



INECUACIONES I.

→ DESIGUALDAD:

$$> ; < ; \geq ; \leq$$

→ CONJUNTO SOLUCIÓN:

→ INTERVALO
(PORCIÓN RECTA)

0;0: 1. I. ABIERTO: $\langle -1; 3 \rangle$
($> ; <$)



2. I. CERRADO: $[-5; 7]$
($\geq ; \leq$)



PROPIEDADES y OPERACIONES

$$1.- a < b \wedge b < c \\ \Rightarrow a < c \wedge a < b < c$$

$$2.- a^2 \geq 0 \Rightarrow a \in \mathbb{R}$$

$$3.- a^2 < 0 \Rightarrow a \in \emptyset$$

$$4.- a < b \wedge c < 0 \\ \Rightarrow -b < -a ; bc < ac$$

$$5.- a < b \Rightarrow \frac{1}{b} < \frac{1}{a}$$

$$6.- a < x < b \\ \Rightarrow -b < -x < -a$$

$$7.- a < x < b \\ \Rightarrow \frac{1}{b} < \frac{1}{x} < \frac{1}{a}$$

$$8.- a < x < b$$

$$a^2 < x^2 < b^2$$

$$9.- -a < x < b$$

$$0 \leq x^2 < \text{MAYOR}^2$$

$$10.- -a < x < -b$$

$$b^2 < x^2 < a^2$$

→ RESOLUCIÓN DE UNA INECUACIÓN

1. VARIANTE

① I. POLINOMIO:

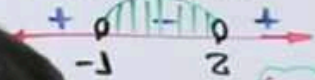
$$x^2 - x < 2$$

$$x^2 - x - 2 < 0$$

$$(x-2)(x+1) < 0$$

$$x-2=0 \quad x+1=0$$

$$x=2 \quad x=-1$$



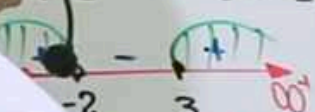
C.I.: $\langle -1; 2 \rangle$

RACCIÓN: (DEN $\neq 0$)

$$\frac{x-3}{x+2} \geq 0 ; x+2 \neq 0 ; x \neq -2$$

$$x-3=0 ; x+2=0$$

$$x=3 \quad x=-2$$



C.I.: $\langle -\infty; -2 \rangle \cup [3; \infty)$

INECUACIONES I.

→ DESIGUALDAD

$>$; $<$; \geq ; \leq

→ CONJUNTO

→ INTERVALO
(PORCIÓN REC)

OjO: J-I. AA

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$$5.- a < b \Rightarrow \frac{1}{b} < \frac{1}{a}$$

$$a < x < b$$

$$-x < -a$$

$$b$$

$$< \frac{1}{a}$$

$$8.- a < x < b$$

$$a^2 < x^2 < b^2$$

$$9.- -a < x < b$$

$$0 \leq x^2 < \text{MAYOR } a^2$$

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$$b^2 < x^2 < a^2$$

RESOLUCIÓN DE UNA INECUACIÓN

1.- VARIABLES EN UN LADO DE LA INECUACIÓN (+)

2.- FACTORIZAMOS

3.- PUNTOS CRÍTICOS EN LA RECTA REAL

① I. POLINOMIO:

$$x^2 - x < 2$$

$$x^2 - x - 2 < 0$$

$$(x-2)(x+1) < 0$$

$$x-2=0 \quad x+1=0$$

$$x=2 \quad x=-1$$



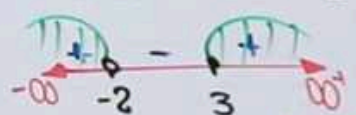
C.S.: $\langle -1, 2 \rangle$

② I. FRACCIÓN: (DEN $\neq 0$)

$$\frac{x-3}{x+2} \geq 0; x+2 \neq 0; x \neq -2$$

$$x-3=0; x+2=0$$

$$x=3; x=-2$$



C.S.: $\langle -\infty, -2 \rangle \cup [3, \infty)$



① $x \in [1; 2] \Rightarrow \frac{1}{a} \leq \frac{1}{5x+3} \leq \frac{1}{b}$

→ "a-b"

$$1 \leq x \leq 2$$

$$5 \leq 5x \leq 10$$

$$8 \leq 5x+3 \leq 13$$

$$\frac{1}{13} \leq \frac{1}{5x+3} \leq \frac{1}{8}$$

→ $a=13; b=8$

$a-b=5$

② $x \in [-1; 3] ; \frac{3x+4}{x+2}$

$$\downarrow$$

$$3 - \frac{2}{x+2}$$

$$+2 \left(\begin{array}{l} -1 \leq x < 3 \\ 1 \leq x+2 < 5 \end{array} \right.$$

