UNIVERSITY OF CAMBRIDGE

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ENGINEERING TRIPOS PART II A

EIETL **MODULE EXPERIMENT 3F3**

RANDOM VARIABLES and RANDOM NUMBER GENERATION
Short Report Template

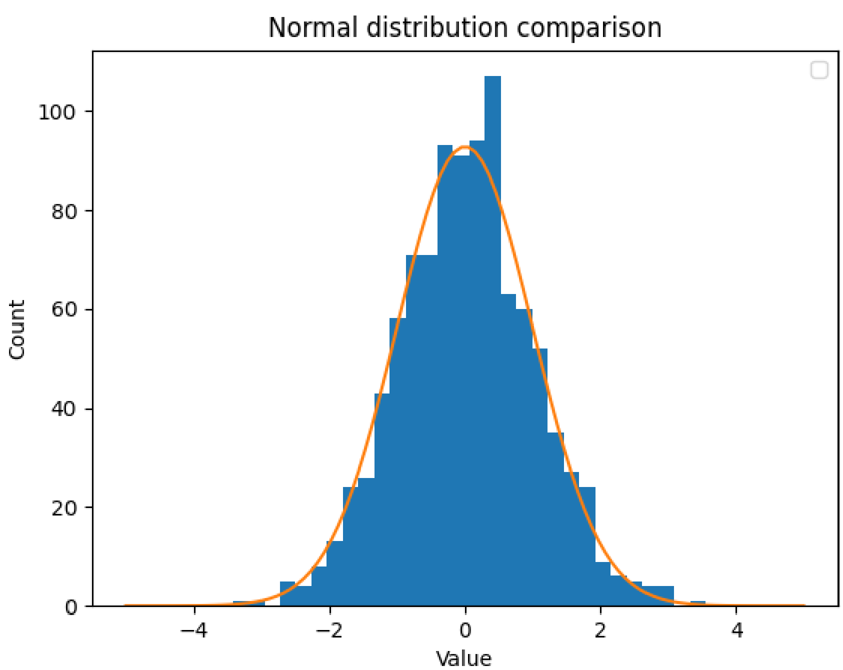
Name: Jack Henry

College: Churchill

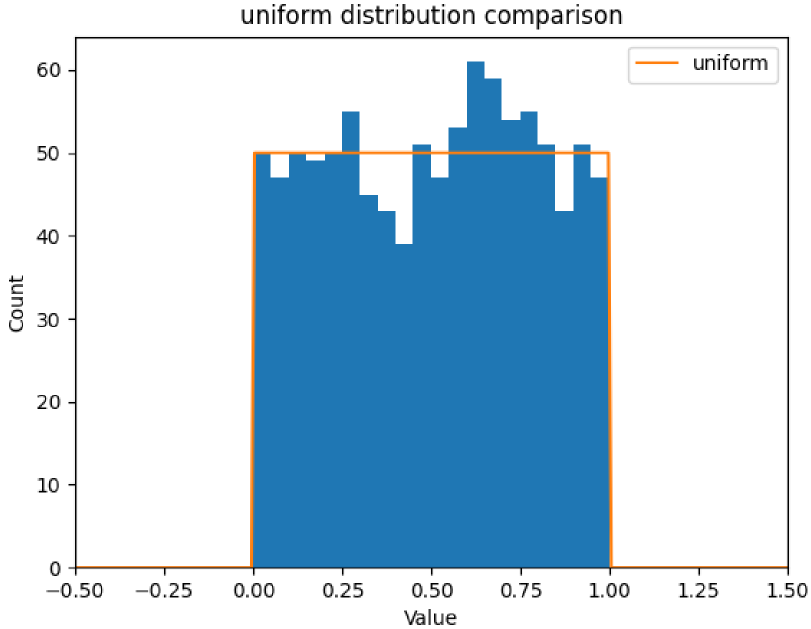
Lab Group Number: 15th November 2024 Student 10

1. **Uniform and normal random variables.**

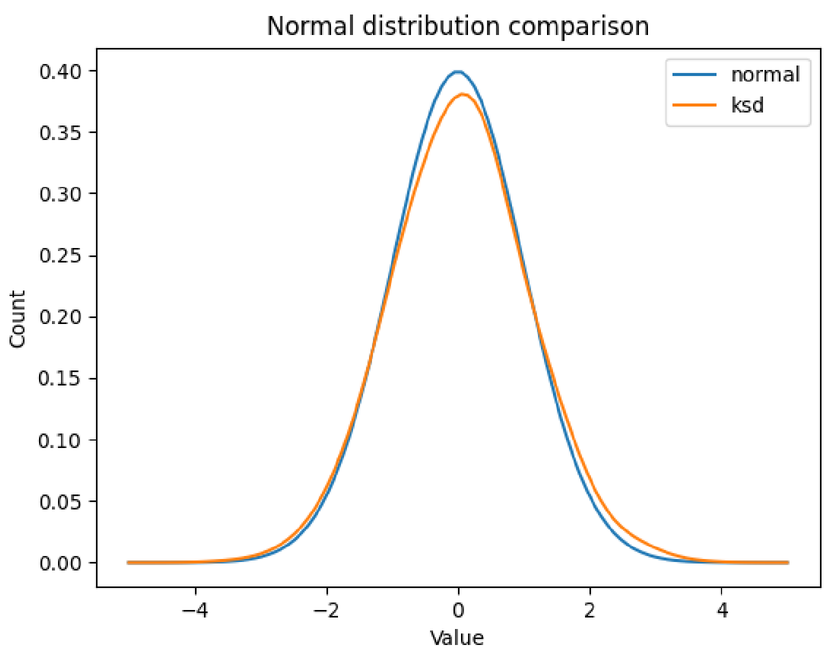
   Histogram of Gaussian random numbers overlaid on exact Gaussian curve (scaled):



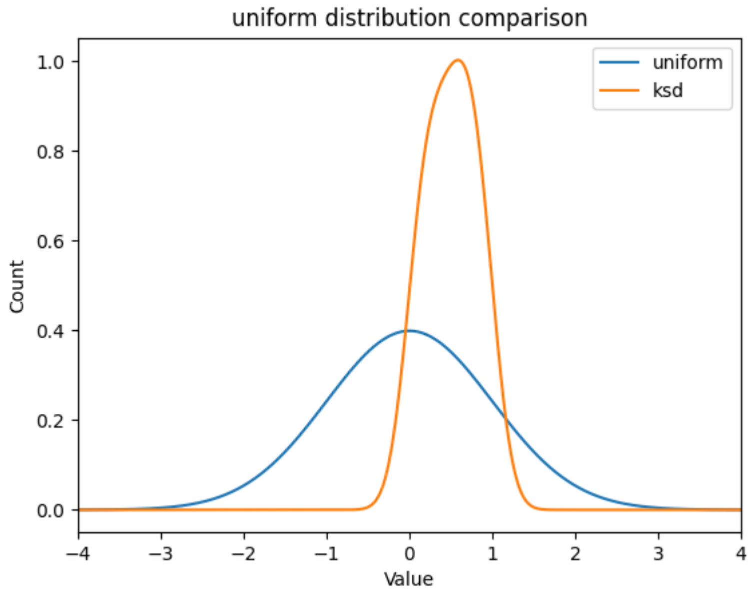
Histogram of Uniform random numbers overlaid on exact Uniform curve (scaled):



Kernel density estimate for Gaussian random numbers overlaid on exact Gaussian curve:



Kernel density estimate for Uniform random numbers overlaid on exact Gaussian curve:



Comment on the advantages and disadvantages of the kernel density method compared with the histogram method for estimation of a probability density from random samples:

The kernel density method shows a smooth curve that uses an average of the counts for each value. This gives a good overall shape when estimating a pdf of that is also smooth, however will also tend to smooth out any sharper features of the pdf. This can be seen from the slight smoothing of the peak of the normal distribution, and especially in the corners of the uniform distribution. The uniform distribution can still be identified from this method however, as the count goes up to 1.0, and the distribution centres on 0.5 and generally spans from 0 to 1. It also does not show the variation of the counts, which the histogram method does. The histogram method shows both the overall shape, and variation can be inferred through the visuals of each bin count. These bins however reduce the resolution of the estimation.

