MATH 4820 - Homework 4

Jack Reilly Goldrick

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0.1 Problem 1

0.1.1 Part A

- Using the Euclidean Algorithm for the continued fraction of $\frac{34}{21}$ is

$$[1; \overline{1, 1, 1, 1, 1, 2}]$$

0.1.2 Part B

- The continued fraction reveals the following possible solutions:

$$\frac{21}{13}$$

$$\frac{13}{8}$$

$$\frac{8}{5}$$

$$\frac{5}{3}$$

$$\frac{3}{2}$$

- the optimal solution is to be revealed as $x = 13$ and $y = -8$

0.2 Problem 2

- Beginning with guess $(n, 1, n^2 - d)$
- Using Brahmagupta's Composition we have:

$$(\frac{nm + d}{n^2 - d},\ \frac{n + m}{n^2 - d},\ \frac{m^2 - d}{n^2 - d})$$

- setting $m = n$ we have:

$$(\frac{2n^2 + 1}{-1},\ -2n,\ 1)$$

- thus $x = -(2n^2 + 1)$ and $y = -2n$

0.3 Problem 3

- Making the first substitution provided in the problem statement and squaring both sides we have:

$$\frac{(n+1)(2n+1)}{6} = M^2$$

$$(n+1)(2n+1) = 48M^2$$

- Distributing and completing the square we have:

$$(4n+3)^2 - 1 = 48M^2$$

$$(4n+3)^2 - 48M^2 = 1$$

- Using the continued fraction expansion we have

$$\sqrt{48} = [6; \overline{1, 12}]$$

- testing possible solutions we have:

$$4n + 3 = 1351 \text{and} M = 195$$

- Thus we have:

$$n = 337$$