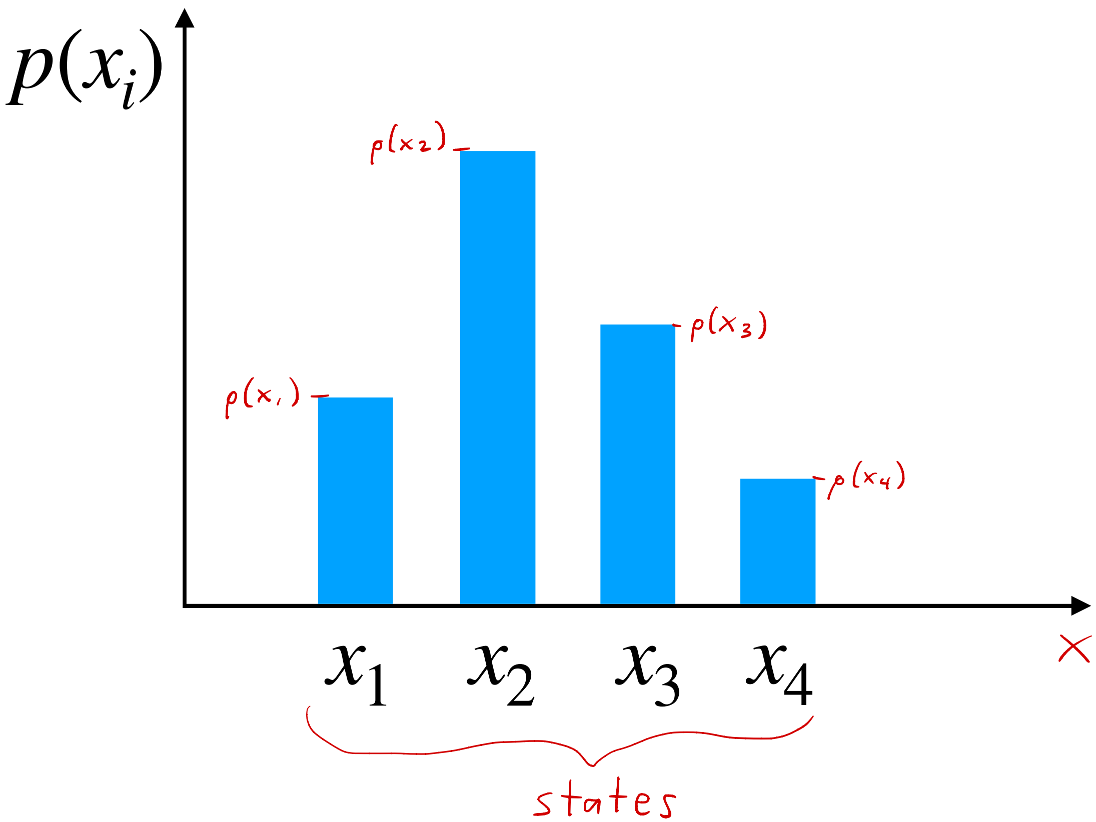
Let X be a discrete random variable. The *probability mass function (pmf) of X is:*

$p(X = x) =$ Probability that the random variable X takes the value x

When there is no ambiguity:

$$\boxed{p(x) \equiv p(X = x)\,.}$$

Visualization of the probability mass function



Properties of the probability mass function

- The probability mass function is nonnegative:

$$p(x) \geq 0.$$

- The probability mass function is normalized:

$$\sum_x p(x) = 1,$$

where the summation is over all the possible values of X.

at least one x is realized

Properties of the probability mass function

- Let X be a discrete random variable.

- The probability of X taking either the value x_1 or the value x_2 (assuming $x_1 \neq x_2$) is:

$$p(X = x_1 \text{ or } X = x_2) \equiv p(X \in \underbrace{\{x_1, x_2\}}_{\text{set}}) = p(X = x_1) + p(X = x_2)$$

$$= p(x_1) + p(x_2) \quad \text{(shorthand)}$$

PREDICTIVE
SCIENCE LABORATORY

Properties of the probability mass function

- More generally, the probability that the random variable X takes any value in a set A is given by:

$$\boxed{p(X \in A) = \sum_{x \in A} p(x)}$$

Functions of random variables

- Consider a function $g(x)$.

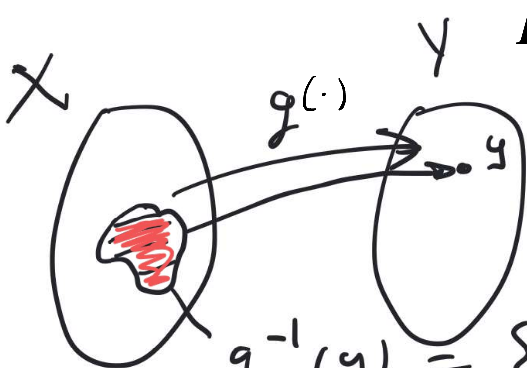
- We can now define a new random variable:

$$Y = g(X)\,.$$

- It has its own probability mass function (pmf):

$$p(y) = p(Y=y) = \sum_{\underbrace{x\in g^{-1}(y)}_{\text{pre-image (A)}}} p(x)$$

where



$$g^{-1}(y) = \{x : g(x) = y\}$$