User's Manual

Composite Compressive Strength Modeller

A Windows™ based composite design tool for engineers
Version 2.0

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Composite Compressive Strength Modeller

User's Manual

1. Introduction

Welcome to the Composite Compressive Strength Modeller (CCSM) – a design tool for deformation analysis and failure prediction of composite materials.

CCSM incorporates the following features:

1. Classical laminate theory for the prediction of laminate elastic properties;
2. Stress and strain analysis when in-plane forces and/or bending moments are applied;
3. Unnotched failure prediction by conventional failure criteria and the Budiansky-Fleck compressive failure criterion;
4. Compressive failure prediction for notched composite plates.
5. A user-expandable database to store material and geometrical properties.

The program is a tool to predict laminate failure, once the loads on a section of the laminate are known. For simple geometries it may be clear what the loading is, while for more complicated geometries the program may be used as part of a larger calculation to check for failure at critical points in the structure.

CCSM is written in Microsoft® Visual Basic™ language using Visual Studio 2012and it runs in the Microsoft® Windows™ operating system. It is structured so that the user, with a basic knowledge of composite mechanics, can use it with little reference to the manual. However, the user would benefit from reading through this manual, in particular the Quickstart section, chapter 3 and the detailed guide, chapter 4. CCSM 2.0 updates and simplifies Version 1.4.

This manual consists of the following chapters:

1. Introduction.	
2. Installation:	Instructions on how to install CCSM
3. Quick Start:	A 'quick start', self-sufficient chapter illustrating how to use the CCSM package.
4. Detailed guide	Detailed guide to all aspects of the program with tutorials
Appendix A	Theoretical Background: underlying principles of CCSM

2. Installation

2.1 System requirements

CCSM uses Microsoft .NET Framework 4 (or above) which needs to be installed before running the application.

2.2 The CCSM package

The CCSM deployment file contains the following items:

* User's manual;
* A Quickstart manual;
* Application and data files.

2.5 Setting up CCSM

Simply place the files from the deployment zipped file in an appropriate directory. If you want to change the database then you will need write permission for CCSM.MDB.

3. Quick Start

This chapter contains an introductory example of the use of CCSM. The chapter is intended to be followed at a computer running CCSM: commands and data to type in to CCSM are listed.

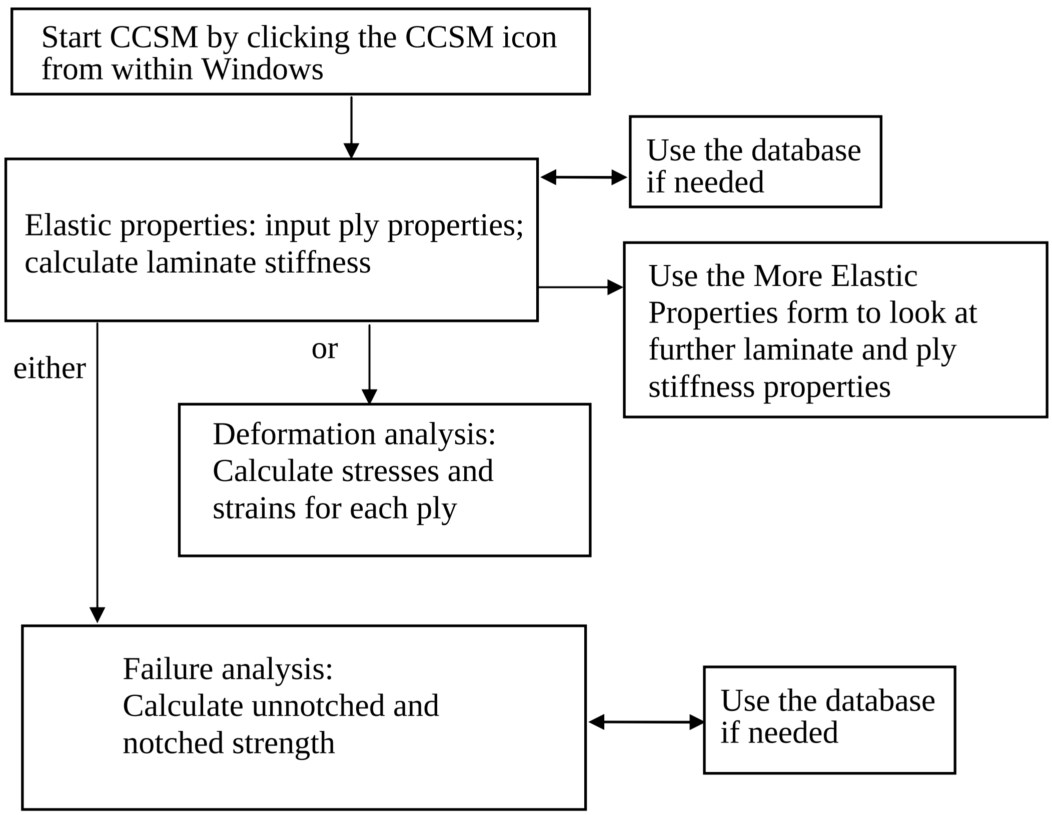
CCSM is written in Visual Basic (VB), a package designed to produce especially user-friendly graphical interfaces. The user should work through the various forms in CCSM by following the logic of a problem. For example, in order to calculate the stiffness of the laminate, sufficient information about each lamina should be provided first. Similarly, analysis of stress and strain would be meaningless without previously calculating the laminate properties and specifying the applied loads.

CCSM contains a number of forms for each stage of the analysis. Details at each stage are filled in using text boxes containing data, option buttons, and command buttons.

3.1 How to use CCSM

The following flow chart shows what to do in CCSM. In each form in CCSM corresponding to each box in the flow chart, there is an information box providing information about what to do next. Boxes in which to input data have a white background.

Further help can be obtained from the ? buttons and from the manual. Details of the program authors are included using the About button.



A flow diagram showing the structure of CCSM

3.2 A quick start example (Example file: QCKSTART.CSM)

This section goes through a simple analysis to illustrate the essential features of CCSM. More advanced tutorials are given in chapter 4.

The problem. Determine the stiffness and compliance matrices for a $[0^o / \pm 45^o / 0^o]_s$ symmetric laminate consisting of 0.1 mm thick unidirectional AS/3501 carbon fibre – epoxy laminae. Also find the stresses and strains for each lamina when the laminate is subjected to a single uniaxial force per unit length N_X=200 MN/mm. Use the Tsai-Wu failure criterion to decide the load level corresponding to first ply failure.

The following lamina stiffness and strength data are given: longitudinal modulus E_{11}=138 GPa, transverse modulus E_{22}=9 GPa, shear modulus G_{12}=6.9 GPa, Poisson's ratio ν_{12}=0.3, longitudinal tensile strength, denoted as SL(+)=1448 MPa, longitudinal compressive strength, SL(-)=1172 MPa, Transverse tensile strength, ST(+)=48.3 GPa, Transverse compressive strength, ST(-)=248 GPa, in-plane shear strength, SLT=62.1 GPa (taken from R. F. Gibson's Principles of Composite Material Mechanics, Table 2.2, P.48, 1994).

A step by step illustration is given below:

Step 1: Starting CCSM
After starting Windows, launch the Compressive Composite Strength Modeller program. The *Geometry and elastic analysis:* form is then loaded automatically and the cursor will be blinking in the C*omposite name* text box.

Step 2: Entering elastic properties data.
In this step laminate data and elastic properties are entered. All white text boxes are input boxes, and the light yellow boxes are output or information boxes. Now type in the following data, according to the problem:

Which input box	**What you type or do**	**Note**
Composite:	AS/3501	Optional
Comments:	Quickstart example	Optional
Total number of plies	8	
Ply No.	1	Type the current ply number into this box
Angle	0	Type the angle into this box (in degrees)

Thickness	0.1	**In mm** The total thickness box displays the total thickness of the laminate, based on the entered thicknesses for each ply.
E11	138	Lamina's Young's modulus in first (fibre) direction E_{11} (in local lamina co-ordinates), **in GPa**
E22	9	Lamina's Young's modulus in second (transverse to fibre) direction E_{22}
Nu12	0.3	Poisson's ratio ν_{12}
G12	6.9	In-plane shear modulus G_{12}
	(click Save Ply Data)	At this point, all necessary data for ply No. 1 have been input. Click the Save Ply Data button to save the input. The ply arrangement grid is filled for each ply where data has been saved. For ply Nos. 2 to No 4, the input procedures will be similar.
Ply No.	2	
Angle	45	
	(click Save Ply Data)	Notice that after inputting data for ply No. 1, the Lamina properties and thickness text boxes still hold the data for Ply No.1. Those data, because expressed in local lamina co-ordinate, are valid for other plies as long as they are for the same material. Different properties for each lamina are allowed in CCSM – you just type in the corresponding data for each lamina.
Ply No.	3	
Angle	-45	
	(click Save Ply Data)	
Ply No.	4	
Angle	0	
	(click Save Ply Data)	

Ply Nos. 5-8		Ply data for plies 5-8 have been automatically filled in, because the **laminate type option** is by default **symmetric**. The ply geometry and property data are symmetrical about the centre line.

Step 3. Calculating the stiffness of the laminate

At this point, data for all laminae have been entered. Now click the **Calculate** button to calculate the laminate stiffness. The first 5 components represent E_x, E_y, G_{xy}, ν_{xy} and ν_{yx}, for the laminate, using standard notation for an orthotropic laminate. The sixth component, E', is the appropriate elastic modulus for an orthotropic material such that $G = K^2 / E'$, where G is the elastic energy release rate and K is the mode I stress intensity factor. Further details of the meanings of these symbols are given in the Theory section, Appendix B.

Section 4.2.1 covers in more detail the various controls on the Elastic analysis form. In particular it explains in detail how the **Ply Input**, **Ply Editing** and **Change All** tools can be used to speed up the input and modifications to the laminate geometry. In brief, the different columns in the Ply Input tool refer to different angles required for the Previous, Current (=) and Next ply. Click Save Ply Data to store changes after clicking on the appropriate button.

Step 4 Deformation analysis

In our example problem, we wish to find the stresses and strains in each ply. This is done, after calculating the laminate stiffness, using the Deformation Analysis, clicking on the appropriate button under the GoTo tool.

On the *Deformation analysis:* form, first input the applied load, in this case 0.2 in the *Force resultant* N_x text box (converting from MN/mm to MN/m). The other text boxes can be left empty as these components are zero.

The input text boxes should be:

Force resultants on laminate	N x	N y	N xy	M x	M y	M xy
	0.2					

This is the only input needed for the deformation analysis in this problem. Now click the **Calculate** button. The mid-plane strains and curvatures will be calculated, and shown as:

Mid-plane strains and curvatures	E x	E y	Gamma xy	K x	K y	K xy
	3075	1934	~0.	~0.	~0.	~0.

where ~0 is a very small value (of the order of rounding errors). The mid-plane shear strain γ_{xy} (Gamma xy) and all curvatures are zero, because the laminate is symmetric and there are no bending components.

Though the stresses for all plies have been calculated, only stresses in one ply will be shown in the *Stress State* grid (the first ply by default). Stresses at the top, middle and bottom of each selected ply are shown; this takes into account the possibility of linear variation in the stress in the presence of bending. At the present example, the stresses in each ply is constant. The bottom row shows the average stress through the full thickness of the laminate. The results are shown as:

Through thickness t of selected ply	Stress state for ply No. 1		
	Sigma_x	Sigma_y	Tau_xy
top	421.697	-9.161	0.
middle	421.697	-9.161	0.
bottom	421.697	-9.161	0.
Laminate stresses	250	0.	0.

These stresses are in global coordinates (i.e. running along the x and y directions of the laminate). To see the ply stresses in local coordinates (i.e. running along and transverse to the fibre direction in each ply), click on the appropriate **Output Option**.

To display the ply stresses of other plies, navigate through the ply arrangement grid either clicking with a mouse or using cursor keys. The current ply is highlighted in this grid.

Notice that in the **Output Option** box, the "stress" option is set as the default. Clicking the "strain" option will display strain data giving:

Through thickness t of selected ply	Strain state for ply No. 1		
	Epsilon_x	Epsilon_y	Gamma_xy
top	3075	-1934	~0.
middle	3075	-1934	~0.
bottom	3075	-1934	~0.
Laminate strains			

Note that strains are output in microstrain. It can be seen that there are stress discontinuities at the ply interfaces, while the strains are continuous across the interface:- this reflects the basic assumption of the classical laminate theory.

Step 5: Failure analysis

To predict laminate failure, click on the **Failure Analysis** button, once the laminate properties have been entered and the laminate stiffness calculated.

There are five failure analysis criteria available in CCSM: the maximum stress, the maximum strain, the Tsai-Hill, the Tsai-Wu and the Budiansky-Fleck-Soutis (BFS) compressive failure criteria. All these criteria are **lamina** failure criteria. Select the "Tsai-Wu" option for the present problem.

Now type in the following strength data for the material AS/3501:

Which input box:	**What you type:**	**Note**
Longitudinal tensile strength SL(+)	1448	In MPa.
Longitudinal compressive strength SL(-)	1172	
Transverse tensile strength ST(+)	48.3	
Transverse compressive strength ST(-)	248	
In-plane shear strength SLT	62.1	

Click on the **Save Data - All Plies** button, to store this data.

Instead of typing the above data in each input text box, you can make use of the Database menu. Click on the **Database** button. All relevant data which have been stored in the database will appear in the list box on the left side of the form. Click on the required material's name in this list, which is 'AS/3501' in the present example, then click the **Take record as input** button. You will return to the *Failure analysis* form with the selected strength data displayed in the appropriate input text boxes. Now click on the **Save Data - All Plies** button as before.

Finally we need to input the Force pattern which is applied to the laminate. Enter the appropriate data in the force input boxes, e.g.

Force pattern applied to laminate	N x	N y	N xy	M x	M y	M xy
	0.2					

Note that it is only the ratio of forces that is required here, so that a pattern

Force pattern applied to laminate	N x	N y	N xy	M x	M y	M xy
	1					

would convey exactly the same information.

