

Lecture 3: Discrete Random Variables

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Variance of a discrete random variable

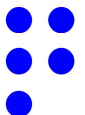
Expectation of a random variable

- The variance of a random variable is:

$$V[X] := \mathbb{E} \left[(X - \mathbb{E}[X])^2 \right]$$

$$= \sum_x (x - \mathbb{E}[X])^2 p(x)$$

- You can think of the variance as the spread of the random variable around its expectation.
- However, do not take this too literally for discrete random variables.



Properties of the variance

- Take any constant c :

$$V[X + c] = V[X]$$

Proof:

$$\begin{aligned} V[X + c] &= E[(X + c - E[X + c])^2] \\ &= E[(X - E[X])^2] \\ &= V[X] \end{aligned}$$

c's cancel

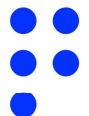
Properties of the variance

- Take any constant λ :

$$V[\lambda X] = \lambda^2 V[X]$$

Proof:

$$\begin{aligned} V[\lambda X] &= E[(\lambda X - E[\lambda X])^2] \\ &= E[(\lambda X - \lambda E[X])^2] \\ &= E[\lambda^2 (X - E[X])^2] \\ &= \lambda^2 E[(X - E[X])^2] \\ &= \lambda^2 V[X] \quad \blacksquare \end{aligned}$$



from quiz: see utility theory

Properties of the variance

- It holds that:

used later for
sampling average
approximations

$$V[X] = E[X^2] - (E[X])^2$$

Proof: $V[X] = E[(X - E[X])^2]$

$$= E[\underbrace{X^2}_{\text{r.v.}} - \underbrace{2XE[X]}_{\text{const.}} + \underbrace{(E[X])^2}_{\text{const.}}]$$
$$= E[X^2 - 2XE[X] + (E[X])^2]$$
$$= E[X^2] - 2(E[X])^2 + (E[X])^2$$

linearity

□