Decision making

Contents

```python
import numpy as np
np.set_printoptions(precision=3)
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
sns.set(rc={"figure.dpi":100, "savefig.dpi":300})
sns.set_context("notebook")
sns.set_style("ticks")

import scipy
import scipy.stats as st
import urllib.request
import os

def download(
    url : str,
    local_filename : str = None
):
    """Download a file from a url.

    Arguments
    url            -- The url we want to download.
    local_filename -- The filemame to write on. If not
                      specified
    """
    if local_filename is None:
        local_filename = os.path.basename(url)
    urllib.request.urlretrieve(url, local_filename)
```

Objectives

- To demonstrate how to use the results of a binary classifier to make decisions

High melting explosives sensitivity

Let’s repeat what we did for the HMX example, but after splitting the dataset into training and validation subsets. We will be making predictions on the validation subset.

```python
url = "https://github.com/PredictiveScienceLab/data-analytics-
se/raw/master/lecturebook/data/hmx_data.csv"
download(url)

import pandas as pd
data = pd.read_csv('hmx_data.csv')
x = data['Height'].values
label_coding = {'E': 1, 'N': 0}
y = np.array([label_coding[r] for r in data['Result']])
data['y'] = y
data.head()
```

	Height	Result	y
0	40.5	E	1
1	40.5	E	1
2	40.5	E	1
3	40.5	E	1
4	40.5	E	1

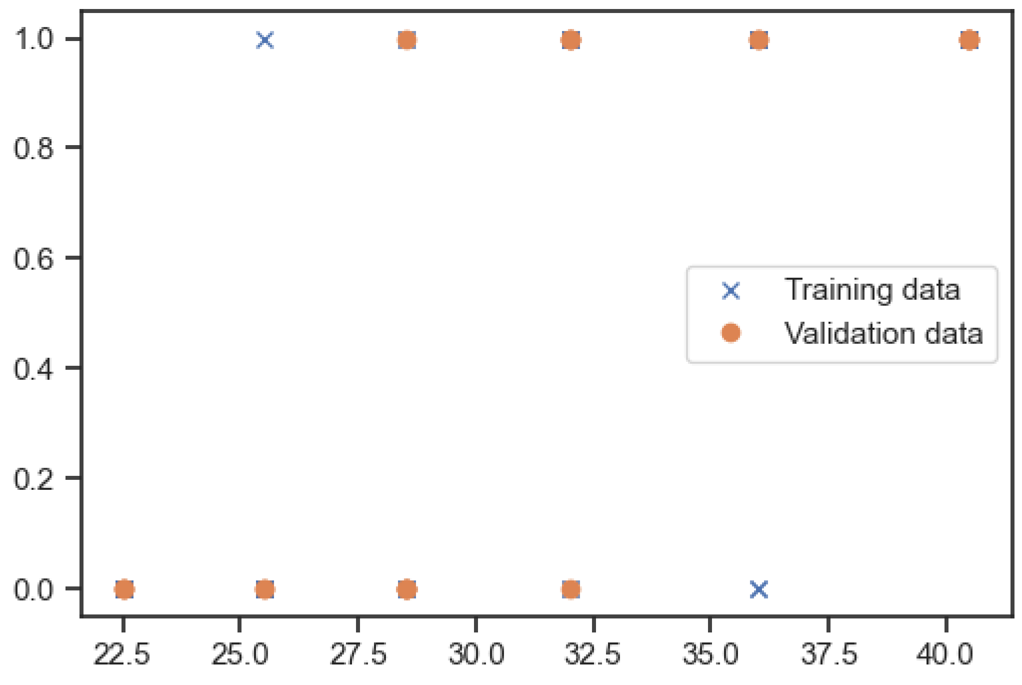
Separate data into training and validation:

```
from sklearn.model_selection import train_test_split

x_train, x_valid, y_train, y_valid = train_test_split(x, y, test_size=0.33)

num_obs = x.shape[0]

fig, ax = plt.subplots()
ax.plot(x_train, y_train, 'x', label='Training data')
ax.plot(x_valid, y_valid, 'o', label='Validation data')
plt.legend(loc='best');
```



Train the model:

```
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LogisticRegression

# Design matrix
poly = PolynomialFeatures(2)
Phi = poly.fit_transform(x[:, None])

# Fit
model = LogisticRegression(
    penalty='none',
    fit_intercept=False
).fit(Phi, y)
```

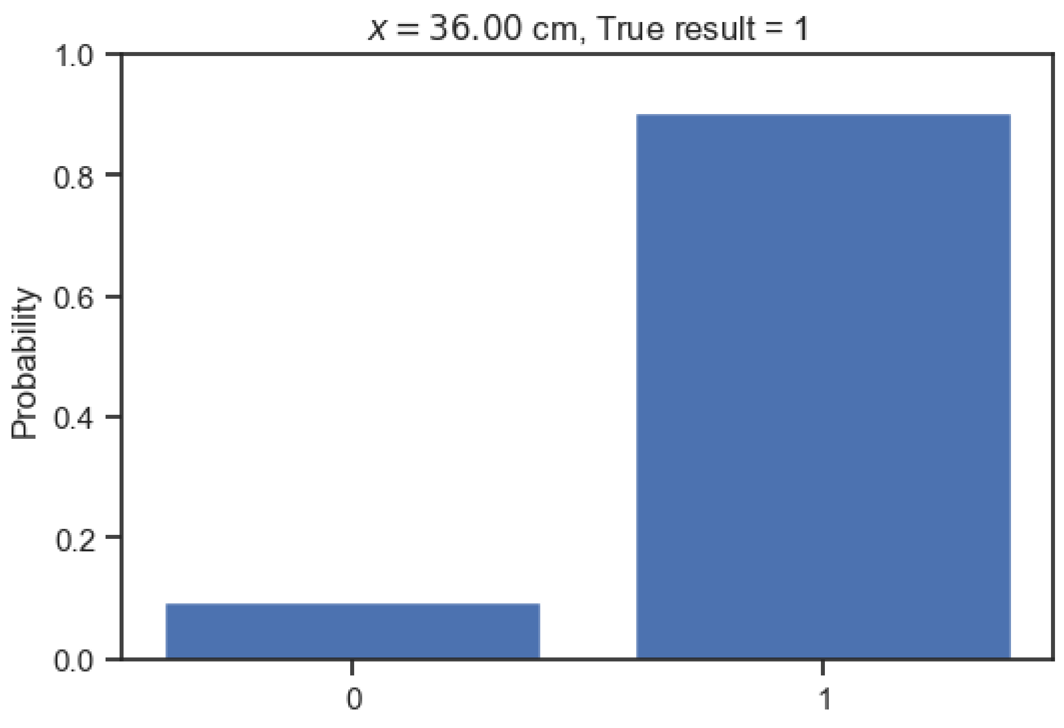
Make probabilistic predictions on the validation data:

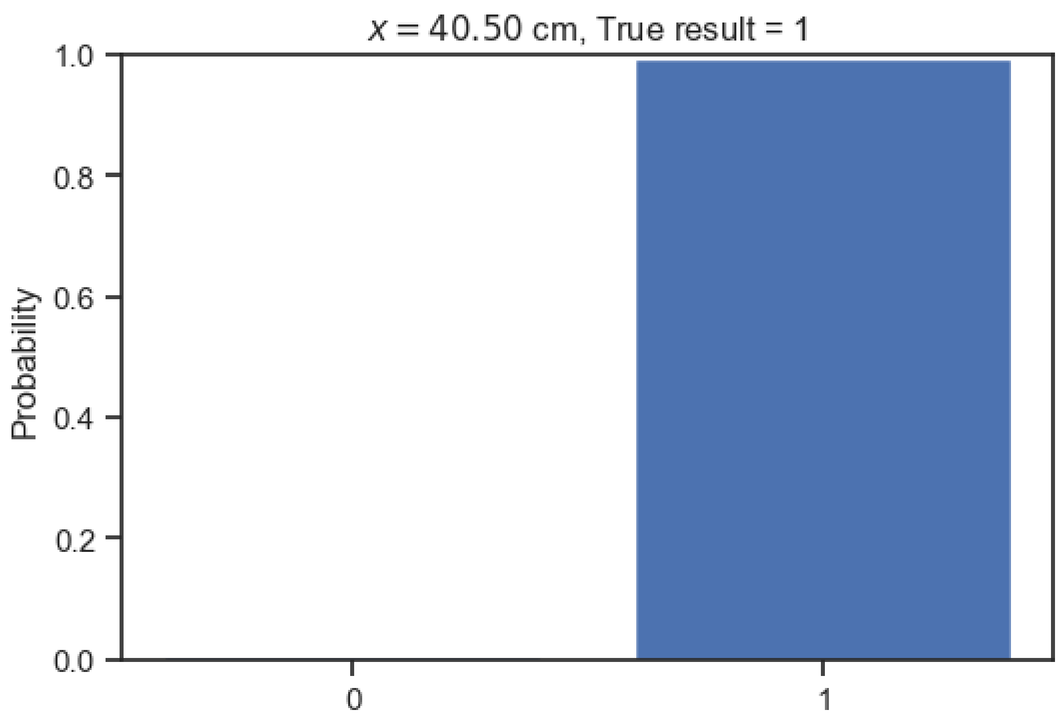
```
Phi_valid = poly.fit_transform(x_valid[:, None])
predictions = model.predict_proba(Phi_valid)
print('x\tp(y=0|x)\tp(y=1|x)\tTrue label')
print('-' * 80)
for i in range(x_valid.shape[0]):
    print(
        f"{x_valid[i]:1.2f}\t"
        + f"{predictions[i, 0]:1.2f}\t\t"
        + f"{predictions[i, 1]:1.2f}\t\t"
        + f"{y_valid[i]:d}"
    )
```

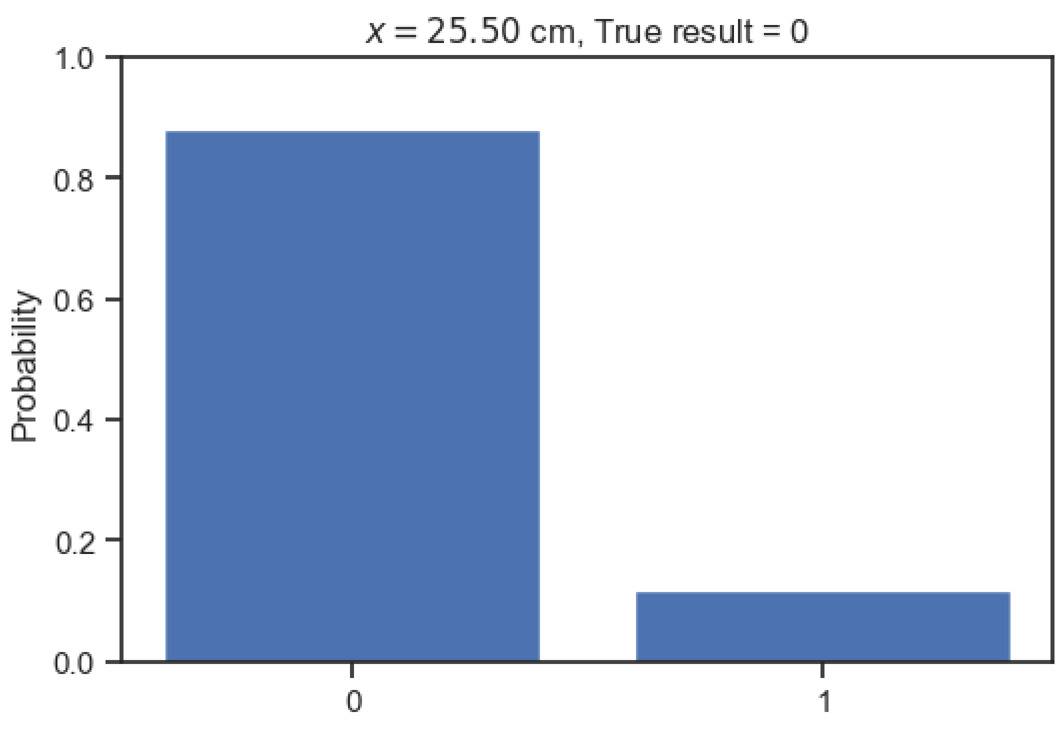
```
x       p(y=0|x)        p(y=1|x)        True label
--------------------------------------------------------------------------------
36.00   0.10            0.90            1
40.50   0.01            0.99            1
25.50   0.88            0.12            0
22.50   0.93            0.07            0
28.50   0.75            0.25            0
28.50   0.75            0.25            0
22.50   0.93            0.07            0
32.00   0.43            0.57            1
40.50   0.01            0.99            1
40.50   0.01            0.99            1
28.50   0.75            0.25            0
32.00   0.43            0.57            0
28.50   0.75            0.25            0
22.50   0.93            0.07            0
40.50   0.01            0.99            1
25.50   0.88            0.12            0
32.00   0.43            0.57            1
36.00   0.10            0.90            1
28.50   0.75            0.25            1
32.00   0.43            0.57            1
```

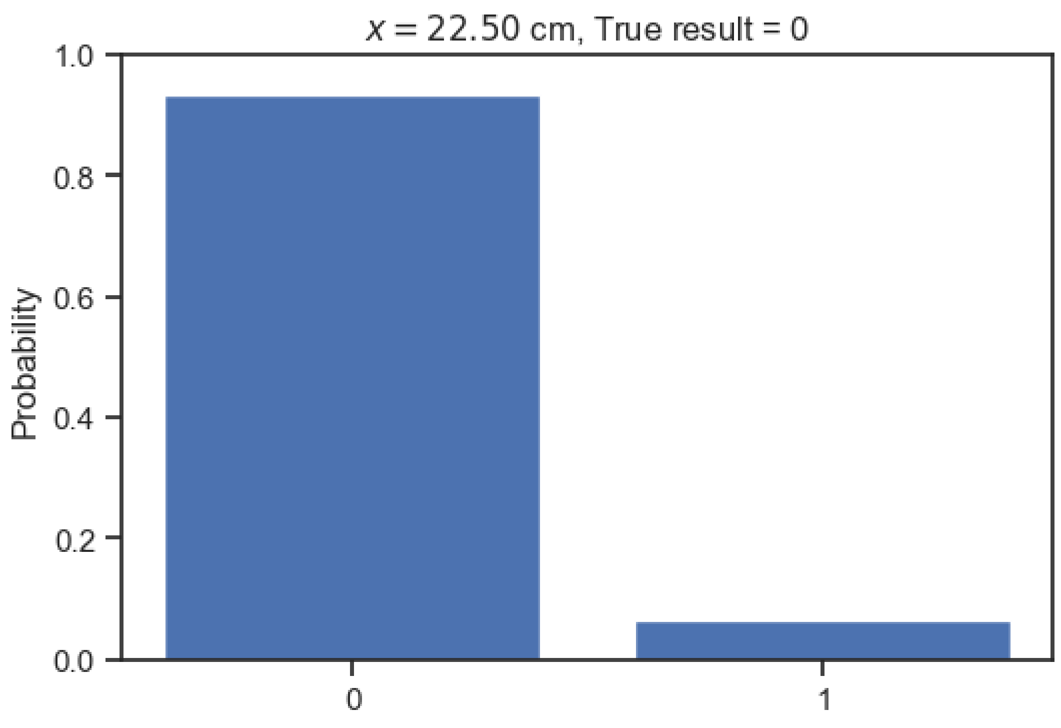
And here is a nice way to visualize these probabilities:

