

PROF. FISHER DUKE.

### Formas Compuestas

#### A Identidades Fundamentales

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

#### B Identidades Auxiliares

$$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$$

$$\sin(A+B) - \sin(A-B) = 2 \sin B \cos A$$

$$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$$

$$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$$

$$\sin(A+B) \sin(A-B) = \sin^2 A - \sin^2 B$$

$$\cos(A+B) \cos(A-B) = \cos^2 A - \sin^2 B$$

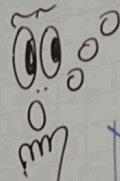
$$\checkmark \tan A + \tan B = \frac{\tan(A+B)}{1 - \tan A \tan B}$$

### ARCOS DOBLES

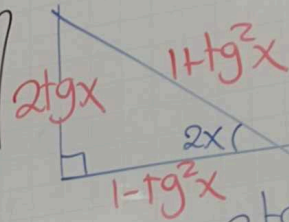
$$\sin 2x = 2 \sin x \cos x$$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \sin^2 x \\ &= 2 \cos^2 x - 1 \end{aligned}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$



Tambien....

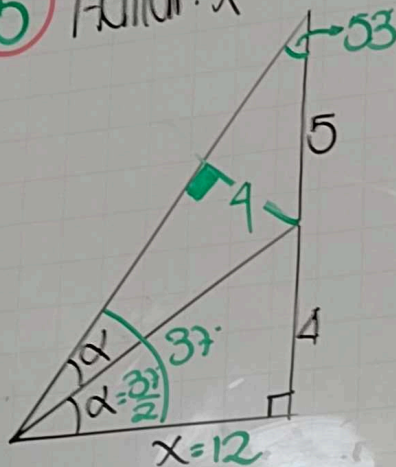


$$\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

MACIA: GAY

15) Hallar:  $x$



$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\frac{4}{12} = \frac{2 \left( \frac{4}{x} \right)}{1 - \left( \frac{4}{x} \right)^2}$$

$$\frac{1}{3} = \frac{8}{x^2 - 16}$$

$$x = 12$$

16

70

$$\frac{x + \sin \alpha}{x + 1}$$

$$\alpha + 1$$

$$\frac{\cos \alpha + \sin \alpha}{\cos \alpha + 1}$$

$$\cos \alpha + 1$$

$$\frac{(\cos \alpha + 1) \sin \alpha}{\cos \alpha + 1} = \sin \alpha$$

$$= 0,6$$

$$\frac{25}{25} = 1$$

$$\frac{-7}{25}$$



$$\frac{\tan 10^\circ - \tan 50^\circ}{1 + \tan 10^\circ \tan 50^\circ}$$

$$\frac{\tan 50^\circ - \tan 70^\circ}{1 + \tan 50^\circ \tan 70^\circ}$$

$$\frac{\tan 70^\circ - \tan 90^\circ}{1 + \tan 70^\circ \tan 90^\circ}$$

$$\frac{\tan 90^\circ - \tan 10^\circ}{1 + \tan 90^\circ \tan 10^\circ}$$

5) Hallar:

$$E = \sec 28^\circ \sec 17^\circ + \sqrt{2} \tan 28^\circ \tan 17^\circ$$

$$E = \frac{2}{2 \cos 28^\circ \cos 17^\circ} + \frac{\sqrt{2} \sin 28^\circ \sin 17^\circ}{2 \cos 28^\circ \cos 17^\circ}$$

$$\begin{aligned} * \cos(A+B) + \cos(A-B) &= 2 \cos A \cos B \\ * \cos(A+B) - \cos(A-B) &= -2 \sin A \sin B \end{aligned}$$

$$* \cos(A-B) - \cos(A+B) = 2 \sin A \sin B$$

$$E = \frac{2 + \sqrt{2} (\cos 11^\circ - \cos 45^\circ)}{\cos 45^\circ + \frac{\cos 11^\circ \cdot \sqrt{2}}{\sqrt{2}}}$$

$$E = \frac{2 + \sqrt{2} \cos 11^\circ - 1}{1 + \sqrt{2} \cos 11^\circ} = \frac{\sqrt{2} (1 + \sqrt{2} \cos 11^\circ)}{1 + \sqrt{2} \cos 11^\circ}$$

$$E = \sqrt{2} \quad \#$$

13) Reducir:

$$H = \frac{\sin 2\alpha + \sin \alpha}{2 \cos \alpha + 1}$$

$$H = \frac{2 \sin \alpha \cos \alpha + \sin \alpha}{2 \cos \alpha + 1}$$

$$H = \frac{\sin \alpha (2 \cos \alpha + 1)}{2 \cos \alpha + 1} = \sin \alpha$$

14) Si:  $\cos u = 0,6$

$$\begin{aligned} \cos 2u &= 2 \cos^2 u - 1 \\ &= 2(0,6)^2 - 1 \\ &= 2\left(\frac{3}{5}\right)^2 - 1 = \frac{-7}{25} \quad \# \end{aligned}$$

