# Engineering Part IIB

## Module 4C2 – Designing with Composites

## Design Practicalities
## Design methodology and laminate selection

**Professor Michael Sutcliffe**

### Aims of this course

To describe the design of laminates using **engineering models** of laminates.

To introduce other **practicalities** governing the design of composite components.

To describe a **case study** which illustrates the design process.

### Selected bibliography

Quinn, J.A. Composites - Design Manual. A very practical guide. James Quinn Associates Ltd, 415 Woolton Rd, Liverpool, L25 4SY, JH 53

Shenoi, RA and Wellicome, JF. Composite materials in maritime structures. Lots of specific details and interesting background, Cambridge, TW 65

Bader, MG. Materials selection, preliminary design and sizing for composite laminates. Journal paper: Composites Part A, **27A**, (1996), pp65-70. Details of using carpet plots for practical laminate design.

Mayer, RM. Design with reinforced plastics. Good on practical details with some interesting case studies. The Design Council (Available from Chapman Hall) JG 206

# Course contents

**Introduction/Manufacturing - Michael Sutcliffe**

**Stiffness – Athina Markaki**

**Strength - Michael Sutcliffe**

**Design Practicalities - Michael Sutcliffe**

## Contents



**Coursework - Michael Sutcliffe**

Design of a composite component

**Why use composites?**

*Mechanical properties*

- High specific strength and stiffness

*Functional properties*

- Corrosion resistance. Resins are good here. GFRP pipes or tanks.
- Wear resistance. Can be good, for example carbon-carbon brake liners.
- Low radar signature. Intrinsic to GFRP and CFRP. Eurofighter, stealth.
- Low thermal expansion. Cooling tower drive shafts.
- Low heat transmission. Phenolic resin composites in rocket nozzle components.
- Good electrical insulation. GFRP, but CFRP is conductive along the fibres.
- Low sound transmission. Due to high mechanical damping. Helicopter shells.
- Textured surfaces. Smooth hulls, textured climbing walls, rough boat decks.
- Self-colouring. Use dye in the polymer or in a surface gel coat.

*Operational*

- Low cost. Especially with glass fibres and net shape manufacture of parts with a complex shape.
- Low maintenance.

*Why not to use composites?*

- High cost. High material cost of CFRP and high design costs.
- Limited design data. Lack of confidence.
- Properties not established until manufacture.
- Complicated load paths and failure mechanisms. e.g. use of stiffeners.
- Low damage tolerance. Low toughness. Sudden failure, e.g. bike forks.
- Difficult to repair.
- Difficult damage evaluation
- Lack of codes and standards. Some codes for boats and pressure vessels.
- Poor recycling. Especially of thermoset resins.
- Poor public acceptance. Poor for critical structures; good for sports goods.
- Fire, toxicity and smoke. Most important in aircraft design. Phenolic resin used in Airbus floor panels.

## 1 An outline design method for composites.

A design will include geometrical, mechanical and materials information. In this course we will focus on the mechanical and materials aspects, since it in these areas that composite design differs from conventional materials design.

**Conceptual Design**

Many candidate materials
Many candidate manufacturing routes

↓ *One or two materials and lay-ups*

**Preliminary Design**

More specific material properties
Approximate lay-up

↓ *Single material and lay-up*

**Detailed Design**

Single material properties
Lay-up optimisation
Coupon and sub-component testing

↓

**Prototyping**

Component testing

↓

**Production**

## 1.1 Conceptual design

In principle the range of available materials and lay-ups is very wide for a composite designer. In the conceptual design stage inappropriate manufacturing routes and materials are eliminated. One or two materials and very approximate lay-ups are chosen to take to the next design stage.

- Specify design objectives and constraints
  - Mechanical
    - e.g. Strength, stiffness, loading spectrum, geometrical constraints.
  - Functional
    - e.g. Environmental hazards.
  - Operational
    - e.g. Cost, available manufacturing routes

- Use formal methods, for example merit indices (e.g. $E/\rho$), or 'experience' to select the best materials, approximate lay-ups and manufacturing routes.

  Identify the modes of failure, in particular finding which are critical.

  Use back-of-the envelope calculations and approximate materials data.

  Include cost and product value (e.g. a price premium for a lighter component).

- It may be possible to draw on existing comparable designs to short-circuit the conceptual design stage.

- A material substitution, e.g. composite for metal, will require a re-think of the whole component.

- See earlier in the course for more details of manufacturing routes.

## 1.2 Preliminary design

This stage aims to use better materials data and more sophisticated calculations to identify a single material and to refine the lay-up. The thickness of the laminate should be minimised to reduce the weight and cost.

## 1.3 Detailed design

A preliminary design has been chosen. This should be checked and optimised using laminate analysis. More sophisticated failure criteria may be used, and more complicated loading patterns can be considered.

- Use coupon tests to characterise the material and lay-up mechanical behaviour.
- Perform global and local stress analyses, using industry codes, finite element analysis and theoretical models.
- Use any existing codes and standards.
- Mechanical design of features (e.g. lugs, attachment points).
- Detailed design of joints.
- Testing of sub-components (e.g. stiffeners, joints, lugs).
- Detailed tooling design.
- Quality assurance.
- Refined cost estimates.

**Prototyping**

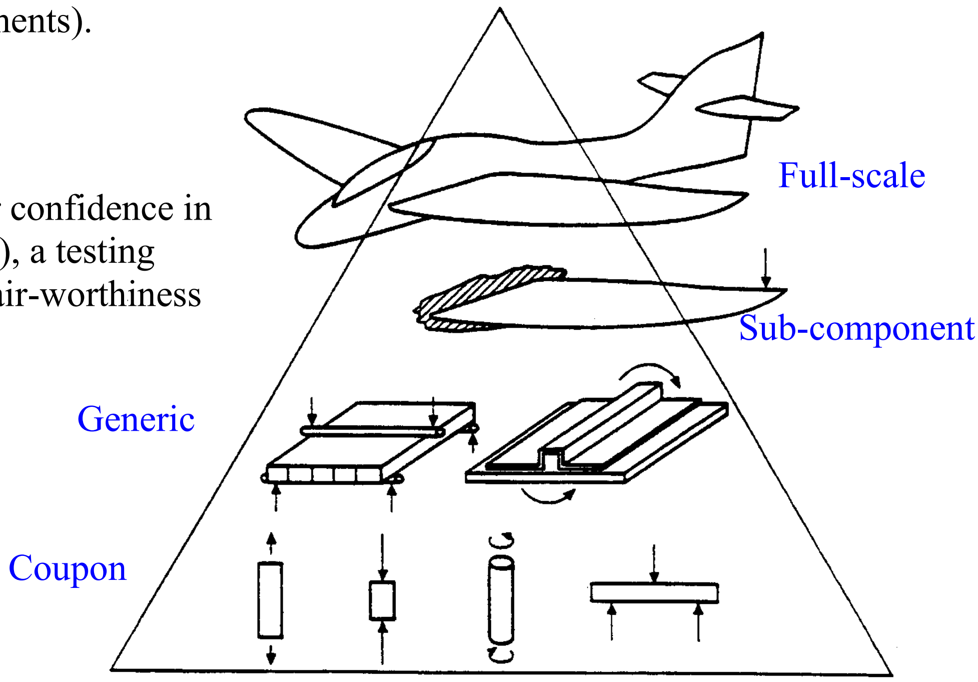
- Proving details of design and manufacture.
- Testing larger sections and the final component.