Theoretical mean and standard deviation calculation for uniform density as a function of N:

The probability that a sample $x^{(i)}$ lies within a bin of the histogram is:

$$p_j = \int_{c_j-\delta/2}^{c_j+\delta/2} p(x)dx \tag{1}$$

For a uniform distribution for values for $a < x < b$:

$$p(x) = \frac{1}{b-a} \tag{2}$$

The integral in equation (1) evaluated using equation (2) gives:

$$p_j = \frac{\delta}{b-a} = \frac{bin\ width}{range\ of\ distribution} = \frac{1}{number\ of\ bins} \tag{3}$$

Each bin can be modelled as a binomial distribution as $X_j \sim B(N, p_j)$
Therefore, the mean of the count data for each bin is:

$$\mu = Np_j = \frac{N}{number\ of\ bins} \tag{4}$$

And the standard deviation of the count data for each bin is:

$$\sigma = \sqrt{Np_j(1-p_j)} = \sqrt{\frac{N}{number\ of\ bins}(1 - \frac{1}{number\ of\ bins})} \tag{5}$$

Explain behavior as N becomes large:

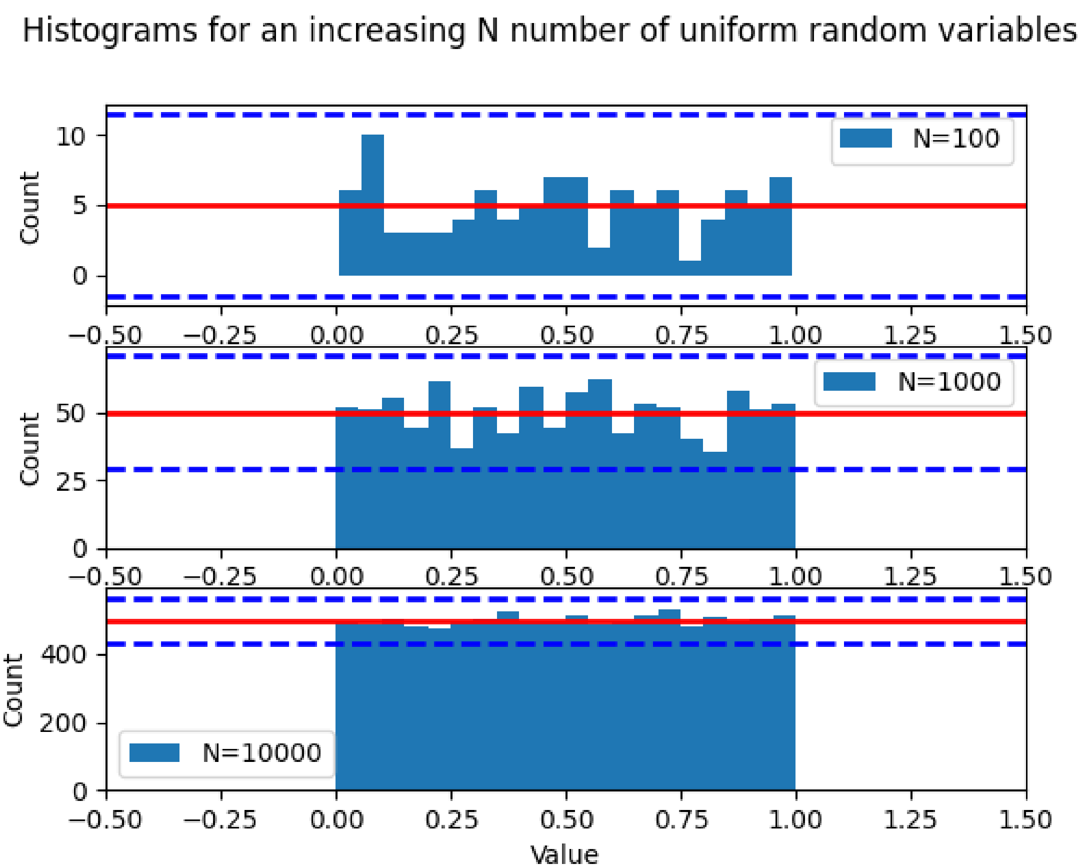
From equations (4) and (5):

$$\mu \propto N \qquad \text{and} \qquad \sigma \propto \sqrt{N} \tag{6}$$

This means as N increases, the mean will increase in proportion by the same amount, whilst the standard deviation will only increase through a square root relationship.

Effectively, the relative value of the mean compared to the number of generated random variables remains constant, whilst the standard deviation decreases at a square root rate. This can be seen in the figures below.

Plot of histograms for $N = 100$, $N = 1000$ and $N = 10000$ with theoretical mean and ±3 standard deviation lines:



Red line = theoretical mean
Blue dotted line = theoretical mean ± 3 standard deviations

Are your histogram results consistent with the multinomial distribution theory?

Yes, the results are consistent. It can visually be seen from the histograms that the mean of the entire histogram is the theoretical mean, with some variation for each bin. This variation is contained within the ±3 standard deviation range, for all 3 subplots, which is where we would expect over 99% of results.

2. **Functions of random variables** For normally distributed $N(x|0,1)$ random variables, take $y = f(x) = ax + b$. Calculate $p(y)$ using the Jacobian formula:

If $X \sim N(0,1)$ and $y = f(x) = ax + b$, we can calculate $p(y)$ using

$$p(y) = \sum_{k=1}^{K} \frac{p(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{x=x_k(y)} \tag{7}$$

where $x_k(y) = f^{-1}(y)$ for each possible function of $f^{-1}(y)$.

