Lecture 20: State-space models - Kalman filters

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How do we estimate the parameters of the model?

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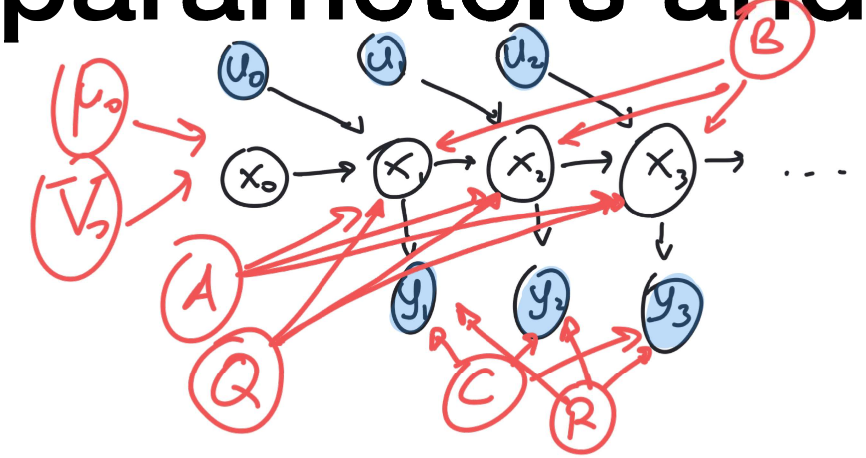
Joint posterior over parameters and states



$$\vartheta = (\underbrace{A, B, C}_{\text{matrices that govern the dynamics}}, \underbrace{Q, R}_{\text{process \& measurement covariance matrices}}, \overbrace{\mu_0, V_0}^{\text{initial condition parameters}})$$

prior: $\vartheta \sim p(\vartheta)$

$$\text{parameter posterior: } p(\vartheta \mid y_{1:n}, u_{0:n-1}) \overset{\text{sum rule}}{=} \int \underbrace{p(\vartheta, x_{0:n} \mid y_{1:n}, u_{0:n-1})}_{\text{joint posterior}} \, \overset{\text{marginalize}}{d x_{0:n}}$$

$$p(\vartheta, x_{0:n} \mid y_{1:n}, u_{0:n-1}) \propto \overbrace{p(x_{0:n} \mid y_{1:n}, u_{0:n-1}, \vartheta)}^{\text{state posterior}} \overbrace{p(\vartheta)}^{\text{parameter prior}}$$

$$\propto \overbrace{p(y_{1:n} \mid x_{0:n}, u_{0:n-1}, \vartheta)}^{\text{likelihood}} \overbrace{p(x_{0:n} \mid u_{0:n-1}, \vartheta)}^{\text{states prior}} \overbrace{p(\vartheta)}^{\text{parameter prior}}$$

$$= p(\vartheta)\, p(x_0 \mid \vartheta) \prod_{t=1}^{n} p(x_t \mid x_{t-1}, u_{t-1}, \vartheta)\, p(y_t \mid x_t, \vartheta)$$

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Maximum likelihood estimate of the parameters

$$p(\theta \mid y_{1:n}, u_{0:n-1}) = \int p(\theta, x_{0:n} \mid y_{1:n}, u_{0:n})\, dx_{0:n}$$

$$\Downarrow$$

Expectation Maximization

(for linear case w/ Gaussian)

Bishop 9.3

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