Lecture 15: Advanced topics in Bayesian linear regression

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Diagnostics for posterior predictive

↳ to see if it is correct

Standarized errors

Post. Pred. $p(y|x, data) = N(y|m(x), \sigma^2(x))$

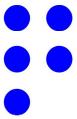
Validation data : $x_i, y_i, \quad i=1, \dots, N^v$

model says: $y_i | x_i \sim N(m(x_i), \sigma^2(x_i))$ ← this is what we want to test

Standarized Error : $z_i := \dfrac{y_i - m(x_i)}{\sigma(x_i)} \sim N(0,1)$ (If model correct ✱)

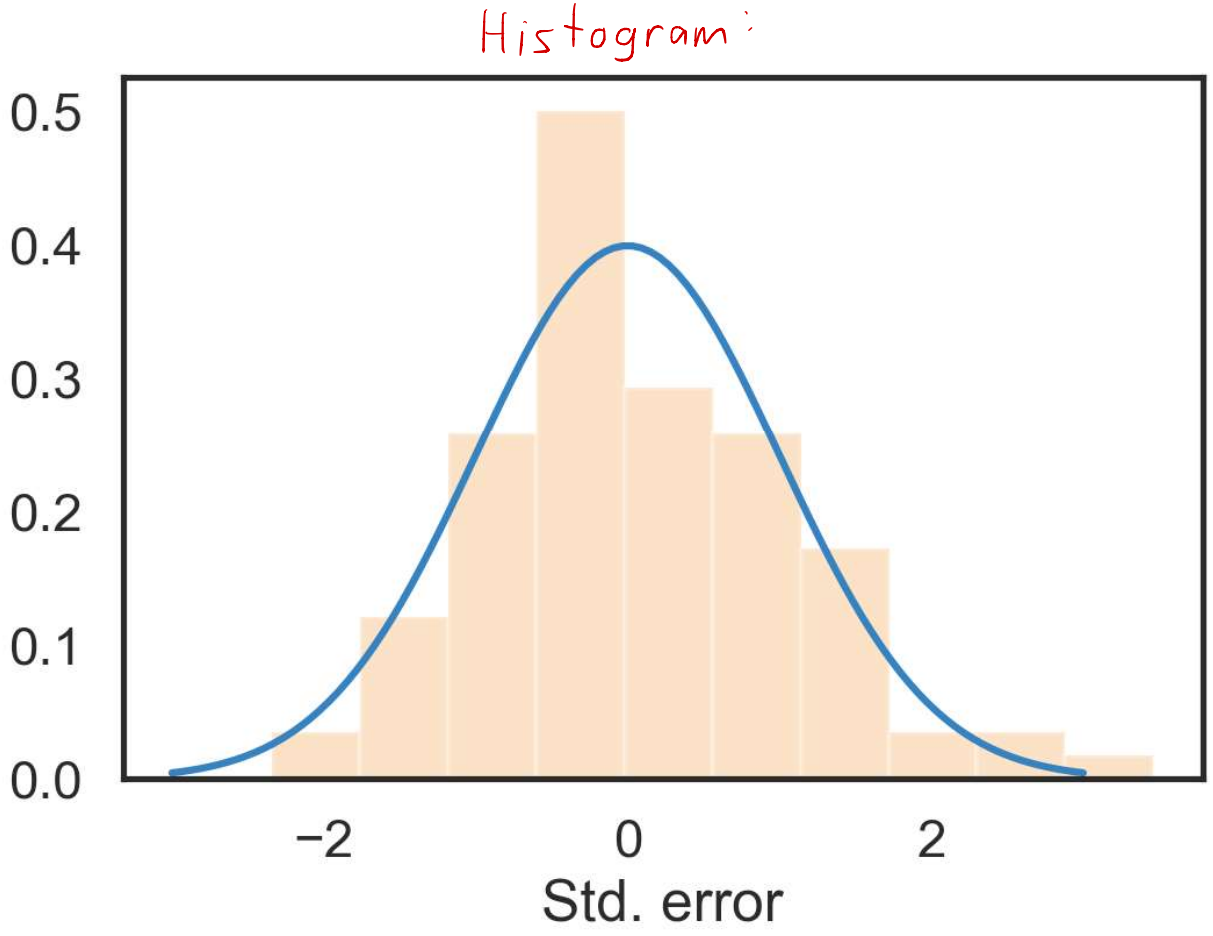
$\mathbb{E}[z_i] = \mathbb{E}\left[\dfrac{y_i - m(x_i)}{\sigma(x_i)}\right] = \dfrac{\{\mathbb{E}[y_i] - m(x_i)\}}{\sigma(x_i)} = 0$ ($\mathbb{E}[y_i] = m(x_i)$)

