Minimum Energy Filtering for Collaborative Localisation

Thesis Proposal Review

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What is the future of mobile robotics?

- Mobile robots are already in use across many industries
- Unmanned Aerial Vehicles (UAV)
- Unmanned Ground Vehicles (UGV)
- Safer, faster, cheaper, more reliable, more accurate
- What does the future hold?



DRONE TEAM
intel
DRONE TEAM
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DRONE TEAM
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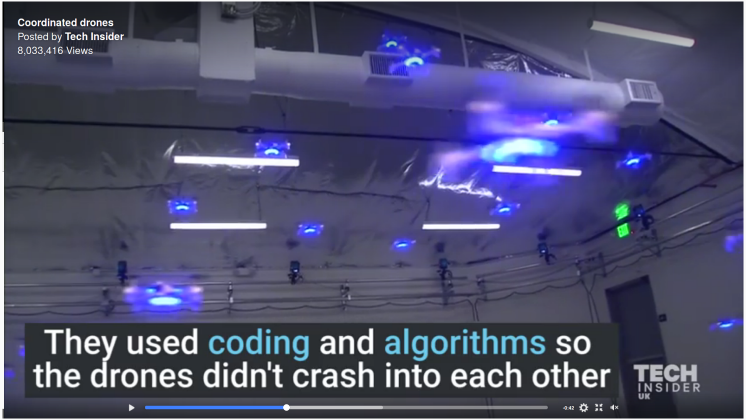
Swarm Robotics

Multiple independent robots working together with a common objective

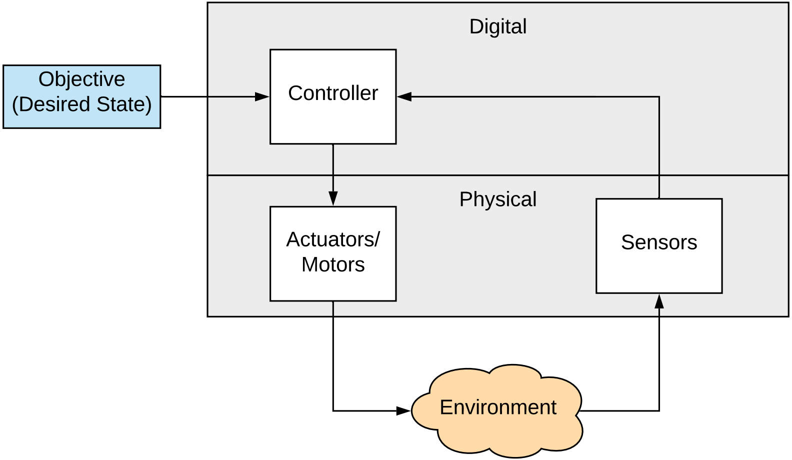
- Robust, Resilient, Redundant
  - Losing a single vehicle is not the end of the mission
- Distributed Sensing
  - Spatial distribution of sensors
  - Faster collection of information
  - Different perspectives of a single target
- Information sharing
  - Heterogeneous networks
  - Lower cost
  - Diversity of information

If only it was this simple

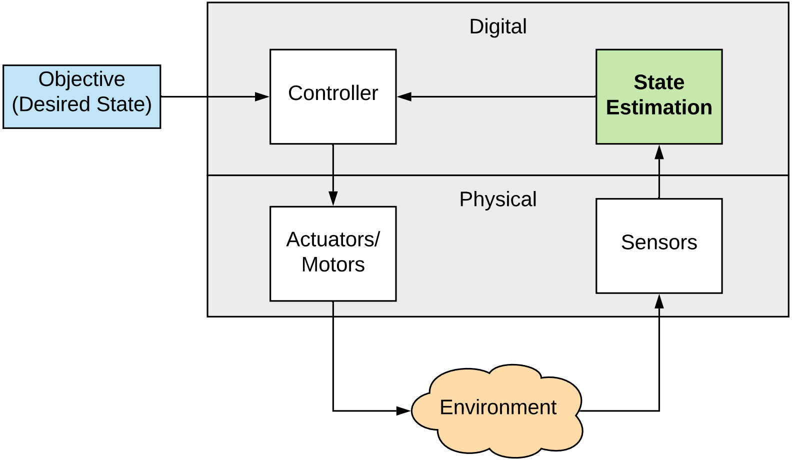



```
if(goingToCrashIntoEachOther){
    dont();
}
```

