MATH 4820 - Homework 5

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0.1 Problem 1

- We can begin by finding the representations of each point A, C and P.

$$A = (2\cos\theta,\ 2\sin\theta)$$

$$C = (\cos\theta,\ 2)$$

$$P = (x, y) = (x, x^2)$$

- Since the circle centered at C intersects both P and the Origin, we can use Pythagorean Theorem to equate the distance of C to the origin to the distance from C to P.

$$(x - \cos\theta)^2 + (x^2 - 2)^2 = (\cos\theta)^2 + 4$$

- This results in the equation:

$$x^4 - 3x^2 - 2x\cos\theta = 0$$

- Applying the restriction $x \neq 0$ we have:

$$x^3 - 3x - 2\cos\theta = 0$$

- Applying $x = 2\cos\frac{\theta}{3}$ we have:

$$2(4\cos^2\frac{\theta}{3} - 3\cos\frac{\theta}{3}) = 2\cos\theta$$

- Using Cosine Triple Angle Identity we have:

$$2\cos\theta = 2\cos\theta$$

$$1 = 1$$

- since the resulting relation reduced to $1 = 1$ we can confirm that this is a valid construction.

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0.2 Problem 2

$$\sqrt{7} = [2; \overline{1, 1, 1, 4}]$$

$$\sqrt{11} = [3; \overline{3, 6}]$$

$$\sqrt{13} = [3; \overline{1, 1, 1, 1, 6}]$$

0.3 Problem 3

All equations follow:

$$x^2 - py^2 = 1$$

0.3.1 Computations of Approximations

$$\sqrt{7} = [2; \overline{1, 1, 1, 4}]$$

- Let $x = 127,\ y = 48,\ p = 7$
- $127^2 - 7 * 48^2 = 1$
- $16129 - 16128 = 1$
- $1 = 1$

$$\sqrt{11} = [3; \overline{3, 6}]$$

- Let $x = 199,\ y = 60,\ p = 11$
- $199^2 - 11 * 60^2 = 1$
- $39601 - 39600 = 1$
- $1 = 1$

$$\sqrt{13} = [3; \overline{1, 1, 1, 1, 6}]$$

- Let $x = 649,\ y = 180,\ p = 13$
- $649^2 - 13 * 180^2 = 1$
- $421201 - 421200 = 1$
- $1 = 1$