

Lecture 6: Random Vectors

Professor Ilias Bilonis

Random vectors

Joint pdf of many random variables

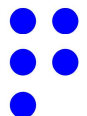
- Take N random variables X_1, \dots, X_N .
- $\mathbf{X} = (X_1, \dots, X_N)$ is called a random vector.
- We will refer to their joint pdf as:

$$p(\mathbf{x}) = p(x_1, \dots, x_N)$$

usual
properties
hold

$$p(\mathbf{x}) \geq 0$$
$$\int p(\mathbf{x}) dx_1 dx_2 \dots dx_N = 1$$

$$p(x_i) = \int p(\mathbf{x}) dx_2 \dots dx_{i-1} dx_{i+1} \dots dx_N$$



Expectation of a random vector

- The expectation of a random vector is the vector of expectations of each component:

$$\mathbb{E}[\mathbf{X}] = \begin{pmatrix} \mathbb{E}[X_1] \\ \vdots \\ \mathbb{E}[X_N] \end{pmatrix}$$

\uparrow
 \mathbb{R}^N

Covariance matrix of two random vectors

- Let \mathbf{X} be an N -dimensional random vector.
- Let \mathbf{Y} be an M -dimensional random vector.
- The covariance of \mathbf{X} and \mathbf{Y} is the $N \times M$ matrix consisting of all covariances between the components of \mathbf{X} and \mathbf{Y} , i.e.,

$$\mathbb{C}[\mathbf{X}, \mathbf{Y}] = \left(\mathbb{C}[x_i, y_j] \right)_{ij}$$

Covariance matrix of two random vectors

- Let \mathbf{X} be an N -dimensional random vector.
- Let \mathbf{Y} be an M -dimensional random vector.
- We can easily show that:

$$\mathbb{C}[\mathbf{X}, \mathbf{Y}] = \mathbb{E} \left[\underbrace{(\mathbf{x} - \mathbb{E}[\mathbf{X}])}_{N \times 1} \cdot \underbrace{(\mathbf{y} - \mathbb{E}[\mathbf{Y}])^T}_{1 \times M} \right]$$

$N \times M$

outer product

Self-covariance of a random vector

- Let \mathbf{X} be an N -dimensional random vector.
- The self-covariance of a random vector is the $N \times N$ matrix:

$$\mathbb{C}[\mathbf{X}, \mathbf{X}] = \mathbb{E} \left[(\mathbf{X} - \mathbb{E}[\mathbf{X}]) (\mathbf{X} - \mathbb{E}[\mathbf{X}])^T \right]$$