Lecture 2: Basics of Probability Theory

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The sum rule

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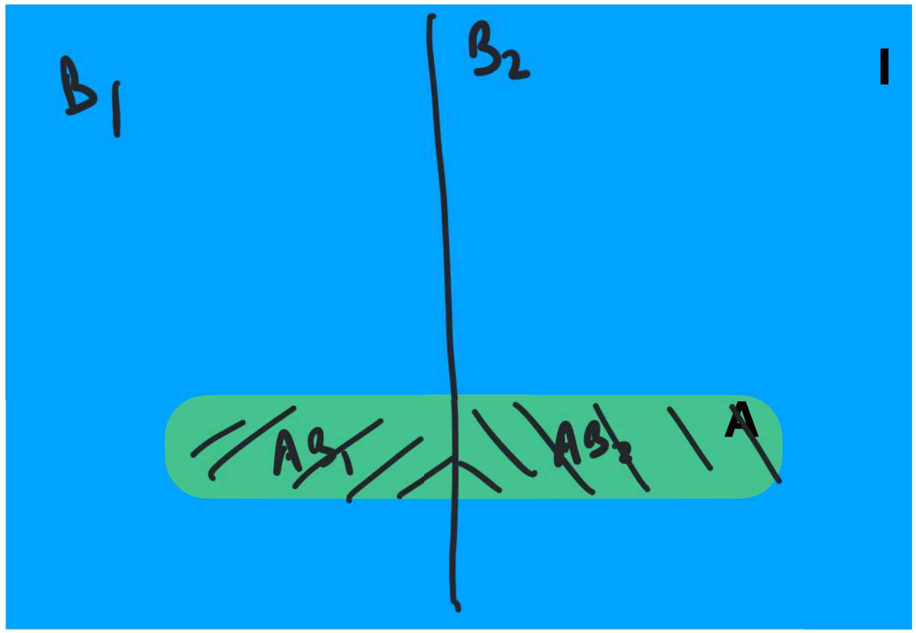
Motivation of the sum rule

B is a partition of I *



$$p(B_1 + B_2 | I) = 1$$

$$p(B_1 B_2 | I) = 0$$

$$\boxed{\begin{aligned} p(A|I) &= p(AB_1|I) + p(AB_2|I) \\ &\overset{\text{Bayes}}{=} p(A|B_1 I)\,p(B_1|I) + p(A|B_2 I)\,p(B_2|I) \end{aligned}}$$

!

Example: Drawing balls from a box Without replacement

We have found that:

$$p(B_1 | I) = \frac{2}{5} \quad p(R_1 | I) = \frac{3}{5} \quad p(R_2 | B_1, I) = \frac{2}{3} \quad p(R_2 | R_1, I) = \frac{5}{9}$$

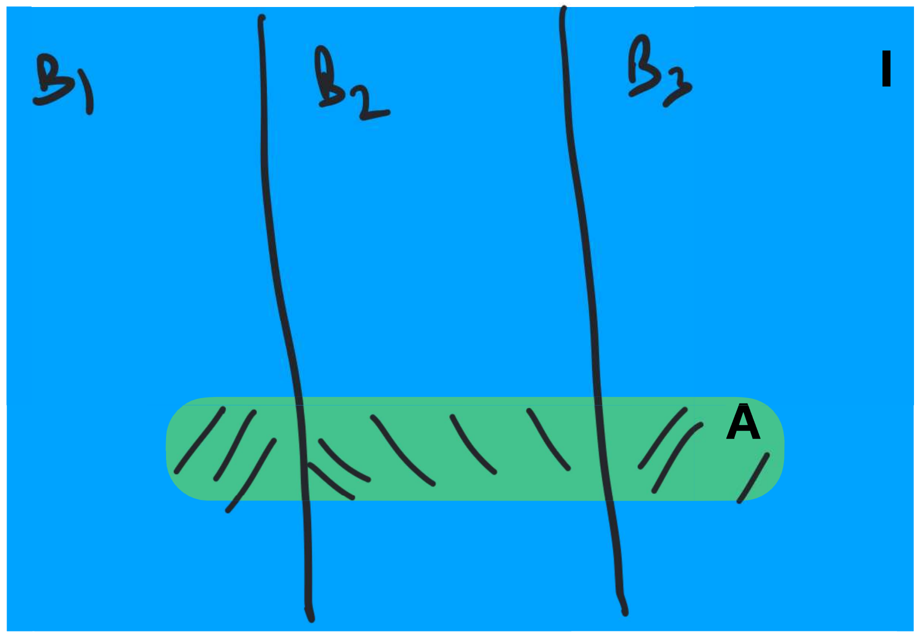
What is the probability of getting a red ball that the second draw independently of what we got in the first one?

