Homework 4

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Problem 1

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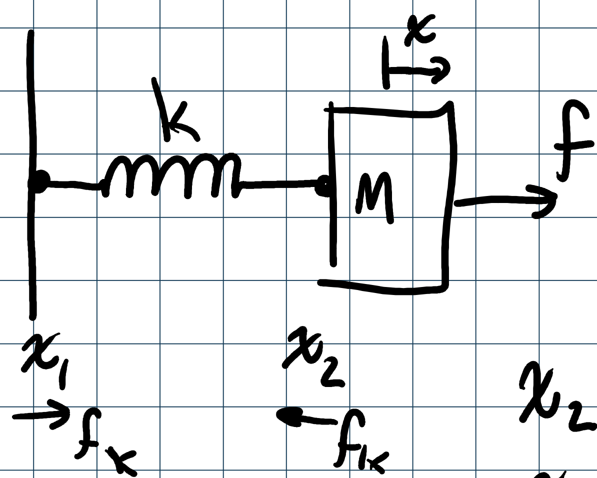
a.)

$\vec{f} = k\Delta\vec{x}$



$\Sigma F = m\ddot{x} = f_{in} + f_k$

$$\boxed{f_{in} = m\ddot{x} + kx}$$

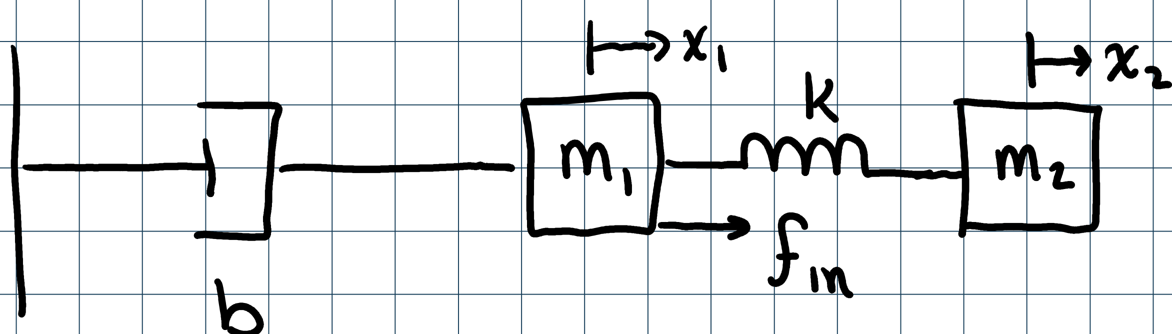
$x_2 = x$

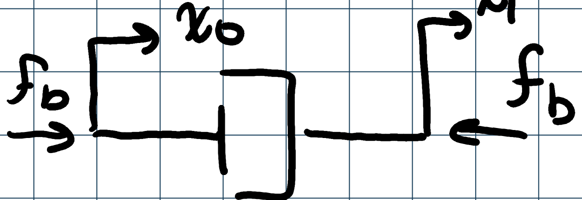
$x_1 = 0 \;\forall t$

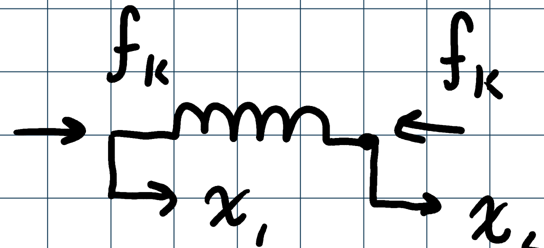
$f_k = k(x_1 - x_2)$

$\Rightarrow f_k = -kx$

b.)



