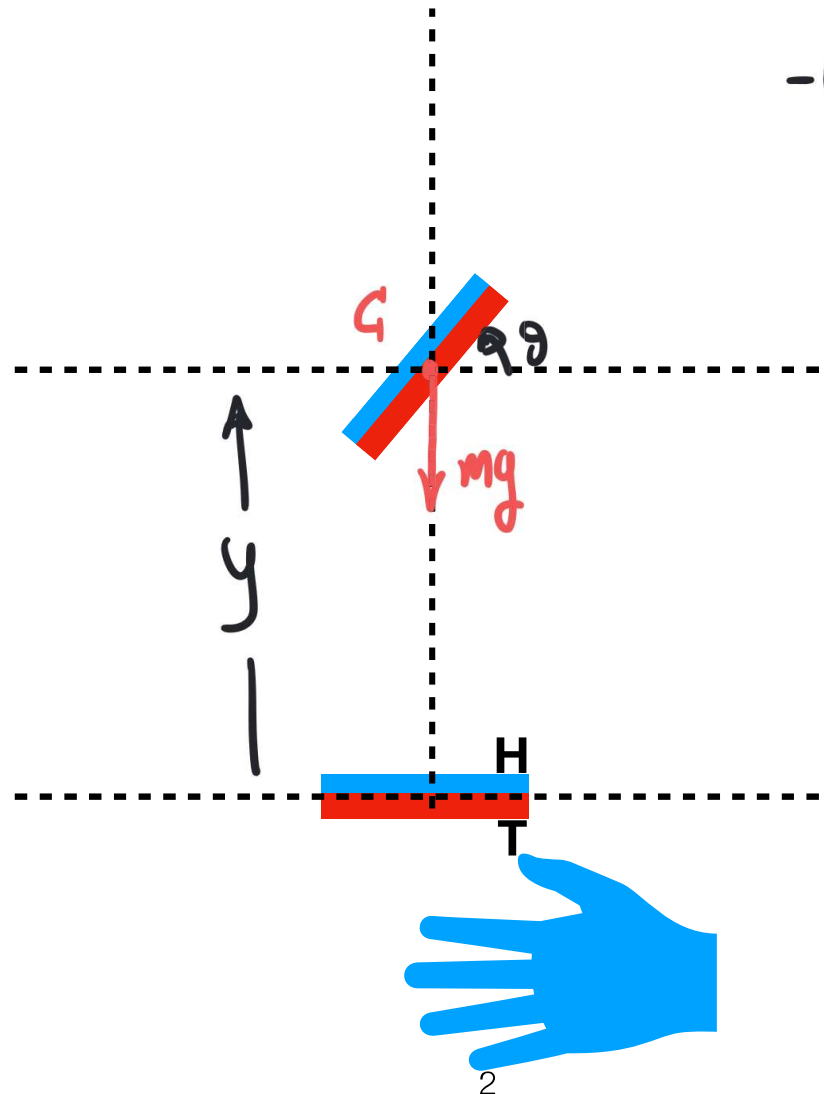


# Lecture 2: Basics of Probability Theory

Professor Ilias Bilonis

## Dynamics of a coin toss

# Dynamics of a coin toss



$$\sum F_y = ma_y$$

$$-mg = m\ddot{y}$$

$$\Rightarrow \ddot{y} = -g \quad (1)$$

$$\sum \mathcal{M}_c = I\alpha$$

$$0 = I\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = 0 \quad (2)$$

# Dynamics of a coin toss

$$\begin{aligned}\ddot{y} &= -g, \\ \ddot{\theta} &= 0, \\ \underline{y}(0) &= \underline{0}, \\ \underline{\theta}(0) &= \underline{0}, \\ \underline{\dot{y}}(0) &= v_0, \\ \underline{\dot{\theta}}(0) &= \omega_0.\end{aligned}$$

System of 2nd order ODE's

init. vel.

init. ang. vel.

