Sampling the categorical

Contents

```
import numpy as np
np.set_printoptions(precision=3)
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
sns.set(rc={"figure.dpi":100, 'savefig.dpi':300})
sns.set_context('notebook')
sns.set_style("ticks")
```

Objectives

- Demonstrate how we can sample from the Categorical distribution using uniform samples.

Readings

- These notes.

Sampling the Bernoulli distribution

The Bernoulli distribution arises from a binary random variable representing the outcome of an experiment with a given probability of success. Let us encode success with 1 and failure with 0. It is a special case of the Categorical (2 labels). Then, we say that the random variable

$$X \sim \text{Bernoulli}(\theta),$$

is a Bernoulli random variable with parameter θ if:

$$X = \begin{cases} 1, & \text{with probability } \theta, \\ 0, & \text{otherwise.} \end{cases}$$

To sample from it, we do the following steps:

- Sample a uniform number u (i.e., a number of $U([0,1])$).
- If $u \leq \theta$, then set $x = 1$.
- Otherwise, set $x = 0$.

Let's see if this process does indeed produce the desired result. Here is the code:

```
def sample_bernoulli(theta : float):
    """Sample from the Bernoulli.

    Arguments:
        theta -- The probability of success.
    """
    u = np.random.rand()
    if u <= theta:
        return 1
    return 0
    return 0
```

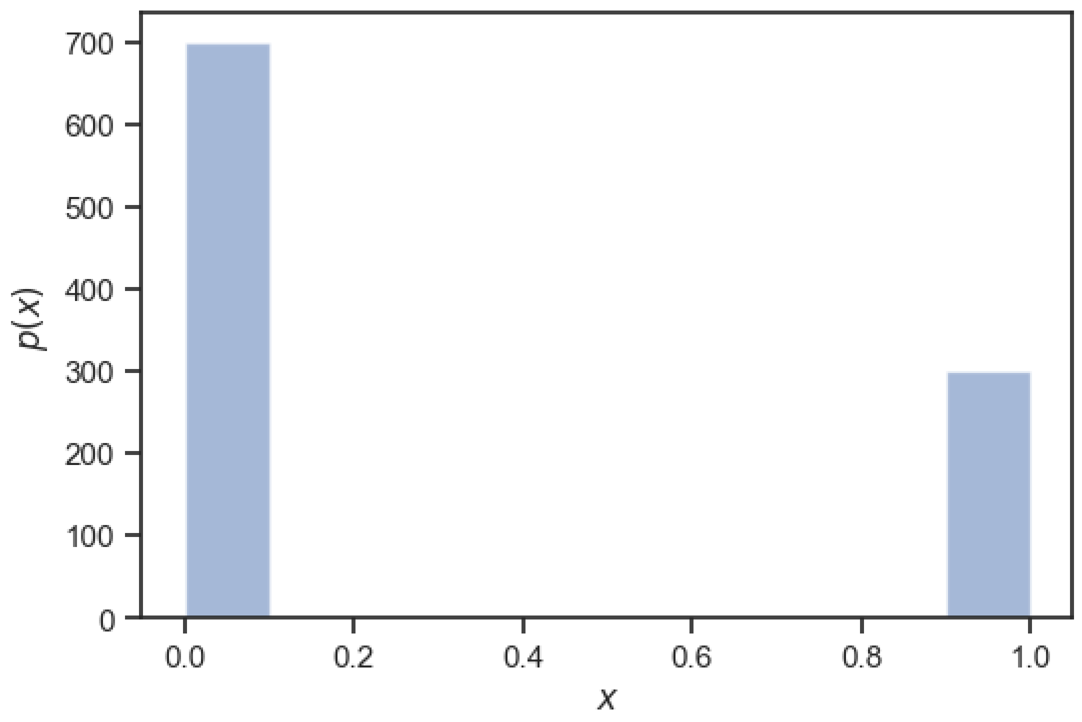
And here is how to use it:

```
for _ in range(10):
    print(sample_bernoulli(0.5))
```

```
0
1
1
1
0
0
0
0
1
0
```

Let's do a histogram of a huge number of samples:

```
N = 1000
X = np.array(
    [sample_bernoulli(0.3) for _ in range(N)]
)
fig, ax = plt.subplots()
ax.hist(X, alpha=0.5)
ax.set_xlabel(r"$x$")
ax.set_ylabel(r"$p(x)$");
```



Ok, it looks fine. About θN samples went to 1 and $(1-\theta)N$ samples went to 0.