$$p(R_2 | I) = p(R_2 B_1 | I) + p(R_2 R_1 | I)$$

$$= p(R_2 | B_1 I)\, p(B_1 | I) + p(R_2 | R_1 I)\, p(R_1 | I)$$

$$= \frac{2}{3} \cdot \frac{2}{5} + \frac{5}{9} \cdot \frac{3}{5} = \dots = \#$$

Generalization of the sum rule to three sets



$$p(A|I) = p(AB_1|I) + p(AB_2|I) + p(AB_3|I)$$
$$= p(A|B_1 I)p(B_1|I) + p(A|B_2 I)p(B_2|I) + \dots$$

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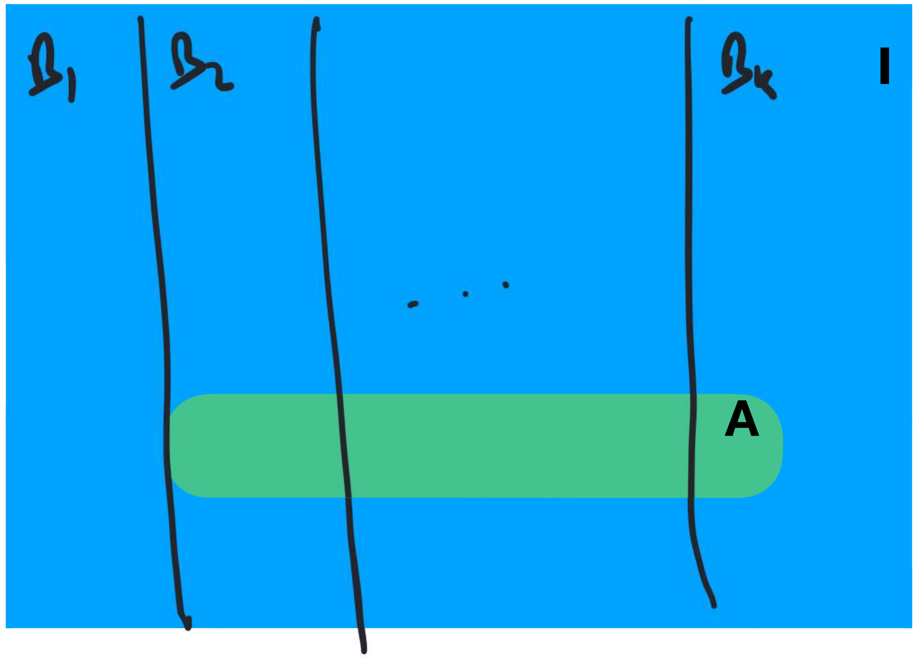

$$p(B_1 + \dots + B_k | I) = 1$$

$$p(B_i B_j | I)$$

$$\boxed{p(A|I) = \sum_{i=1}^{k} p(AB_i|I) = \sum_{i=1}^{k} p(A|B_i I)\, p(B_i|I)}$$

given: $p(R_2|I) = p(R_2 B_1|I) + p(R_2 R_1|I)$

$= p(R_2|B_1 I)\,p(B_1|I) + p(R_2|R_1 I)\,p(R_1|I) = 0.6$

$p(B_2|I) = ?$

$p(B_2|I) = p(B_2 B_1|I) + p(B_2 R_1|I)$

$= p(B_2|B_1 I)\,p(B_1|I) + p(B_2|R_1 I)\,p(R_1|I)$

$= \frac{3}{9} \cdot \frac{4}{10} + \frac{4}{9} \cdot \frac{6}{10} = \frac{2}{5} = 0.4$

$p(R_2 + B_2|I) = p(R_2 \cup B_2) = 1 = p(R_2) + p(B_2)$ (disjoint)

$0.6 + 0.4 = 1$ ✓