Lecture 10: Quantifying uncertainties in Monte Carlo estimates

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The central limit theorem

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The Central Limit Theorem

- Take $X_1, X_2, \ldots$ to be iid random variables with mean μ and variance σ^2.

- Consider their average:

$$S_N = \frac{X_1 + \ldots + X_N}{N}$$

- The Central Limit Theorem States that:

$$S_N \sim N\left(\mu, \frac{\sigma^2}{N}\right) \text{ for large } N.$$

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Example of the Central Limit Theorem

- Take $X_i \sim \text{Exp}(r)$ with r fixed.

- Define the average of N such variables:

$$S_N = \frac{X_1 + \ldots + X_N}{N}$$

- Characterize probability density function of the average via samples. - repeatedly sample $X_1, \ldots, X_N$

- Compare to the CLT prediction.

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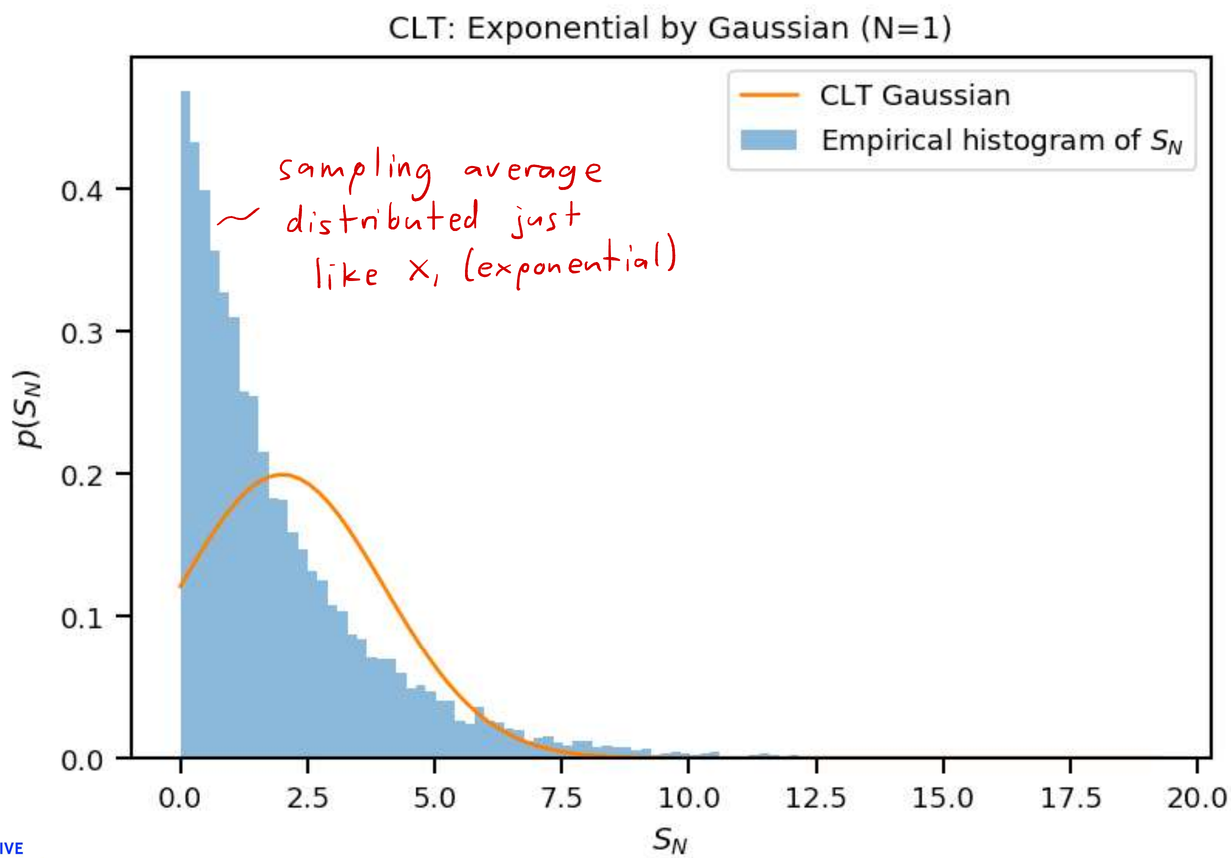
N=1