$\mathbb{V}[z_i] = \mathbb{V}\left[\dfrac{y_i - m(x_i)}{\sigma(x_i)}\right] = \dfrac{1}{\sigma^2(x_i)} \mathbb{V}[y_i] = \dfrac{\sigma^2(x_i)}{\sigma^2(x_i)} = 1$

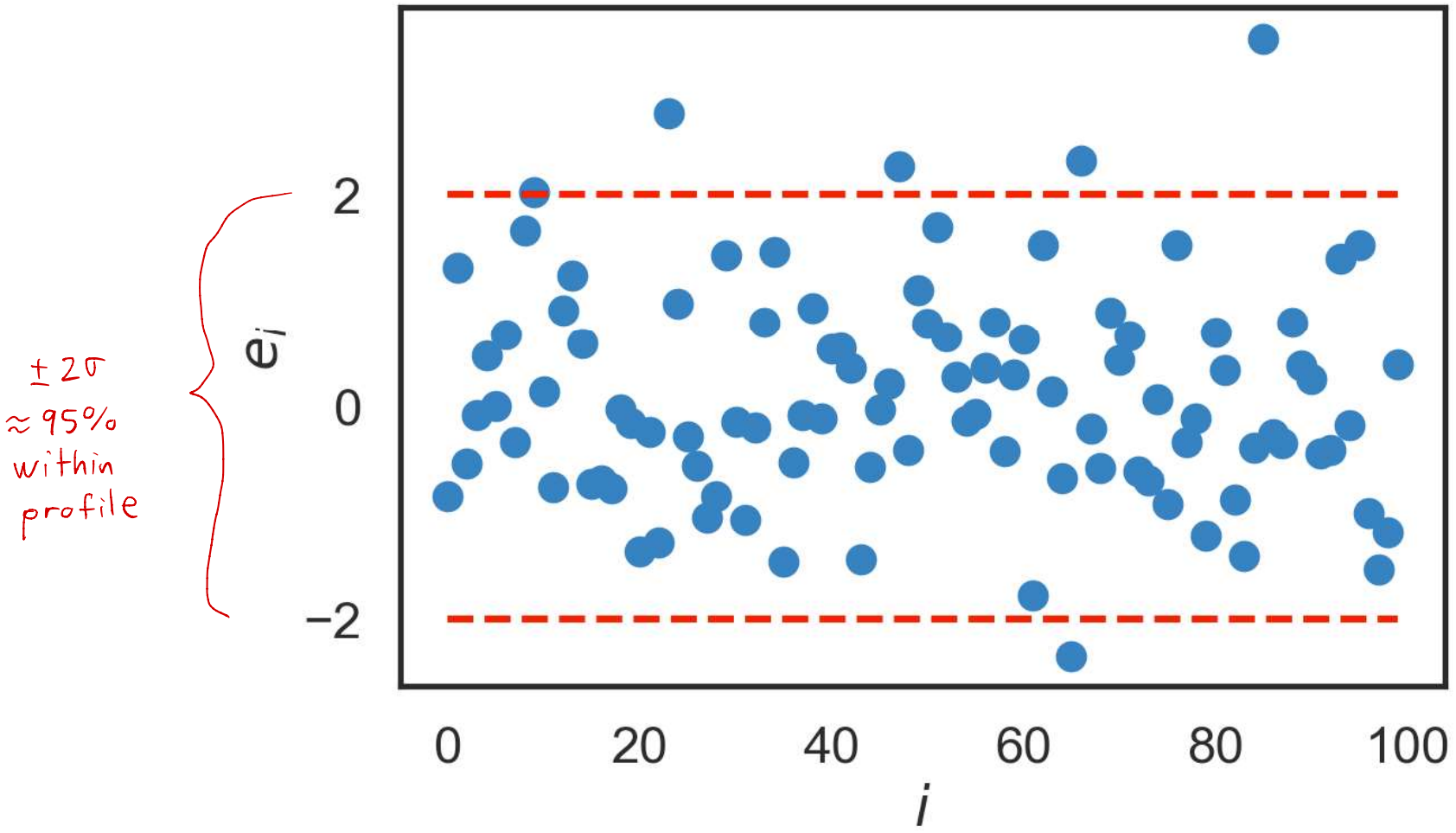


PREDICTIVE
