Geometria Analitica

I: Plano Cartesiano:
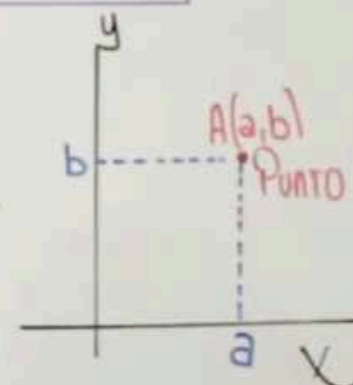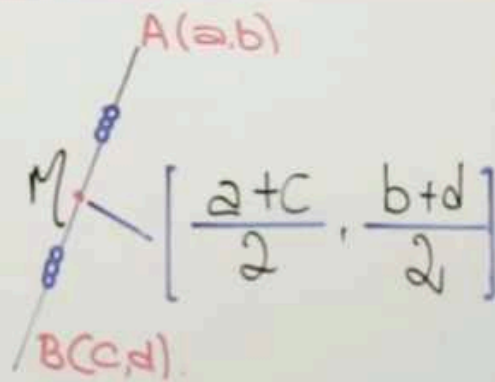

II C

I C

Eje de las Absisas.

Eje X

Origen (0,0)

Eje de las Ordenadas.

III C

IV C
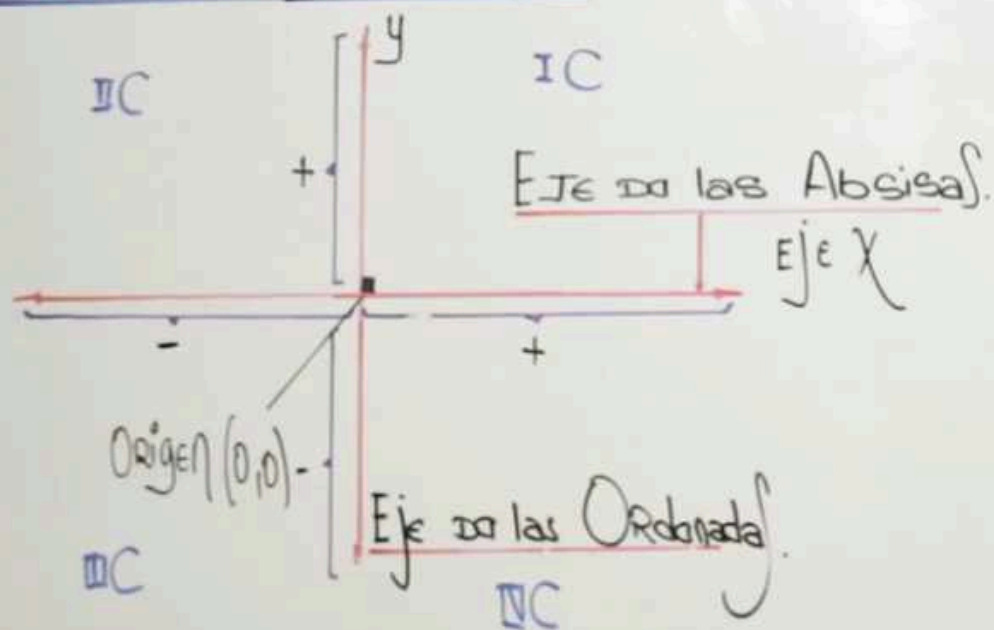
II: Distancia Entre 2 Puntos:
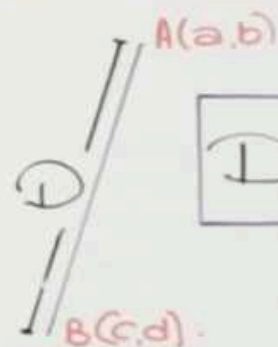
A(a,b)

D

B(c,d)

$$D = \sqrt{(a-c)^2 + (b-d)^2}$$

III: Punto Medio:

A(a,b)

M $\left[\dfrac{a+c}{2}, \dfrac{b+d}{2} \right]$

B(c,d)

A(a,b) Punto

b

a

CASO II:

$$M = Tg\,\theta$$

CASO III:

$A(a,b)$

$$M = \frac{b-d}{a-c}$$

$B(c,d)$

Forma de Hallar E.R.