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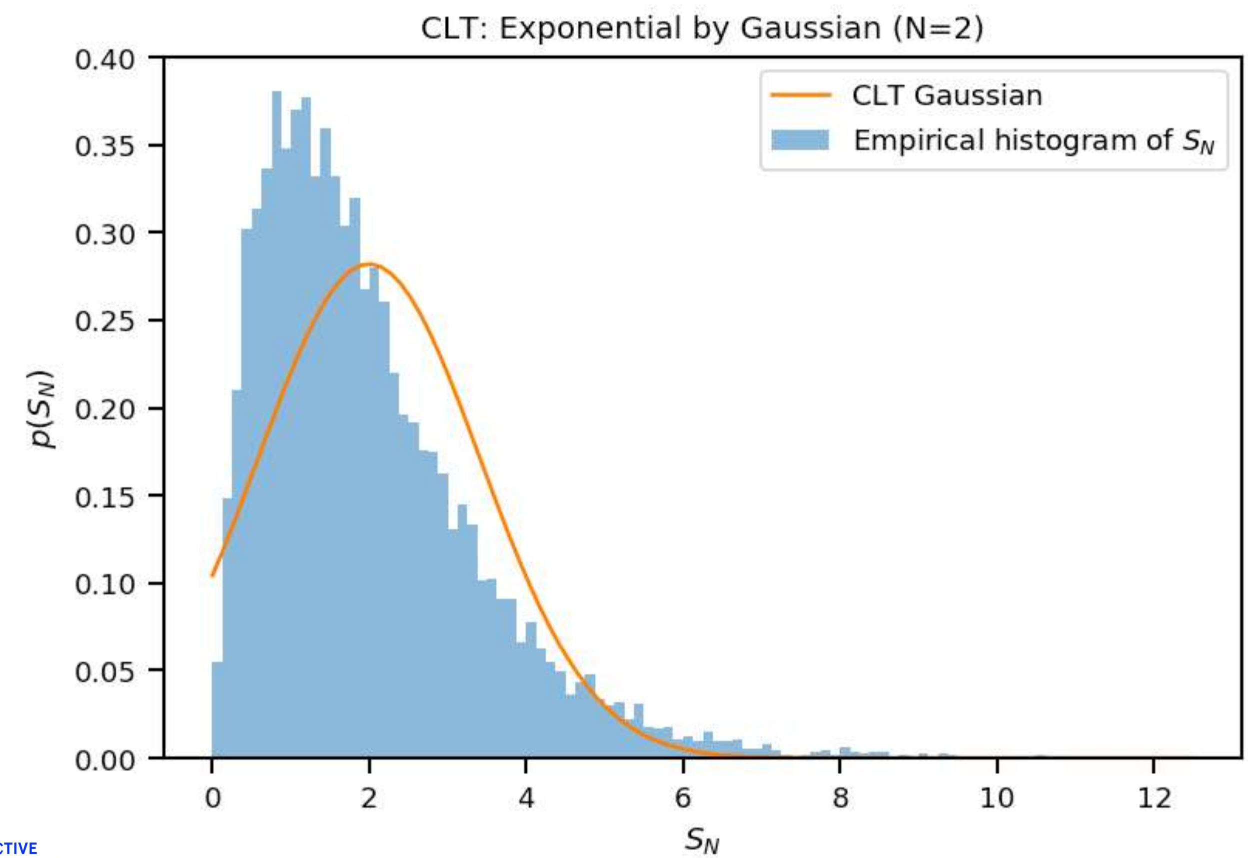
N=2



