MATH 4820 - Homework 8

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0.1 Problem 1: Exercise 6.5.2

- The obvious solution $y_0 = \sqrt[3]{2}$
    i. Using the Roots of Unity n = 3 we have:
        1. $\omega_1 = e^{\frac{2\pi}{3}i}$
        2. $\omega_2 = e^{\frac{4\pi}{3}i}$
    - This gives us another solution $y_2 = \omega_1 y_0 = \sqrt[3]{2}e^{\frac{2\pi}{3}i}$
    - Using the Final Root of Unity we have: $y_3 = \omega_2 y_0 = \sqrt[3]{2}e^{\frac{4\pi}{3}i}$

0.2 Problem 2

- Since this Cubic is depressed, we can already identify p and q:

$$p = 15 \wedge q = 4$$

- Using Cardono's Formula we have:

$$u = \sqrt[3]{2 + 11i} = 2 + i$$

$$v = \sqrt[3]{2 - 11i} = 2 - i$$

- Thus the solution that is $x = 2$ is

$$x = \sqrt{y} = \sqrt{u + v} = \sqrt{4} = 2$$

- using the roots of unity for n = 3 we have:
    - $\omega_1 = e^{\frac{2\pi}{3}i}$
    - $\omega_2 = e^{\frac{4\pi}{3}i}$
    - Permuting the solution x with the first and second root we have

$$y_2 = \omega_1 u + \omega_2 v = -2 - \sqrt{3}$$

$$y_3 = \omega_2 u + \omega_1 v = \sqrt{3} - 2$$

0.3 Problem 3

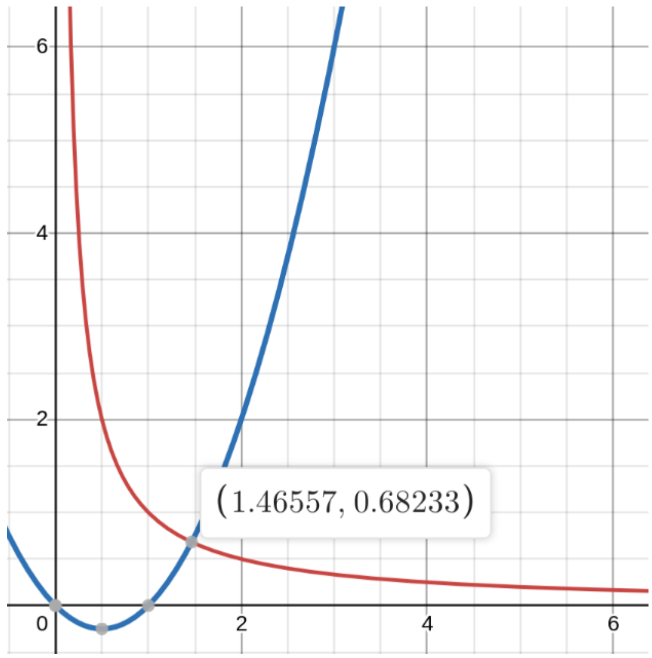

Figure 1:

- Equating the two continuous relations we have the following polynomial of x:

$$0 = x^3 - x^2 - 1$$

- Depressing the cubic with the substitution $x = t + \frac{1}{3}$ we have:

$$0 = t^3 - \frac{1}{3}t - \frac{29}{27}$$

- Using Cardano's formula for $p = \frac{1}{3} \wedge q = \frac{29}{27}$ we have:

$$u = 2^{\frac{2}{3}} \frac{\sqrt[3]{3\sqrt{93} + 29}}{6}$$

$$v = 2^{\frac{2}{3}} \frac{\sqrt[3]{29 - 3\sqrt{93}}}{6}$$

- Thus the desired real solution is $x = u + v + \frac{1}{3} = 1.1322378985432 + \frac{1}{3} = 1.46557123188$ Which is consistent with the graph above.

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