

# **Lecture 18:**

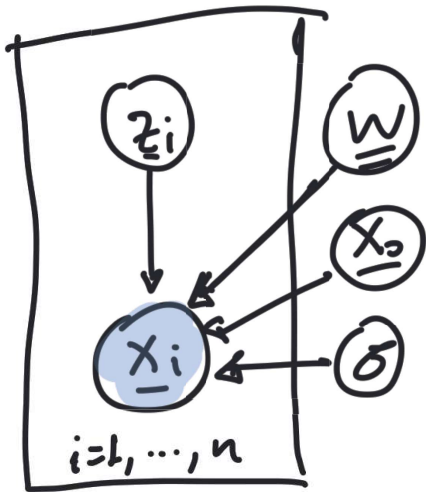
# **Dimensionality Reduction**

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## **Probabilistic principal component analysis**

# Probabilistic interpretation

$$\underline{X}_i = \underbrace{\underline{W} \cdot \underline{z}_i + \underline{x}_0}_{\text{PCA}} + \underbrace{\text{noise}}_{\sigma \cdot \underline{\varepsilon}_i} ; \quad \underline{\varepsilon}_i \sim \mathcal{N}_D(0, \underline{I})$$



$$\underline{z}_i \sim p(\underline{z}_i) = \mathcal{N}(\underline{z}_i | 0, \underline{I})$$

$$p(\underline{z}_i | \underline{x}_{1:n}, \underline{W}, \underline{x}_0, \sigma) = p(\underline{z}_i | \underline{x}_i, \underline{W}, \underline{x}_0, \sigma)$$

Bayes'  
&  
Rule  
Matching  
&  
Squares

$$p(\underline{x}_i | \underline{z}_i, \underline{W}, \underline{x}_0, \sigma) p(\underline{z}_i)$$

two Gaussians

$$\mathcal{N}(\underline{z}_i | (\underline{W}^T \underline{W} + \sigma^2 \underline{I})^{-1} \underline{W}^T (\underline{x}_i - \underline{x}_0), (\underline{W}^T \underline{W} + \sigma^2 \underline{I})^{-1})$$

# Maximum likelihood solution

$$\max_{\underline{w}, \sigma, \underline{x}_0} \log p(\underline{x}_{1:n} | \underline{w}, \underline{x}_0, \sigma)$$

$$\log \int p(\underline{x}_{1:n} | \underline{z}_{1:n}, \underline{w}, \underline{x}_0, \sigma) d\underline{z}_{1:n}$$

Gaussian

standard

Results are identical to PCA for  $\underline{x}_0$   
and  $\underline{w}$ ,

$$\sigma^2 = \frac{1}{D-d} \sum_{i=d+1}^D \lambda_i$$

Bishop 2006 Ch. 12.2.