2) Calcular:

$$E = \frac{\tan 80^\circ - \tan 20^\circ}{\sqrt{3} \tan 80^\circ \tan 20^\circ}$$

$$E = \frac{\frac{\tan(60-20) + \tan(20-20)}{\sqrt{3}} \cdot \tan 80^\circ \tan 20^\circ}{\sqrt{3} \tan 80^\circ \tan 20^\circ}$$

$$E = \sqrt{3}$$

3) Calcular:

$$E = \frac{\cot 88^\circ}{\cot 44^\circ - \cot 46^\circ}$$

$$E = \frac{\tan 2^\circ}{\tan 46^\circ - \tan 44^\circ}$$

$$E = \frac{1}{2}$$

$$E = \frac{1}{2}$$

$$\alpha + \theta = 90^\circ$$

$$\sin \alpha = \cos \theta$$

$$\tan \alpha = \cot \theta$$

$$\sec \alpha = \csc \theta$$

$$\sin \alpha \sec \theta = 1$$

$$\tan \alpha \cot \theta = 1$$

$$\cot \alpha \cot \theta = 1$$

$$\csc \alpha \csc \theta = 1$$

$$\tan \alpha = \frac{1}{\tan \theta}$$

$$\frac{\tan 46^\circ - \tan 44^\circ}{1 + \tan 46^\circ \tan 44^\circ} = 1$$

$$\frac{\tan 46^\circ - \tan 44^\circ}{1} = 1$$

4) Calcular:

$$E = \frac{\tan 70^\circ - \tan 50^\circ}{\tan 20^\circ + \tan 50^\circ}$$

$$E = \frac{1}{\frac{1}{\tan 70^\circ} + \frac{\tan 50^\circ \tan 70^\circ}{\tan 70^\circ}}$$

$$E = \frac{(\tan 70^\circ - \tan 50^\circ) \tan 70^\circ}{1 + \tan 50^\circ \tan 70^\circ}$$

$$E = \tan 20^\circ \tan 70^\circ = 1$$

ien....

## PROBLEMAS DE CLASE

1 Calcular:  $(45^\circ + 25^\circ)$

$$\frac{\sqrt{3} \cos 70^\circ}{\cos 25^\circ - \sin 25^\circ} = \frac{\sqrt{3} \cos(45^\circ + 25^\circ)}{\cos 25^\circ - \sin 25^\circ}$$

$$\frac{\sin 45^\circ \cos x + \sin x \cos 45^\circ}{\frac{\sqrt{2}}{2}}$$

$$\frac{\sqrt{2}}{2} (\cos x + \sin x)$$

$$\star \sin(45^\circ + x) = \frac{\sqrt{2}}{2} (\cos x + \sin x)$$

$$\star \cos(45^\circ + x) = \frac{\sqrt{2}}{2} (\cos x - \sin x) \checkmark$$

$$= \sqrt{3} \left[ \frac{\sqrt{2}}{2} (\cos 25^\circ - \sin 25^\circ) \right]$$

$$\cos 25^\circ - \sin 25^\circ$$

$$E = \frac{\sqrt{6}}{2} \#$$



→ 53

5

16) Reduir:  $1 - 2\sin^2 \alpha$

$$J = \frac{1 - \cos 2\alpha}{\sin 2\alpha} \rightarrow 2\sin \alpha \cos \alpha$$

$$J = \frac{1 - (1 - 2\sin^2 \alpha)}{2\sin \alpha \cos \alpha}$$

$$J = \frac{\sin^2 \alpha}{\sin \alpha \cos \alpha} = \frac{\sin \alpha}{\cos \alpha}$$

$$J = \tan \alpha \quad \#$$

17) Reduir:

$$A = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$$

$$A = \tan^2 \alpha \quad \#$$

$$A = \frac{1 - (1 - 2\sin^2 \alpha)}{1 + (2\cos^2 \alpha - 1)} = \frac{2\sin^2 \alpha}{2\cos^2 \alpha}$$

$$A = \tan^2 \alpha \quad \#$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

18 Simplificar:

$$Q = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} = \frac{\frac{c^2}{e^2} - \frac{s^2}{e^2}}{\frac{c^2}{e^2} + \frac{s^2}{e^2}}$$

$$Q = \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos^2 \alpha + \sin^2 \alpha} = \cos 2\alpha$$