$$y - \quad = \quad (x - \quad)$$
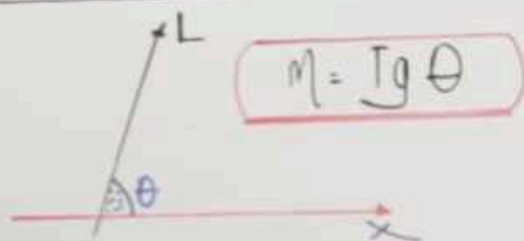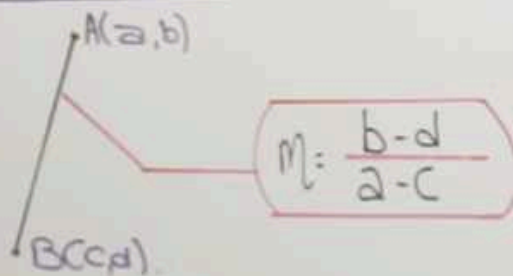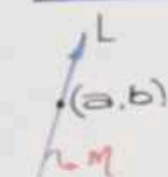
$$y - b = m(x - a)$$

Propiedades.

Paralelas:

$M_1 \quad L_1 \quad L_2$

M_2
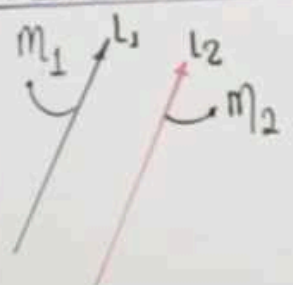
$$L_1 \parallel L_2 \Rightarrow m_1 = m_2$$

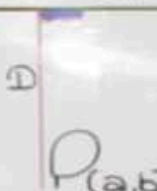**Ortogonales.**

$L_2 \quad L_1$

M_1

M_2
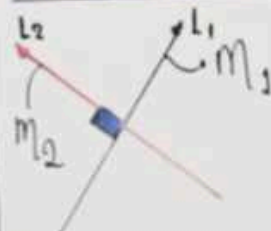
$$L_1 \perp L_2 \Rightarrow m_1 \cdot m_2 = -1$$

Distancia de un Punto a una Recta

$$AX + BY + C = 0$$
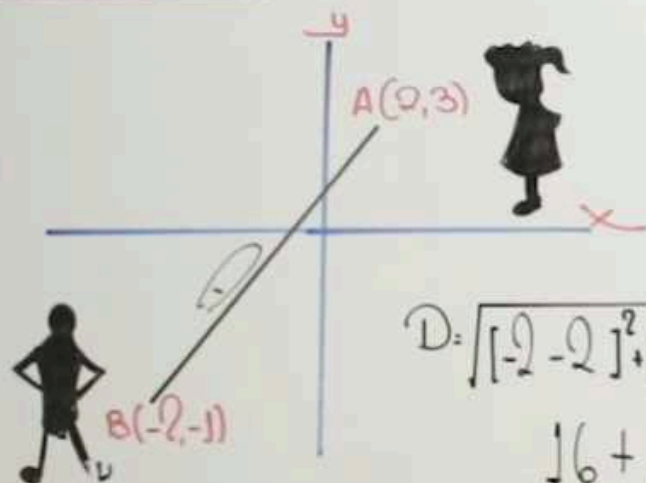
D

$P_{(a,b)}$

$$D = \frac{|Aa + Bb + C|}{\sqrt{A^2 + B^2}}$$

Distancia Entre 2 Rectas.

