Sampling from continuous distributions - Inverse sampling

Contents


```
import numpy as np
np.set_printoptions(precision=3)
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
sns.set(rc={"figure.dpi":100, 'savefig.dpi':300})
sns.set_context('notebook')
sns.set_style("ticks")
```

Objectives

- Demonstrate how we can sample from continuous distributions using the method of inverse sampling.

Readings

- These notes.

Inverse sampling

How do you sample an arbitrary univariate continuous random variable X with CDF $F(x)$. In this scenario, *inverse sampling* is the way to go. It relies on the observation that the random variable

$$Y = F^{-1}(U),$$

where F^{-1} is the inverse of the CDF of X and $U \sim \mathcal{U}([0,1])$ has exactly the same distribution as X.

We will demonstrate this by example. To this end, let us consider an exponential random variable:

$$X \sim \text{Exp}(r),$$

where $r > 0$ is known as the *rate parameter*. The exponential distribution describes the time it passes between random events that occur at a constnat rate r. The CDF of the Exponential is:

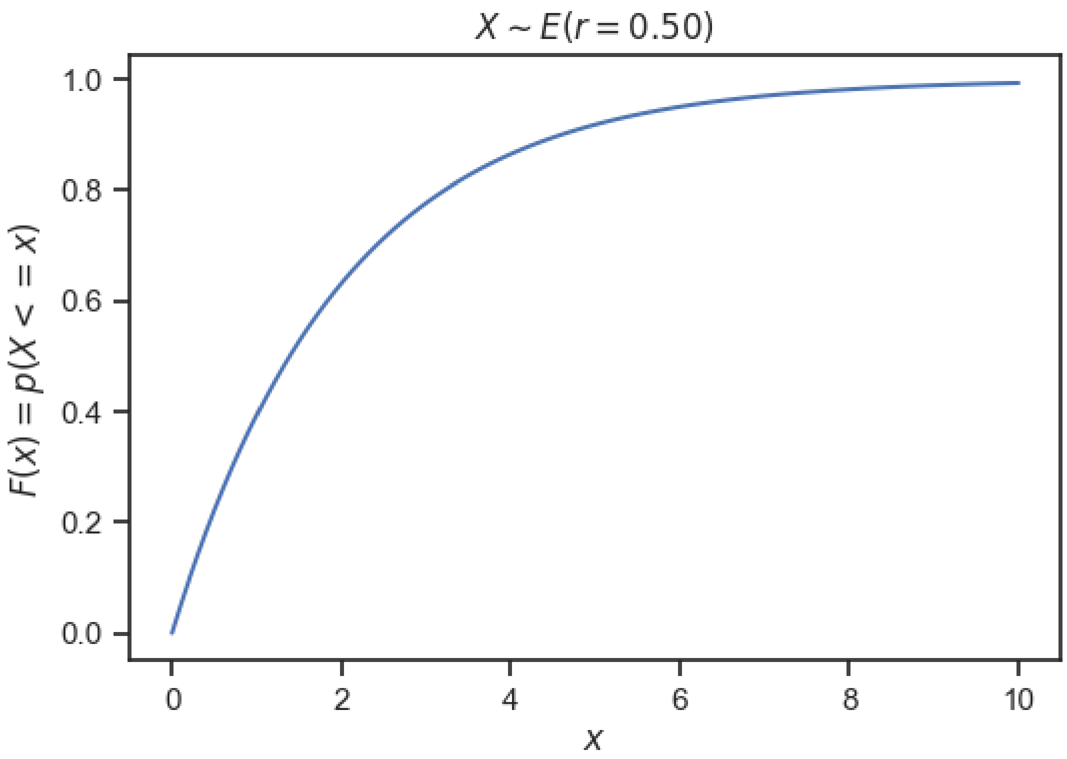
$$F(x) = p(X \le x) = 1 - e^{-rx}.$$

Let's plot it for $r = 0.5$.

```
import scipy.stats as st

r = 0.5
X = st.expon(scale=1.0 / r)
fig, ax = plt.subplots()
x = np.linspace(0., 5. / r, 100)
ax.plot(x, X.cdf(x))
ax.set_xlabel(r"$x$")
ax.set_ylabel(r"$F(x) = p(X <= x)$")
ax.set_title(f"$X\sim E(r={r:1.2f})$");
```

