

TRANSFORMACIONES TRIGONOMETRICAS

* DE SUMA O DIFERENCIA A PRODUCTO

$$\text{SENA} + \text{SENB} = 2 \text{SEN}\left(\frac{A+B}{2}\right) \cdot \text{COS}\left(\frac{A-B}{2}\right)$$

$$\text{SENA} - \text{SENB} = 2 \text{SEN}\left(\frac{A-B}{2}\right) \cdot \text{COS}\left(\frac{A+B}{2}\right)$$

$$\text{COSA} + \text{COSB} = 2 \text{COS}\left(\frac{A+B}{2}\right) \cdot \text{COS}\left(\frac{A-B}{2}\right)$$

$$\text{COSA} - \text{COSB} = -2 \text{SEN}\left(\frac{A+B}{2}\right) \cdot \text{SEN}\left(\frac{A-B}{2}\right)$$

* DE PRODUCTO A SUMA O DIFERENCIA

$$2 \text{SENA} \cdot \text{COSB} = \text{SEN}(A+B) + \text{SEN}(A-B)$$

$$2 \text{COSA} \cdot \text{COSB} = \text{COS}(A+B) + \text{COS}(A-B)$$

$$2 \text{SENA} \cdot \text{SENB} = \text{COS}(A-B) - \text{COS}(A+B)$$

①

$$K = \frac{\text{SEN } 75^\circ + \text{SEN } 15^\circ}{\text{SEN } 75^\circ - \text{SEN } 15^\circ}$$

$$= \frac{2 \text{SEN } 45^\circ \cdot \text{COS } 30^\circ}{2 \text{SEN } 30^\circ \cdot \text{COS } 45^\circ}$$

$$K = \frac{\text{COS } 30^\circ}{\text{SEN } 30^\circ} = \text{COT } 30^\circ$$

$$\therefore K = \sqrt{3}$$

②

$$\frac{\text{COS } 4\beta + \text{COS } 2\beta}{\text{COS } 2\beta - \text{COS } 4\beta} = \frac{\text{COT } 3\beta}{m}$$

$$\frac{2 \text{COS } 3\beta \cdot \text{COS } \beta - 2 \text{SEN } 3\beta \cdot \text{SEN } (-\beta)}{-2 \text{SEN } 3\beta \cdot \text{SEN } (-\beta)} = \frac{\text{COT } 3\beta}{m}$$

$$\frac{\text{COS } 3\beta \cdot \text{COS } \beta}{\text{SEN } 3\beta \cdot \text{SEN } \beta} = \frac{\text{COT } 3\beta}{m}$$

$$\text{COT } 3\beta \cdot \text{COT } \beta = \frac{\text{COT } 3\beta}{m}$$

$$m = \text{TAN } \beta$$

③ $\text{COS } 88^\circ + \text{SEN } 87^\circ = 0$

$$M = \frac{\text{COS } 28^\circ - \text{COS } 32^\circ}{-2 \text{SEN } 30^\circ \cdot \text{SEN } (-2^\circ)} + \frac{\text{COS } 57^\circ + \text{COS } 63^\circ}{2 \text{COS } 60^\circ \cdot \text{COS } (3^\circ)}$$

$$M = \frac{\text{SEN } 30^\circ \cdot \text{SEN } 2^\circ}{1} + \frac{\text{COS } 60^\circ \cdot \text{COS } 3^\circ}{1}$$

$$M = \frac{\text{SEN } 2^\circ}{\text{COS } 88^\circ} + \frac{\text{COS } 3^\circ}{\text{SEN } 87^\circ}$$

$$\therefore M = 0$$

④

$$E = \frac{\text{SEN } 5x + \text{SEN } 6x + \text{SEN } 7x}{\text{COS } 5x + \text{COS } 6x + \text{COS } 7x}$$

$$= \frac{2 \text{SEN } 6x \cdot \text{COS } (x) + \text{SEN } 6x}{2 \text{COS } 6x \cdot \text{COS } (x) + \text{COS } 6x}$$

$$= \frac{\text{SEN } 6x (2 \text{COS } x + 1)}{\text{COS } 6x (2 \text{COS } x + 1)}$$

$$\therefore E = \text{TAN } 6x$$

TRIGONOMETRICAS

PRODUCTO

$$\cos\left(\frac{A-B}{2}\right)$$

$$\cos\left(\frac{A+B}{2}\right)$$

$$\cos\left(\frac{A-B}{2}\right)$$

$$\sin\left(\frac{A-B}{2}\right)$$

SENCIA

$$\sin(A-B)$$

$$\sin(A+B)$$

$$\sin(A+B)$$

16

$$P = \frac{2\sin 40^\circ \cdot \cos 10^\circ - 2\cos 20^\circ \cdot \sin 10^\circ}{2\cos 20^\circ \cdot \cos 10^\circ - 2\sin 40^\circ \cdot \sin 10^\circ}$$

