Lecture 3: Discrete Random Variables

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The Categorical distribution

Example: The Categorical distribution

- Models an experiment with K outcomes.

$$X = \begin{cases} \underbrace{c_1,}_{} & \text{with probability } p_1, \\ \vdots & \\ c_K, & \text{with probability } p_K, \end{cases}$$

each outcome has a probability

labels could differ

- Notation:

$$\boxed{X \sim \text{Categorical}(p_1, \ldots, p_K)}$$

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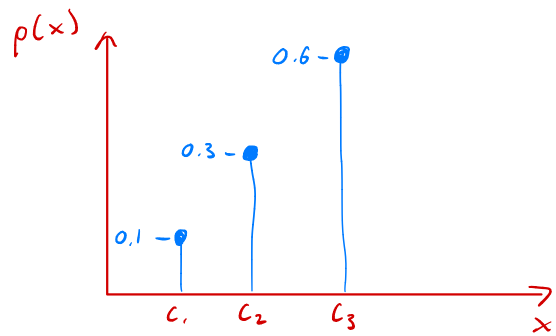
Example: PMF of a Categorical

- Assume $X \sim$ Categorical$(0.1, 0.3, 0.6)$.
- We have $K = 3$ possible outcomes, say c_1, c_2, c_3.
- The PMF is:

$$p(X=c_1) = 0.1$$
$$p(X=c_2) = 0.3$$
$$p(X=c_3) = 0.6$$



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Example: PMF of a Categorical

- Assume $X \sim$ Categorical$(0.1, 0.3, 0.6)$.
- We have $K = 3$ possible outcomes, say c_1, c_2, c_3 .
- The probability that X is either c_1 or c_3.

$$
\begin{aligned}
p(X = c_1 \text{ or } X = c_3) &= p(X \in \underbrace{\{c_1, c_3\}}_{\text{set}}) \\
&= p(X = c_1) + p(X = c_3) \quad \text{decompose the set} \\
&= 0.1 + 0.6 \\
&= 0.7
\end{aligned}
$$

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Example: PMF of a Categorical

- Assume $X \sim$ Categorical$(0.1, 0.3, 0.6)$.
- We have $K = 3$ possible outcomes.
- The expectation is:

$$\mathbb{E}[X] = \sum_x x\, p(x) = c_1 \cdot 0.1 + c_2 \cdot 0.3 + c_3 \cdot 0.6$$

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Example: PMF of a Categorical

- Assume $X \sim$ Categorical$(0.1,0.3,0.6)$.

- We have $K = 3$ possible outcomes.

- The variance is: $\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

where: $\mathbb{E}[X^2] = \sum_x x^2 p(x) = c_1^2 \cdot 0.1 + c_2^2 \cdot 0.3 + c_3^2 \cdot 0.6$

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