MCEN 3030

12 March 2024

HW#5 Due Wednesday

HW#6 Due Friday

$$\frac{\partial}{\partial n}\left(x^n\right) = x^n \log(x)$$

ln

Last time: 1D optimization

Today: 2D optimization

Multidimensional Unconstrained Optimization

Idea: Robot looks for yellow in image field.
Seek peak in yellow intensity, $f(x,y)$

Challenge: We need to respond quickly in order to hit the ball $\longrightarrow$ several images per second? Each with many thousands of pixels

One Method: <u>Brute Force</u>

Evaluate $f(x,y)$ at all points and pick the max.

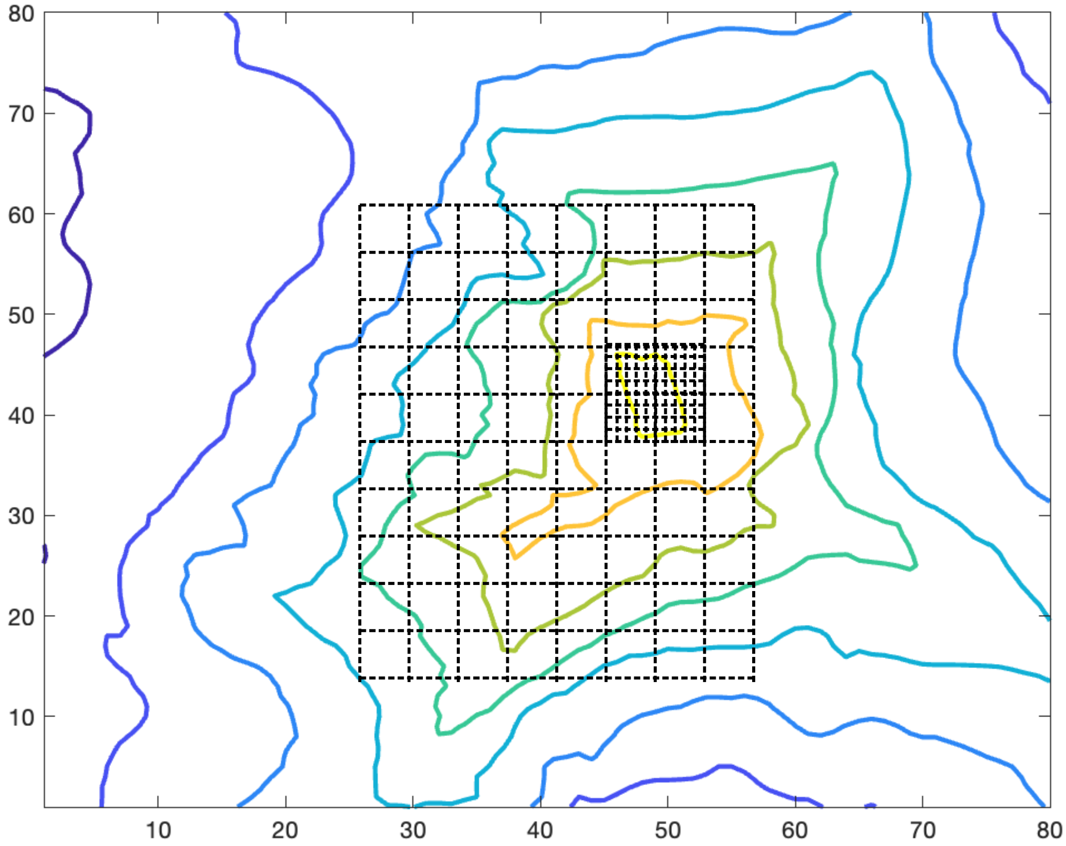
Ex: 2 Megapixel camera

1600 pixels - by - 1200 pixels = 1.9 million nodes

Hopefully there are more efficient ways! ...

