MCEN 3030

1 Feb 2024

HW & Quiz #1 Due Sun 11:59PM

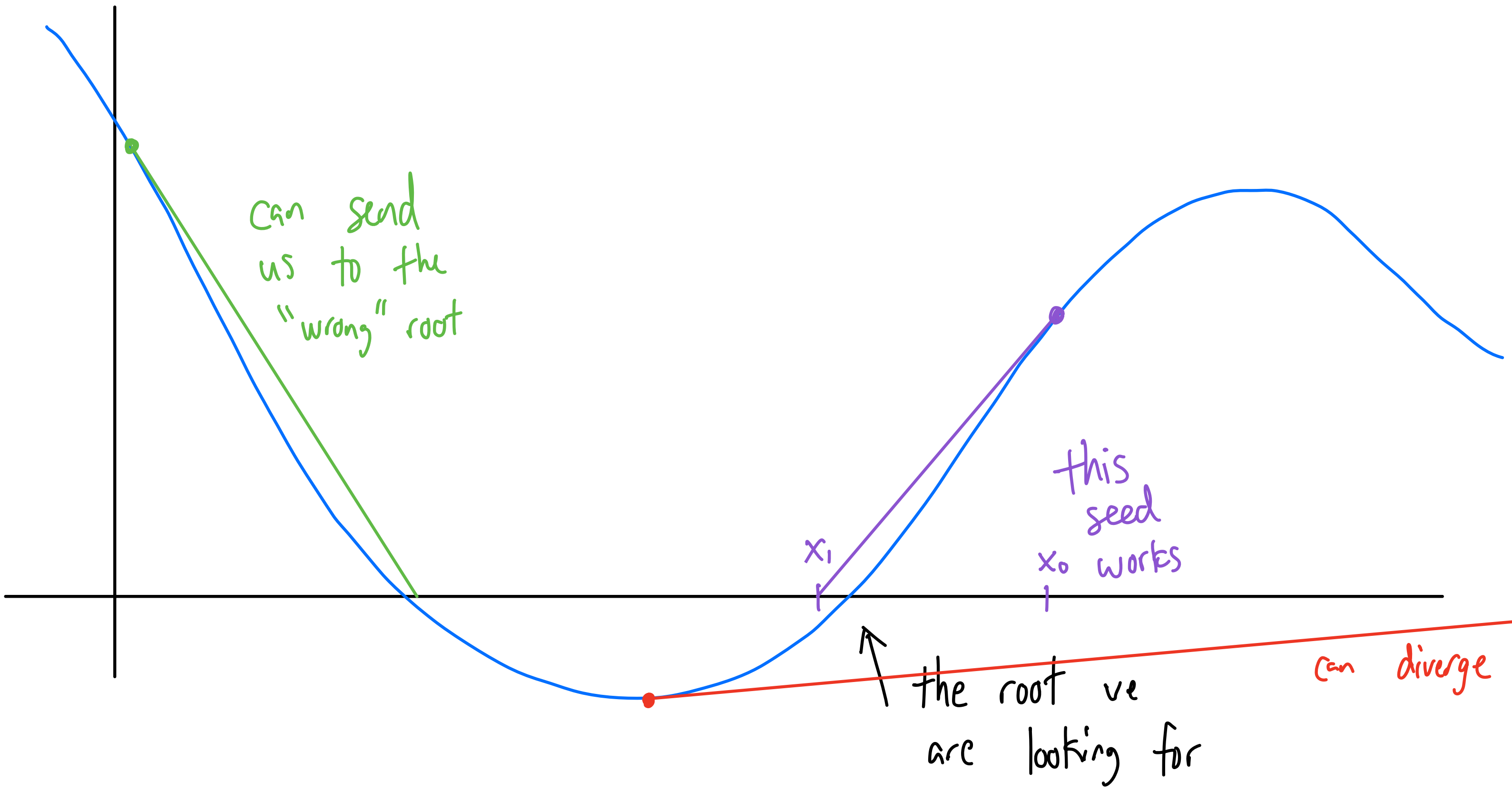
runme & pdf details added after class

Last time: Root Finding – Newton's Method "open method"

Today: Root finding – Bisection & False-position methods "closed method"

We will probably do another "open" method next week but I wanted to contrast one open and one bracketed method this week.

