MCEN - 3047

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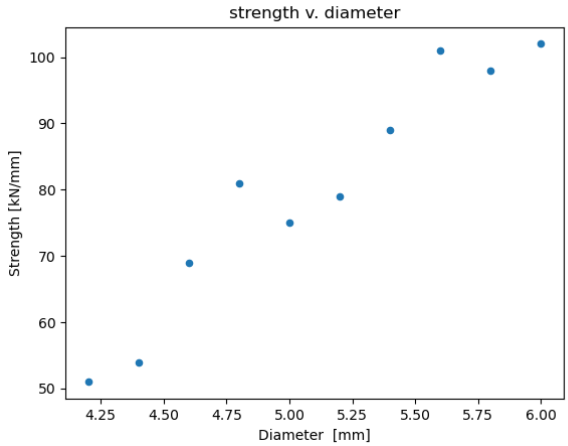
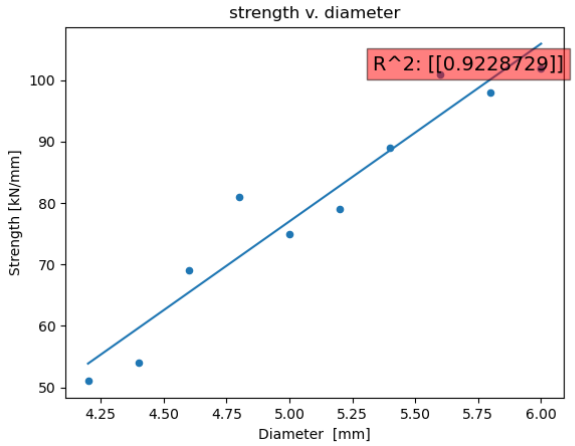
0.1 Problem 1

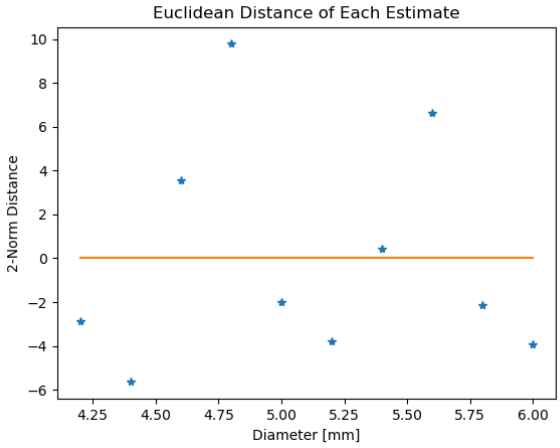
0.1.1 Part A



- A linear Model may work here. However, the plot has a general linear trend, I believe the R^2 should be at least 80

0.1.2 Part C





0.1.3 Part E

0.1.4 Part F

- 8.68175868988037 kN

0.1.5 Part G

- 91.62211608886719 kN

0.1.6 Part H

- While we can calculate a value. Given the lack of knowledge, we cannot use this local behavior to predict global behavior. Thus it is not advised as a valid methodology.