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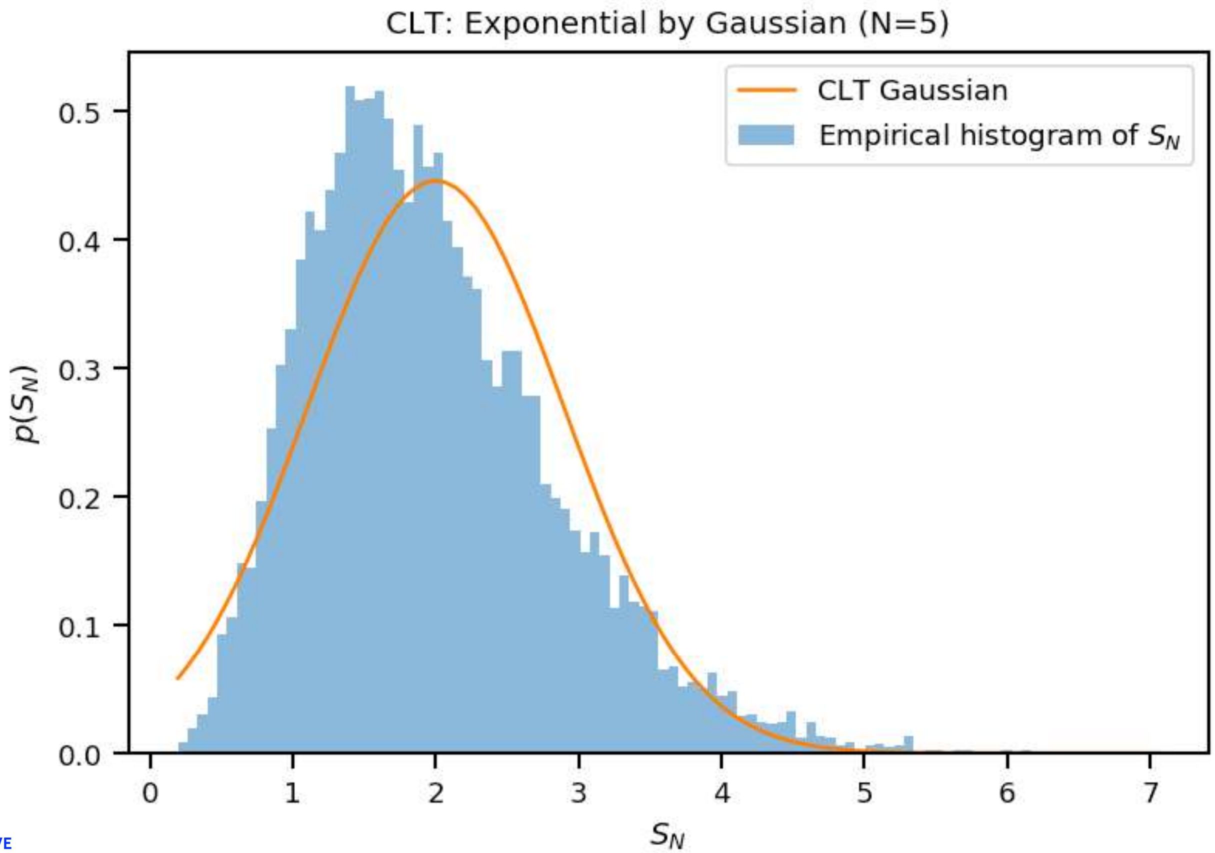
N=5



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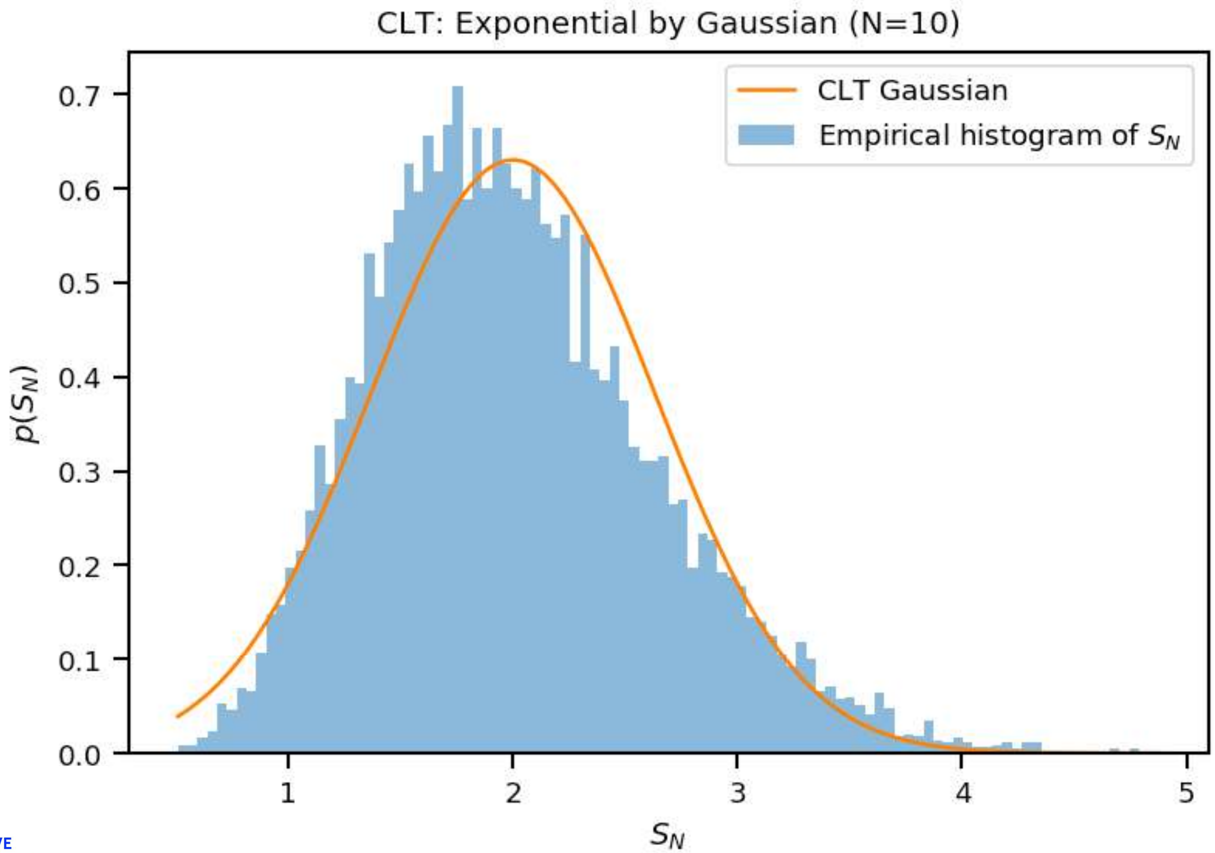
N=10