$$P = \frac{\cancel{S_{50}} + \cancel{S_{30}} - (\cancel{S_{30}} + \cancel{S_{(-10)}})}{\cancel{C_{30}} + \cancel{C_{10}} - (\cancel{C_{30}} - \cancel{C_{50}})}$$

$$P = \frac{\sin 50^\circ + \sin 10^\circ}{\cos 50^\circ + \cos 10^\circ}$$

$$P = \frac{\cancel{2} \sin 30^\circ \cdot \cancel{\cos 20^\circ}}{\cancel{2} \cos 30^\circ \cdot \cancel{\cos 20^\circ}}$$

$$P = \tan 30^\circ$$

$$\therefore P = \frac{\sqrt{3}}{3}$$

18

ΔPQR :

$$\rightarrow P + Q + R = 180^\circ$$

$$\sin P - \cos Q = \cos P - \sin Q$$

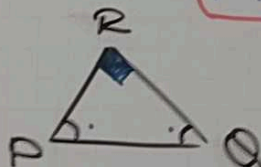
$$\sin P + \sin Q = \cos P + \cos Q$$

$$2\sin\left(\frac{P+Q}{2}\right) \cdot \cos\left(\frac{P-Q}{2}\right) = 2\cos\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right)$$

$$\sin\left(\frac{P+Q}{2}\right) = \cos\left(\frac{P+Q}{2}\right)$$

SUMAN 90°

$$\rightarrow P + Q = 90^\circ$$



$\therefore \Delta PQR$: RECTANGULO

21

$$\tan x = \frac{2\cos 20^\circ - \sin 50^\circ}{\sin 40^\circ}$$

$$= \frac{\cos 20^\circ + \cos 20^\circ - \cos 40^\circ}{\sin 40^\circ}$$

$$= \frac{\cos 20^\circ - 2\sin 30^\circ \sin 10^\circ}{\sin 40^\circ}$$

$$= \frac{\cancel{\sin 70^\circ} + \sin 10^\circ}{\sin 40^\circ}$$

$$= \frac{2\sin 40^\circ \cdot \cos 30^\circ}{\sin 40^\circ}$$

$$\rightarrow \tan x = 2\cos 30^\circ$$

$$\tan x = \sqrt{3}$$

$$\therefore x = 60^\circ$$

9

SEN 8

$$T = \frac{2\sin}{2\cos}$$

SEN

$$T = \frac{\sin}{\sin}$$

$$T = \frac{\sin}{\sin}$$

$$T = \frac{2}{2}$$

\therefore

SEN 2x

SEN

$$9 \quad T = \frac{\cancel{\text{SEN } 8\theta} + \cancel{\text{SEN } 6\theta} + 2\text{SEN } 7\theta \cdot \text{COS } \theta - \cancel{\text{SEN } 6\theta}}{2\text{COS } 3\theta \cdot \text{SEN } \theta + \text{SEN } 2\theta + \cancel{\text{SEN } 4\theta} + \cancel{\text{SEN } (-2\theta)}}$$

$$T = \frac{\text{SEN } 8\theta}{\text{SEN } 4\theta - \cancel{\text{SEN } 2\theta} + \cancel{\text{SEN } 2\theta}}$$

$$T = \frac{\overbrace{\text{SEN } 8\theta}^{2 \times 4\theta}}{\text{SEN } 4\theta}$$

$$T = \frac{2\text{SEN } 4\theta \cdot \text{COS } 4\theta}{\cancel{\text{SEN } 4\theta}}$$

$$\therefore T = 2\text{COS } 4\theta$$

$$\text{SEN } 2x = 2\text{SEN } x \cdot \text{COS } x$$

$\frac{\theta}{2}$

$$\text{SEN } \theta = 2\text{COS } \frac{\theta}{2} \cdot \text{SEN } \frac{\theta}{2}$$

$$20 \quad \text{COS } 2\theta = 2\text{COS}^2 \theta - 1 \quad 1 + \text{COS } 2\theta = 2\text{COS}^2 \theta$$

