Newton Raphson 2D ∧ Non-Linear:

Linear: $@ x_i, y_i \rightarrow \begin{bmatrix} \partial f/\partial x & \partial f/\partial y \\ \partial g/\partial x & \partial g/\partial y \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} -f(x_i, y_i) \\ -g(x_i, y_i) \end{bmatrix}$

use Cramer's Rule.

Univariant: 2D Golden

Search where each var, x, y, z, is checked one at a time.

- checking x means we use $f(x_1, y_0, z_0) \wedge f(x_2, y_0, z_0)$

Non-Linear Least Squares:

- error: $\left| \frac{\tau_{1+i} - \tau_i}{\tau_{1+i}} \right| < \text{error}$

add all params' error

$R :=$ Resid, $J_i :=$ Jacobian w/r param

$B :=$ Δ Parameters' value

$$B_i = (J_i^T J_i)^{-1} J_i^T R_i$$

$$R_{ij} = y_j - f(\text{Params})$$

- Solving for paramchange.

Steepest Assent:

- $\nabla f = \langle g F_x, g F_y \rangle = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$
- $x_{i+1} = x_i + k^* \left(\frac{\partial f}{\partial x} \Big|_{x_i, y_i} \right)$
- $y_{i+1} = y_i + k^* \left(\frac{\partial f}{\partial y} \Big|_{x_i, y_i} \right)$
- $k^* = \text{goldsearch}(x_i, y_i)$

$\rightarrow k_{low} = \left[\frac{(x_L - x_i)}{\partial f/\partial x |_{x_i, y_i}}, \frac{(y_L - y_i)}{\partial f/\partial y |_{x_i, y_i}} \right]$

$\rightarrow k_{upp} = \left[\frac{(x_u - x_i)}{\partial f/\partial x |_{x_i, y_i}}, \frac{(y_u - y_i)}{\partial f/\partial y |_{x_i, y_i}} \right]$

$\rightarrow l = \min(\text{abs}(k_{low})),\ l \in \mathcal{I}$

$\rightarrow u = \min(\text{abs}(k_{upp})),\ u \in \mathcal{I}$

$\rightarrow k_u = k_{low}(l);\ k_l = k_{up}(u)$

$\rightarrow$ if $k_l > k_u$ * Checks $F(x(k), y(k))$

$\rightarrow k_t = k_l$

$\rightarrow k_l = k_u$

$\rightarrow k_u = k_t$

$\rightarrow k^*$ is a parameterization like t

Golden Search: $\boxed{d = \frac{1}{2}(\sqrt{5} - 1)}$

$x \in [x_L, x_u]$

1.) $x_1 = x_L + d(x_u - x_L)$

$x_2 = x_u - d(x_u - x_L)$

2.) Evaluate $f(x_1)$ & $f(x_2)$

3.) • $f(x_1) > f(x_2)$

• erase $[x_L, x_2]$

$\rightarrow x_1 = x_L + d(x_u - x_L)$

$\rightarrow$ Eval new $f(x_1)$, iterate

- $f(x_1) < f(x_2)$

• erase $[x_1, x_u]$

$\rightarrow x_u = x_1$

$\rightarrow x_1 = x_2$

$\rightarrow x_1 = x_u - d(x_u - x_L)$

$\rightarrow$ Eval $f(x_2)$, iterate...

3.) • $f(x_1) > f(x_2)$

• erase $[x_L, x_2]$

→ $x_L = x_2$; $x_2 = x_1$

• $x_1 = x_u - \delta(x_u - x_L)$

→ Eval $f(x_2)$, iterate...