Lecture 14: Bayesian Linear Regression

Professor Ilias Bilionis

Probabilistic interpretation of leas squares - Estimating the measurement noise

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Reminder: Generalized linear model and least squares fit

$$x_{1:N} = (x_1, \dots, x_N) \;;\; y_{1:N} = (y_1, \dots, y_N)$$

$$y = w_1 \varphi_1(x) + \dots + w_m \varphi_m(x) = \sum_{j=1}^{m} w_j \varphi_j(x) = \overbrace{\underline{\varphi}(x)^T \underline{w}}^{\text{inner product}}$$

features

basis functions

$(w_1, \dots, w_m)$

$(\varphi_1(x), \dots, \varphi_m(x))$

$$\min \; L(\underline{w}) = \sum_{i=1}^{N} \left(y_i - \underline{\varphi}^T(x_i)\underline{w}\right)^2$$

$(y_1, \dots, y_N)$ observed outputs

$$\nabla_{\underline{w}} L(\underline{w}) = 0 \Rightarrow \underline{\underline{\Phi}}^T \underline{\underline{\Phi}} \cdot \underline{w} = \underline{\underline{\Phi}}^T \cdot \underline{y}$$

Design matrix $N \times m$ (# obs., # features)

$$\underline{\underline{\Phi}} = \begin{pmatrix} \varphi_1(x_1) & \dots & \varphi_m(x_1) \\ \vdots & & \vdots \\ \varphi_1(x_N) & & \varphi_m(x_N) \end{pmatrix}$$

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