Homework 10

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Problem 1

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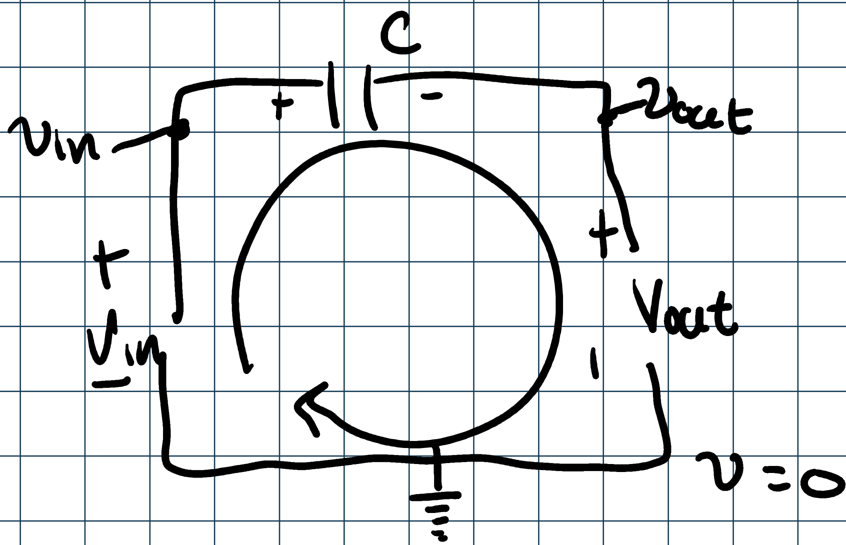
a.



$$\dot{i} = i_c = i_R = C(\dot{v}_{in} - \dot{v}_{out}) = \frac{v_{out}}{R}$$

$$RC(\dot{v}_{in} - \dot{v}_{out}) = v_{out}$$

$$\mathcal{L}\{v_{out} + RC\dot{v}_{out} = \dot{v}_{in}\}$$

all I.C.'s are zero:

$$V_o + RCsV_o = sV_{in}$$

$$V_o(1+RCs) = sV_{in}$$

$$\boxed{G(s) = \frac{V_o}{V_{in}} = \frac{s}{1+RCs}}$$

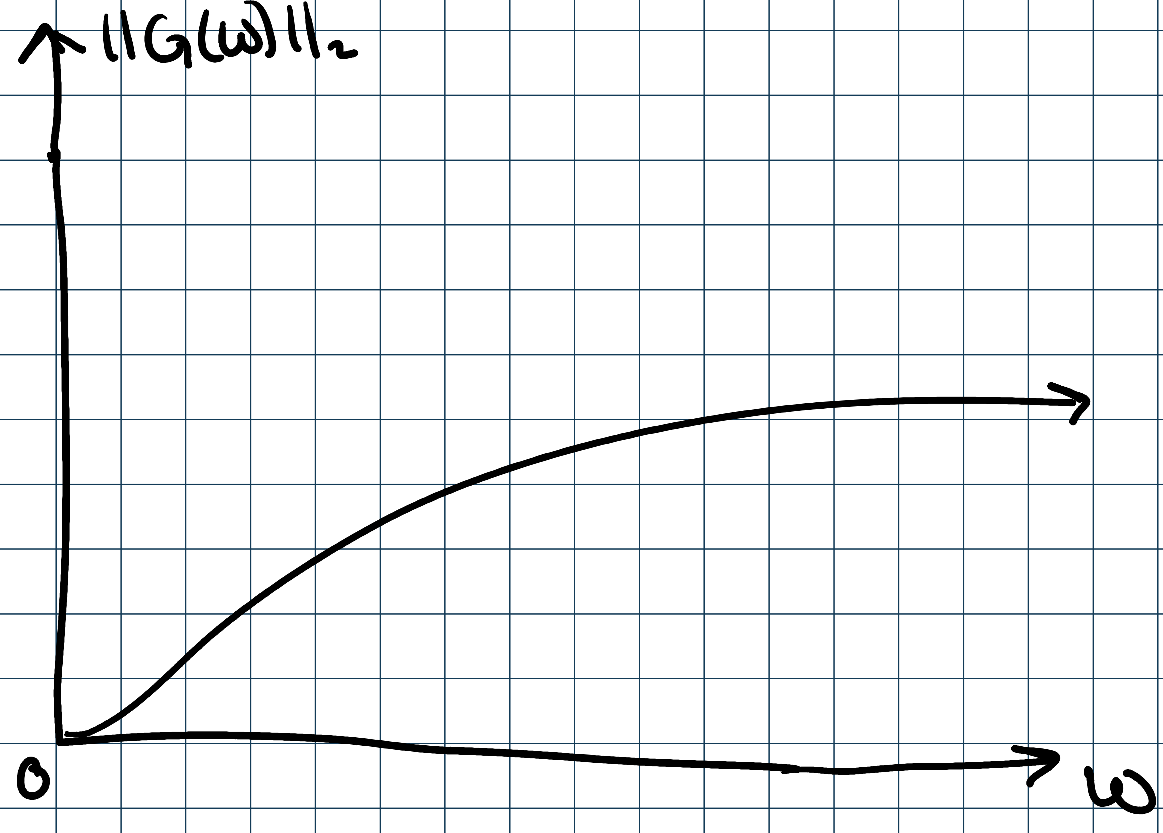
b.