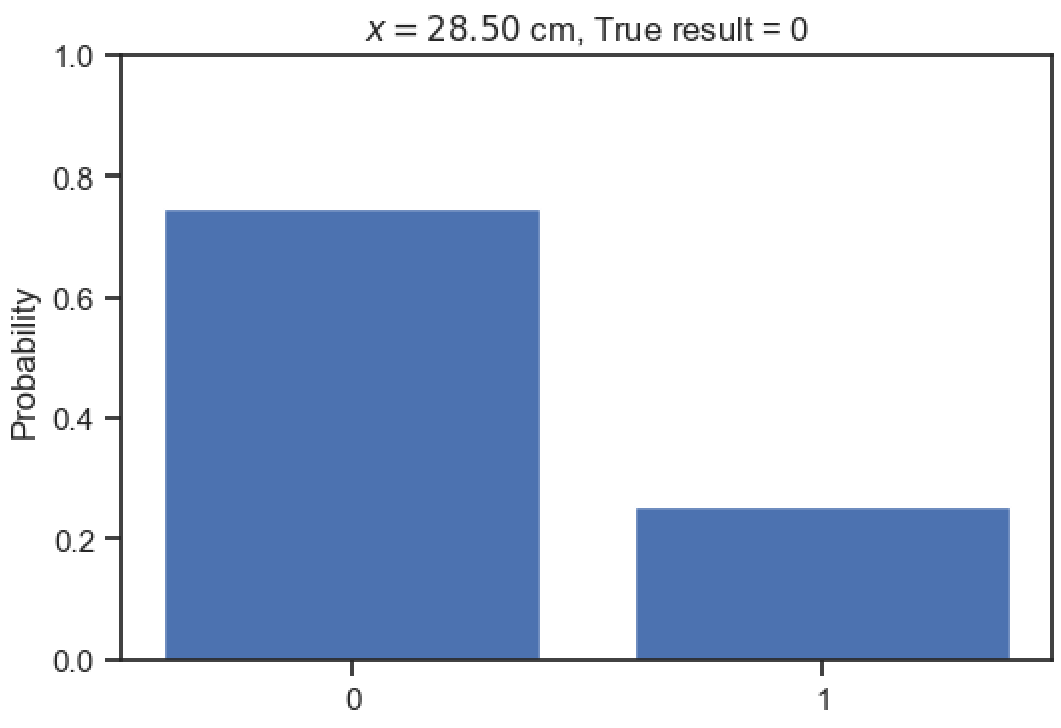
```python
for i in range(x_valid.shape[0]):
    fig, ax = plt.subplots()
    ax.bar(np.arange(2), predictions[i])
    ax.set_title(f'$x={x_valid[i]:1.2f}$ cm, True result = {y_valid[i]:d}')
    ax.set_ylim([0, 1.0])
    ax.set_xticks([0, 1])
    ax.set_xticklabels(model.classes_)
    ax.set_ylabel('Probability')
```

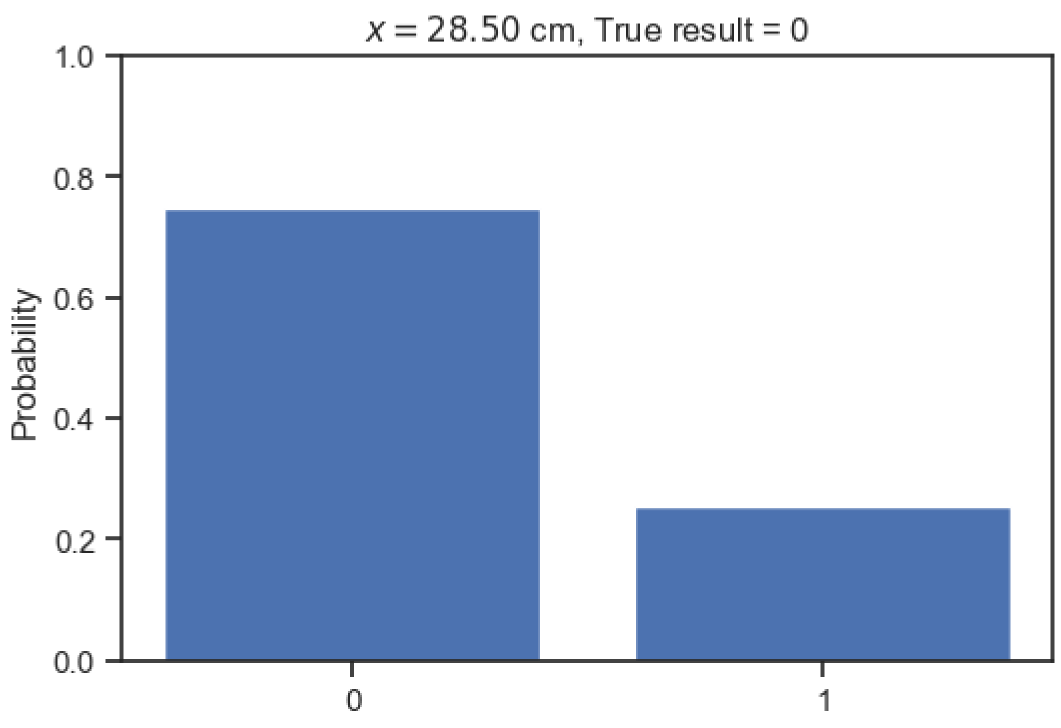


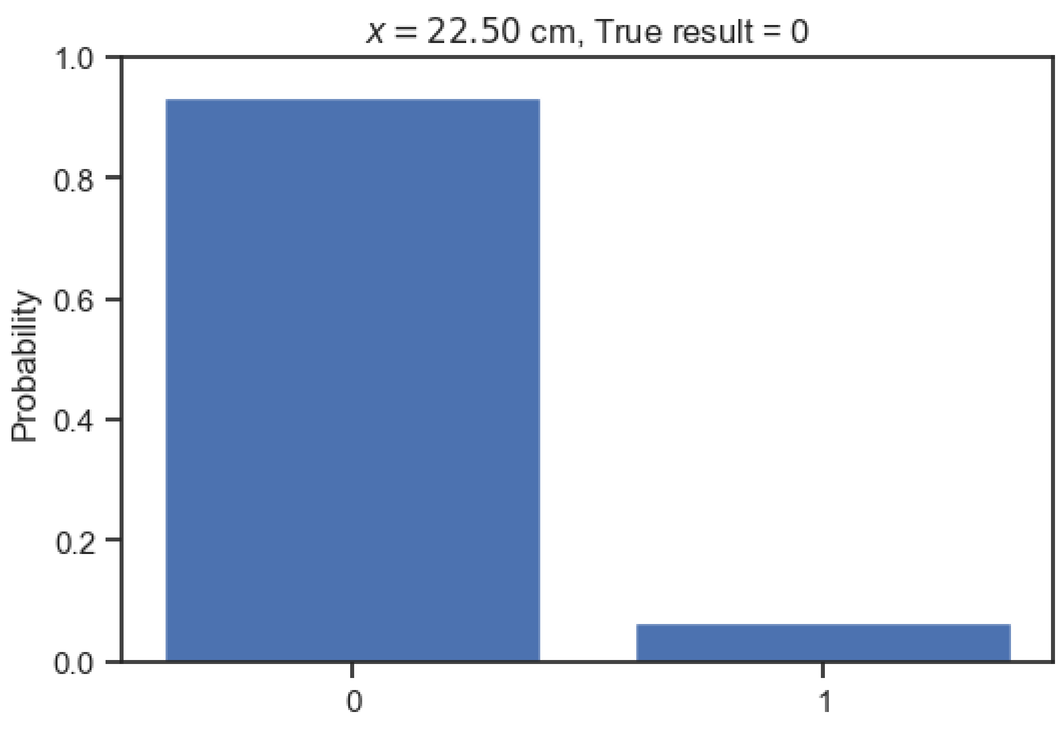


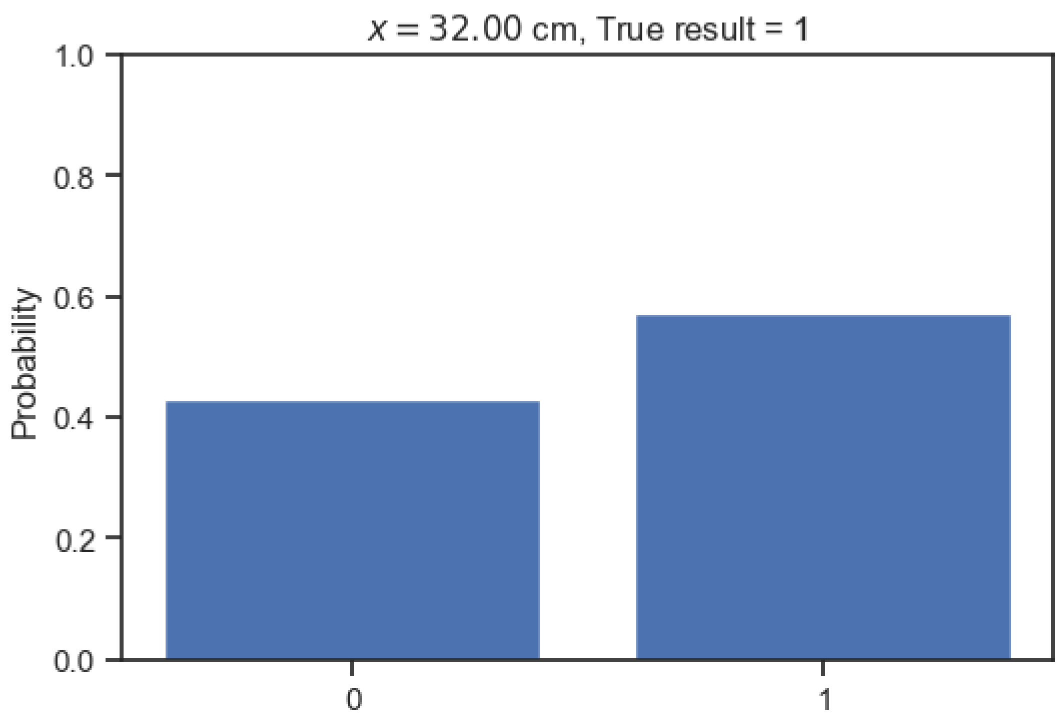

$x = 25.50$ cm, True result = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

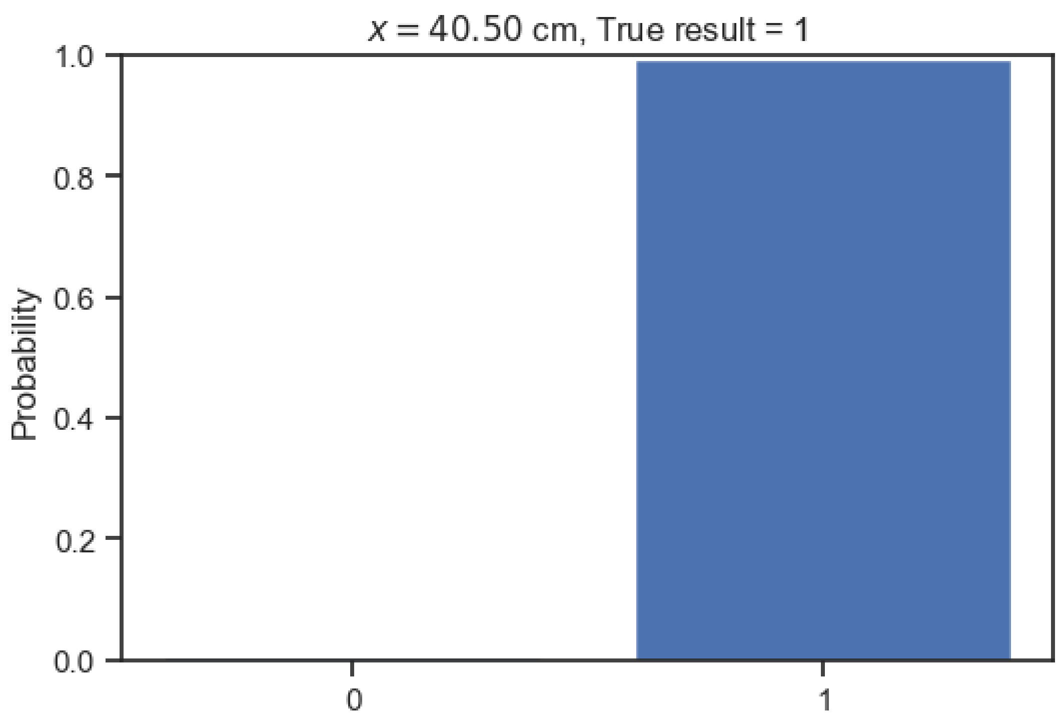

$x = 22.50$ cm, True result = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