When the necessary data for the failure analysis have been input, click the **Calculate** button. The output boxes below the input force pattern show the failure loads,

Forces to give failure

N x	N y	N xy	M x	M y	M xy
0.327	0	0	0	0	0

telling us that the force to cause the first ply failure, when the Tsai-Wu failure criterion is applicable, will be N_x=0.327 MN/m with $N_y=N_{xy}=M_x=M_y=M_{xy}=0$. The magnitude of the input load N_x=0.2 MN/m will not affect the actual value of the failure load N_x. The relative magnitudes of the load components are determined by the input values of N_x: N_y: N_{xy}: M_x: M_y: M_{xy} ; the failure loads scales with this load pattern.

Observe also that in the ply arrangement grid on the left side, the fourth column marked "Fails first?" shows which plies have failed. For the present case, all the 45/-45 degree plies fail at the same time. Hence the grid appears as:

No.	Angle	Thick.	Fails first?
1	0	0.1	
2	45	0.1	Yes
3	-45	0.1	Yes
4	0	0.1	
5	0	0.1	
6	45	0.1	Yes
7	-45	0.1	Yes
8	0	0.1	

In this section we have covered the Tsai-Wu failure criterion for unnotched strength. The advanced tutorials in Section 4.2 give further information on the failure analysis, including an illustration using the Budiansky-Fleck-Soutis model for compressive failure, and the prediction of both the unnotched and the notched strength of laminates.

4. Detailed guide and advanced tutorials

This section is a detailed guide to the use of CCSM. Section 4.1 contains information about the nomenclature used in CCSM. Section 4.2 contains further information and advanced tutorials for each form in CCSM. To help use the full capability of CCSM, it is suggested that you go through these tutorials. Finally section 4.3 contains details of the databases used by CCSM.

4.1 Nomenclature convention in CCSM

This section describes in detail the nomenclature used in CCSM.

Global co-ordinate: the two co-ordinates of the global (laminate) system are denoted by x and y. The first axis of the global co-ordinate system, the x axis, coincides with the fibre direction of the 0 degree plies.

Local co-ordinate: the two axial directions of the local co-ordinates of a lamina are denoted by 1 and 2. The first axis of the local (lamina) co-ordinate system, the 1 axis, coincides with the fibre direction of the lamina.

The sign convention for lamina orientation with relation to the global co-ordinate is illustrated in Fig. 1.

The stress resultants used for the laminate analysis are defined in Fig. 2.

The laminated plate geometry and ply numbering system is illustrated in Fig. 3.

The nomenclature for the Budiansky-Fleck-Soutis compressive failure theory is illustrated in Fig. 4. For the compressive failure case, the stress in the longitudinal fibre direction σ_L will be negative.

The definitions of a or R, b, and w for the three types of specimens for the bridging analysis are shown in Fig. 5. Note that for the centre notch panels a and w are the semi-notch length and the semi-width of the specimen.

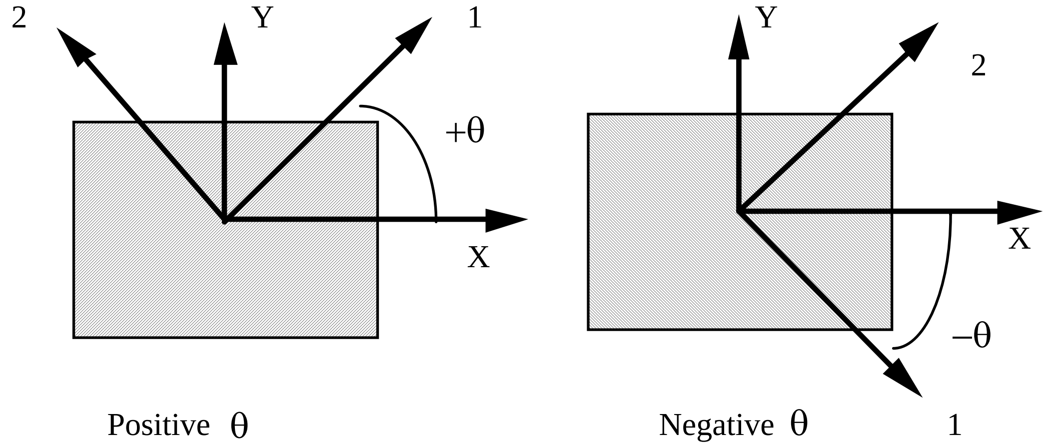


Fig. 1 Sign convention for lamina orientation.

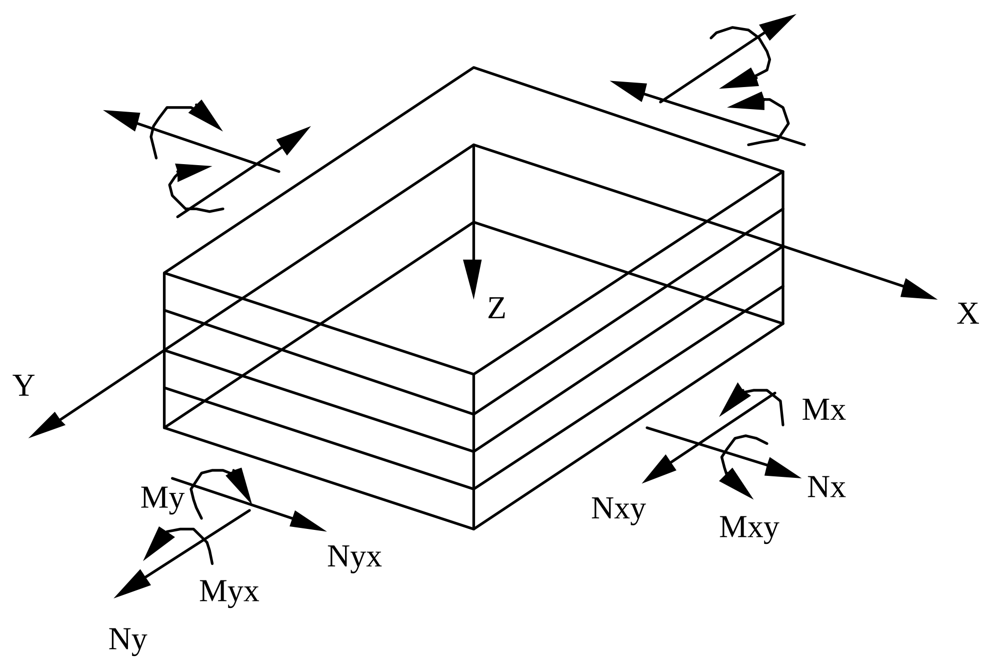


Fig. 2 Coordinate system and stress resultants for laminates plate.

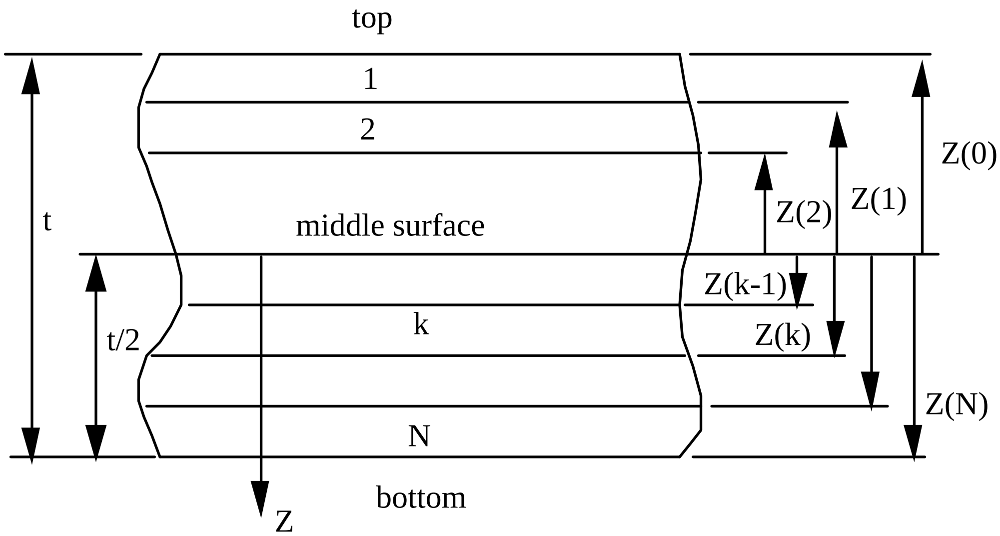


Fig. 3 Laminate plate geometry and ply numbering system.

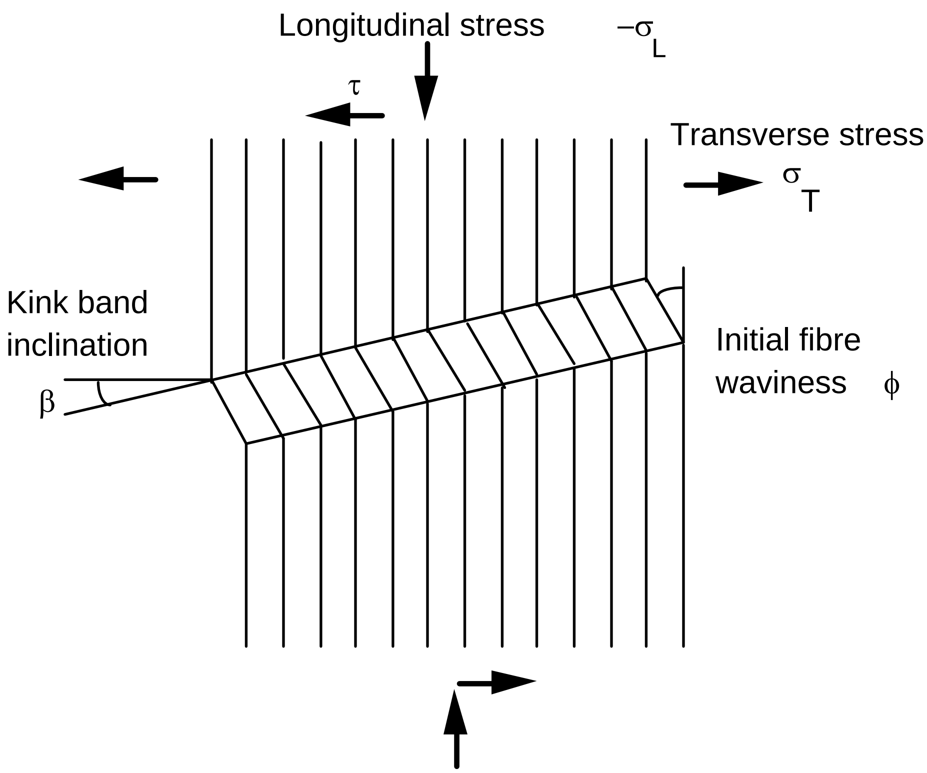


Fig. 4 Infinite band model of microbuckling

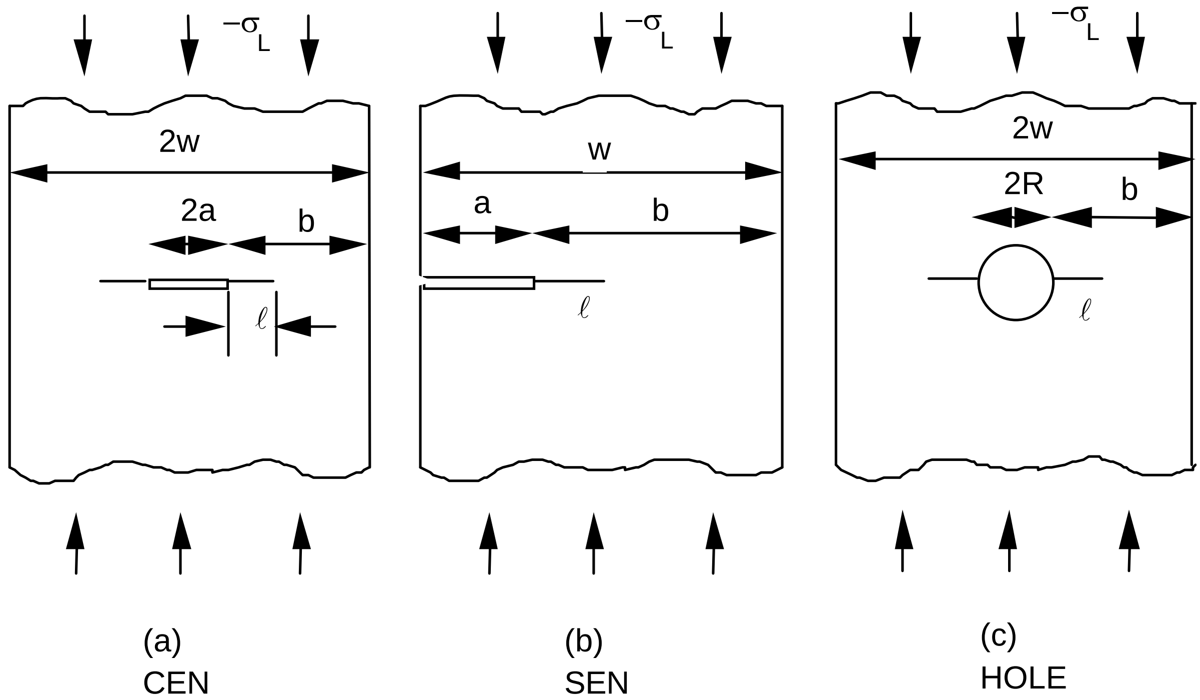


Fig. 5 Geometry of specimens (a) centre notch, (b) single edge notch (c) central hole, and the definitions of a, R, b and w.

4.2 Details for each form

More detailed information on each of the separate forms are grouped together in this section, with associated tutorials.

4.2.1 Laminate elastic properties forms

In this section further features of the Geometry and Elastic Analysis and More Elastic Laminate Properties forms are explained.

