MCEN 3030

30 Jan 2024

HW & Quiz #1 Due Sunday

- Formatting guidelines posted soon

Last time: Error

Today: Root-Finding:

Newton-Raphson Method

Last time: Error

- Comparison against experiment/validation result
  - significant digits might inform our "acceptable error" in calculated results

- Numerical errors
  - round-off errors: related to the finite storage size of numbers

$$\begin{array}{r} 154.1 \\ +\ 1.541 \\ \hline 155.6\_\,\_ \end{array}$$

  - truncation error: related to the fact that we have to cut-off an iterative/series calculation at some point

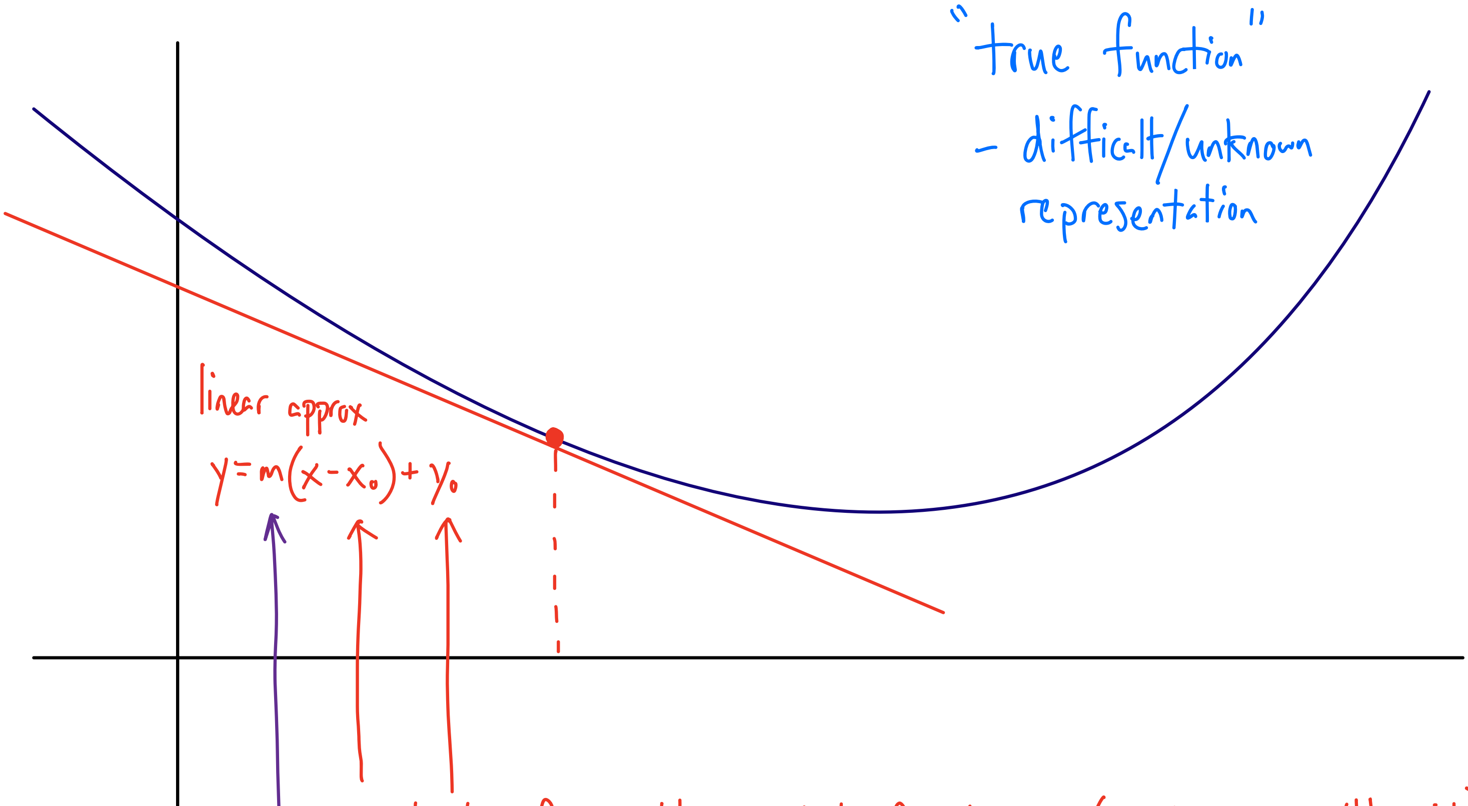
Definitions of error (calculation)

$$\left|(\text{calculated value}) - (\text{trusted value})\right| \rightarrow \text{error}$$

$$\frac{\left|(\text{calculated value}) - (\text{trusted value})\right|}{(\text{trusted value})} \rightarrow \text{percent error}$$

$$\frac{\left|(\text{new value}) - (\text{old value})\right|}{(\text{new value})} \rightarrow \text{related to "convergence"}$$

