

Semana: #06  
FACTORIZACIÓN EN  $\mathbb{Q}$  <sup>(B)</sup>

Def:

polinomios

x factores primos.

Factor primo:

(No desintegrada)

- $x^2 - 1 = (x+1)(x-1)$  NO F.P
- $x^3 + 1 = (x+1)(x^2 - x + 1)$  NO F.P
- $x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$  NO F.P
- $x^2 - 2 = (x + \sqrt{2})(x - \sqrt{2})$  (SI F.P)

Teorema:

$$P(x) = 3x^2(x+1)(x-2)$$

- # factor primos = 3
- # divisores =  $3 \cdot 4 \cdot 5 = 60$
- # fact. algebraicos =  $60 - 1 = 59$



F. Primos = F. Irreducible

F. Lineal = F. primo.  
 $(ax+b)$

multiplicidad = repetición.

$$(x+1)^3(x+2)^2(x-1)$$

## Métodos:

① Factor común: (menor exp.)  
(Agrupación de términos)

② Aspas:  $\left\{ \begin{array}{l} \text{A. simple: } 3 \\ \text{A. Doble: } 6 \\ \text{A.D. Especial: } 5 \end{array} \right.$

③ Identidades:  $\left\{ \begin{array}{l} \text{- Argand} \\ \text{- dif. cuad.} \\ \text{- Syd. cubos.} \end{array} \right.$

④ Divisores binómicos: (Paolo Ruffini)

$$\text{V.N} = \frac{\text{Div. T. IND}}{\text{Div. Coef. princ.}}$$

⑤ Artificios:  $\left\{ \begin{array}{l} \text{- Quita y pon} \\ \text{- Descomp.} \\ \text{- Cambio variable.} \end{array} \right.$

01:

$$P_{(x,y)} = x^2 y (x+y^2) (x^2+1)$$

$$\# \text{ f. primos} = 3$$

$$\# \text{ div} = 3 \cdot 2 \cdot 2 = 12$$

$$\# \text{ fact. alg} = 11$$

Rpta: ①

02:

$$x^2 y (x+1)(x+3)(x-y)(x+z^2)(x^2+1)$$

$$\# \text{ f. primos} = 6$$

$$\# \text{ f. p. cuadráticos} = 1$$

$$x; y; x+3; x-y; x+z^2; x^2+1$$



Q3:

$$= x^5 y + x^4 y z + x^3 y + x^2 y z$$

$$= x^2 y (x^3 + x^2 z + x + z)$$

$$= x^2 y \{ x^2 (\underline{x+z}) + 1(\underline{x+z}) \}$$

$$= x^2 y (x+z)(x^2+1)$$

# fact. primos = (4)

Q4:

$$= 4x^4 y - 4x^3 y^2 - 24x^2 y^3$$

$$= 4x^2 y (x^2 - xy - 6y^2)$$

$$\begin{array}{r} x \\ x \end{array} \begin{array}{l} \diagup \\ \diagdown \end{array} \begin{array}{l} -3y = -3xy \\ +2y = 2xy \\ -xy \end{array}$$

$$= 4x^2 y (x-3y)(x+2y)$$

FFF (V) //

$$x^2 y^3$$

$$y^2)$$

$$y = -3xy$$

$$y = \underline{2xy}$$

$$-xy$$

$$\times$$

05:

$$x-y=a //$$

$$12(x-y)^2 + 7(x-y) - 12$$

$$12a^2 + 7a - 12$$

$$4a$$

$$3a$$

$$-3 = -9a$$

$$+4 = \underline{16a}$$

$$7a$$

$$= (4a-3)(3a+4)$$

$$= (4x-4y-3)(3x-3y+4)$$

$$\# \text{ f. primos: } (2)$$

06:

$$ab(x^2-y^2) + xy(a^2-b^2)$$

$$abx^2 - aby^2 + xy a^2 - xy b^2$$

$$(abx^2 + xy a^2) - (aby^2 + xy b^2)$$

$$ax(bx+ay) - by(ay+bx)$$

$$(ay+bx)(ax-by)$$

$$\underline{2 \text{ f. primos: } (2)}$$



Q7:

$$P = x^8 - 1$$

$$P = (x^4 + 1)(x^4 - 1)$$

$$P = (x^4 + 1)(x^2 + 1)(x^2 - 1)$$

$$P = (x^4 + 1)(x^2 + 1)(x + 1)(x - 1) /$$

# fact. primos = ①  
Cuadráticos

Q8:

$$x^6 - x^2 - 80x - 16$$

$$x^6 - (x^2 + 80x + 16)$$

$$(x^3)^2 - (x + 4)^2$$

$$(x^3 + x + 4)(x^3 - x - 4) /$$

# f. primos = ②

12:

$$3^{m+1} = b$$

$$R(m) = 3^{2m+2} - 3^{m+1} - 30$$

$$R = (3^{m+1})^2 - 3^{m+1} - 30$$

$$R = b^2 - b - 30$$
$$\begin{array}{r} b \\ b \end{array} \begin{array}{l} \downarrow \\ \times \\ \downarrow \end{array} \begin{array}{l} -6 \\ +5 \end{array}$$

$$R = (b-6)(b+5)$$

$$R = (3^{m+1} - 6)(3^{m+1} + 5)$$

# fact. primos = 2

14:

$$x^2 - 2x = A$$

$$(x+1)(x-3)(x+4)(x-6) + 38$$

$$(x^2 - 2x - 3)(x^2 - 2x - 24) + 38$$

$$(A-3)(A-24) + 38$$

$$A^2 - 27A + 110$$

$$\begin{array}{r} A \\ A \end{array} \begin{array}{l} \downarrow \\ \times \\ \downarrow \end{array} \begin{array}{l} -22 \\ -5 \end{array}$$

$$(A-22)(A-5)$$

$$(x^2 - 2x - 22)(x^2 - 2x - 5)$$

es? primos (152)

-6)  
+  
88

+38

38

15:

$$5x^2 + 16xy + 3y^2 + 11x + 5y + 2$$

$$= (5x + y + 1)(x + 3y + 2)$$

\* A. DOBLE: (3 A. Simples)

①

$x^2$

②

$xy$

③

$y^2$

④

$x$

⑤

$y$

⑥

2. IND

17: (A.O. Especial)

$$2x^4 + 5x^3 + 3x^2 + 5x - 3$$

$$= (2x^2 + x + 3)(x^2 - 2x - 1)$$

$$= (2x^2 + x + 3)(x^2 - 2x - 1)$$



L)

$$\begin{array}{r}
 2x^2 + 5x - 3 \\
 + 3 = 3x^2 \\
 - 1 = 2x^2 \\
 \hline
 x^2
 \end{array}$$

$$\begin{array}{r}
 -1 \\
 \hline
 \end{array}$$

19:

$$x^4 + 2x^2 + 9$$

$$\begin{array}{r}
 x^4 + 0x^3 + 2x^2 + 0x + 9 \\
 x^2 \quad \quad \quad -2x \quad \quad \quad 3 \\
 \hline
 x^2 \quad \quad \quad 2x \quad \quad \quad 3
 \end{array}$$

$$= (x^2 - 2x + 3)(x^2 + 2x + 3)$$

$$2 \text{ pairs } (x^2)$$

$$2x^2 + 3 = p$$

$$8(x^2 + 1)^2(x^2 + 3)$$

$$43 = 4$$

29:

$$\begin{aligned}
 &= (2x^2 + 3)^3 + 4x^2(x^2 + 3) - 10x^2 - 3 \\
 &= (2x^2 + 3)^3 + 4x^4 + 12x^2 - 10x^2 - 3 \\
 &= (2x^2 + 3)^3 + 4x^4 + 2x^2 - 3 \\
 &= (2x^2 + 3)^3 + (2x^2 + 3)^2 - 5(2x^2 + 3) + 3 \\
 &= p^3 + p^2 - 5p + 3
 \end{aligned}$$

1	1	-5	3
1	1	2	-3
1	2	-3	0
1	1	3	
	1	3	0

$$\begin{aligned}
 &= (p-1)^2(p+3) \\
 &= (2x^2+2)(2x^2+6)
 \end{aligned}$$