Homework 5

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Group 1

Problem 1

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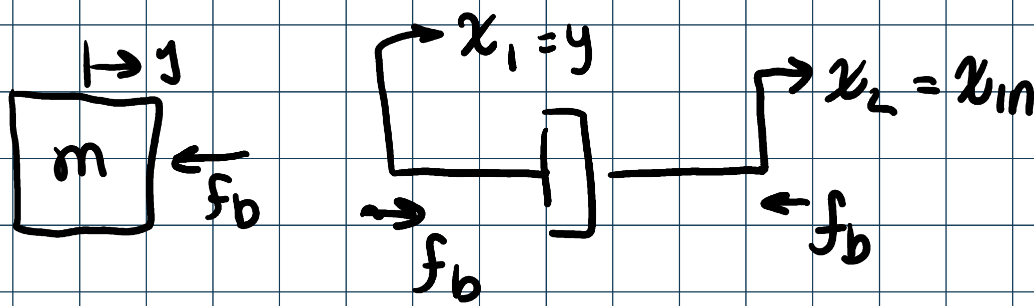
a. Second Order

b.



$$\Sigma F = m\ddot{y} = -f_b \qquad f_b = b(\dot{y} - \dot{x}_{in})$$

$$m\ddot{y} = b\dot{x}_{in} - b\dot{y}$$

$$\boxed{\text{all I.C.} = 0}$$

$$m\ddot{y} + b\dot{y} = b\dot{x}_{in}$$

$$m\left(s^2 Y(s)\right) + b(sY(s) = bsX(s)$$

$$Y(s)\left(ms^{\cancel{2}\,1} + b\cancel{s}\right) = (b\cancel{s})X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{bs}{ms^2 + bs}$$

$$H(s) = \frac{b}{ms + b}$$

$$\boxed{H(s) = \frac{\frac{b}{m}}{s + b/m}}$$

1st Order based on the T.F.

Problem 2

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a.)

$$G(s) = \frac{\sigma}{s+\sigma}\check{k}$$

$$G(s) = (2)\frac{4}{s+4} \Rightarrow \check{k} = 2$$

$$t_R = \frac{2.2}{4} = \frac{1.1}{2} = \frac{11}{20}$$

$$\boxed{t_R = 11/20 \text{ sec}}$$

b.)

$$G(s) = 2A\frac{4}{s+4}$$

$$\lim_{t\to\infty}\left(y(t) = 2A\left(1 - e^{-4t}\right)\right) = 2A$$

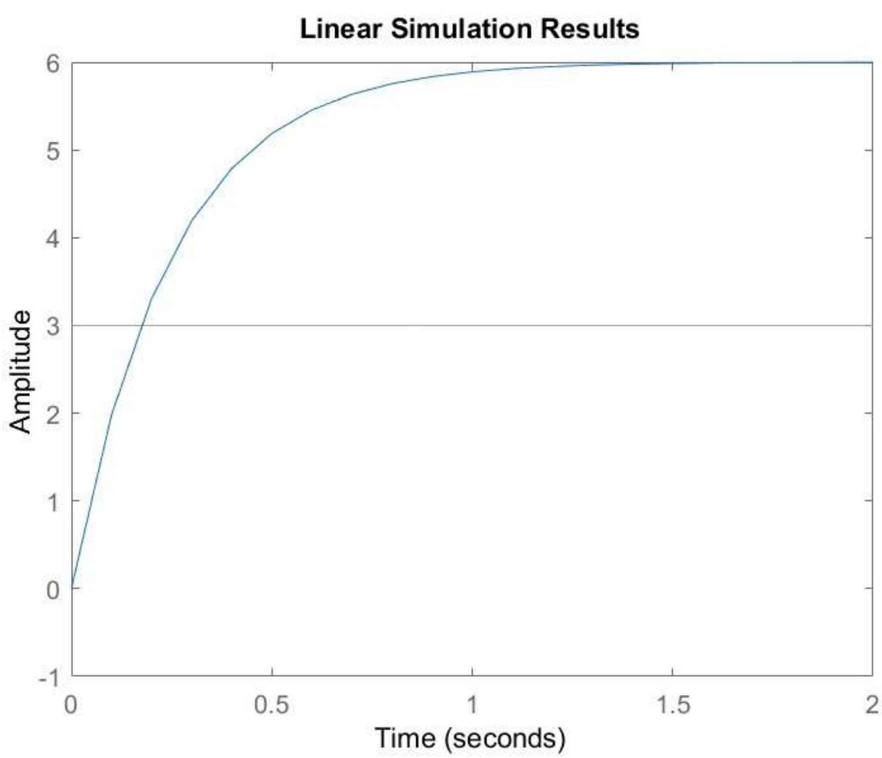
$$\boxed{\lim_{t \to \infty}(y(t)) = 6}$$