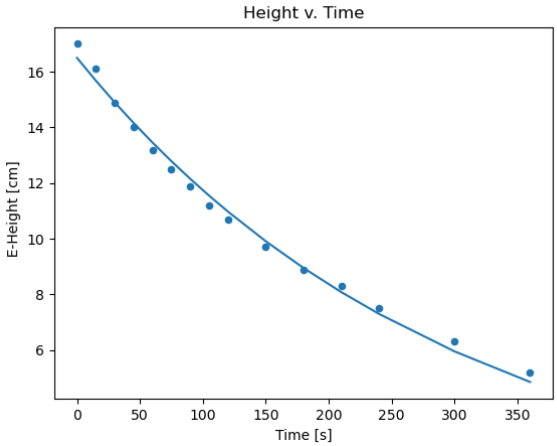
0.2 CODE

0.2.1 Part I

- 5.6218mm

0.3 Problem 2

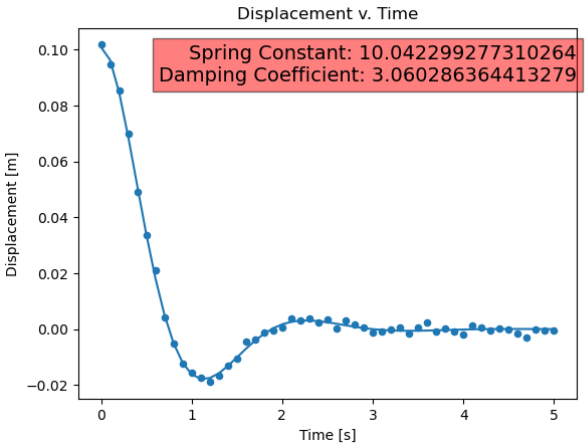
0.3.1 Part A



0.3.2 Part B

- 294.4462890625

0.4 Problem 3




```
import torch as tc
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy.optimize import curve_fit


def get_df(file_path):
df = pd.read_csv(file_path)
return df

def quick_scatter(df, curve=None, title=None, text=
    ↪ None):
graph = df.plot.scatter(x=0, y=1, title=title)
if curve is not None:
graph.plot(curve[0], curve[1])
if text is not None:

plt.text(1, .9, s=text, fontsize = 14, ha='right',
    ↪ va='center', transform=plt.gca().transAxes,
bbox = dict(facecolor = 'red', alpha = 0.5))
plt.show()

def resid_scatter(resids, input):
plt.plot(input, resids, '*')
plt.plot(input, np.zeros(len(input)))
```

```
plt.title("Euclidean Distance of Each Estimate")
plt.ylabel("2-Norm Distance")
plt.xlabel("Diameter [mm]")
plt.show()

# Create a tensor from a csv file
def tensorize_df(df):
# print(data.head())
# Convert the data to a numpy array
data = df.to_numpy()
x1 = data[:, 1]
x0 = data[:, 0]

return tc.tensor(x1, dtype=tc.float), tc.tensor(x0,
    ↪  dtype=tc.float)


# Exponential decay function
def non_linear_least_squares(input, output, h_0=15,
    ↪  tau=100, tol = 1e-7):

error = 100
error_seq = 100

resid = output - h_0 * tc.exp(- input / tau)

while error_seq > tol:


dh_0 = tc.exp(-input / tau)
dtau = h_0 * input * tc.exp(-input / tau) * tau
    ↪ **(-2)

J = tc.stack([dh_0, dtau], dim=1)

J_dual = J.mT

del_coeff = tc.inverse(J_dual @ J) @ J_dual @ resid
# print(del_coeff)

h_0 = h_0 + del_coeff[0]
tau = tau + del_coeff[1]

resid = output - h_0 * tc.exp(- input / tau)
# import pdb; pdb.set_trace()
error = tc.sqrt(resid.unsqueeze(0) @ resid)
```

```
error_seq = tc.abs(del_coeff[0] / h_0) + tc.abs(
    ↪ del_coeff[1] / tau)




final_coeff = [h_0, tau]
output = h_0 * tc.exp(- input / tau)

return final_coeff, output, error, error_seq

def calc_error(tensor, output, coeff, resid=False):

# output = output.to(tc.float)
error_vect = output - (tensor @ coeff)


if resid:
r_squared = error_vect @ error_vect
r = tc.sqrt(r_squared)
return r, r_squared


dim = len(output)
output = output.unsqueeze(0)
tensor_dual = tensor.mT

t_gram = tensor_dual @ tensor

tensor_gram_inv = tc.inverse(t_gram)

P = tensor @ tensor_gram_inv @ tensor_dual

M = tc.eye(dim, dtype=tc.float) - P

L = tc.eye(dim, dtype=tc.float) - tc.ones(dim, dim,
    ↪  dtype=tc.float) / dim

# import pdb; pdb.set_trace()
rss = output @ M @ output.mT
# import pdb; pdb.set_trace()
tss = output @ L @ output.mT

r_squared = 1 - (rss / tss)
```

```
return r_squared

# r_squared = 1 - (tc.norm(error_vect) / tc.norm(
    ↪ output))


def least_squares(input, output, order=1, resid=
    ↪ False, time_span=None):

"""
-- The alg represents the equation Ax = b