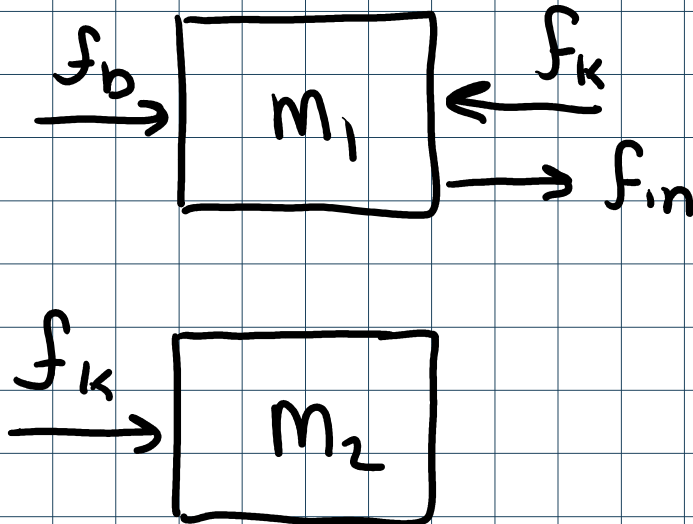




$f_b = b(\cancel{\dot{x}_0}^{\,0} - \dot{x}_1)$

$f_k = k(x_1 - x_2)$

$$f_b = b(\cancel{\dot{x}_0} - \dot{x}_1) \qquad f_k = k(x_1 - x_2)$$



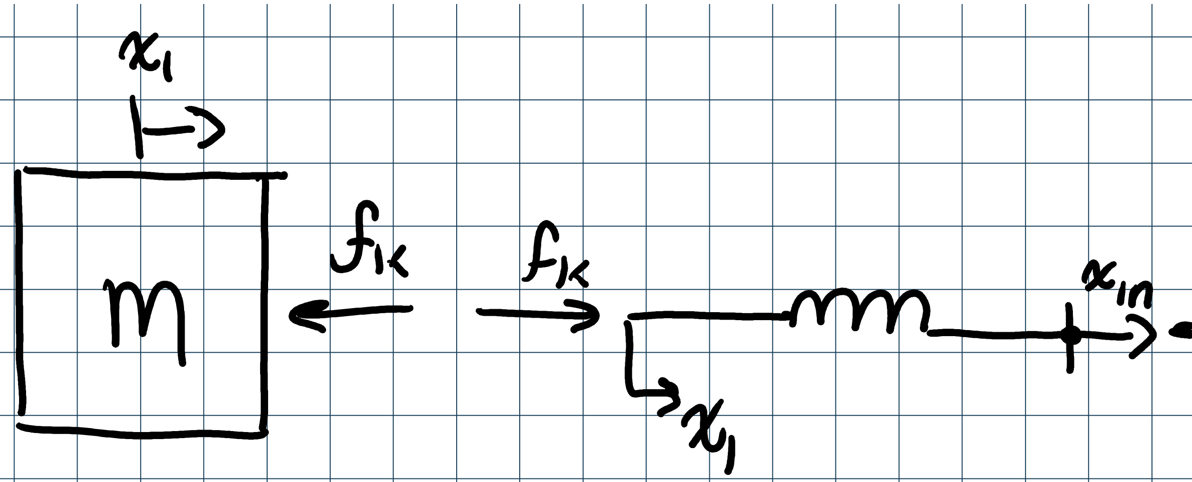
1. $$\Sigma F = m_1\ddot{x}_1 = -b\dot{x}_1 - k(x_1 - x_2) + f_{in}$$

$$\boxed{m_1\ddot{x}_1 + b\dot{x}_1 + k(x_1 - x_2) = f_{in}}$$

2. $$\Sigma F = m_2\ddot{x}_2 = f_k = k(x_1 - x_2)$$

$$\boxed{m_2\dot{x}_2 + kx_2 = kx_1}$$

c.)



$$f_k = k(x_1 - x_{in})$$

$$\Sigma F = m\ddot{x}_1 = -f_k$$

$$m\ddot{x}_1 = kx_{in} - kx_1$$

$$\boxed{m\ddot{x}_1 + kx_1 = kx_{in}}$$

Problem 2

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1. $m_1\ddot{x}_1 + b\dot{x}_1 + k(x_1 - x_2) = f_{in}$

2. $m_2\ddot{x}_2 + kx_2 = kx_1$

1.) $F_{in}(s) = m(s^2X_1(s) - 0 - 0)$

$+ b(sX_1(s) - 0)$

$G_1(s) = \dfrac{X_2(s)}{F_{in}(s)}$

$+ k(X_1(s) - X_2(s))$

$F_{in}(s) = m_1s^2X_1 + bsX_1 + kX_1 - kX_2$

2.) $kX_1(s) = m_2(s^2X_2(s) - 0 - 0)$

$+ kX_2$

$kX_1 = m_2s^2X_2 + kX_2$

$x_1 = \dfrac{m_2}{k}s^2X_2 + X_2$

$$\frac{m_2}{k} s^2 X_2 + X_2$$

$$F_{in} = m_1 s^2 \left(\frac{m_2}{k} s^2 X_2 + X_2 \right)$$

$$+ bs \left(\frac{m_2}{k} s^2 X_2 + X_2 \right)$$

$$+ k \left(\frac{m_2}{k} s^2 X_2 + X_2 \right) - kX_2$$

$$F_{in} = X_2 (m_1 s^2) \left(\frac{m_2}{k} s^2 + 1 \right)$$

$$+ X_2 (bs) \left(\frac{m_2}{k} s^2 + 1 \right)$$

$$+ X_2 (k) \left(\frac{m_2}{k} s^2 + 1 \right)$$

$$- X_2 (k) (1) = x_2 (m_1 s^2 + bs + k) \left(\frac{m_2}{k} s^2 + 1 \right) - kx$$

