Lecture 4: Continuous Random Variables

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The uniform distribution

PREDICTIVE
SCIENCE LABORATORY

The uniform distribution

- Models a random variable that takes values within an interval $[a, b]$ all with equal probability.

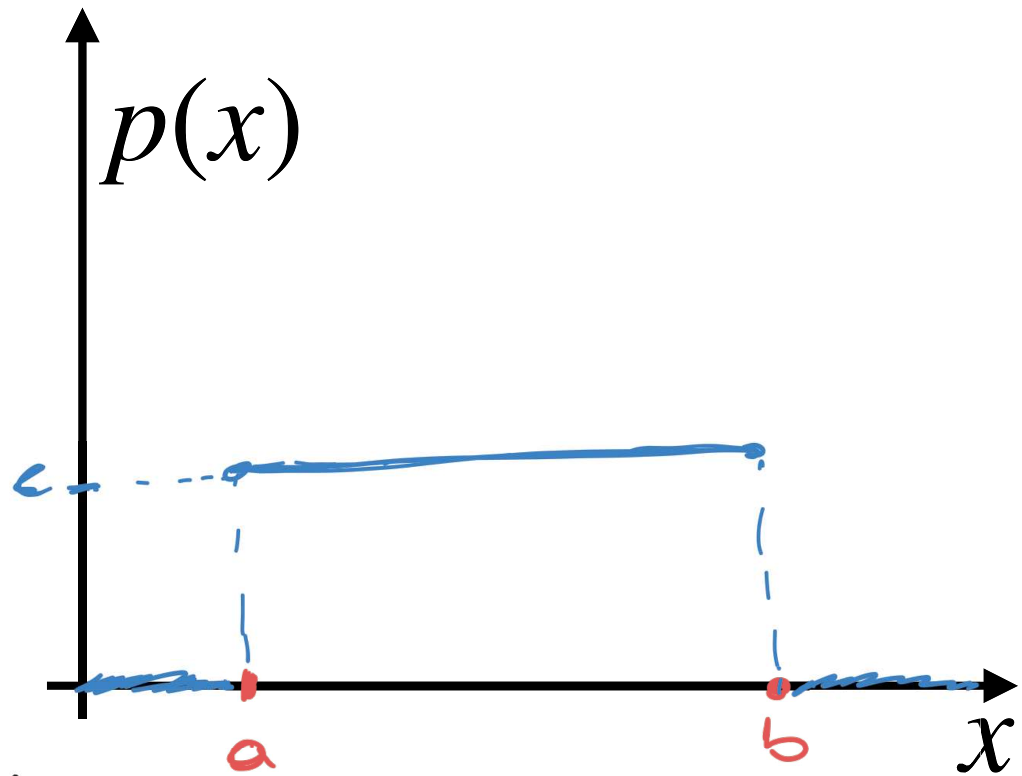
- We write:

follows

$$X \sim \mathcal{U}([a,b])$$

- The probability density is:

$$p(x) = \begin{cases} c, & x \in [a,b] \\ 0, & \text{otherwise} \end{cases}$$



$$\int_a^b p(x)dx = 1 \Rightarrow c(b-a) = 1$$

$$\Rightarrow c = \frac{1}{b-a}$$

normalization constant

The CDF of the uniform distribution

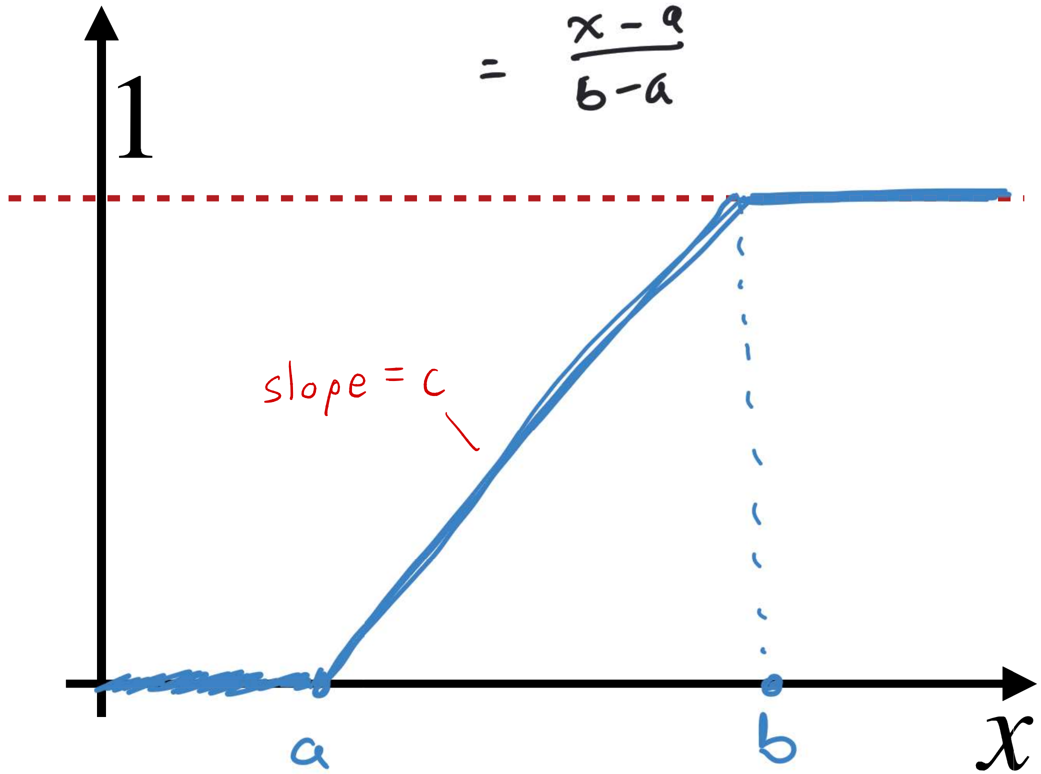
$$p(x) = F'(x) \Rightarrow F(x) = \int_a^x p(x)\,dx = \int_a^x \frac{1}{b-a}\,dx$$
$$= \frac{x-a}{b-a}$$

- Consider:

$$X \sim U([a,b])$$

- The CDF is:

$$F(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{b-a}, & \text{if } a \le x \le b \\ 1, & \text{otherwise} \end{cases}$$



The expectation of the uniform distribution

$$X \sim U([a, b])$$

$$p(x) = \begin{cases} \frac{1}{b-a}, & x \in [a,b] \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbb{E}[X] = \int xp(x)dx = \int_a^b x \frac{1}{b-a} dx = \frac{x^2}{(b-a)2}\Big|_a^b = \boxed{\frac{a+b}{2}}$$

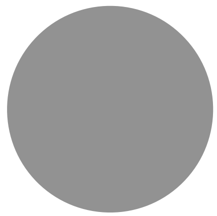
$$\mathbb{E}[X^2] = \int_a^b x^2 p(x) dx = \frac{x^3}{3(b-a)}\Big|_a^b = \dots$$

$$\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \boxed{\frac{(b-a)^2}{12}}$$

Example of a uniform random variable

- Take:

$$X = \textbf{mass of}$$



- You are told that the manufacturer guarantees that the mass is between 9.99 and 10.01 grams?

- If this is all the information we have, we would assign a uniform:

$$X \sim U([9.99, 10.01])$$