$$\frac{1}{5} < \frac{1}{x+2} \leq 1$$

$$\frac{2}{5} < \frac{2}{x+2} \leq 2$$

$$-2 \leq -\frac{2}{x+2} < -\frac{2}{5}$$

$$1 \leq 3 - \frac{2}{x+2} < \frac{13}{5}$$

→ $[1; \frac{13}{5}]$

③ $3 \leq x \leq 7 ; M = \frac{2x+1}{2x-5}$

$$\downarrow$$

$$1 + \frac{6}{2x-5}$$

$$\times 2 \left(\begin{array}{l} 3 \leq x \leq 7 \\ 6 \leq 2x \leq 14 \end{array} \right.$$

$$-5 \left(\begin{array}{l} 1 \leq 2x-5 \leq 9 \end{array} \right.$$

$$\times 6 \left(\begin{array}{l} \frac{1}{9} \leq \frac{1}{2x-5} \leq 1 \end{array} \right.$$

$$+1 \left(\begin{array}{l} \frac{6}{9} \leq \frac{6}{2x-5} \leq 6 \end{array} \right.$$

$$\frac{5}{3} \leq 1 + \frac{6}{2x-5} \leq 7$$

→ $[\frac{5}{3}; 7]$

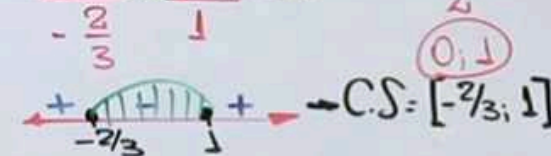
④ ¿CUANTOS ENTEROS?

$$3x^2 \leq x+2$$

$$3x^2 - x - 2 \leq 0$$

$$\begin{array}{r} 3x^2 - x - 2 = 2x \\ x - 1 = -3x \\ -x \end{array}$$

$$(3x+2)(x-1) \leq 0$$



⑤ $x^2 + 4x + 3 < 0$

$$x^2 + 4x + 3 < 0$$

$$\begin{array}{r} x^2 + 4x + 3 = 3x \\ x - 1 = x \\ 4x \end{array}$$

$$(x+3)(x+1) < 0$$

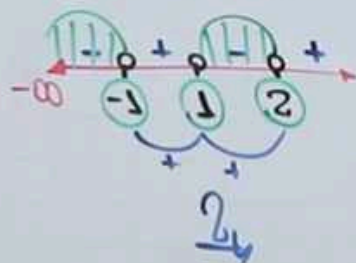




⑥ $1x^5 - 2x^4 + 2x^3 - 4x^2 - 3x + 6 \leq 0$

| | | | | | | |
|---|--------|----|----|----|----|----|
| | 1 | -2 | 2 | -4 | -3 | 6 |
| ① | ↓ | 1 | -1 | 1 | -3 | -6 |
| | 1 | -1 | 1 | -3 | -6 | 0 |
| ② | ↓ | -1 | 2 | -3 | 6 | |
| | 1 | -2 | 3 | -6 | 0 | |
| ③ | ↓ | 2 | 0 | 6 | | |
| → | $1x^2$ | 0 | 3 | 0 | | |

→ $(x-1)(x+1)(x-2)(x^2+3) < 0$
 $x=1 \quad x=-1 \quad x=2$

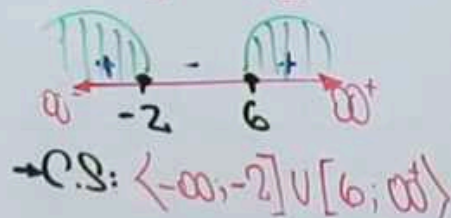


⑦ $(m+3)x^2 - 2mx + 4 = 0$
 DATO: $ax^2 + bx + c = 0$
 $\Delta > 0 \rightarrow \mathbb{R} \neq$
 $\Delta = 0 \rightarrow \mathbb{R} =$
 $\Delta < 0 \rightarrow \text{IMAGINARIOS}$
 $\Rightarrow \Delta \geq 0$

$(-2m)^2 - 4(m+3)(4) \geq 0$
 $4m^2 - 16m - 48 \geq 0$

$m^2 - 4m - 12 \geq 0$
 $m \begin{matrix} \nearrow -6 = -6m \\ \searrow 2 = \frac{2m}{4m} \end{matrix}$

