COMS30127 Coursework 3

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Part A: Integrate-and-fire neurons

Question 1

Euler's method was used with a timestep (δt) of 0.25ms on the differential equation below (Equation 1). It was assumed that the neuron spiked and is reset at time $t = 0$, therefore the voltage is initiated with $V(0) = E_L$. The result of this is the plot found in Figure 1.

$$\frac{dV}{dt} = \frac{E_L - V + R_m I_e}{\tau_m} \tag{1}$$

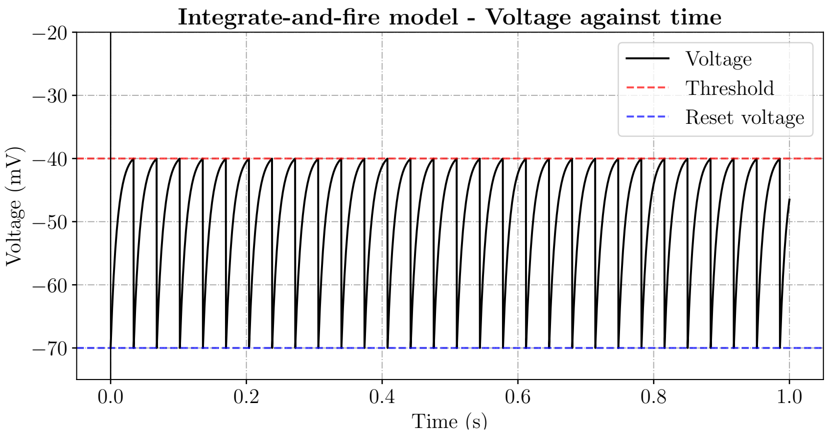


Figure 1: Simple integrate-and-fire model simulated for 1 second of time, assuming no refractory period.

Question 2

In Figures 2 and 3, the two model neurons (which have the same parameters) have synaptic connections between each other. Equation 2 shows the model of the neuron as an electrical circuit, where the synapse is represented by a time-dependant conductor in series with a battery. Figures 2 and 3 show the neurons modelled using excitatory and inhibitory synapses respectively. Euler's method was again used for Equation 2, and for $\frac{ds}{dt} = -\frac{s}{\tau_s}$.

$$\frac{dV}{dt} = \frac{E_L - V + R_m I_e + R_m \bar{g}_s s(t)(E_s - V)}{\tau_m} \tag{2}$$

When a spike occurs in one neuron, the excitatory synapse causes an increase in the rate of change of the voltage of the connected neuron. This occurs as, in Equation 2, E_s holds a value of 0mV, meaning the bracket $(E_s - V)$ is always positive. Therefore, when a spike arrives, causing $s(t)$ to increase by Δs, this leads to an increase in the left-hand side, $\frac{dV}{dt}$.

Alternatively, the inhibitory synapse causes a decrease in the rate of change of the voltage of the connected neuron. Here, E_s holds a value of -80mV, meaning the bracket $(E_s - V)$ is always either negative or zero. Therefore, when a spike arrives for an inhibitory synapse there is a decrease in the left-hand side, $\frac{dV}{dt}$.

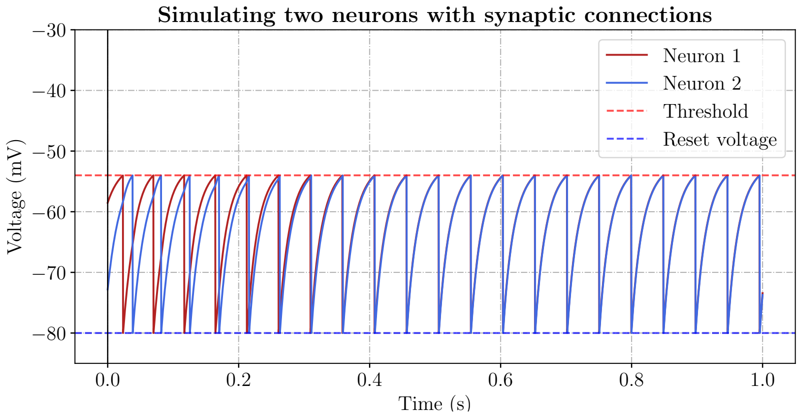


Figure 2: Two integrate-and-fire neurons which have synaptic connections between each other. Synapses are excitatory with $E_s = 0$mV.