**Production**

- Feedback to the design team.
- Quality control.
- Incremental improvement (but difficult for aircraft due to certification requirements).

## 1.4 The testing pyramid

As well as being needed for confidence in the design (product liability), a testing pyramid is required by the air-worthiness authorities for certification.



## 2 Laminate design calculations

The above section gives an outline methodology for composite design. In this section a practical implementation for designing laminates is described, as proposed by Bader (see the bibliography), based on experience. These are deliberately simple to ensure that a sensible laminate is identified. The rules can be broken with the help of more sophisticated design calculations.

### 2.1 Laminate design - rules of thumb

1. Simplify

2. Use a balanced symmetric lay-up. Prevents coupling between direct, shear and bending loads and the associated deflections. This coupling may be a problem, particularly for thin laminates, and in any case it is difficult to quantify. For tubes the need for a symmetrical lay-up is less important.

3. Use the same **material** in all plies. Hybrid laminates, with more than one type of material - e.g. CFRP and GFRP, are common, but this makes the analysis more complicated.

4. Use either a ±θ lay-up (for filament winding) or a sequence of 0, ±45 and 90 plies (for pre-preg lay-up).

5. Have at least 10% total of off-axis ±45 or 90° plies. This prevents splitting and avoids the problems of high Poisson's ratio for a simple ±45° array. Ideally both 45 or 90° orientations should be included, but with 45° plies needing to be used in sets of four, this may be inappropriate for thin laminates. The difficulty of optimising with thin laminates is driving manufacturers to make thinner laminae.

A woven 0/90° cloth (e.g. kevlar or GFRP) is commonly incorporated to provide a tough surface layer, providing impact resistance. This can be incorporated into a laminate analysis using a knock-down factor for strength and stiffness. Just one outer layer will make the laminate unsymmetric, although this may not be such a great problem for a thick laminate.

### 2.2 Laminate building blocks

The above rules give us only a limited number of ply building blocks, which can be repeated to build up a thick laminate.

| **Lay-up** | **Minimum number of plies** | **Remarks** |
|---|---|---|
| $[\pm\theta]_s$ | 4 | Angle ply |
| $[0_m, \pm 45]_s$ | 6 | Angle ply |
| $[0_m, 90]_s$ | 3 | Cross ply. Use a single central 90 ply for 3 plies |
| $[0_m, 90_n, \pm 45]_s$ | 8 | $[0,90,\pm45]$s is quasi-isotropic |

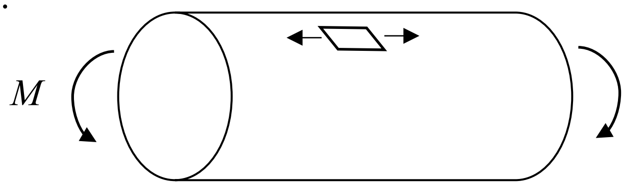
## 2.3 Conceptual design

For this and subsequent laminate calculations we assume that the loads in the structure can be expressed in terms of line loads $N_x$, $N_y$ and $N_{xy}$ and bending moments $M_x$, $M_y$ and $M_{xy}$ on the laminate at critical points in the structure.

**Local plate loads** are given from structural calculations (e.g. FE or hand calculations).

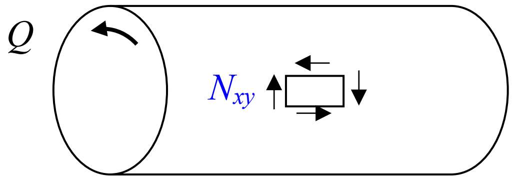
Beam with local wall thickness $t$ in bending:

$$\frac{\sigma}{y} = \frac{M}{I} \text{ and } N_x = \sigma t$$




Tube of radius $R$ and thickness $t$ in torsion:

$$N_{xy} = \frac{Q}{2\pi R^2}$$




**Local plate curvatures and strains** are related to the local loads via the composite lay up and material properties using laminate plate theory:

$$\begin{Bmatrix} N \\ \cdots \\ M \end{Bmatrix} = \begin{bmatrix} A & \vdots & B \\ \cdots & \cdot & \cdots \\ B & \vdots & D \end{bmatrix} \begin{Bmatrix} \varepsilon^o \\ \cdots \\ \kappa \end{Bmatrix}$$

If $N$ and $M$ are known, this matrix can be inverted to solve for the strains and curvatures.

Conversely strains and curvatures might be imposed, from which the line loads can be found.

Or in some cases a mixture of loads and strains/curvatures might be applied, with a mixture of loads and strains/curvatures as the unknowns.

In any case the strains in the different plies through the thickness vary with the ply location $z$ through the laminate thickness in a linear manner,

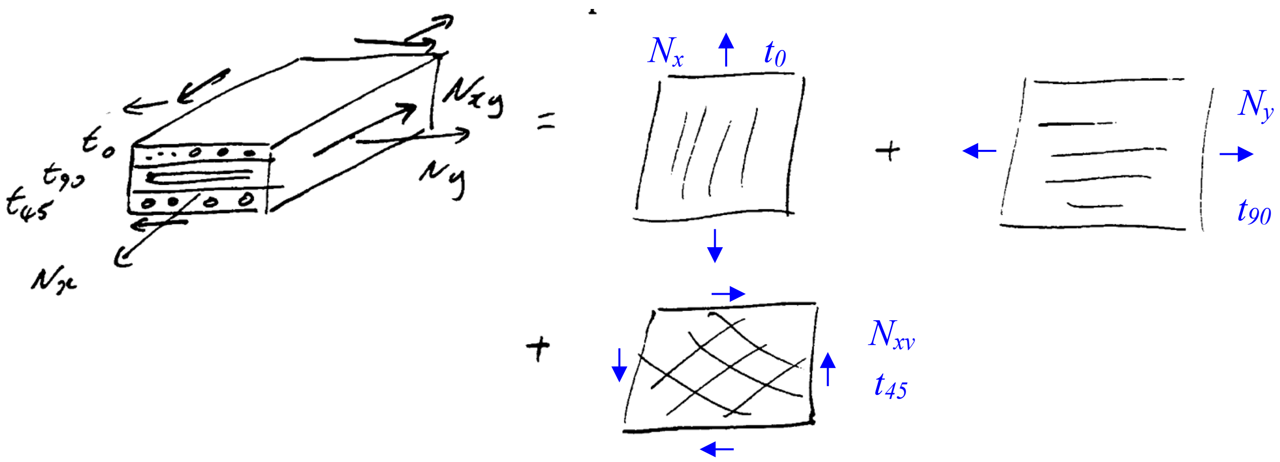
i.e. $\varepsilon_x = \varepsilon_x^o + z\kappa_x$ etc.

where $\varepsilon_x^o$ is the mid-ply strain (i.e. the laminate extensional strain)

### 2.3.1 Conceptual design – stiffness

In the conceptual stage of laminate design it is helpful to think of the plies running in the 0, 90 and ±45 directions as acting independently, each taking the load in the axial, transverse or shear directions.