$$AX + BY + C_1 = 0 \quad L_1$$

D

$$AX + BY + C_2 = 0 \quad L_2$$
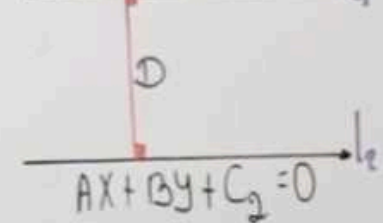
$$D = \frac{|C_2 - C_1|}{\sqrt{A^2 + B^2}}$$
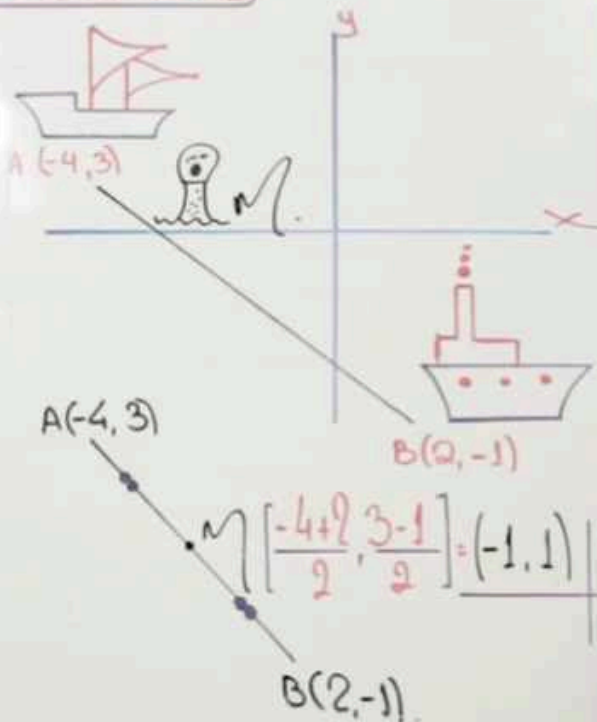
Problema 01

$A(2,3)$

$B(-2,-1)$

$$D = \sqrt{[-2-2]^2 + [-1-3]^2}$$
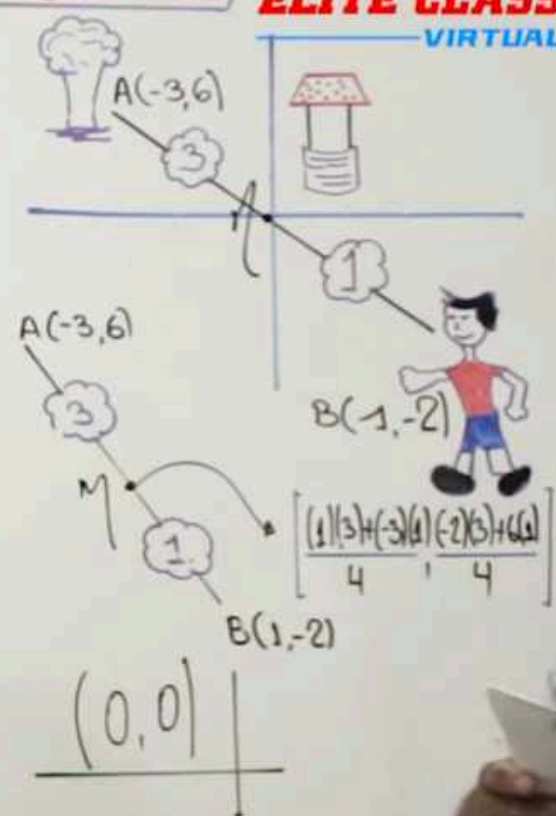
$$16 + 16$$

$$2(16)$$

$$D = 4\sqrt{2}$$

Problema 02

$A(-4,3)$

$B(2,-1)$

$A(-4,3)$

$B(2,-1)$

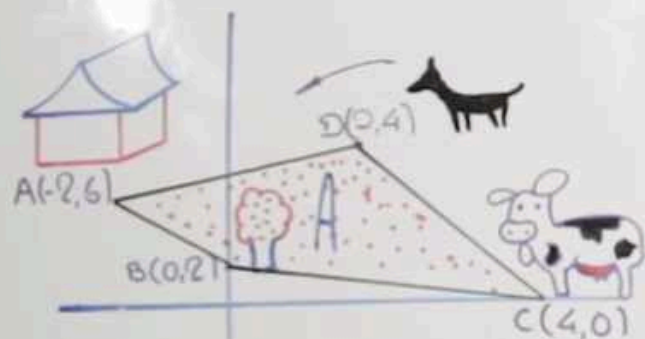$$M\left[\frac{-4+2}{2}, \frac{3-1}{2}\right] = (-1,1)$$