$$G(\omega) = \frac{\omega j}{1 + RC\omega j}$$

$$\|G(\omega)\|_2 = \frac{\omega}{\sqrt{1 + R^2C^2\omega^2}}$$

$$\angle G(\omega) = \frac{\pi}{2} - \mathrm{Tan}^{-1}(RC\omega)$$

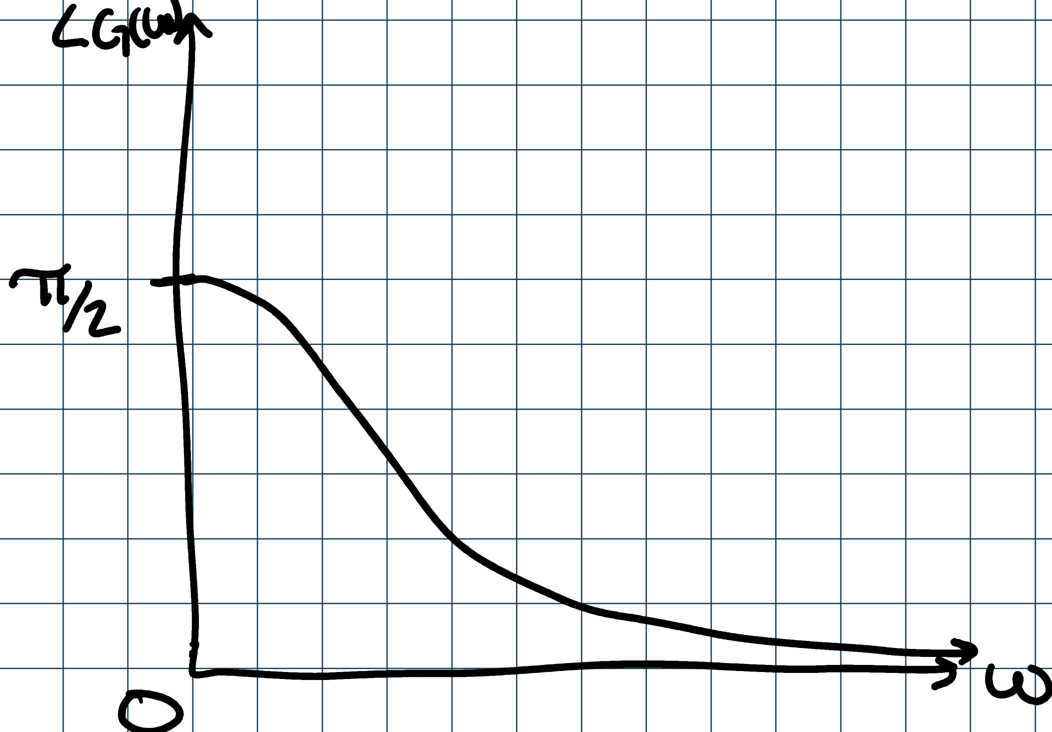
c.)