The thicknesses of the plies in the 0, 90 and ±45 directions are $t_0$, $t_{90}$ and $t_{45}$ respectively and the total thickness of the laminate is $t$.




The axial stiffness $E_x$ of the laminate is given from the stiffness of the 0° plies:

$$tE_x = t_0 E_1 \Rightarrow E_x = E_1 \frac{t_0}{t}$$

$E_1$ is the axial stiffness of the unidirectional laminate.

.

Similarly the transverse $E_y$ and the shear stiffness $G_{xy}$ of the laminate can be estimated by the contribution of the appropriate ply component

$$E_y = t_{90} E_1 / t$$
$$G_{xy} = t_{45} \mathrm{G} / t$$

The shear stiffness $G$ of the ±45 plies is *approximately* equal to $E_1/4$ (and **not** $G_{12}$). Typical values are given in Table 1 below.

### Stiffness constraints

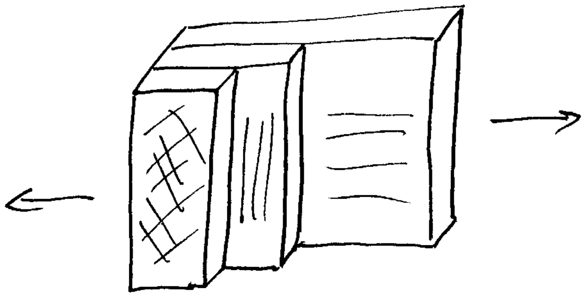
These are imposed using the laminate stiffness in the appropriate direction.

For example, if the end deflection $\delta$ is specified for a thin-walled tube of radius $R$ cantilevered with an end load $W$ ($I = \pi R^3 t$), this will impose a constraint on the product of the stiffness $E_x$ along the axis of the tube, and the wall thickness $t$ of the tube, e.g.

$$\delta \le \frac{WL^3}{3EI} = \frac{WL^3}{3E_x \pi R^3 t} \Rightarrow E_x t \ge \frac{WL^3}{3\pi\delta R^3}$$

### 2.3.2 Conceptual design – strength (strain allowables)

**Strength constraints** on the laminate can most easily be modelled at the conceptual design stage using strain allowables in the laminate.



This approach has the advantage that the stresses in individual plies do not need to be calculated.

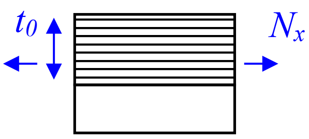
For this approach it is assumed that 0, 45 and 90° plies are present in the laminate, to prevent splitting failure. The strain is the same in each ply of the laminate, and failure in the **laminate** occurs at the lowest failure strain for any of the ply orientations. In other words, the failure strain (the 'strain allowable') is determined by the weakest failure mechanism in the laminate.

This will normally be due to failure in the ply transverse to the load direction (with the exception of axial compression of Kevlar).

The strain allowable for loads $N_x$ and $N_y$ depends on whether the load is tensile or compressive. Shear loading $N_{xy}$ can also be modelled by a shear strain allowable. Failure may occur either by shear of the 0 and 90° plies or by the induced transverse tension in one of the 45° directions. Suggested strain allowables are given in Table 1 below.

The required thickness of the laminate is determined by putting the strains $\varepsilon$ in each direction equal to the strain allowables $e^+$, $e^-$ and $e_{\mathrm{LT}}$, using compressive, tensile or shear strain allowables as appropriate.

For **conceptual** design, Poisson's strains can be ignored and the strain in any direction is given by the laminate stiffness model presented above, assuming that the stiffness in a given direction is only due to plies in the corresponding direction.




$$\varepsilon_x = \frac{N_x}{tE_x} = \frac{N_x}{t_0 E_1}. \quad \text{Putting } \varepsilon_x = e \;\Rightarrow\; t_0 = \frac{N_x}{E_1 e}$$

$t_0 E_1 = tE_x$

$$\text{Similarly} \quad t_{90} = \frac{N_y}{E_1 e}, \quad t_{45} = \frac{N_{xy}}{Ge_{LT}}$$

The total laminate thickness is the sum of the thicknesses of the individual ply components:

$$t = t_0 + t_{90} + t_{45}$$

## 2.4 Preliminary design

### 2.4.1 Preliminary design – stiffness (carpet plots)

For **preliminary** design, **carpet plots** can be used to find the laminate stiffness and Poisson's ratio.