$$F_{in} = X_2 \left(\left(\frac{m_2}{k} s^2 + 1 \right) (m_1 s^2 + bs + k) - k \right)$$

$$G(s) = X_2 \quad = \frac{1}{\quad}$$

$$G(s) = \frac{X_2}{F_{in}} = \frac{1}{\left(\frac{m_2}{k}s^2+1\right)\left(m_1 s^2+bs+k\right)-k}$$

Problem 3

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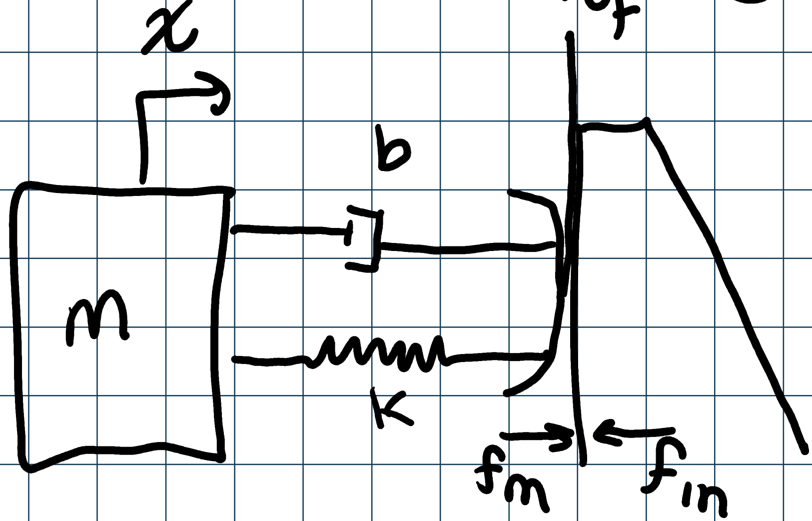
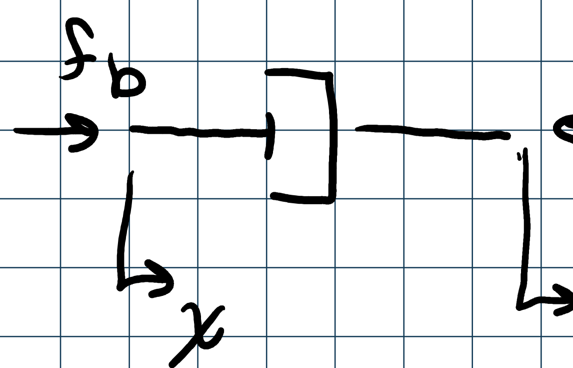
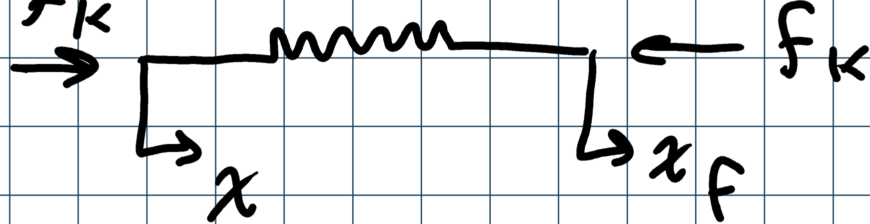
$$x_f = 0 \quad \forall t \in \mathbb{R}$$





