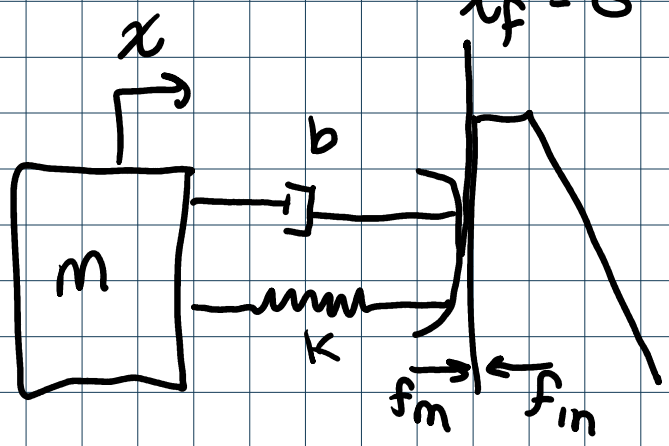
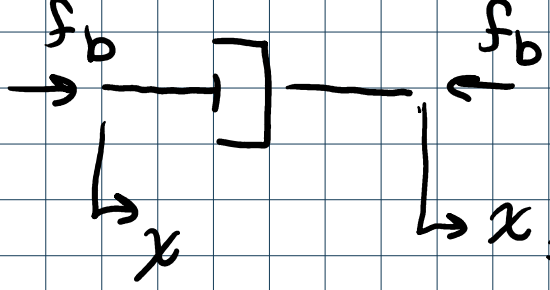
Problem 3

Wednesday, February 14, 2024 6:02 PM

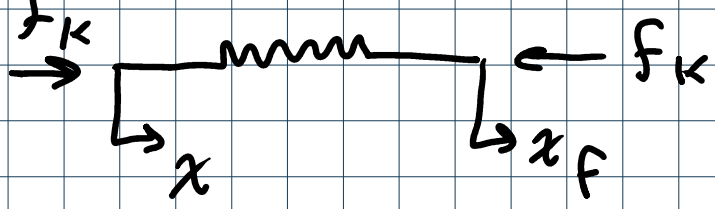
$x_f = 0 \quad \forall t \in \mathbb{R}$





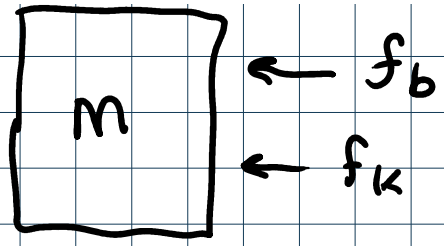
$x_f = 0 \Rightarrow \dot{x}_f = 0$

$$f_b = b(\dot{x} - \dot{x}_f) = b\dot{x}$$



$$f_k = k(x - \cancel{x_f}^{0})$$

$$f_k = kx$$



$$\Sigma F = m\ddot{x} = -f_b - f_k$$

$$m\ddot{x} = -b\dot{x} - kx$$

$$\therefore \quad m\ddot{x} + b\dot{x} + kx = 0 =$$

$$0 = m\left(s^2X(s) - s\cancel{x(0)}^{0} - \dot{x}(0)\right) =$$

$$+ b\left(sX(s) - \cancel{x(0)}^{0}\right)$$

$$+ kX(s)$$

$$0 = ms^2X - mv_{x0} + bsX + kX$$

$$mv_{ox} = (X(s))(ms^2 + bs + k)$$

$$\boxed{\frac{mv_{ox}}{ms^2+bs+k} = X(s)}$$

$$\frac{m v_{ox}}{m s^2 + b s + k} = X(s)$$

$$\frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

$$\frac{-80 \cdot 10^3 \pm 2 \cdot 10^3 \sqrt{622}}{2 \cdot 18 \cdot 10^3}$$

$$\frac{-44 \pm \sqrt{622}}{18} = s_{1,2} \Rightarrow \quad X(s) = m v_0 \left(\frac{1}{} \right.$$

$$x(t) = m v_0 \left(C_1 e^{-\frac{44 - \sqrt{622}}{18} t} + C_2 e^{-\frac{44 + \sqrt{622}}{18} t} \right)$$

$$0 = m v_0 (C_1 + C_2)$$

$$\frac{13}{10} = mv_0\left(C_1 r_1 + C_2 r_2\right)$$

$$\boxed{C_1 = -C_2}$$

$$\frac{13}{10} = mv_0 C_2\left(r_2 - r_1\right)$$

$$\frac{44}{18} + \frac{\sqrt{622}}{18} - \frac{44}{18} + \frac{\sqrt{622}}{18}$$

$$\frac{13}{10} = mv_0 C_2\left(\frac{\sqrt{622}}{9}\right)$$

$$\frac{117}{10\sqrt{622}}\,\frac{1}{mv_0} = C_2$$

$$C_1 = = \frac{-117}{mv_0 \cdot 10\sqrt{622}}$$

$$\boxed{x_{max} = .2077\,m}$$