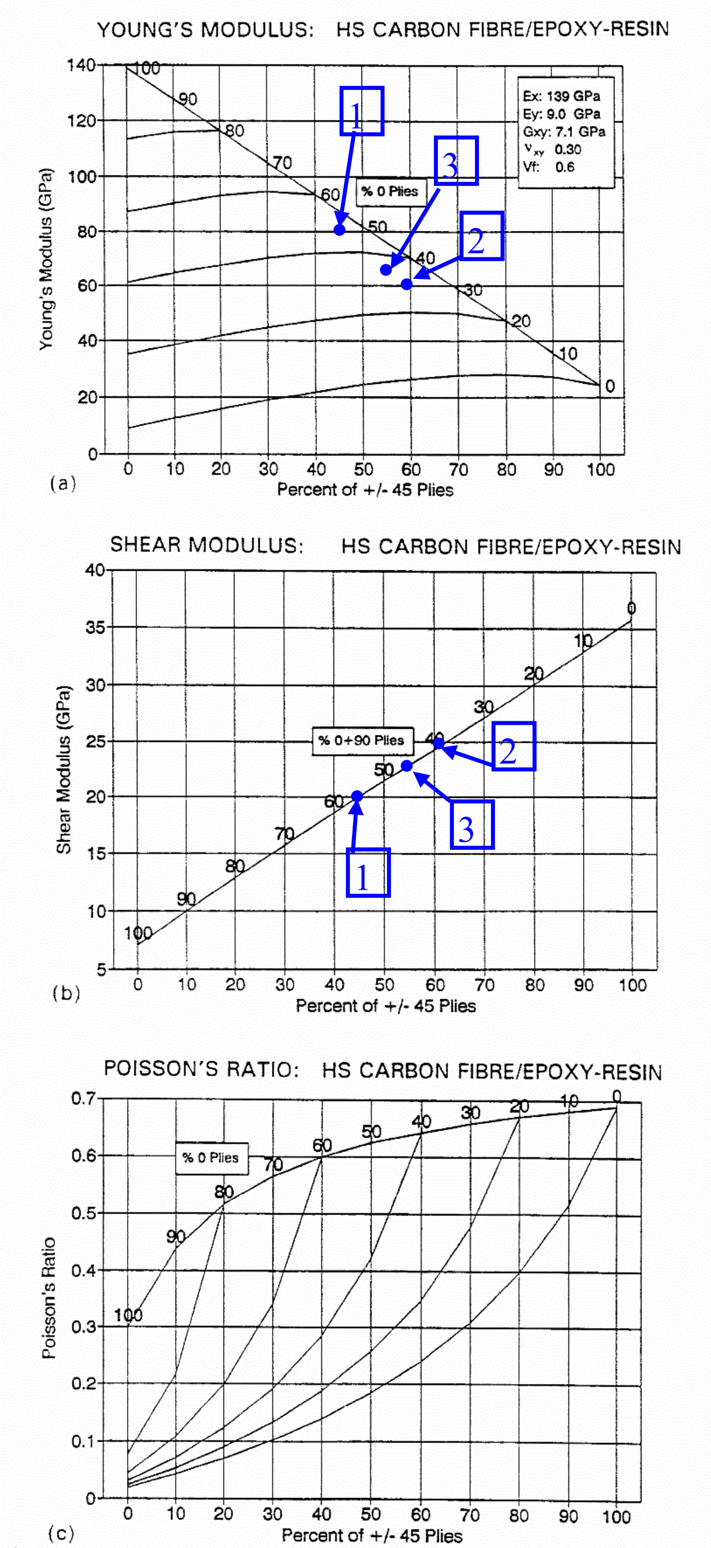
Carpet plots for carbon, glass and Kevlar (aramid) fibres in epoxy are given here [Bader].

The plots give the laminate stiffness as a function of the percentage of 0° and ±45 plies. The percentage of 90° plies is found by the difference between 100% and the percentage of the 0 and ±45 plies.

For $E_y$ swap the 0 and 90° plies, e.g. treat a [0,90,0] laminate as a [90,0,90] laminate for the purposes of finding $E_y$ .

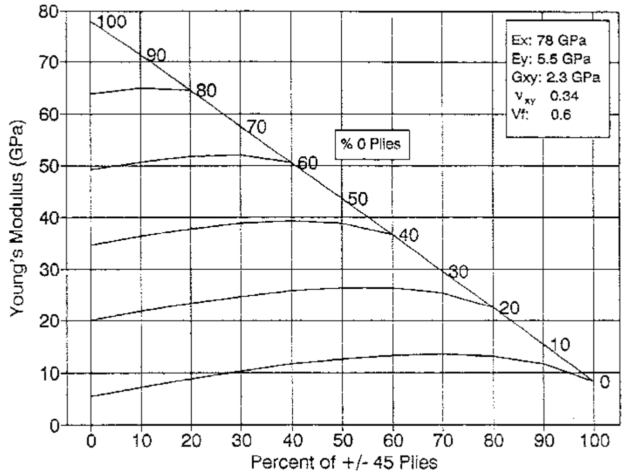
The Poisson's $\nu_{xy}$ is given.

For $\nu_{yx}$ use $E_x / \nu_{xy} = E_y / \nu_{yx}$

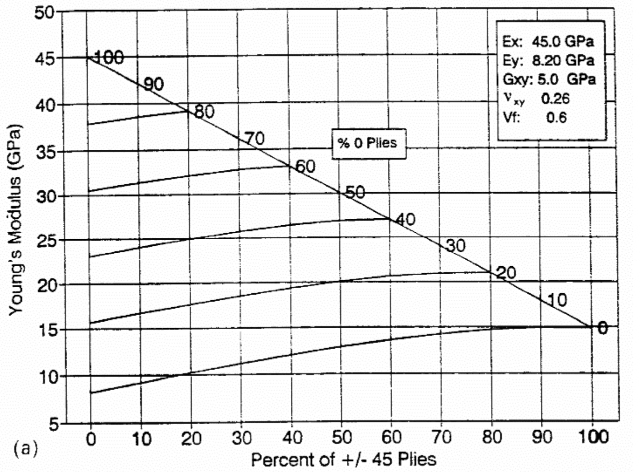


# Kevlar

YOUNG'S MODULUS: ARAMID FIBRE/EPOXY-RESIN

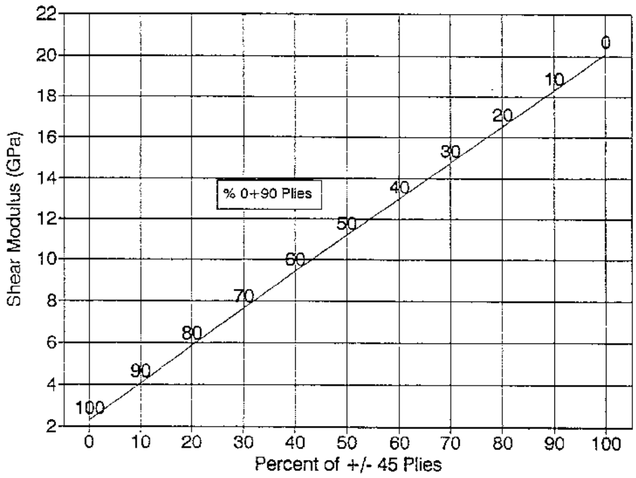



YOUNG'S MODULUS: E-GLASS FIBRE/EPOXY-RESIN

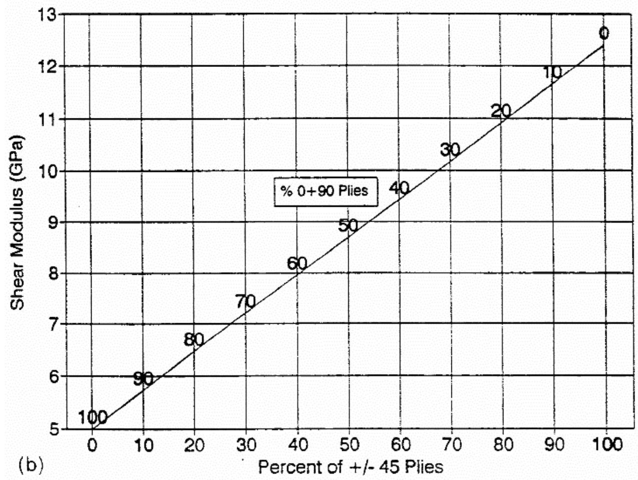



(a)