$$\lim_{\omega \to 0} \left(\| G(\omega) \|_2 \right) = 0$$

$$\frac{\cancel{\pi}}{\cancel{2}} \cdot \frac{\cancel{180}^{\,90}}{\cancel{\pi}}$$

$$\lim_{\omega \to \infty} \left(\| G(\omega) \|_2 \right) = \frac{1}{|RC|} \text{ radians} = \frac{180}{\pi |RC|} \deg$$



$$\lim_{\omega \to 0} \angle G(\omega) = \pi/2 = 90^\circ$$

$$\lim_{\omega \to \infty} \angle G(\omega) = 0$$

Problem 2

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a.

$$G(\omega) = \frac{K}{\underbrace{-a\omega^3 j} - b\omega^2 + \underbrace{c\omega j} + d}$$

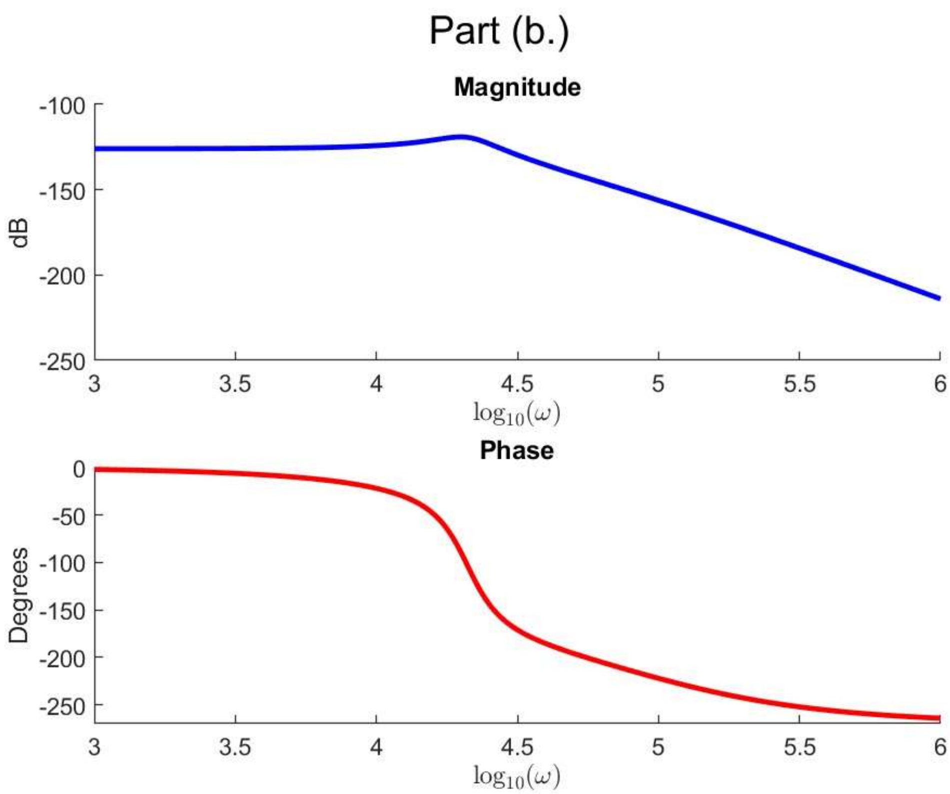
$$= \frac{K}{(d - b\omega^2) + ((c - a\omega^2)\omega)j}$$