Grid search

Idea: Coarse search over a region suspected of having a max, and then refine

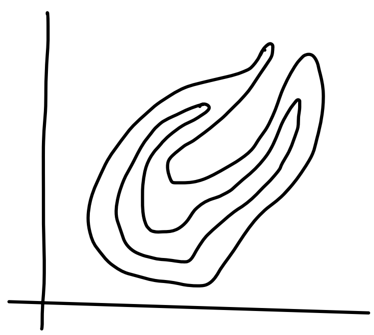


Random Search

0) Define a range over which you know there is a max

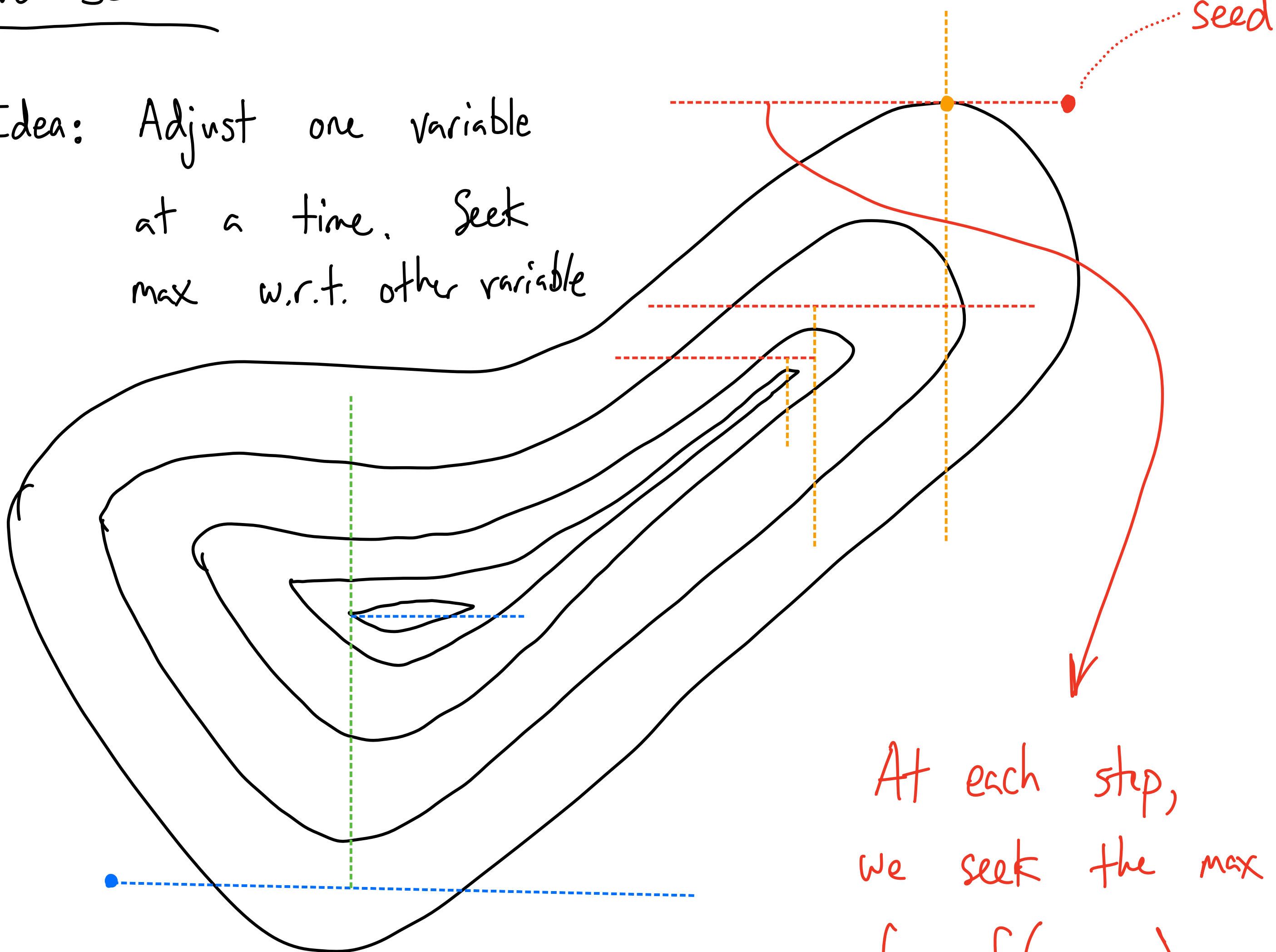
1) Randomly (uniform distribution) choose a point and compare to the previously found max. Update if the new one is better

2) Continue until haven't updated the "best" in a while



Univariate Search

Idea: Adjust one variable at a time. Seek max w.r.t. other variable



At each step, we seek the max of, $f(x, y_0)$ or $f(x_0, y)$

const

fxn of 1 var

Golden Search

Benefits:

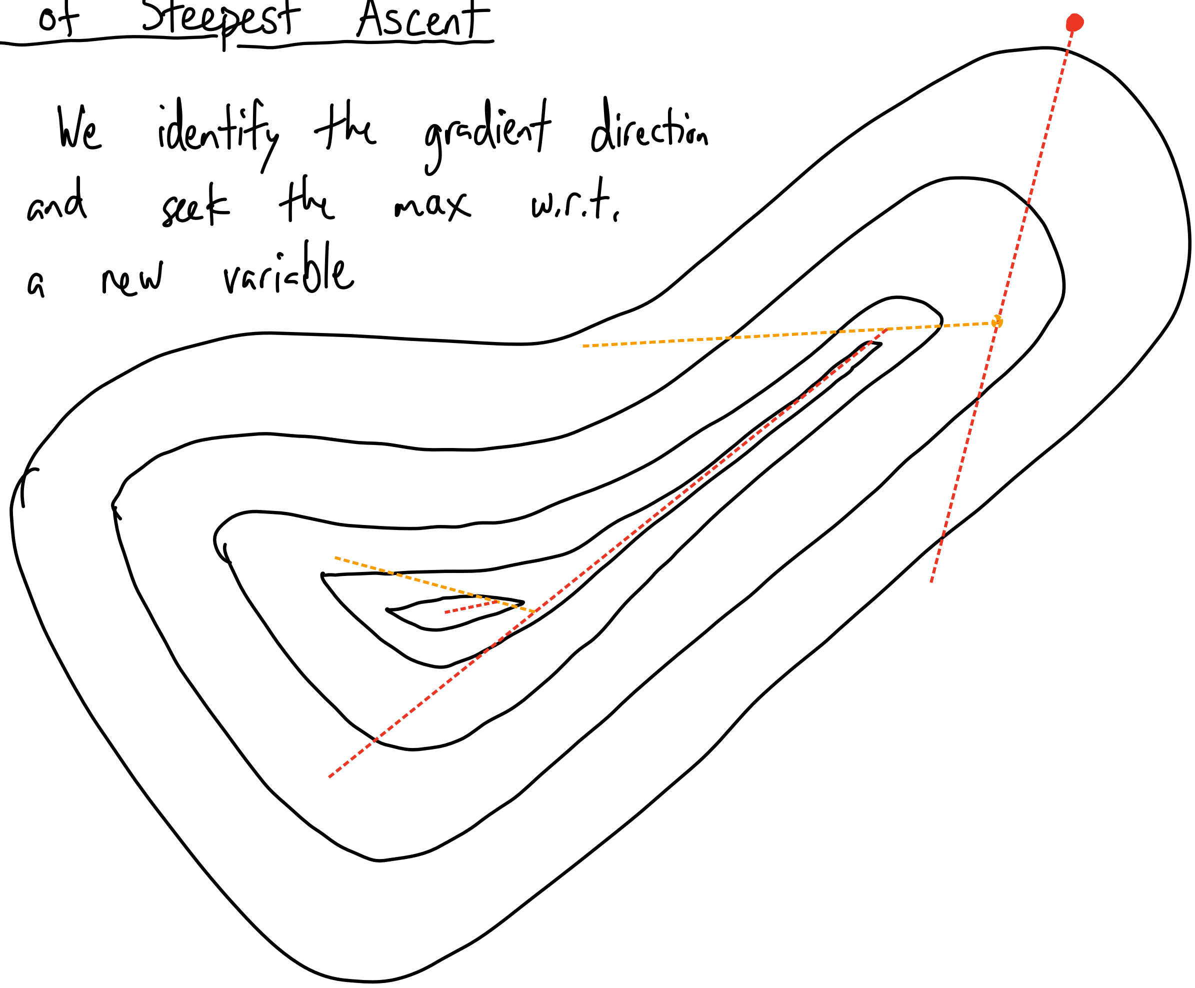
- We didn't have to evaluate at every point, just did a series of Golden Searches
- We didn't need to calculate derivatives
- $f(x,y) \rightarrow$ easy to set, e.g. $x = \text{const}$

Drawback:

- Thin contours $\rightarrow$ slow convergence

Method of Steepest Ascent

Idea: We identify the gradient direction and seek the max w.r.t. a new variable



Benefits:

- faster convergence than the univariate method

Drawback:

- Must be able to calculate ∇f

The algorithm + math:

0) Pick a seed and evaluate

$$\underline{\nabla} f(x_0, y_0) = \left\langle \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)}, \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} \right\rangle$$

1) Invent a new "progress" variable, say k, to consider points

$$\left(\underset{\text{const}}{x_0} + k \underset{\text{const}}{\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)}}, \underset{\text{const}}{y_0} + k \underset{\text{const}}{\left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)}} \right)$$

$\Longrightarrow$ Write $f(x,y) \to f(k)$

2) Use 1D search, e.g Golden Search Method, to identify the max in $f(k) \to k^*$

3) Convert $k^* \to (x_1, y_1)$ & evaluate the gradient at this location.

$$x_1 = x_0 + k^* \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} \quad \& \quad y_1 = y_0 + k^* \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)}$$

Iterate until convergence, e.g. $\left| \frac{x_{i+1} - x_i}{x_{i+1}} \right|$ & $\left| \frac{y_{i+1} - y_i}{y_{i+1}} \right|$ are small