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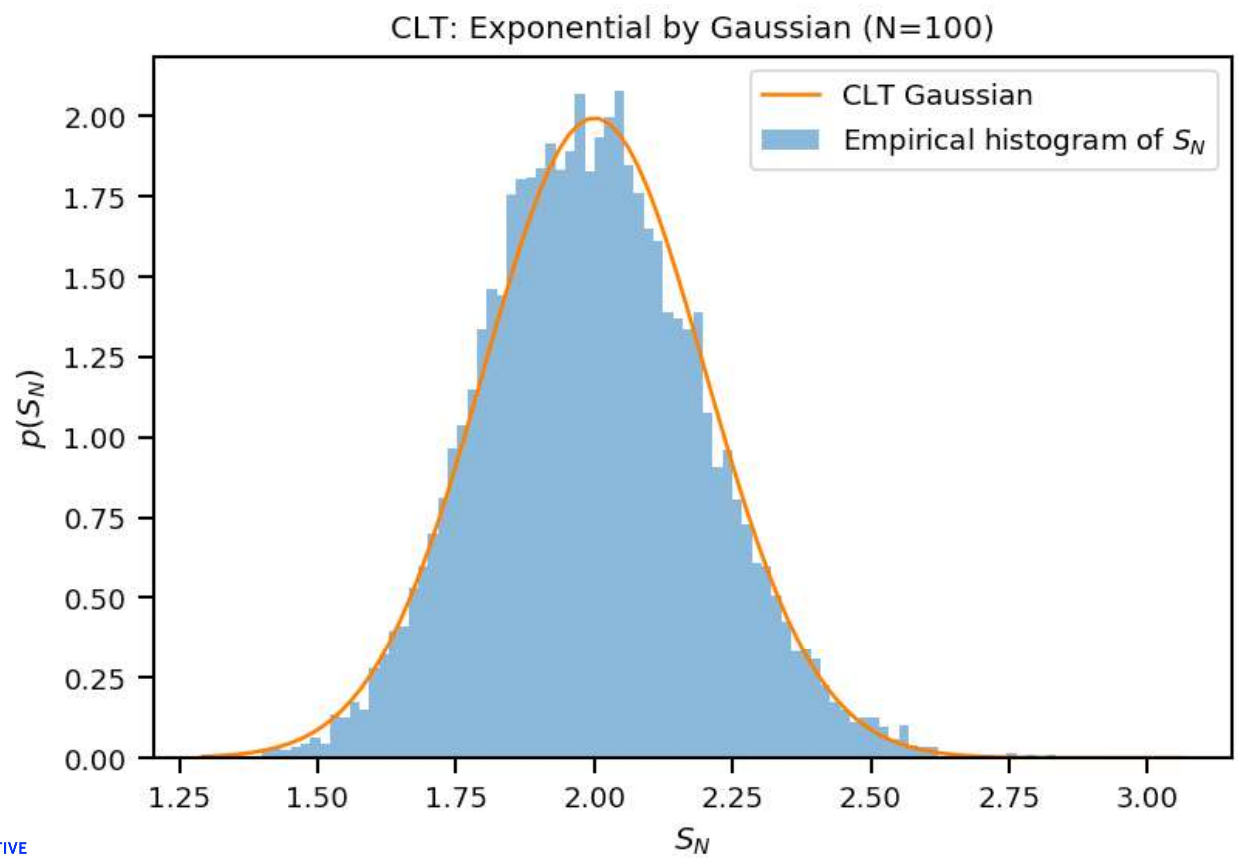
N=100