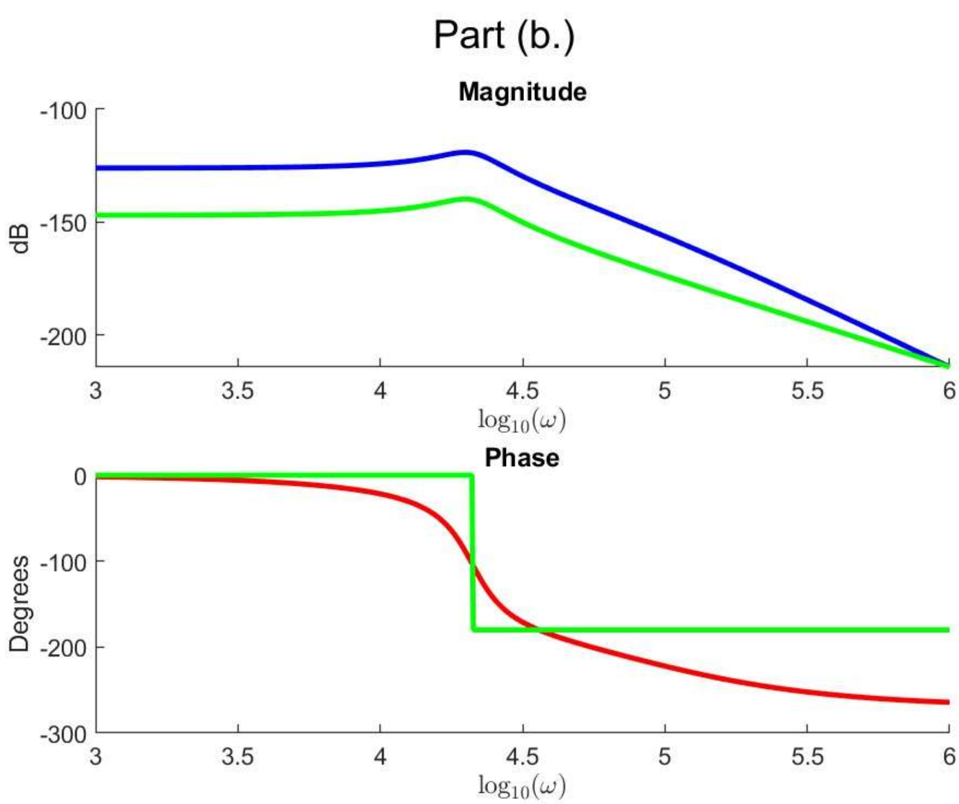
$$\|G(\omega)\|_2 = \frac{|K|}{\sqrt{(d-b\omega^2)^2 + (c-a\omega^2)^2\omega^2}}$$