0.55s

c.) From fig: $t_{.9} = .58s$

$t_{.1} = .0303s$

$$\boxed{t_R = .5497s \approx .55s \approx t_R \text{ from(a.)}}$$



Problem 3

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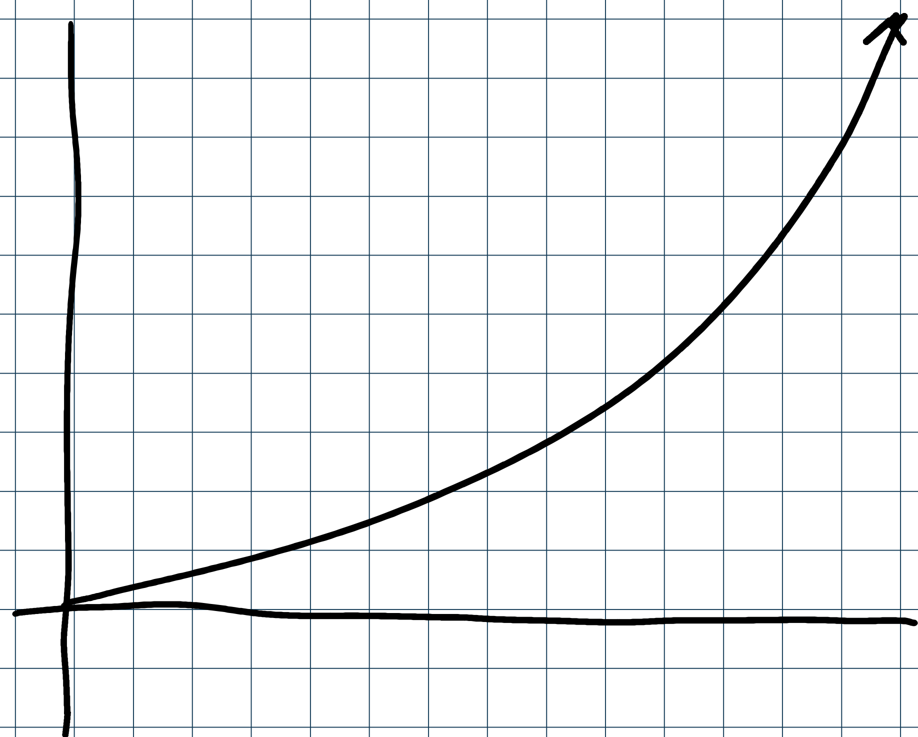

- This balloons to ∞ because according the one unstable pole

<u>Proof:</u> Suppose $y(t) = c_1 e^{-r_1 t} + c_2 e^{-r_2 t}$ is a solution to the system with transfer function $G: \mathbb{R} \to \mathbb{R}$ $\forall t, c_1, c_2 \in \mathbb{R}$

- Thus by def we have, $\exists r_1, r_2, \hat{k} \in \mathbb{R}$

$$G(s) = \hat{k} \frac{1}{(s+r_1)(s+r_2)}$$

$$G(s) = K \frac{}{(s+r_1)(s+r_2)}$$

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Problem 4

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$$G(s) = \check{K}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For underdamped:

- $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

  $= -\sigma \pm j\omega_d$

- $\omega_n = \sqrt{k/m}$, $\zeta = \dfrac{b}{2\omega_n m} = \dfrac{b}{2\sqrt{mk}}$

- $\omega_d = \omega_n\sqrt{1-\zeta^2}$

a.) $$G_g(s) = \frac{\omega_n^2}{s^2 + \omega_n s + \omega_n^2}$$

$G_g(s) = \dfrac{36}{\quad} = \dfrac{1/m}{\quad}$

$$G(s) = \frac{36}{s^2+6s+36} = \frac{1/m}{s^2+\frac{b}{m}s+\frac{k}{m}}$$

$$G(s) = \frac{1}{\frac{1}{36}s^2+\frac{1}{6}s+1} = \frac{1}{k}\frac{k/m}{s^2+b/m\, s+k/m}$$

$m = 1/36 \quad b = 1/6 \quad k = 1$

$\boxed{\omega_n = 6} = \sqrt{k/m}$

$$\zeta = \frac{\frac{1}{6}}{2(6)\,1/36} = \frac{\cancel{36}}{2\cancel{(6)}\cancel{(6)}} = 1/2$$