Solve the initial value problem:

$$\begin{aligned}y(t) &= -\frac{1}{2}gt^2 + v_0t, \\ \theta(t) &= \omega_0t.\end{aligned}$$

# Dynamics of a coin toss

$$\begin{aligned}y(t) &= -\frac{1}{2}gt^2 + v_0t, \\ \theta(t) &= \omega_0t.\end{aligned}$$

Time it takes to hit the hand:

$$\underline{y(t_1) = 0} \implies t_1 = \frac{2v_0}{g}.$$

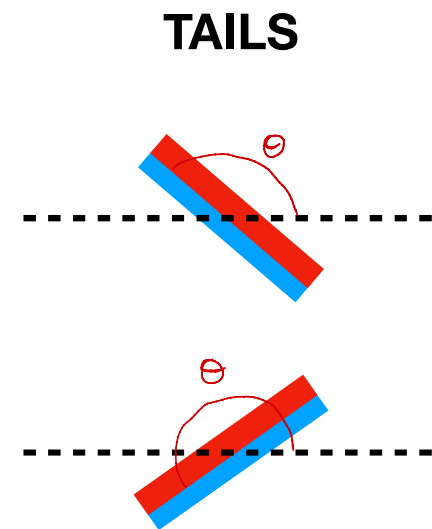
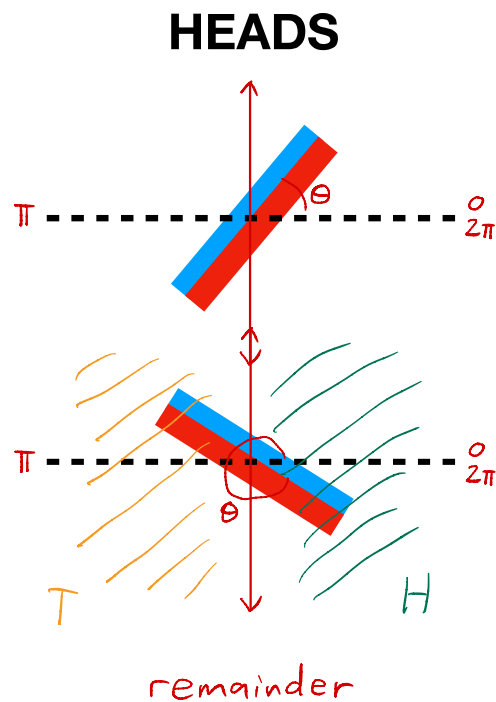
Angle when the coin hits the hand again:

$$\theta(t_1) = \frac{2v_0\omega_0}{g}$$

# Dynamics of a coin toss

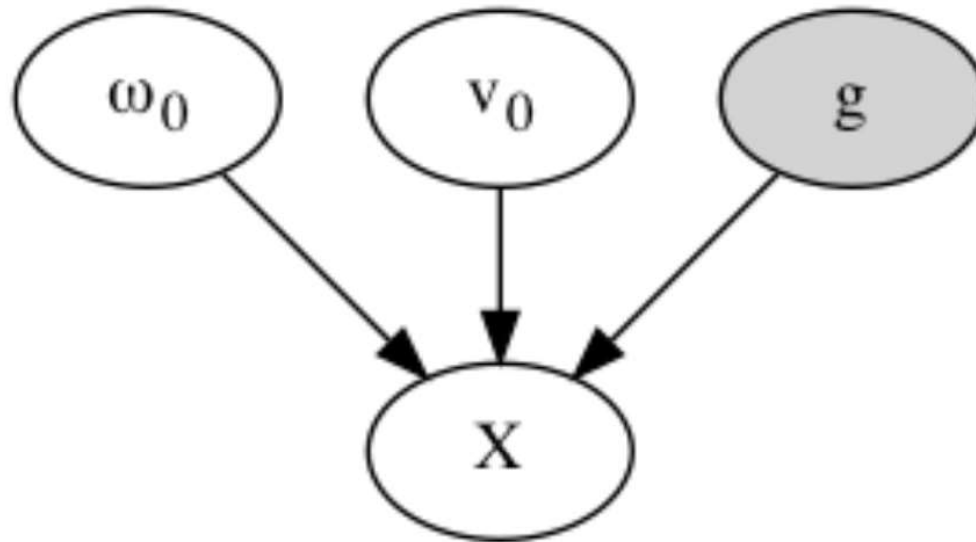
Angle when the coin hits the hand again:

$$\theta(t_1) = \frac{2v_0\omega_0}{g}$$



# Graphical causal model representation

$$\textcolor{red}{X} = \begin{cases} T, & \text{if } \frac{2v_0\omega_0}{g}(\bmod 2\pi) \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right), \\ H, & \text{otherwise.} \end{cases}$$



# Dynamics of a coin toss: Rise of uncertainty