# Prelude: Taylor Series




evaluate from the original function (just go with it)

evaluate from a hopefully calculable derivative of the original function

$$\underbrace{f(x)}_{\text{orig function}} = \underbrace{f(x_0) + \left.\frac{df}{dx}\right|_{x_0} \cdot (x - x_0)}_{\text{"linear approximation of } f(x) \text{ near } x_0\text{"}} + \frac{1}{2}\left.\frac{d^2 f}{dx^2}\right|_{x_0} \cdot (x - x_0)^2 + \text{H.O.T.}$$

("higher-order terms")

Computationally (in a lot of applications) the linear approximation is going to be useful & interesting

# Newton - Raphson Method

Idea: Linear approximation $f(x) \approx f(x_0) + \left.\frac{df}{dx}\right|_{x_0}(x-x_0)$

use this as part of an iterative scheme to find where $f(x) = 0$

e.g.: $e^x = x \longrightarrow e^x - x = 0 = f(x)$

$$f(x) = 0 = f(x_0) + \left.\frac{df}{dx}\right|_{x_0}(x-x_0)$$

rearrange to solve for $x$, i.e. "the root"

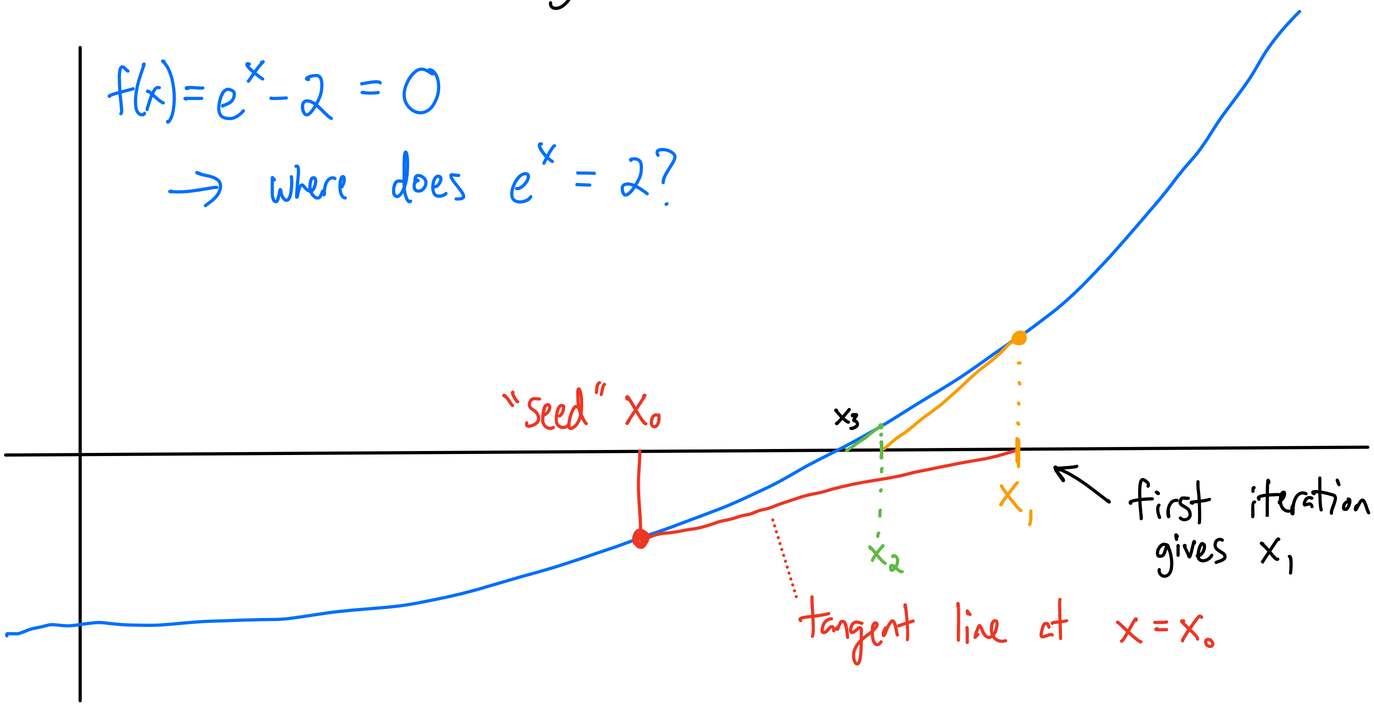
$$\Longrightarrow \quad x = x_0 - \frac{f(x_0)}{\left.(df/dx)\right|_{x_0}}$$

