Lecture 6: Random Vectors

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The multivariate normal - conditioning

PREDICTIVE
SCIENCE LABORATORY

Marginalization

- Assume that you have a random vector $\mathbf{X}$ made out of two sub-random vectors $\mathbf{X}_1$ and $\mathbf{X}_2$, i.e.:

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix}$$

- Assume that:

$$\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix} \qquad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12}^T & \boldsymbol{\Sigma}_2 \end{pmatrix}$$

- What is the PDF of $\mathbf{X}_1 | \mathbf{X}_2 = \mathbf{x}_2$?

PREDICTIVE
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Marginalization

- Assume that:

$$\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix} \qquad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12}^T & \boldsymbol{\Sigma}_2 \end{pmatrix}$$

- We conditional PDF is:

$$p(\mathbf{x}_1 \,|\, \mathbf{x}_2) = \frac{p(x_1, \underline{x_2})}{p(\underline{x_2})} \propto p(x_1, \underline{x_2}) = N\left(\begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} \middle| \begin{pmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{pmatrix}, \begin{pmatrix} \underline{\underline{\Sigma}}_1 & \underline{\underline{\Sigma}}_{12} \\ \underline{\underline{\Sigma}}_{12}^T & \underline{\underline{\Sigma}}_2 \end{pmatrix} \right)$$

completing the square

$$= \dots = N\left(\underline{x}_1 \,\middle|\, \underbrace{\underline{\mu}_{1|x_2}, \; \underline{\underline{\Sigma}}_{1|x_2}}_{\text{need to specify these}} \right)$$

↳ see next slide

PREDICTIVE
SCIENCE LABORATORY

Marginalization

- Assume that:

$$\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

uses

$$\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix} \qquad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12}^T & \boldsymbol{\Sigma}_2 \end{pmatrix}$$

- We get that:

$$\mathbf{X}_1 \,|\, \mathbf{X}_2 = \mathbf{x}_2 \sim N\Big(\overset{\text{prior mean}}{\boldsymbol{\mu}_1} + \underbrace{\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_2^{-1}(\overset{\text{data}}{\mathbf{x}_2} - \boldsymbol{\mu}_2)}_{\text{correction}}, \overset{\text{prior covariance}}{\boldsymbol{\Sigma}_1} - \underbrace{\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_2^{-1}\boldsymbol{\Sigma}_{12}^T}_{\text{correction}}\Big)$$