SHEAR MODULUS: ARAMID FIBRE/EPOXY-RESIN

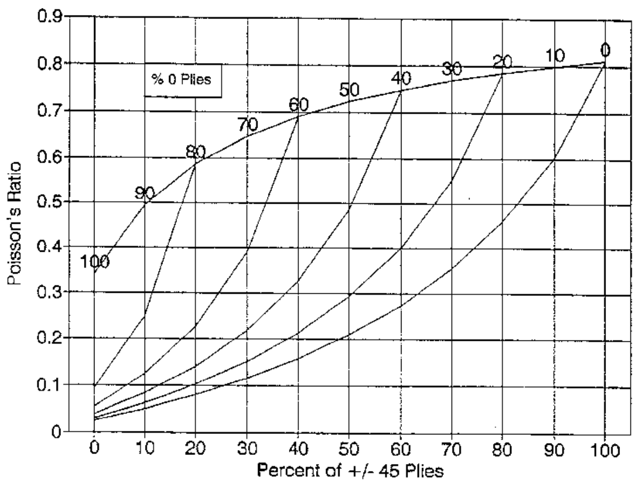



SHEAR MODULUS: E-GLASS FIBRE/EPOXY-RESIN

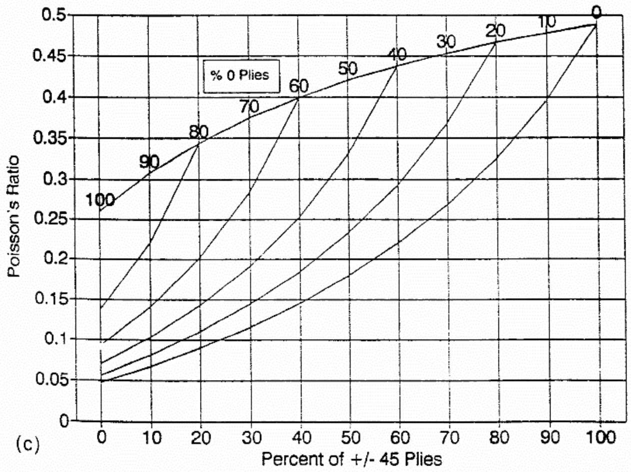



(b)

POISSON'S RATIO: ARAMID FIBRE/EPOXY-RESIN




POISSON'S RATIO: E-GLASS FIBRE/EPOXY-RESIN




(c)

### 2.4.2 Preliminary design – strength

Again we use **strain allowables**, as for conceptual design. However now the laminate stiffness properties are taken from the carpet plots as described above. We include Poisson's ratio effects in the laminate strain calculations to give:

$$\varepsilon_x = \frac{1}{E_x t}\left(N_x - N_y \nu_{xy}\right), \quad \varepsilon_y = \frac{1}{E_y t}\left(N_y - N_x \nu_{yx}\right), \quad \gamma_{xy} = \frac{N_{xy}}{G_{xy} t}$$

Putting each of these strains equal to a strain allowable, in turn, gives us three different expressions for the required **total** thickness of the laminate $t_x$, $t_y$ and $t_{xy}$

$t_x$, $t_y$, $t_{xy}$

Three expressions for **total** thickness based on $N_x$, $N_y$, $N_{xy}$

$$t_x = \frac{1}{E_x e}\left(N_x - N_y \nu_{xy}\right) \qquad \boldsymbol{N_x} \textbf{ load}$$

$$t_y = \frac{1}{E_y e}\left(N_y - N_x \nu_{yx}\right) \qquad \boldsymbol{N_y} \textbf{ load}$$

$$t_{xy} = \frac{N_{xy}}{G_{xy} e_{LT}} \qquad \boldsymbol{N_{xy}} \textbf{ load}$$

The required laminate thickness is the maximum of these three thicknesses. The laminate thickness is **NOT** the sum of these three thicknesses.

To optimise the laminate these three thicknesses should be made roughly equal by adjusting the ply mix.

### 2.5 Material properties for conceptual and preliminary design

| | **Steel** | **Aluminium** | **CFRP** | **GFRP** | **Kevlar** |
|---|---|---|---|---|---|
| Cost C (£/kg) | 3 | 20 | 150 | 8 | 80 |
| $E_1$ (GPa) | 210 | 70 | 140 | 45 | 80 |
| G (GPa) | 80 | 26 | ≈38 | ≈11 | ≈20 |
| ρ (kg/m$^3$) | 7800 | 2700 | 1500 | 1900 | 1400 |
| $e^+$ (%) | 0.1-0.8 | 0.1-0.8 | 0.4 | 0.3 | 0.5 |
| $e^-$ (%) | 0.1-0.8 | 0.1-0.8 | 0.5 | 0.7 | 0.1 |
| $e_{LT}$ (%) | 0.15-1 | 0.15-1 | 0.5 | 0.5 | 0.3 |

Use in coursework

**Table 1. Approximate material constants for conceptual and preliminary design**

- The shear stiffness contribution G of the ±45 plies for composites can be *approximately* estimated by $E_1/4$ (c.f. laminate plate theory or Mohr's circle).
- Strain allowables are for a laminate containing plies in the 0, 45 and 90° directions.
- Note the low shear and compressive strain allowables for Kevlar.
- Steel and aluminium properties vary widely with alloy content and processing.
- Costs are based on online shopping for parts, see below, including manufacturing costs and deducting an assumed level of profit and are **very** approximate.

## 2.6 Cost case study

- Online shopping price of various components
- Range of different materials, similar types of structure (except kevlar)
- Price reflects a range of factors apart from raw material including:
  - complexity of part
  - manufacturing route
  - market sensitivity to price and profit margin

| Part | Mass (kg) | Cost (£) | Cost/mass (£/kg) |
|---|---|---|---|
| **Steel** structural beam | 23 | 88 | 3.8 |
| **Steel** aerial mast | 1.1 | 9 | 8.0 |
| **Aluminium** yacht mast top section | 3.0 | 222 | 74 |
| **Aluminium** structural beam | 2.3 | 88 | 38 |
| **GFRP** yacht aerial mast | 1.6 | 36 | 22 |
| **GFRP** pultruded structural beam | 21 | 209 | 10 |
| **Kevlar** bullet proof vest | 2.7 | 423 | 160 |
| **CFRP** pultruded small box 10mm | 0.029 | 15 | 530 |
| **CFRP** yacht mast | 2.0 | 460 | 240 |
| **CFRP** mast top section | 2.5 | 744 | 300 |

**Table 2. List prices of various components parts (November 2022)**

https://www.fibregrid.com/grp-grating-structures/pultruded-grp-profiles/i-beam-profile/

https://www.elite-armor.com/shop/elite-armor-rx2-bulletproof-vest-stab-resistant-vest-106019p.html

https://www.tridentuk.com/gb/ilca-4-6-7-upper-mast-top-section-carbon-laser-ilc1925.html
https://www.tridentuk.com/gb/ilca-4-6-7-upper-mast-top-section-laser-alloy-ilc1920.html

## 3 Summary of laminate design

Consider each constraint in turn and draw up a table of equations specifying the required material properties and ply thicknesses. Identify any critical constraints. Then calculate the laminate thickness required in the three ply directions to give the minimum cost (or weight), using increasingly accurate methods and material properties.

