Lecture 2: Basics of Probability Theory

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Interpretation of probability

PREDICTIVE
SCIENCE LABORATORY

Knowledge-based interpretation

The probability of 

= how sure I am about the outcome of a coin toss experiments given all the information I have about the initial conditions of the tossing process

actual uncertainty in this case comes from uncertainty in the initial conditions prior to the tossing process

Arguing logical/scientific propositions

- A: a logical sentence
- B: another logical sentence
- I: all the information we know (sample space)

No other restriction apart that A and B are not contradictions.

Notation shortcuts

$$\textbf{not } A \equiv \neg A \equiv \bar{A}$$

$$A \textbf{ and } B \equiv A, B \equiv AB \equiv A \cap B$$

$$A \textbf{ or } B \equiv A + B \equiv A \cup B$$

Talking about probabilities

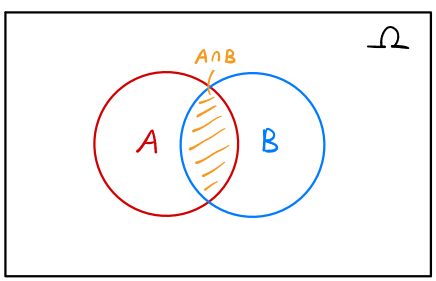
$p(A \mid BI) \;=\;$ the probability of A being true given that we know that B and I are true

or (assuming I is implied)

$=$ the probability of A being true given that we know that B is true

or (assuming arguments about truth are implied)

$=$ the probability of A given B



$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{}{} \quad ; \quad P(A) = \frac{}{}$$

↳ it <u>IS</u> possible for $P(A|B) > P(A)$!!!

Interpretation

$p(A \mid B, I)$ **in** $[0,1]$

quantifying the degree of plausibility that A is true given that B and I are true.

$p(A \mid B, I) = 1$ we are certain that A is **true** if B is true (and I)

↳ $p(\bar{A} \mid B, I) = 0$

$p(A \mid B, I) = 0$ we are certain that A is **false** if B is true (and I)

↳ $p(\bar{A} \mid B, I) = 1$

$0 < p(A \mid B, I) < 1$ we are uncertain about A if B is true (and I)

$p(A \mid B, I) = \frac{1}{2}$ we are completely ignorant about A if B is true (and I)