SCIENCE LABORATORY

Standardized Errors

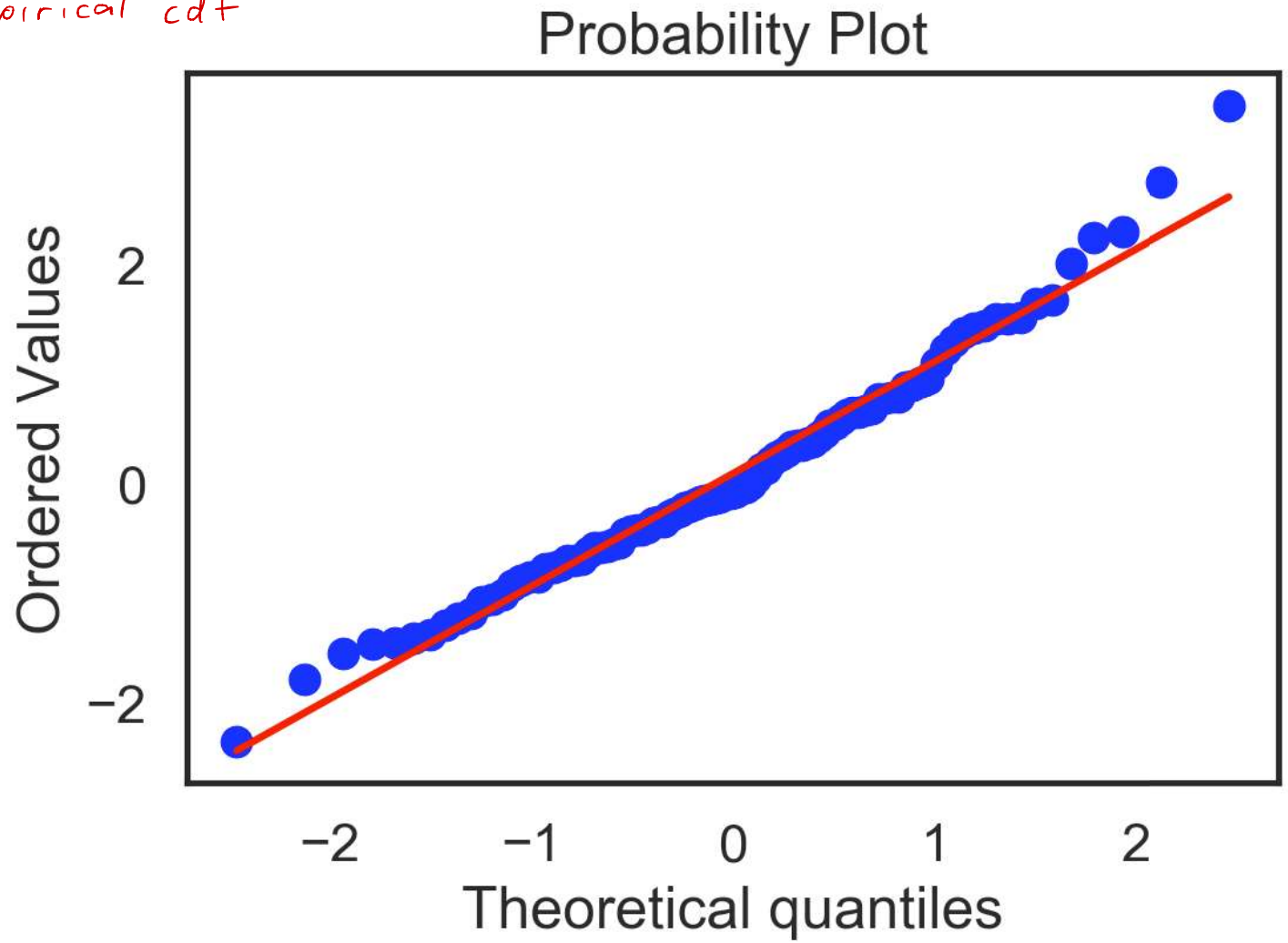


Standardized Errors



Standardized Errors

determine empirical quantiles using empirical cdf



empirical

quantile-quantile plots: Q-plots