Problema 03

$A(-3,6)$

3

1

$B(1,-2)$

$A(-3,6)$

3

M

1

$B(1,-2)$

$$\left[\frac{(1)(3)+(-3)(1)}{4}, \frac{(-2)(3)+(6)(1)}{4}\right]$$

$$(0,0)$$

Circunferencia

Problema 04

$A(-2,6)$

$D(0,4)$

$B(0,2)$

$C(4,0)$

$A = \frac{1}{2} |24 - 0|$

$A = 12$
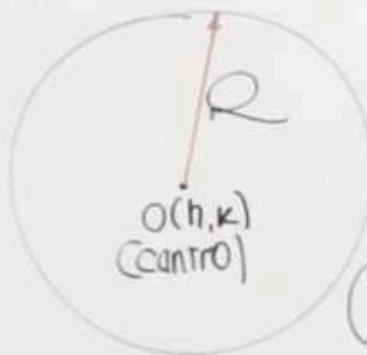
Canonica $O(0,0)$

$$x^2 + y^2 = R^2$$

Ordinaria $O(h,k)$

$$(x-h)^2 + (y-k)^2 = R^2$$

General

$$x^2 + y^2 + Cx + Dy + F = 0$$

Propiedad:

$$h = \frac{-C}{2} \qquad K = \frac{-D}{2}$$

$$R = \sqrt{h^2 + k^2 - F}$$

$O(h,k)$
(centro)

Problema 05

5

$O(-2,3)$

$$(x+2)^2 + (y-3)^2 = 25$$

Problema 06

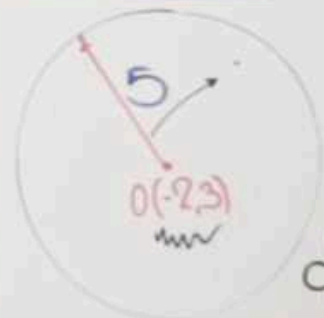$$x^2 + y^2 - 2x + 4y + 1 = 0$$

$$h = \frac{-(-2)}{2} = 1 \qquad k = \frac{-4}{2}$$

$$k = -2$$

$$R = \sqrt{1^2 + (-2)^2 - 1}$$

$$O(1, -2) \qquad R = 2$$

Parábola.

Elementos de la Parábola:

F:	FOCO O PUNTO FIJO
V:	VÉRTICE
LD:	LÍNEA DIRECTRIZ.
p:	PARÁMETRO
ST:	CUERDA.

L_D

2p

M

T

2p

eje Focal

p

p

R

2p

F

N — CUERDA focal

S

LADO RECTO |4p|

RADIO focal

Ecuación de la Parábola.

CASO I

$$x^2 = 4p\,y$$

CASO II

V(0,0)

$$x^2 = -4p\,y$$

CASO III

V(0,0)

$$y^2 = 4p\,X$$

CASO IV

V(0,0)

