Lecture 20: State-space models - Kalman filters

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Derivation of Kalman filter - Update

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Derivation of Kalman filter - Update

PREDICT:

If $p(x_{n-1} \mid y_{1:n-1}, u_{0:n-2}) = N(x_{n-1} \mid \mu_{n-1}, V_{n-1})$

Then Predict:

$$p(x_n \mid y_{1:n-1}, \underbrace{u_{0:n-2}, u_{n-1}}) = N(x_n \mid \underbrace{A\mu_{n-1} + Bu_{n-1}}_{\mu_n^p \text{ - for prediction}}, \underbrace{AV_{n-1}A^T + Q}_{V_n^p})$$

use prediction

UPDATE:

new observation included

all previous controls

$$\text{posterior: } \underbrace{p(x_n \mid y_{1:n}, u_{0:n-1})}_{G} \underset{\text{Bayes}}{\propto} \underbrace{p(y_n \mid x_n)}_{\substack{\text{prior} \\ \text{Emission} \\ N(Cx_n, R)}} \underbrace{p(x_n \mid y_{1:n-1}, u_{0:n-1})}_{\substack{\text{likelihood} \\ N(\mu_n^p, V_n^p)}}$$

Complete the Square

$$p(x_n \mid y_{1:n}, u_{0:n-1}) = N(x_n \mid \mu_n, V_n)$$

$$\mu_n = \underbrace{\mu_n^p}_{d} + \underbrace{\underbrace{K_n}_{d \times q} \underbrace{(y_n - C\mu_n^p)}_{q}}_{d}$$ - correction term

$$V_n = V_n^p - \underbrace{\underbrace{K_n}_{d \times q} \underbrace{C}_{q \times d} \underbrace{V_n^p}_{d \times d}}_{d \times d}$$ - correction term

Kalman gain:

$$K_n = \underbrace{\underbrace{V_n^p}_{d \times d} \underbrace{C^T}_{d \times q} \underbrace{\boxed{\left(\underbrace{\underbrace{C}_{q \times d} \underbrace{V_n^p}_{d \times d} \underbrace{C^T}_{d \times q}}_{q \times q} + \underbrace{R}_{q \times q}\right)^{-1}}}_{q \times q}}_{d \times q}$$

way to carry this out so you don't need to invert (computationally expensive)

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