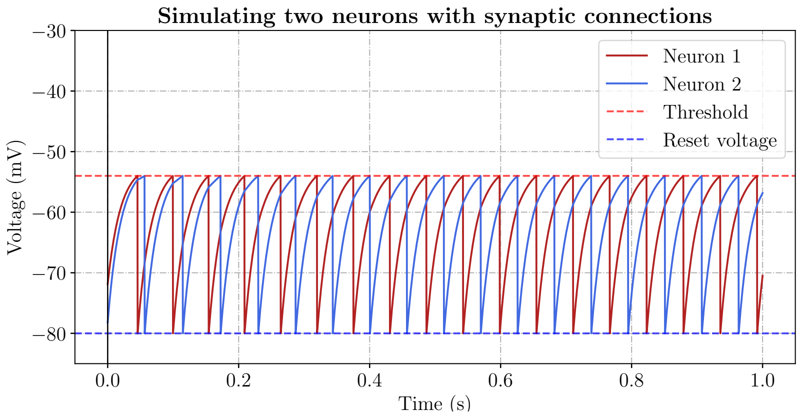


Figure 3: Two integrate-and-fire neurons which have synaptic connections between each other. Synapses are inhibitory with $E_s = -80$mV.

The neurons with the excitatory synapse connection (Figure 2) eventually produce spikes simultaneously, as a spike in one neuron activates the connected neuron, and with each spike the neurons' spike timings become closer until they are identical, and reach the threshold voltage synchronously. On the other hand, the neurons with the inhibitory synapse connection (Figure 3) will eventually produce spikes perfectly out of phase, as a spike in one neuron drags the voltage of the other back towards rest.

Part B: STDP

Question 1

The model depicted in Figure 4 is a single leaky-integrate-and-fire neuron with 40 incoming excitatory synapses. Each incoming synapse has a constant average input firing rate of 15Hz, and strength equal to the maximal conductance (4nS). The result of this is that the neuron fires irregularly at a rate of roughly 20 Hz.

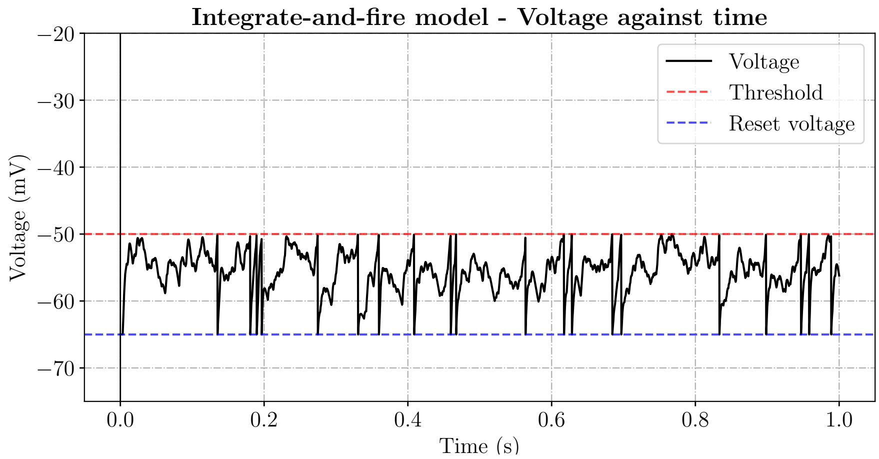


Figure 4: Single leaky-integrate-and-fire neuron with 40 incoming excitatory synapses, each with constant synapse strength of 4nS.

Question 2

Implementing spike-timing-dependent plasticity (STDP) with independent homogeneous Poisson inputs, the behaviour of the model was recorded for a longer period of 300s. The synaptic strength distribution in Figure 5 can be seen to converge towards a bimodal distribution. The two peaks lie at 0nS and 4nS (the maximal conductance). This suggests that over time the synaptic weights will tend towards one of the bounds of the synaptic strength range.