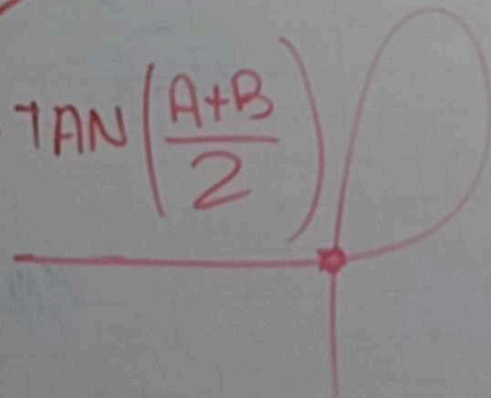
$$W = \frac{\text{SEN } A + \text{SEN } B + \text{SEN } (A+B)}{1 + \text{COS } (A+B) + \text{COS } A + \text{COS } B}$$

$$2\text{COS}^2 \left(\frac{A+B}{2} \right) \quad 2\text{COS} \left(\frac{A+B}{2} \right) \cdot \text{COS} \left(\frac{A-B}{2} \right)$$

$$W = \frac{\cancel{2}\text{SEN} \left(\frac{A+B}{2} \right) \cdot \text{COS} \left(\frac{A-B}{2} \right) + \cancel{2}\text{SEN} \left(\frac{A+B}{2} \right) \cdot \text{COS} \left(\frac{A+B}{2} \right)}{\cancel{2}\text{COS}^2 \left(\frac{A+B}{2} \right) + \cancel{2}\text{COS} \left(\frac{A+B}{2} \right) \cdot \text{COS} \left(\frac{A-B}{2} \right)}$$

$$W = \frac{\text{SEN} \left(\frac{A+B}{2} \right) (\text{COS} \left(\frac{A-B}{2} \right) + \text{COS} \left(\frac{A+B}{2} \right))}{\text{COS} \left(\frac{A+B}{2} \right) (\text{COS} \left(\frac{A+B}{2} \right) + \text{COS} \left(\frac{A-B}{2} \right))}$$

$$\therefore W = \text{TAN} \left(\frac{A+B}{2} \right)$$



$\frac{E}{\cos \angle ANG}$

DMK

③ $\cos 88^\circ + \sin 87^\circ = 0$

$$M = \frac{\cos 28^\circ - \cos 32^\circ}{-2 \sin 30^\circ \cdot \sin(-2^\circ)} + \frac{\cos 57^\circ + \cos 63^\circ}{2 \cos 60^\circ \cdot \cos(3^\circ)}$$

$$M = \frac{\cancel{2} \sin 30^\circ \cdot \sin 2^\circ}{\cancel{2}} + \frac{\cancel{2} \cos 60^\circ \cdot \cos 3^\circ}{\cancel{2}}$$

$$M = \frac{\sin 2^\circ}{\cos 88^\circ} + \frac{\cos 3^\circ}{\sin 87^\circ}$$

$\therefore M = a$

④
$$E = \frac{\sin 5x + \sin 6x + \sin 7x}{\cos 5x + \cos 6x + \cos 7x}$$

$$= \frac{2 \sin 6x \cos(x) + \sin 6x}{2 \cos 6x \cos(x) + \cos 6x}$$

$$= \frac{\sin 6x (2 \cos x + 1)}{\cos 6x (2 \cos x + 1)}$$

$\therefore E = \tan 6x$

⑤ $\cos^2 \theta - \sin^2 \theta = \cos 2\theta$

$$L = \frac{\sin 52^\circ + \cos 22^\circ}{\cos^2 4^\circ - \sin^2 4^\circ}$$

$$L = \frac{\cos 38^\circ + \cos 22^\circ}{\cos 8^\circ}$$

$$L = \frac{2 \cos 30^\circ \cdot \cos 8^\circ}{\cos 8^\circ}$$

$$L = \cancel{2} \left(\frac{\sqrt{3}}{\cancel{2}} \right)$$

$\therefore L = \sqrt{3}$

⑨ \sin

$$T = \frac{2 \sin}{2 \cos}$$

$$T = \frac{\sin}{\sin}$$

$$T = \frac{\sin}{\sin}$$

$$T = \frac{\sin}{\sin}$$

\sin

\sin

$$\sin 40^\circ + \sin 20^\circ = \underbrace{2 \sin 30^\circ}_{1} \cdot \cos 10^\circ = \cos 10^\circ$$

$$\cos 30^\circ + \cos 50^\circ = 2 \cos 40^\circ \cdot \cos(\times 10^\circ) = 2 \cos 40^\circ \cdot \cos 10^\circ$$

$$2 \sin 30^\circ \cdot \cos 50^\circ = \sin 80^\circ + \sin(-20^\circ) = \sin 80^\circ - \sin 20^\circ$$

$$2 \cos 40^\circ \cdot \cos 5^\circ = \cos 45^\circ + \cos 35^\circ //$$