27 Reduzir:

$$K = \frac{\cos \phi - \frac{1}{2} \tan(\frac{\phi}{2})}{\frac{1}{2} \tan(\frac{\phi}{2}) + \cos \phi}$$

$$K = \frac{2 \cos \phi - \tan(\frac{\phi}{2})}{\tan(\frac{\phi}{2}) + 2 \cos \phi}$$

$$\tan \frac{x}{2} = \frac{\cos x - \cos x}{\cos x + \cos x}$$

$$\cos \frac{x}{2} = \frac{\cos x + \cos x}{\cos x + \cos x}$$

$$K = \frac{2 \cos \phi - (\cos \phi - \cos \phi)}{\cos \phi - \cos \phi + 2 \cos \phi} = 1$$

30 Simplificar:

$$\pm = \sin 2\phi \cdot \frac{1 - \tan^4 \phi}{4 \tan^2 \phi}$$

$$\pm = \sin 2\phi \cdot \frac{\sec^2 \phi (1 + \tan^2 \phi)(1 - \tan^2 \phi)}{2 \tan \phi \cdot 2 \tan \phi} = \frac{\sec^2 \phi}{\tan 2\phi}$$

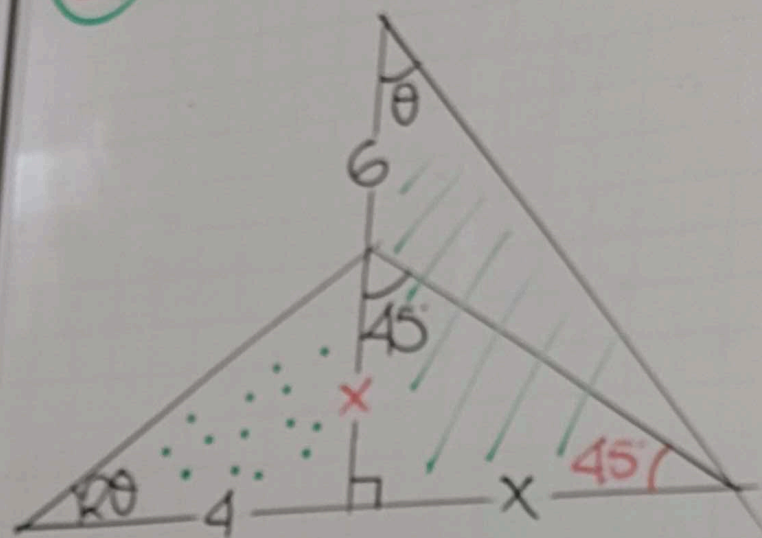
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\pm = \frac{\cancel{\sin 2\phi} \cos 2\phi}{\cancel{\sin 2\phi} \cdot \frac{\cos \phi \cos \phi}{2 \sin \phi \cos \phi}} = \frac{\cos 2\phi}{\sin 2\phi} = \cot 2\phi$$

$$\pm = \frac{\cos 2\phi}{\sin 2\phi} = \cot 2\phi$$



32 Calculate:  $x$



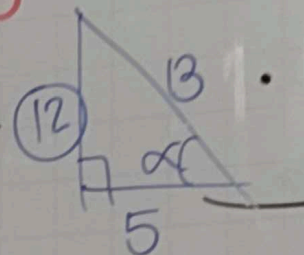
$$\tan 20^\circ = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\frac{x}{4} = \frac{2 \left( \frac{x}{6+x} \right)}{1 - \left( \frac{x}{6+x} \right)^2} \Rightarrow x = 3$$

(31) Si:  $\cos \alpha = -\frac{5}{13}$

16

$\cos^2 \alpha = \frac{25}{169}$ ;  $\alpha \in \text{IIIC}$

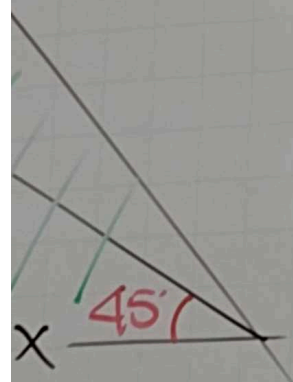


Calcular:  $\tan\left(\frac{\alpha}{2}\right) = \frac{\cos \alpha - \operatorname{ctg} \alpha}{1 + \sin \alpha}$

$= \frac{13}{12} - \left(-\frac{5}{12}\right)$

$= \frac{13}{12} + \frac{5}{12}$

$= \frac{18}{12} = \frac{3}{2}$



3  
#

#