Of course, we have already seen that this is implemented in scipy.stats. Here is a quick reminder of that code.

```
import scipy.stats as st
X = st.bernoulli(0.3)
X.rvs(size=10)
```

```
array([1, 0, 0, 0, 0, 1, 0, 0, 1, 0])
```

Sampling the K-label Categorical

Consider a generic discrete random variable X taking K different values. Without loss of generality, you may assume that these values are integers $\{0,1,2,\dots,K-1\}$ (they are just the labels of the discrete objects anyway). Let us assume that

$$p(X=k)=p_k,$$

where, of course, we must have:

$$p_k\geq 0,$$

and

$$\sum_{k=0}^{K-1} p_k = 1.$$

Remember, that an succinct way to write this is using the Dirac delta:

$$p(x) = \sum_{k=0}^{K-1} p_k \delta(x-k).$$

In any case, here is how you sample from such a distribution:

- Draw a uniform sample u.
- Find the index $j \in \{0, 1, \dots, K-1\}$ such that:

$$\sum_{k=0}^{j-1} p_k \le u < \sum_{k=0}^{j} p_k.$$

Once u is less than the ongoing sum, return j

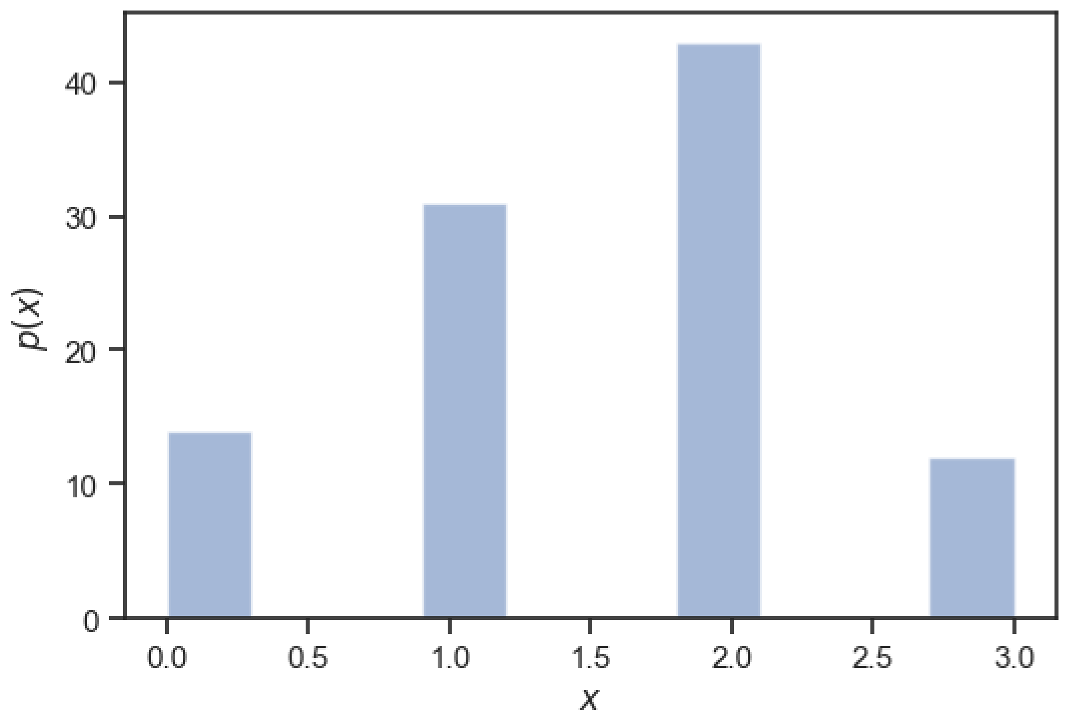
- Then, your sample is j.

