Semana: #05

Cocientes Notables (CN)

Def: forma: $\dfrac{A^n \pm B^n}{A \pm B}$

- N° términos $= n$
- $T_K = (\pm)\, A^{n-K} \cdot B^{K-1}$
- $\underset{\leftarrow}{T_K} = (\pm)\, A^{K-1} \cdot B^{n-K}$
- signo $\begin{cases} (+): \text{Lugar impar.} \\ (-): \text{lugar par.} \end{cases}$

La otra:

forma: $\dfrac{A^n \pm B^m}{A^p \pm B^q}$

- $N^o T = \dfrac{n}{p} = \dfrac{m}{q}$

(regla general C.N)

- $T_K = (\pm)\, (A^p)^{NT-K} \, (B^q)^{K-1}$
- $\underset{\leftarrow}{T_K} = (\pm)\, (A^p)^{K-1} \, (B^q)^{NT-K}$
- $NT = \underset{\in \mathbb{Z}}{NTR} + \underset{\in \mathbb{Q}}{NTI}$

$NTR = \underset{exp \geq 0}{NTRE} + \underset{exp < 0}{NTRF}$

$\pm B^m$

$\overline{\pm B^q}$

$\frac{m}{q}$

(C.N)

$(B^q)^{k-1}$

$(B^q)^{m-k}$

F

<0

Dato:

$$1+x+x^2+\dots+x^{10} = \frac{x^{11}-1}{x-1}$$

$$1+x^2+x^4+\dots+x^{20} = \frac{x^{22}-1}{x^2-1}$$

$$1+x^5+x^{10}+\dots+x^{55} = \frac{x^{60}-1}{x^5-1}$$

$$1+x^n+x^{2n}+\dots+x^{pn} = \frac{x^{(p+1)n}-1}{x^n-1}$$

OJO

$\frac{+}{+}$: $+ - + - \dots (+)$

$\frac{+}{-}$: NO CN DIV. INEX

$\frac{-}{+}$: $+ - + - \dots (-)$

(IMPAR) $\frac{-}{-}$: $+ + + \dots (+)$

OJ:

06:

$$\frac{2^9 - 2^8 + \dots - 1}{2^9 + 2^8 + \dots + 1}$$

$$\frac{\left[\dfrac{\cancel{2^{10}-1}}{2+1}\right]}{\left[\dfrac{\cancel{2^{10}-1}}{2-1}\right]} = \frac{1}{3}$$

Rpta

07:

$$\frac{x^{14} + x^{12} + \dots + x^2 + 1}{x^6 + x^4 + \dots + 1}$$

$$\frac{\left[\dfrac{x^{16}-1}{\cancel{x^2-1}}\right]}{\left[\dfrac{x^8-1}{\cancel{x^2-1}}\right]} = \frac{x^{16}-1}{x^8-1}$$

$$= \frac{(x^8+1)\cancel{(x^8-1)}}{\cancel{x^8-1}}$$

$$= x^8+1$$

Rpta

$\ldots + 1$

$\sqrt{T} = \frac{21}{3} = 7$

05:

$$\frac{2^9 + 2^8 + 2^7 + \ldots + 1}{2^4 + 2^3 + 2^2 \ldots + 1}$$

$$\frac{\dfrac{2^{10}-1}{\cancel{2-1}}}{\dfrac{2^5-1}{\cancel{2-1}}} = \frac{2^{10}-1}{2^5-1}$$

$$= \frac{1023}{31}$$

$$= 33$$

Rpta.

$2x$

$\frac{x}{3x}$

03:

$$\frac{x^{5n-10} - y^{20}}{x^5 - y^2}$$

$N^o T = \frac{20}{2} = 10$

04:

$x^{18} - x^{15} + x^{12} - \dots + 1$

$$\frac{x^{21} + 1}{x^3 + 1}$$

$NT = \frac{21}{3} = 7$

01:

$$\frac{x^R - y^S}{x^3 - y^4}$$

$N^\circ T = 8$

$$\frac{R}{3} = \frac{S}{4} = 8$$

$R = 3 \times 8 = 24$

$S = 4 \times 8 = 32$

$\therefore R + S = 56$

RPTA.