$$y^2 = -4p\,X$$

CASO I

$V(h,k)$

$(0,0)$

$$(X-h)^2 = 4p(Y-K)$$
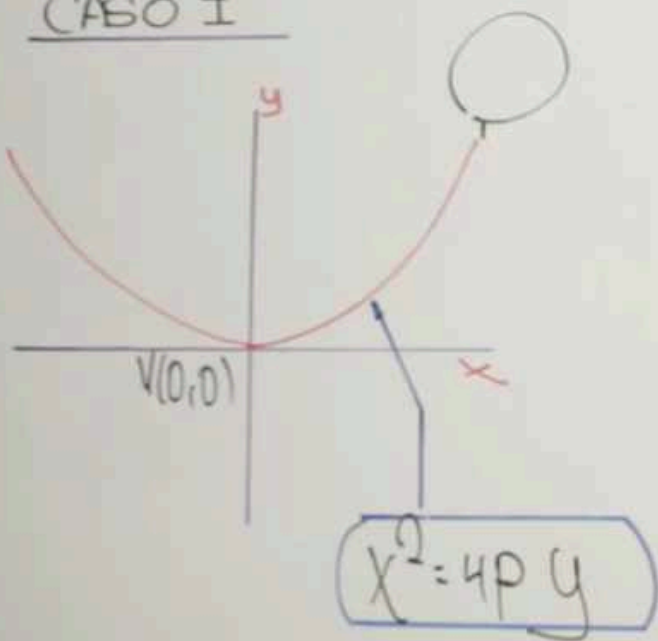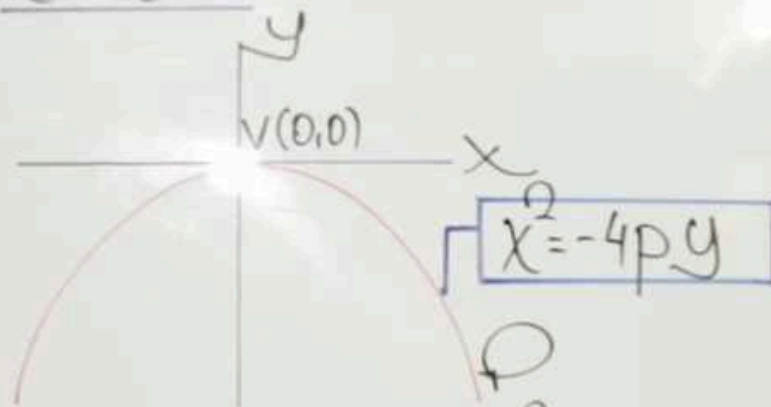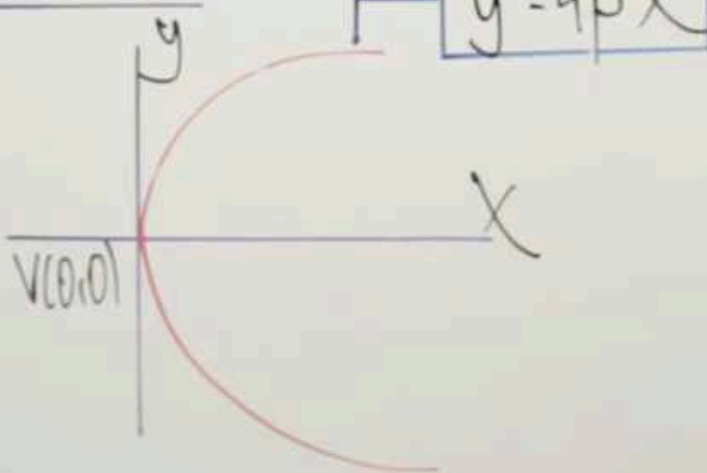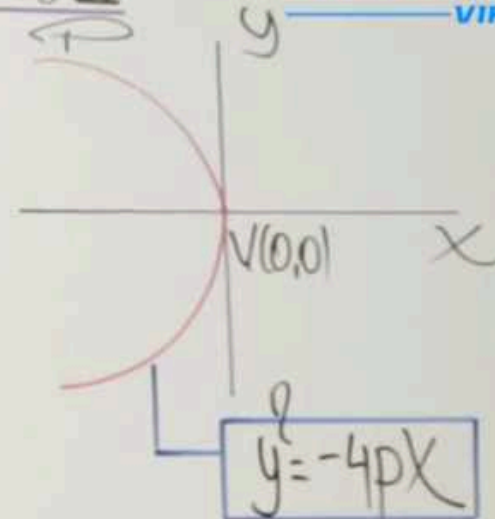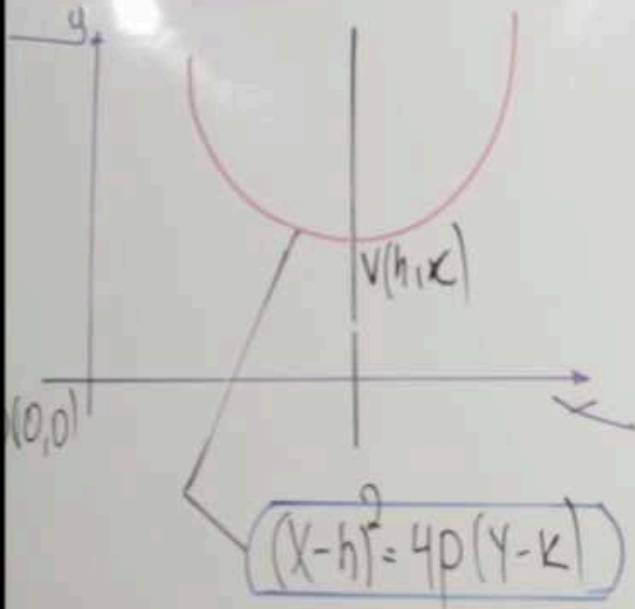
Problema 07