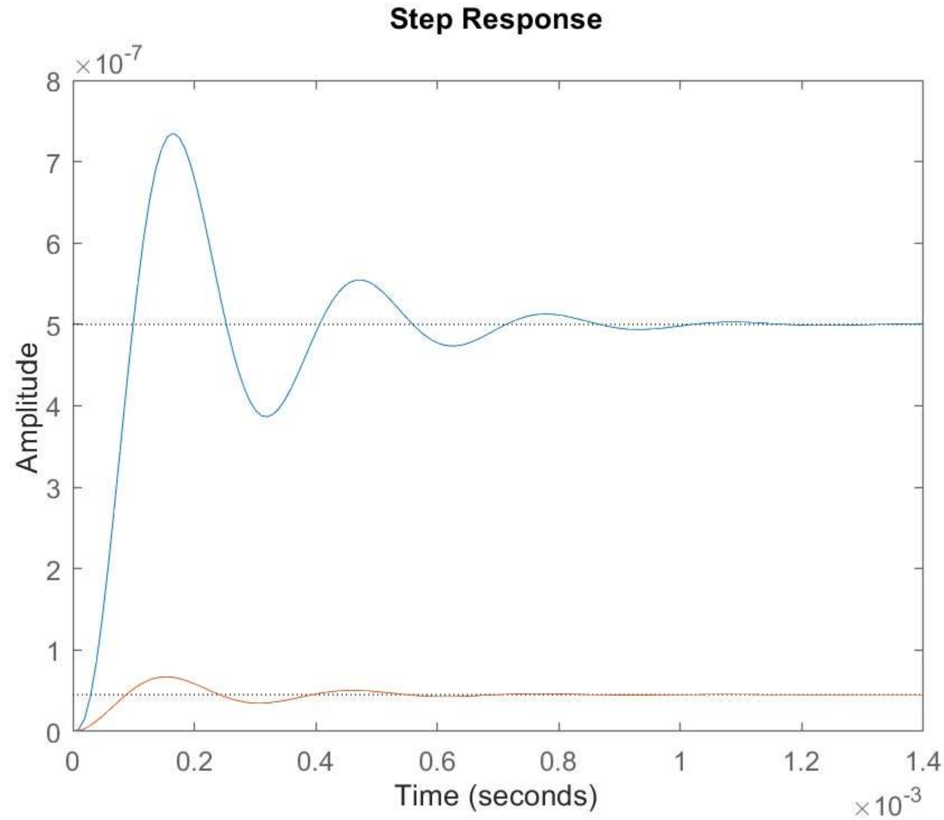
$$\angle G(\omega) = -\tan^{-1}\left(\frac{(c-a\omega^2)\omega}{d - b\omega^2}\right)$$

$\omega_n = 19869$ Hz

$\omega_{3dB} = 12504$ Hz


Part (b.)
Magnitude
-100
-150
-200
-250
dB
3
3.5
4
4.5
5
5.5
6
$\log_{10}(\omega)$
Phase
0
-50
-100
-150
-200
-250
Degrees
3
3.5
4
4.5
5
5.5
6
$\log_{10}(\omega)$


Part (b.)
Magnitude
-100
-150
-200
dB
3
3.5
4
4.5
5
5.5
6
$\log_{10}(\omega)$
Phase
0
-100
-200
-300
Degrees
3
3.5
4
4.5
5
5.5
6
$\log_{10}(\omega)$


Step Response
×10^-7
8
7
6
5
4
3
2
1
0
Amplitude
0
0.2
0.4
0.6
0.8
1
1.2
1.4
Time (seconds)
×10^-3

3.

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$$G = \frac{\hat{k}\sigma}{\sqrt{\sigma^2+\omega^2}}$$ estimating params σ $\hat{k}$

$$\frac{\partial G}{\partial \hat{k}} = \frac{\sigma}{\sqrt{\sigma^2+\omega^2}} \quad \begin{matrix} f \\ g \end{matrix}$$

$$\frac{f'g - fg'}{g^2}$$

$$\frac{\partial G}{\partial \sigma} = \hat{k}\left(\frac{\sqrt{\sigma^2+\omega^2} - \sigma\frac{2\sigma}{2\sqrt{\sigma^2+\omega^2}}}{\sigma^2+\omega^2}\right)$$

$$= \hat{k}\left(\frac{\frac{\cancel{\sigma^2}+\omega^2-\cancel{\sigma^2}}{\sqrt{\sigma^2+\omega^2}}}{\sigma^2+\omega^2}\right)$$

$$= \frac{\hat{k}\omega^2}{(\sigma^2+\omega^2)^{3/2}}$$

$$J_i = \begin{bmatrix} \frac{\sigma}{\sqrt{\sigma^2+\omega^2}} & \frac{k\omega^2}{(\sigma^2+\omega^2)^{3/2}} \\ \downarrow \omega_I & \downarrow \end{bmatrix} \underbrace{\begin{bmatrix} \Delta \hat{k} \\ \Delta \sigma \end{bmatrix}}_{B}$$

