MATH 4820 - Homework 4

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0.1 Problem 1

0.1.1 Exercise 2.3.3

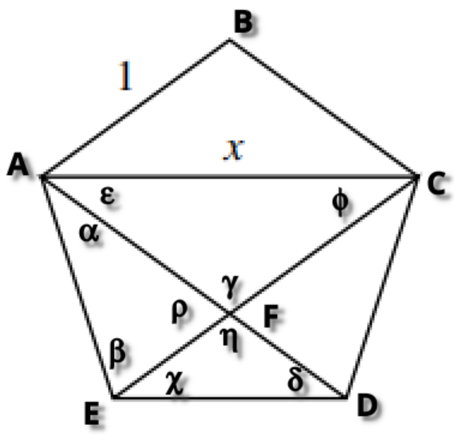

Figure 2.6: The regular pentagon

- We can see that angles $\alpha = \delta = 36°$ since $\angle AED = 108°$ and $\triangle AED$ is isosceles.

  Using Opposite interior Angles of Parallel lines we can deduce that $\epsilon = \delta$

- We can repeat this process for $\triangle EDC$ resulting in $\epsilon = \delta = \chi = \phi = \alpha$
- Thus looking at $\triangle AEC$ we can find β by solving the equation for β:

$$180 = 3 * x + \beta \text{ Such that } x = 36°$$

- this results in $\beta = \phi = 72°$ and $\triangle AEF \wedge \triangle AEC$ being isosceles.
- From this we can deduce that segment EC is the same length as AC which is x and that the segment AF has the same length as the sides of the pentagon, 1.
- thus we have the following relation:

$$x - 1 = FD$$

- given the angles and segments making $\triangle AFD \wedge \triangle AFC$ similar we can say:

$$\frac{1}{FD} = \frac{x}{1}$$

$$\frac{1}{x-1} = \frac{x}{1}$$

- Thus we have the desired relation :

$$x = \frac{1}{x-1}$$

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0.2 Problem 2

0.2.1 Exercise 2.3.4

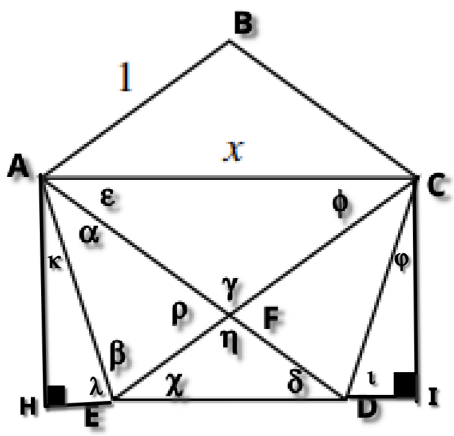

Figure 2.6: The regular pentagon

- The construction can be achieved via creating two new triangles AHE and DCI. Thus we can say:

$$x = HE + ED + DI$$

- We can use the facts of constructible right triangles to show that segments HE and DI are constructible.
- Then using the fact that a sum of constructible segments is also cosntructible we can say that the length x is a result of the sum of three constructible numbers.
- Therefore x is constructible.

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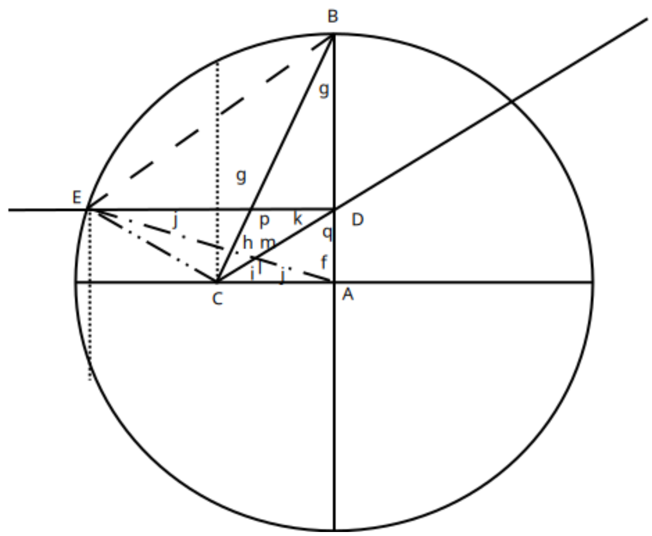
0.3 Problem 3



- To verify the Constructibility of the Pentagon we will show that $\angle i$ divides the circle into 5 equal parts.

1. Using the equilateral triangles we can construct the midpoint of $\overline{AR}$ called C.
2. Forming $\overline{CB}$ we create a right triangle ABC.
3. Given that the ratios of side lengths are constructible numbers then angles $g, (h+i), \wedge(j+f)$ are as well through cosine.
4. Knowing the sum of constructible numbers must also be a constructible number, we can say that angles $h, i, j \wedge f$ are constructible as well.
5. Given that $\overline{CD}$ is the angle bisector of $\angle(h+i)$ and i is opposite interior of k, we can conclude that $h = i = k$.
6. Exterior Angles and Triangles can be used to derive the equivalence of m and l

$$m = 180 - g - f$$

$$l = q + f$$

- Making the appropriate substitution using the fact that k and i are complementary we have:

$$f = \frac{90 + g - i}{2}$$

- 90, 2, g and I are all constructicble numbers, the process can be repeated 4 more times to validly complete the regular pentagon
- Therefore this is a valid construction.

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