In the case of $y = f(x) = ax + b$,

$$\left|\frac{dy}{dx}\right| = a \tag{8}$$

and

$$x_k(y) = f^{-1}(y) = \frac{y-b}{a} \tag{9}$$

Therefore,

$$p(y) = \frac{p(\frac{y-b}{a})}{a} \tag{10}$$

and as $p(x)$ is a standard gaussian distribution, we can write

$$p(y) = \frac{1}{\sqrt{2\pi a^2}} e^{\left(-\left(\frac{(y-b)^2}{2a^2}\right)\right)} \tag{11}$$

It is clear that the distribution for Y can be modelled as:

$$Y \sim N(b, a^2)$$

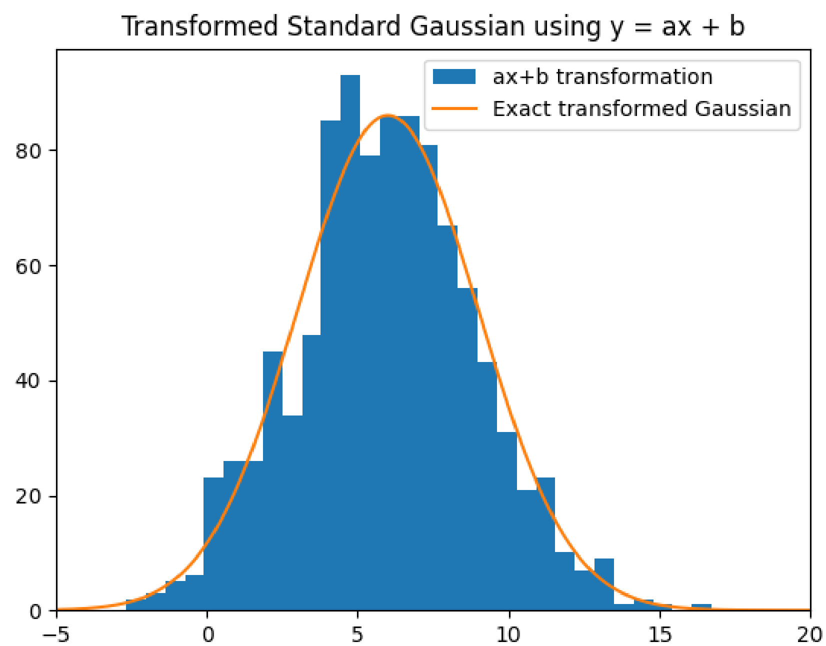
Explain how this is linked to the general normal density with non-zero mean and non-unity variance:

Doing the above calculations for a general distribution of $X \sim N(\mu, \sigma^2)$, we get equation (12)

$$p(y) = \frac{1}{\sqrt{2\pi a^2 \sigma^2}} e^{\left(-\left(\frac{(y-(a\mu+b))^2}{2a^2\sigma^2}\right)\right)} \tag{12}$$

From this we can see that the transformation of $\mathrm{f}(x) = ax + b$ translates the mean and standard deviation for a general normal density by $a\mu + b$ and $a\sigma$ respectively.

Verify this formula by transforming a large collection of random samples $x^{(i)}$ to give $y^{(i)} = \boldsymbol{f}(x^{(i)})$, histogramming the resulting y samples, and overlaying a plot of your formula calculated using the Jacobian:



Now take $p(x) = N(x|0, 1)$ and $\boldsymbol{f}(x) = x^2$. Calculate $p(y)$ using the Jacobian formula:

Using the same method as before and equation (7), for $X \sim N(0,1)$ and $y = f(x) = x^2$, it is now found that:

$$\left|\frac{dy}{dx}\right| = |2x| \tag{13}$$

and

$$x_k(y) = f^{-1}(y) = \pm\sqrt{y} \tag{14}$$

Therefore,

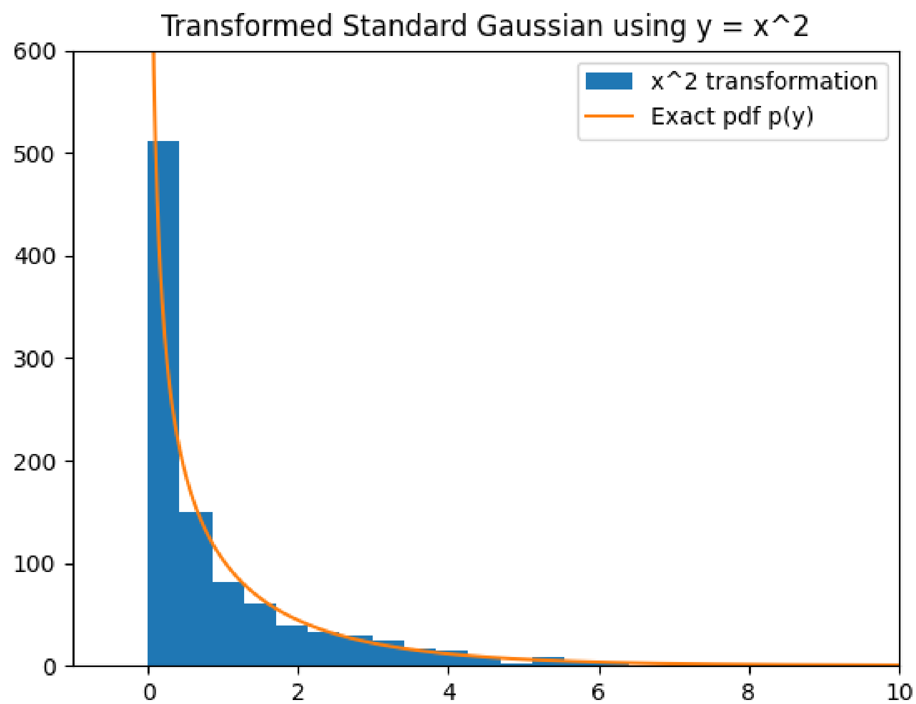
$$p(y) = \sum_{k=1}^{2} \frac{p(\pm\sqrt{y})}{|2\sqrt{y}|}\bigg|_{x=x_k(y)} = \frac{p(+\sqrt{y})}{|2\sqrt{y}|} + \frac{p(-\sqrt{y})}{|2\sqrt{y}|} = \frac{p(+\sqrt{y})}{|\sqrt{y}|} \tag{15}$$

And so,

$$p(y) = \frac{1}{\sqrt{2\pi y}} e^{(-\frac{1}{2}y)} \tag{16}$$

This is an exponentially distributed pdf.

Verify your result by histogramming of transformed random samples:



3. **Inverse CDF method**

Calculate the CDF and the inverse CDF for the exponential distribution:

For an exponential distribution

$$p(y) = e^{-y}, \quad y \geq 0 \tag{17}$$

The CDF is the integral of this,

$$F(y) = \int_0^y e^{-y} dy = 1 - e^{-y} \tag{18}$$

The inverse CDF for $x = F(y)$ is then

$$F^{-1}(x) = y = -\ln(1 - x) \tag{19}$$

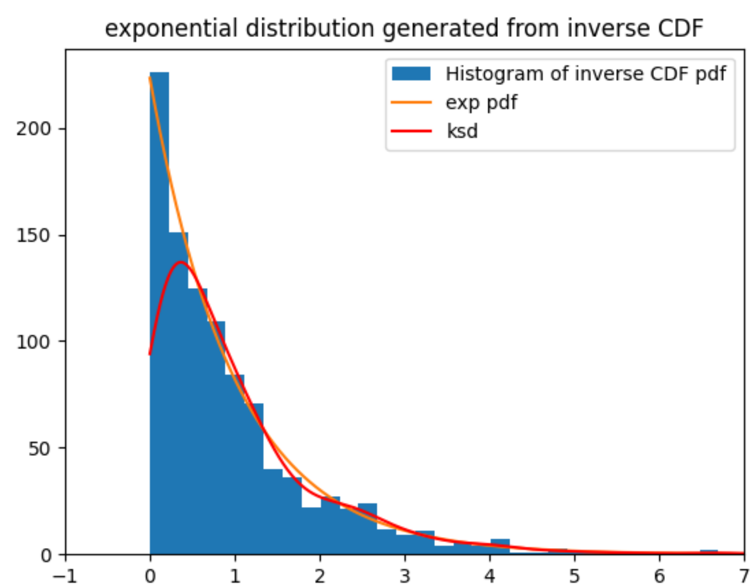
If we apply this to a uniform distribution of $X \sim U(0,1)$, we can generate an exponential distribution by transforming each uniformly distributed random variable. Due to the symmetry and range limit of the uniform distribution used we can also simply equation (19) to