$$R_{i+1} =$$

$$R_i = J_i B$$

$$J_i' R_i = J_i J_i B$$

$$(J_i' J_i)^{-1} J_i' = B$$

<u>1 norm error</u>

$$\hat{k} = 5.5196$$

$$\Rightarrow \quad G(s) = \frac{(5.5196)(.7843)}{.7843 + s}$$

$$\sigma = .7843$$

<u>2. Norm error</u>

Same

Sindy Norm w 1 norm.

Same

$$\boxed{G(s) \overset{10^{-9}}{\approx} \frac{4.328798491}{.78425928 + s}}$$

Problem 5

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a.)

$$a_0 = \frac{2}{T_0}\int_0^{T_0} f(t)\,dt$$

$$a_0 = \frac{\cancel{2}}{\cancel{T_0}}\left(\cancel{2}\left(\frac{1}{\cancel{2}}\left(\frac{\cancel{T_0}}{\cancel{2}}\right)A\right)\right.$$

$$\boxed{a_0 = A_0}$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} f(t)\cos\left(\frac{2\pi n}{T_0}t\right)dt$$

$$f(t) = \frac{2A}{T_0}t$$

$$a_{n_1} = \frac{2}{T_0}\int_0^{\frac{1}{2}T_0} \frac{2At}{T_0}\cos\left(\frac{2\pi n}{T_0}t\right)dt$$

D I

$+ \quad \frac{2At}{T_0} \qquad \cos\left(\frac{2\pi n}{T_0}t\right)$

$$- \frac{2A}{T_0} \quad \frac{T_0}{2\pi n}\sin\left(\frac{2\pi n\, t}{T_0}\right)$$

$$+ \quad 0 \leftarrow -\left(\frac{T_0}{2\pi n}\right)^2 \cos\left(\frac{2\pi n}{T_0}t\right)$$

$$a_n = \frac{4}{T_0}\left[\frac{2A}{T_0}t\left(\frac{T_0}{2\pi n}\right)\sin\left(\frac{2\pi n\, t}{T_0}\right) + \frac{2A}{T_0}\left(\frac{T_0^2}{4\pi^2 n^2}\right)\cos\left(\frac{2\pi n}{T_0}t\right)\right]_0^{\frac{1}{2}T_0}$$

$$= \frac{4}{T_0}\left[\frac{\cancel{2}A\cancel{T_0}}{\cancel{2}\cancel{T_0}}\left(\frac{T_0}{2\pi n}\right)\cancel{\sin\left(\frac{2\pi n T_0}{2T_0}\right)}^{\,0}\right.$$

$$+ \frac{\cancel{2}A}{\cancel{T_0}}\left(\frac{T_0^{\cancel{2}\,1}}{\cancel{4}_{2}\pi^2 n^2}\right)\cos\left(\frac{2\pi n}{\cancel{T_0}}\frac{1}{\cancel{2}}\cancel{T_0}\right)$$

$$-$$

$$\left.\frac{\cancel{2}A}{\cancel{T_0}}\left(\frac{T_0^{\cancel{2}\,1}}{\cancel{4}_{2}\pi^2 n^2}\right)\right]$$

$$= \frac{\cancel{4}^{\,2}}{\cancel{T_0}}\left[\frac{A\cancel{T_0}}{\cancel{2}(\pi n)^2}\cos(\pi n) - \frac{A\cancel{T_0}}{\cancel{2}(\pi n)^2}\right]$$

$$= \frac{2A}{(\pi n)^2}(\cos(\pi n) - 1)$$

$(-1)^n$ $\overset{1}{-\frac{1}{\pi}}$ $\overset{2}{\frac{1}{2\pi}}$ $\overset{3}{\frac{-1}{3\pi}}$

$b_n = 0$ bc even

b.