*Conceptual Design*

- Use approximate material properties (Table 1) and treat each ply independently.
- Use merit indices to aid material selection if there is only one constraint. With multiple constraints it will be necessary to make direct estimates of e.g. cost or weight.

*Preliminary Design*

- Choose a trial laminate from the laminate building blocks, based on the conceptual design.
- Use carpet plot data to find the stiffnesses for your trial laminate and strength based on strain allowables (i.e. **not** using detailed material properties in each direction).
- Calculate the thickness of laminate determined by each constraint.
- Update your laminate to make the laminate thickness imposed for each constraint approximately equal.

*Detailed design*

- Use laminate theory to check mixed failure criteria and various loading patterns, now using detailed material properties.
- Optimise as necessary.

With more than one constraint a **tabular** approach is helpful, e.g.

***Conceptual Design***

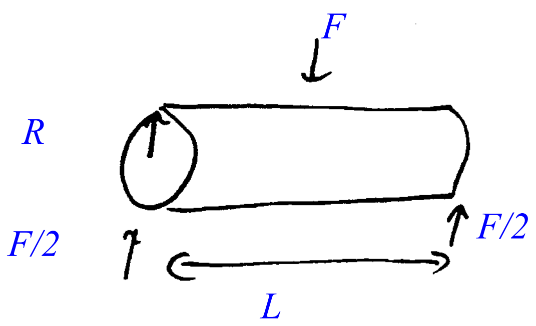
| | 0° ply | 90° ply | 45° plies |
|---|---|---|---|
| Strength | | | |
| Stiffness #1 | | | |
| Stiffness #2 | | | |
| Max | | | |

***Preliminary Design***

| | Thickness |
|---|---|
| Strength | t1 |
| Stiffness #1 | t2 |
| Stiffness #2 | t3 |
| Thickness | max (t1, t2, t3) |

Thickness = $t_0 + t_{90} + t_{45}$

**4 Single constraint example – loading of a high-tech canoe hull**




The hull is modelled as a circular beam in bending, with dimensions shown. Assumed that there is a premium of £50 per kg of weight saved.

We will simplify the problem sufficiently to be able to illustrate the merit index approach. We do this by assuming that the hull is strength critical (neglecting stiffness constraints) and also by neglecting the shear flow due to the bending loads.

First the load constraint needs to be expressed in terms of laminate loading.

These are maximum at the top and bottom of the hull.

$$'\frac{\sigma}{y} = \frac{M}{I}' \Rightarrow N_x = \frac{FL}{4\pi R^2}$$

The laminate will contain 0, 90 and ±45° plies, but we will ignore the 45 and 90° plies in the conceptual design calculations.

Strain allowable

$$t = t_0 = \frac{N_x}{E_1 e} = \frac{FL}{4\pi R^2 E_1 e}$$

$$\text{Mass} = \text{Volume} \times \rho = 2\pi R t L \rho$$

To find the best material to minimise mass, eliminate the free variable $t$

$$\text{Mass} = \frac{FL^2}{2R}\frac{\rho}{E_1 e}$$

Hence, to minimise mass, we need to minimise $\rho/(E_1 e)$. Use the smaller of the tensile or compressive allowable strains ***e*** for each composite.

This merit index is calculated in Table 2. CFRP is a clear winner.

In fact we wish to maximise profit. We will calculate the 'lost profit' relative to a hull of zero cost and mass

$$\text{Lost profit} = \text{Premium} \times \text{Mass} + \text{Cost} = (\text{Premium} + \text{C}) \times \text{Mass}$$

where C is material cost per kg.

$$\text{Lost profit} = (50 + C) \times \text{Mass} = (50 + C)\frac{FL^2}{2R}\frac{\rho}{E_1 e}$$

where the 50 is associated with the weight premium.

To maximise profit, minimise $(50+C)\rho/(E_1e)$. See Table 2. Steel is the best choice, closely followed by CFRP. Aluminium is a little worse followed by GFRP. Kevlar is the worst choice on account of its poor compressive strength.

In practice toughness, manufacturing costs, durability and process details may alter the balance considerably.

In addition we should look much more closely at the mechanical design assumptions. For example buckling can become an issue for thin steel or CFRP sheets.

| | **Steel** | **Aluminium** | **CFRP** | **GFRP** | **Kevlar** |
|---|---|---|---|---|---|
| Cost C (£/kg) | 3 | 20 | 150 | 8 | 80 |
| $E_1$ (GPa) | 210 | 70 | 140 | 45 | 80 |
| $\rho$ (kg/m$^3$) | 7800 | 2700 | 1500 | 1900 | 1400 |
| Min of $e^-$ or $e^+$ (%) | 0.4 | 0.4 | 0.4 | 0.3 | 0.1 |
| $\rho/E_1 e$ | 93 | 96 | 27 | 140 | 175 |
| (50+C) $\rho/E_1 e$ | 4900 | 6800 | 5400 | 8200 | 23000 |

**Table 2. Conceptual design merit index calculations**

## 5 Multiple Constraint Case Study - A drive shaft for the Renault Espace

The full design process will be illustrated by going through the design of a composite drive shaft for a Renault Espace car.

An existing design of Espace is to be modified to allow for four-wheel drive. Preliminary studies showed that a steel drive shaft would require major redesign of the underbody at considerable expense.

A key constraint is **'whirling'** vibration of the shaft. One solution would be a steel shaft with a central support bearing. However there is a huge cost associated with a **central bearing**, because of body redesign, fitting costs, extra components, extra mass and loss of space.

Could a composite drive shaft be made at reasonable cost to fit into the existing space?

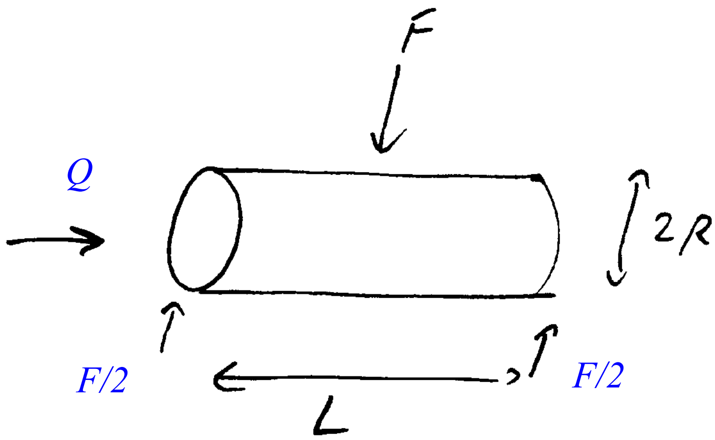
https://www.netcarshow.com/renault/1987-espace_quadra/

### 5.1 Objective

Minimise cost-weight function - say a premium of £50 per kg plus a penalty of £500 for a central bearing.

