

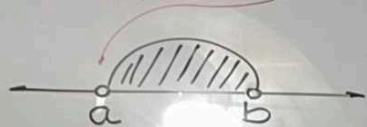
## Semana: #11

### Inecuaciones en $\mathbb{R}$

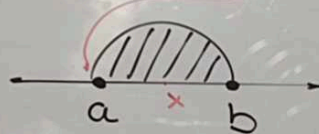
- Desigualdad:  $A \leq B$
- Inecuación:  $A \leq B_x$   
(condición)
- Intervalo:  $I \subset \mathbb{R}$ 
  - ✓ Acotado: extremos  $\mathbb{R}$   
 $\langle a, b \rangle$ ;  $[a, b]$ ;  $\langle a, b]$
  - ✓ No acotado: extremos  $\infty$   
 $\langle -\infty, a \rangle$ ;  $\langle a, +\infty \rangle$   
 $\langle -\infty, +\infty \rangle \equiv \mathbb{R}$

$\sqrt{a^2} = |a|, \forall a \in \mathbb{R}$

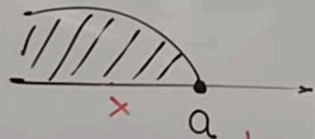
- $x \in \langle a, b \rangle \rightarrow a < x < b$



- $x \in [a, b] \rightarrow a \leq x \leq b$



- $x \in \langle -\infty, a \rangle \rightarrow x < a$



$\langle a, +\infty \rangle \equiv x > a$

$[a, +\infty) \equiv x \geq a$

$\langle -\infty, -a \rangle \equiv x < -a$

$\langle -\infty, -a] \equiv x \leq -a$

### Prop:

$\wedge \equiv \cap$   
 $\vee \equiv \cup$   
 $\sim \equiv ( )'$

- $A > 0 \rightarrow A$  es positivo (+)
- $A < 0 \rightarrow A$  es negativo (-)
- $A \geq B \rightarrow A > B \vee A = B$
- $B < A < C \rightarrow B < A \wedge A < C$
- $A^2 < B \rightarrow -\sqrt{B} < A < \sqrt{B}$
- $A \leq B \wedge \underbrace{C > 0}_{+} \rightarrow AC \leq BC$
- $A \leq B \wedge \underbrace{C < 0}_{-} \rightarrow AC \geq BC$

### OJO Caso I:

$x^2 < 25 \rightarrow -5 < x < 5$   
 $x^2 < 8 \rightarrow -2\sqrt{2} < x < 2\sqrt{2}$   
 $x < 3 \rightarrow -x > -3$   
 $-x \geq -4 \rightarrow x \leq 4$

### Caso II: (P. INGA)

$-4 < x < 3 \rightarrow 0 \leq x^2 < 9$   
 $-5 < x < 1 \rightarrow 0 \leq x^2 < 25$   
 $-4 < x \leq -2 \rightarrow 0 \leq x^2 < 16$



03:

$$P = \{x \in \mathbb{R} / -3 \leq x+4 \leq 9\}$$

$$T = \{x \in \mathbb{R} / 3x < \frac{x}{4} - 1 < 2\}$$

$$P \cap T = [a, b] \quad \begin{cases} a = -8/11 \\ b = 44 \end{cases}$$

$$\begin{aligned} -3 \leq x+4 \leq 9 & \quad 3x < \frac{x}{4} - 1 < 2 \\ -7 \leq x \leq 5 & \quad 3x < \frac{x}{4} - 1 \wedge \frac{x}{4} - 1 < 2 \\ 0 \leq x^2 \leq 49 & \quad 12x < x-4 \wedge \frac{x}{4} < 3 \\ -5 \leq x^2 \leq 44 & \quad 11x < -4 \wedge x < 12 \\ & \quad x < -\frac{4}{11} \wedge x < 12 \end{aligned}$$

$$P = [-5, 44]$$

$$\therefore P \cap T = \left[-\frac{8}{11}, 44\right]$$

$$T = \left(-\infty, -\frac{4}{11}\right)$$

04:

$$x \in [1, 2] + \frac{1}{a} \leq \frac{1}{5x+3} \leq \frac{1}{b}$$

$$1 \leq x \leq 2$$

$$5 \leq 5x \leq 10$$

$$8 \leq 5x+3 \leq 13$$

$$\frac{1}{13} \leq \frac{1}{5x+3} \leq \frac{1}{8} \quad \begin{matrix} 13=a \\ 8=b \end{matrix}$$

$$CS = \left[\frac{1}{13}, \frac{1}{8}\right]$$



$$a-b = 13-8 = 5$$

05:

$$3 \leq x \leq 7$$

$$M = \frac{3x+1}{3x-5} = \frac{3x-5}{3x-5} + \frac{6}{3x-5} = 1 + \frac{6}{3x-5}$$