$$y = -\ln(x) \tag{20}$$

Python code for inverse CDF method for generating samples from the exponential distribution:

```
def task_3():
    x = np.random.rand(1000) # generate uniform random variables
    x_values = np.linspace(0, 7, 1000)
    exp_pdf = np.exp(-x_values)
    y = -np.log(x) # Inverse CDF of x. x is uniformly distributed
between 0 and 1 so can simplify 1-x to x

    Area_transformed_exp = (1000*(max(y)-min(y))/30)

    plt.figure()
    plt.hist(y, bins=30, label = 'Histogram of inverse CDF pdf')

    plt.plot(x_values, Area_transformed_exp*exp_pdf, label='exp pdf')

    ks_density = ksdensity(y, width=0.3)
    plt.plot(x_values, Area_transformed_exp*ks_density(x_values),
label='ksd', color='r')

    plt.title('exponential distribution generated from inverse CDF')
    plt.legend()
    plt.xlim(-1,7)
    plt.savefig('Inverse CDF Method')
    plt.show()
    plt.close()
```

Plot histograms/ kernel density estimates and overlay them on the desired exponential density:



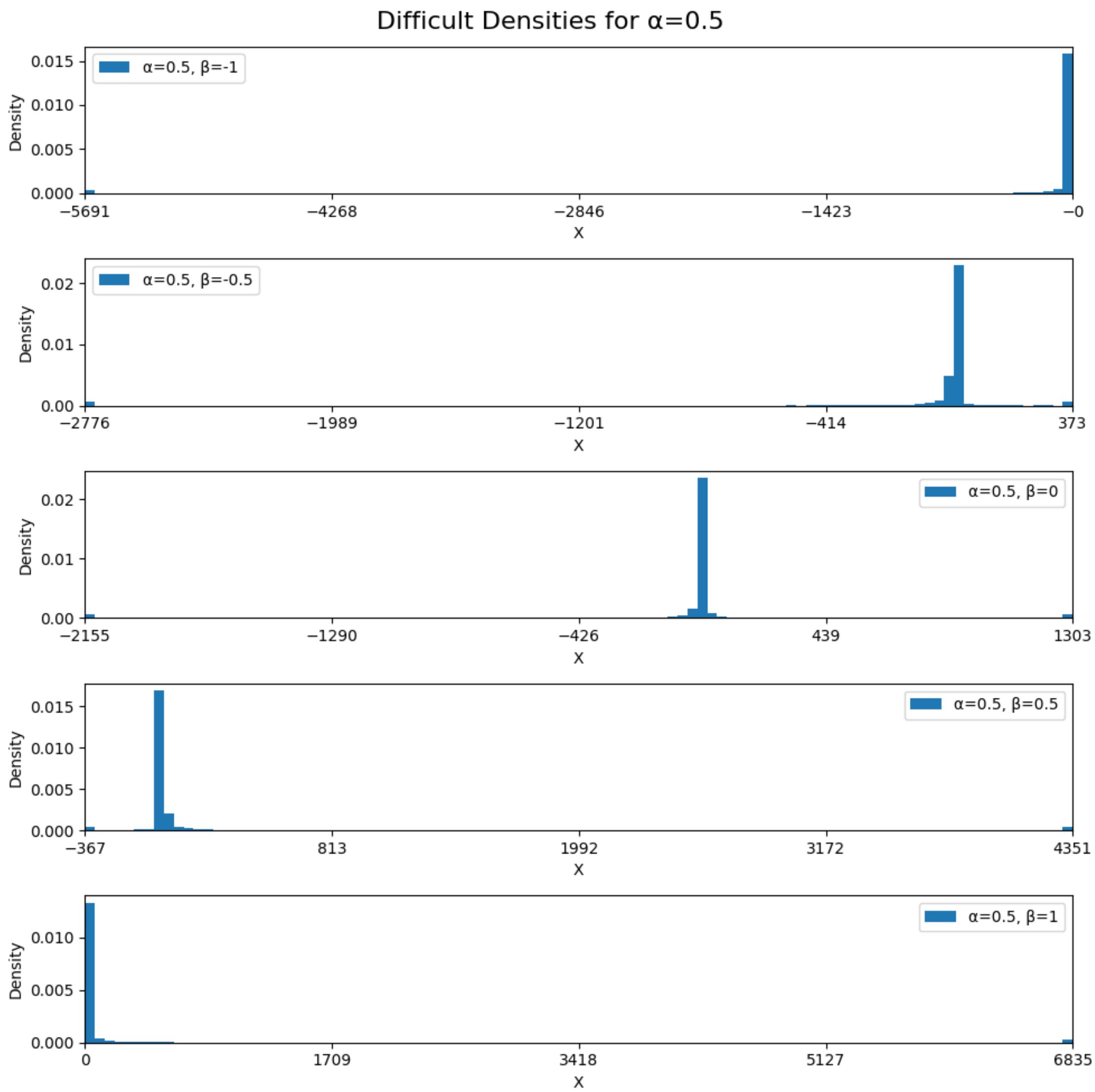
4. Simulation from a `difficult' density.