02:

$$\frac{x^{4n+4} - y^{5n}}{x^{n+1} + y^{2n-3}}$$

$$\frac{4n+4}{n+1} = \frac{5n}{2n-3} = N^\circ T$$

$(4n+4)(2n-3) = 5n(n+1)$

$8n^2 - 12n + 8n - 12 = 5n^2 + 5n$

$\cancel{3}^{1}n^2 - \cancel{9}^{3}n - \cancel{12}^{4} = 0$

$n^2 - 3n - 4 = 0$

$n \quad -4 = 0 \rightarrow n = 4$

$n \quad +1 = 0 \rightarrow n = -1$

08. $\dfrac{x^6 - a^6}{x - a}$

$T_4 = x^{6-4} \cdot a^{4-1}$

$T_4 = x^2 a^3$

09. $\dfrac{x^{60} - y^{78}}{x^5 + y^6}$

$NT = \dfrac{60}{5} = 12$

$T_8 = -(x^5)^{12-8} (y^6)$

$T_8 = -x^{20} \cdot y^{42}$

y^{72}

6

(12)

(8) (y^6) (8-1)

10: $\frac{x^{40}+y^{10}}{x^4+y}$ NT=10

$T_9 = +(x^4)^{10-9}(y)^{9-1}$

$T_9 = x^4 \cdot y^8$

(10)

$\frac{x^{40}+y^{10}}{x^4+y} = x^{36} - x^{32}y + x^{28}\cdot y^2 - x^{24}\cdot y^3 + x^{20}\cdot y^4 - x^{16}\cdot y^5 + x^{12}\cdot y^6 - x^8\cdot y^7 + x^4\cdot y^8 - y^9$

11: $\frac{x^a - y^b}{x^5 - y^7}$

$\frac{a}{5} = \frac{b}{7} = 40$

$T_{17} = x^{115} \cdot y^{112}$

$T_{17} = (x^5)^{NT-17} (y^7)^{17-1}$

$T_{17} = x^{5NT-85} \cdot y^{112}$

5NT-85=115

5NT=200

NT=40

a=200, b=280 → b-a=80

12:

$\frac{3n+9}{n-1}$

(3n+9)(2n

$6n^2-9n+18$

9n-

4

12:

$$\frac{x^{3n+9}+y^{6n+11}}{x^{n-1}+y^{2n-3}}$$

$$\frac{3n+9}{n-1}=\frac{6n+11}{2n-3}=\frac{35}{5}=7$$

$(3n+9)(2n-3)=(6n+11)(n-1)$

$\cancel{6n^2}-9n+18n-27=\cancel{6n^2}-6n+11n-11$

$9n-27=5n-11$

$4n=16$

$n=4$

$T_4=-(x^3)^{7-4}(y^5)^{4-1}$

$T_4=-x^9 \cdot y^{15}$

13:

$$\frac{(x+y)^{22}-(x-y)^{22}}{4xy}$$

$$\frac{(x+y)^{22}-(x-y)^{22}}{(x+y)^2-(x-y)^2}$$ NT = 11

$T_6=\{(x+y)^2 \cdot (x-y)^2\}^5$

$T_6=(x^2-y^2)^{10}=(9-7)^{10}$

3 $\sqrt{7}$

$=2^{10}$

$=1024$

(17-1)

a = 80

2x

$\frac{x}{2x}$

29:

$$\frac{x^{a+b}\cdot y^{ab} - y^{a^3+b^3+ab}}{(xy)^{ab} - y^{a^2+b^2}}$$

$$\frac{\cancel{y^{ab}}\left(x^{a+b} - y^{a^3+b^3}\right)}{\cancel{y^{ab}}\left(x^{ab} - y^{a^2+b^2-ab}\right)}$$

$$\frac{\cancel{a+b}}{ab} = \frac{\cancel{(a+b)}\cancel{(a^2-ab+b^2)}}{\cancel{a^2-ab+b^2}}$$

$ab=1$

07:

$$\frac{x^{14}+\ldots}{x\ldots}$$

$$\frac{x^{16}\ldots}{x^{2}\ldots}$$

$$\frac{x^{8}-\ldots}{x^{2}\ldots}$$