**Nomenclature**

F - Central transverse force
m - Mass
L - Length
Q - Torque
R - Tube radius
t - Wall thickness



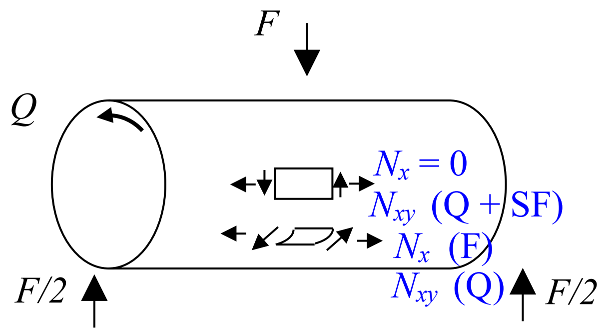

### 5.2 Constraints

- Geometric: a length L of 1.6 m, a radius R of 0.045 m.
- Static strength: Torque Q of 2800 Nm at 25°C, 2400 Nm at 100°C
  Central load F of 200 N
- Fatigue strength: 600,000 cycles at ±700 Nm
- Vibrational: No whirl at a rotational speed of 100 Hz
- Operating temperature of -40°C to 100°C
- Durability: Ageing for 25 days in a salt spray, 40 °C, 95% R. Humidity
- Environmental: Chippings, brake fluid, rust remover, liquid manure

The mechanical constraints again need to be expressed in terms of laminate properties – loads or stiffnesses.

### 5.2.1 Torque loading

$$N_{xy} \times R \times 2\pi R = Q \Rightarrow N_{xy} = \frac{Q}{2\pi R^2}$$




### 5.2.2 Bending loads

*Bending stresses*

$$'\frac{\sigma}{y} = \frac{M}{I}' \Rightarrow N_x = \frac{FL}{4\pi R^2}$$

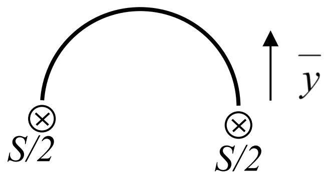
| | |
|---|---|
| $M = FL/4$ | $y = R$ |
| $I = \pi R^3 t$ | $N_x = \sigma t$ |

For more sophisticated failure criteria we should consider the interaction between $N_x$ due to bending and $N_{xy}$ for the torque loading. Conceptual and preliminary design neglects this.

*Shear flow*

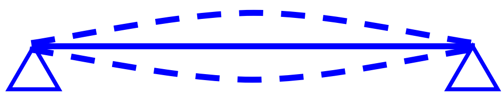
The maximum shear stress due to shear flow occurs at the sides of the tube. On one side the shear stress adds to the torque stress, on the other side it subtracts.

$$'S = \frac{FA_s\bar{y}}{I}' \Rightarrow N_{xy} = \frac{F}{2\pi R}$$




| | |
|---|---|
| $\bar{y} = 2R/\pi$ | $'F' = F/2$ |
| $A_s = \pi R t$ | $N_{xy} = S/2$ |

### 5.2.3 Vibrational constraint



The problem is whirl, where the shaft vibrates in a bending mode as illustrated. To avoid this we need to ensure that the resonant frequency $f$ of the fundamental mode is above the operating frequency of the shaft. Standard texts give $f$ for a beam in bending, with free ends, as

$$'f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}' \Rightarrow f = \frac{\pi R}{2L^2}\sqrt{\frac{E_x}{2\rho}}$$

$EI(R, L)$

$m(R, L)$
mass/unit length

(MF Ashby, Materials selection in mechanical design)

Putting in numbers, $\frac{E_x}{\rho} > 8\left(\frac{fL^2}{\pi R}\right)^2 = 0.027 \times 10^9$ Nm/kg

NB $t$ cancels out

A safety factor of 1.5 might reasonably be added to this.

## 5.3 Conceptual design

Use 'experience' to recognise that the shaft will be a thin-walled tube.

Relevant available manufacturing routes for tubes are pre-preg lay-up and filament winding.

The fatigue endurance limit of materials is generally about half the ultimate strength of materials, suggesting that the static strength requirement will be more critical.

Environmental hazards do not eliminate composites.

Because there is more than one constraint, it is not straightforward to use merit indices here. Instead we have to consider all the constraints in turn.

We will only consider a pre-preg lay-up with a composite composed of 0's, 90's and 45's. The full process would also look at a ±θ filament-wound design.

*Whirling constraint*
For conceptual design purposes, assume that only the 0° plies contribute to the axial stiffness.

safety factor

$$E_x / \rho > 1.5 \times 0.027 = 0.0405 \text{ GNm/kg}$$

$$\frac{t_0}{t} \approx \frac{E_x}{E_1} > \frac{\rho \times 1.5 \times 0.027 \times 10^9 \text{ Nm/kg}}{E_1} \Rightarrow \begin{array}{l} 43\% \text{ for CFRP} \\ 71\% \text{ for Kevlar} \end{array}$$

It is not possible to meet this constraint using GFRP, steel or aluminium (see Table 4). These materials will need a central bearing to reduce the effective whirl length L.

| | Steel | Aluminium | CFRP | GFRP | Kevlar |
|---|---|---|---|---|---|
| $E_1$ (GPa) | 210 | 70 | 140 | 45 | 80 |
| ρ (kg/m$^3$) | 7800 | 2700 | 1500 | 1900 | 1400 |
| $E_1/\rho$ (GNm/kg) | 0.027 | 0.026 | 0.093 | 0.024 | 0.057 |

**Table 4. Whirling constraint calculations**

*NB In this case the constraint only imposes a limit on material properties. In general (e.g. with a stiffness constraint), there will be a limit on a combination of material properties and ply thicknesses.*

*Strength constraints*

For conceptual design calculations we estimate the thickness of the 0° plies needed to take the axial load due to bending and the thickness of the ±45° plies needed for the shear load associated with the torque and shear flow.

*Axial*

$$t_0 = \frac{N_x}{E_1 e} = \frac{FL}{4\pi R^2 E_1 e} = \begin{matrix} 22\,\mu\text{m for CFRP} & e^+ \\ 157\,\mu\text{m for Kevlar} & e^- \end{matrix}$$

*Shear*

small here

0's > 43% ⇒ > 22 μm

$$t_{45} = \frac{N_{xy}}{G e_{LT}} = \left( \frac{Q}{2\pi R^2} + \cancel{\frac{F}{2\pi R}} \right) \frac{1}{G e_{LT}} \approx \frac{Q}{2\pi R^2} \frac{1}{G e_{LT}} = \begin{matrix} 1250\,\mu\text{m for CFRP} \\ 3670\,\mu\text{m for Kevlar} \end{matrix}$$

Note that the axial loading is redundant – with the whirling constraint and shear loading this will automatically be met.