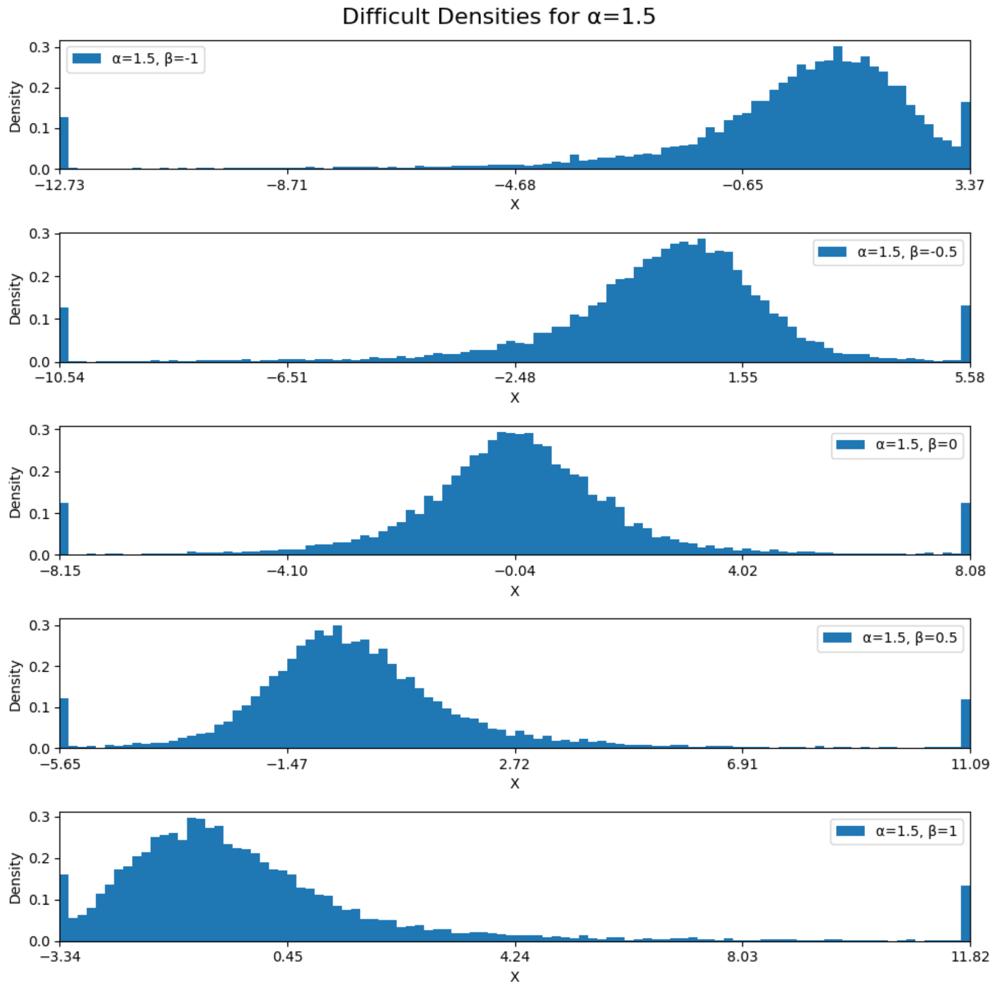
Python code to generate N random numbers drawn from the distribution of X:

```
def task_5():
    # a) choose parameters
    alphas = [0.5, 1.5]  # Different alpha values to test
    betas = [-1, -0.5, 0, 0.5, 1]  # Different beta values

    # Function to generate X based on the given recipe
    def calc_X(alpha, beta, N):
        b = (1 / alpha) * np.arctan(beta * np.tan(np.pi * alpha /
2))
        s = (1 + beta**2 * np.tan(np.pi * alpha / 2)**2)**(1 / (2 *
alpha))
        U = np.random.uniform(-np.pi / 2, np.pi / 2, N)
        V = np.random.exponential(1, N)
        X = (
            s
            * (np.sin(alpha * (U + b)) / (np.cos(U))**(1 / alpha))
            * ((np.cos(U - alpha * (U + b))) / V)**((1 - alpha) /
alpha)
        )
        return X

    N = 10000  # Number of samples
    bins = 100  # Higher resolution for histograms

    for alpha in alphas:
        fig, axs = plt.subplots(len(betas), figsize=(10, len(betas)
* 2), sharex=False)
        for i, beta in enumerate(betas):
            ax = axs[i] if len(betas) > 1 else axs
            X = calc_X(alpha, beta, N)

            # Dynamically determine truncation range based on
percentiles
            x_min, x_max = np.percentile(X, [1, 99])

            # Apply truncation only if outliers are few
            X_truncated = np.clip(X, x_min, x_max)
            counts, bin_edges = np.histogram(X_truncated,
bins=bins, range=(x_min, x_max), density=True)

            # Adjust first and last bins for outliers
            counts[0] += np.sum(X < x_min) / (N * (x_max - x_min) /
bins)
            counts[-1] += np.sum(X > x_max) / (N * (x_max - x_min)
/ bins)

            # Plot histogram
```

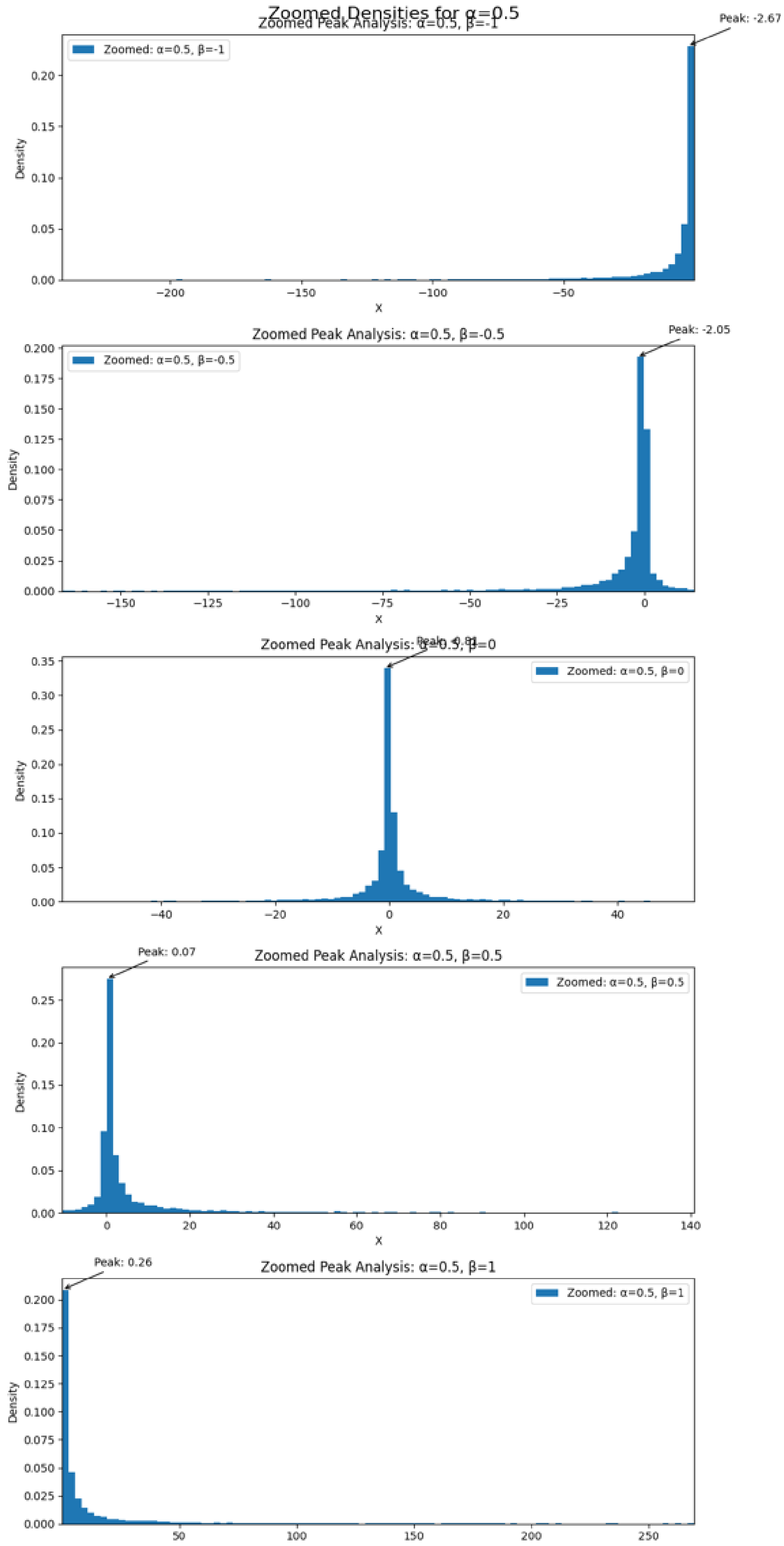
```
            ax.bar(bin_edges[:-1], counts, width=(bin_edges[1] -
bin_edges[0]), align='edge', label=f'Œ±={alpha}, Œ≤={beta}')
            ax.set_xlabel('X')
            ax.set_ylabel('Density')
            ax.set_xlim(x_min, x_max)
            ax.legend()
            ax.set_xticks(np.linspace(x_min, x_max, 5))  # Add x-
axis ticks

        fig.suptitle(f'Difficult Densities for Œ±={alpha}',
fontsize=16)
        plt.tight_layout()
        plt.savefig(f'Difficult_densities_alpha_{alpha}.png')
        plt.show()
```