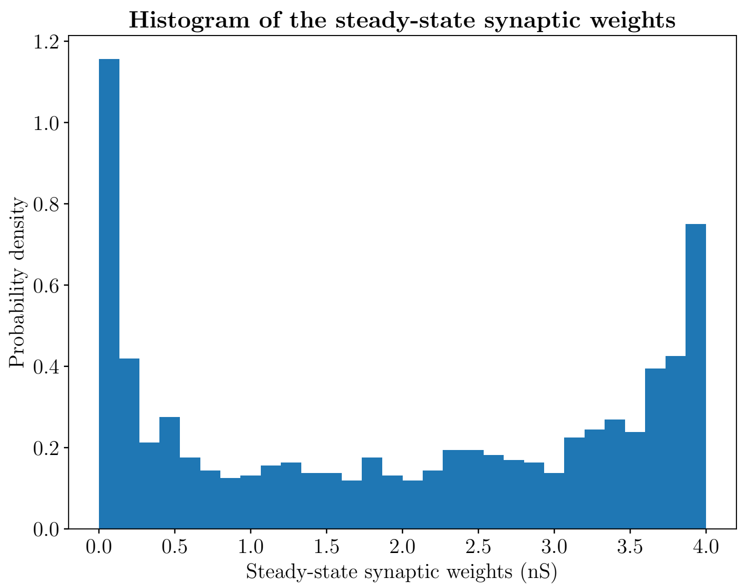


Figure 5: Histogram of the steady-state synaptic weights using an average input firing rate of 15 Hz. Simulation mode STDP is switched on.

The figure below (Figure 6) represents the simulation run over a 300s window. The 300s window was split into 10s bins, and the average firing rate over each bin was plotted against the respective bin centre.

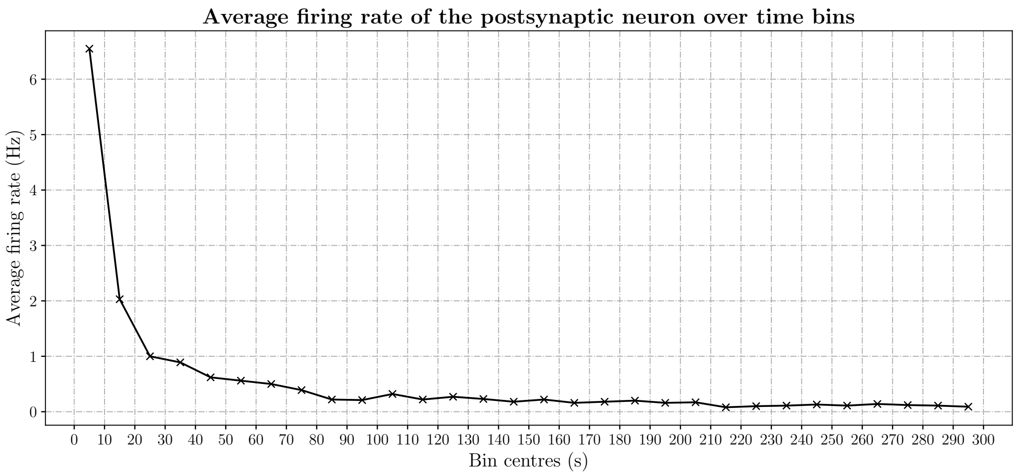


Figure 6: Average firing rate of the postsynaptic neuron across a 300 second simulation, using 10-second time bins. Simulation mode STDP is switched on, and the average input firing rate is 15Hz.

The steady-state firing can be seen to be reached when the gradient of the curve reaches roughly zero, as any fluctuation after is caused only by noise, which is not time dependent. Therefore, it is possible to take the mean firing rate of the last 3 windows (270 - 300s) to report the steady-state firing rate. When the simulation mode STDP is switched on, the steady-state firing rate is 0.12Hz, whereas when this is switched off, it is 21.4Hz.