$(m-6)(m+2) \geq 0$
 $\frac{m-6}{6} \quad \frac{m+2}{-2}$



⑧ $(x-4)(2-x)(x+1) \geq 0$
 $\frac{(x-4)}{4} \frac{(x-2)}{2} \frac{(x+1)}{-1} \leq 0$

C.S: $\langle -\infty, -1 \rangle \cup [2, 4]$
 $\frac{4}{4}$

⑨ $\frac{6x-3}{2} - (2x-6) > \frac{x+1}{4}$
 $2(6x-3) - 4(2x-6) > x+1$
 $12x-6-8x+24 > x+1$
 $4x-6 > x+1$
 $3x > 7$
 $x > \frac{7}{3}$

⑩ $x \in \langle -2, 3 \rangle$, $0 \leq x^2 + 2x + 3$
 $0+b=??$
 $-2 < x < 3$
 $-1 < x+1 < 4$
 $0 \leq (x+1)^2 < 16$
 $0 \leq (x+1)^2 + 2 < 18$
 $2; b=18$

$a+b=20$

⑥ $1x^5 - 2x^4 + 2x^3 - 4x^2 - 3x + 6 < 0$

| | | |
|----|----|----|
| 1 | -2 | 6 |
| 1 | | -6 |
| 1 | | 0 |
| -1 | | |
| 1 | | |
| 2 | 2 | |
| 1 | 0 | |

$\rightarrow 1x^2 - 3x + 6 < 0$

⑦ $(m+3)x^2 - 2mx + 4 = 0$
 DATO: $ax^2 + bx + c = 0$
 $\Delta > 0 \rightarrow \mathbb{R} \neq$
 $\Delta = 0 \rightarrow \mathbb{R} =$
 $\Delta < 0 \rightarrow \text{IMAGINARIOS}$
 $\Rightarrow \Delta \geq 0$

$(-2m)^2 - 4(m+3)(4) \geq 0$

$4m^2 - 16m - 48 \geq 0$

$m^2 - 4m - 12 \geq 0$

$m \begin{matrix} -6 \\ 2 \end{matrix} = \begin{matrix} -6m \\ 2m \end{matrix}$

$\frac{(m-6)(m+2)}{6 \cdot -2} \geq 0$

$\rightarrow \text{C.S.}: \langle -\infty, -2 \rangle \cup [6, \infty)$

⑧ $(x-4)(2-x)(x+1) \geq 0$

$\frac{(x-4)(x-2)(x+1)}{4 \cdot 2 \cdot -1} \leq 0$

$\infty \quad -1 \quad 2 \quad 4 \quad \infty$

C.S.: $\langle -\infty, -1 \rangle \cup [2, 4]$

\downarrow

⑨ $\left(\frac{6x-3}{2} - (2x-6) \right) > \frac{x-3}{4} \cdot 4$

$2(6x-3) - 4(2x-6) > x-3$

$12x-6-8x+24 > x-3$

$3x > -21$

$x > -7$

$\langle -7, \infty \rangle$

⑩ $x \in \langle -2, 3 \rangle ; a \leq x^2 + 2x + 3 \leq b$

$a+b = ??$

$-2 < x < 3$

$-1 < x+1 < 4$

$0 \leq (x+1)^2 < 16$

$2 \leq (x+1)^2 + 2 < 18$

$a=2; b=18$

$a+b = 20$

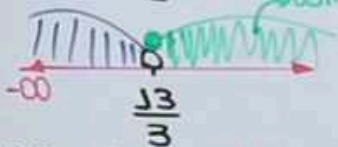
11) $\frac{4x+1}{5} > \frac{3x-2}{3}$

$$3(4x+1) > 5(3x-2)$$

$$12x+3 > 15x-10$$

$$13 > 3x$$

$$\frac{13}{3} > x$$



$$C.S: \left\langle -\infty; \frac{13}{3} \right\rangle$$

Complemento:

$$\left[\frac{13}{3}; \infty \right)$$

12) $\frac{2x^2-2mx-n}{x} < 0$
 $x \in \langle -3; 5 \rangle$



$$(x+3)(x-5) < 0$$

$$x^2-2x-15 < 0$$

$$\times 2 \rightarrow 2x^2-4x-30 < 0$$

$$\begin{aligned} \checkmark +2mx &= -4x \\ 2m &= -4 \\ m &= -2 \end{aligned}$$

$$\checkmark +n = -30$$

$$n = -30$$

$$\Rightarrow M \cdot n = 60$$

13) $\frac{2}{3} < \frac{x+1}{x+3} < \frac{7}{9}$

$$\frac{2}{3} < 1 - \frac{2}{x+3} < \frac{7}{9}$$

$$-\frac{1}{3} < -\frac{2}{x+3} < -\frac{2}{9}$$

$$\frac{2}{9} < \frac{2}{x+3} < \frac{1}{3}$$

$$\frac{1}{9} < \frac{1}{x+3} < \frac{1}{6}$$

$$6 < x+3 < 9$$

$$3 < x < 6$$

$$C.S: \langle 3; 6 \rangle$$

15) $\left(\frac{x}{4} + 1 < \frac{x-5}{2} < \frac{x}{3} + 2 \right)$

$$\rightarrow m.c.m(4; 2; 3) = 12$$

$$3x+12 < 6x-30 < 4x+24$$

$$3x+12 < 6x-30$$

$$42 < 3x$$

$$14 < x$$

$$6x-30 < 4x+24$$

$$2x < 54$$

$$x < 27$$

$$14 < x < 27$$

$$\rightarrow \underline{26}$$

16) $(1-x)(x-3)(x^2+1) > 0$

⑪ $\frac{4x+1}{5} > \frac{3x-2}{3}$

$3(4x+1) > 5(3x-2)$
 $12x+3 > 15x-10$

⑫ $\frac{2x^2-2mx-n}{x} < 0$
 $x \in \langle -3; 5 \rangle$



$(x+3)(x-5) < 0$
 $x^2-2x-15 < 0$

$\times 2$
 $2x^2-4x-30 < 0$

$\checkmark +2mx = -4x$
 $2m = -4$
 $m = -2$

$\checkmark +n = -30$
 $n = -30$
 $\Rightarrow M \cdot n = 60$

⑬ $\frac{2}{3} < \frac{x+1}{x+3} < \frac{7}{9}$
 $\frac{2}{3} < 1 - \frac{2}{x+3} < \frac{7}{9}$

$-\frac{1}{3} < -\frac{2}{x+3} < -\frac{2}{9}$

$\frac{2}{9} < \frac{2}{x+3} < \frac{1}{3}$

$\frac{1}{9} < \frac{1}{x+3} < \frac{1}{6}$

$6 < x+3 < 9$

$3 < x < 6$

C.S: $\langle 3; 6 \rangle$

⑮ $\left(\frac{x}{4} + 1 < \frac{x-5}{2} < \frac{x}{3} + 2 \right)$
 $\rightarrow m.c.m(4; 2; 3) = 12$
 $3x+12 < 6x-30 < 4x+24$

$3x+12 < 6x-30$
 $42 < 3x$
 $14 < x$

$6x-30 < 4x+24$
 $2x < 54$
 $x < 27$

$14 < x < 27$

$\rightarrow 26$

⑯ $(1-x)(x-3)(x^2+1) > 0$
 $(x-1)(x-3)(x^2+1) < 0$



C.S: $\langle 1; 3 \rangle$

⑰ $2x-5 < x+8 < 3x-7$

$2x-5 < x+8$
 $x < 13$

$x+8 < 3x-7$
 $15 < 2x$
 $7.5 < x$

$7.5 < x < 13$



17) $(x^2+7)(x^2+25)(x^2-9) \leq 0$
 $\Delta < 0$
 $(x^2+7)(x^2+25)(x+3)(x-3) \leq 0$
 $\Delta < 0$ -3 3

C.S: $[-3, 3]$

18) $(x^2-x+6)(x^2-x-6)(x-1)^2(x+4)^2 \leq 0$
 $\Delta < 0$ $=0$ $=0$
 $x^2-x-6 \leq 0$
 $(x-3)(x+2) \leq 0$
 3 -2

$x-1=0$ $x+4=0$
 $x=1$ $x=-4$

C.S: $[-2, 3] \cup \{-4\}$

19) $(3x+1)(x-2)^2(x+5)(x-2)(4-x) \leq 0$
 $=0$ $=0$
 $(3x+1)(x+5)(4-x) \leq 0$
 $(3x+1)(x+5)(x-4) \geq 0$
 $-\frac{1}{3}$ -5 4

$x-2=0$
 $x=2$

C.S: $[-5, -\frac{1}{3}] \cup [4, \infty) \cup \{2\}$

20) $\frac{(x-1)(x+2)}{x-5} > 0$
 1 -2
 $x-5$
 5

C.S: $\langle -2, 1 \rangle \cup \langle 5, \infty \rangle$