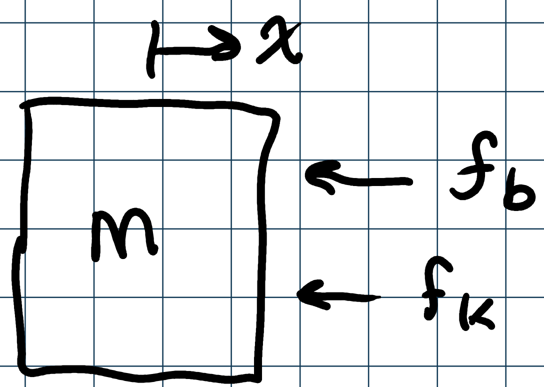
$$x_f = 0 \Rightarrow \dot{x}_f = 0$$

$$f_b = b(\dot{x} - \dot{x}_f) = b\dot{x}$$



$$f_k = k(x - \cancel{x_f}^{\,0})$$

$$f_k = kx$$



$$\sum F = m\ddot{x} = -f_b - f_k$$

$$m\ddot{x} = -b\dot{x} - kx$$

$$\therefore \quad m\ddot{x} + b\dot{x} + kx = 0 =$$

$$0 = m\left(s^2 X(s) - s\,\cancelto{0}{x(0)} - \dot{x}(0)\right) =$$

$$+\, b\left(sX(s) - \cancelto{0}{x(0)}\right)$$

$$+\, kX(s)$$

$$0 = ms^2 X - mv_{x0} + bsX + kX$$

$$mv_{0x} = (X(s))(ms^2 + bs + k)$$

$$\boxed{\frac{mv_{0x}}{ms^2 + bs + k} = X(s)}$$

Problem 4

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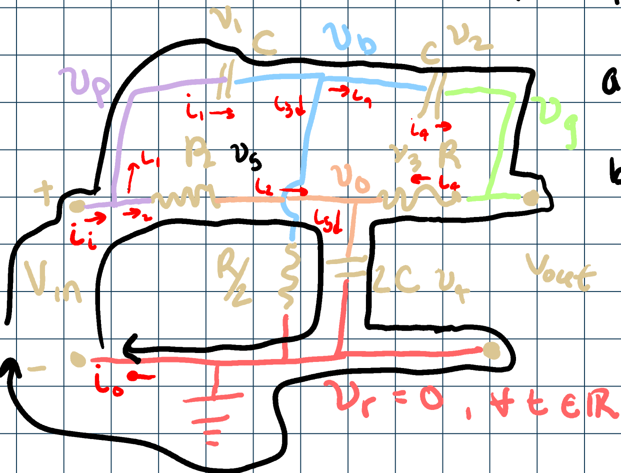
$\frac{1}{C}\int i_1\,dt = v_{in} - \frac{1}{C}\int i_4\,dt - v_{out}$

$v_1 = v_p - v_b \qquad v_2 = v_b - v_g$

$v_g + v_2 = v_1$



a.) $i_i(t) = i_1 + i_2$

b.) $i_1(t) = i_3 + i_4$

c.) $i_5(t) = i_4 + i_2$

d.) $i_o(t) = i_3 + i_5$

*) $i_C = C\left(\underset{L}{\dot{v}_1} - \underset{R}{\dot{v}_2}\right)$

**) $i_R = \frac{1}{R}\left(\underset{L}{v_1} - \underset{R}{v_2}\right)$

• $v_{in} = v_p - v_r$

$$0 = -v_{in} + \overbrace{(v_p - v_b)}^{v_1} + \overbrace{(v_b - v_g)}^{v_2}$$

$$+ \underbrace{(v_g - v_o)}_{v_3} + \underbrace{(v_o - v_r)}_{v_4}$$

$$\boxed{v_{in} = v_1 + v_2 + v_3 + v_4}$$

• $0 = -v_{in} + (v_p - v_o) + (v_o - v_r)$

1. $i_o = C(\dot{v}_p - \dot{v}_b) + \frac{1}{R}(v_p - v_o)$

2. $i_1 = \frac{1}{R/2}(v_b - 0) + C(\dot{v}_b - \dot{v}_g)$

$\rightarrow i_1 = \frac{1}{R/2}v_b + \frac{1}{R}(v_g - v_o)$

3. $i_5 = \frac{1}{R}(v_g - v_o) + \frac{1}{R}(v_p - v_o) = 2C(\dot{v}_o - \dot{v}_r)$

4. $i_o = \frac{1}{R/2}(v_b - 0) + 2C(\dot{v}_o - \dot{v}_r)$

5. $v_{in} = v_p - v_r = v_p$

6. $v_{out} = v_g - v_r = v_g$

3.) $2RC(\dot{v}_o - \cancel{\dot{v}_r}^{\,0}) = v_g - v_o + v_p - v_o$

$$2RC\dot{v}_o = v_g + v_p - 2v_o$$

$$\boxed{2RC\dot{v}_o + 2\dot{v}_o = v_{out} + v_{in}}$$

2.) $\cancel{\frac{1}{R/2} v_b} + C(\dot{v}_b - \dot{v}_g) = \cancel{\frac{1}{R/2} v_b} + \frac{1}{R}(v_g - v_o)$

$$RC(\dot{v}_b - \dot{v}_g) = v_g - v_o$$

$$\boxed{\begin{aligned} v_o &= v_{out} - RC(\dot{v}_b - \dot{v}_{in}) \\ \dot{v}_o &= \dot{v}_{out} - RC(\ddot{v}_b - \ddot{v}_{in}) \end{aligned}}$$

