Problem 1

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a.) $f(t) = \left|A\sin\left(\frac{2\pi}{T_0}t\right)\right|$

$$\frac{1}{2}a_0 = \int_0^{T_0} A\sin\left(\frac{2\pi}{T_0}t\right)dt$$

$$= 2A\int_0^{T_0/2} \sin(2\pi/T_0\, t)\,dt$$

$$= \cancel{2}A\left(\frac{-T_0}{\cancel{2}\pi}\right)\left[\cos\left(\frac{2\pi}{\cancel{T_0}}t\right)\right]_0^{T_0/2}$$

$$= -\frac{AT_0}{\pi}\left[\cos(\pi) - \cos(0)\right]$$

$$= \frac{-AT_0}{\pi}\left[-2\right] = \frac{2AT_0}{\pi}$$

$$a_0 = \frac{4AT_0}{\pi} = \frac{8A}{\omega_0}$$

$$\frac{1}{2}a_0 = \frac{\overset{2}{\cancel{4}}AT_0}{\cancel{2}\pi} = \frac{2AT_0}{\pi}$$

$$\boxed{\frac{1}{2}a_0 = \frac{4A}{\omega_0} \Rightarrow a_0 = \frac{8A}{\omega_0}}$$

$$a_n = \frac{4A}{T_0}\int_0^{T_0/2} \sin(\omega_0 t)\cos(\omega_0 n t)\,dt =$$

$$= \frac{2A}{T_0}\int_0^{T_0/2} \sin((\omega_0 t)(1+n)) + \frac{2A}{T_0}\int_0^{T_0/2} \sin((\omega_0 t)(1-n))\,dt$$

$$= -\frac{2A}{(T_0)(\omega_0)(1+n)}\left[\cos(\omega_0(1+n)t)\right]_0^{T_0/2} - \frac{2A}{(T_0)(\omega_0)(1-n)}\left[\cos(\omega_0(1-n)t)\right]_0^{T_0/2}$$

$$= \frac{-2A}{T\omega_0}\left(\left(\cos\left(\frac{\cancel{2}\pi}{\cancel{T_0}}(1+n)\frac{\cancel{T_0}}{\cancel{2}}\right)\right)\left(\frac{1}{1+n}\right) + \frac{1}{1-n}\cos\left(\frac{\cancel{2}\pi}{\cancel{T_0}}(1-n)\frac{\cancel{T_0}}{\cancel{2}}\right)\right)\right.$$

$$= \frac{-2A}{T\omega_0}\left(\left(\cos\left(\frac{\cancel{2}\pi}{\cancel{T_0}}(1+n)\frac{\cancel{T_0}}{\cancel{2}}\right)\left(\frac{1}{1+n}\right) + \frac{1}{1-n}\cos\left(\frac{\cancel{2}\pi}{\cancel{T_0}}(1-n)\frac{\cancel{T_0}}{\cancel{2}}\right)\right.\right.$$

$$\left.\left. - \left(\frac{1}{1+n} + \frac{1}{1-n}\right)\right)\right) = \frac{-2A}{T_0\omega_0}\left(\frac{(-1)^{n+1}}{n+1} + \frac{(-1)^{n+1}}{1-n} - \ldots\right.$$

$$= \frac{-2A}{T_0\omega_0}\left(\frac{2}{1-n^2}\right)\left(-1^{n+1} - 1\right) = \frac{2A\cancel{T_0}}{\cancel{2}\pi\cancel{T_0}}\left(\frac{\cancel{2}}{1-n^2}\right)\left(1+(-1)^n\right)$$

$$= \frac{2A}{\pi}\frac{1}{1-n^2}\left(1+(-1)^n\right)$$

$$a_n = \frac{4A}{\omega_0 T_0}\frac{1}{1-n^2}\left(1+\cos(\pi n)\right)$$

$-$ 1 $+$ $0, 2$

$$\boxed{a_n = \frac{2A}{\pi}\frac{1}{1-n^2}\left(1+(-1)^n\right)}$$

$$x(t) = \frac{2AT_0}{\pi} + \sum^{\infty}\frac{2A(1+(-1)^n)}{\pi(1-n^2)}\cos(\omega_0 n t)$$

$$x(t) = \frac{2AT_0}{\pi} + \sum_{n=2} \frac{2A(1+(-1)^n)}{\pi(1-n^2)} \cos(\omega_0 n t)$$

b. Rectifiers are not linear Since mathematically, the absolute Value is not Linear.

$f(x+y) \neq |f(x)| + |f(y)|$. in all cases.

Problem 4

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$$R(s) = \frac{\omega_0}{s^2+\omega_0^2}$$

- $$T(s) = \frac{Y}{R} = \frac{CG}{1+CG}, \quad E = R - Y = \left(1 - \frac{CG}{1+CG}\right)R = \frac{1}{1+CG}R$$

$$\mathcal{E} = sE(s)\Big|_{s=0} := \frac{sR}{1 + \frac{K(s+\alpha)^2}{(s^2+\omega_0^2)(s)(s^2+2s+2)}}\Bigg|_{s=0}$$

$$:= \frac{(s)^2(s^2+\omega_0^2)(s^2+2s+2)R}{(s)(s^2+\omega_0^2)(s^2+2s+2)+K(s+\alpha)^2}\Bigg|_{s=0}$$

$$= 0$$

$$\boxed{\mathcal{E} = 0}$$

Problem 5

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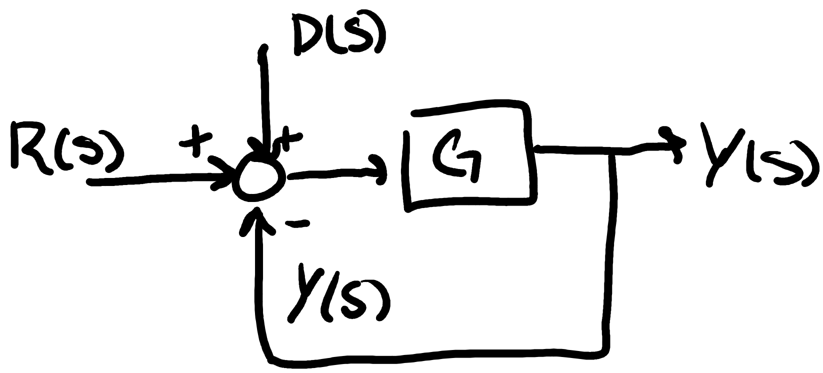

$$G\left(R(s) + D(s) - Y(s)\right) = Y(s)$$

$$G\left(R(s) + D(s)\right) = (1+G)\left(Y(s)\right)$$

$$\frac{G\left(R(s) + D(s)\right)}{1+G} = Y(s)$$

Assuming $R(s) = 0 \ \forall s \in \mathbb{C}$

$$\frac{G\,D(s)}{1+G} = Y(s) \Rightarrow \frac{Y(s)}{D(s)} = \frac{G}{1+G}$$

$$E(s) = R - Y = 0 - Y = \frac{-G}{1+G} D(s)$$

$$E(s) = R - Y = 0 - Y = \frac{-G}{1+G} D(s)$$

let $L(s) = G(s) = \dfrac{1}{(s)(s^2+2s+2)}$

Thus $L(s) \equiv 1$

- So for:

a.) $\varepsilon_{step} = 0$

b.) $\varepsilon_{ramp} = \dfrac{1}{s\,L(s)}\Big|_{s=0} = \dfrac{1}{s^2+2s+2}\Big|_{s=0}$

$$= \frac{1}{2}$$

$$\varepsilon_{ramp} = 1/2$$