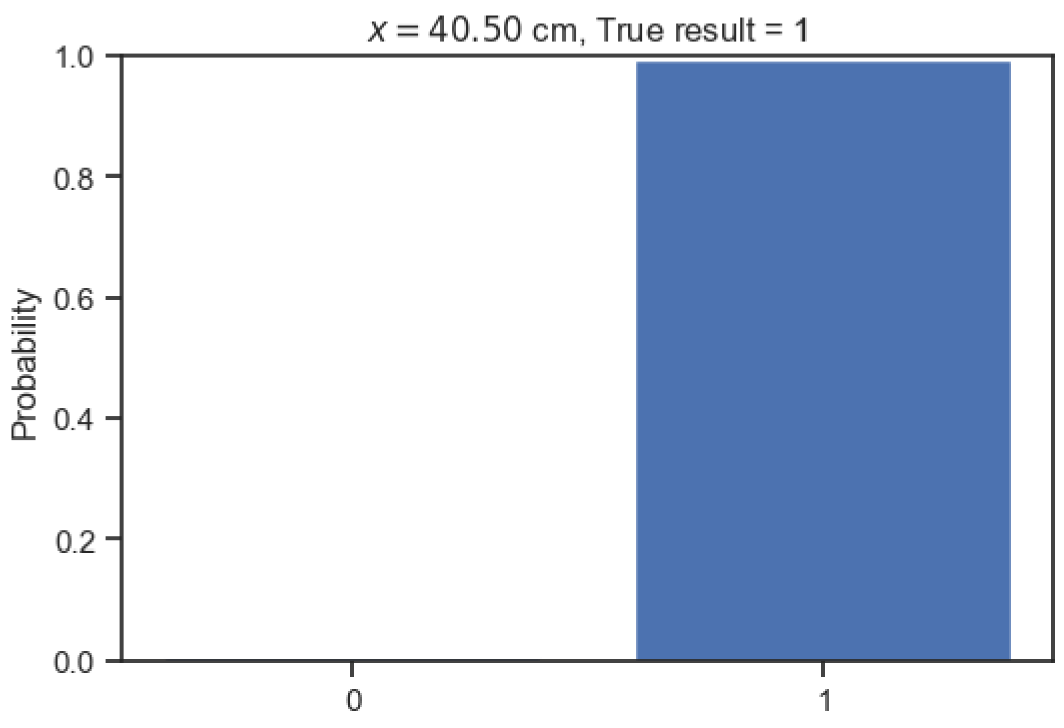

$x = 28.50$ cm, True result = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

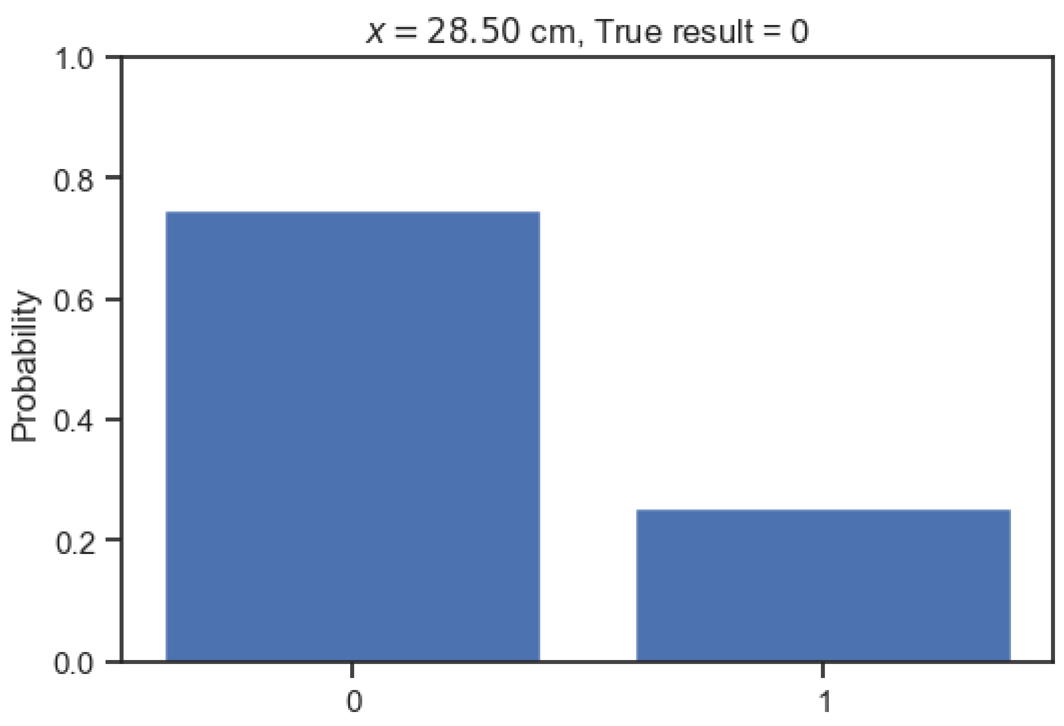

$x = 28.50$ cm, True result = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

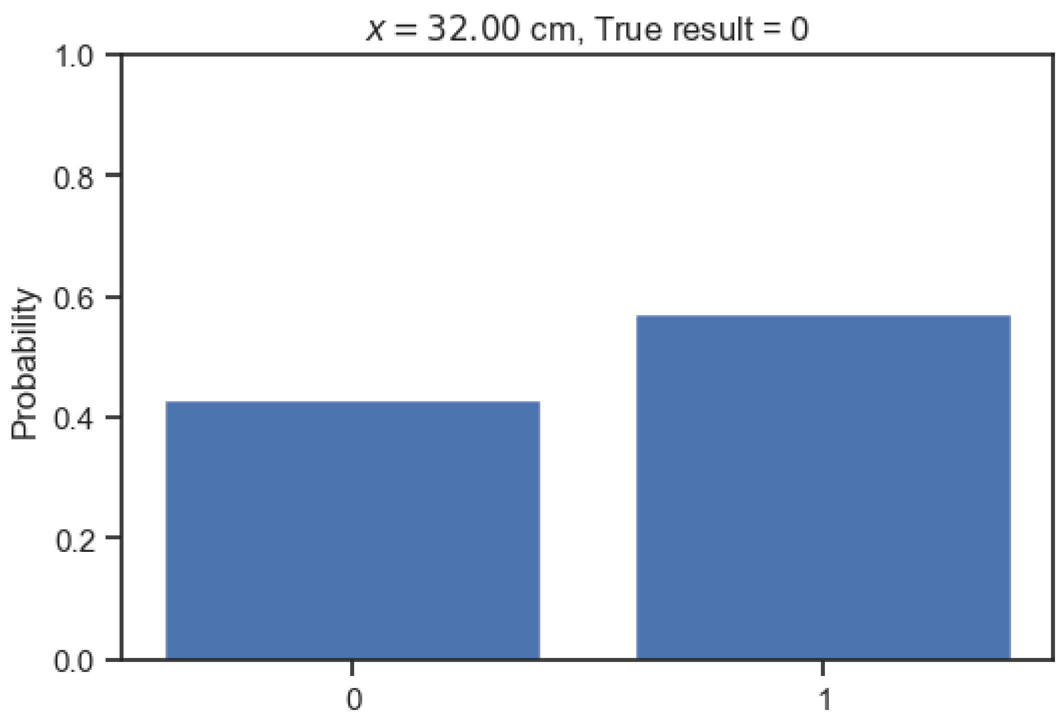

$x = 22.50$ cm, True result = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

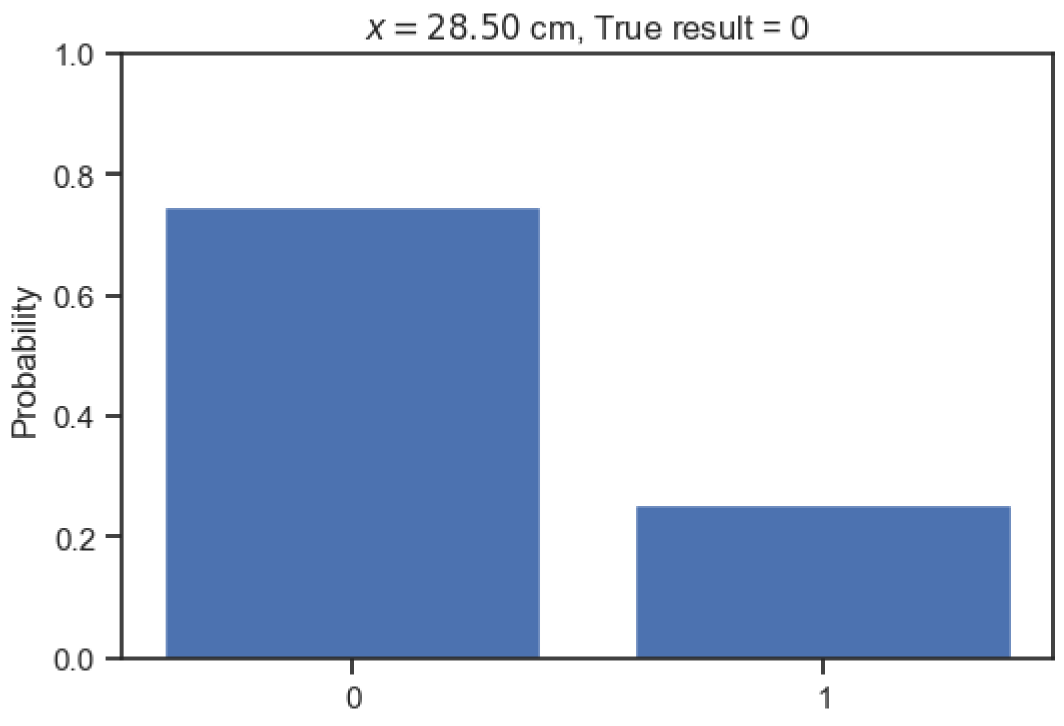

$x = 32.00$ cm, True result = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

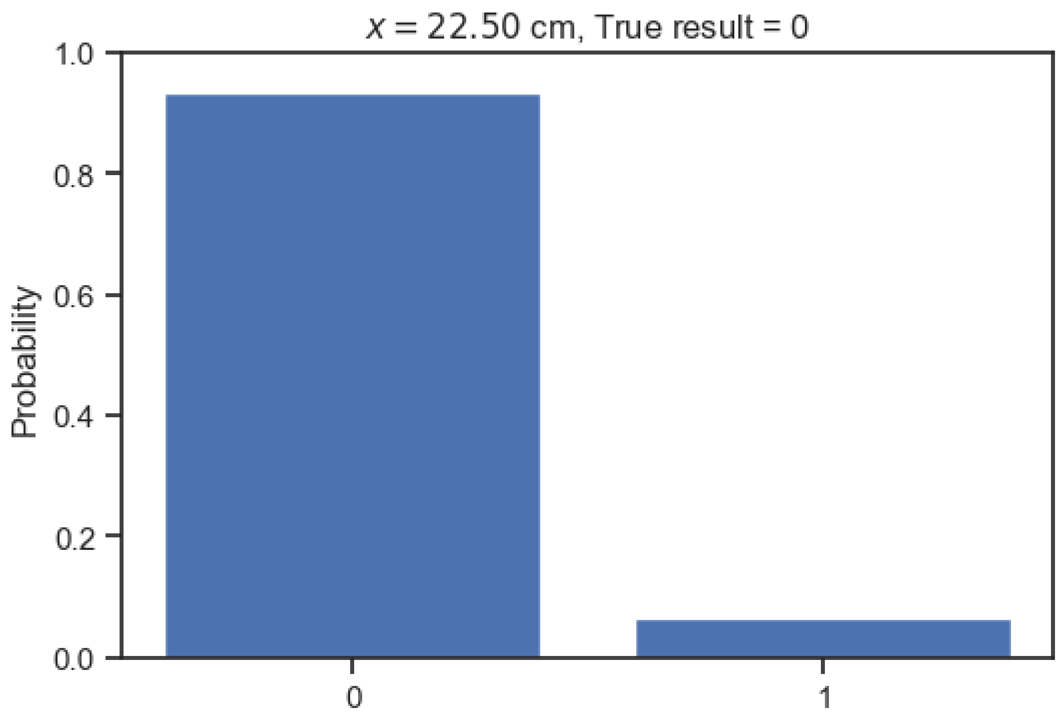

$x = 40.50$ cm, True result = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

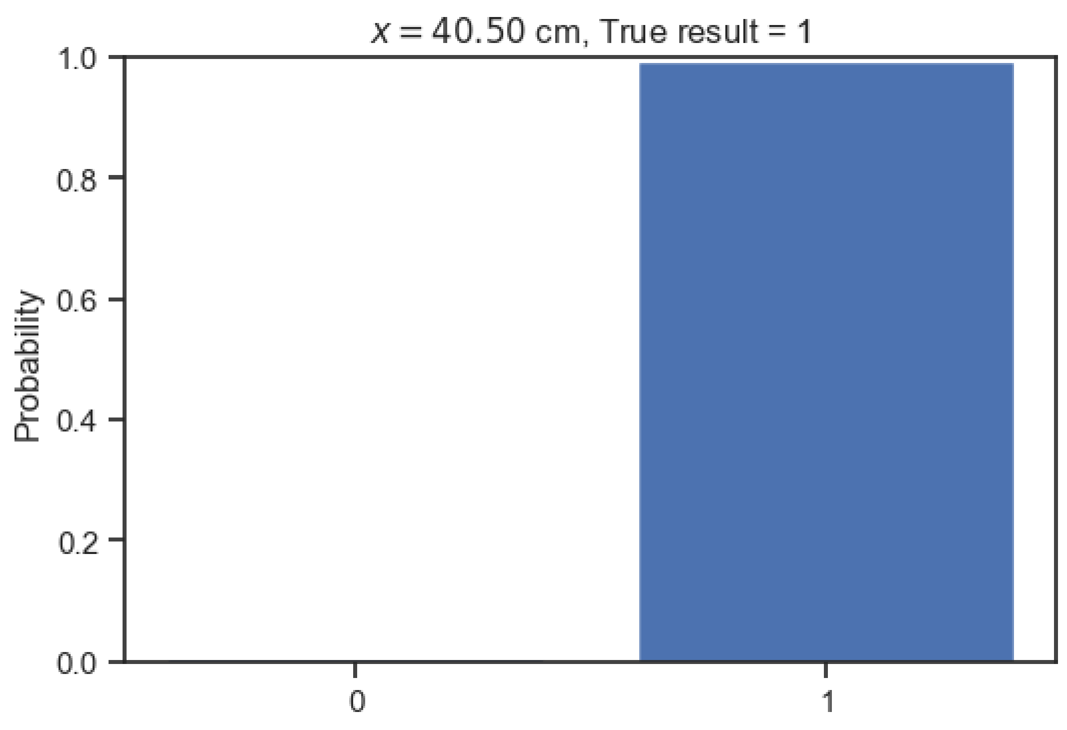

$x = 40.50$ cm, True result = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