Robot Control Loop



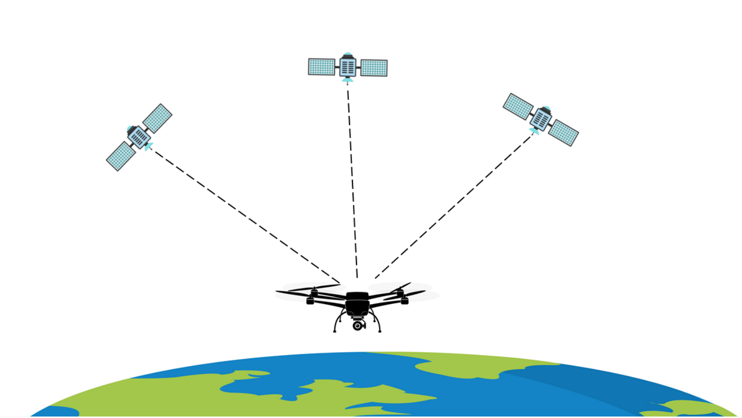
Robot Control Loop



Robot Localisation

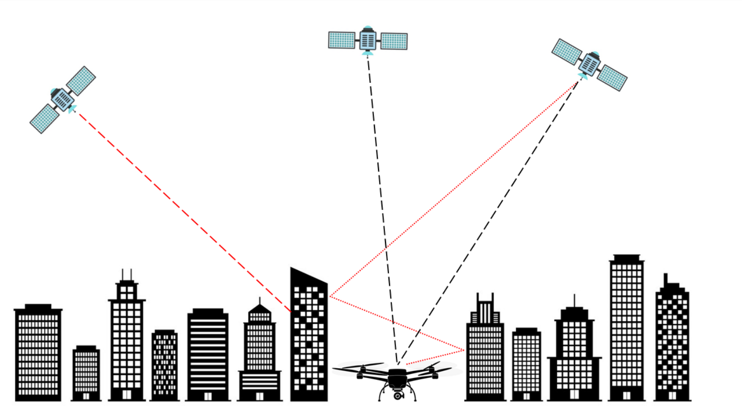
Traditional Approach to Localisation — GNSS

1. GNSS — Global Navigation Satellite System (eg. GPS)
2. GNSS augmentation can give centimetre level accuracy
3. Accessible anywhere on Earth*



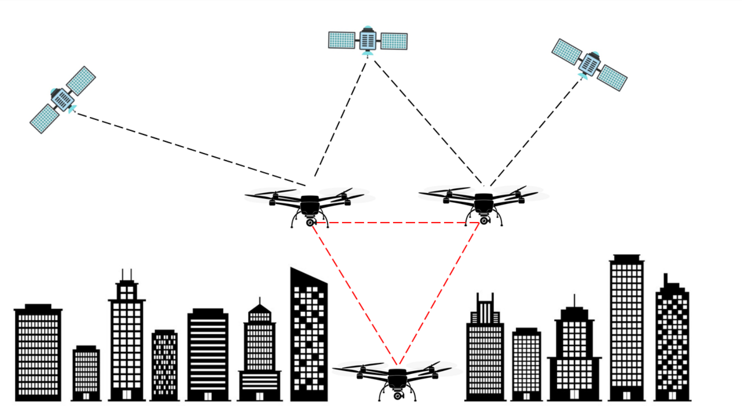
Problems with GNSS

- ‘Urban Canyon’ — Blocked signals and multipath
- Poor reception indoors and underground
- Interference from other sources — accidental or deliberate



Potential Solution: Collaborative Localisation (CL)

- Vehicles communicate and share information
- Vehicles can take measurements to other vehicles



Stochastic Filtering

"All models are wrong, but some are useful."

— George Box

Problem Definition

Continuous Time Model

$$\dot{x} = f(x, u) + \delta$$
$$y = h(x) + \epsilon$$

Discrete Time Model

$$x_{k+1} = f(x_k, u_k) + \delta_k$$
$$y_k = h(x_k) + \epsilon_k$$

- x - System State
- u - Control input
- f - System model
- y - Sensor measurement
- h - Measurement model
- δ - Model error
- ϵ - Measurement error

We want to find the ‘best’ estimate of the state, x, given only the model and the measurements, y