Introduction

The laminate geometry and lamina elastic properties are input in this form. Boxes in which to input data have a white background.

First enter the number of plies in the laminate. Click on the appropriate option button if the laminate is unsymmetric. Then enter the elastic properties, thickness and ply angle of the first ply and click **Save Ply Data**. It is assumed by default that all plies have the same properties and ply thickness, although this can subsequently be overwritten by typing in data for each ply and saving the ply data. A composite name and comments are optional and can be entered at any time. The database button accesses a database of lamina elastic properties. Use the ply arrangement box to navigate through the plies, either by clicking on the relevant ply or using cursor keys, to see the saved material properties. When all the ply data have been saved, the laminate properties are calculated using the **Calculate** button. The total thickness box displays the total laminate thickness, based on the ply thicknesses entered.

Materials Data Input Option

There are two choices for defining the elastic data for each ply. The Engineering option requires conventional stiffness moduli for the lamina, such as might be obtained from tests on a unidirectional laminate. The stiffnesses are defined in local co-ordinates (i.e. along and transverse to the fibre direction), so do not change with the ply orientation. The Micromechanics option requires data stiffness about the constituent fibres and matrix, and the fibre volume fraction. These are used to estimate laminate properties using standard equations, as detailed in Appendix B.1.1 and in Gibson, 1994.

Laminate Type

By default when starting CCSM, it is assumed that the laminate is symmetric. For a symmetric option, only data for plies at or above the centreline can be directly input. If an unsymmetric laminate is required, then the appropriate laminate type option should be chosen. Subsequently clicking on the symmetric button will cause all data below the centre line to be overwritten.

Fast input

Once data for the first ply has been filled in, the lamina elastic properties and thickness become the default for subsequent plies. However the ply angle needs to be entered for each ply, and all the data needs to be saved. The **Ply Input** buttons give a fast method of inputting this data. The columns refer to the ply position, relative to the current one, as highlighted in the ply arrangement grid. The row denotes the ply angle. Thus clicking on the **Next** column and the **90** row increases the number in the **Ply Number** data box by one, and puts 90 in the **Ply Angle** data box. Now the data can be saved, either by clicking the **Save Ply Data** button, or by hitting the **Enter** key on the keyboard (the default action for the Enter key, in this case the Save Ply Data button, is highlighted on the screen). The data boxes for ply number and angle can speedily be changed using the Previous, current or Next columns, repeatedly clicking on the appropriate column to increment or decrease the ply number as required, and keeping track of the current ply using the ply arrangement highlighting. When the required ply number and angle have been put in the input data boxes this information, plus the materials and thickness data, can be saved using the **Save Ply Data** button.

Ply Editing

Use the ply arrangement box to navigate through the plies, using either the mouse or cursor keys. The saved material properties for the highlighted ply are displayed. Several plies can be selected for cutting or deleting by dragging with the mouse. Plies are pasted above the selected ply. If strength properties have already been defined for the laminate, these properties are associated with each ply and are cut and paste with the plies. The total number of plies is automatically updated when plies are cut or pasted. The number of plies can also be changed by entering a new number in the Total Number of Plies box.

Change All

The materials properties and thickness of all plies can be changed at once, by entering the new data in the input data box, and then clicking on the appropriate button.

Laminate Elastic Properties

Laminate stiffnesses E_x etc have the normal definitions for an elastic orthotropic laminate, when the laminate is symmetric, see Appendix B1.1 and B.3 for details.

For unsymmetric matrices, due to coupling between different loads, it is not possible to define stiffnesses such as E_x for the laminate in the normal sense. However, the inverse of the appropriate element of the compliance matrix can give an effective stiffness where there is only loading in the relevant direction. These are the values that are quoted. Refer to Appendix B.3 for more details.

E' is an effective elastic modulus, for use in the relation $GE'=K^2$ between the strain energy release rate G and the stress intensity factor K, where mode one loading is considered and a crack runs in the y direction.

More Elastic Laminate Properties Form
In this form further derived elastic properties of the laminate are output. The laminate compliance and stiffness matrix are given in the format detailed in Appendix B.1.1. In addition, the transformed stiffness matrix $\overline{Q}$ for each ply (see Appendix B.1.2) can be viewed by clicking on the corresponding row in the ply arrangement grid. The selected ply is highlighted in this grid.

4.2.2 Elastic Properties Tutorial

Problem: Determine the stiffness and compliance matrices for a [+45/-45/-45/+45] symmetric angle-ply laminate consisting of 0.25 mm thick T300/934 carbon fibre – epoxy laminae. Find out also the transformed lamina stiffness for each ply.

Step 1. Activate CCSM by double clicking the CCSM icon in the Windows environment.

Step 2. Input data into the text boxes as following:

Input text box:	**What you type:**	**Note**
Composite:	T300/934	Optional
Comments:	Tutorial 1.	Optional
Total number of plies	4	
Ply No.	1	
Angle	45	
Thickness	0.25	in mm

E11	Click the **Database** button. Click on **T300/934** in the name list to the left of the form. Click the **Take record as Input** button.	See Section 4.3 for details on changing database entries.
E22	(automatically filled)	
Nu12	(automatically filled)	
G12	(automatically filled)	
	click **Save Ply Data**	Note the change in the ply arrangement grid.
Ply No. 2	click **–45** button in the **Next** column	
Angle	Automatically filled	Uses the same data as for the previous ply
Thickness	Automatically filled	
	Click on **Save Ply Data**	This should be the default action when **Enter** is hit on the keyboard
Ply Nos. 3-4		Automatically filled in for a symmetric laminate

Step 3. Calculate the stiffness by clicking the **Calculate** button. The effective laminate engineering constants are calculated, and the **More Elastic Properties** Button is enabled. To view the stiffness and compliance of the laminate, click on this button.

The stiffness matrix is viewed by default, as:

43.497	29.697	~0.	~0.	~0.	~0.
29.697	43.497	~0.	~0.	~0.	~0.
~0.	~0.	34.322	~0.	~0.	~0.
~0.	~0.	~0.	3.62E-6	2.47E-6	1.89E-6
~0.	~0.	~0.	2.47E-6	3.62E-6	1.89E-6
~0.	~0.	~0.	1.89E-6	1.89E-6	2.86E-6

with units MPa.m, MPa.m^2 and MPa.m^3 for extensional, coupling and bending terms respectively.

Click the **Laminate Compliance** button to view the compliance matrix, as:

0.043	-0.029	~0.	~0.	~0.	~0.
-0.029	0.043`	~0.	~0.	~0.	~0.
~0.	~0.	0.029	~0.	~0.	~0.
~0.	~0.	~0.	5.74E5	-2.96E5	-1.84E5
~0.	~0.	~0.	-2.96E5	5.74E5	-1.84E5
~0.	~0.	~0.	-1.84E5	-1.84E5	5.95E5

where again units are in MPa and m. In this table ~0 represents a very small value (in the order of 1.E-17 ~ 1.E-20) which is due to numerical rounding and should be practically taken as zero. The default output format is to have up to three digits after the decimal point. Very small or large numbers can be displayed in scientific notation. In the Elastic Properties form, choose the "Data format/Scientific4:   +1.2345E00" option to change the output data format.

4.2.3 Deformation analysis

Further details of the deformation analysis form are investigated in this section. In this form the deformation of the laminate, and stresses and strains in the individual plies of the laminate, are calculated. You must enter the load applied to the laminate either in terms of applied line loads and bending moments, or in terms of applied (micro)strains and curvatures. Use the input option box to switch between these. Deformation data for each ply is displayed either in the global (x-y) co-ordinates of the laminate or in terms of the local (1-2) co-ordinates running along the fibre direction for each ply, and in terms of stresses or strains. Again these options are controlled by output option boxes. The ply arrangement table is used to display data for each of the plies, navigating using the mouse or arrow keys.

When changing input options, the equivalent load for the new input option is automatically calculated. This may cause small loads of the order of rounding errors being put into the input boxes – these can safely be ignored.

4.2.4 Deformation Analysis Tutorial

Problem. A [+45/-45/-45/+45] symmetric angle-ply laminate consisting of 0.25 mm thick AS/3501 carbon fibre – epoxy laminae is subjected to a single uniaxial force per unit length Nx=50 MN/mm. Determine the mid-plane strain and the resulting stresses along the x and y axes in each lamina.

Step 1 Elastic properties

The calculation of elastic properties is as described in the previous section and will not be repeated here. Navigate back to the Elastic Form. To change the material properties, use the **Database** button to put the properties for AS/3501 in the material input boxes, and then use the **Elastic Properties** button in the **Change All** tool to give the required laminate. Now re-**Calculate** the elastic properties.

Step 2. Deformation analysis

Click the **Deformation Analysis** button in the *Elastic Properties* form to invoke the deformation analysis form. Input the applied force N_x=0.05 MN/m (converting from mm to m)

Force resultants applied to laminate	N x	N y	N xy	M x	M y	M xy
	0.05					

Now click **Calculate**. The mid-plane strain and the resulting stresses and strains in each ply will be calculated as

Mid-plane strain	E0 x	E0 y	Gamma xy	K x	K y	K xy
	2137	–1485	~0.	~0.	~0.	~0.

By default the stresses in the first ply are also shown at the bottom of the form as:

Location through the thickness t of selected ply	Stress state for ply No. 1 (MPa)		
	Sigma_x	Sigma_y	Tau_xy
top	50.	0.	21.1614
middle	50.	0.	21.1614
bottom	50.	0.	21.1614
Average over laminate	50.	0.	0.

Note that data are shown at the top, middle and bottom of each ply in order to consider the possibility that the stresses are not constant through the thickness. Since the curvatures are zero here, the stresses do not depend on the through-thickness location. The last row shows the laminate stresses, which are the averaged stresses of the corresponding stress components over the whole laminate thickness.

These stresses are in global co-ordinates (i.e. running along the x and y directions of the laminate). To see the ply stresses in local co-ordinates (i.e. running along and transverse to the fibre direction in each ply), click on **Local Coordinates** in the **Output Option** box, to give

Location through the thickness t of selected ply	Stress state for ply No. 1 (MPa)		
	Sigma_1	Sigma_2	Tau_12
top	46.1614	3.8386	-25.0
middle	46.1614	3.8386	-25.0
bottom	46.1614	3.8386	-25.0
Average over laminate			

To display the ply stresses of other plies, navigate through the ply arrangement grid either clicking with a mouse or using cursor key. The current ply is highlighted in this grid.

To view strains in local coordinates, click on **Strain** in the **Output Option** box, to give the microstrains in the first ply,

Location through thickness t of selected ply	Strain state for ply No. 1		
	Epsilon_x	Epsilon_y	Gamma_xy
top	326.1583	326.1583	-3623.1884
middle	326.1583	326.1583	-3623.1884
bottom	326.1583	326.1583	-3623.1884
Average over laminate			

4.2.5 Failure analysis

Further details of the laminate failure analysis form are described in this section. As well as conventional failure criteria, the Budiansky-Fleck-Soutis criterion for compressive failure of unnotched and notched laminates is included. Sections 4.2.6 and 4.2.7 give Tutorial examples for conventional and compressive failure analysis.

4.2.5.1 Conventional Failure Criteria

The prediction of first ply failure due to in-plane stresses is a straightforward application of the appropriate multiaxial lamina strength criterion in combination with the lamina stress analysis from the classical lamination theory. Details of the various failure criteria are described in Appendix B. Since a laminate generally has plies at several orientations, the ultimate load-carrying capacity of the laminate may be higher than the first ply failure. The analysis of subsequent ply failure is not implemented in CCSM.

After choosing your failure criterion, ply data should first be entered in the appropriate data boxes, or using the **Database**. Where the failure strengths are the same for each ply, click the **Save Data - All Plies** button. For a laminate made up of different materials, individual strength data for each ply can be entered using the **Save Data - This Ply** button. The strength of each ply can be examined by clicking and navigating through the ply arrangement table.

You must specify the load pattern – the ratio of all non-zero force components. The absolute values of these components are not important. The failure analysis is performed by clicking **Calculate**. CCSM will find out the proportionality factor for failure, scaling the forces accordingly. The failure loads are given below the input force pattern. The ply grid identifies plies which fail first (i.e. at the failure load)(.

4.2.5.2 BFS Failure Criteria

The model of **unnotched** strength used in CCSM assumes that failure occurs in the 0° plies by plastic microbuckling. The **notched** strength of the composite is then predicted using the Fleck-Soutis model of microbuckle growth, giving the longitudinal (axial) stress or strain of the laminate at failure and the length of the microbuckled region emanating from the end of the notch at this critical peak failure load.

While the analysis predicts the failure due to plastic microbuckling, the user should also be aware that other modes of failure may occur; for example elastic microbuckling, splitting, fibre crushing or matrix failure. This criterion should not be used if off-axis plies could fail first (this could be checked using conventional failure criteria). Refer to the Appendix B for a more detailed explanation, references and comments on the validity of these models.

Unnotched strength

The BFS failure model for unnotched strength is used in a similar way to the conventional failure analysis. The unnotched strength of the laminate can be predicted based on either a micromechanics model or using strength data for each lamina, such as might be obtained from unidirectional tests. The **Strength Input Options** are used to change this. Further details of these strength inputs are given in Appendix B, section B.6.

After choosing the **BFS failure criterion** and the **Input ply strengths** from the **Strength Input Options**, ply strength data should be entered in the appropriate data boxes, either directly or using the **Database**. Where the failure strengths are the same for each ply, click the **Save Data - All Plies** button. For a laminate made up of different materials, individual strength data for each ply can be entered using the **Save Data - This Ply** button. The strength of each ply can be examined by clicking and navigating through the ply arrangement table.

In the BFS criterion, the stress pattern, rather than the force pattern is used as input. Because the BFS criterion is a compressive one, the axial stress "Sigma_L" must be negative. Again the absolute values of these components are not important. A typical input pattern would be Sigma_L=–1, Sigma_T=0, Tau=0. Components of bending are not modelled in the BFS compressive failure theory.

