Trigonometría

Identidades Trigonométricas

Fundamentales

I. Recíprocas:

- $\operatorname{sen}\theta \cdot \csc\theta = 1$
- $\cos\theta \cdot \sec\theta = 1$
- $\tan\theta \cdot \cot\theta = 1$

II. Cociente:

- $\tan\theta = \dfrac{\operatorname{sen}\theta}{\cos\theta}$
- $\cot\theta = \dfrac{\cos\theta}{\operatorname{sen}\theta}$

III. Pitagóricas:

- $\operatorname{sen}^2\theta + \cos^2\theta = 1$
- $\sec^2\theta = \tan^2\theta + 1$
- $\csc^2\theta = \cot^2\theta + 1$

Auxiliares

$$\operatorname{sen}^4\theta + \cos^4\theta = 1 - 2.\operatorname{sen}^2\theta.\cos^2\theta$$

$$\operatorname{sen}^6\theta + \cos^6\theta = 1 - 3.\operatorname{sen}^2\theta.\cos^2\theta$$

$$\tan\theta + \cot\theta = \sec\theta.\csc\theta$$

$$\sec^2\theta.\csc^2\theta = \sec^2\theta + \csc^2\theta$$

$$(1 \pm \operatorname{sen}x \pm \cos x)^2 = 2(1 \pm \operatorname{sen}x)(1 \pm \cos x)$$

$$\frac{1 \pm \operatorname{sen}\theta}{\cos\theta} = \frac{\cos\theta}{1 \mp \operatorname{sen}\theta}$$

$$\frac{1 \pm \cos\theta}{\operatorname{sen}\theta} = \frac{\operatorname{sen}\theta}{1 \mp \cos\theta}$$

Además:

Si: $a.\operatorname{sen}x + b.\cos x = c \wedge a^2 + b^2 = c^2$

$\operatorname{sen}x = \dfrac{a}{c}$ $\cos x = \dfrac{b}{c}$

$\tan x = \dfrac{a}{\ldots}$

01) DATO:

$$\text{SEN}\,x + \text{COS}\,x = a$$

PIDEN:

$$M = \frac{\text{SEN}\,x}{1 - \text{COT}\,x} + \frac{\text{COS}\,x}{1 - \text{TAN}\,x}$$

$$\text{COT}\,x \rightarrow \frac{\text{COS}\,x}{\text{SEN}\,x} \qquad \text{TAN}\,x \rightarrow \frac{\text{SEN}\,x}{\text{COS}\,x}$$

$$\frac{\text{SEN}^2 x}{\text{SEN}\,x - \text{COS}\,x} + \frac{\text{COS}^2 x}{\text{COS}\,x - \text{SEN}\,x}$$

$$\frac{\text{SEN}^2 x}{\text{SEN}\,x - \text{COS}\,x} - \frac{\text{COS}^2 x}{\text{SEN}\,x - \text{COS}\,x}$$

$$\frac{\cancel{(\text{SEN}\,x - \text{COS}\,x)}(\text{SEN}\,x + \text{COS}\,x)}{\cancel{\text{SEN}\,x - \text{COS}\,x}}$$

$$//.$$

$$(a^2 - b^2) = (a-b)(a+b)$$

$\cdot\cos^2\theta$

$\cdot\cos^2\theta$

θ

$n x)(1 \pm \cos x)$

04) DATO:

$$\sqrt{5} + \sqrt{2}.\cot\theta = \sqrt{7}.\csc\theta$$

$$\sqrt{5} + \sqrt{2}\,\frac{\cos\theta}{\text{sen}\,\theta} = \sqrt{7}.\frac{1}{\text{sen}\,\theta}$$

$$\underset{a}{\sqrt{5}}\,\text{sen}\,\theta + \underset{b}{\sqrt{2}}.\cos\theta = \underset{c}{\sqrt{7}}$$

$$\hookrightarrow \tan\theta = \frac{\sqrt{5}}{\sqrt{2}}$$

PIDEN:

$$A = \frac{1+\text{sen}\,\theta}{\cos\theta} - \frac{\cos\theta}{1+\text{sen}\,\theta}$$

$$A = \frac{1+\text{sen}\,\theta}{\cos\theta} - \frac{1-\text{sen}\,\theta}{\cos\theta}$$

$$\frac{2\,\text{sen}\,\theta}{\cos\theta} = 2.\tan\theta$$

$$2.\frac{\sqrt{5}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{10}$$

07)