where A is the input matrix, x is the unknown
    ↪ vector
of linear coefficients and b is the output vector.
-- We are attempting to minimize the max error
    ↪ between
points in a metric space represented by the Feature
Space.
-- The function returns the vector x.
"""


# Create the input matrix
A = tc.tensor([[i**j for j in range(order+1)] for i
    ↪  in input], dtype=tc.float)

A_dual = A.mT

A_gram = A_dual @ A

A_gram_inv = tc.inverse(A_gram)

coeff = A_gram_inv @ A_dual @ output

# print(coeff)

if time_span is None:
out_est = A @ coeff
else:
time = tc.linspace(start=time_span[0],steps=
    ↪ time_span[1] , end=time_span[2])

if resid:
return coeff.numpy(), out_est.numpy(), calc_error(
    ↪ tensor=A, output=output, coeff=coeff, resid=
```

```
    ↪ True)

return coeff.numpy(), out_est.numpy(), calc_error(
    ↪ tensor=A, output=output, coeff=coeff)


def model(t, A, phi, b, omega_d):


return (A * np.exp(-b *.5 * t) * np.cos(omega_d * t
    ↪  + phi))



def estimate_spring(x, t, initial_guess=None):
if initial_guess is None:
initial_guess = [1, 1, 1, 1]
# import pdb; pdb.set_trace()
params, covariance = curve_fit(model, xdata=t.numpy
    ↪ (), ydata=x.numpy(), p0=initial_guess)

# Extract estimated parameters
A_est, phi_est, beta_est, omega_est = params
# print(params)

return A_est, phi_est, beta_est, omega_est


def problem_1(order=1):
df = get_df("../data/p1.csv")
quick_scatter(df, title="strength v. diameter")
stre, dia = tensorize_df(df)
coeff, output, error = least_squares(output=stre,
    ↪ input=dia, order=order)
notes = f"R^2: {error.numpy()}"
quick_scatter(df, title="strength v. diameter",
    ↪ curve=[dia, output], text=notes)
time = tc.linspace(start=5, steps=100, end=6)

A = tc.tensor([[i**j for j in range(order+1)] for i
    ↪  in time], dtype=tc.float)
est_out = A @ coeff
resid = stre.numpy() - output
resid_scatter(input=dia.numpy(), resids=resid)
# print(time[50])
print(f"Strength change for dx = .3mm: {coeff[1] *
```

```
    ↪ .3 }")
print(f"Strength at 5.5mm: {est_out[50]}")




def problem_2():
df = get_df("../data/beer.csv")

height, time = tensorize_df(df)
final_coeff, output, error, _ =
    ↪ non_linear_least_squares(input=time, output=
    ↪ height)


quick_scatter(df, curve=[time, output], title="
    ↪ Height v. Time")

print(f"Time constant: {final_coeff[1].numpy()}")


def problem_3():
df = get_df("../data/p3.csv")

x, t = tensorize_df(df)

# print(t)
# import pdb; pdb.set_trace()

_, _, b_est, omega_d_est = estimate_spring(x=x, t=t
    ↪ )

k_est = omega_d_est**2 + .25 * b_est**2
notes = f"Spring Constant: {k_est}\nDamping
    ↪ Coefficient: {b_est}"
quick_scatter(df, title="Displacement v. Time",
    ↪ curve=[t, model(t, *estimate_spring(x, t))],
    ↪  text=notes)




def main():
problem_1()
```

```
problem_2()
problem_3()
main()
```