Failure can be output in terms of stresses or (micro)strains using the **Output Option**. The failure analysis is performed by clicking **Calculate**. The laminate strength is given when the stresses in a 0° ply exceed the lamina failure stresses. The laminate unnotched strength is printed on the right, in the middle of the form. The ply grid identifies the plies which fail first.

Notched Strength

The Fleck-Soutis model of notched strength assumes that a microbuckle and associated delamination damage grows from the edge of a sharp notch or hole. The resistance to damage can be modelled using the unnotched strength and a compressive 'fracture toughness'. The laminate unnotched strength can be input directly, after changing the **Strength Input Options**, or predicted as described in the previous section. The fracture toughness K_c is measured using centre-notched coupon specimens. Typical values for CFRP composites are in the range 40 – 50 MPa√m.

Notched strength inputs

The notch geometry type is chosen using the **Geometries** option. Centre or single edge notches, and open, equivalent, countersunk or filled holes are allowed. The equivalent hole model suggested by Soutis and Curtis, 1996 is used to model post-impact compressive strength. The format for defining the lengths of the specimen is changed by clicking on the appropriate **Geometry input** option. The notch size is defined by the notch length or semi length a or the hole radius R depending on the geometry option. The panel size is given by the panel width or semi-width w, or the unnotched ligament length b. These dimensions are illustrated in section 4.1 and further details of the analysis are given in Appendix B.6. The toughness of the laminate can either be input directly in terms of G_c or K_c. This choice is governed by clicking on the apporpriate **Toughness** option.

Prediction of notched strength

Once the notch geometry and toughness have been input, the notched failure analysis can be performed by clicking on the **Calculate** button. Where required, the laminate unnotched strength will be predicted at the same time. The notched strength of the panel is given in the output box at the bottom of the form. For the centre and edge notched geometries and for the open hole, the length of the microbuckle at peak load is also given. This length can be estimated for the equivalent, filled and countersunk holes from the corresponding calculation for an open hole.

4.2.6 Conventional Failure Analysis Tutorial

In this tutorial a conventional failure analysis is worked through.

Problem. A $[90/0/90]_s$ laminate consisting of 0.25 mm thick AS/3501 carbon fibre – epoxy laminae is subjected to a tensile uniaxial loading along the x direction. The ply moduli are E_1=138 GPa, E_2=9 GPa, ν_{12}=0.3, G_{12}=6.9 GPa. Using both the Tsai-Hill and Tsai-Wu criterion, find the loads corresponding to first ply failure. The material failure strength data are as follows: longitudinal tensile strength SL(+)=1448 MPa, longitudinal compressive strength SL(-)=1172 MPa, transverse tensile strength ST(+)=48.3 MPa, transverse compressive strength ST(-)=248 MPa, in-plane shear SLT=62.1 MPa.

Step 1 Elastic properties

Follow the procedures described in Tutorial 1 to input the laminate material properties and geometry. Click **Calculate** to calculate the elastic properties of the laminate.

Step 2 Failure Analysis input

Select the Tsai-Hill **Failure Criterion option**. Either type in the strength data for SL(+), SL(-), ST(+), ST(-) and SLT or use the **Database** button to input the strength data for AS/3501, clicking on this material in the material list, and then clicking on **Take record as Input**. Use the **Save Data – All Plies** button to store this data.

Now enter the force pattern. Remember that the absolute numbers are irrelevant, it is the ratio of forces that matters. In the present case, the only non-zero force is Nx, therefore an appropriate force pattern would be:

Force pattern applied to laminate	N x	N y	N xy	M x	M y	M xy
	1					

Step 3. Now click **Calculate** to perform the failure analysis. The applied force pattern is scaled up or down to give the applied loads at failure in the output grid at the bottom of the form. In this case we have:

Applied loads at failure

N x	N y	N xy	M x	M y	M xy
0.422					

and the failed plies will also be marked by "Fail" in the ply arrangement grid:

No.	Angle	Thick	Fails first?
1	90	0.25	Yes
2	0	0.25	
3	90	0.25	Yes
4	90	0.25	Yes
5	0	0.25	
6	90	0.25	Yes

Therefore, according to the Tsai-Hill criterion first ply failure will occur in the 90° plies with Nx=0.422 MN/m.

Step 4. Now choose the Tsai-Wu failure criterion option, and re-**Calculate** the failure loads as:

Applied loads at failure

N x	N y	N xy	M x	M y	M xy
0.420					

According to the Tsai-Wu criterion, first ply failure will occur at Nx=0.420 MN/m, close to the result obtained using the Tsai-Hill criterion in this case.

4.2.7 BFS Compressive Failure Analysis Tutorial

Details of the Budiansky-Fleck-Soutis failure analysis are investigated in this tutorial.

Problem. A $[(\pm 45/0/90)_3]_s$ laminate consisting of 0.125 mm thick T800/924C carbon fibre – epoxy laminae is subjected to a compressive uniaxial loading along the x direction. Find the compressive failure stress corresponding to 0^o ply failure by the Budiansky-Fleck compressive failure criterion, using the lamina elastic properties from the data base of E_{11}=161 GPa, E_{22}=9.25 GPa, ν_{12}=0.34, G_{12}=6 GPa, and assuming the following material properties: matrix shear strength k=62.35 MPa, initial fibre waviness $\overline{\phi} = 3^o$, microbuckle band inclined angle β=25°.

If a panel of total width 50 mm is made of this laminate with a central hole of radius 2.5 mm find the notched failure strength and corresponding microbuckle length at failure, using the measured value of fracture toughness of the laminate of 42.5 MPa$\sqrt{\text{m}}$. Find the variation in strength with hole size for a fixed panel width of 50 mm.

Step 1. Elastic properties

Follow the procedure described in Tutorial 1 to construct the 24-ply laminate and click on **Calculate** (this file has the correct ply geometry and material properties). Lamina elastic properties for T800/924C are stored in the property database. The laminate should have a stiffness Ex = 61.707 GPa.

Step 2. Predicting the unnotched strength

Go to the **Failure Analysis**, and click the **BFS** compressive option. Check that the Strength Input Option is set to the **Micromechanics model**. The data input appearance changes to a suitable layout for this failure criterion. Now type the following input data:

Matrix shear strength k	62.35
Fibre waviness phi ϕ	3
Kink band inclination angle β	25

Click on the **Save Data - All Plies**. Put a stress Sigma_L equal to -1 in the stress pattern panel at the top of the form then on the **Calculate** button to perform an unnotched strength analysis (ignore the warning re notched strength predictions). The output box half way down the screen on the right shows that the laminate unnotched strength (Sigma_L) is 566.6 MPa. In this case the output is the composite failure **stress**, rather than the force per unit length. This compares with the measured value by Soutis et al (1993) quoted in the references in section B11 of 568 MPa.

Step 3. Predicting the notched strength

To proceed to the calculation of the notched strength, data about the geometry of the notched panel must be input, together with the toughness of the laminate.

First check that the Notched strength by clicking on **Open/equivalent hole** in the **Geometry option**. To input the geometry in a convenient form, select **R and w** from the **Geometry Input Options** box of the notched strength part of the form. Input the radius 2.5 mm and the **semi-**width 25 mm, in the input data boxes, as required for the problem. To input the known fracture toughness, select **Kc given** in the **Toughness Option** box and type in the required value of 42.5 in the **Kc** input data box.

To perform the notched analysis, click the **Calculate** button. The remote compressive stress σ_L and the critical microbuckle length at failure lc are given by:

Remote stress_L	361.928
Critical microbuckle length lc	3.7619

Note that the results show us that the remote stress of 362 MPa is substantially less than the unnotched strength of 568 MPa but that the microbuckle can grow to a length of 3.7 mm, longer than the hole radius in this case, before the maximum load is reached and failure occurs.

4.3 Materials Databases

One of the powerful features of CCSM is its connection to a user-maintainable database. The materials database file for CCSM is called CCSM.MDB; this resides in the CCSM directory. To access the appropriate database click on the **Database** button in the relevant forms.

4.3.1 DataBase (Elastic Property Data)

The two **input options** (Engineering or Micromechanics) are explained in section 4.2.1. There is only a database for the Engineering option. The name list on the left side of the form lists the all the relevant materials data stored in the database. Navigate through the database using a mouse or arrow keys. The current record is highlighted. Click **Take record as input** to fill in the appropriate input boxes in the Elastic Analysis form from this record and to go back to this form.

To add a new data set, click the **Add Record** button first, then type in the material name (which should be unique) and the related properties data. Finally click the **Save Data** button if you want to store this record permanently in the database. If the save fails (for example if you don't have write access to the database file CCSM.mdb) then this will be indicated by a pop-up window. Click the **Delete** button to delete the current record in the database. To **change** an existing data record edit the appropriate cells, then save the data. Once completing editing of the database either take the current record as input to the elastic form or quit without taking the record.

4.3.2 DataBase (Strength):

The DataBase (Strength) form is very similar to the DataBase (Elastic Properties) form described above. The text boxes now require strength data for the material, of course. The database is only available for the conventional failure criteria.

4.3.3 Database for the bridging analysis

The large scale bridging analysis (see Appendix B.8) is the underlying theory used by CCSM to predict the notched composite compressive strength and microbuckle length at failure. To ensure robustness and run time efficiency, CCSM uses the strategy of interpolating from look-up tables rather than carrying out a real-time bridging analysis. Details about the interpolation/extrapolation are explained in sections B.8 and B.9. The look-up tables for the bridging analysis are stored in text files which are accessed by CCSM when needed, in a way which is transparent to the user. The look-up table files are: CENLENG.DAT, CENSTRE.DAT, SENLENG.DAT, SENSTRE.DAT, HOLELENG.DAT, and HOLESTRE.DAT. These files should not be changed or deleted.

Appendix A Theoretical Background

Appendix A describes the theoretical background behind the CCSM. First the classical laminate theory is introduced (A.1-A.4). This theory is used for the calculation of the stresses and strains of the laminated composite. Four conventional failure criteria for the orthotropic lamina (A.5), and the Budiansky-Fleck compressive failure criteria (A.6) are then described, which, when combined with the laminate theory, leads to the failure analysis of a laminate on the ply-by-ply basis (A.7). The bridging analysis, which deals with the failure analysis for the notched laminate or laminate with a hole, is described in section A.8. The numerical implementation of the bridging analysis results is described in A.9. A list of the references applicable to each of the theory sections is given in A.10. In particular, most of the material about the laminate theory comes from Gibson's book *Principles of Composite Material Mechanics*, referenced in A.10.

A.1 Lamina stress-strain relationships

A.1.1 The orthotropic lamina

As shown in Fig. A1, a unidirectional composite lamina has three orthogonal planes of symmetry (i.e. the 12, 23, and 13 planes) and is called an orthotropic material. The coordinate axes in Fig. A1 are referred to as the principal material coordinates since they are associated with the reinforcement directions.

Expressed in terms of 'engineering constants', the stress-strain relationship for a three-dimensional state of stress is:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} \qquad \text{(A.1)}$$

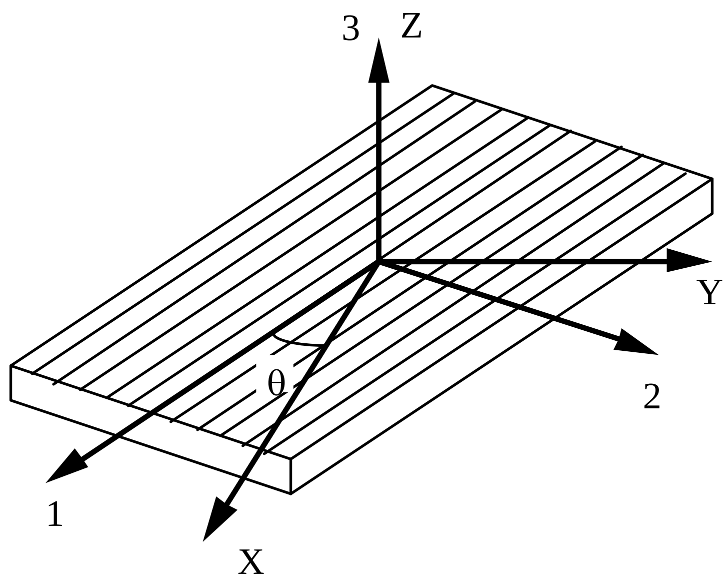


Fig. A1 Orthotropic lamina with principal and non-principal coordinate system

where E_1, E_2 and E_3 are the elastic moduli and $\nu_{ij} = -\varepsilon_j / \varepsilon_i$ are the Poisson ratios. Note that the following relationship holds:

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j} \qquad \text{(no sum on i,j)} \tag{A.2}$$

In practice the lamina is often assumed to be in a simple two-dimensional state of stress. In this case the orthotropic stress-strain relationships in Eq. (A.1) can be simplified to:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{A.3}$$

where the compliances S_{ij} and the engineering constants are related by:

$$S_{11} = \frac{1}{E_1}, \quad S_{22} = \frac{1}{E_2}, \quad S_{12} = S_{21} = -\frac{\nu_{21}}{E_2} = -\frac{\nu_{12}}{E_1}, \quad S_{66} = \frac{1}{G_{12}} \tag{A.4}$$

Thus, there are five non-zero compliances and only four independent compliances for the specially orthotropic lamina. The lamina stresses are given in terms of strains by:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{Bmatrix} \tag{A.5}$$

where the Q_{ij} are the components of the lamina stiffness matrix, which are related to the compliances and the engineering constants by:

$$
\begin{aligned}
Q_{11} &= \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \\
Q_{12} &= -\frac{S_{12}}{S_{11}S_{22} - S_{12}^2} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = Q_{21} \\
Q_{22} &= \frac{S_{11}}{S_{11}S_{22} - S_{12}^2} = \frac{E_2}{1 - \nu_{12}\nu_{21}} \\
Q_{66} &= \frac{1}{S_{66}} = G_{12}
\end{aligned} \tag{A.6}
$$

The lamina properties are calculated using the above formulae with the engineering properties E_{11}, E_{22} etc.

