MATH 4820 - Homework 2

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0.1 Problem 1 using Area of Squares

- Assume $\sqrt{3} = \frac{m}{n} : m > n \ \forall m, m \in \mathbb{Z}$, and m,n are co-prime
- squaring both sides and multiplying them by n^2 we have:

$$3n^2 = m^2$$

- Since the area of a square is the side-length squared, this equation can be interpreted as the area of the sum three squares with side-length n is equal to the area of a square with side-length m.
  - This implies the larger area is divisible by three, thus, we have

$$m = 3k, \forall k \in \mathbb{Z}$$

- Substituting 2k for m we have:

$$3n^2 = 9k^2$$

$$n^2 = 3k^2$$

  - This implies that n is also divisible by three as well.

- The original assumption was that these numbers m and n were co-prime to satisfy $\sqrt{3}$ as a rational number. However, we have just shown that m and n are not co-prime for the numbers m and n to remain integers. Thus we have raised a contradiction arising from the square root of three's rationality; therefor, the square root of three is irrational.

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0.2 Problem 2

- Let $a = 270 \land b = 168$
- Following the Euclidean Algorithm we have:

$$270 = q_1(168) + r_1 \implies q_1 = 1 \land r_1 = 102$$

$$168 = q_2(102) + r_2 \implies q_2 = 1 \land r_2 = 66$$

$$102 = q_3(66) + r_3 \implies q_3 = 1 \land r_3 = 36$$

$$66 = q_4(36) + r_4 \implies q_4 = 1 \land r_4 = 30$$

$$36 = q_5(30) + r_5 \implies q_5 = 1 \land r_5 = 6$$

$$30 = q_6(6) + r_6 \implies q_6 = 5 \land r_6 = 0$$

- This implies the gcd(270, 168) = 6
- Now to find the integer solution we have:

$$\begin{aligned} 6 &= 36 - (1 * 30) \\ &= 2(36) - 66 \\ &= 5(102) - 1(168) \\ &= 5(270) - 6(168) \end{aligned}$$

- This implies $x = 5 \land y = -6$

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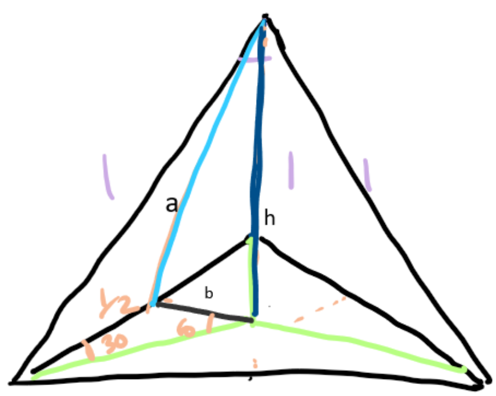
0.3 Problem 3



- Using my Diagram and the properties of 30-60-90 Right Triangles, we have:

$$h = \sqrt{\frac{27}{36} - \frac{3}{36}} = \frac{\sqrt{6}}{3}$$

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