Lecture 5: Collections of Random Variables

Professor Ilias Bilionis

Example: Drawing two balls from a box without replacement

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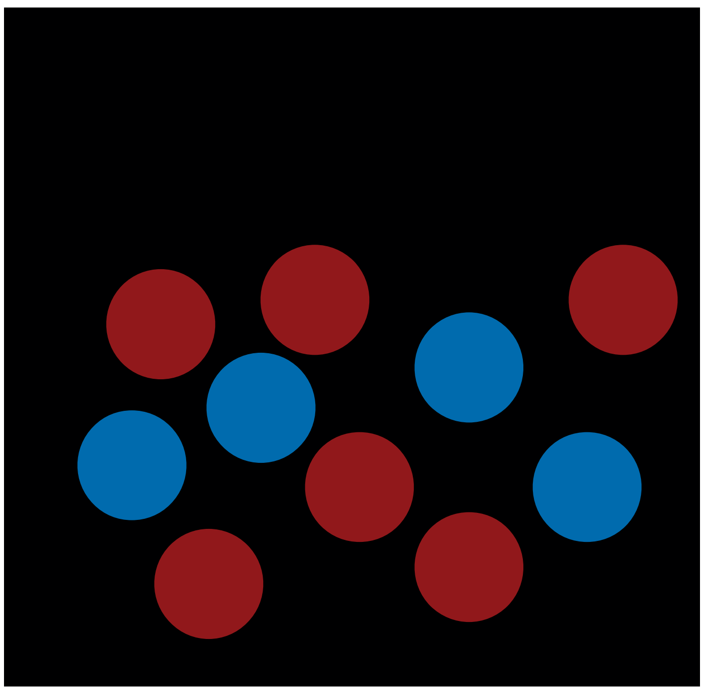
Example: Drawing balls from a box without replacement

Consider the following example of prior information I:

We are given a box with 10 balls 6 of which are red and 4 of which are blue. The box is sufficiently mixed so that when we get a ball from it, we don't know which one we pick. When we take a ball out of the box, we do not put it back.



- Assume that we represent red balls with a 0 and blue balls with 1.
- $X =$ be the random variable corresponding to the outcome of the first draw and Y of the second.
- Let’s find the joint pdf $p(x, y)$.

Example: Drawing balls from a box without replacement

Using results from previous lectures.

$$p(X=0,Y=0)=p(X=0)p(Y=0|X=0)=\frac{3}{5}\cdot\frac{5}{9}\approx 0.33$$

$$p(X=0,Y=1)=p(X=0)p(Y=1|X=0)=\frac{3}{5}\cdot\frac{4}{9}\approx 0.27$$

$$p(X=1,Y=0)=p(X=1)p(Y=0|X=1)=\frac{2}{5}\cdot\frac{2}{3}\approx 0.27$$

$$p(X=1,Y=1)=p(X=1)p(Y=1|X=1)=\frac{2}{5}\cdot\frac{1}{3}\approx 0.13.$$

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- Here is how we marginalize:

second draw

$$p(Y=0) = \sum_x p(X=x, Y=0) = p(X=0, Y=0) + p(X=1, Y=0) \approx 0.33 + 0.27 = 0.6$$

$$p(Y=1) = \sum_x p(X=x, Y=1) = p(X=0, Y=1) + p(X=1, Y=1) \approx 0.27 + 0.13 = 0.4.$$

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- Let's find the covariance of the two random variables.

$$\mathbb{E}[X] = 0 \cdot p(X=0) + 1 \cdot p(X=1) = 0.4,$$
$$\mathbb{E}[Y] = 0 \cdot p(Y=0) + 1 \cdot p(Y=1) = 0.4.$$

$$\begin{aligned}
\mathbb{C}[X,Y] &= \mathbb{E}\left[(X-\mathbb{E}[X])\,(Y-\mathbb{E}[Y])\right] \\
&= \sum_{x,y} p(X=x, Y=y)(x-0.4)(y-0.4) \\
&\approx 0.33 \cdot (0.16) + 0.27 \cdot (-0.24) + 0.27 \cdot (-0.24) + 0.13 \cdot (0.36) \\
&= -0.03.
\end{aligned}$$