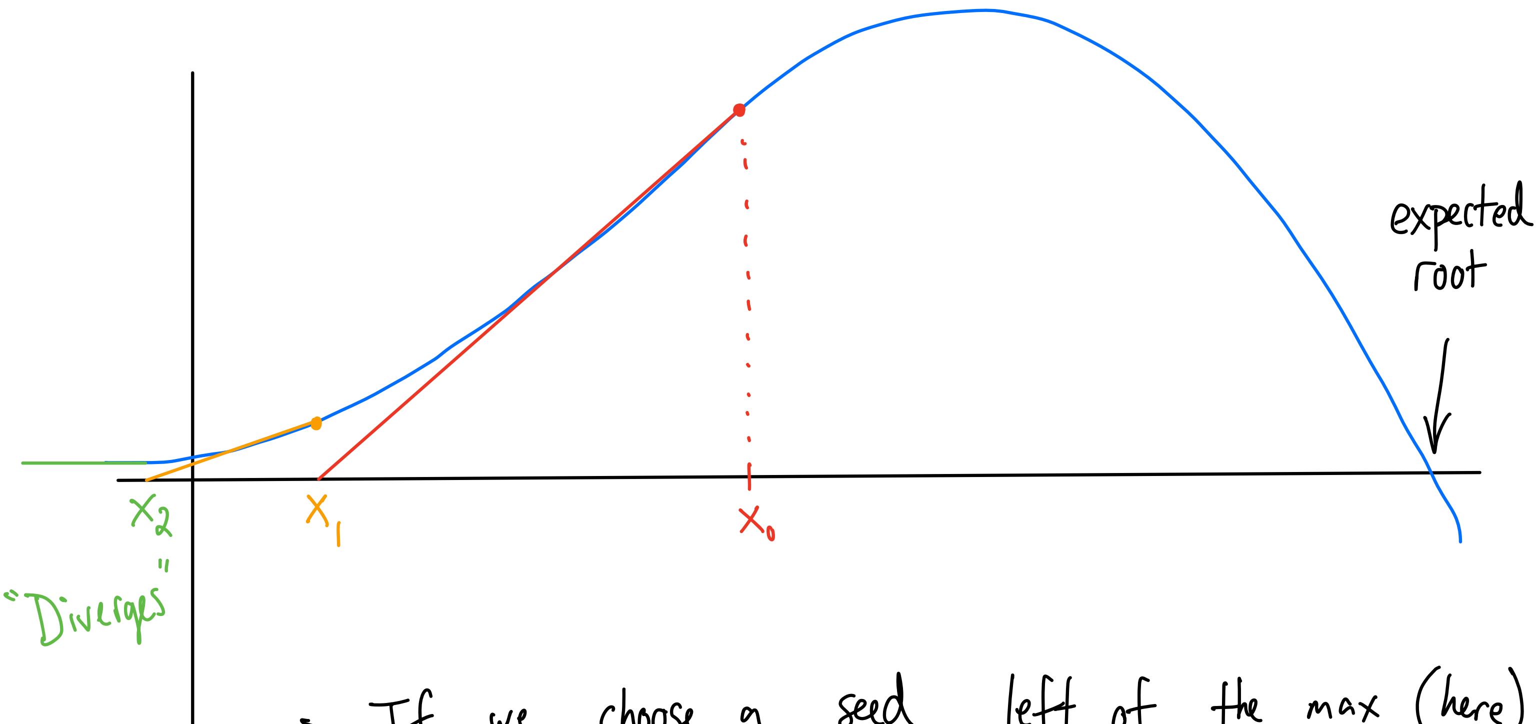
actually, use as an iterative scheme

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{\left.(df/dx)\right|_{x_i}}}$$

1) Start with a guess for the root, $x_0$, called the "seed" (probably a function input)
2) Often (but not always), the above can be used to improve the estimate of the root location
3) Iterate through: $x_0 \rightarrow x_1 \longrightarrow x_2 \longrightarrow x_3 \longrightarrow \ldots$ until an acceptable error is reached $(f(x_i) \approx 0)$ or until convergence $x_{i+1} \approx x_i$

Let's do a few graphically




→ If we choose a seed left of the max (here) the algorithm will <u>not</u> converge on the root of the equation. Problem!

→ This method is an "open method" meaning it is not bounded to a certain interval