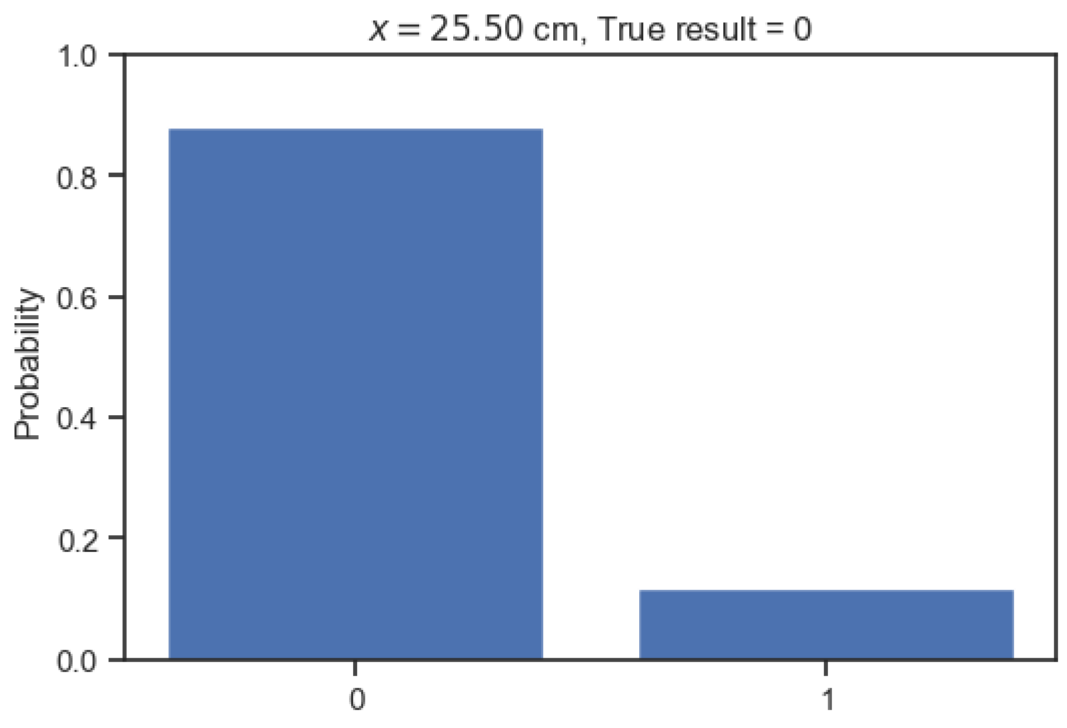

$x = 28.50$ cm, True result = 0
Probability
0.0
0.2
0.4
0.6
0.8
1.0
0
1

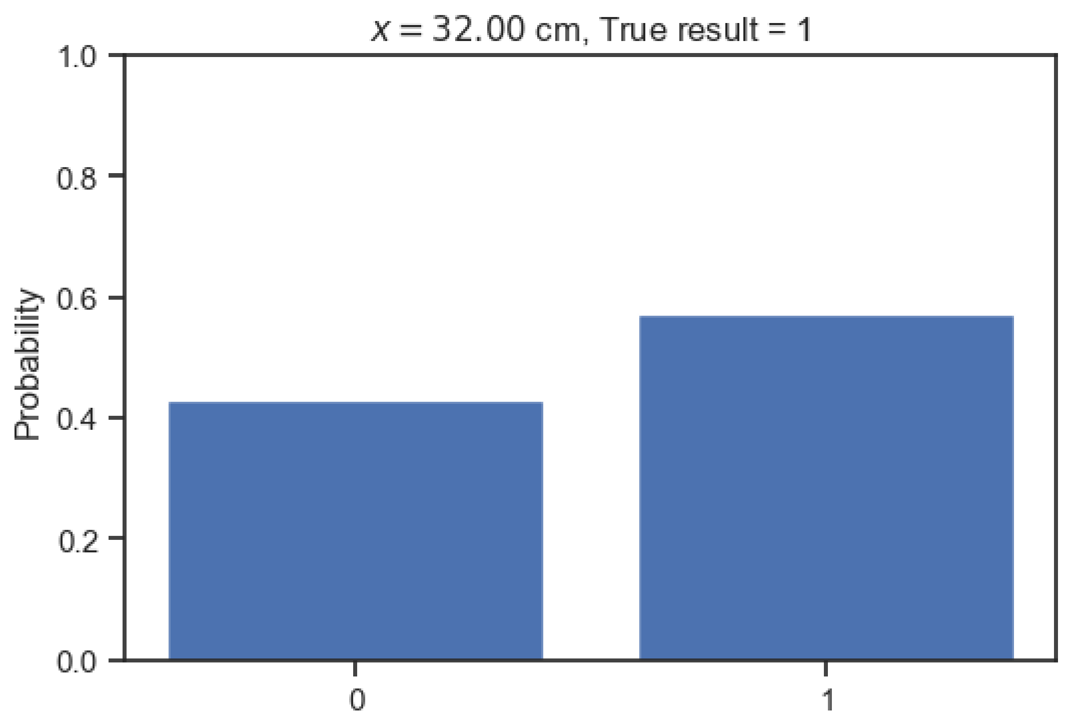

$x = 32.00$ cm, True result = 0
Probability
0.0
0.2
0.4
0.6
0.8
1.0
0
1

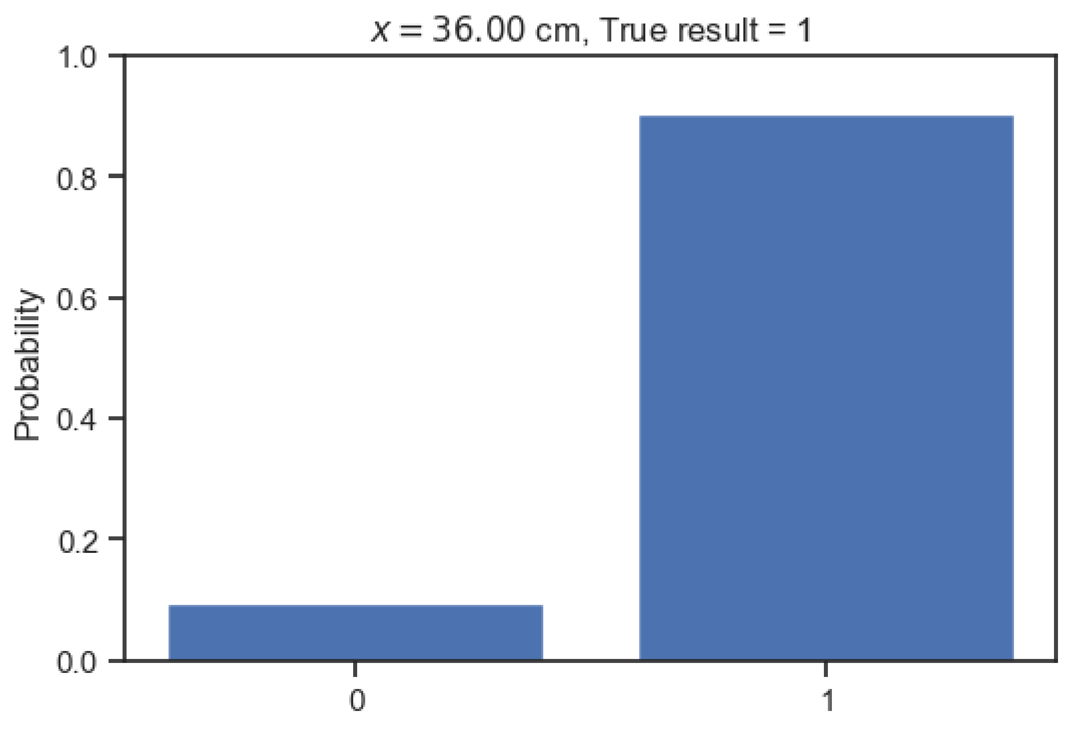

$x = 28.50$ cm, True result = 0
Probability
0.0
0.2
0.4
0.6
0.8
1.0
0
1

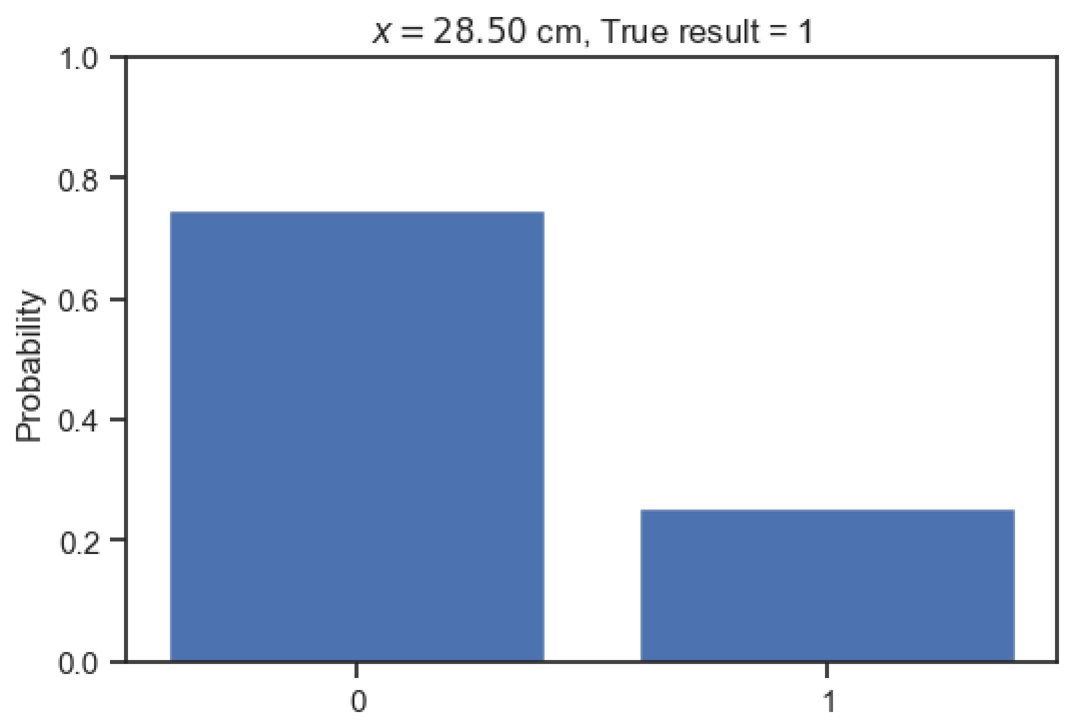

$x = 22.50$ cm, True result = 0
Probability
0.0
0.2
0.4
0.6
0.8
1.0
0
1

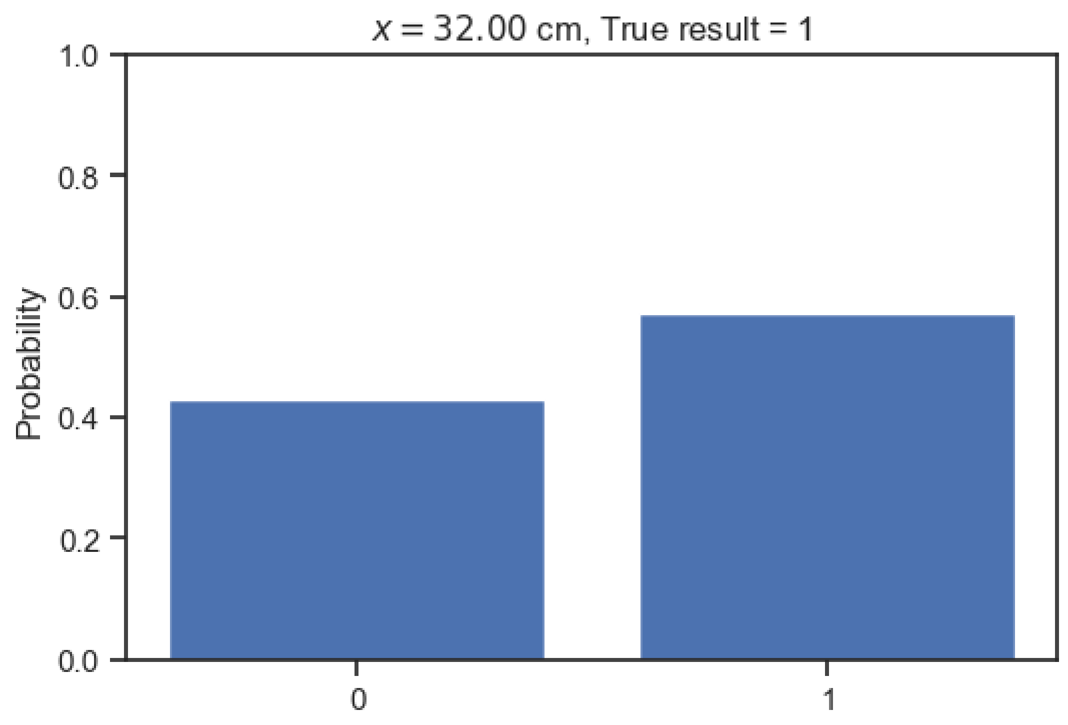

$x = 40.50$ cm, True result = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1


$x = 25.50$ cm, True result = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1


$x = 32.00$ cm, True result = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1


$x = 36.00$ cm, True result = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1





Now we are ready to pose and solve the decision-making problem. We just need to define a cost matrix:

```
# c_00 = cost of correctly picking 0 when 0 is true
# c_01 = cost of wrongly picking 0 when 1 is true
# c_11 = cost of correctly picking 1 when 1 is true
# c_10 = cost of wrongly picking 1 when 0 is true
cost_matrix = np.array(
    [
        [0.0, 1.0],
        [1.0, 0.0]
    ]
)
```

Here is some code that computes the expected cost of each choice given the predicted probabilities:

```
def expected_cost(cost_matrix, prediction_prob):
    """Calculate the expected cost of each decision.

    Arguments
    cost_matrix     --  A D x D matrix. `cost_matrix[i, j]`
                        is the cost of picking `i` and then
                        `j` happens.
    prediction_prob --  An array with D elements containing
                        the probability that each event
                        happens.
    """
    assert cost_matrix.ndim == 2
    D = cost_matrix.shape[0]
    assert cost_matrix.shape[1] == D
    assert prediction_prob.ndim == 1
    assert prediction_prob.shape[0] == D
    res = np.zeros((2,))
    for i in range(2):
        res[i] = (
            cost_matrix[i, 0] * prediction_prob[0]
            + cost_matrix[i, 1] * prediction_prob[1]
        )
    return res
```

checking for errors in the inputs

each element of res is the expected cost of a particular decision

As a demonstration, here is the expected cost of each decision for the first few validation points. We will put a star (*) next to the choice with minimum cost.