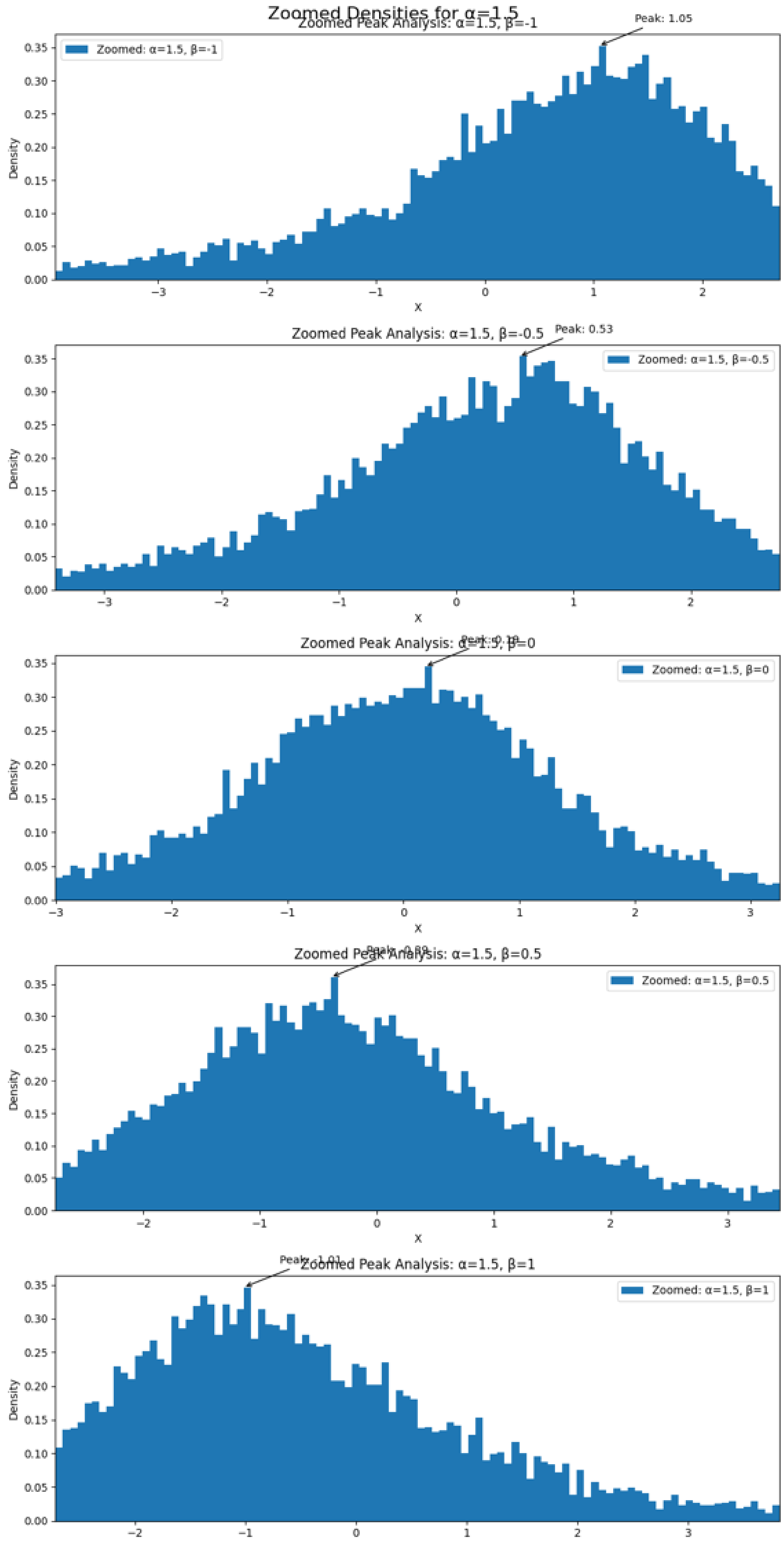
Plot some histogram density estimates with   alpha= 0.5, 1.5 and several values of beta :





Zoomed in versions have also been provided.


Zoomed Densities for α=0.5
Zoomed Peak Analysis: α=0.5, β=-1
Zoomed: α=0.5, β=-1
Peak: -2.67
Density
X
Zoomed Peak Analysis: α=0.5, β=-0.5
Zoomed: α=0.5, β=-0.5
Peak: -2.05
Density
X
Zoomed Peak Analysis: α=0.5, β=0
Zoomed: α=0.5, β=0
Density
X
Zoomed Peak Analysis: α=0.5, β=0.5
Zoomed: α=0.5, β=0.5
Peak: 0.07
Density
X
Zoomed Peak Analysis: α=0.5, β=1
Zoomed: α=0.5, β=1
Peak: 0.26
Density
X



Zoomed Densities for α=1.5
Zoomed Peak Analysis: α=1.5, β=-1
Peak: 1.05
Zoomed: α=1.5, β=-1
Zoomed Peak Analysis: α=1.5, β=-0.5
Peak: 0.53
Zoomed: α=1.5, β=-0.5
Zoomed Peak Analysis: α=1.5, β=0
Zoomed: α=1.5, β=0
Zoomed Peak Analysis: α=1.5, β=0.5
Zoomed: α=1.5, β=0.5
Zoomed Peak Analysis: α=1.5, β=1
Zoomed: α=1.5, β=1
Density
X

Hence comment on the interpretation of the parameters alpha and beta:

The figures are set to truncate the graphs based on percentiles. From this we can see that the 98th percentile range for alpha = 0.5 is far greater than alpha = 1.5, suggesting a larger alpha reduces the overall range of the distribution. The magnitude of beta also seems to affect this range, with high magnitudes corresponding to higher ranges.

Despite the wider range of the alpha = 0.5 graphs, it seems that they also have a greatly concentrated centre of the distribution, with sharper peaks and what looks like a quickly decaying distribution, possibly suggesting a small range for around 80% of data points. This suggests a smaller alpha creates a higher concentrated centre. Beta seems to have minimal effect on this concentration.

Beta seems to mainly affect the skew of the distribution, with Beta values of 0 corresponding to a symmetrical distribution, and negative and positive values corresponding to the opposite direction skewness.