Question 3

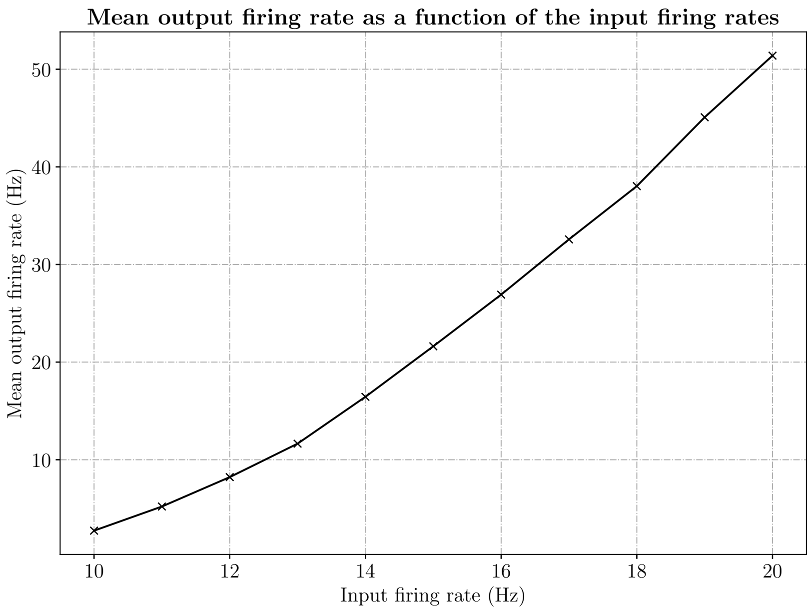


Figure 7: Steady-state firing rate of the postsynaptic neuron over a range of input firing rates. STDP is switched off.

The input firing rates were varied over the range of 10Hz to 20Hz in 1Hz increments. When the simulation was run with STDP switched off (Figure 7), the input firing rates and the mean output firing rate were positively correlated. STDP 'off' mode means that synaptic strengths are fixed and remain constant throughout the simulation. Therefore, as $s(t)$ increases by $\Delta s = 0.5$ for each presynaptic spike, $\frac{dV}{dt}$ also increases. The increase of the rate of change of the voltage causes the postsynaptic neuron to spike with a greater output rate. Therefore, the more $s(t)$ increases due to a higher input firing rate, the more the mean output firing rate increases.

As seen in Figure 8, the input firing rates and the mean output firing rate are negatively correlated when STDP is switched on. STDP 'on' mode means that every spike (pre- and postsynaptic) triggers changes in the relative strengths of activated synapses. These strengths can increase whenever a post-synaptic spike occurs or decrease at the synapses that receive a pre-synaptic spike. In this case, increasing the input firing rates leads to an increase in the frequency of presynaptic spikes. This leads directly to depression in the strengths of the respective synapses. This in turn causes $\frac{dV}{dt}$ of the postsynaptic neuron to fall, and therefore its firing rate.

