Homework 7

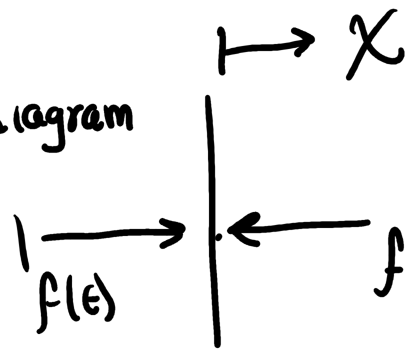
Monday, March 4, 2024 11:51 AM

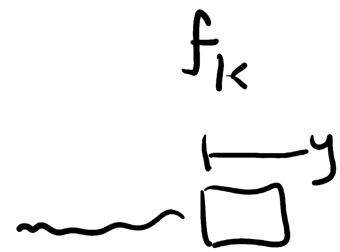
Problem

Monday, March 4, 2024 12:06 PM

$m\ddot{x} = f(t) - f$

$$\underbrace{m\ddot{y} + b\dot{y} + ky}_{\text{Hom'g must exist}} = \underbrace{kx_{in}}_{\text{by diagram}}$$





$f_K = kx - k(y)$

$$f(t) = (m_s)\ddot{x}$$

$$F(s) = m_s s^2 X(s), \ \forall\ \dot{x}(0) = 0,\ x(0) = 0$$

$$X(s) = \frac{F(s)}{m_s s^2}$$

$$ms^2 Y(s) + bsY(s) + kY(s) = kX(s)$$

$$(Y(s))(ms^2 + bs + k) = k(X(s))$$

$$\boxed{\frac{Y(s)}{F(s)} = \frac{k/m_s}{s^2(ms^2 + bs + k)}}$$

Problem 2

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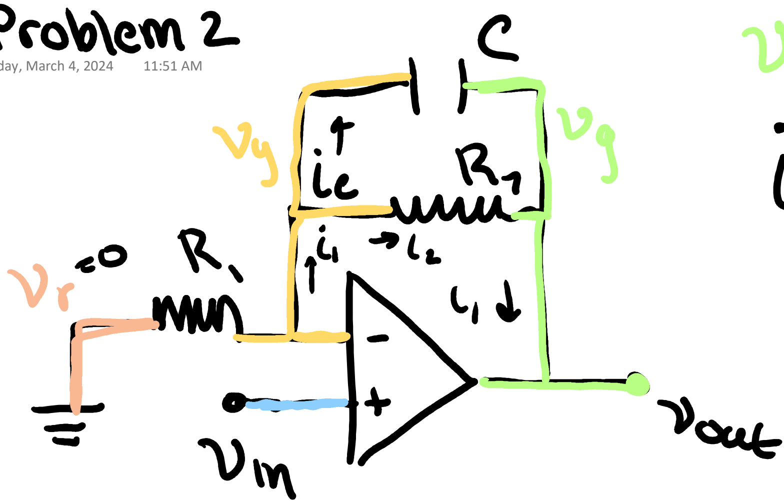

$v_g = v_{out}$

$V = IR$

$$i_1 = i_c + \frac{v_y - v_{out}}{R_2}$$

$$i_1 = C(\dot{v}_y - \dot{v}_{out}) + \frac{v_y - v_{out}}{R_2}$$

$$i_1 = \frac{v_y}{R_1}$$

1. $C(\dot{v}_y - \dot{v}_{out}) = i_c$

2. $v_{out} = G(v_{in} - v_y)$

3. $v_y = i_1 R_1$

4. $$\boxed{\frac{v_{in}}{R_1} = C(\dot{v}_{in} - \dot{v}_{out}) + \frac{v_{in} - v_{out}}{R_2}}$$

$$R_2 v_{in} = R_1 R_2 C \frac{d}{dt}(v_1 - v_0) + R_1 v_{in} - R_1 v_0$$

$$R_2 V_1 = R_1 R_2 C s(V_1 - V_0) + R_1 V_1 - V_0 R_1$$

$$(R_2 - R_1 R_2 C s - R_1) V_1 = -(R_1 R_2 C s + R_1) V_0$$

$$\boxed{\frac{(R_1 - R_2 + R_1R_2Cs)}{R_1(1+R_2Cs)} = V(s)} = \frac{V_o(s)}{V_1(s)}$$

Problem 3

Monday, March 4, 2024 8:19 PM

$$\frac{s}{s^2+1}$$

$$v(t) = \cos(t)$$

$$w(t) = \sin t$$

$$y(t) = \int_0^t \sin(\underset{\alpha}{\tau})\cos(\underset{B}{t-\tau})\,d\tau$$

$$\tau - t$$

$$= \frac{1}{2}\int_0^t \sin(\cancel{\tau} + t - \cancel{\tau}) + \sin(\underset{2\tau - t}{\tau - t + \tau})\,d\tau$$

$$u = 2\tau - t$$

$$= \frac{1}{2}\left(\int_0^t \sin(t)\,d\tau + \int_{-t}^{t} \frac{1}{2}\sin(u)\,du\right):$$

$$u = 2\tau - t, \ \forall\ \tau, t \in \mathbb{R}$$

$$\left.\begin{aligned} u &= 2\tau - t \\ u(0) &= -t \\ u(t) &= t \end{aligned}\right\} \Rightarrow \int_{-t}^{t} \frac{1}{2}\sin(u)\,du = 0$$

$$= \frac{1}{2}\int_0^t \sin(t)\,d\tau$$

$$= \frac{1}{2}\tau \sin t \Big|_0^t$$

$$y(t) = \frac{1}{2} t \sin t$$