Where the Micromechanics input option is used, the elastic properties of the lamina are calculated from the Elastic modulus, Poisson's ratios of the fibres and matrix, E_f, ν_f, E_m and ν_m, and the fibre volume fraction V_f, as follows. The volume fraction of matrix V_m and the shear moduli of fibres and matrix E_f, and E_m are given by

$$
\begin{aligned}
V_m &= 1 - V_f \\
G_f &= \frac{E_f}{2(1 + \nu_f)} \\
G_m &= \frac{E_m}{2(1 + \nu_m)}
\end{aligned} \tag{A7}
$$

and the elastic moduli and Poisson’s ratio according to the law of mixtures as

$$
\begin{aligned}
E_{11} &= E_f V_f + E_m V_m \\
E_{22} &= \frac{1}{V_f / E_f + V_m / E_m} \\
\nu_{12} &= \nu_f V_f + \nu_m V_m \\
G_{12} &= \frac{1}{V_f / G_f + V_m / G_m}
\end{aligned} \tag{A8}
$$

A.1.2 The generally orthotropic lamina

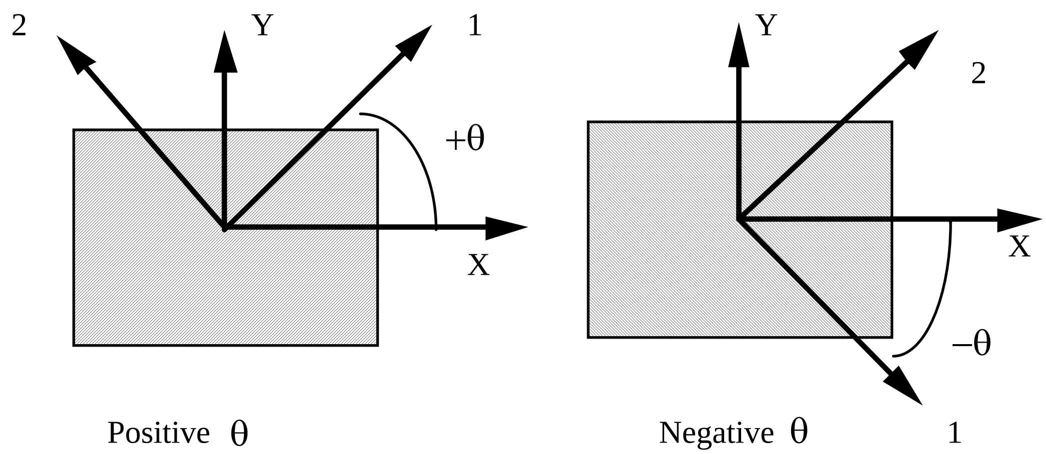


Fig. A2 Sign convention for lamina orientation.

In the analysis of laminates having multiple laminae, it is often necessary to know the stress-strain relationships for the generally orthotropic lamina in non-principal coordinates (or 'off-axis' coordinates) such as x and y in Fig. A1. Consider a lamina which is rotated by an angle θ with respect to the 1-2 axes, as shown in Fig. A2. The sign convention for the lamina orientation angle, θ, is given in Fig. A2. The relationships are found by combining the equations for transformation of stress and strain components from the 12 axes to the xy axes, and the final results are:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{A.9}$$

where the $\overline{Q}_{ij}$ are the components of the transformed lamina stiffness matrix which are defined as follows:

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}C^4 + Q_{22}S^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})S^2C^2 + Q_{12}(C^4 + S^4) \\
\overline{Q}_{22} &= Q_{11}S^4 + Q_{22}C^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})C^3S - (Q_{22} - Q_{12} - 2Q_{66})CS^3 \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})CS^3 - (Q_{22} - Q_{12} - 2Q_{66})C^3S \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})C^2S^2 + Q_{66}(S^4 + C^4)
\end{aligned} \tag{A.10}$$

with $C = \cos\theta$ and $S = \sin\theta$. It should be noted that the number of independent coefficients in (A.10) is still four.

In CCSM, the matrix $[\overline{Q}_{ij}]$ is called Qbar. The Qbar matrix for each ply can be viewed in the

More elastic properties form.

The strains can be expressed in terms of the stresses as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{A.11}$$

where the $\bar{S}_{ij}$ are the components of the transformed lamina compliance matrix which are defined by equations similar to, but not exactly the same form as, Eqs. (A.10) (see the reference listed in A.10 for details).

A.2 Classic laminate theory (with bending)

Fig. A3 defines the coordinate system to be used in the description of laminate theory used in CCSM. The xyz coordinate system is assumed to have its origin on the middle surface of the plate, so that the middle surface lies in the xy plane. The displacements at a point in the x, y, z directions are u, v, and w, respectively. The basic assumptions are:

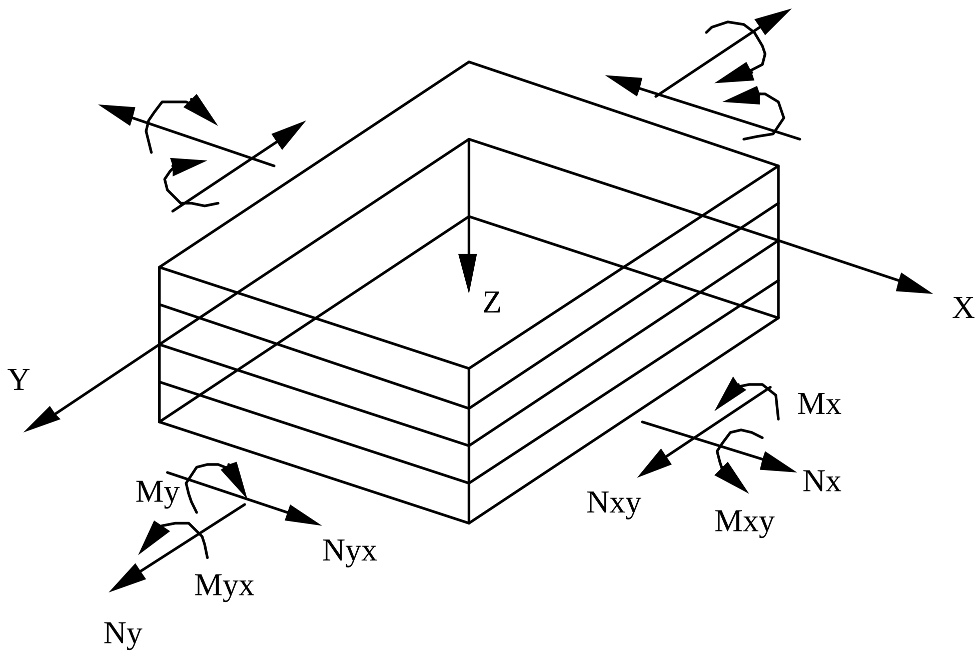


Fig. A3 Coordinate system and stress resultants for laminates plate.

1. The plane consists of orthotropic laminae bonded together, with the principal material axes of the orthotropic laminae oriented along arbitrary directions with respect to the xy axes.
2. The thickness of the plate, t, is small compared to the lengths along the plate edges, a and b.
3. The displacement u, v, and w are small compared with the plate thickness.
4. The in-plane strains ε_x, ε_y, and γ_{xy} are small compared with unity.
5. Transverse shear strains γ_{xz} and γ_{yz} are neglected.
6. Tangential displacements u and v are linear functions of the through-thickness z coordinate.
7. The transverse normal strain ε_x is neglected.
8. Each ply is linear elastic.
9. The plate thickness t is constant.
10. The transverse shear stresses τ_{xz} and τ_{yz} vanish on the plate surfaces defined by $z = \pm t/2$.

Assumption 5 is a result of the assumed state of plane stress in each ply, whereas assumptions 5 and 6 together define the Kirchhoff deformation hypothesis that normals to the middle surface remain straight and normal during deformation. According to assumptions 6 and 7, the displacements can be expressed as:

$$
\begin{aligned}
u &= u^{o}(x,y) + z\,F_1(x,y) \\
v &= v^{o}(x,y) + z\,F_2(x,y) \\
w &= w^{o}(x,y) = w(x,y)
\end{aligned}
\tag{A.12}
$$

where u^{o} and v^{o} are the tangential displacements of the middle surface along the x and y directions, respectively. Due to assumption 7, the transverse displacement at the middle surface, $w^{o}(x,y)$, is the same as the transverse displacement of any point having the same x and y coordinates, so $w^{o}(x,y) = w(x,y)$.

Substituting Eqs. (A.12) in the strain-displacement equations for the transverse shear strain and using assumption 5, we find that

$$
\begin{aligned}
\gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = F_1(x,y) + \frac{\partial w}{\partial x} = 0 \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = F_2(x,y) + \frac{\partial w}{\partial y} = 0
\end{aligned}
\tag{A.13}
$$

and that

$$
\begin{aligned}
F_1(x,y) &= -\frac{\partial w}{\partial x} \\
F_2(x,y) &= -\frac{\partial w}{\partial y}
\end{aligned}
\tag{A.14}
$$

Substituting Eqs. (A.12) and (A.14) in the strain-displacement relations for the in-plane strains, we find that:

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} = \varepsilon_x^{o} + z\kappa_x \\
\varepsilon_y &= \frac{\partial v}{\partial y} = \varepsilon_y^{o} + z\kappa_y \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy}^{o} + z\kappa_{xy}
\end{aligned}
\tag{A.15}
$$

where the strains on the middle surface are:

$$
\varepsilon_x^{o} = \frac{\partial u^{o}}{\partial x} \qquad \varepsilon_y^{o} = \frac{\partial v^{o}}{\partial y} \qquad \gamma_{xy}^{o} = \frac{\partial u^{o}}{\partial y} + \frac{\partial v^{o}}{\partial x} \tag{A.16}
$$

and the curvatures of the middle surface are:

$$
\kappa_x = -\frac{\partial^2 w}{\partial x^2} \qquad \kappa_y = -\frac{\partial^2 w}{\partial y^2} \qquad \kappa_{xy} = -\frac{\partial^2 w}{\partial x \partial y} \tag{A.17}
$$

Here κ_x is the bending curvature associated with bending of the middle surface in the xz plane, κ_y is the bending curvature associated with bending of the middle surface in the yz plane, and

κ_{xy} is the twisting curvature associated with out-of-plane twisting of the middle surface, which lies in the xy plane before deformation.

Since Eqs. (A.15) give the strains at any distance z from the middle surface, the stresses along arbitrary xy axes in the k-th lamina of a laminate may be found by substituting Eqs. (A.13) into the lamina stress-strain relationships from Eqs. (A.9) as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^o + z\kappa_x \\ \varepsilon_y^o + z\kappa_y \\ \gamma_x^o + z\kappa_{xy} \end{Bmatrix} \tag{A.18}$$

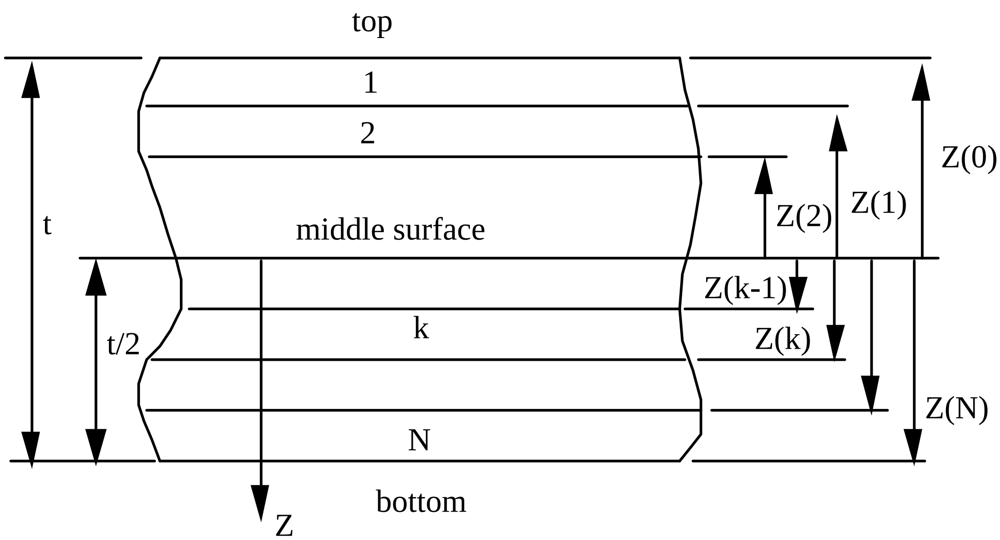


Fig. A4 Laminated plate geometry and ply numbering system.

It is convenient to use forces and moments per unit length in the laminated plate analysis. The magnitude of these forces may be clear where the geometry of the component is relatively simple. Where the component is more complex, CCSM should be used as part of a larger calculation which finds the forces throughout the structure to assess failure of critical sections. The forces and moments per unit length shown in Fig. A3 are referred to as stress resultants.

The force per unit length in the i-th direction, N_i, is given by (i=x, y, z):

$$N_i = \int_{-t/2}^{t/2} \sigma_i dz = \sum_{k=1}^{N} \left\{ \int_{z_{k-1}}^{z_k} (\sigma_i)_k dz \right\} \tag{A.19}$$

and the moment per unit length, M_i, is given by:

$$(\sigma_i)_k M_i = \int_{-t/2}^{t/2} \sigma_i z dz = \sum_{k=1}^{N} \left\{ \int_{z_{k-1}}^{z_k} (\sigma_i)_k z dz \right\} \tag{A.20}$$

where t=laminate thickness

$(\sigma_i)_k$=i-th stress component in the k-th lamina

z_{k-1}=distance from middle surface to inner surface of the k-th lamina

z_k=distance from middle surface to outer surface of the k-th lamina, as shown in Fig. A4

Upon substituting the lamina stress-strain relationships from Eqs. (A.18) in Eqs. (A.19) and (A.20), respectively, the following relationship is obtained:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \qquad \text{(A.21)}$$

where the laminate extensional stiffness, A_{ij}, are given by:

$$A_{ij} = \int_{-t/2}^{t/2} (\bar{Q}_{ij})_k \, dz = \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k - z_{k-1}) \qquad \text{(A.22)}$$

the laminate coupling stiffnesses are given by:

$$B_{ij} = \int_{-t/2}^{t/2} (\bar{Q}_{ij})_k \, z dz = \frac{1}{2} \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2) \qquad \text{(A.23)}$$

and the laminate bending stiffness are given by:

$$D_{ij} = \int_{-t/2}^{t/2} (\bar{Q}_{ij})_k \, z^2 dz = \frac{1}{3} \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \qquad \text{(A.24)}$$

with the subscripts i,j=1,2, or 6.