Both alpha and beta affect the mode value, and likely the mean as well. The position of this seems to be a balance between the two variables as the graphs for alpha = 0.5 are different in polarity of the mode compared to alpha = 1.5.

Appendix: full python code

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import uniform


def ksdensity(data, width=0.3):
    """Returns kernel smoothing function from data points in data"""
    def ksd(x_axis):
        def n_pdf(x, mu=0., sigma=1.):  # normal pdf
            u = (x - mu) / abs(sigma)
            y = (1 / (np.sqrt(2 * np.pi) * abs(sigma)))
            y *= np.exp(-u * u / 2)
            return y
        prob = [n_pdf(x_i, data, width) for x_i in x_axis]
        pdf = [np.average(pr) for pr in prob]  # each row is one x
value
        return np.array(pdf)
    return ksd

def norm_pdf(x, mu=0, sigma=1):  # normal pdf
    """Returns normal distribution"""
    u = (x - mu) / abs(sigma)
```

```
    y = (1 / (np.sqrt(2 * np.pi) * abs(sigma)))
    y *= np.exp(-u * u / 2)
    return y

def uni_pdf(x, b=1, a=0):
    """Returns uniform distribution"""
    #y = np.linspace(1, 1, Length)
    y = uniform.pdf(x, loc=a, scale=b-a)
    return y


def task_1():
    def normal_task():
        #Plot histogram against normal

        # np.linspace(start, stop, number of steps)
        x_values = np.linspace(-5., 5., 100)

        # Plot normal distribution
        x = np.random.randn(1000)
        plt.figure(1)
        plt.hist(x, bins=30) # number of bins

        # Area of histogram = number of variables (N) times range
divided by number of bins. This is the scale factor
        Area_norm = 1000*(max(x)-min(x))/30 # This is a slight
approximation but is sufficient

        #plot exact normal distribution
        plt.plot(x_values, Area_norm*norm_pdf(x_values))


        plt.legend()

        plt.title('Normal distribution comparison')
        plt.xlabel('Value')
        plt.ylabel('Count')
        plt.legend()

        plt.savefig('Normal hist.png')
        plt.show()
        plt.close()

        #Plot normal against kernel density

        plt.figure(2)

        plt.plot(x_values, norm_pdf(x_values), label='normal')
        ks_density = ksdensity(x, width=0.4)
        plt.plot(x_values, ks_density(x_values), label='ksd')

        plt.legend()

        plt.title('Normal distribution comparison')
        plt.xlabel('Value')
        plt.ylabel('Count')
        plt.legend()

        plt.savefig('Normal kernel.png')
```

```
        plt.show()
        plt.close()

    def uniform_task():
        # Plot uniform distribution against histogram
        x = np.random.rand(1000)
        plt.figure(1)
        plt.hist(x, bins=20)
        x_values = np.linspace(-4, 4, 1000)

        #plot exact uniform distribution
        Area_uni=1000*1/20  #Area of histogram = number of variables
(N) times range divided by number of bins. This is the scale factor

        plt.plot(x_values, Area_uni*uni_pdf(x_values),
label='uniform')

        plt.xlim(-0.5,1.5)
        plt.legend()

        plt.title('uniform distribution comparison')
        plt.xlabel('Value')
        plt.ylabel('Count')
        plt.legend()

        plt.savefig('Uniform hist.png')
        plt.show()
        plt.close()

        #Plot normal against kernel density

        plt.figure(2)

        ks_density = ksdensity(x, width=0.2)

        plt.plot(x_values, norm_pdf(x_values), label='uniform')

        plt.plot(x_values, ks_density(x_values), label='ksd')

        plt.xlim(-4,4)
        plt.legend()

        plt.title('uniform distribution comparison')
        plt.xlabel('Value')
        plt.ylabel('Count')
        plt.legend()

        plt.savefig('Uniform kernel.png')
        plt.show()
        plt.close()

    def N_task():
        N_list=[100, 1000, 10000]
        n_bins = 20 # number of histogram bins
        fig, ax = plt.subplots(3)

        for N in N_list:
            # Generalising uniform histograms
```

```
            mean = N/n_bins # N is number of generated RVs. Mean for
numbers only in range of interest.
            sd = np.sqrt(N*(1-1/n_bins)/n_bins)
            x = np.random.rand(N)
            p = ax[N_list.index(N)]
            p.hist(x, bins=n_bins, label = 'N={}'.format(N))

            p.axhline(y=mean, color='r', linestyle='-', linewidth=2)
            p.axhline(y=mean+3*sd, color='b', linestyle='--',
linewidth=2)
            p.axhline(y=mean-3*sd, color='b', linestyle='--',
linewidth=2)
            p.set_xlim(-0.5,1.5)


            p.set_xlabel('Value')
            p.set_ylabel('Count')
            p.legend()
        fig.suptitle('Histograms for an increasing N number of
uniform random variables')
        plt.savefig('Uniform histogram for N=X.png')
        plt.show()
        plt.close()

    normal_task()
    uniform_task()
    N_task()

def task_2():

    x = np.random.randn(1000) # generate gaussian random variable

    def linear_transformation():

        a = 3
        b = 6

        x_values = np.linspace(-5, 20, 1000)
        y = a*x+b  #linear transform
        y_pdf = (1/(np.sqrt(2*np.pi*a**2)))*np.exp(-((x_values-
b)**2)/(2*a**2))


        Area_transformed_norm = (1000*(max(y)-min(y))/30)


        plt.hist(y, bins=30, label = 'ax+b transformation')
        plt.plot(x_values, Area_transformed_norm*y_pdf, label='Exact
transformed Gaussian')
        plt.title('Transformed Standard Gaussian using y = ax + b')
        plt.legend()
        plt.xlim(-5, 20)
        plt.savefig('linear transformation')
        plt.show()
        plt.close()

    def parabolic_transformation():
        x_values = np.linspace(-1, 10, 1000)
        y = x**2
```

```
        y_pdf = (1/(np.sqrt(2*np.pi*x_values)))*np.exp(-x_values/2)

        Area_transformed_norm = (1000*(max(y)-min(y))/30)

        plt.figure()
        plt.hist(y, bins=30, label = 'x^2 transformation')
        plt.plot(x_values, Area_transformed_norm*y_pdf, label='Exact
pdf p(y)')
        plt.title('Transformed Standard Gaussian using y = x^2')
        plt.legend()
        plt.xlim(-1, 10)
        plt.ylim(0,600)
        plt.savefig('parabolic transformation')
        plt.show()
        plt.close()


    linear_transformation()
    parabolic_transformation()

def task_3():
    x = np.random.rand(1000) # generate uniform random variables
    x_values = np.linspace(0, 7, 1000)
    exp_pdf = np.exp(-x_values)
    y = -np.log(x) # Inverse CDF of x. x is uniformly distributed
between 0 and 1 so can simplify 1-x to x