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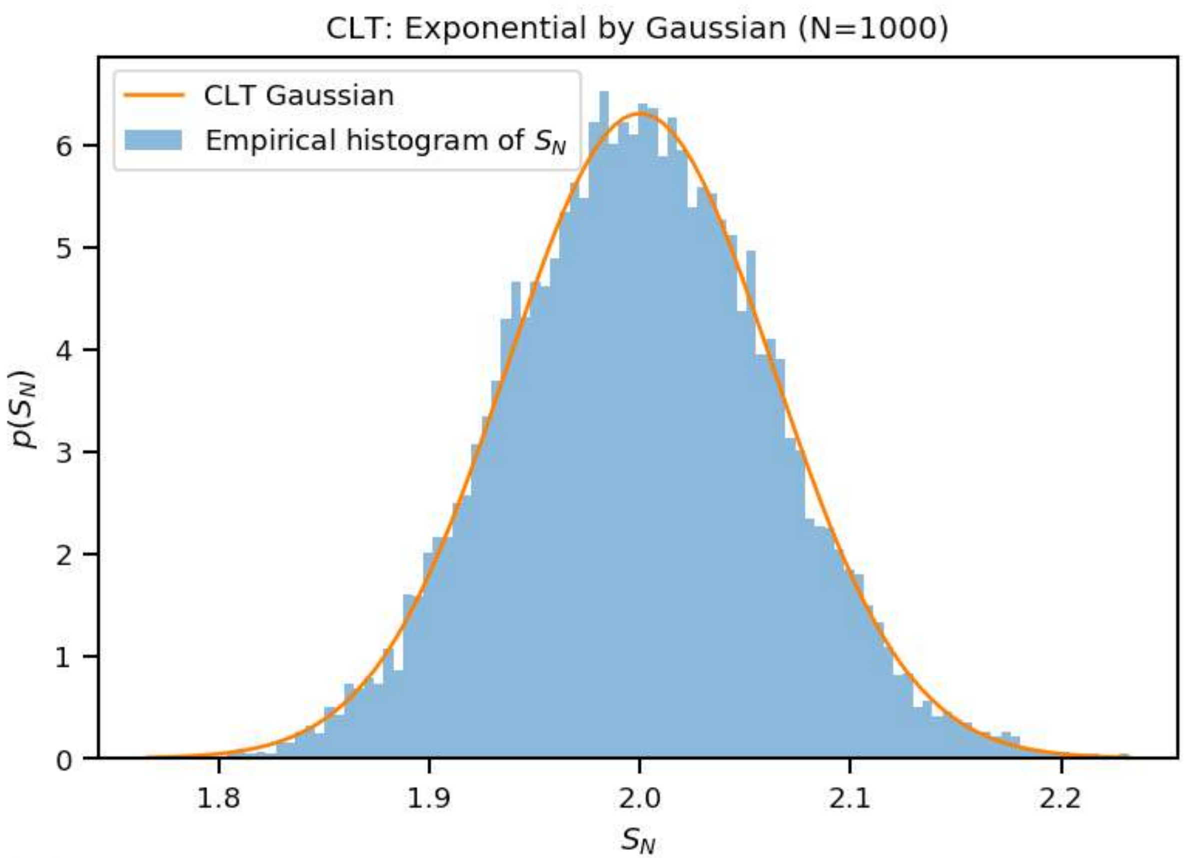
CLT holds for any set of i.i.d. random variables

N=1000



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