Linear System Model

$$\dot{x} = Fx + Gu + \delta$$
$$y = Hx + \epsilon$$

Noise Model

$$\delta \sim \mathcal{N}(0, R) \quad \text{i.i.d.}$$
$$\epsilon \sim \mathcal{N}(0, Q) \quad \text{i.i.d.}$$

Minimum Variance State Estimate

$$\hat{x}(t) := \arg\min_{x'} \int_{-\infty}^{\infty} \left\| x - x' \right\|^2 p(x|y_{[0,t]})dx$$
$$\hat{x}(t) = E[x|y_{[0,t]}]$$

We could choose a different metric eg. Maximum likelihood, maximum aposteriori

Kalman Filter Properties

- For linear systems with Gaussian noise, the Kalman Filter is optimal*
- The Kalman filter is recursive i.e. $\hat{x}_k = f(\hat{x}_{k-1}, u_k, y_k)$

Can we use the same approach for non-linear systems?

1. Linearise the system around the state estimate
2. Apply a linear Kalman Filter
3. Repeat

- This is the **Extended Kalman filter (EKF)**
- Not optimal anymore

Collaborative Localisation [1]

- We need to estimate the state of multiple robots
- In the literature, almost everyone is using an EKF
- But pose estimation is highly non-linear!
- Need to carefully manage double-counting of measurements
- Trade-off between communication complexity and filter performance

[1] Roumeliotis, 2003. Bahr, 2009. Luft, 2018. Zamani 2019

An Introduction to Minimum Energy Filtering

Deterministic System Model

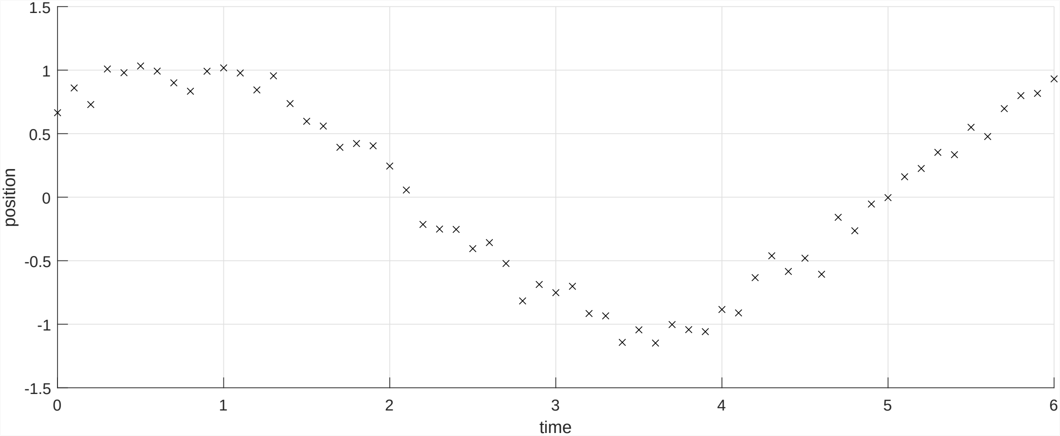
Recall the same general system model

$$\dot{x} = f(x, u) + \delta$$
$$y = h(x) + \epsilon$$

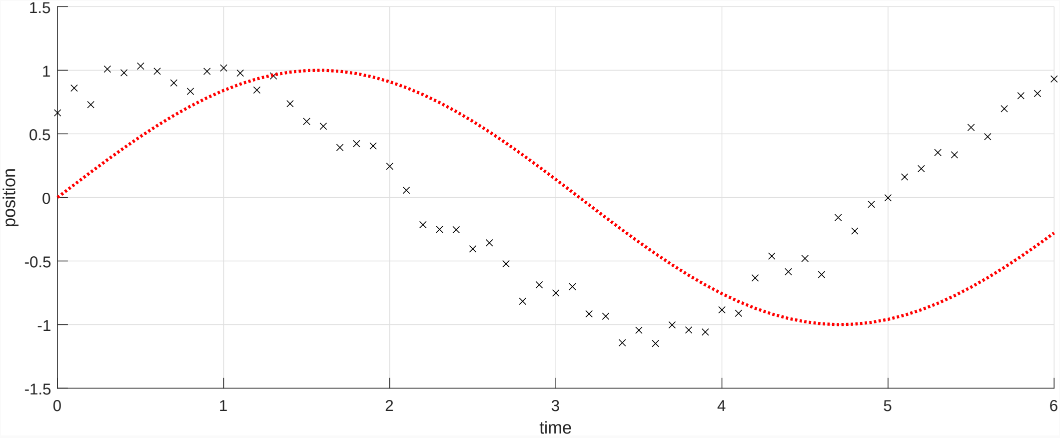
Deterministic Error model

- Consider δ and ϵ as deterministic, but unknown error signals, *NOT* random variables.
- Given a known trajectory, x', and a set of measurements, y', we could determine δ and ϵ.
- There are many different trajectories that are compatible with the measurements and the model