pick the y that minimizes the expected loss, so:

```
print('x\tCost of 0\tCost of 1\tTrue label\tChoice')
print('-' * 80)
for i in range(x_valid.shape[0]):
    exp_c = expected_cost(cost_matrix, predictions[i])
    line = f'{x_valid[i]:1.2f}\t{exp_c[0]:1.2f}'
    tmp = f'\t\t{exp_c[1]:1.2f}'
    correct_choice = True
    if exp_c[0] < exp_c[1]:
        line += '*'
        if y_valid[i] == 1:
            correct_choice = False
    else:
        tmp += '*'
        if y_valid[i] == 0:
            correct_choice = False
    line += tmp + f'\t\t{y_valid[i]}'
    if correct_choice:
        line += '\t\tCORRECT'
    else:
        line += '\t\tWRONG'
    print(line)
```

we want to pick y = 0 <-- if the cost of picking 0 is less

add marker to line so it is added before tmp

if the validation target is 1

we want to pick y = 1 <-- if the cost of picking 1 is less

add * to temp to mark the second label

if the validation target is 0

* IT ALL BOILS DOWN TO MINIMIZING THE EXPECTED COST *

Printing the result, using the above function expected_cost()

```
x       Cost of 0       Cost of 1       True label      Choice
--------------------------------------------------------------------------------
36.00   0.90            0.10*           1               CORRECT
40.50   0.99            0.01*           1               CORRECT
25.50   0.12*           0.88            0               CORRECT
22.50   0.07*           0.93            0               CORRECT
28.50   0.25*           0.75            0               CORRECT
28.50   0.25*           0.75            0               CORRECT
22.50   0.07*           0.93            0               CORRECT
32.00   0.57            0.43*           1               CORRECT
40.50   0.99            0.01*           1               CORRECT
40.50   0.99            0.01*           1               CORRECT
28.50   0.25*           0.75            0               CORRECT
32.00   0.57            0.43*           0               WRONG
28.50   0.25*           0.75            0               CORRECT
22.50   0.07*           0.93            0               CORRECT
40.50   0.99            0.01*           1               CORRECT
25.50   0.12*           0.88            0               CORRECT
32.00   0.57            0.43*           1               CORRECT
36.00   0.90            0.10*           1               CORRECT
28.50   0.25*           0.75            1               WRONG
32.00   0.57            0.43*           1               CORRECT
```

Notice that most of the choices are correct. But there are some wrong choices. The particularly bad wrong choices are the ones where we predict 0 (no explosion), but there is actually an explosion. Are there any such cases? there appears to be one such case

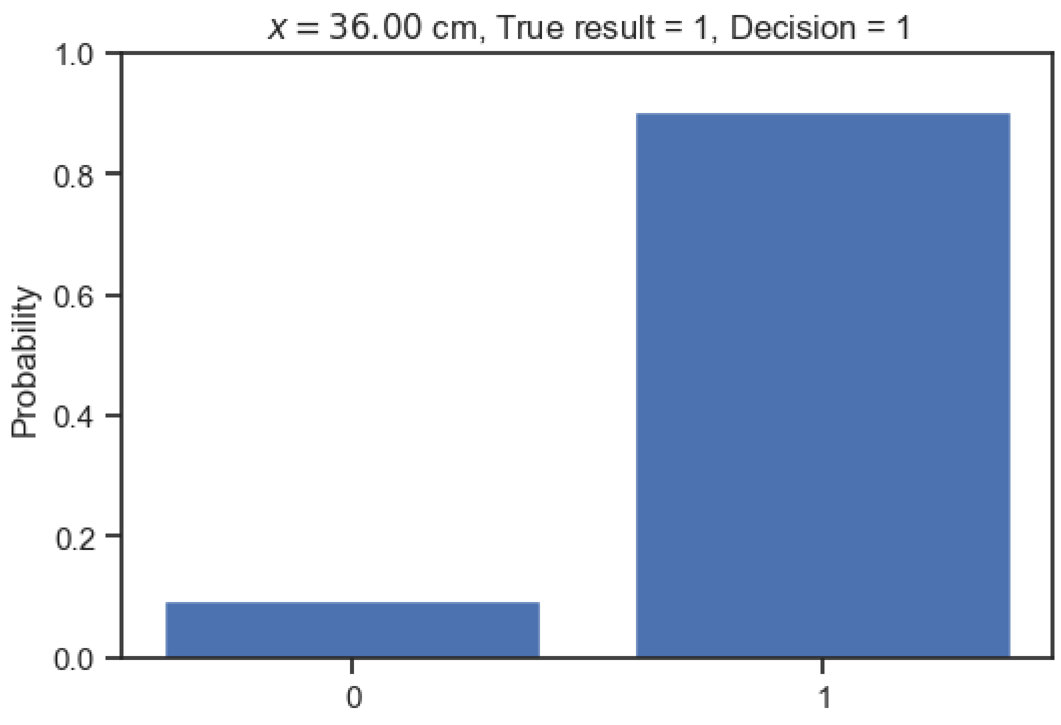
Let me now show you another very nice way to compute the expected cost for all the validation points in one line. This is using the einsum function (Einstein summation convention). It takes a while to understand what it does, but if you do you can shorten by a lot your linear algebra code. The idea is that repeated indices are summed over.

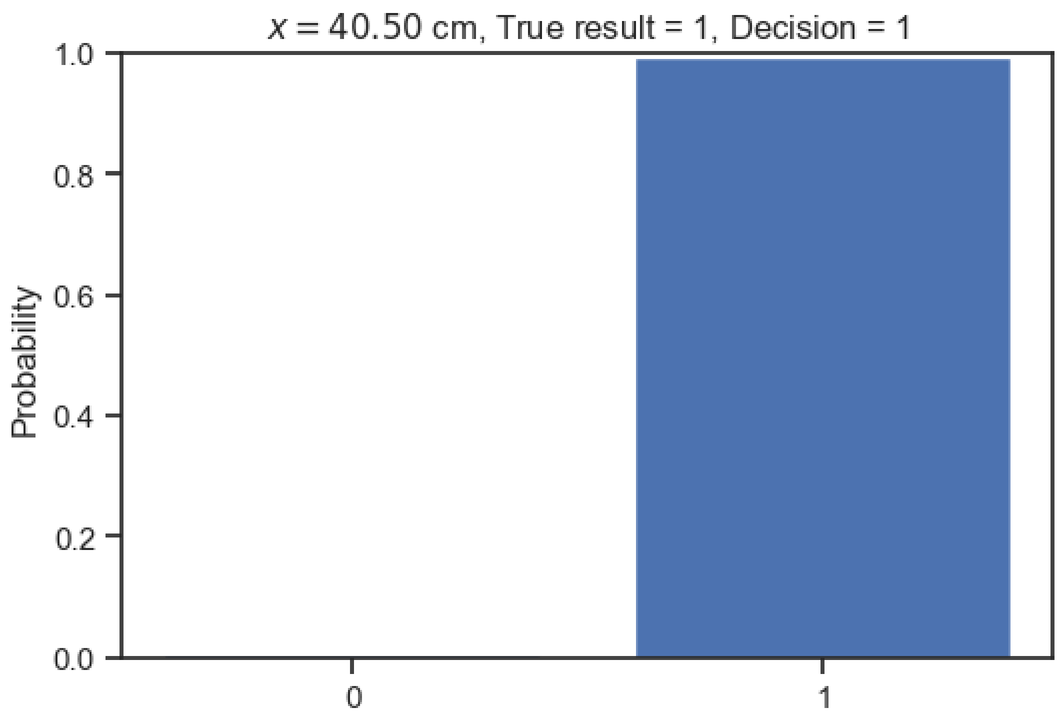
```
exp_cost = np.einsum('ij,kj->ki', cost_matrix, predictions)
print(exp_cost)
```

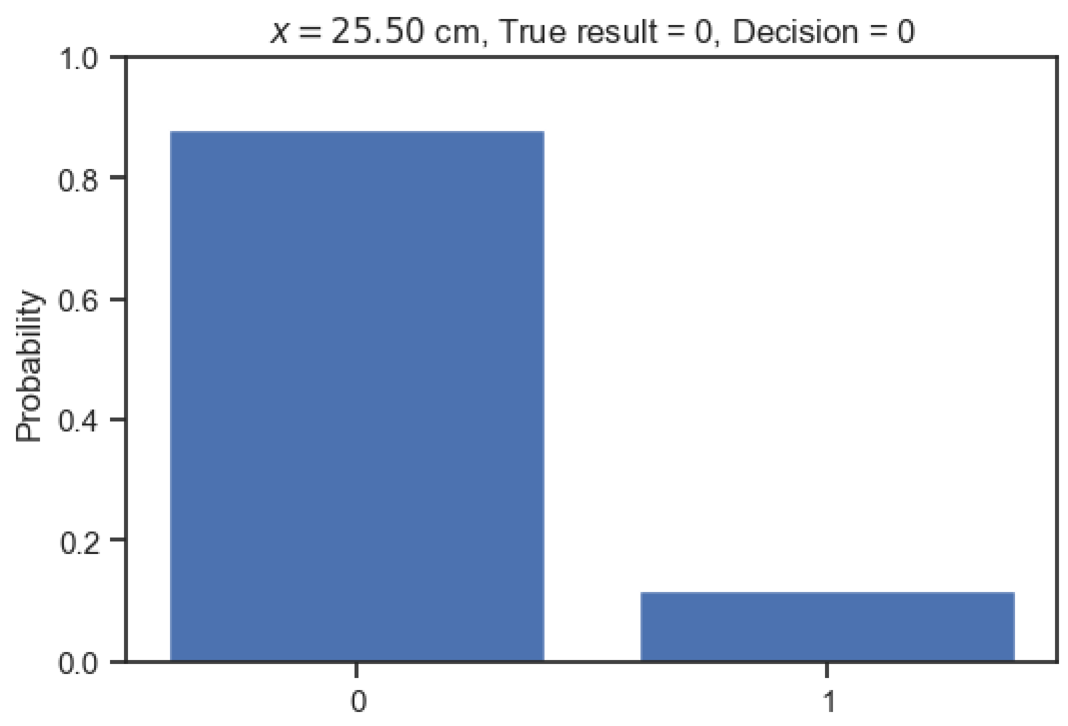
```
[[0.904 0.096]
 [0.993 0.007]
 [0.12  0.88 ]
 [0.065 0.935]
 [0.253 0.747]
 [0.253 0.747]
 [0.065 0.935]
 [0.571 0.429]
 [0.993 0.007]
 [0.993 0.007]
 [0.253 0.747]
 [0.571 0.429]
 [0.253 0.747]
 [0.065 0.935]
 [0.993 0.007]
 [0.12  0.88 ]
 [0.571 0.429]
 [0.904 0.096]
 [0.253 0.747]
 [0.571 0.429]]
```

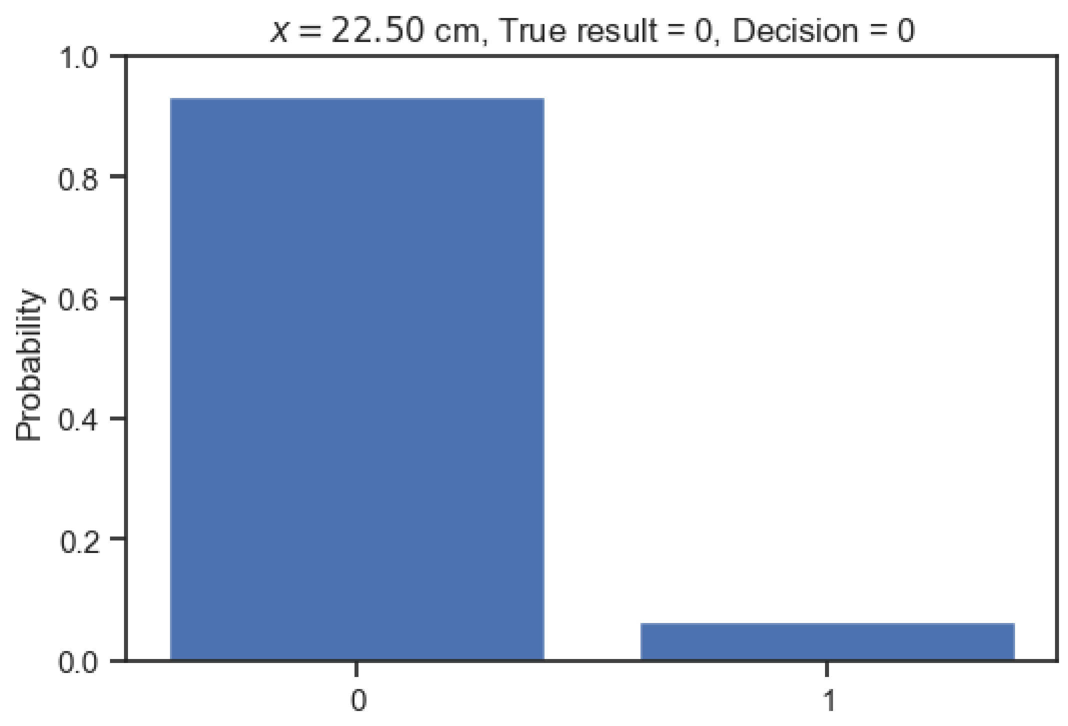
Here is yet another way to visualize the decisions of binary classification:

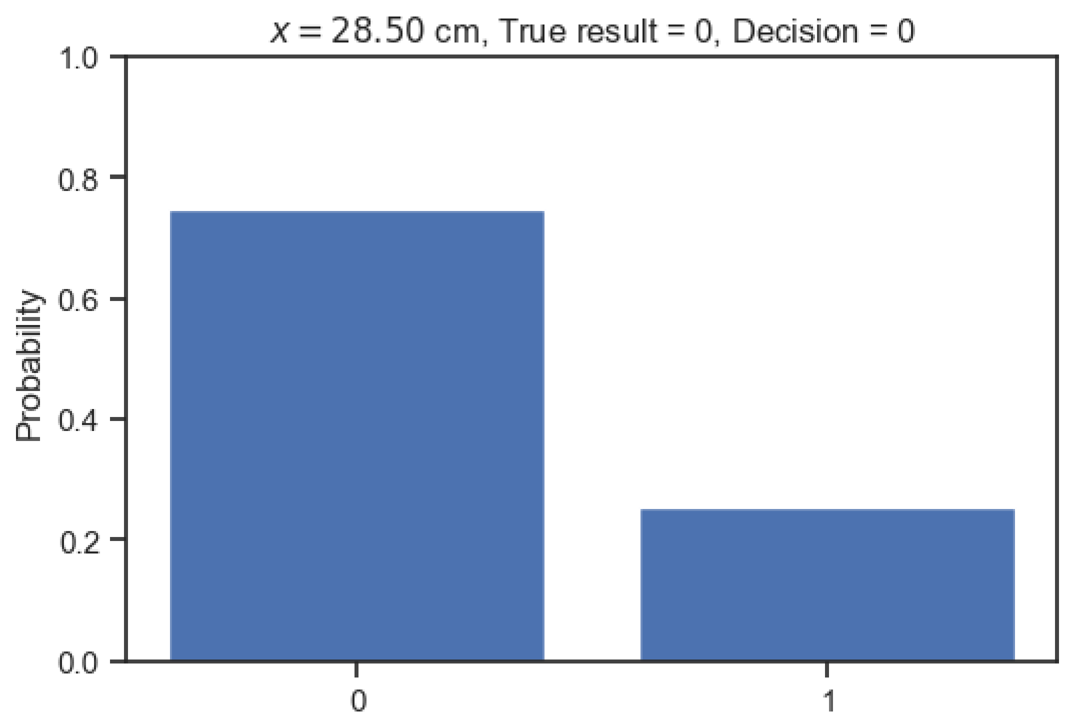
```
for i in range(x_valid.shape[0]):
    # Make decision
    decision = model.classes_[np.argmin(exp_cost[i])]
    fig, ax = plt.subplots()
    ax.bar(np.arange(2), predictions[i])
    ax.set_title(
        f'$x={x_valid[i]:1.2f}$ cm, '
        + f'True result = {y_valid[i]:d}, '
        + f'Decision = {decision:d}'
    )
    ax.set_ylim([0, 1.0])
    ax.set_xticks([0, 1])
    ax.set_xticklabels(model.classes_)
    ax.set_ylabel('Probability');
```

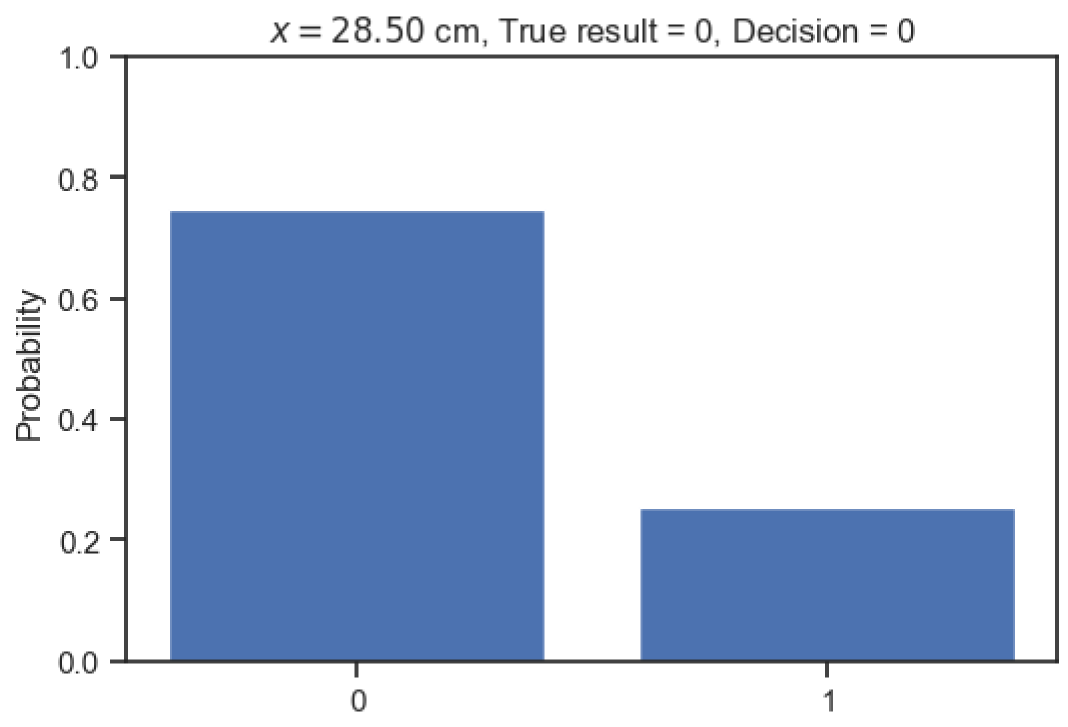


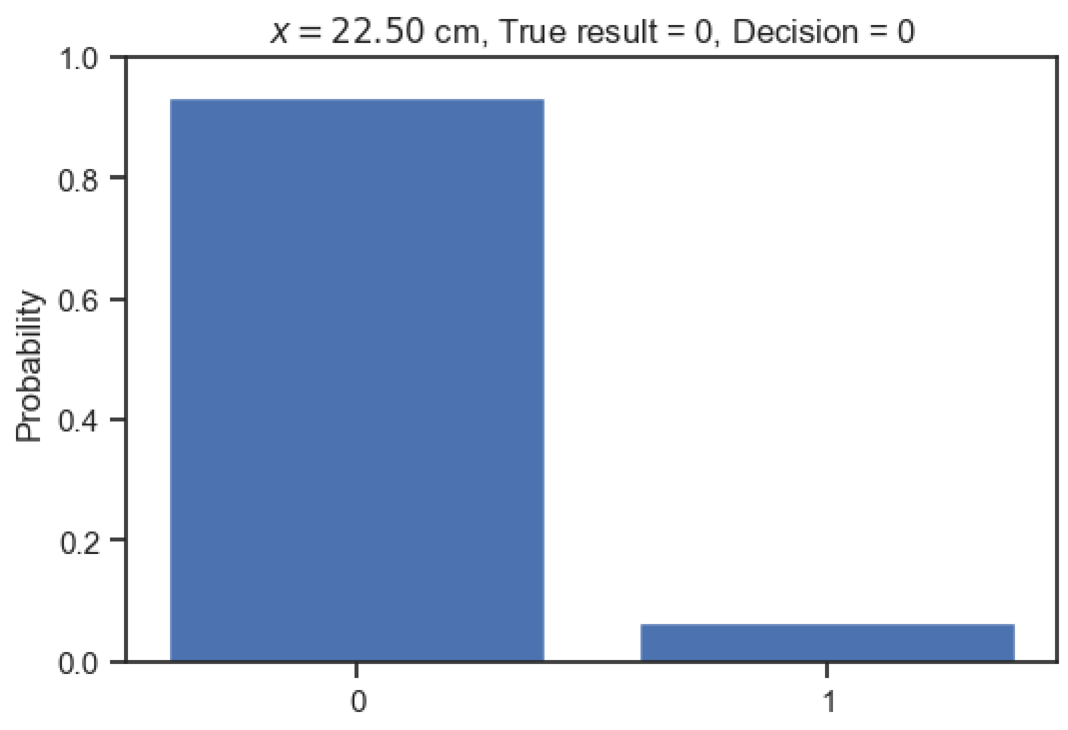


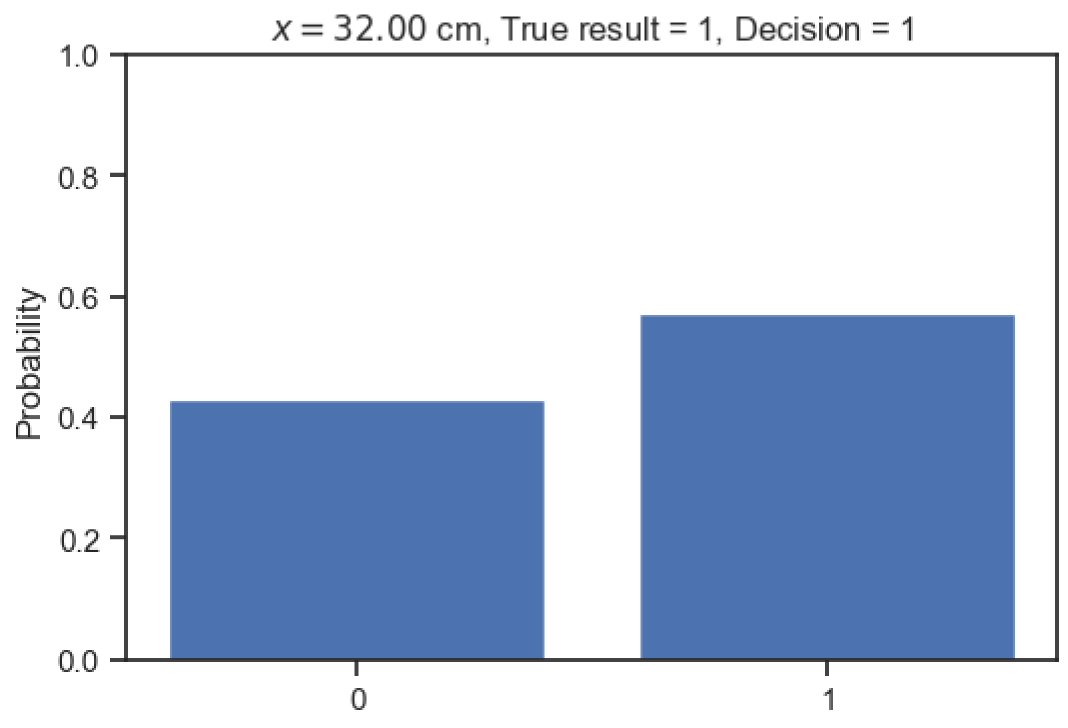

$x = 25.50$ cm, True result = 0, Decision = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

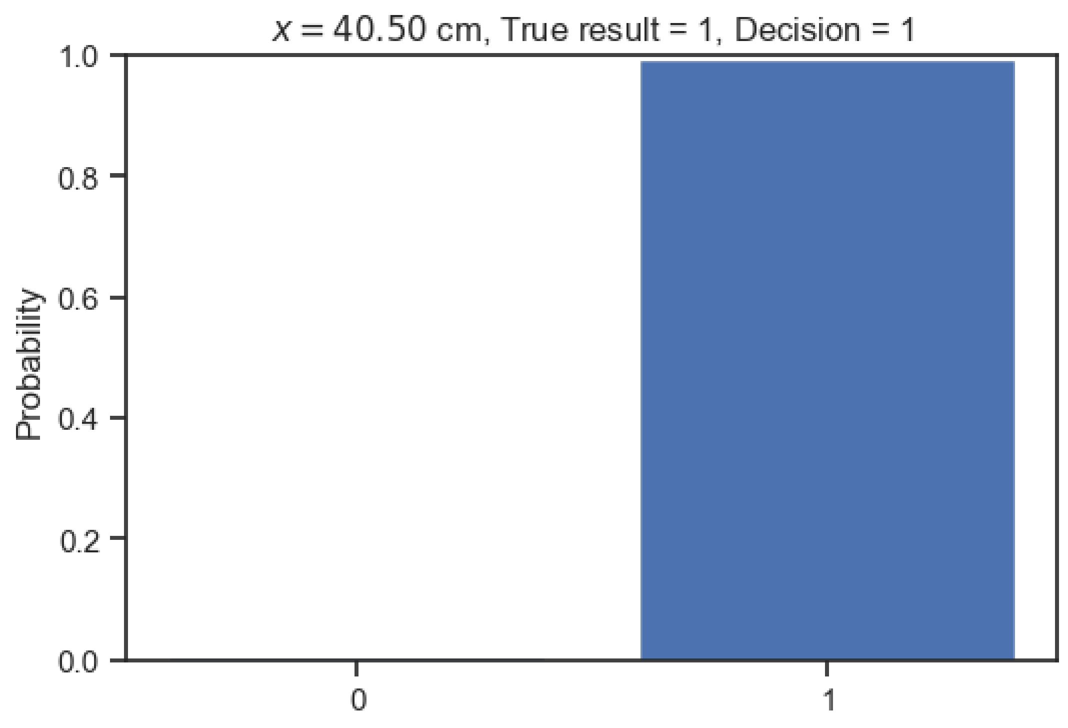

$x = 22.50$ cm, True result = 0, Decision = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

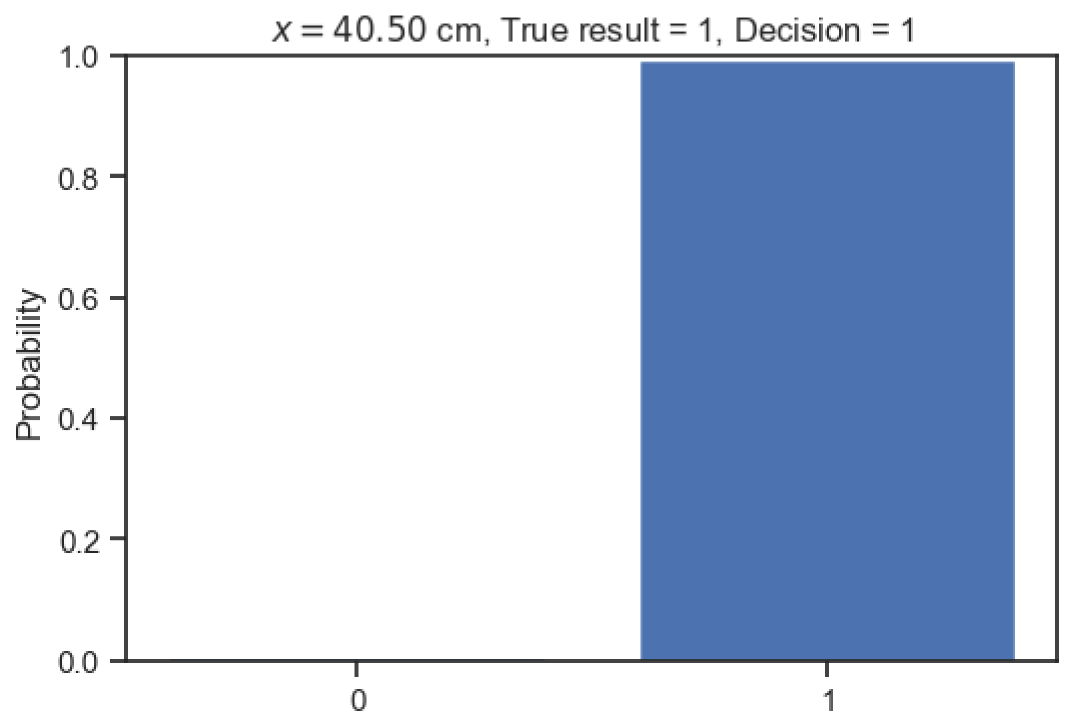

$x = 28.50$ cm, True result = 0, Decision = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