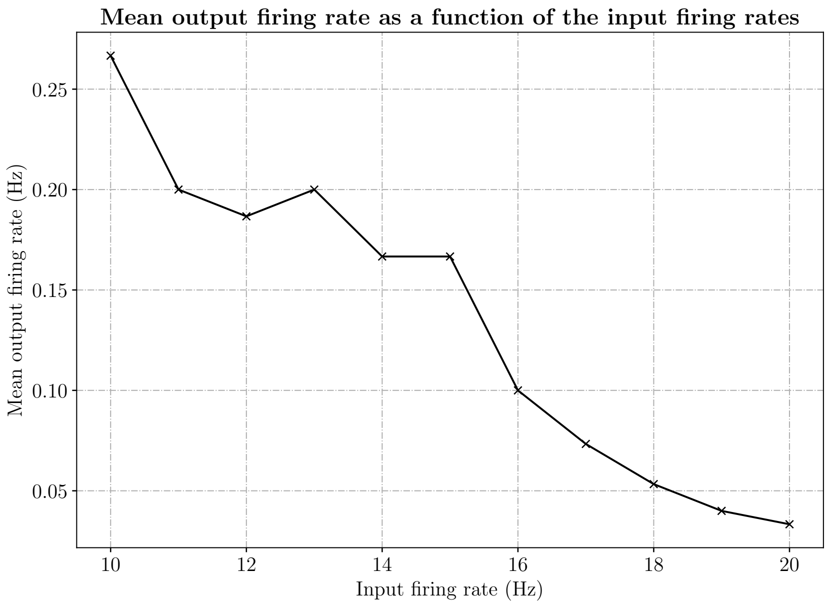
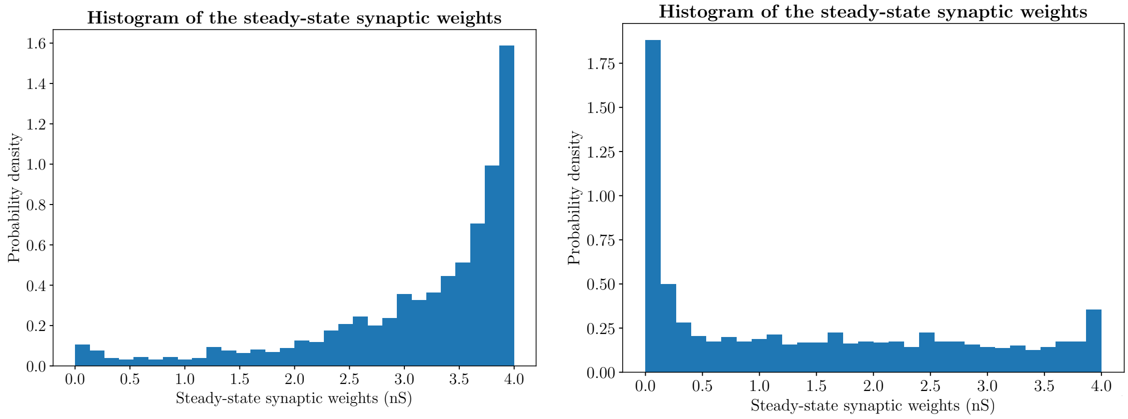


Figure 8: Steady-state firing rate of the postsynaptic neuron over a range of input firing rates. STDP is switched on.

In Figure 9 (left), an average input firing rate of 10Hz means a decrease in presynaptic activity from the initial case shown in Figure 5. Presynaptic spikes cause depression in the strengths of relevant synapses. Gradual depression can be seen in the synapse strengths away from their initial strength of 4nS, however a distinct mode at 4nS remains. Some depression has occurred in all synapses, however postsynaptic activity can follow presynaptic activity in stronger synapses, causing potentiation and creating positive feedback. This process leads to the second peak at the maximum synaptic strength (4nS) and a large proportion of synapses maintaining a weight close to this value.



$\langle r \rangle = 10\text{Hz}$ $\langle r \rangle = 20\text{Hz}$

Figure 9: Histograms of the steady-state synaptic weights with STDP switched on.

An increase in presynaptic activity from the initial case can be seen in Figure 9, with an average input firing rate of 20Hz. The higher input firing rate leads to an increase in presynaptic activity. It is also known from Figure 8 that higher input firing rates decrease the mean output firing rate of the postsynaptic neuron. The peak at zero is caused by any uncorrelated activity, where the plasticity rule causes a weakening of the synaptic strength (depression). As the incoming synapses fire more regularly, depression occurs in the incoming synapses at a faster rate, and the rate of postsynaptic spikes falls. Therefore, a greater proportion of presynaptic activity is uncorrelated with the activity of the postsynaptic neuron, and there is less potentiation at the synapses. This leads to the synaptic strengths falling towards zero at a greater rate. This can be seen as there is a significant mode of steady-state synaptic weights at a strength of 0nS, with few reaching the maximum synaptic strength.