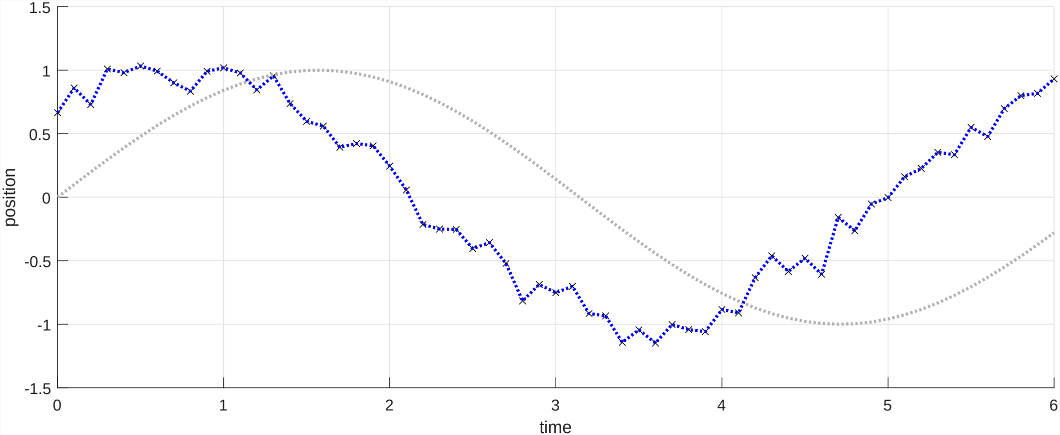
Example Scenario



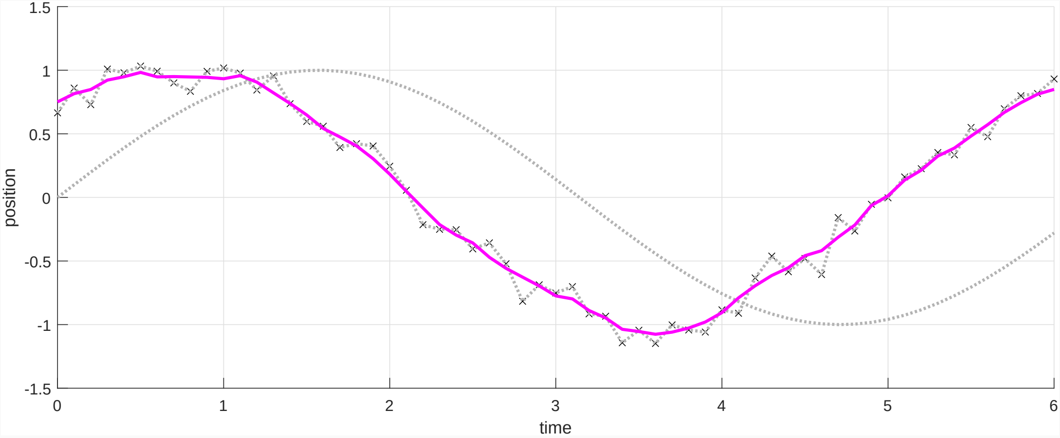
Example Scenario - Potential Model I



Example Scenario - Potential Model II



Example Scenario - Potential Model III



Minimum Energy Filter [2]

Occam's Razor: "the simplest solution is most likely the right one"

Minimum Energy Cost Functional

$$J_t(x', \delta_{[0,t]}, \epsilon_{[0,t]}) = c_0(x') + \int_0^t ||\delta(\tau)||_R^2 + ||\epsilon(\tau)||_Q^2 d\tau$$

$$\hat{x}(t) = \arg\min_x J_t$$

- This is an infinite-dimensional optimisation problem
- We can use the techniques from optimal control theory to find a solution
- For linear systems, the resulting filter is the same as the Kalman filter

[2] Mortensen, 1968. Hijab 1980. Willems, 2004

Research Proposal

“Research is what I’m doing when I don’t know what I’m doing.” — Wernher von Braun