$$3 \leq x \leq 7$$

$$9 \leq 3x \leq 21$$

$$4 \leq 3x-5 \leq 16$$

$$\frac{1}{16} \leq \frac{1}{3x-5} \leq \frac{1}{4}$$

$$\frac{7}{8} \leq \frac{6}{3x-5} \leq \frac{7}{2}$$

$$\frac{11}{8} \leq M \leq \frac{13}{2}$$

06:

$$\frac{x}{4} + 1 < \frac{x-5}{2} < \frac{x}{3} + 2$$

$$\frac{x}{4} + 1 < \frac{x-5}{2} \quad \wedge \quad \frac{x-5}{2} < \frac{x}{3} + 2$$

$$\frac{x+4}{4} < \frac{x-5}{2} \quad \wedge \quad \frac{x-5}{2} < \frac{x+6}{3}$$

$$x+4 < 2x-10 \quad \wedge \quad 3x-15 < 2x+12$$

$$14 < x \quad \wedge \quad x < 27$$

$$14 < x < 27$$

$$f = \left(\frac{14}{27}, 27\right)$$



$$\begin{array}{ccc|c}
 & 1 & -2 & 2 & 40 \\
 \hline
 5 & 1 & -2 & 2 & 40 \\
 \hline
 & 1 & -2 & -8 & -40 \\
 \hline
 4 & 1 & 4 & 8 & 0 \\
 \hline
 & 1 & 2 & 0 & 
 \end{array}$$



09:  $\mathbb{C}S = \mathbb{R}$   $\Delta = B^2 - 4AC$

$$3 + 36x - 12x^2 \leq M$$

$$0 \leq 12x^2 - 36x + (M-3)$$

$$\Delta \leq 0$$

$$(-36)^2 - 4 \cdot 12 \cdot (M-3) \leq 0$$

$$36^2 \leq 4 \cdot 12 \cdot (M-3)$$

$$27 \leq M-3$$

$$30 \leq M$$

10:  $\textcircled{Pc}$

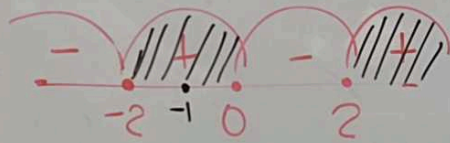
$$x^3 > 4x$$

$$x^3 - 4x > 0$$

$$x(x^2 - 4) > 0$$

$$x(x+2)(x-2) > 0$$

PC: 0, -2, 2



$$J = [-2, 0] \cup [2, +\infty)$$

$\therefore \textcircled{-3}$

11:

$$-2x^2 - x + 10 \leq 0$$

NUNCA: Aspa.

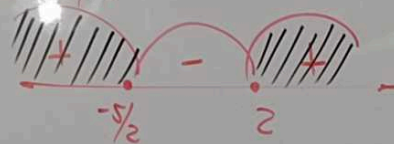
TACHAR: Positivo

ABAJO: Abierto.

$$\therefore 2x^2 + x - 10 > 0$$

$$(2x+5)(x-2) > 0$$

PC:  $-5/2, 2$



$$J = \langle -\infty, -5/2 \rangle \cup [2, +\infty)$$

12:

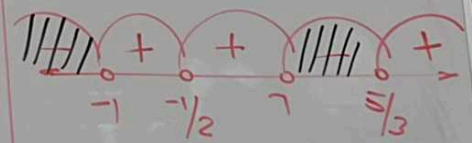
$$(6x+3)^2(x-1)^3(3x-5)^7$$

I) PAR: NO alternar

II) IMPAR: SI alternar.

$$(2x+1)^2(x+1)^3(x-1)^3(3x-5)^7$$

PC:  $-1/2, -1, 1, 5/3$



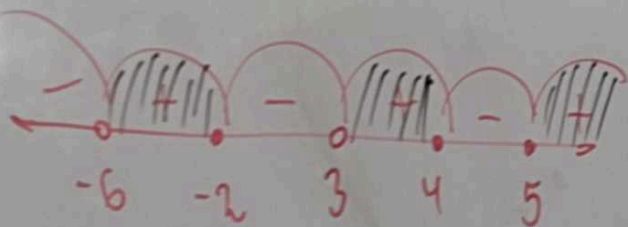
$$J = \langle -\infty, -1 \rangle \cup \langle 1, 5/3 \rangle$$

16:

$$\frac{x^3 - x^2 + 2x + 40}{10 - 5x - x^2} \leq 0$$

$$\frac{(x-5)(x-4)(x+2)}{(x+6)(x-3)} \geq 0$$

PC: 5, 4, -2, -6, 3



$$S = (-6, -2] \cup (3, 4] \cup [5, +\infty)$$

18:

$$\frac{x^2 + 1}{-x} \leq x - \frac{1}{x}$$

$$-\frac{x^2 + 1}{x} \leq \frac{x^2 - 1}{x}$$

$$0 \leq \frac{x-1}{x} + \frac{x^2+1}{x}$$

$$0 \leq \frac{2x^2}{x}$$

$$0 \leq 2x$$

$$0 < x$$

$$S = \mathbb{R}^+$$

(0, +infinity)