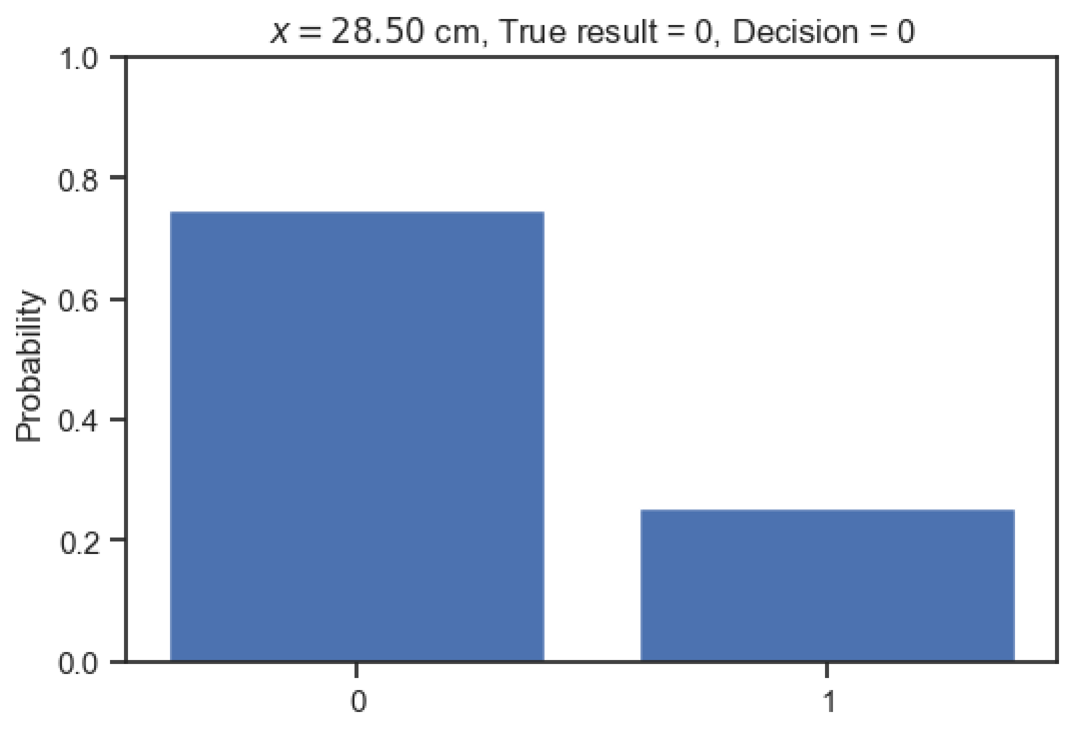

$x = 28.50$ cm, True result = 0, Decision = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

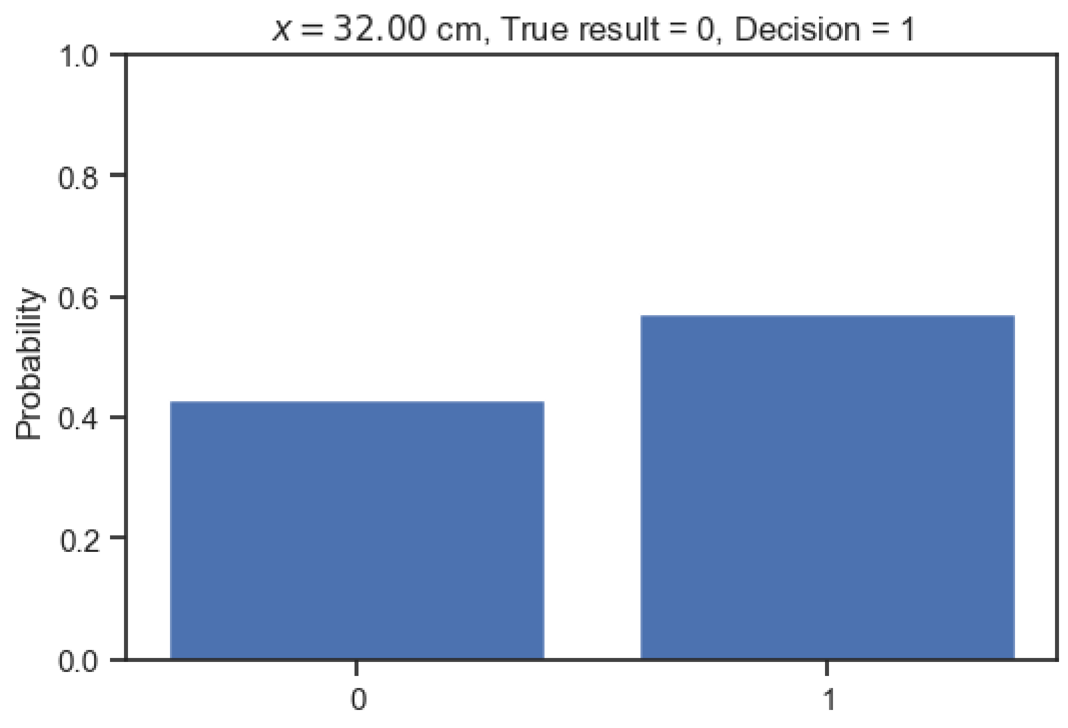

$x = 22.50$ cm, True result = 0, Decision = 0
Probability
0
1

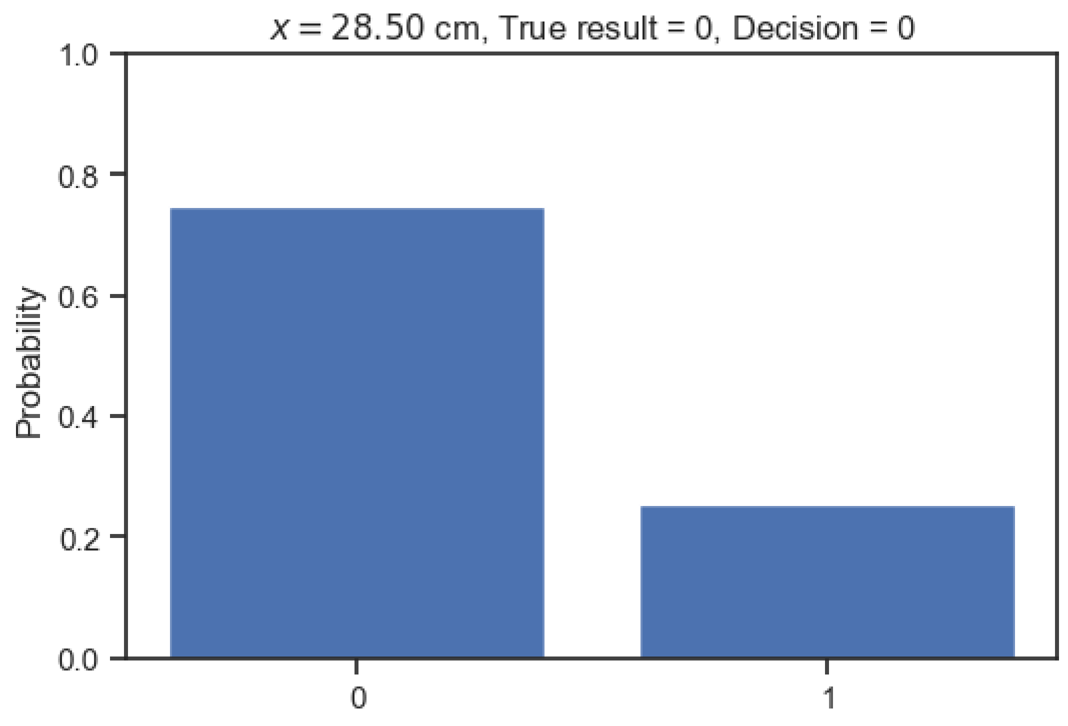

$x = 32.00$ cm, True result = 1, Decision = 1
Probability
0
1

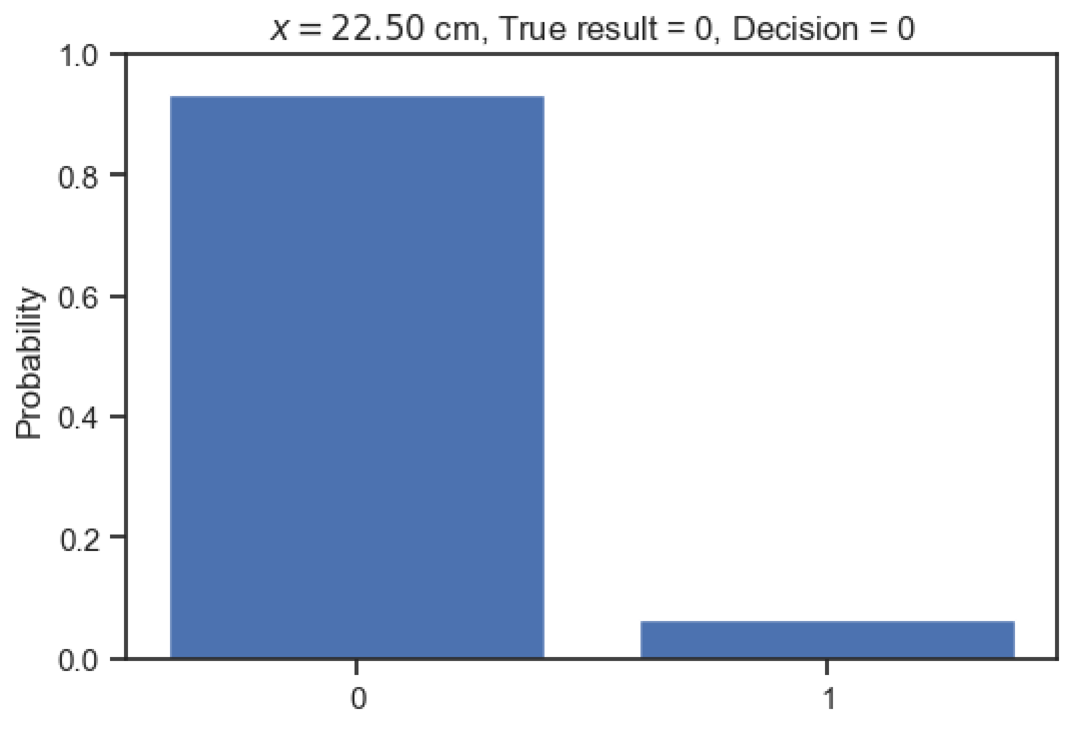

$x = 40.50$ cm, True result = 1, Decision = 1
Probability
0
1

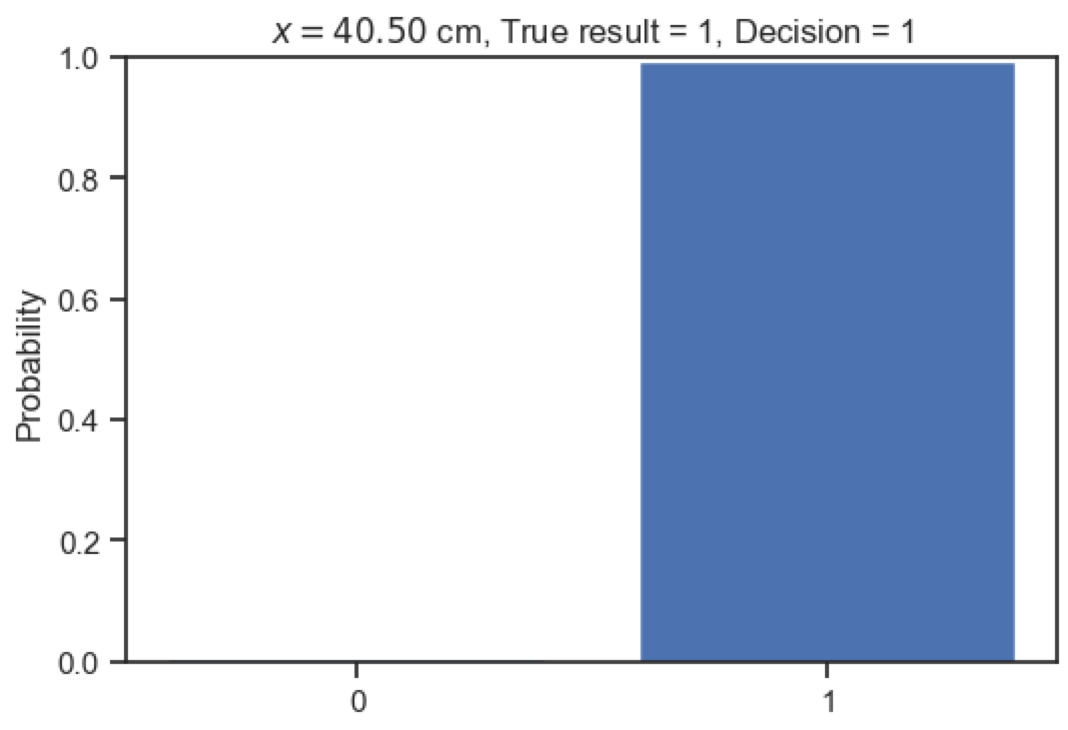

$x = 40.50$ cm, True result = 1, Decision = 1
Probability
0
1

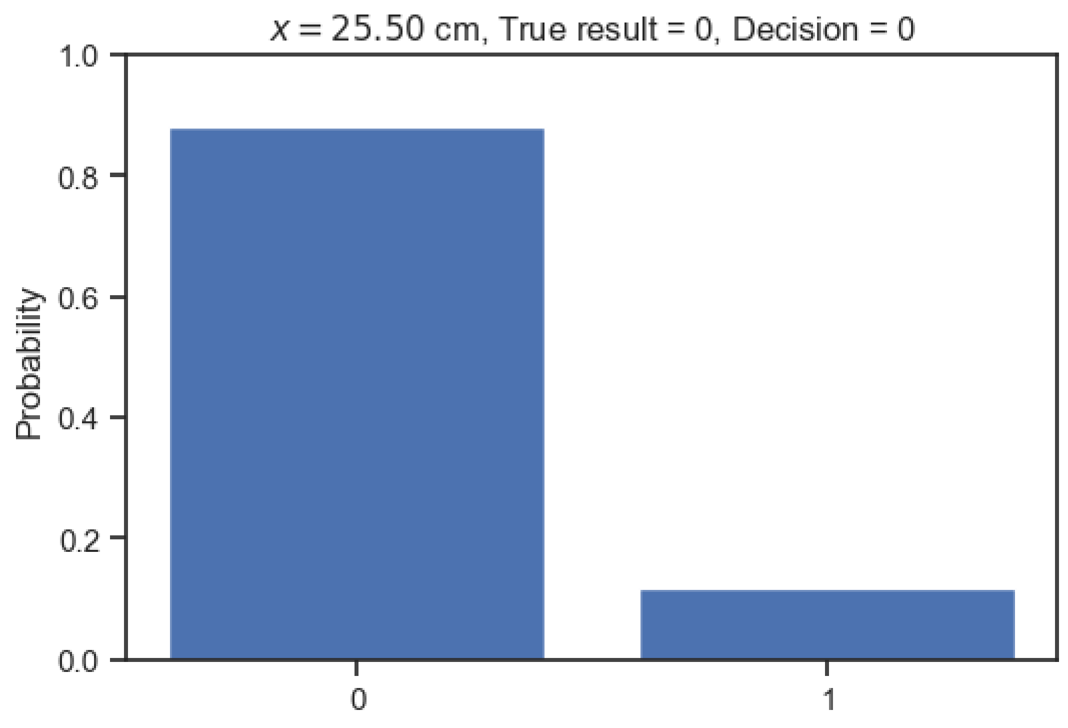

$x = 28.50$ cm, True result = 0, Decision = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