Question 4

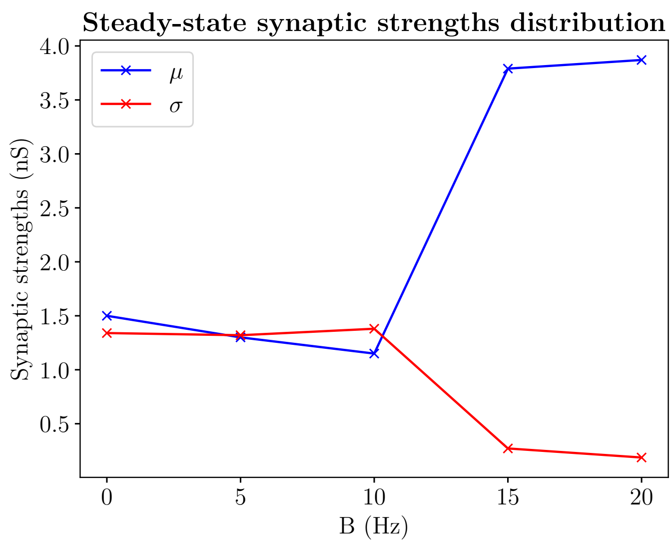
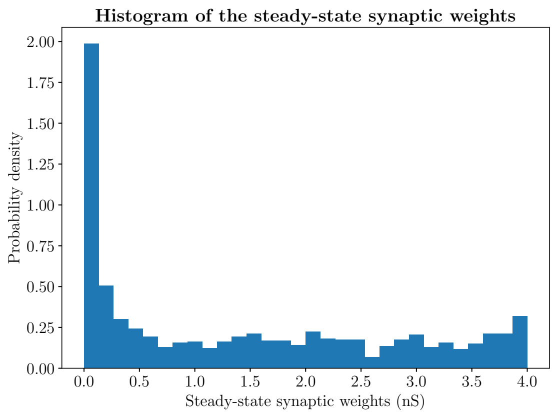
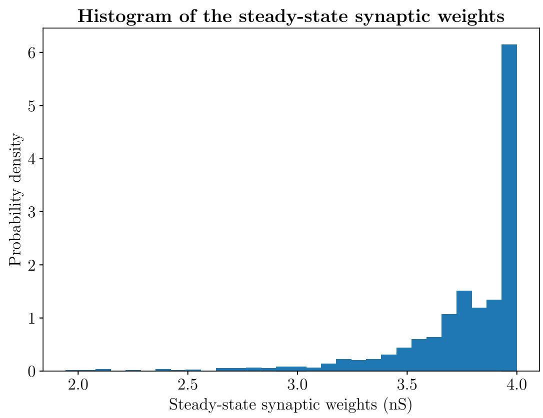


Figure 10: Mean and standard deviation of the steady-state synaptic strengths over varying values of the correlation parameter B.



$B = 0\text{Hz}$



$B = 20\text{Hz}$

Figure 11: Histograms of the steady-state synaptic weights with STDP switched on, changing the correlation parameter B.

When the correlation parameter B is set to zero (Figure 11, left) the distribution of the steady-state synaptic weights follows the same distribution as Figure 9 (right) as assigning zero to B sets all spike trains as independent. The same explanation applies here.

Giving B a value greater than zero ($B = 20$Hz in Figure 11, right), means the input firing rates co-vary in time, and the spike trains are correlated. As the correlation parameter B increases from 0Hz to 10Hz, the mean of the steady-state synaptic strengths is low and decreases gradually (Figure 10). However, it's during this period when the standard deviation of the strengths is at its highest, at a value roughly equal to the mean (1.4nS), implying the weights are widely dispersed either side of the mean. This suggests that between $B = 0$Hz and $B = 10$Hz, the activity in the system is largely uncorrelated, and synaptic strengths tend towards zero over time due to presynaptic spikes causing depression without a postsynaptic response. At these values of B the correlation between the presynaptic spike trains is not high enough to engage the post synaptic neuron sufficiently in order to maintain the mode at 4nS through positive feedback. This range produces a distribution similar to that visualised in Figure 11 (left).

After this, when B is set to 15-20Hz, the mean increases rapidly to around 3 times its value at $B = 10$Hz, roughly 3.8nS. This suggests the postsynaptic neuron responds to the more highly correlated presynaptic activity, and potentiation occurs alongside positive feedback. The standard deviation of the synaptic strengths decreases at this point to a value of roughly 0.2nS, suggesting the strengths are more densely packed around the mean. This distribution is visualised in Figure 11 (right). Here, the synaptic weights have tended to the maximum synaptic strength at their steady-state values.