See documentation for scipy.stats.expon, parametrizing with the rate parementer requires scale = 1 / rate parameter



To sample T using inverse sampling, we need the inverse of the CDF. This is easily shown to be:

$$F^{-1}(u) = -\frac{\ln(1-u)}{r}.$$

Let's see if this is going to give us the right samples. We will compare the empirical histogram obtained by inverse sampling to the actual PDF $p(x)$. Here is the code for inverse sampling:

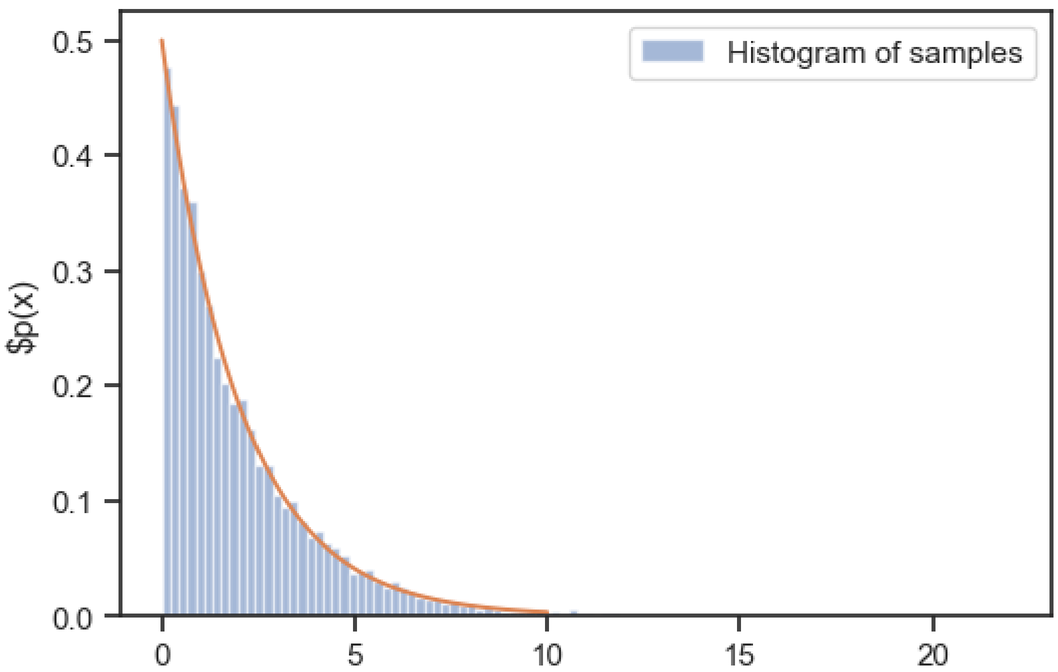
```
def sample_exp(r : float):
    """Sample from an exponential.

    Arguments:
    r  --  The rate parameter.
    """
    u = np.random.rand()
    return -np.log(1. - u) / r
```

And here is the histogram of some samples:

```
N = 10000
x_samples = np.array(
    [sample_exp(r) for _ in range(N)]
)

fig, ax = plt.subplots()
ax.hist(
    x_samples,
    alpha=0.5,
    density=True,
    bins=100,
    label="Histogram of samples"
)
ax.plot(x, X.pdf(x))
ax.set_xlabel(r"$x$")
ax.set_ylabel(r"$p(x)")
plt.legend(loc=r"best");
```



x

Questions

- Modify the code above to implement inverse sampling for a univariate Gaussian with zero mean and unit variance. Use `scipy.stats` to find the inverse CDF of the Gaussian (It is `st.norm.ppf`). Here is how to use it:

```
# Standard normal random variable
Z = st.norm(loc=0.0, scale=1.0)
# The inverse CDF of the standard normal, say at 0.7, can be evaluated by:
Z.ppf(0.7)
```

```
0.5244005127080407
```

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