Equation (A.21) may be written in partitioned form as

$$\begin{Bmatrix} N \\ \cdots \\ M \end{Bmatrix} = \begin{bmatrix} A & \vdots & B \\ \cdots & \cdot & \cdots \\ B & \vdots & D \end{bmatrix} \begin{Bmatrix} \varepsilon^o \\ \cdots \\ \kappa \end{Bmatrix} \qquad \text{(A.25)}$$

for convenience.

In CCSM, the stiffness matrix in (A.21) for the laminate is shown in the *More Elastic properties* form, by clicking the Laminate Stiffness button.

A.3 Laminate compliances

The inverse of the stiffness matrix (A.21) or (A.25) gives the compliances of the laminate:

$$\begin{Bmatrix} \varepsilon^o \\ \cdots \\ \kappa \end{Bmatrix} = \begin{bmatrix} A & \vdots & B \\ \cdots & . & \cdots \\ B & \vdots & D \end{bmatrix}^{-1} \begin{Bmatrix} N \\ \cdots \\ M \end{Bmatrix} = [S] \begin{Bmatrix} N \\ \cdots \\ M \end{Bmatrix} \tag{A.26}$$

The above relation is used to calculate the lamina stresses and strains associated with prescribed laminate loads.

In CCSM, the compliance matrix in (A.26) for laminate is shown in the *More Elastic properties* form by clicking the **Laminate Compliance** button.

For a balanced symmetric laminate, the B sub-matrix is zero, indicating that there is no coupling between in-plane and bending terms. The upper-left quarter of equation A.26 is now in the same form as the equivalent equation A3 for an orthotropic lamina. Hence it is appropriate to define **laminate** engineering constants E_x, E_y, G_{xy} ν_{xy} and ν_{yx}, using the compliance matrix S in equation A.26, with equivalent expressions to those given in equation A.4, and converting from forces to stresses via the thickness t;

$$E_x = \frac{1}{tS_{11}}, \quad E_y = \frac{1}{tS_{22}}, \quad \frac{E_x}{\nu_{xy}} = \frac{E_y}{\nu_{yx}} = -\frac{1}{tS_{12}} = -\frac{1}{tS_{21}}, \quad G_{xy} = \frac{1}{tS_{33}} \tag{A.27}$$

For an unsymmetric matrix, there is coupling between the in-plane and bending terms, so that this decomposition is no longer valid. However, the compliance matrix S can still give an effective stiffness where there is only loading in the relevant direction, for example $1/tS_{11}$ gives an effective value for E_x where there is only an N_x load term. These are the values that are quoted.

A.4 Determination of lamina stresses and strains

Calculation of lamina stresses and strains is a straightforward procedure. Making use of Eqs. (A.9), the stresses in the k-th lamina, when written in abbreviated matrix notation, are given by:

$$\{\sigma\}_k = [\overline{Q}]_k \left(\{\varepsilon^o\} + z\{\kappa\}\right) \tag{A.28}$$

where $\{\varepsilon^{o}\}$ and $\{\kappa\}$ are the midplane strains and the curvatures respectively. Here the subscript k refers to the k-th ply.

A.5 Conventional lamina failure criteria

Five lamina strengths are relevant in the lamina failure analysis. They are:

$S_L^{(+)}$: the longitudinal tensile strength

$S_L^{(-)}$: the longitudinal compressive strength

$S_T^{(+)}$: the transverse tensile strength

$S_T^{(-)}$: the transverse tensile strength

S_{LT} : the in-plane shear strength

A.5.1 Maximum stress

This criterion predicts failure when any principal material axis stress component exceeds the corresponding strength, i.e. failure occurs whenever one of the following holds:

$$\sigma_1 \le -S_L^{(-)}$$

or $$\sigma_1 \ge S_L^{(+)}$$

or $$\sigma_2 \le -S_T^{(-)}$$

or $$\sigma_2 \ge S_T^{(+)}$$

or $$|\tau_{12}| \ge S_{LT} \tag{A.29}$$

The maximum stress in each ply is used in equation A.29 and in corresponding equations for the other conventional failure criteria (or maximum strain where appropriate). This maximum stress or strain need not be at the centre of the ply.

A.5.2 Maximum strain

This criterion predicts failure when any principal material axis strain component exceeds the corresponding ultimate strain, i.e. failure occurs whenever one of the following holds:

$$\varepsilon_1 \le -e_L^{(-)}$$

or $$\varepsilon_1 \ge e_L^{(+)}$$

or $$\varepsilon_2 \le -e_T^{(-)}$$

or $$\varepsilon_2 \ge e_T^{(+)}$$

or $$|\gamma_{12}| \ge e_{LT} \tag{A.30}$$

Assuming linear elastic behaviour, the ultimate strains can be calculated by:

$$e_L^{(+)} = \frac{S_L^{(+)}}{E_1}, \quad e_L^{(-)} = \frac{S_L^{(-)}}{E_1}, \quad e_T^{(+)} = \frac{S_T^{(+)}}{E_2}, \quad e_T^{(-)} = \frac{S_T^{(-)}}{E_2}, \quad e_{LT} = \frac{S_{LT}}{G_{12}} \quad \text{(A.30)}$$

A.5.3 Tsai-Hill

The Tsai-Hill criterion states that failure occurs when the following relation satisfies:

$$\frac{\sigma_1^2}{S_L^2} - \frac{\sigma_1\sigma_2}{S_L^2} + \frac{\sigma_2^2}{S_T^2} + \frac{\tau_{12}^2}{S_{LT}^2} \geq 1 \quad \text{(A.32)}$$

The Tsai-Hill criterion assumes that the material has equal strengths in tension and compression. When tensile and compressive strengths are different, modification can be made by using the appropriate value of S_L and S_T for the corresponding stress components. For example, if σ_1 is positive and σ_2 is negative, the values of $S_L^{(+)}$ and $S_T^{(-)}$ would be used in (A.32).

A.5.4 Tsai-Wu

The Tsai-Wu criterion states that failure occurs when the following relation satisfies:

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 \geq 1 \quad \text{(A.33)}$$

where

$$F_{11} = \frac{1}{S_L^{(+)}S_L^{(-)}}, \qquad F_{22} = \frac{1}{S_T^{(+)}S_T^{(-)}}, \qquad F_1 = \frac{1}{S_L^{(+)}} - \frac{1}{S_L^{(-)}},$$

$$F_2 = \frac{1}{S_T^{(+)}} - \frac{1}{S_T^{(-)}}, \quad F_{66} = \frac{1}{S_{LT}^2}, \quad F_{12} = -\frac{(F_{11}F_{22})^{1/2}}{2}$$

A.6 BFS compressive failure criterion

The unnotched strength of the laminate can be predicted based on either a micromechanics model or using strength data for each lamina.

The micromechanics model is based on the Budiansky-Fleck compressive failure analysis. In this section we describe its application to a 0° **lamina**. Section A.7 explains how this information is used to find the **laminate** unnotched strength. This criterion assumes that the unnotched compressive strength of the lamina is governed by *imperfection-sensitive plastic microbuckling* with the imperfection in the form of fibre misalignment. Consider microbuckling from an infinite band of uniform fibre misalignment ϕ as shown in Fig. A5 in a **unidirectional** material. The composite is subjected to a remote axial stress σ_L parallel to the fibre direction, an in-plane transverse stress σ_T and an in-plane longitudinal shear stress τ. The infinite band is inclined with respect to the fibre axes such that the normal to the band is rotated by an angle β with respect to the remote fibre direction, as shown in Fig. A5.

For the case where the composite displays a rigid-perfectly plastic in-plane response the compressive strength of the lamina is given from Slaughter et al (1993) by

$$\left|\sigma_L\right| = \frac{\alpha k - \tau - \sigma_T \tan\beta}{\phi} \tag{A.34}$$

where k is the shear yield strength of the composite and $\alpha = \sqrt{1 + R^2 \tan^2 \beta}$. The constant R is taken as 1.5 (Jelf and Fleck 1994).

It may be helpful to review the various input parameters to this model. The dominating influences are the matrix shear strength and the fibre waviness. The matrix strength k can be estimated from the yield strength of the unidirectional composite in shear. Typical values for polymer matrices are in the range 30 - 100 MPa. The estimate of fibre waviness ϕ is not trivial for real composites and is the subject of current research. However, a typical value for ϕ would be in the range of 1-3° for standard polymer matrix composites. Research suggests that a misaligned region of more than say 30-50 fibres will be needed to affect the strength; a few misaligned fibres would not make a difference. Experimentally it is found that microbuckles propagate across the specimen at a fixed orientation to the direction of the 0 degree fibres. Typically this β angle of propagation of the microbuckle lies between 20 and 30°. Sutcliffe and Fleck (1997) give theoretical predictions of the propagation direction. In practice there appears to be little variation in the value of β in composite materials, and the analysis is not sensitive to this parameter, so that the chosen value of β is not critical. The review paper by Fleck (1997) gives further details of these microbuckling models and their inputs.

Although the effect of shear stresses have been verified by Jelf and Fleck, the effect of transverse stresses has not been, and this part of the model should be used with caution. While the analysis predicts the failure due to plastic microbuckling, the user should also be aware that other modes of failure may occur; for example elastic microbuckling, splitting, fibre crushing or matrix failure. A check on the elastic microbuckling limit (equal to G_{12}) is carried out by CCSM, but the user should be careful in checking that these other modes do not occur. Refer to the work of Jelf and Fleck (1992) for further information. The B-F model does not allow for plate bending components, and the stresses at the mid-plane of each ply are used in equation A.34. See Shu and Fleck, 1997, for a discussion of strain hardening effects, which are not modelled here.

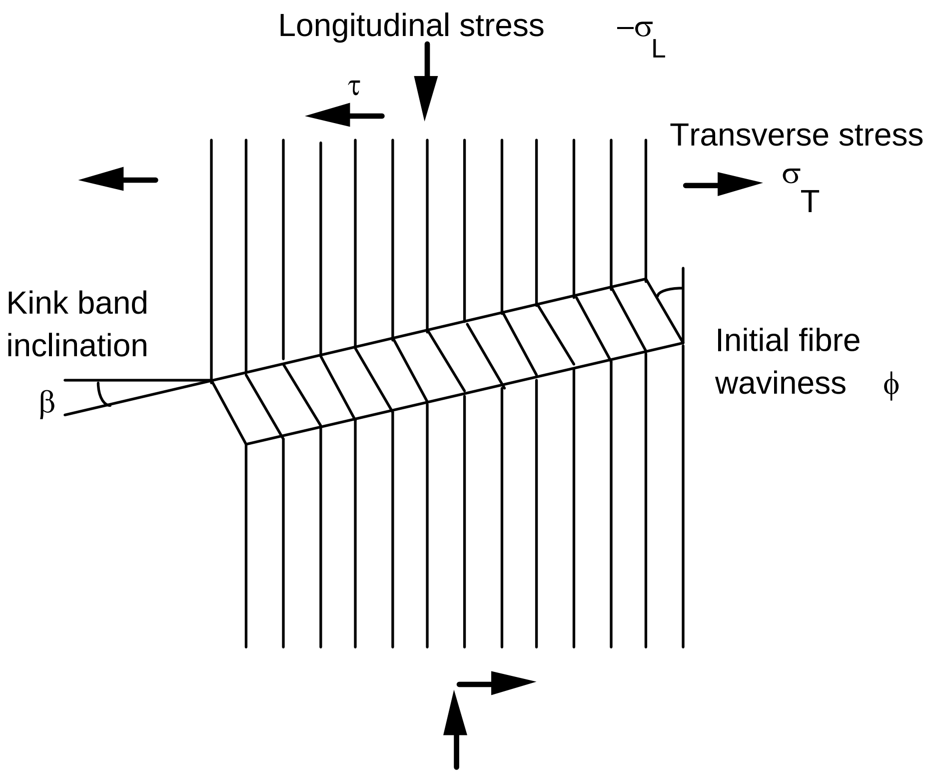


Fig. A5   Infinite band model of microbuckling

The **Input Ply Strengths** option provides an alternative to the micromechanics model for the prediction of unnotched laminate strength. This model, as suggested by Soutis and Edge, 1997, requires as inputs the lamina compressive and shear strengths S_L^-, and S_{LT}, such as might be obtained from unidirectional tests. The strength of each ply under combined stresses σ_1, σ_2 and τ_{12} is estimated by linear interpolation, neglecting the transverse stress σ_2, so that failure occurs when

$$\frac{-\sigma_1}{s_L^-} + \frac{|\tau_{12}|}{s_{LT}} \geq 1 \tag{A.35}$$

Only failure of the 0° plies is considered for both the Micromechanical and Ply Strength failure criteria.

A.7 Laminate failure analysis

The previous two sections discuss several multiaxial failure criteria for estimating the strength of individual lamina under in-plane stresses. Such criteria can be used on a ply-by-ply basis for a laminate to determine which ply fails first under in-plane loads. Interlaminar stresses leading to delamination are not considered in CCSM. For the BFS compressive analysis it is assumed that failure will occur first in the microbuckling 0° plies. The failure of these 0° lamina is examined using equation A.34, taking the stresses in the 0° plies arising from the laminate loading. The laminate strength is given when the stresses on the laminate generates the lamina failure stresses in a 0° ply.

