Lecture 4: Continuous Random Variables

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The probability density function

The probability density function (PDF)

- Consider a continuous random variable X and some value it can take x.

- The probability density function (PDF) $p(x)$ is defined by:

$$p(x) \simeq \frac{p(x \le X \le x + \Delta x)}{\Delta x}$$

for some small Δx.

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The probability density function (PDF)

$$p(x) \approx \frac{p(x \leq X \leq x + \Delta x)}{\Delta x}$$

— probability that X is inside an interval: can write using the cdf

$$p(x) \approx \frac{F(x+\Delta x) - F(x)}{\Delta x}$$

$$p(x) \approx \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x}$$

$$\boxed{p(x) = F'(x) = \frac{dF(x)}{dx}}$$

(whenever the limits exist)

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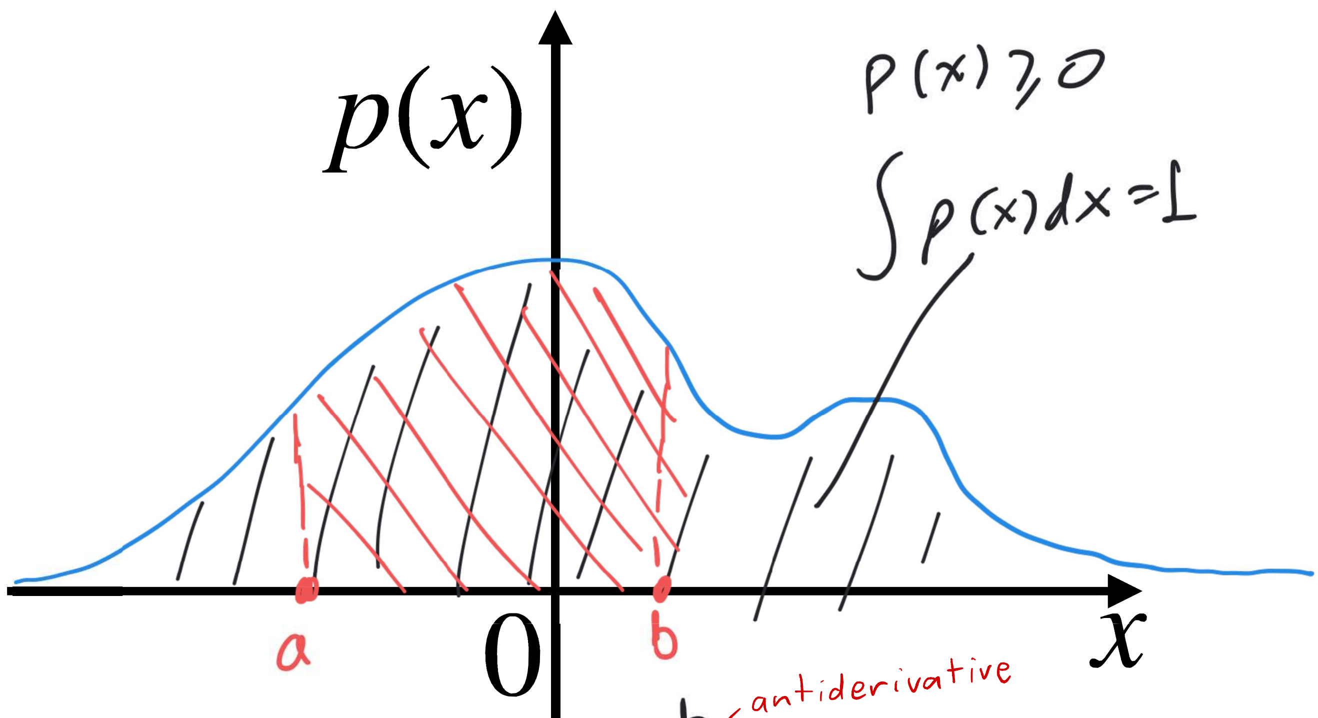
Visualizing the probability density



antiderivative

★ $$p(a \leq X \leq b) = \int_a^b p(x)\,dx = F(b) - F(a).$$

Properties of the PDF

- $p(x) \geq 0$ for all x.
- $\int p(x)dx = 1$
- $\int_a^b p(x)dx = F(b) - F(a) = p(a \leq X \leq b)$

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Another useful property of the PDF

For any "good" subset A of the real numbers:
(Borel)

$$p(X \in A) = \int_A p(x)\,dx$$

This property holds even for random vectors!

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The PDF of a function of a given random variable - The change of variables formula

- Let X be a random variable and $Y = g(X)$ be a random variable defined as a function of it.

- If you have the PDF of X can you find the PDF of Y?

- When g is one to one, there is an analytical answer given by the *change of variables formula*:

$$p(y) = p(x = g^{-1}(y)) \, \underbrace{\left| \frac{d}{dy} \left(g^{-1}(y) \right) \right|}_{\text{Jacobian}})$$