Semana: #12

Logaritmos

(Jhon Nepier)

Def: $A, B > 0 \; ; \; B \neq 1$

Exponente: n — base: B — potencia: A

$$\log_B A = n \longleftrightarrow B^n = A$$

$\log_2 8 = 3$ $\quad$ $\log_2 32 = 5$

$\log_2 1024 = 10$ $\quad$ $\log_5 125 = 3$

$\log_n n = 1$ $\quad$ $\log_n 1 = 0$

Prop:

1. $\log_n (A \cdot B) = \log_n A + \log_n B$
2. $\log_n (A/B) = \log_n A - \log_n B$
3. $\log_B A^n = n \cdot \log_B A = \log_{\sqrt[n]{B}} A$
4. $\log_B \sqrt[n]{A} = \frac{1}{n} \log_B A = \log_{B^n} A$
5. $\log_B A = \log_{\sqrt[3]{B}} \sqrt[3]{A} = \log_{B^5} A^5$
6. $\log_B A = \dfrac{1}{\log_A B}$
7. $\log_A \cancel{B} \cdot \log_{\cancel{B}} \cancel{C} \cdot \log_{\cancel{C}} D = \log_A D$
8. $\log_B A = \dfrac{\log_n A}{\log_n B}$
9. $A^{\log_A B} = B^{\log_A A} = B$
10. $\text{Antilog}_B A = B^A$

$\text{Colog}_B A = -\log_B A$

OBS: $2 < e < 3$

- $\text{Log } A = \log_{10} A$ (vulgar)
- $\text{Ln } A = \log_e A$ (neperiano)

01:

$$E = \log_{3\sqrt{3}} (27 \sqrt[3]{9})$$

$\frac{2}{3} + 3 = 11/3$

$$E = \log_{\sqrt{3}^3} (3^3 \cdot \sqrt[3]{3^2})$$

$$E = \log_{3^{3/2}} 3^{11/3} = \frac{11/3}{3/2} = \frac{22}{9}$$

02:

11/3

02:

$$5^{\log_{11}(x^2-7x+21)} \quad \log_{11} 25$$

$$\cancel{5}^{\log_{11}(x^2-7x+21)} = \cancel{(5)}^{\log_{11} 3^2}$$

$$\cancel{\log_{11}}(x^2-7x+21) = \cancel{\log_{11}} 9$$

$$x^2-7x+12=0$$

$$x_1 = 4 \;//$$

$$x_2 = 3$$

03:

$$10^{\log x^2} = 5x-7$$

$$\underbrace{10^{\log_{10} x^2}} = 5x-7$$

$$x^2 = 5x-7$$

$$x^2-5x+7=0$$

• Suma = 5

04:

$$\alpha+\beta=-10$$

$$\alpha\cdot\beta=10$$

$$x^2+10x+10=0$$

$$x^2-Sx+P=0$$

$\{\alpha, \beta\}$: raíces

$$E=\log\left(\frac{\alpha}{5}\right)+\log\left(\frac{\beta}{2}\right)$$

$$E=\log\left(\frac{\alpha}{5}\cdot\frac{\beta}{2}\right)$$

$$E=\log\left(\frac{\alpha\cdot\beta}{10}\right) \quad \rightarrow 10$$

$$E=\log_{10} 1 = 0 \;//$$

05: Hallar: x^x

$$x+\log(1+2^x) = x\log 5 + \log 6$$

$$\underbrace{\log 10^x + \log(1+2^x)} = \log 5^x + \log 6$$

$$\cancel{\log} 10^x(1+2^x) = \cancel{\log} 5^x\cdot 6$$

$$10^x+20^x = 5^x\cdot 6$$

$$\underbrace{20^x+10^x-6\cdot 5^x}=0$$

$$\cancel{5^x}(4^x+2^x-6)=0$$

$$2^x \quad +3=0$$

$$2^x \quad -2=0$$

$2^x+3=0$

$2^x=-3$

$2^x-2=0$

$2^x=2$

$x=1$

06:

$$\text{antilog}_x\ \text{antilog}_{\sqrt[4]{2}}\ \text{antilog}_2 3 = 81$$

$$\text{antilog}_x\ \text{antilog}_{\sqrt[4]{2}} 8 = 81$$

$$\text{antilog}_x \left(\sqrt[4]{2}\right)^{8} = 81$$

$$\underbrace{\text{antilog}_x 4 = 81}$$

$$x^4 = 81$$

$$x = 3$$

07: $y - z = 8$

$\text{antilog}_2 x$; $\text{antilog}_4 y$; $\text{antilog}_8 z$: PG

Hallar: $x - z$

OBS: 2^x; 4^y; 8^z : PG

$$\therefore\ 2^x \cdot 8^z = (4^y)^2$$

$$2^x \cdot 2^{3z} = 2^{4y}$$

$$x + 3z = 4y$$

$$x = 4y - 3z$$

$$x - z = 4y - 4z$$

$$x - z = 4(y - z)$$

$$x - z = 32$$

08: Calcular: $\sqrt[3]{x^2}$

$$\underbrace{\log_{x} 2} + \log_x 4 = \frac{5}{6}$$

$$\underbrace{\log_{x} \sqrt{2} + \log_{x} 4} = \frac{5}{6}$$

$$\log_{x} (\sqrt{2} \cdot 4) = \frac{5}{6}$$

$2^2 \cdot 2^{1/2}$

$2^{5/2}$

$$x^{\frac{5}{6}} = 2^{\frac{5}{2}}$$

$$x^{\frac{1}{3}} = 2$$

$$x = 8$$

09: $x = 7^{\log_5 a}$

$$E = \sqrt{5^{\log_a x} + 6x^{\log_a 5}}$$

$$E = \sqrt{x^{\log_a 5} + 6 \cdot x^{\log_a 5}}$$

$$E = \sqrt{7 \cdot x^{\log_a 5}}$$

$$E = \sqrt{7 \cdot 7^{\log_5 a \cdot \log_a 5}}$$

$$E = \sqrt{49} = 7$$

10:

Log

= lo

10:

$\log_2 x \cdot \cancel{\log_{4x} 8} \cdot \cancel{\log_x 4} \cdot \cancel{\log_{2x} 8}$
$\cdot \cancel{\log_4 2x} \cdot \cancel{\log_8 4}$

$= \log_x \cancel{4} \cdot \log_{\cancel{4}} \cancel{2x} \cdot \log_{\cancel{2x}} 8 \cdot$
$\log_2 \cancel{4x} \cdot \log_{\cancel{4x}} \cancel{8} \cdot \log_8 x^3$

$= \log_x \cancel{8} \cdot \log_{\cancel{8}} x^9$

$= \log_x x^9 = 9$

11: Hallar: x

$\log_4 x^2 + \log_4 y + \log_4 z = 2$

$\log_9 y^2 + \log_9 z + \log_9 x = 2$

$\log_{16} z^2 + \log_{16} x + \log_{16} y = 2$

$\boxed{\log_4 x^2yz = 2} \rightsquigarrow x^2yz = 2^4 \ (\times)$

$\log_9 y^2zx = 2 \rightsquigarrow y^2zx = 3^4$

$\log_{16} z^2xy = 2 \rightsquigarrow z^2xy = 4^4$

$x^{\cancel{4}} \cdot y^{\cancel{4}} \cdot z^{\cancel{4}} = 2^{\cancel{4}} \cdot 3^{\cancel{4}} \cdot 4^{\cancel{4}}$

$xyz = 24$

$x = \dfrac{2}{3}$

12:

$\log x^2 = a$

$\log y^2 = b$

$E = 20 \cdot \log \sqrt[10]{\dfrac{x}{y}}$

$E = \dfrac{20}{10} \cdot (\log x - \log y)$

$E = \cancel{2} \cdot \left(\dfrac{a}{\cancel{2}} - \dfrac{b}{\cancel{2}}\right) = a - b$

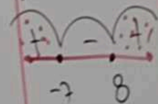
28:

$\sqrt[\text{impar}]{x} \Rightarrow$ elimina todo.

$\sqrt[\text{par}]{x} \Rightarrow x \geq 0$

$\dfrac{(x^2-36)(x+1)}{(x^2+14x+48)} \geq 0$

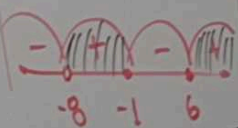
$x^2 - x - 56 \geq 0$
x −8
x +7
PC: 8, −7

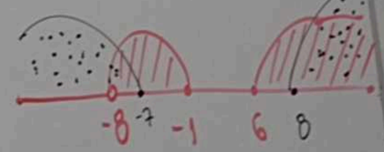


$\cap$ $\dfrac{\cancel{(x+6)}(x-6)(x+1)}{\cancel{(x+6)}(x+8)} \geq 0$

$\dfrac{(x-6)(x+1)}{x+8} \geq 0$

PC: 6, −1, −8





CS: ⟨−8, −7] ∪ [8, +∞⟩