24:

$$\frac{|x|}{x-2006} < 0$$

$x-2006 < 0$   
 $x < 2006$

Valor Absoluto:  $|x| < 2006$

1)  $|A| = B \rightarrow A = B \vee A = -B$

2)  $|A| = |B| \rightarrow (A+B)(A-B) = 0$

3)  $|A| < B \rightarrow -B < A < B$

4)  $|A| > B \rightarrow A > B \vee A < -B$

5)  $|A| < |B| \rightarrow (A+B)(A-B) < 0$

25:

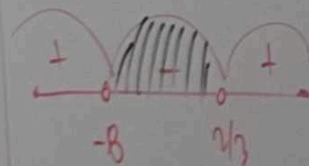
$$\frac{|x-5|}{|2x+3|} > 0$$

$$(x-5)(2x+3) > 0$$

$$(3x-2)(-x-8) > 0$$

$$(3x-2)(x+8) < 0$$

PC:  $\frac{2}{3}, -8$



$$S = (-8, \frac{2}{3})$$



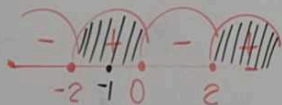
$$x^3 > 4x$$

$$x^3 - 4x > 0$$

$$x(x^2 - 4) > 0$$

$$x(x+2)(x-2) > 0$$

$$\text{PC: } 0, -2, 2 \therefore -2 \text{ is } -3$$



$$CS = [-2, 0] \cup [2, +\infty)$$

11:

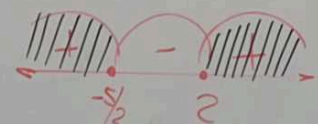
$$-2x^2 - x + 10 \leq 0$$

$$2x^2 + x - 10 \geq 0$$

$$\begin{array}{r} 2x \quad +5 \\ x \quad -2 \end{array}$$

$$(2x+5)(x-2) \geq 0$$

$$\text{PC: } -\frac{5}{2}, 2$$



$$CS = (-\infty, -5/2] \cup [2, +\infty)$$

12:

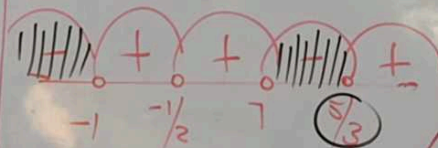
$$(2x+1)^2(x-1)^3(3x-5)^4 < 0$$

$$\bullet \text{ IAP: } \text{NO}$$

$$\bullet \text{ IMPAR: SI alternar.}$$

$$(2x+1)^2(x+1)^3(x-1)^3(3x-5)^4 < 0$$

$$\text{PC: } -\frac{1}{2}, -1, 1, 5/3$$



$$CS = (-\infty, -1) \cup (1, 5/3)$$

20:

$$\sqrt[3]{\frac{5x+1}{3}} < \sqrt[3]{\frac{3(2x+1)}{5}}$$

$$\frac{5x+1}{6} < \frac{3x+3}{5}$$

$$\frac{5x+1}{6} < \frac{3x+3}{5}$$

$$25x+5 < 18x+18$$

$$7x < 13$$

$$x < \frac{13}{7}$$

$$\therefore CS = (-\infty, 13/7)$$

$$\sqrt[5]{\frac{3(x+1)}{5}}$$

3

$$\frac{3x+3}{5}$$

$$x+18$$

3

①

$$\left(\frac{13}{2}\right)$$

22:

$$\sqrt{4-x} + \sqrt{x+2} \geq 0$$

$$4-x \geq 0 \wedge x+2 \geq 0$$

$$4 \geq x \wedge x \geq -2$$

$$-2 \leq x \leq 4$$

$$\therefore \text{Cf} = [-2, 4]$$

25: Puntos Críticos (PC)

$$|x-5| > |2x+3|$$

$$(3x-2)(-x-8) > 0$$

$$(3x-2)(x+8) < 0$$

$$\text{PC: } \frac{2}{3}; -8$$



$$\text{C.S.} = (-8, \frac{2}{3})$$

Valor Absoluto:

$$|-3| = -(-3) = 3$$

$$|-2| = -(-2) = 2$$

$$|5| = 5$$

$$① |A| < B \rightarrow -B < A < B$$

$$② |A| > B \rightarrow A > B \vee A < -B$$

$$③ |A| < |B| \rightarrow (A+B)(A-B) < 0$$

$$|x| < 3 \rightarrow -3 < x < 3$$

$$|x| < 2 \rightarrow -2 < x < 2$$

$$|x| > 1 \rightarrow x > 1 \vee x < -1$$

$$|x| > \sqrt{2} \rightarrow x > \sqrt{2} \vee x < -\sqrt{2}$$