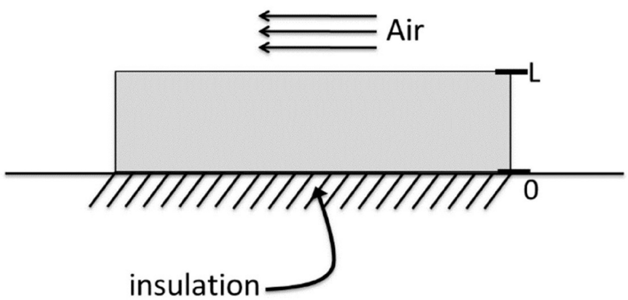
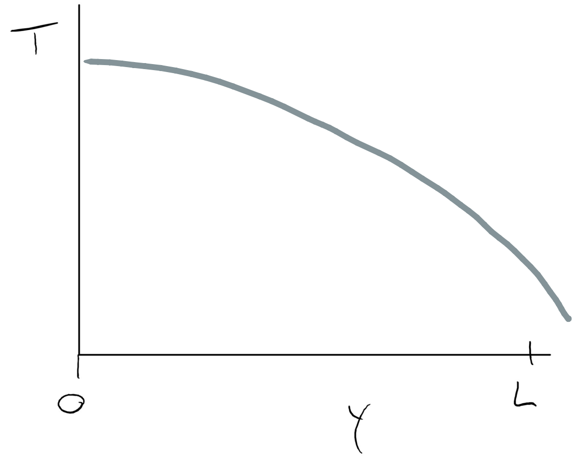
Workshop 5

Monday, September 9, 2024 8:55 AM

A computer chip generates thermal energy uniformly. The top of the chip is cooled by air flowing over it. The chip's base is insulated. It is at steady state. Draw T as a function of y.





1. Simplify the heat diffusion equation. Indicate why you crossed out each term.

$$\cancel{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)} + \dot{q} = \cancel{\rho c_p \frac{\partial T}{\partial t}}$$

2. Describe why you need two boundary conditions and no initial conditions.

Then the diff eq can model many different systems. Unique sols vs many solutions

3. Integrate the simplified heat diffusion equation to find T as a function of y, with two unknown constants of integration.

$\frac{\partial^2 T}{\partial y^2} = -\frac{\dot q}{k}$, $\frac{\partial T}{\partial y} = -\frac{\dot q}{k} y$

4. Write down the two boundary conditions you need to find the two constants of integration, C_1 and C_2

BC1: $-k\frac{\partial T}{\partial y} = 0$ @ $y=0 \Rightarrow \frac{\partial y}{\partial T} = \frac{-\dot q}{k} y$

BC2: $-k\frac{\partial T}{\partial y} = h(T(y) - T_0) \Rightarrow$
@ $y=L$

5. Use the two BC's to solve for C_1 and C_2.

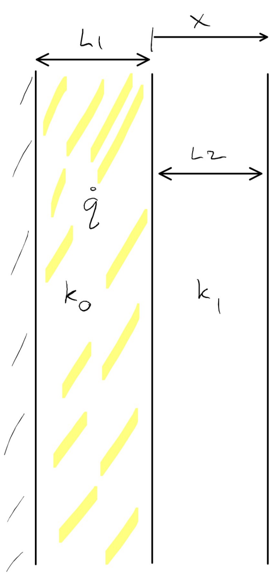
$\frac{-\dot q L}{k} = h(T(y) - T_0) \Rightarrow \frac{-\dot q L}{kh} = T(L) - T_0 \Rightarrow T_L = T_0 - \frac{\dot q L}{kh}$

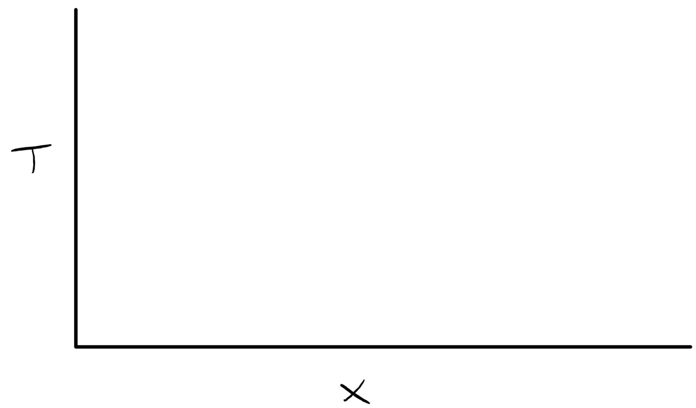
6. With knowledge of C_1 and C_2 you can proceed to solve for T as a function of y.

$T = \frac{-\dot q}{2k} y^2 + T_0$

$T_L = T_0 - \frac{\dot q L}{kh}$

Thermal energy is generated in the highlighted layer. The goal is to determine the temperature profile across the layer on the right. Note that x = 0 occurs at the interface between the two layers. The very left side is insulated. The ambient temperature, T_∞ is known. The heat transfer coefficient, h, is known. The surface temperature on the right side of the right layer is not known. There is no contact resistance between the two layers (they are perfectly bonded).





1. Simplify the heat diffusion equation. Indicate why you crossed out each term.

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

2. Write down the two boundary conditions you need to find the two constants of integration, C_1 and C_2

   BC1:

   BC2:

3. Use the two BC's to solve for C_1 and C_2.

4. With knowledge of C_1 and C_2 you can proceed to solve for T as a function of y.