Newton Raphson:

Linear: * use taylor for higher order.

- $f(x) \simeq f(x_0) + \frac{df}{dx}\Big|_{x_0}(x - x_0)$, find $f: f(x)=0$

$\Rightarrow x_{i+1} = x_0 - f(x_0) / \left(df/dx\big|_{x_0}\right)$

Bisection:

1. $x_m = \frac{x_L + x_u}{2}$
2. $\rightarrow f(x_L)f(x_m) < 0$

   $\Rightarrow$ Root between $x_L \le x \le x_m$

   $\Rightarrow x_u = x_m$ to reset

   $\rightarrow f(x_m)f(x_u) < 0$

   $\Rightarrow$ Root betwee $x_m \le x \le x_u$

   $\Rightarrow x_L = x_m$ to reset

3. Recalculate x_m with new $x_L \vee x_u$

   $\rightarrow$ The recalculate $f(x_m), f(x_L), f(x_u)$

False Position:

- $x_{i+1} = g(x_i)$ $\rightarrow$ note that we want $g(x): |g'(x)| < 1$
- $g(x) = f(x) + x$

$\rightarrow$ to get $f(x)=0$ we want to find $g(x) = x$

$\rightarrow$ error $= \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right|$

$\rightarrow$ $g(x)$ can come from

$Ax = b$, m-rows, n-cols

1. Exactly one solution, if rank is equal to num columns, square, non-singular

2. ∞ #-solutions,

   $\rightarrow$ Compatable (row of zeros with a compatability eq from img.

   $\rightarrow$ $\# \ge 1$ free vars

   $\rightarrow$ rank $< n$

3. No solution if

   $\rightarrow$ Non Compatable

   $\rightarrow \left[\begin{array}{cccc|c} \cdot & \cdots & & \vdots & a \\ \vdots & \ddots & & \vdots & \vdots \\ 0 & 0 & 0 \cdots & 0 & z \end{array}\right] \forall, z \neq 0$

   $\hookrightarrow$ This incompatable.

→ $g(x)$ can come from solving $f(x)=0$ for x