MATH 4820 - Homework 7

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0.1 Problem 1

- Since x is congruent to 1 modulo $m : m \in \{2, 3, 4, 5, 6\}$, we can combine these congruence to the following using their least common multiple:

$$x \equiv 1 \mod 60$$

- The following two equations are left over:

$$x \equiv 1 \mod 60$$

$$\&$$

$$x \equiv 0 \mod 7$$

- Thus $x = 61$

0.2 Problem 2

- This problem requires two cases to be examined in order to see if the system is compatible:

  [1.] $\gcd(a, b) = 1$

  - Since the difference of a and b is always an integer and every integer is divisible by the multiplicative identity element, thus this case is trivially compatible $\forall a, b \in \mathbb{Z} : \gcd(a, b) = 1$

  [2.] $\gcd(a, b) \in \mathbb{Z}$

  - To begin we have the following equalities for a, b, with the $\gcd(a, b) = z$:

$$a = zc, \ \forall c \in \mathbb{Z}$$

$$\&$$

$$b = zd, \ \forall c \in \mathbb{Z}$$

  - substituting these values into the congruence compatibility relation we have:

$$zd - zc = k_1 zd - k_2 zc$$

  - Factoring both sides we have:

$$z(d - c) = z(k_1 d - k_2 c)$$

  - Since Both sides are divisible by the gcd, we can conclude that the system is compatible.

0.3 Problem 3

- Yes the solution can be determined from the least significant digits provided. The reason this is possible is due to the uniqueness provided by the Chinese Remainder Theorem. All these bases are pairwise-compatible since pairs can be generated such that the gcd of any pair is either 1 or a common factor between moduli like 2, 3, and 5. Thus we can conclude that this entire system of congruences is compatible. Since we know this system is compatible, the CRT will guarantee a unique solution up to the lcm of the moduli. Since the lcm is 2520 and $1000 < 2520$ we can conclude that we can find the specific number in question since it will be a unique solution to the Chinese Remainder Theorem which our problem can be interpreted through.