1-4)

$$C(\dot{v}_p - \dot{v}_b) + \frac{1}{R}(v_p - v_o) = \frac{2}{R} v_b + 2C(\dot{v}_o)$$

$$RC\dot{v}_{in} - RC\dot{v}_b + v_{in} - v_o = 2v_b + 2RC\dot{v}_o$$

$$RC\dot{v}_{in} - RC\dot{v}_b + v_{in} - v_o = 2v_b + 2RC\dot{v}_o$$

$$RC\dot{v}_{in} + v_{in} = 2v_b + 2RC\dot{v}_o + RC\dot{v}_b + v_o$$

$$\cancel{RC\dot{v}_{in}} + v_{in} = 2v_b + 2RC\dot{v}_o + \cancel{RC\dot{v}_b} + v_{out}$$

$$\cancel{-RC\dot{v}_b}$$

$$v_{in} = 2v_b + 2RC\dot{v}_o + v_{out} \qquad \cancel{+RC\dot{v}_{in}}$$

1. $\mathcal{L}\{i_o(t)\} = I_o(s)$

- $I_o(s) = C\left(sV_{in}(s) - sV_b(s)\right) + \frac{1}{R}\left(V_{in}(s) - V_o(s)\right)$

- $I_o(s) = \frac{2}{R}V_b(s) + 2C\left(sV_o(s)\right)$

- $$\boxed{RCs\left(V_{in}(s) - V_b(s)\right) + V_{in}(s) - V_o(s) = 2V_b(s) + 2RCsV_o(s)}$$

2. $$I_1(s) = \frac{2}{R}V_b(s) + C\left(sV_b(s) - sV_{out}(s)\right)$$

$$I_1(s) = \frac{2}{R}V_b(s) + \frac{1}{R}\left(V_{out}(s) - V_o(s)\right) = Cs\left(V_{in} - V_b\right)$$

$$\boxed{RCs\left(\bar{V}_b(s) - \bar{V}_{out}(s)\right) = \bar{V}_{out}(s) - \bar{V}_o(s)}$$

3.

$$2RC\left(s\bar{V}_o(s)\right) = \bar{V}_{out}(s) - \bar{V}_o(s) + \bar{V}_{in}(s) - \bar{V}_o(s)$$

$$(2RCs)\bar{V}_o + 2\bar{V}_o = \bar{V}_{out} + \bar{V}_{in}$$

$$2\bar{V}_o(RCs+1) = \bar{V}_{out} + \bar{V}_{in}$$

$$\boxed{\bar{V}_o = \frac{\bar{V}_{out} + \bar{V}_{in}}{2(RCs+1)}}$$

4.

$$2V_b + RCSV_b - RCSV_{out} = RCSV_{in} - RCSV_b$$

$$RCSV_{in} - RCSV_b = 2V_b + CSRV - RCSV_{out}$$

$$2V_b + 2RCSV_b = RCS(V_{out} + V_{in})$$

$$2V_b(RCS+1) = RCS(V_{out} + V_{in})$$

$$RCS(V_{in} + V_{out}) = 2(RCS+1)V_b$$

$$\boxed{V_b = \frac{RCS(V_{out} + V_{in})}{2(RCS+1)}}$$

5. $RCs\left(\bar{V}_b(s) - \bar{V}_{out}(s)\right) = V_{out}(s) - \bar{V}_o(s)$

$$RCs\,V_b + V_o = (RCs+1)V_{out}$$

$$\frac{(RCs)^2(V_{out}+V_{in})}{2(RCs+1)} + \frac{(V_{out}+V_{in})}{2(RCs+1)} = (RCs+1)(V_{out})$$

$$(V_{out}+V_{in})\left(\frac{R^2C^2s^2+1}{2(RCs+1)^2}\right) = V_{out}$$

$$V_{out}C + CV_{in} = V_{out}$$

$$CV_{in} = (1-c)V_{out} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{c}{1-c}$$

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{(R^2C^2s^2+1)\cancel{(2)(RCs+1)^2}}{\cancel{2(RCs+1)^2}(R^2C^2s^2+4RCs+1)}$$

$$\boxed{H(s) = \frac{R^2C^2s^2+1}{R^2C^2s^2+4RCs+1}}$$