    Area_transformed_exp = (1000*(max(y)-min(y))/30)

    plt.figure()
    plt.hist(y, bins=30, label = 'Histogram of inverse CDF pdf')

    plt.plot(x_values, Area_transformed_exp*exp_pdf, label='exp pdf')

    ks_density = ksdensity(y, width=0.3)
    plt.plot(x_values, Area_transformed_exp*ks_density(x_values),
label='ksd', color='r')

    plt.title('exponential distribution generated from inverse CDF')
    plt.legend()
    plt.xlim(-1,7)
    plt.savefig('Inverse CDF Method')
    plt.show()
    plt.close()

def task_4():
    # a) choose parameters
    alphas = [0.5, 1.5]  # Different alpha values to test
    betas = [-1, -0.5, 0, 0.5, 1]  # Different beta values



    # Function to generate X based on the given recipe
    def calc_X(alpha, beta, N):
        # Calculate constants
        b = (1/alpha)*np.arctan(beta*np.tan(np.pi*alpha/2))
        s = (1+beta**2*np.tan(np.pi*alpha/2)**2)**(1/(2*alpha))

        # b) Generate U and c) V
```

```
        U = np.random.uniform(-np.pi/2, np.pi/2, N)
        V = np.random.exponential(1, N)

        # d) Calculate X
        X =
s*(np.sin(alpha*(U+b))/(np.cos(U))**(1/alpha))*((np.cos(U-
alpha*(U+b)))/V)**((1-alpha)/alpha)

        return X


    N = 10000  # Number of samples
    count = 0
    # Simulate for different alphas and betas
    for alpha in alphas:
        fig, ax = plt.subplots(len(betas), sharex=False)
        count = 0
        for beta in betas:
            p = ax[count]
            X = calc_X(alpha, beta, N)
            outliers = 200
            X_2 = (np.sort(X, axis=1))[outliers/2:-outliers/2]
            p.hist(X_2, bins=500, label=f'Histogram (Œ±={alpha},
Œ≤={beta})')
            p.set_xlabel('X')
            p.set_ylabel('Density')
            p.set_xlim(np.min(X_2),np.max(X_2))
            #p.set_ylim(0,100)
            p.legend()
            count += 1
        fig.suptitle('Histograms of a difficult desnity for
Œ±={}'.format(alpha))
        plt.savefig('Difficult densities for alpha =
{}.png'.format(alpha))
        plt.show()
        plt.close()

def task_5():
    # a) choose parameters
    alphas = [0.5, 1.5]  # Different alpha values to test
    betas = [-1, -0.5, 0, 0.5, 1]  # Different beta values

    # Function to generate X based on the given recipe
    def calc_X(alpha, beta, N):
        b = (1 / alpha) * np.arctan(beta * np.tan(np.pi * alpha / 2))
        s = (1 + beta**2 * np.tan(np.pi * alpha / 2)**2)**(1 / (2 *
alpha))
        U = np.random.uniform(-np.pi / 2, np.pi / 2, N)
        V = np.random.exponential(1, N)
        X = (
            s
            * (np.sin(alpha * (U + b)) / (np.cos(U))**(1 / alpha))
            * ((np.cos(U - alpha * (U + b))) / V)**((1 - alpha) /
alpha)
        )
        return X

    N = 10000  # Number of samples
    bins = 100  # Higher resolution for histograms
```

```
    for alpha in alphas:
        fig, axs = plt.subplots(len(betas), figsize=(10, len(betas) *
2), sharex=False)
        for i, beta in enumerate(betas):
            ax = axs[i] if len(betas) > 1 else axs
            X = calc_X(alpha, beta, N)

            # Dynamically determine truncation range based on
percentiles
            x_min, x_max = np.percentile(X, [1, 99])

            # Apply truncation only if outliers are few
            X_truncated = np.clip(X, x_min, x_max)
            counts, bin_edges = np.histogram(X_truncated, bins=bins,
range=(x_min, x_max), density=True)

            # Adjust first and last bins for outliers
            counts[0] += np.sum(X < x_min) / (N * (x_max - x_min) /
bins)
            counts[-1] += np.sum(X > x_max) / (N * (x_max - x_min) /
bins)

            # Plot histogram
            ax.bar(bin_edges[:-1], counts, width=(bin_edges[1] -
bin_edges[0]), align='edge', label=f'Œ±={alpha}, Œ≤={beta}')
            ax.set_xlabel('X')
            ax.set_ylabel('Density')
            ax.set_xlim(x_min, x_max)
            ax.legend()
            ax.set_xticks(np.linspace(x_min, x_max, 5))  # Add x-axis
ticks

        fig.suptitle(f'Difficult Densities for Œ±={alpha}',
fontsize=16)
        plt.tight_layout()
        plt.savefig(f'Difficult_densities_alpha_{alpha}.png')
        plt.show()

def task_4_peak_focus():
    # a) choose parameters
    alphas = [0.5, 1.5]  # Different alpha values to test
    betas = [-1, -0.5, 0, 0.5, 1]  # Different beta values

    # Function to generate X based on the given recipe
    def calc_X(alpha, beta, N):
        b = (1 / alpha) * np.arctan(beta * np.tan(np.pi * alpha / 2))
        s = (1 + beta**2 * np.tan(np.pi * alpha / 2)**2)**(1 / (2 *
alpha))
        U = np.random.uniform(-np.pi / 2, np.pi / 2, N)
        V = np.random.exponential(1, N)
        X = (
            s
            * (np.sin(alpha * (U + b)) / (np.cos(U))**(1 / alpha))
            * ((np.cos(U - alpha * (U + b))) / V)**((1 - alpha) /
alpha)
        )
        return X
```

```
    N = 10000  # Number of samples
    bins = 100  # Higher resolution for histograms

    for alpha in alphas:
        fig, axs = plt.subplots(len(betas), figsize=(10, len(betas) *
4), sharex=False)
        for i, beta in enumerate(betas):
            ax = axs[i] if len(betas) > 1 else axs
            X = calc_X(alpha, beta, N)

            # Main range for histogram
            x_min, x_max = np.percentile(X, [5, 95])  # Focus closer
on the peak
            X_main = X[(X >= x_min) & (X <= x_max)]

            # Compute histogram for the main range
            counts, bin_edges = np.histogram(X_main, bins=bins,
range=(x_min, x_max), density=True)

            # Plot zoomed histogram
            ax.bar(bin_edges[:-1], counts, width=(bin_edges[1] -
bin_edges[0]), align='edge', label=f'Zoomed: Œ±={alpha}, Œ≤={beta}')

            # Annotations for the peak
            peak_bin = np.argmax(counts)
            peak_value = bin_edges[peak_bin]
            ax.annotate(f'Peak: {peak_value:.2f}', xy=(peak_value,
counts[peak_bin]),
                        xytext=(peak_value + (x_max - x_min) * 0.05,
counts[peak_bin] * 1.1),
                        arrowprops=dict(facecolor='black',
arrowstyle='->'), fontsize=10)

            # Set axis labels and limits
            ax.set_xlim(x_min, x_max)
            ax.set_xlabel('X')
            ax.set_ylabel('Density')
            ax.legend()
            ax.set_title(f'Zoomed Peak Analysis: Œ±={alpha},
Œ≤={beta}')

        fig.suptitle(f'Zoomed Densities for Œ±={alpha}', fontsize=16)
        plt.tight_layout()
        plt.savefig(f'Zoomed_densities_alpha_{alpha}.png')
        plt.show()




#task_1()
#task_2()
#task_3()
#task_4()
#task_5()
#task_4_peak_focus()
```