Finally include 10% of 90° plies and calculate the ratio of ply orientations. The total thickness is calculated from the value for t45 found above.
The lost profit $= (50 + C) \times \text{Mass} = (50 + C) 2\pi R L t \rho$.

| | 0° | 45° | 90° | Total thickness | Lost Profit |
|---|---|---|---|---|---|
| CFRP | 43% | 47% | 10% | 2.7 mm | £370 |
| Kevlar | 71% | 19% | 10% | 19 mm | £1600 |

**Table 5. Final conceptual design calculations**

CFRP is now a clear winner. The calculation indicates that it will be significantly cheaper than a steel shaft with a central bearing.

## 5.4 Preliminary design.

Conceptual design indicates a CFRP shaft. The ratio of ply orientations suggested by the conceptual design is 0° : 45° : 90° = 43% : 47% : 10% with a total thickness of 2.7 mm. In the preliminary design this laminate is optimised further to try to reduce the laminate thickness.

With a standard ply thickness of 0.125 mm plies, the total thickness of 2.7 mm amounts to 21.6 plies. If we were able to use fractions of plies, we would have the numbers of lamina in each ply orientation as 0° : 45° : 90° = 9.3 : 10.1 : 2.2.

*First iteration*

In practice we have to choose integer numbers of laminae.
Two 90° plies should suffice. 45° plies come in fours (or pairs if balanced, not symmetric for tubes), so we could try 12 of these. Finally we need to ensure that the proportion of 0° plies is above about 43%, so try 12 of these, giving a final ply mix of 0° : 45° : 90° = 45% : 45% : 9% and a total thickness of 3.25mm.

Now find the laminate stiffnesses from the carpet plots, section 2.4.

$$E_x = 80 \text{ GPa}$$
$$G_{xy} = 20 \text{ GPa}$$

zero here

$\boxed{\varepsilon = e}$

$$t_x = \frac{1}{E_x e}\left(N_x - N_y \nu_{xy}\right) = \frac{FL}{4\pi R^2 E_x e^+} = 39\mu\text{m}$$
$$t_{xy} = \frac{N_{xy}}{G_{xy} e_{LT}} \approx \frac{Q}{2\pi R^2}\frac{1}{G_{xy} e_{LT}} = 2.2\text{mm}$$

Clearly the bending load is not critical, as expected. The axial stiffness needs only to be greater than 60 GPa (using $E_x/\rho > 0.0405$ GNm/kg), so it looks as though the number of 0's could be reduced.

*Second iteration*

Looking at the carpet plot, the minimum percentage of 0's which meets the axial stiffness criterion is around 30 %, assuming that there are still 10 % of 90° plies, leaving 60% of ±45° plies. Note that it is not always possible to take such an approach.

For this ply mix $G_{xy}$ = 24 GPa. The shear strength constraint gives a laminate thickness txy of 1.8 mm, i.e. 14.7 plies, leaving ply numbers of
0° : 45° : 90° = 4.4 : 8.8 : 1.5.

Although this iteration is probably the best we are going to get, we need a further iteration using integer ply numbers

*Third iteration*

Maybe eight plies (53%) of 45's, five plies of 0's (33 %) and two 90 plies (13%) would suffice, for a total laminate thickness of 1.875 mm.

For this ply mix,

$E_x = 62$ GPa

$G_{xy} = 22$ GPa

The shear strain constraint requires $t_{xy} > 2.00$ mm, so the actual laminate thickness of 1.875 mm is slightly less than it should be.

However this iteration is probably good enough at this stage, given that we will be using more accurate failure calculations in the next phase of design.

A possible lay-up with the required ply numbers is $\left[(0\ ,\pm 45)_2, 90, \bar{0}\right]_s$ where the overbar indicates a central ply (to keep things symmetric). The estimated weight and lost profit for this tube are 1.3 kg and £250. The optimisation has saved £120 per shaft.

## 5.5 Detailed design

We will use a laminate software package to check the proposed laminate for failure using more sophisticated failure criteria, and see if we can further optimise the design by reducing the laminate thickness.

Calculations are done using the Composite Compressive Strength Modeller and the following data for a carbon fibre-epoxy composite AS/3501.

| E1 (GPa) | E2 (GPa) | ν12 | G12 (GPa) | $s_L^+$ (MPa) | $s_L^-$ (MPa) | $s_T^+$ (MPa) | $s_T^-$ (MPa) | $s_{LT}$ (MPa) |
|---|---|---|---|---|---|---|---|---|
| 138 | 9.0 | 0.3 | 6.9 | 1448 | 1172 | 48.3 | 248 | 62.1 |

**Table 6 Detailed material properties**

The laminate requirements are

(i) an axial stiffness $E_x > 60$ GPa

(ii) failure line loads $N_x > \pm 0.0126$ MN/m, $N_{xy} > 0.220$ MN/m at the top and bottom of the tube

(iii) $N_{xy} > 0.221$ MN/m at the side of the tube.

We will use Tsai-Hill and maximum strain failure criteria for condition (ii) and Tsai-Hill for condition (iii).

First try the laminate $\left[(0\ ,\pm45)_2,90,\bar{0}\right]_s$ suggested by the preliminary design, then try to improve on this by trial and error.

| Lay-up | Number of plies | $E_x$ | (ii) T-H $N_{xy}$ | (ii) Max ε $N_{xy}$ | (iii) $N_{xy}$ |
|---|---|---|---|---|---|
| $\left[(0\ ,\pm45)_2,90,\bar{0}\right]_s$ | 15 | 64.5 | 0.375 | 0.375 | 0.375 |
| $\left[0_2,\pm45,90,\bar{0}\right]_s$ | 11 | 77 | 0.215 | | |
| $\left[0_3,\pm45,90\right]_s$ | 12 | 82 | 0.222 | | |

The strain allowables for the preliminary design were very conservative in this case.

Trial and error optimisation is greatly helped by appropriate software, with a further reduction in weight and lost profit of 20 % (£50).

Optimisation is limited by the discrete thickness of the plies, particularly with the need to have 45° plies in sets of 4.

### 5.6 Other practicalities

- A protective woven Kevlar or GFRP cloth should be added to protect against impact.
- Testing of coupons and/or a small section of shaft under the fatigue, ageing and environmental conditions is needed.
- **Testing** of a prototype is needed to confirm the stiffness and strength of the tube.
- The filament wound route with a ±θ lay-up should be investigated.

Availability of the alternative production methods will be important, as well as lay-up optimisation.

- **Hybrid** lay-ups could be considered.
- Cost.

### 5.7 Actual design

- Filament wound tube, ±10° winding angle (i.e. almost along the axis).
- Hybrid material, 75% CFRP, 25% GFRP
- Wall thickness = 2.2 mm.
- Price 'off-the-shelf' ≈ £1000,
- Weight of tube = 1.5 kg
- Weight including end fittings = 5 kg