# Lecture 20: State-space models - Kalman filters

Professor Ilias Bilionis

# Beyond linear models and Gaussian noise

PREDICTIVE
SCIENCE LABORATORY

# Non-linear systems with non-linear observations with Gaussian noise

disturbance

$$x_{n+1} = f(x_n, u_n, z_n), \quad z_n \sim p(z_n)$$

noise

$$y_n = h(x_n, w_n), \quad w_n \sim p(w_n)$$

The ~~most general~~ case.

$\Downarrow$

process noise is "additive & Gaussian"

$$x_{n+1} = f(x_n, u_n) + z_n, \quad z_n \sim N(0, Q)$$

measurement noise is

$$y_n = h(x_n) + w_n, \quad w_n \sim N(0, R)$$

Extended Kalman Filter (Unscented KF)

$\Downarrow$

Idea: Linearize Everything.

$n$: $p(x_{n-1} | y_{1:n-1}, u_{0:n-2}) \simeq N(x_{n-1} | \mu_{n-1}, V_{n-1})$

Predict: $\mu_n^p = f(\mu_{n-1}, u_{n-1})$

center

linearize $f(x, u) \underset{\text{Exp.}}{\overset{\text{Taylor}}{\simeq}} f(\mu_n^p, u) + \nabla_x f(\mu_n^p, u) \cdot (x - \mu_n^p) + \dots$

~~Matrix~~ $d \times d$

$A_n(u)$

$$h(x) \simeq h(\mu_n^p) + \nabla_x h(\mu_n^p) \cdot (x - \mu_n^p)$$

matrix $m \times d$

$C_n$ different at each time step

$$V_n^p = \nabla_x f(\mu_n^p, u_{n-1})\, Q\, \nabla_x f(\mu_n^p, u_{n-1})^T$$

= new

Update: Same but $A_n(u) = \nabla_x f(\mu_n^p, u_{n-1})$ instead of $A$

and $C_n = \nabla_x h(\mu_n^p)$ instead of $C$.

PREDICTIVE
SCIENCE LABORATORY