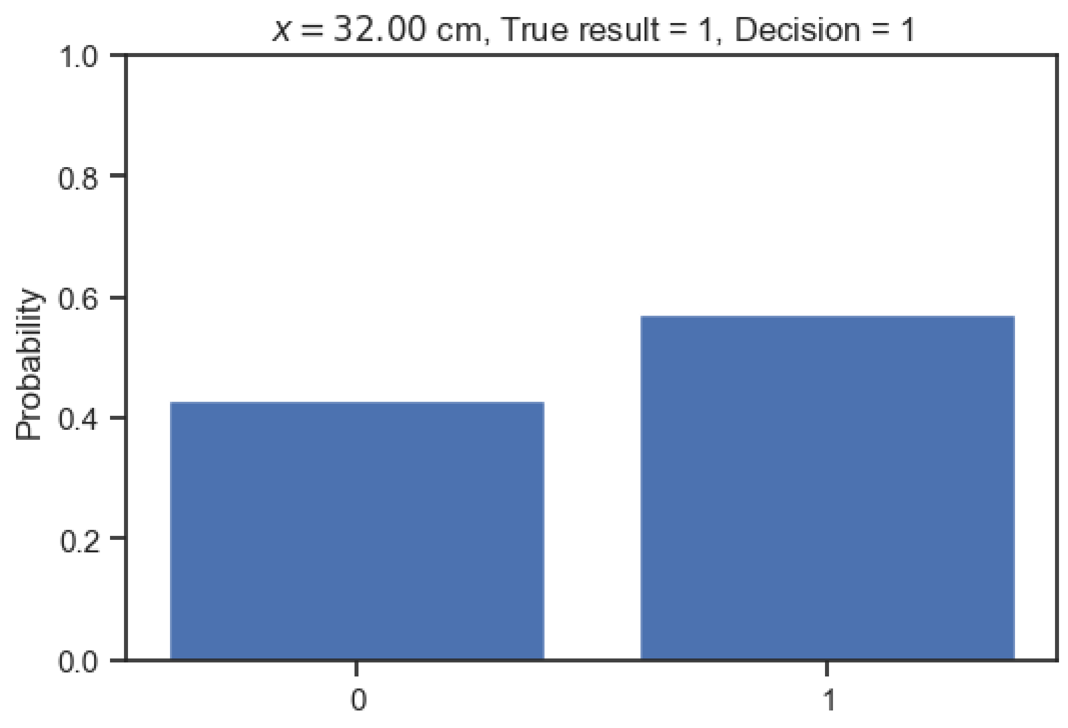

$x = 32.00$ cm, True result = 0, Decision = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

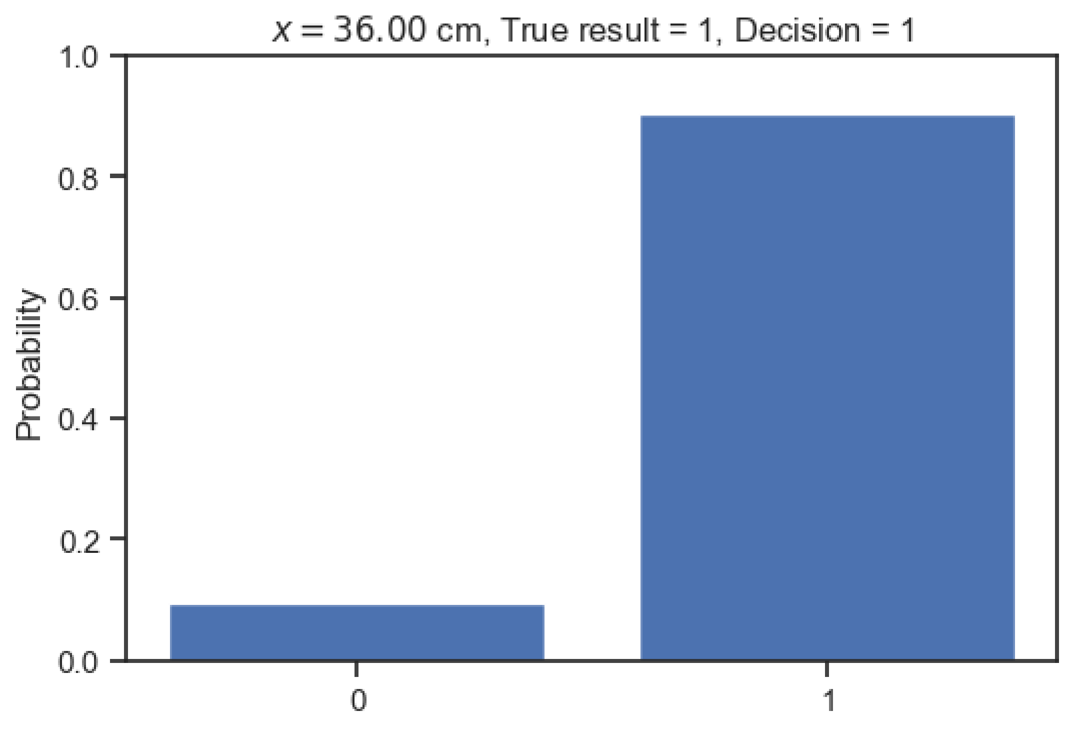

$x = 28.50$ cm, True result = 0, Decision = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

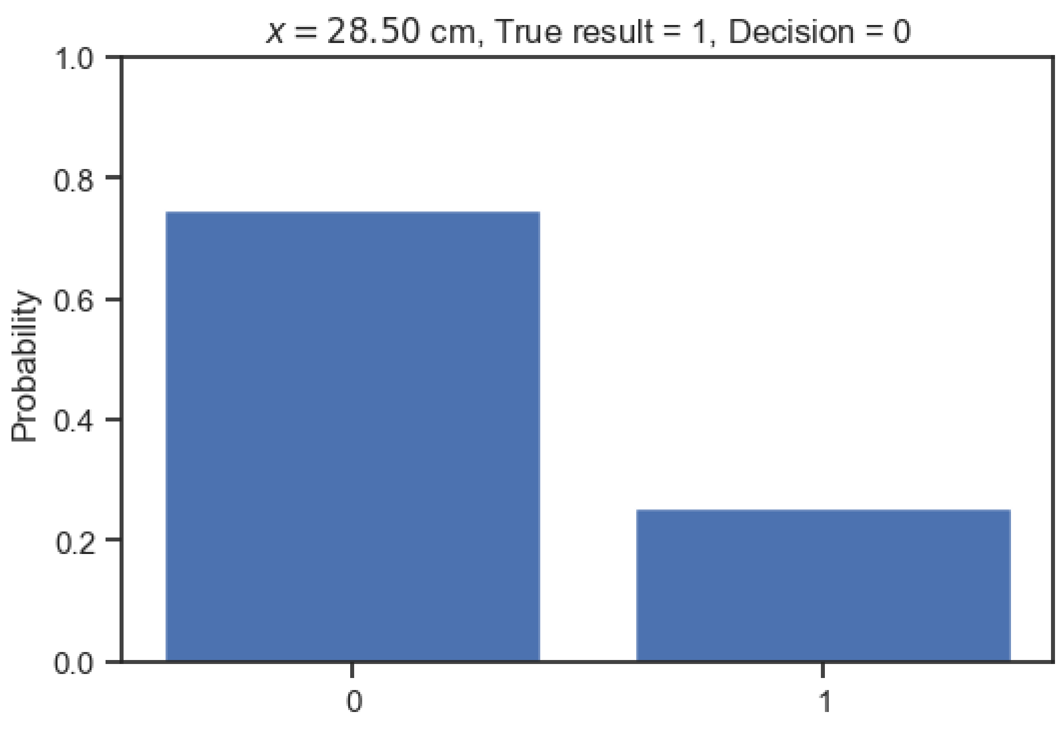

$x = 22.50$ cm, True result = 0, Decision = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

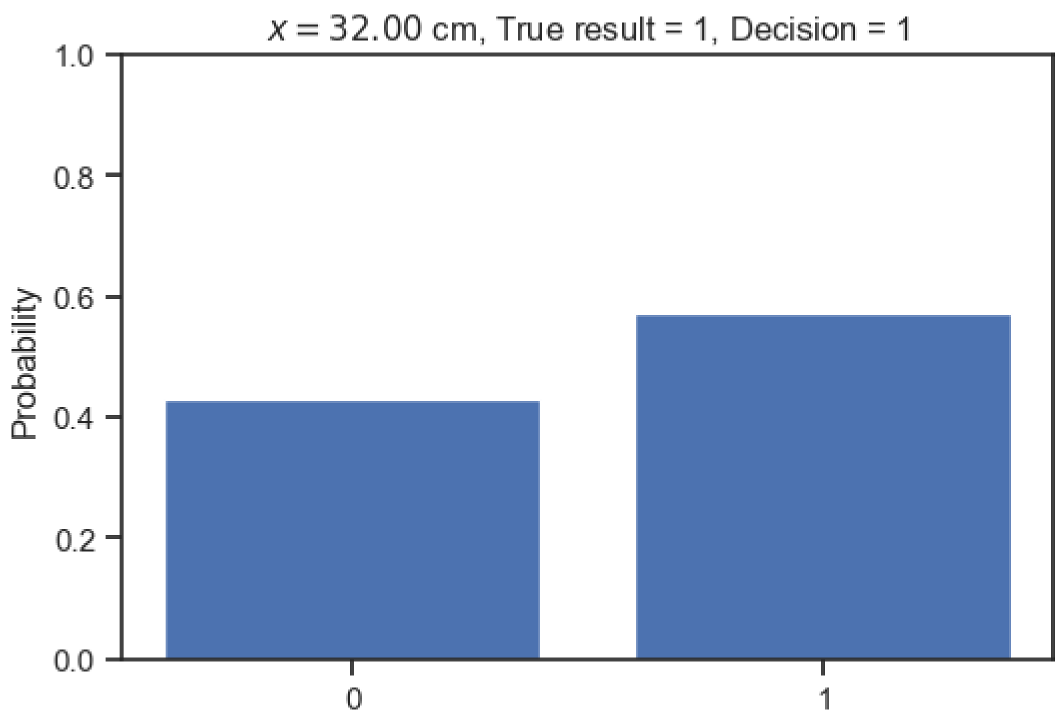

$x = 40.50$ cm, True result = 1, Decision = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1


$x = 25.50$ cm, True result = 0, Decision = 0
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1


$x = 32.00$ cm, True result = 1, Decision = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1


$x = 36.00$ cm, True result = 1, Decision = 1
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1

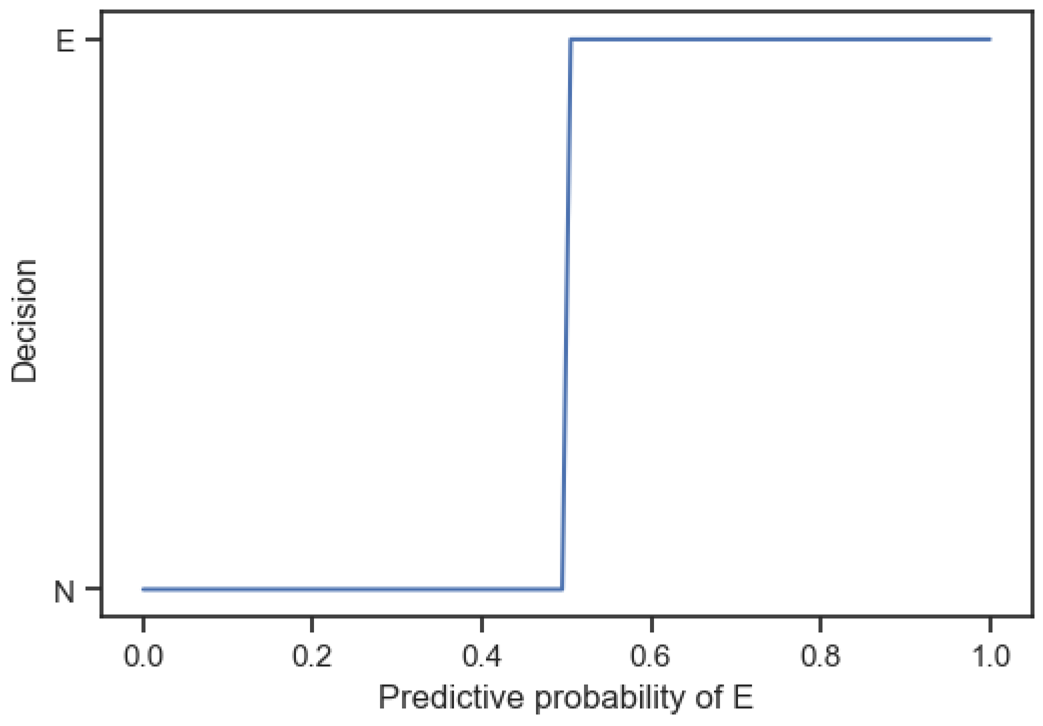




Now let's plot the decision boundary of our model:

```
fig, ax = plt.subplots()
pE = np.linspace(0, 1, 100)
pN = 1.0 - pE
probs = np.hstack([pN[:, None], pE[:, None]])
exp_cost = np.einsum('ij,kj->ki', cost_matrix, probs)
decision_idx = np.argmin(exp_cost, axis=1)
ax.plot(pE, decision_idx)
ax.set_yticks([0, 1])
ax.set_yticklabels(['N', 'E'])
ax.set_ylabel('Decision')
ax.set_xlabel('Predictive probability of E');
```

this line is where we decide based N/E on the minimum loss/cost (index of either 0 or 1)



Questions

- Repeat the analysis above with a different cost matrix that penalizes more calling a non-explosion when there is an explosion.
