MATH 4820 - Homework 1

Jack Reilly Goldrick

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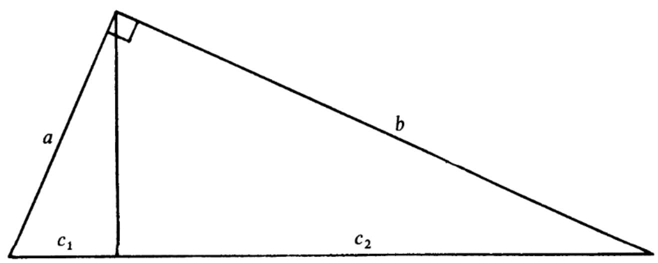
0.1 Problem 1

0.1.1 Exercise 1.4.2



Proof

- Let $c = c_1 + c_2$, h denote the vertical segment, and A, B, C, H denote the vertexes across from a respective segment such that C is the vertex that connects $a \wedge b$, A connects c_2 to b and H connects c_1 and c_2 to h.
- Let $\triangle ABC, \triangle ACH, and \triangle BCH$ all be right triangles since they all have one right angle
- $\triangle ABC$ shares a unique vertex, thus angle, other than the right angle, $\angle CAB$ with $\triangle ACH$ and a separate unique angle that is not right, $\angle CBA$, with $\triangle BCH$
  - The larger triangle ABC has two angles that are equal to either two of the smaller triangles' angles. Thus given the properties of a triangle, the third angle must be equal to either of the larger triangle's remaining angle with respect to the smaller subdivision, meaning that all triangles are composed of the same combination of angles.
    * Thus all three triangles are similar.
- Given that all the triangles are similar the following equations hold true:

$$\frac{a}{c_1} = \frac{c_1 + c_2}{a} \tag{1}$$

$$\frac{b}{c_2} = \frac{c_1 + c_2}{a} \tag{2}$$

- Cross-multiplying the equations we have:

$$a^2 = (c_1 + c_2)(c_1) = c * c_1 \tag{3}$$

$$b^2 = (c_1 + c_2)(c_2) = c * c_2 \tag{4}$$

- Adding equations (3) and (4) we have:

$$a^2 + b^2 = c * c_1 + c * c_2 \tag{5}$$

- Rearranging Equation (5) we have:

$$\begin{aligned} a^2 + b^2 &= c * c_1 + c * c_2 \\ &= c * (c_1 + c_2) \\ &= c * c \\ &= c^2 \end{aligned}$$

- Since $a^2 + b^2 = c^2$ for right triangle ABC, we have proven the Pythagorean Theorem.

0.2 Problem 2

0.2.1 Infinitely Many Triples

Since we know from the parameterization of triples in class that all we need is a pair of numbers (m, n) that are coprime with opposite parity. Since there are infinitely many co-prime integer pairs with opposite parity, we can conclude that there exists infinitely many triples since infinitely many primitive triples exist.

0.3 Problem 3

0.3.1 Finitely many Triples for a=100

- Constructing the Theorem from the variables we have:

$$a^2 + b^2 = c^2 = 10^4 + b^2 \tag{6}$$

$$c^2 - b^2 = 10^4 \tag{7}$$

From equation (7) we have:

$$c^2 - b^2 = 10^4$$
$$(c - b) * (b + c) = (2^4) * (5^4)$$

- Thus from the prime factorization formula, we know a^2, in this case, has 25 factors resulting in 12 pairs of factors.
- Let $x = c - b \wedge y = c + b$
- rearranging we have $c = x + b \wedge b = y - c$. This results in the following two equations:

$$c = \frac{x + y}{2} \tag{8}$$

$$b = \frac{y - x}{2} \tag{9}$$

- Since this construction does not generate only primitive triples since b is always even, and a^2 has finitely many factor pairs, we can conclude that there are only finitely many triples for $a = 100$