Let's code it:

```
def sample_categorical(p):
    """Sample from a discrete probability density.

    Arguments:
        p -- An array specifying the probability of each possible state.
             The number of states ``m=len(p)``.
    """
    K = len(p)
    u = np.random.rand()
    c = 0.
    for j in range(K):
        c += p[j]
        if u <= c:
            return j
```

Let's test it with a four-state discrete random variable with probabilities:

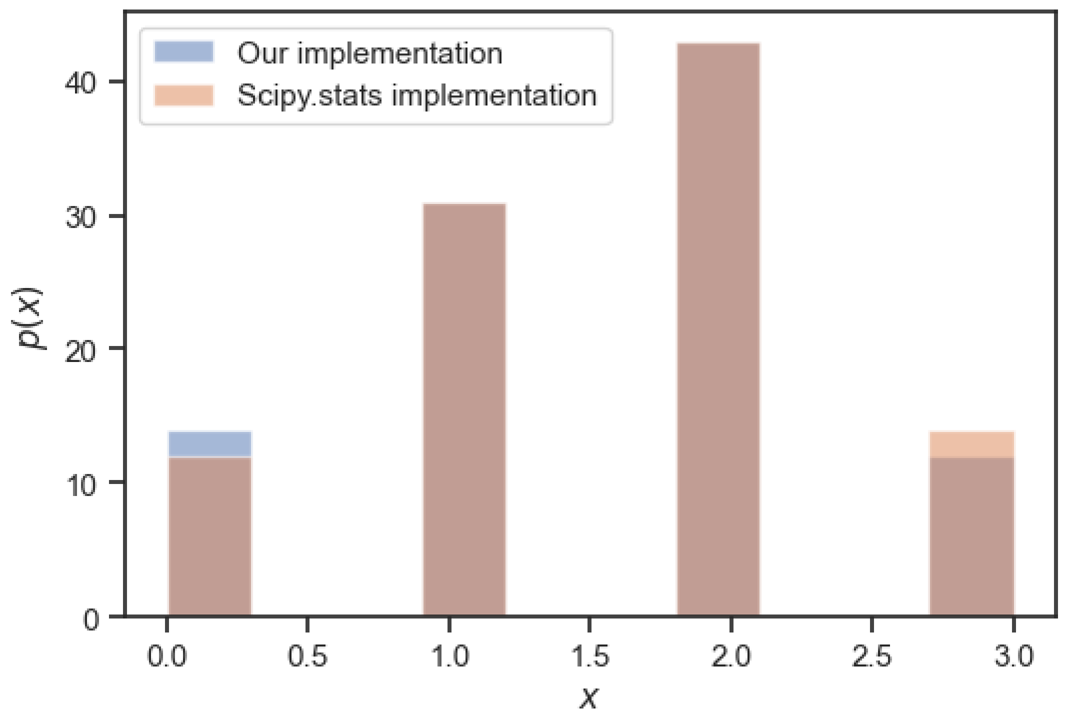
```
p = [0.2, 0.3, 0.4, 0.1]

N = 100
X = np.array(
    [sample_categorical(p) for _ in range(N)]
)

fig, ax = plt.subplots()
ax.hist(X, alpha=0.5)
ax.set_xlabel(r"$x$")
ax.set_ylabel(r"$p(x)$");
```



Of course, `scipy.stats` already implements this functionality. Let's compare.

```
K = len(p)
X_st = st.rv_discrete(values=(np.arange(K), p))
x_st_samples = X_st.rvs(size=N)

# Let's compare the two histograms
fig, ax = plt.subplots()
ax.hist(X, alpha=0.5, label="Our implementation")
ax.hist(x_st_samples, alpha=0.5, label="Scipy.stats implementation")
ax.set_xlabel(r"$x$")
ax.set_ylabel(r"$p(x)$")
plt.legend(loc="best")
```

```
<matplotlib.legend.Legend at 0x118890e80>
```



Questions

- It looks like there is a lot of variability every time you run the results. You need to go back to the code and increase the number of samples N until the results stop changing. Then you should be able to observe that our code does exactly the same thing as `scipy.stats.rv_discrete`.

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