$$\sqrt{(\text{SEN}\theta + \text{SEC}\theta)^2 + (\cos\theta + \text{CSC}\theta)^2} - \frac{\text{SEN}\theta + \cos\theta}{\cos\theta}$$

$$\left(\frac{1}{\text{SEC}\theta} + \frac{1}{\text{SEN}\theta}\right)^2$$

$$\left(\frac{\text{SEN}\theta + \text{SEC}\theta}{\text{SEN}\theta \cdot \text{SEC}\theta}\right)^2$$

$$\frac{\text{TAN}^2\theta \cdot (\text{SEN}\theta + \text{SEC}\theta)^2}{\text{TAN}^2\theta} + \frac{(\text{SEN}\theta + \text{SEC}\theta)^2}{\text{TAN}^2\theta}$$

$$\sqrt{\frac{(\text{SEN}\theta + \text{SEC}\theta)^2 \overbrace{(\text{TAN}^2\theta + 1)}^{\text{SEC}^2\theta}}{\text{TAN}^2\theta}}$$

$$\frac{(\text{SEN}\theta + \text{SEC}\theta) \cdot \text{SEC}\theta}{\text{TAN}\theta}$$

$$\frac{\text{TAN}\theta}{\text{TAN}\theta} + \frac{\text{SEC}^2\theta}{\text{TAN}\theta}$$

$$1 + \overbrace{\frac{\text{TAN}^2\theta}{\text{TAN}\theta} + \frac{1}{\text{TAN}\theta}} - \left(\frac{\text{SEN}\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}\right)$$

$$\cancel{1} + \cancel{\text{TAN}\theta} + \text{COT}\theta - \cancel{\text{TAN}\theta} - \cancel{1} = \text{COT}\theta$$

08)

$\frac{\ldots\theta + \cos\theta}{\ldots\cos\theta}$

$\frac{\cos\theta}{\cos\theta}$

$\ldots T\theta$ //.

(08)

$(\sqrt{x} - \sqrt{15}) . \tan\theta = 1 - \sec\theta$

$(\sqrt{x} + \sqrt{15}) . \tan\theta = 1 + \sec\theta$ ×

$(x - 15) . \tan^2\theta = \underbrace{1 - \sec^2\theta}_{1 - (\tan^2\theta + 1)}$

$(x-15) . \cancel{\tan^2\theta} = -\cancel{\tan^2\theta}$

$x = 14$ //.

(12) $(\tan\theta + \cot\theta)^2 = (c)^2$

$\tan^2\theta + \cot^2\theta + 2 . \underbrace{\tan\theta . \cot\theta}_{1} = c^2$

$\tan^2\theta + \cot^2\theta = c^2 - 2 \quad \ldots (I)$

$(\operatorname{sen}\theta + \csc\theta)^2 = (a)^2$

$\operatorname{sen}^2\theta + \csc^2\theta = a^2 - 2 \quad \ldots (II)$

$(\cos\theta + \sec\theta)^2 = (b)^2$

$\cos^2\theta + \sec^2\theta = b^2 - 2 \quad \ldots (III)$

(I): $\tan^2\theta + \cot^2\theta = c^2 - 2$

(III): $\cos^2\theta + \sec^2\theta = b^2 - 2$ (−)

$\cos^2\theta + 1 - \cot^2\theta = b^2 - \cancel{2} - c^2 + \cancel{2}$

$\cos^2\theta - \cot^2\theta = b^2 - c^2 - 1$

: 2x

$\frac{x}{3x}$ ✓

(−3)

$\cos^2\theta - \cot^2\theta = b^2 - c^2 - 1$

(II): $\operatorname{sen}^2\theta + \csc^2\theta = a^2 - 2$

$1 + 1 = a^2 + b^2 - c^2 - 3$

$5 = a^2 + b^2 - c^2$

14) $\tan^{16}x - 14 = 13 \cdot \tan^2 x$

$\tan^{16}x - 1 = 13\tan^2 x + 13$

$13(\tan^2 x + 1)$

$13 \cdot \sec^2 x$

$\underbrace{\underbrace{\underbrace{(\tan^2 x + 1) \cdot (\tan^2 x - 1)}_{(\tan^4 x - 1)} (\tan^4 x + 1)}_{(\tan^8 x - 1)} (\tan^8 x + 1)}_{(\tan^{16} x - 1)} \cdot \dfrac{1}{(\tan^2 x + 1)}$

$(\tan^{16}x - 1) \cdot \dfrac{1}{\sec^2 x}$

$13 \cdot \cancel{\sec^2 x} \cdot \dfrac{1}{\cancel{\sec^2 x}}$

$\underline{13}$ //.

