Lecture 3: Discrete Random Variables

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Expectation of a discrete random variable

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Expectation of a random variable

- The expectation of a random variable is:

$$\boxed{\mathbb{E}[X] := \sum_x x p(x)}$$

weighted average

- You can think of the expectation as the value of the random variable that one should "expect" to get.

- However, take this interpretation with a grain of salt because it may be a value that the random variable has a zero probability of getting...

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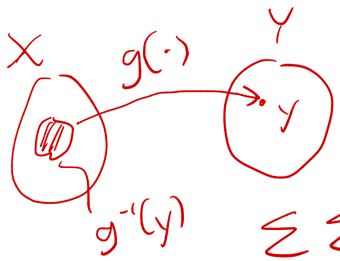
Properties of the expectation



$\sum_y \sum_{x \in g^{-1}(y)} = \sum_x$ (assuming $g(\cdot)$ one-to-one)

- For any function $g(x)$:

$$\boxed{\mathbb{E}[g(X)] = \sum_x g(x)p(x)}$$

$Y = g(x)$

$$E[g(x)] = E[Y] = \sum_y y\,p(y) = \sum_y y \sum_{x \in g^{-1}(y)} p(x)$$

$$= \sum_y \sum_{x \in g^{-1}(y)} y\,p(x) = \sum_x g(x)p(x) \quad \blacksquare$$

Properties of the expectation

- Take any constant c:

$$\mathbb{E}[X+c] = \mathbb{E}[X] + c$$

Proof:

$$\mathbb{E}[X+c] = \sum_x (x+c)\,p(x) = \sum_x x\,p(x) + \sum_x c\,p(x)$$

$$= \mathbb{E}[X] + c \cdot \underbrace{\sum_x p(x)}_{1} \qquad \blacksquare$$

Properties of the expectation

- For any λ real number:

$$\mathbb{E}[\lambda X] = \lambda \mathbb{E}[X]$$

Proof: $\mathbb{E}[\lambda X] = \sum_x \lambda x \, p(x) = \lambda \cdot \sum_x x \cdot p(x) = \lambda E[X]$

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