The aim of my PhD is to improve the accuracy and robustness of localisation algorithms for a group of heterogeneous autonomous vehicles, with a specific focus on GPS-denied environments

Research Track 1: Single-vehicle minimum-energy filtering

- Develop a robust minimum-energy filter for a single autonomous vehicle
- Must be compatible with sensors present on the physical platform
- Not aiming for groundbreaking performance, but must be competitive
- Will be the foundation for multi-vehicle extensions

Research Track 2: Minimum-energy for collaborative localisation

Investigate existing CL algorithms and adapt techniques to minimum-energy

- Most existing approaches to CL use a basic EKF
- Will be able to confirm if minimum-energy yields any advantages over stochastic filtering

Research Track 3: Exploring communications constraints

Investigate the structure and relationship between the information network of the system and the communication network of the vehicles

- Will help to minimise communications overhead
- Trade-off between accuracy of localisation and communication bandwidth

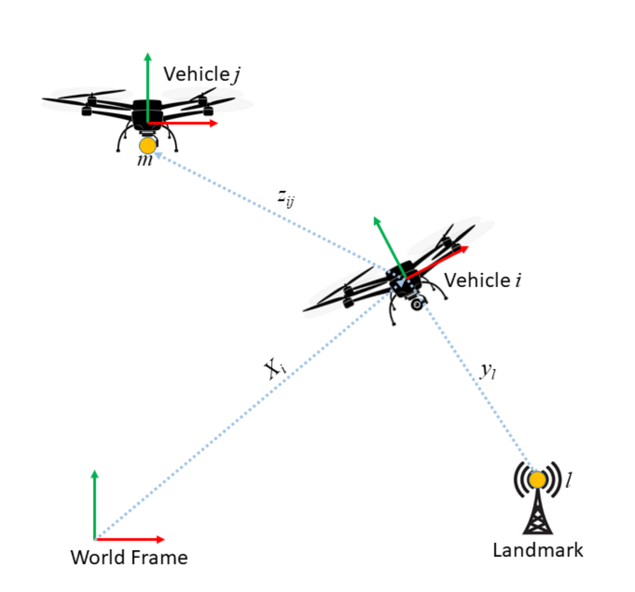
Aiming to demonstrate the collaborative localisation algorithm on real hardware

UAV Platform

UGV Platform

Minimum Energy Filtering for Collaborative Localisation

Problem Setup - Diagram



Problem Setup

State representation

$$X_i := \begin{bmatrix} R_i & p_i \\ 0 & 1 \end{bmatrix} \in \mathrm{SE}(3)$$

Velocity

$$\Omega_i := \begin{bmatrix} \omega_i \\ v_i \end{bmatrix}^{\wedge} \in \mathfrak{se}3$$

Kinematics

$$\dot{X}_i = X_i \Omega_i$$

Velocity Measurement

$$u_i := \begin{bmatrix} \omega_i \\ v_i \end{bmatrix} + \epsilon_i$$

Landmark Measurement

$$\bar{y}_{il} := X_i^{-1} \bar{l} + \mathring{\delta}_{il}$$

Robot to Robot measurement

$$\bar{z}_{ij} := X_i^{-1} X_j \bar{m}_j + \mathring{\eta}_{ij}$$

Cost Functional

Combined state representation

$$\boldsymbol{X} := (X_1, \ldots, X_n) \in \mathrm{SE}(3)^n$$

Cost Functional

$$J_t(\boldsymbol{X}_0', \boldsymbol{\epsilon}, \boldsymbol{\delta}, \boldsymbol{\eta}) := \frac{1}{2} d_{P_0}^2 \left(\boldsymbol{X}_0', \hat{\boldsymbol{X}}_0 \right) + \frac{1}{2} \sum_{i \in N} \int_0^t \left[\|\epsilon_i\|_B^2 + \sum_{l \in L} \|\delta_{il}\|_C^2 + \sum_{j \in N} \|\eta_{ij}\|_D^2 \right] d\tau$$

This assumes that we always have landmark and robot measurements

Discrete Update Filter — Intermittent measurement model

$$J_t(\boldsymbol{X}_0', \boldsymbol{\epsilon}) := \frac{1}{2} d_{P_0}^2 \left(\boldsymbol{X}_0', \hat{\boldsymbol{X}}_0 \right) + \frac{1}{2} \sum_{i \in N} \int_0^t \|\epsilon_i\|_B^2 \, d\tau$$