$$x^2 = -16y$$

$V(0,0)$ $4p = -16$

$F(0,-4)$ $p = -4$

$L_D: Y = 4 \quad \wedge \quad Y - 4 = 0$ ✓

Problema 08

$V(0,4)$ $(3,h)$

$3m$ $4m$

$4m$ $4m$ h

$8m$ $(4,0)$

$$X^2 = -4p(y-4)$$

$16 = -4p(-4)$

$\boxed{4 = 4p}$

$X^2 = -4(y-4)$ $h = \dfrac{7}{4}$

$9 = -4(h-4)$

$-\dfrac{9}{4} + 4 = h$ ✓
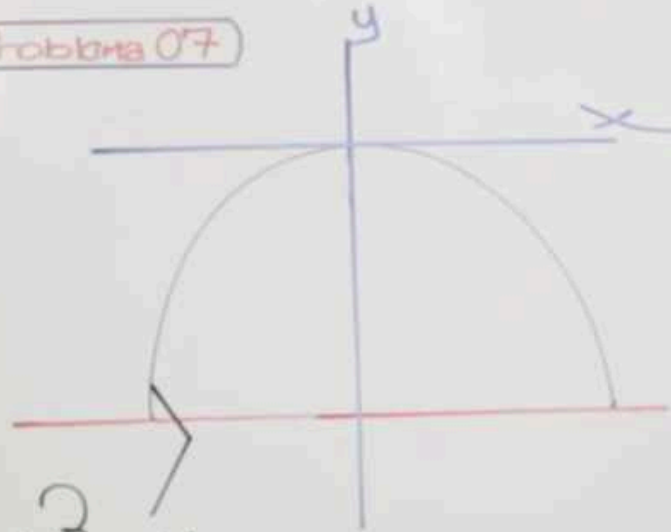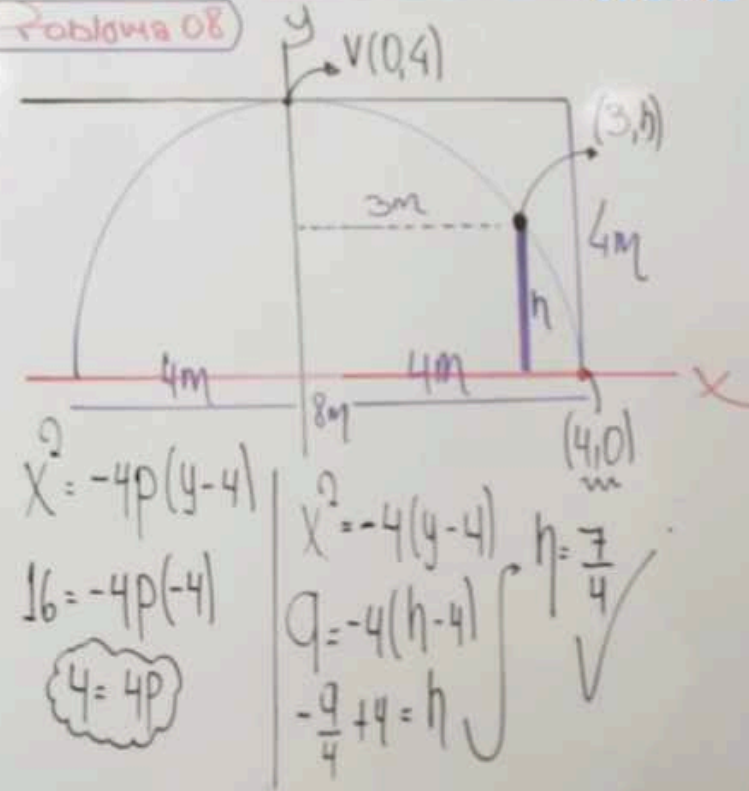
Problema 09

$A(-2,1)$ $B(3,3)$

$$m = \frac{3-1}{3-(-2)} = \frac{2}{5} = m$$

$$Y-1 = \frac{2}{5}(X+2)$$

$$5y-5 = 2x+4$$

$$0 = 2X-5y+9 \checkmark$$

Problema 10

$$D = \sqrt{(-4-2)^2 + (-3-2)^2}$$

$$36 + 25 = \sqrt{61} \checkmark$$

$A(2,2)$ $B(-4,-3)$

$$m = \frac{-3-2}{-4-2} = \frac{-5}{-6} = \frac{5}{6}$$

$$Y-2 = \frac{5}{6}(X-2)$$

$$6y-12 = 5X-10$$

$$0 = 5X-6y+2 \checkmark$$

Problema 11

$$\frac{AP}{PB} = \frac{2k}{3k}$$

$B(5,-1)$

$P\left[\dfrac{-9k+10k}{5k}, \dfrac{12k-2k}{5k}\right]$

$A(-3,4)$ $\left[\dfrac{1}{5}, 2\right]$

Problema 12.