Last time: Newton–Raphsen Method



Benefits:

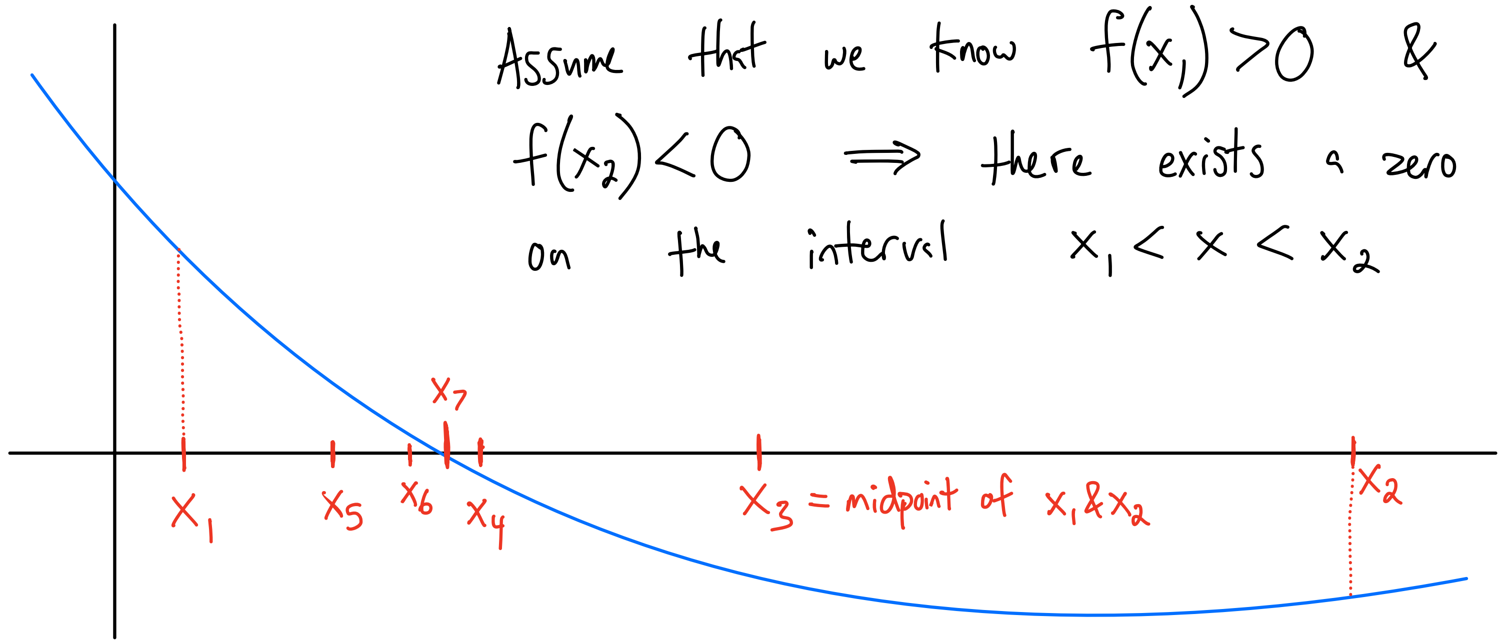
- I like the math
- Tends to be fast(er than other methods)
- Graphical representation
- Multi-variable versions exist

Drawbacks:

- Not guaranteed to converge
- Requires knowledge of $f(x)$ & $f'(x)$
- Requires a seed, and you might have no idea

Bisection Method

Assume that we know $f(x_1) > 0$ & $f(x_2) < 0 \implies$ there exists a zero on the interval $x_1 < x < x_2$



Bisect the interval: $x_3 = \dfrac{x_1 + x_2}{2}$

if $f(x_3) < 0$ we know the zero lies on the left side $x_1 \to x_3$

Because $f(x_1) > 0$

if $f(x_3) > 0$ we know it is on the right size

if $f(x_3) = 0$ we are lucky, & done.

Algorithm:

(0) Assumes you know <u>one</u> root exists between x_L (lower) and x_u (upper). These bracketing values are inputs. (see caveats below.)

(1) Calculate $x_m = \frac{x_L + x_u}{2}$.

(2) If $f(x_L) \cdot f(x_m) < 0$, the zero is between x_L & x_m

→ (3) Reset x_u to x_m (ie. $x_u = x_m$) and go back to (1)

(2) If $f(x_m) \cdot f(x_u) < 0$, the zero is between x_m & x_u

→ (3) Reset x_L to x_m ($x_L = x_m$) and go back to (1)

4) Repeat til an acceptable error is reached. At each iteration, x_m can be thought of as your new "best guess" for the root.