Lecture 4: Continuous Random Variables

Professor Ilias Bilionis

The cumulative distribution function

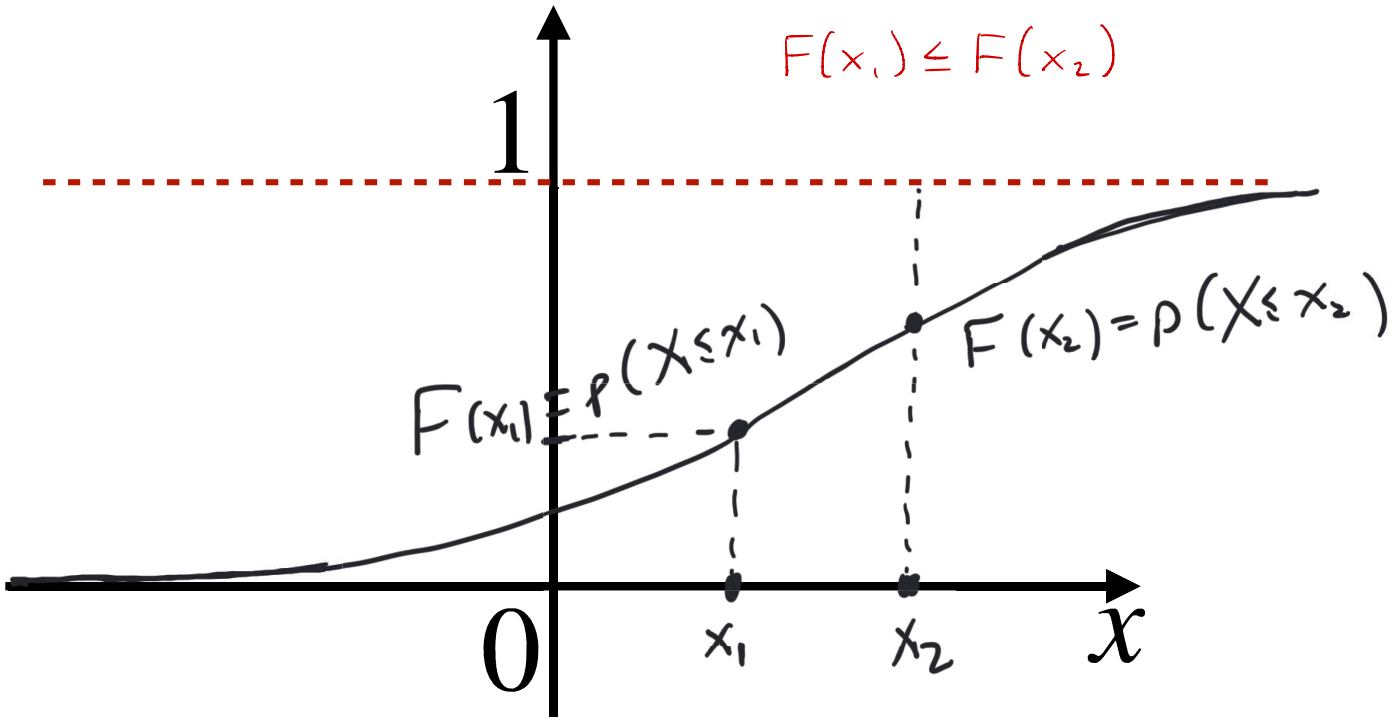
CDF

The cumulative distribution function (CDF)

- Let X be a continuous random variable taking real values.
- Its cumulative distribution function (CDF) $F(x)$ gives the probability that X is smaller than x. By definition:

$$F(x) := P(X \leq x)$$

Visualization of the CDF of a random variable



$$F(x) = p(X \le x)$$

$$F \uparrow$$

$$F(-\infty) = p(X \le -\infty) = 0$$

Properties of the CDF

- $F(x) := \mathbb{P}(X \leq x)$
- $F(x)$ is an increasing function for x.
- $F(-\infty) := \lim_{x \leftarrow -\infty} F(x) = 0$
- $F(+\infty) := \lim_{x \to +\infty} F(x) = 1$

Properties of the CDF

- $F(x) := p(X \leq x)$
- $p(a \leq X \leq b) =$ $F(b) - F(a)$