19)

$$\tan^4 x - \sec^4 x = 1 - \frac{A}{1 - \text{sen}^B x}$$

$$\frac{\text{sen}^4 x}{\cos^4 x} - \frac{1}{\cos^4 x}$$

$$\frac{\text{sen}^4 x - 1}{\cos^4 x}$$

$$\frac{-\overbrace{(1 - \text{sen}^2 x)}^{\cos^2 x}(\text{sen}^2 x + 1)}{\cos^4 x}$$

$$\frac{-\text{sen}^2 x - 1 + 2 - 2}{\cos^2 x}$$

$$\frac{\overbrace{(1 - \text{sen}^2 x)}^{\cos^2 x} - 2}{\cos^2 x}$$

$$\frac{\cos^2 x}{\cos^2 x} - \frac{2}{\cos^2 x}$$

$$1 - \frac{2}{1 - \text{sen}^2 x}$$

$A = 2$ $\quad$ $B = 2$

$A + B = 4$

$1 - \frac{A}{1 - \operatorname{sen}^{B} x}$

$x + 1)$

2

20) $$\frac{(1 + \operatorname{sen} x + \cos x)^2}{(\operatorname{sen} x + \tan x)(\cos x + \cot x)}$$

$$\frac{2(1 + \operatorname{sen} x)(1 + \cos x)}{\left(\frac{\cos x \cdot \operatorname{sen} x}{\cos x} + \frac{\operatorname{sen} x}{\cos x}\right)\left(\frac{\cos x \cdot \operatorname{sen} x}{\operatorname{sen} x} + \frac{\cos x}{\operatorname{sen} x}\right)}$$

$$\frac{2\cancel{(1 + \operatorname{sen} x)}\cancel{(1 + \cos x)}}{\frac{\operatorname{sen} x\cancel{(\cos x + 1)}}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}\cancel{(\operatorname{sen} x + 1)}}{\operatorname{sen} x}}$$

22)

$$\frac{1-\text{SEN}A}{\text{COS}A}=3$$

$$(1-\text{SEN}A)^2=(3.\text{COS}A)^2$$

$$1+\text{SEN}^2A-2\,\text{SEN}A=9.\text{COS}^2A$$

$$-2+2-1-\text{SEN}^2A+2.\text{SEN}A=-9.\text{COS}^2A$$

$$\underbrace{1-\text{SEN}^2A}_{\text{COS}^2A}+2\text{SEN}A-2=-9\,\text{COS}^2A$$

$$2.\text{SEN}A-2=-10.\text{COS}^2A$$

$$\text{SEN}A-1=-5.\text{COS}^2A$$

$$-\frac{1}{\text{COS}A}.\underbrace{\frac{(1-\text{SEN}A)}{\text{COS}A}}_{3}=-5$$

$$\boxed{\frac{3}{5}=\text{COS}A}$$

$A=53°$

PIDEN:

$$\frac{1+\text{COS}A}{\text{SEN}A}=\frac{1+\frac{3}{5}}{\frac{4}{5}}=\frac{\frac{8}{5}}{\frac{4}{5}}=2$$

$2x$

$\frac{x}{3x}$

(-3)

26)

$$\frac{\tan^4 x - \cot^4 x}{\tan^4 x + \cot^4 x + 2 - 2}$$

$\sec x \cdot \csc x = \sqrt{5}$

$$\frac{(\tan^2 x - \cot^2 x)(\tan^2 x + \cot^2 x)}{(\tan^2 x + \cot^2 x)^2 - 2}$$

$$\frac{(\tan x - \cot x)\overbrace{(\tan x + \cot x)}^{\sqrt{5}}\overbrace{(\tan^2 x + \cot^2 x)}^{3}}{(\underbrace{\tan^2 x + \cot^2 x}_{3})^2 - 2}$$

$$\frac{\sqrt{(\tan x - \cot x)^2} \cdot \sqrt{5} \cdot 3}{7}$$

$$\underbrace{\left(\sqrt{\tan^2 x + \cot^2 x - 2}\right)}_{(1)} \cdot \frac{\sqrt{5} \cdot 3}{7}$$

↳ $(\tan x + \cot x)^2 = (\sqrt{5})^2$

$\tan^2 x + \cot^2 x = 3$