$\boxed{\zeta = 1/2}$

$\hat{k} = 1/k = \boxed{1 = \hat{k}}$

$1 - 1/4$

$y_{ss} = G(0) = 1$

$$M_p = (1)\, e^{-\pi/2 \sqrt{3/4}}$$

$$M_p = e^{-\sqrt{3}\pi/4}$$

$$\boxed{\%O.S \approx 25.66\% \Rightarrow y_{max} \approx 1.2566}$$

$$t_R = \frac{18}{60} = \frac{3}{10}$$

$$\boxed{\therefore \text{Chart b}}$$

b.)

$$G(s) = \frac{36}{100s^2 + 60s + 36}$$

$$= \frac{1}{\frac{100}{36}s^2 + \frac{60}{36}s + 1}$$

$m = 25/9 \quad b = 5/3 \quad k = 1$

$\boxed{\omega_n = 3/5}$

$$\zeta = \frac{5/3}{2 \cancel{5/3}}$$

$\boxed{\zeta = 1/2}$

only t_R, t_s changes betwen part a, b.

$$t_R = \frac{18}{6} = 3s \; ,$$

Thus,

Chart C

c.)

$$G(s) = \frac{1}{\frac{1}{36}s^2 + \frac{1}{2}s + 1}$$

$m = 1/36$ $b = 1/2$ $k = 1$

$\omega_n = 6$ $\zeta = 3/2$

$\hat{k} = 1$ $\zeta\omega_n = \sigma$

$$\boxed{\sigma = 9}$$

- $y_{ss} = 1$
- $t_s = .5\overline{1}\ s$
- $t_R = .3\ s$

- No overshoot

Thus,

$\boxed{\text{Chart D}}$

d.)

Only rise time changes

$t_R = 3s$

Thus,

$\boxed{\text{Chart E}}$

e.)

$$G(s) = \frac{1}{10}\frac{10}{(s+10)} = \hat{k}\,\frac{10}{s+10}$$

$k = 10, \quad \sigma = 10$

$t_r = \frac{22}{100}s = .22\,s$

$t_s = .46s$

$$\boxed{\text{Thus, Chart A}}$$

f.)

$\omega_n = \sqrt{10} \qquad \sigma = 11/2$

$$b = \frac{11\sqrt{10}}{20}$$

Thus,

Chart F

Problem 5

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$y_{ss} = 1.2 = C\hat{k} \qquad C = 3/5 \Rightarrow \hat{k} = 2$

$t_s = 2.75s \Rightarrow \sigma \approx 1.673$

$\%OS \approx 16/12 \approx 1.3\overline{3} \Rightarrow \zeta = 1/3$

$\sigma = \omega_n \zeta \Rightarrow 3\sigma = \omega_n \Rightarrow \omega_n \approx 5.018 \approx 5$

Thus, $\boxed{G(s) \approx \dfrac{50}{s^2 + \frac{10}{3}s + 25}}$

Problem 6

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- $G(s) = \dfrac{1}{\text{Char}(x)} = \dfrac{1}{9s^2 + cs + 4}$

- $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\hat{k}$

$$= \frac{1}{4}\,\frac{4/9}{s^2 + \frac{c}{9}s + 4/9} \Rightarrow \hat{k} = 1/4,\ \omega_n = 2/3$$

$$2\omega_n\zeta = \frac{c}{9} \qquad \zeta = \frac{c}{2\sqrt{9\cdot 4}} = \frac{c}{12}$$

$$\boxed{t_R \approx \frac{1.8}{2/3} = 2.7s < 3s \checkmark}$$

$$-\pi\zeta \qquad -\frac{\frac{c\pi}{12}}{\quad}$$

$$\frac{3}{10} = e^{-\frac{\pi b}{\sqrt{\frac{144}{144} \quad \frac{c^2}{144}}}} = e^{-\frac{\frac{}{\cancel{12}}}{\frac{\sqrt{144-c^2}}{\cancel{12}}}} =$$

$$\ln(3/10)^2 = \left(-\frac{c\pi}{\sqrt{144-c^2}}\right)^2$$

$$\ln(3/10)^2 = \frac{c^2\pi^2}{144-c^2}$$

let $\ell = \ln(3/10)^2$

$$144 - c^2 = \frac{\pi^2}{\ell} c^2$$

$$144 = \left(\frac{\pi^2}{\ell} + 1\right) c^2$$

$$\boxed{c = \frac{12}{\sqrt{\pi^2 \ln(3/10)^{-2} + 1}}}$$