We introduce the value function

$$V(\boldsymbol{X}, t) := \min_{\epsilon[0,t]} J_t(\boldsymbol{X}, \epsilon)$$

The minimum-energy state estimate is then

$$\hat{\boldsymbol{X}}(t) := \arg\min_{\boldsymbol{X}} V(\boldsymbol{X}, t)$$

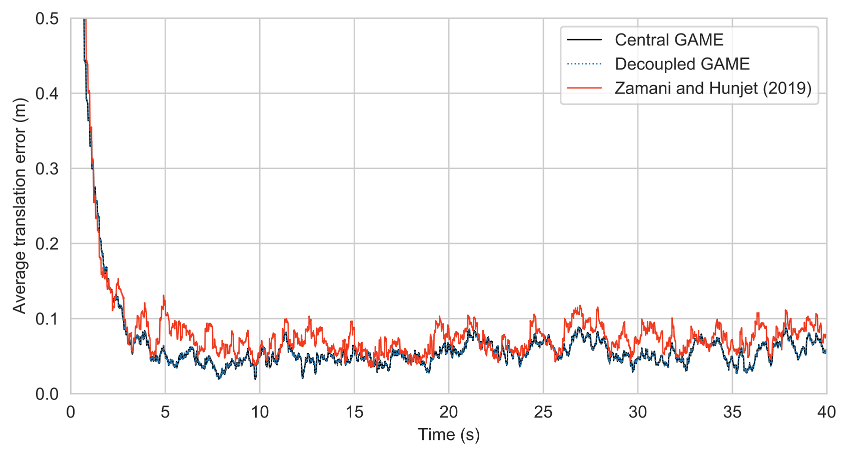
When a measurement is received, add to the value function

$$V^+(\boldsymbol{X}, t) := V(\boldsymbol{X}, t) + \frac{1}{2} \|\delta_{il}\|_C^2 , \qquad \hat{\boldsymbol{X}}^+(t) := \arg\min_{\boldsymbol{X}} V^+(\boldsymbol{X}, t)$$

Results

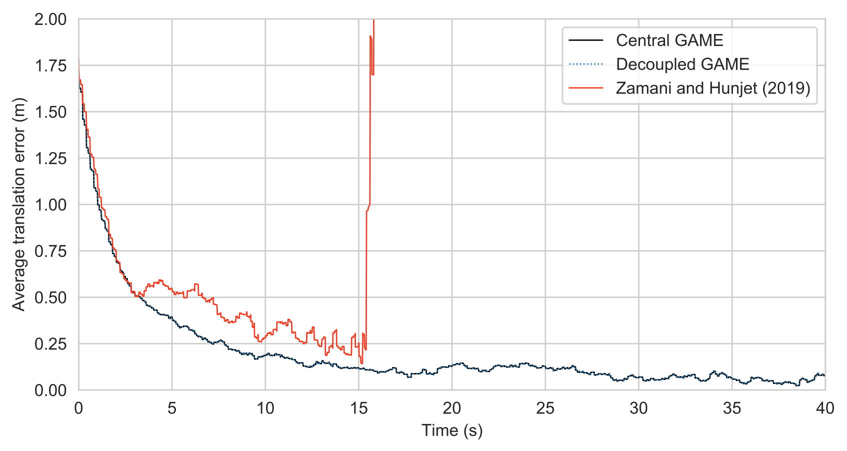
- The resulting filter is recursive
- Jointly estimates the optimum trajectory for all robots based on all measurements
- We can compute centrally or distribute the filter equations among the robots
- $O(n^2)$ communication complexity
- Paper accepted for publication (IFAC World Congress 2020)

Simulation Results



4 Robots in formation. All robots and 4 landmarks visible to each robot at 10Hz.

Simulation Results - Limited Information



4 robots in formation. Each robot can only see one landmark and one other robot.

Next Steps

1. Implementable minimum-energy filter for a single robot
2. Comparison to current EKF approaches
3. Numerical integration and stability
4. Hardware platform development and demonstration

Acknowledgements

Thanks to everyone who has helped to get me this far

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- Alex Martin
- James Russell
- My friends and family

Questions?

Image Credits

- https://im-mining.com/2019/02/19/bhp-looks-phased-rollout-autonomous-trucks-wa-following-jimblebar-success/
- https://www.ifpo.org/wp-content/uploads/2018/09/abjdrones.jpg
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