Lecture 2: Basics of Probability Theory

Professor Ilias Bilionis

The product rule of probability

PREDICTIVE
SCIENCE LABORATORY

The product rule

$$p(A|B) = \frac{p(A \cap B)}{p(B)} \longrightarrow p(A \cap B) = p(A,B)\,p(B)$$

The **product rule** (Bayes' rule, Bayes' theorem):

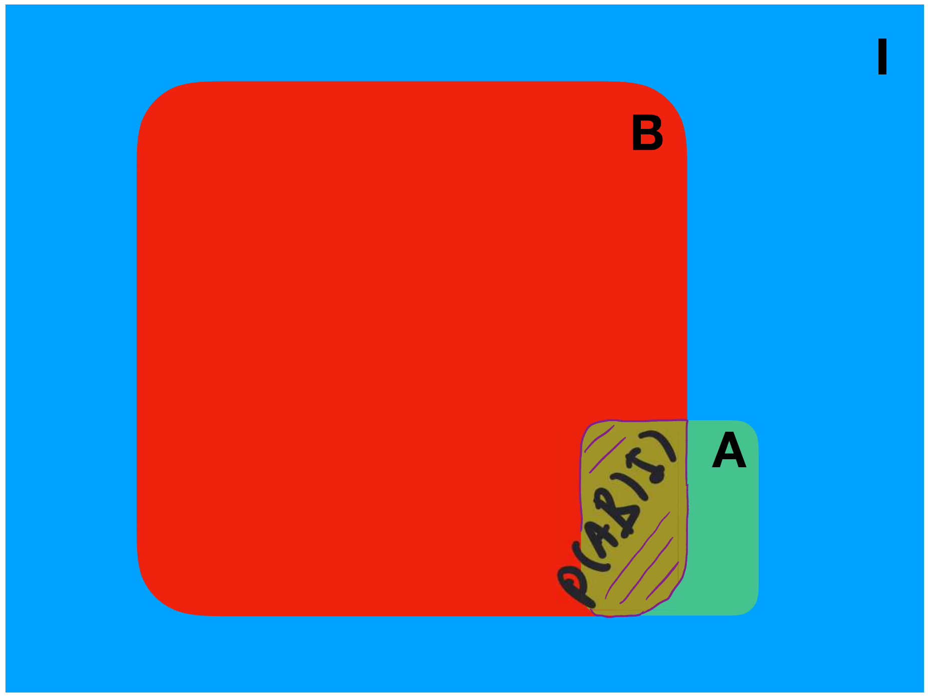
$$p(A, B \mid I) = \underline{p(A \mid B, I)}\,\underline{p(B \mid I)}$$

Other common for of this rule:

$$p(A,B \mid I) = p(B \mid A, I)\, p(A \mid I)$$

$$p(A \mid B, I) = \frac{p(A, B \mid I)}{p(B \mid I)}$$

Venn diagram interpretation of Bayes' rule

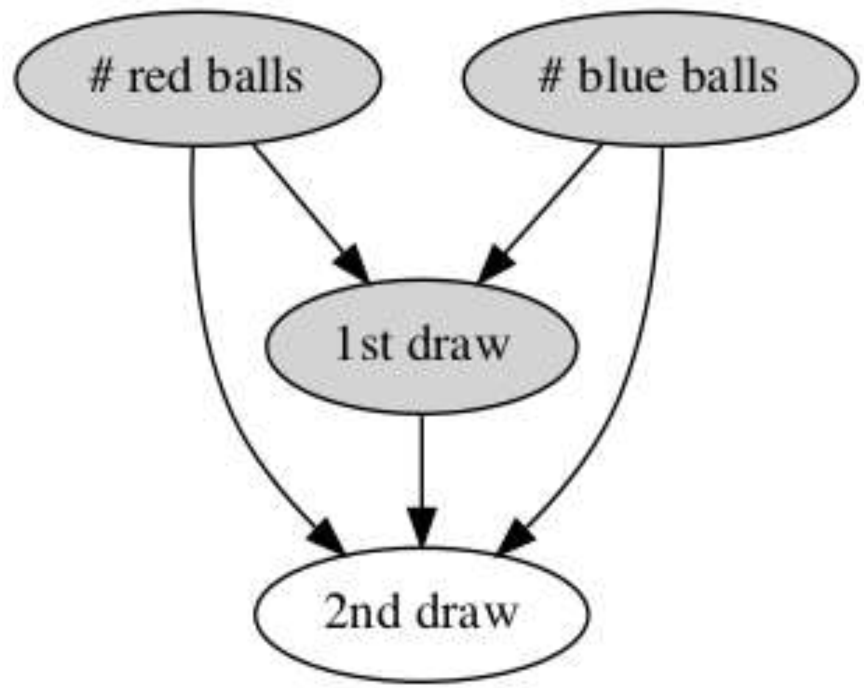


$$p(A\,|\,B,I) = \frac{p(AB\,|\,I)}{p(B\,|\,I)} = \frac{\text{area of } AB}{\text{area of } B}$$

Example: Drawing balls from a box without replacement

Let R_2 be the sentence:

The second ball we draw is red.

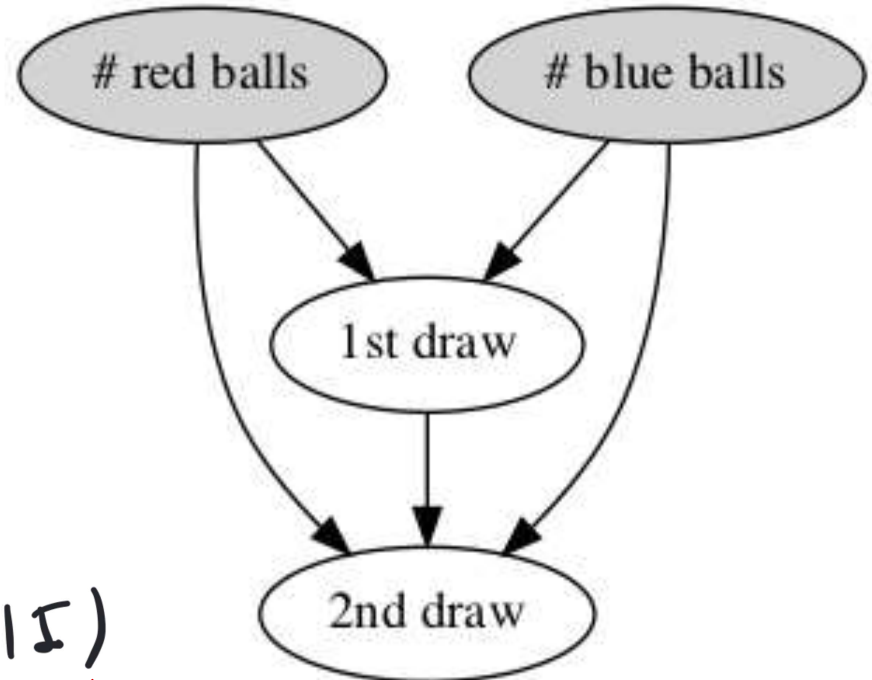


What is the probability of R_2 given that B_1 is true?

- We had 10 balls, 6 red and 4 blue.
- Since B_1 is true, we now have 6 red and 3 blue balls.
- Therefore: $p(R_2|B_1, I) = \frac{6}{9}$

Example: Drawing balls from a box without replacement

Let's find the probability that we draw a blue ball in the first draw B_1 and a red ball in the second draw R_2.



We have to use the **product rule**:

$$p(B_1, R_2 \mid I) = p(R_2 \mid B_1, I)\,\underbrace{p(B_1 \mid I)}_{\text{in this case, easier to find this value first}}$$

$$= \frac{6}{9} \cdot 0.4 = \boxed{0.26}$$

$$p(R_1 B_2 | I) = p(R_1 \cap B_2) = p(B_2 \cap R_1)$$

$$= p(B_2 | R_1)\, p(R_1)$$

$$= \frac{4}{9} \cdot \frac{6}{10} = \frac{24}{90} \checkmark$$