The prediction of first ply failure due to in-plane stresses is a straightforward application of the appropriate multiaxial lamina strength criterion in combination with the lamina stress analysis from the classical lamination theory. Since a laminate generally has plies at several orientations, the ultimate load-carrying capacity of the laminate may be higher than the first ply failure. The analysis of subsequent ply failure is not implemented in CCSM.

A.8 Failure of notched laminates

In this section we describe in detail the Fleck-Soutis model used to estimate the notched strength of laminates. The model assumes that a microbuckle and associated delamination damage grow from the edge of a sharp notch or hole. The resistance to damage can be modelled using the unnotched strength and a compressive 'fracture toughness' of the **laminate**. The unnotched strength may either be input directly, or the Budiansky-Fleck or ply strength failure criteria described in section A.6 can be used to predict this. The laminate compressive fracture toughness K_c, which is derived from the failure load for a panel with a sharp notch, may either be taken directly from experiments, or it may be predicted from data on laminates of the same material as that under consideration, but with other lay-ups. Typical values for CFRP composites are in the range 40 - 50 MPa√m.

For countersunk and filled holes, a simple knockdown or strengthening factors of 0.85 and 1.21 respectively are applied, as suggested by Soutis and Edge, 1997. These factors are based on limited data, and should be used only as guidelines. The equivalent hole is used to model post-impact compressive strength, as described by Soutis and Curtis, 1996. They observe that, when a panel is loaded in uniaxial compression, damage propagates from regions of delamination (e.g. arising from impacts) in a similar way to that observed with open holes. Further details, in particular describing how to estimate the diameter of this equivalent hole, are given in this reference. The length of the microbuckle at peak load for the equivalent, filled and countersunk holes may be estimated from the equivalent calculation for an open hole.

Carpet plots, which give the effect of ply mix on notched strength for a symmetric laminate, are calculated from the data in the failure form for strength, toughness and geometry type. It is assumed that the composite is composed only of 0, 90 and ±45° plies. Several curves, each corresponding to a constant proportion of 0° plies, are produced, showing the variation of strength with the change in ±45° plies. The proportion of 90° plies is found using the fact that the total proportion of plies sums to 100%. The axial compressive stress is plotted on the graph. The stress pattern in the failure form is used for these plots. The other stress components will be in the proportions specified by the load pattern specified here and shown in the Plots form. The unnotched strength for each ply mix is predicted based on the lamina strength option and geometry type chosen in the failure form. As for the failure analysis, only failure in the 0 degree plies is considered. The Soutis-Fleck bridging analysis is used to calculate the notched strength. The toughness is assumed to be independent of ply mix, and is taken from the Failure Analysis form. The percentage of 0 degree plies is varied from 10 to 90%. Elastic and strength data for the first ply are used throughout the laminate. As with the standard BFS compressive failure analysis, care should be taken in interpreting results at extremes of ply mix, as the BFS compressive failure criterion may be inappropriate. Soutis and Edge, 1997, give further details of carpet plot calculations.

In the rest of this section, the details of the method used to calculate the notched strength are outlined. The method involves crack bridging models, which have had notable success in the prediction of damage from notches in engineering materials under remote tension. They have also been used to estimate the development of microbuckling from a hole in a composite under compression. The usual strategy is to concentrate the inelastic deformation associated with plasticity, cracking, microbuckling and so on within a crack and to assume some form of traction-displacement bridging law across the crack faces. As a simple example, in Dugdale's analysis of plastic yielding in metals from the root of a notch the bridging normal traction across the crack faces is assumed to equal the tensile yield strength of the solid. The material response elsewhere in the cracked specimen or structure is assumed to be linear elastic. Other sophisticated examples adopt more realistic, consequently more complicated, bridging laws. One such example is the bridging law derived from an infinite band calculation of fibre microbuckling: the crack traction T versus crack overlap 2v law is assumed to equal the remote stress σ_r versus extra remote displacement Δv response of an infinite microbuckle band under remote compression (the extra remote displacement is the end shortening minus the contribution to shortening associated with elastic axial straining).

Details about the bridging analysis, which is incorporated in the current version of CCSM, can be found in Sutcliffe and Fleck's (1993) paper. Only a brief outline is given below. Although the effect of compressive loading has been well tested experimentally (e.g. Soutis et al, 1993, Soutis and Edge, 1997), recent work by Fleck, Liu and Shu, 1998, suggest that the predicted effects of transverse and shear loading should be treated with caution for notched geometries.

Considered the geometries shown in Fig. A6. The stress intensity factor K_σ due to a distribution of normal compressive stresses $\sigma(x)$ along the microbuckle at the tip of a microbuckle of length ℓ is given by the integral:

$$K_\sigma = \int_0^\ell \sigma(x) m(\ell, x) dx \tag{A.36}$$

where the weight function m(ℓ, x) can be conveniently found from the point load solution given by published results for a crack emanating from a central hole and for a single edge notch and a centre notched panel, which are the three geometries dealt with in the current version of CCSM.

Using the weight function method, the crack closing displacement $u_\sigma(x)$ on the microbuckle face (where u is half the relative displacement of the two faces of the microbuckle) due to $\sigma(x)$ is given by:

$$u_\sigma(x) = -\frac{1}{E'} \int_o^\ell m(\ell', x') d\ell \int_0^{\ell'} \sigma(x') m(x, x') dx' \tag{A.37}$$

where the orthotropic equivalent elastic modulus E' is defined by:

$$\frac{1}{E'} = \left(\frac{1}{2E_{xx}E_{yy}} \right)^{1/2} \left[\left(\frac{E_{yy}}{E_{xx}} \right)^{1/2} - \nu_{yx} + \frac{E_{yy}}{2G_{xy}} \right]^{1/2} \tag{A.38}$$

The weight function method is also used to find the stress intensity factor at the microbuckle tip K_{rm} and the displacements across the microbuckle $u_r(x)$ due to the remote stress for the notched panels, using Bueckner's rule, briefly, that is (A.36) and (A.37) are used again to calculated K_{rm} and $u_r(x)$ but the traction on the crack faces is that along the crack line for a specimen containing no crack. For specimens with a central hole Newman, 1982, gives the stress intensity factor at the end of the microbuckle and the displacements due to a remote uniform stress.

The net stress intensity factor at the microbuckle tip K_m and the displacements along the microbuckle $u(x)$ are then:

$$
\begin{aligned}
&K_m = K_\sigma + K_{rm} \\
&u(x) = u_\sigma(x) + u_r(x)
\end{aligned}
\tag{A.39}
$$

The compressive traction $\sigma(x)$ across the microbuckle is related to the closing displacement $u(x)$ across the microbuckle by a functional relationship. For the current situation, a simple linear softening law is used:

$$
\frac{\sigma}{\sigma_{un}} = \begin{cases} 1 - \dfrac{u}{u_c} & \text{for } 0 \le u \le u_c \\ 0 & \text{for } u > u_c \end{cases}
\tag{A.40}
$$

where u_c is a critical microbuckle overlap displacement. The appropriate value of u_c is found from the measured toughness G_{IC}, using the assumed variation of microbuckle load with displacement,

$$
G_{IC} = \int_0^{u_c} 2\sigma(u)du = \sigma_{un}u_c
\tag{A.41}
$$

To solve for the remote stress, the cohesive zone is divided into N elements of equal length. The traction distribution within the cohesive zone is taken to be piecewise linear, with triangular distributions of crack face traction with peak value centred on the i-th node. The integrals in equations (A.36) and (A.37) are calculated numerically at the node points for triangular distributions of traction of unit magnitude at the N node points. N-1 simultaneous equations concerning the non-zero displacements at the 1 to N-1 discretised nodes (except the node at the tip where the displacement is zero) are then obtained. An additional equation is that the stress intensity factor at the tip of the microbuckle K_m due to the remote stress and microbuckle traction is zero:

$$
K_m = 0
$$

therefore the system governing equations are fully decided and solved numerically.

The calculations give a relationship between the microbuckle length and the applied remote stress. The failure stress is given by the maximum applied stress ; the calculations also give the corresponding critical microbuckle length l_C at the maximum stress. CCSM presents results at failure in terms of the remote stress σ_L, the average stress over the unnotched ligament b and the critical microbuckle length.

The bridging analysis used in CCSM does not strictly apply with applied shear and transverse loads. However an estimate of the failure load under these conditions can be made by using the notched results for purely compressive loading, with the compressive strength knocked down by an amount given by a failure analysis for unnotched laminates. This is the approach used in CCSM.

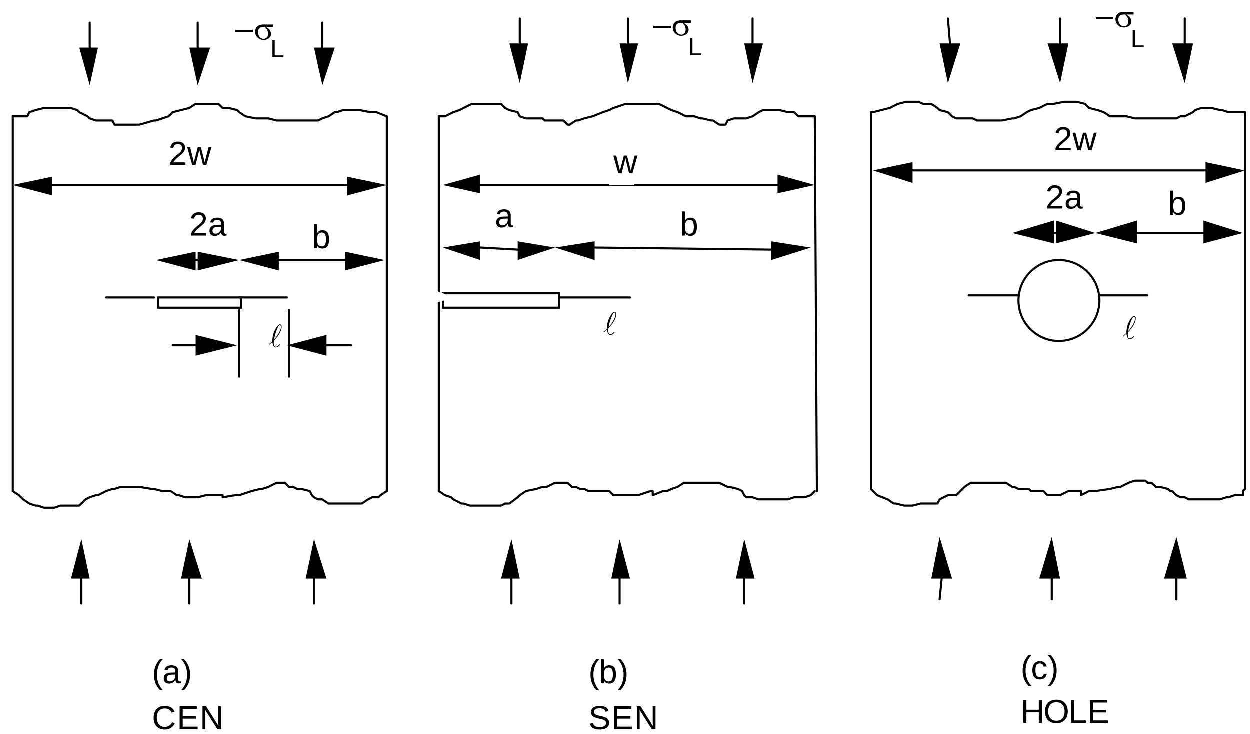


Fig. A6 Geometry of specimens (a) centre notch, (b) single edge notch (c) central hole, and the definitions of a, b, and w.

In general the bridging analysis can be described by the following functional relationship:

$$\frac{\ell_c}{r_p} = f\left(\frac{a}{r_p}, \frac{b}{r_p}\right) \quad \text{(A.42)}$$

$$\frac{\sigma_r}{r_p} = g\left(\frac{a}{r_p}, \frac{b}{r_p}\right) \quad \text{(A.43)}$$

where $r_p = \frac{E' G_{IC}}{\sigma_{un}^2} = \frac{K_{IC}^2}{\sigma_{un}^2}$ is the bridging length scale, σ_{un} is the unnotched strength of the laminate, ℓ_c is the critical microbuckle length, σ_L is the remote stress of the notched specimen at failure, a is the half crack length (CEN), or hole radius (HOLE), or total crack length (SEN) and b is the ligament length.

Equations (A.42) and (A.43) are the basis for the interpolation and extrapolation for the bridging analysis in CCSM described in the following section.

As noted in Sutcliffe and Fleck's (1993) paper, the weight functions used are for isotropic materials; they show that errors in using them for the orthotropic materials are not too large.

A.9 Interpolation and extrapolation of the bridging analysis data

A number of bridging analyses have been carried out for three types of commonly used specimens, a centre-cracked panel (CEN), a single edge cracked panel (SEN) and a plate with a hole (HOLE), and for a range of crack and ligament sizes (Sutcliffe and Fleck, 1993). Results are expressed as functions of a/r_p and b/r_p (where a is the half crack size for CEN, crack size for SEN, and hole radius for HOLE, b is the ligament, $r_p = (K_c/\sigma_{un})^2$ is a characteristic length scale, K_c is the toughness and σ_{un} the unnotched laminate strength). Dimensionless results for the variation of the notched strength and critical microbuckle length with a/r_p and b/r_p between 10^{-3} and 10^{+3} for the three notched geometries[1] are stored as look-up tables. Use of a look-up table is more robust and faster than performing the calculation in real time. The look-up table files are: CENLENG.DAT, CENSTRE.DAT, SENLENG.DAT, SENSTRE.DAT, HOLELENG.DAT, and HOLESTRE.DAT. Data in these files should not be changed. If the geometry to be analysed is located at one of the calculated data grid points, the results will be retrieved directly from the look-up table. If the geometry to be calculated lies in-between or outside the grid points an interpolation or extrapolation scheme is used to calculate results from data at the nearest grid points. The rest of this section explains in detail the procedures used.