$A(-1,4)$

$C(3,3)$

$B(2,-1)$

$$\begin{vmatrix} 2 & -1 \\ -3 & 3 & 3 \\ -3 & -4 & 4 \\ 8 & 2 & -1 \\ 2 \end{vmatrix} \begin{matrix} -6 \\ 12 \\ 1 \\ 19 \end{matrix}$$

$$S = \frac{1}{2} | 19 - 2 |$$

$$S = \frac{17}{2}$$

Problema 13

$L_1 \parallel L_2$

$L_1: 2x + 6y + 7 = 0 \qquad L_2: 2x + my - 1 = 0$

$$l_1 = -\frac{2}{6} = -\frac{1}{3} \qquad m_2 = -\frac{2}{m}$$

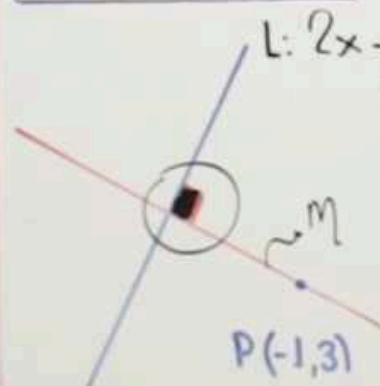$$-\frac{1}{3} = -\frac{2}{m} \rightsquigarrow \{m = 6\}$$

Problema 14:

$L: 2x - y - 3 = 0$

$m_1 = 2$

$m_1: m = -1$

$m = -\frac{1}{2}$

$\sim M$

$P(-1,3)$

$$y - 3 = -\frac{1}{2}(x + 1)$$

$$2y - 6 = -x - 1$$

$$x + 2y - 5 = 0$$

Problema

$C: x^2 + y^2 + 4x + 2y + 1 = 0$

$$h = -\frac{4}{2} = -2 \quad \} \quad K = -\frac{2}{2}$$

$$h = -2 \quad \} \quad K = -1.$$

$$R = \sqrt{4 + 1 - 1} = 2$$

$$R = 2$$

$$L_0 = 2\pi R = 4\pi$$

Titagoras:

$$a^2 = b^2 + c^2$$

$$16 = b^2 + 9$$

$$7 = b^2$$

Problema 18

$V(\pm 4, 0)$

$F(\pm 3, 0)$

$C = 3$

$(-3,0)$ 0 $F(3,0)$ $(4,0)$

$a = 4$

$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

Problema 19

$F(\pm 2, 0)$

$a = \frac{6}{3}$

$e = ccentricidad$

$$e = \frac{c}{a}$$

$$e = \frac{c}{a} = \frac{2}{3}$$

$a = 3$

$F(-2,0)$ $F(2,0)$

$C = 2$
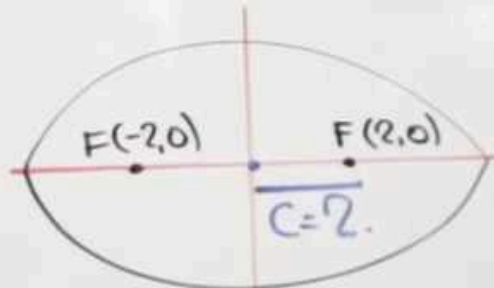
$$a^2 = b^2 + c^2$$

$$9 = b^2 + 4$$

$$5 = b^2$$

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

Problema 20

$V(0, \pm 4)$

$F(0, \pm 3)$

$$a^2 = b^2 + c^2$$

$$16 = b^2 + 9$$

$$7 = b^2$$

$$\frac{y^2}{16} + \frac{x^2}{7} = 1$$

$V(0,4)$

$a = 4$ $F(0,3)$

$C = 3$

$F(0,-3)$

$V(0,-4)$