[1] Data from the bridging analysis for the hole are only available for b/w<0.25 and (a+lc)/w<0.75. Values for geometries outside this range have been estimated, by comparison with the asymptotic values and the centre notch calculations.

A.9.1 For points inside the macro grid of the look-up table

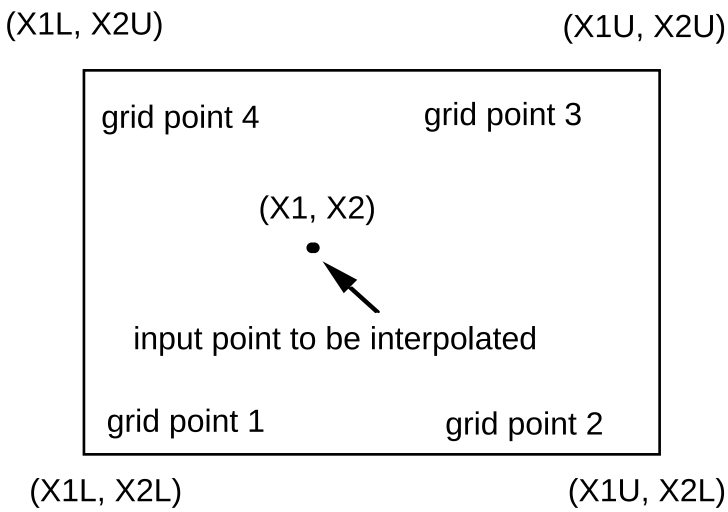


Fig. A7 Schematic figure of a unit grid for the bicubic interpolation

A bicubic interpolation is adopted in CCSM. To do a bicubic interpolation within a unit grid square [(X_{1L}, X_{2L}), (X_{1U}, X_{2L}), (X_{1U}, X_{2U}), (X_{1L}, X_{2U})] (see Fig. A7). The function Y and the derivatives $\frac{\partial Y}{\partial X_1}$, $\frac{\partial Y}{\partial X_2}$, $\frac{\partial^2 Y}{\partial X_1 \partial X_2}$ at each of the four corners of the square are given in the look-up table. The numbering of the corner points starts at the lower left corner, and counts anti-clock-wise.

There are two steps to the interpolation, described in detail in the reference listed in A.10: first obtain the sixteen quantities C_{ij} based on the given functions and its derivatives at the four grid corners, i, j=1, ..., 4. Next, substitute the C's into the following bicubic interpolation formula for the point of interest, or the input point, (X_1, X_2):

$$Y(X_1, X_2) = \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} T^{i-1} U^{j-1} \qquad \text{(A.44)}$$

where $T = \frac{X_1 - X_{1L}}{X_{1U} - X_{1L}}$, $U = \frac{X_2 - X_{2L}}{X_{2U} - X_{2L}}$. (A.45)

As discussed in the previous section, the bridging analysis can be generally described by the following functional relationship:

$$\frac{\ell_c}{r_p} = f(\frac{a}{r_p}, \frac{b}{r_p}) \tag{A.46}$$

$$\frac{\sigma_r}{r_p} = g(\frac{a}{r_p}, \frac{b}{r_p}) \tag{A.47}$$

The bicubic interpolation formula (A.44) is valid for any grid square. Since the precision of the interpolation will be better for smaller grid sizes, the area of interpolation is divided into smaller grids and (A.44) is applied to each of these sub-divided grids. To avoid confusion in the description, we refer to the sub-divided grid as the 'unit grid', while the total area is the 'macro grid'.

The smoother the function over the grid, the better is the precision of the bicubic interpolation. When the functions (A.46) and (A.47) are plotted with log axes for a / r_p and b / r_p, they are smooth. Therefore the interpolation is carried out in the $\log(a / r_p)$ - $\log(b / r_p)$ space rather than as a function of a / r_p and b / r_p.

For the three types of specimens, the macro grid covers the following unit grid points: $\log(a/r_p)$ and $\log(b/r_p)$=–3 to 3.

Function values of (A.46) and (A.47) (f and g), together with their corresponding first order partial derivatives and cross derivative, i.e. $\frac{\partial f}{\partial \log(a / r_p)}$, $\frac{\partial f}{\partial \log(b / r_p)}$, $\frac{\partial^2 f}{\partial \log(a / r_p) \partial \log(b / r_p)}$ and $\frac{\partial g}{\partial \log(a / r_p)}$, $\frac{\partial g}{\partial \log(b / r_p)}$, $\frac{\partial^2 g}{\partial \log(a / r_p) \partial \log(b / r_p)}$ at the unit grid points have been calculated and tabulated in data files. These data files are:

CENLENG.DAT (f and its derivatives for CEN specimen)
CENSTRE.DAT (g and its derivatives for CEN specimen)
SENLENG.DAT (f and its derivatives for SEN specimen)
SENSTRE.DAT (g and its derivatives for SEN specimen)
HOLELENG.DAT (f and its derivatives for HOLE specimen)
HOLESTRE.DAT (g and its derivatives for HOLE specimen)

In the CCSM bridging analysis if the input data point $\log(a / r_p) - \log(b / r_p)$ is within the lower and upper bounds of the macro grid, the program will find the right unit grid square which encloses the input data point ($a / r_p, b / r_p$) and the function values for the remote stress σ_r and the critical microbuckle length ℓ_c at the input point will be interpolated from the unit grid using the bicubic interpolation (A.44).

A.9.2 For points outside the macro grid of the look-up table

When the input data point falls outside the macro grid, an extrapolation technique has to be used. CCSM relies on the asymptotic nature of the solution in these areas to obtain the correct values. Details are given in the references quoted. Three distinct regions are noted:

Region (1): when $a / r_p , b / r_p$ are both very large, loading is in the so-called LEFM situation where the remote applied stress intensity factor at failure will be equal to the fracture toughness of the composite, i.e. $K_r = K_c$, and the microbuckle length is found to be: $\ell_c = 0.75 r_p$ for both CEN and SEN specimens. The remote stress can then be backed out from the K_r value by the LEFM formula. For the HOLE specimen, failure is determined by the stress concentration factor, i.e. $K_t \sigma_r = \sigma_{un}$, while the ℓ_c tends to 0 and becomes unstable.

The limits, in summary are:

when $a / r_p >> 1$ and $b / r_p >> 1$

CEN:
$$K_r \to K_c$$
$$\ell_c \to 0.75 r_p$$
$$\sigma_r = K_r / (Y \sqrt{\pi a}) \tag{A.48}$$

SEN:
$$K_r \to K_c$$
$$\ell_c \to 0.75 r_p$$
$$\sigma_r = K_r / (Y \sqrt{\pi a}) \tag{A.49}$$

HOLE:
$$\ell_c \to 0 \text{ (unstable)}$$
$$\sigma_r = \sigma_{un} / K_t \quad (K_t \text{ is the stress concentration factor}) \tag{A.50}$$

Region (2): when a/r_p is small and b/r_p large, the situation is summarised below:

when $\quad a/r_p \ll 1$ and $b/r_p \gg 1$

CEN: $\quad \ell_c \to 0.45 r_p$

$$\sigma_r \to \sigma_{un} b/w$$

$$K_r = Y\sigma_r\sqrt{\pi a} \qquad \text{(A.51)}$$

SEN: $\quad \ell_c \to 0.4 r_p$

$$\sigma_r \to \sigma_{un}(b/w)^2$$

$$K_r = Y\sigma_r\sqrt{\pi a} \qquad \text{(A.52)}$$

HOLE: $\quad \ell_c \to 0.55 r_p$

$$\sigma_r \to \sigma_{un} b/w \qquad \text{(A.53)}$$

Region (3): when b/r_p is very small (regardless of the value of a/r_p), we find:

when $\quad b/r_p \ll 1$

CEN: $\quad \ell_c \to b$

$$\sigma_r \to \sigma_{un} b/w$$

$$K_r = Y\sigma_r\sqrt{\pi a} \qquad \text{(A.54)}$$

SEN: $\quad \ell_c \to b$

$$\sigma_r \to \sigma_{un}(b/w)^2$$

$$K_r = Y\sigma_r\sqrt{\pi a} \qquad \text{(A.55)}$$

HOLE: $\quad \ell_c \to b$

$$\sigma_r \to \sigma_{un} b/w \qquad \text{(A.56)}$$

In the actual calculation, the above asymptotic solutions are assumed to be reached when $\log(a/r_p))$ or $\log(b/r_p) = \pm 3$.

While there are asymptotic solutions in certain areas outside the interpolation grid, in other areas there is a transition between two types of asymptotic behaviour. The way in which CCSM handles data in this area is explained by way of example.

Consider a centre notched specimen with small a/r_p; there is a transition from LEFM with failure determined by a stress intensity factor criterion, to failure given by the unnotched strength, as b/r_p changes from being very large to very small. In the extrapolation scheme, it is assumed that the form of the transition is not dependent on the value of a/r_p in this area, as long as a/r_p remains small, and varies in the following way with b/r_p

$$\frac{\sigma_r}{\sigma_{un}} = g_1(B) = \lambda(B)\left(w_1 \lim{}_1(B) + w_2 \lim{}_2(B)\right)$$

$$\text{where } B = \left(\log_{10}\left(\frac{b}{r_p}\right)\right), \quad w_1 = 0.5 + 0.5\cos\left(\frac{(B+3)\pi}{6}\right) \text{ and } w_1 + w_2 = 1 \qquad \text{(A.57)}$$

where the functions $\lim_1$ and $\lim_2$ are the expressions for the stress or stress intensity factor K which apply asymptotically at either end of the boundary of the calculated grid with b/r_p small and large respectively (in this case they would be given by equations A.54 and A.51). λ, which is a function of b/r_p , expresses the form of the transition in behaviour between these extremes, and is found from the data computed at the edge of the grid (along $a/r_p = 0.001$ in this case).

Similar expressions are used along any other boundaries where there is a transition in limiting behaviour.

B.10 References

References for each of the above theory sections are given below.

Sections B.1 to B.5, B.7 and B.10:	R. F. Gibson, *Principles of Composite Material Mechanics*, McGraw-Hill, 1994
Section B.6:	B. Budiansky and N. A. Fleck, Compressive failure of fibre composites, *J. Mech. Phys. Solids*, **41**(1), pp.183-211, 1993
Sections B.8 and B.9	M. P. F. Sutcliffe and N. A. Fleck, Effect of geometry upon compressive failure of notched composites, *Int. J. Fracture*, **59**, pp.115-132, 1993
	B. N. Cox and D. B. Marshall, *Acta Metall. et Mater.*, **39**, 59-589, 1991
	J. C. Newman, in *Proceedings of the AGARD Conference on the Behaviour of Short Cracks in Airframe Components*, France, 1982
Section B.9	W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flannery, *Numerical Recipes in FORTRAN*, Second Edition, Cambridge University Press, 1994

Other references and further reading

B. Budiansky and N. A. Fleck, (1994) Compressive kinking of fiber composites: a topical review, *Appl. Mech. Rev.*, **47**(6), part 2, S246-S250W.

B. Budiansky and N. A. Fleck, (1993) Compressive failure of fibre composites, *J. Mech. Phys. Solids*, **41**(1), pp.183-211

B. N. Cox and D. B. Marshall, (1991) *Acta Metall. et Mater.*, **39**, 59-589

N. A. Fleck, L. Deng and B. Budiansky, (1995) Prediction of kind band width in fiber composites, *J. Applied Mech* **62**, pp329-337

NA Fleck (1997) "Compressive Failure of Fibre Composites" *Advances in Applied Mechanics*, Vol. 34, Academic Press, ed. JW Hutchinson and TY Wu, pp 43-118

NA Fleck, D Liu and JY Shu (1998) Microbuckle initiation from a hole and from the free edge of a fibre composite' submitted to Int J Solids and Structures

R. F. Gibson, (1994) *Principles of Composite Material Mechanics*, McGraw-Hill,

P. M. Jelf and N. A. Fleck (1992) Compression failure mechanisms in unidirectional composites. *Journal of Composite Materials* **26** 2706-2726

P. M. Jelf and N. A. Fleck (1994) The failure of composite tubes due to combined compression and torsion. *J. Materials Science* **29** 3080-3086

J. C. Newman, (1982) in *Proceedings of the AGARD Conference on the Behaviour of Short Cracks in Airframe Components*, France

W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flannery, (1994) *Numerical Recipes in FORTRAN*, Second Edition, Cambridge University Press

JY Shu and NA Fleck (1997) Microbuckle initiation in fibre composites under multiaxial loading', Proc Roy Soc A453, 2063-2083

S. Slaughter, N. A. Fleck and B. Budiansky, (1992) Microbuckling of fibre composites: the role of multi-axial loading and creep, *J. Eng. Mater. Tech.*, **115**(5), pp.308-313.

C. Soutis, N.A. Fleck and P.A. Smith (1991) Failure prediction technique for compression loaded carbon fibre-epoxy laminate with open holes, *J Comp. Materials*, **25**, 1476-1498.

C Soutis, PT Curtis and NA Fleck (1993) "Compressive failure of notched carbon fibre composites" Proc R. Soc. London A, Vol. 440, pp 241-256

C Soutis and PT Curtis (1996) "Prediction of the post-impact compressive strength of CFRP laminated composites", *Composites Science and Technology*, **56**, pp 677-684.

C. Soutis and EC Edge (1997), A method for the production of carpet plots for notched compression strength of carbon fibre reinforced plastic multidirectional laminates, *Proc Instn Mech Engrs*, **211** (G), pp 251-261

M. P. F. Sutcliffe and N. A. Fleck, (1993), Effect of geometry upon compressive failure of notched composites, *Int. J. Fracture*, **59**, pp.115-132

MPF Sutcliffe and NA Fleck (1997) "Microbuckle propagation in fibre composites" Acta Metallurgica and Materialia, , pp921-932