Semana: #03

Polinomios - Grados

Def: E.A.R.E ($\exp \in \mathbb{Z}_0^+$)

$P_{(x)} = 5x^3 + 6x^2 + 7x + 8$

- x : variable
- 5, 6, 7, 8 : coef. ó constantes
- 3 : grado de $P_{(x)}$.
- 5 : coef. principal.
- 8 : térm. indep.

Grados: G. Relativo (G.R) (monomio - polin.); G. Absoluto (grado) (G.A)

$M_{(x,y)} = 3x^2 y^3 z^4$

- $GR_x = 2$
- $GR_z = 0$
- $GR_y = 3$
- $GA = 2+3 = 5$

$P_{(x,y)} = \underbrace{2x^2y^3}_{5} - \underbrace{7x^3y}_{4} + \underbrace{2xy^7}_{8}$

- $GR_x = 3$
- $GA = 8$
- $GR_y = 7$

Prop: $GA(\cancel{P^2} \cdot Q + Q^3) = 30$

$P^\circ = 4 \quad Q^\circ = 10$

1. $GA(P \pm Q) = 10$
2. $GA(P \times Q) = 4 + 10 = 14$
3. $GA(P \div Q) = 4 - 10 = -6$
4. $GA(P^3) = 4 \times 3 = 12$
5. $GA(\sqrt[5]{Q}) = \frac{10}{5} = 2$

01:

$P_{(x,y,z)} = ($

30

01: Polinomio = EARE

$P_{(x,y,z)} = (n-1)x^{5}y^{n-2} - 3xz + 3x^{n}y^{5-n}z$

$n-2 \geq 0 \rightarrow n \geq 2$

$5-n \geq 0 \rightarrow 5 \geq n$

$\therefore\ 2 \leq n \leq 5$

$\downarrow$

2,3,4,5

$\sum n = 2+3+4+5 = 14$

02:

$P = 2x^{4} + 6x^{2}y^{2}z^{1} + 3x^{1}y^{4}z^{2} + 6y^{4} + 8x^{5}y^{2}z + 40z^{4}$

$GA(t_1) = 4$

$GA(t_2) = 5$

$GA(t_3) = 7$

$GA(t_4) = 4$

$GA(t_5) = 8$ Rpta

$GA(t_6) = 4$

03: $GR_x = 5$

$R_{(x)} = 3x^{a-1} - 5x^{a} + x^{a+1} - 3x^{a+2}$

$GR_x = a+2$

$5 = a+2$

$3 = a$

$2a = 6$ Rpta

$R_{(x)} = 3x^{2} - 5x^{3} + x^{4} - 3x^{5}$

04:

$(x+xy)^2 - (x-xy)^2$

Identidad Legendre:

- $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$
- $(a+b)^2 - (a-b)^2 = 4ab$

$00 = 4 \cdot x \cdot xy$

$= 4x^{2}y^{1} \rightarrow GA = 3$ Rpta

05: $GA = 33$

$2x^{a}y^{a+1} + 5x^{2a}y^{a+3} - 6x^{a-6}y^{a+7} + 7x^{2a}y^{a+2}$

$GA(t_1) = a + a+1 = 2a+1$

$GA(t_2) = 2a+a+3 = 3a+3$

$GA(t_3) = a-6+a+7 = 2a+1$

$GA(t_4) = 2a+a+2 = 3a+2$

$3a+3 = 33$

$3a = 30$

$a = 10$

$GR_y - GR_x$

$17 - 20 = -3$

Polinomios (Especiales)

- $P_{(x)} = 3x^2+7x+5x^3+2$ $NT = GA+1$

(Polin. completo)

- $P_{(x)} = x^5+3x^2+x-3$

(Polin. ordenado dec.)

- $P_{(x,y)} = \underbrace{x^2y^3}_{5}+\underbrace{x^3y^2}_{5}-\underbrace{5xy^4}_{5}$

(Pol. homogéneo)

- $P_{(x)} = x^3+3x^2+4$

(Pol. mónico) Coef P = 1

- $P_{(x)} = ax^2+bx+c$

$a=b=c=0$ (Pol. ident. nulo)

Valor numérico (VN)

Caso I: $P_{(x)} = x^2+2x+1$

$P_{(3)} = 3^2+2(3)+1 = 16$

$P_{(8)} = 8^2+2(8)+1 = 81$

Caso II: $P_{(x-2)} = x^2+3$

$P_{(5)} = 7^2+3 = 52$ VN

$x-2=5$

$x=7$

Teorema:

1) $T.IND(P) = P_{(0)}$

2) $\Sigma coef_{(P)} = P_{(1)}$

06:

$P_{(x)} = (3x-1)^2(5x+2)^3(x-7)+8250$

$\Sigma coef(P) = P_{(1)}$

$= 2^2 \cdot 7^3 \cdot (-6)+8250$

$= -8232+8250$

$= 18$

07:

$P_{(x)} = (3x-1)^{10}+3(9x-1)^5+x^3-4$

$T.IND(P) = P_{(0)}$

$= (-1)^{10}+3(-1)^5+0-4$

$= 1+(-3)+0-4$

$= -2-4$

$= -6$

Rpta

08: (Homogéneo)

$P_{(x,y)} = x^{m-2n} \cdot y^{m+n} + 15x^{n} \cdot y^{m+2n} - 2x^{m-n} \cdot y^{8}$

$GA(t_1) = m-2n+m+n = 2m-n$

$GA(t_2) = n+m+2n = 3n+m$

$GA(t_3) = m-n+8$

$3n+\cancel{m} = \cancel{m} \ldots 3$

$4n = 8$

$n = 2$

$2m-\cancel{2} = m-\cancel{2}+8$

$m = 8$

$\therefore m^{n} = 8^{2} = 64$

Rpta.

09: (Homogéneo)

$P = 8x^{n^3-3} \cdot y^{7} + 5x^{a} \cdot y^{b} + 7x^{n^3-8} \cdot y^{4n} + 11x^{3a+5} \cdot y^{2}$

$t_1^{\circ} = n^3+4 = 31$

$t_2^{\circ} = a+b = 31$

$t_3^{\circ} = n^3-8+4n = 31$

$t_4^{\circ} = 3a+7 = 31$

$\cancel{n^3}+4 = \cancel{n^3}-8+4n$

$12 = 4n$

$3 = n$

$3a+7 = 31$

$3a = 24$

$a = 8$

$8+b = 31$

$b = 23$

Rpta.

10: (Completo)

$P_{(x)} = x^{n+1} + 3x^{n+2} + x^{n+3} + 5$

$NT = GA+1$

$4 = GA+1$

$3 = GA \sim \boxed{n=0}$

$P_{(x)} = x + 3x^{2} + x^{3} + 5$

$P_{(-1)} = (-1) + 3(-1)^{2} + (-1)^{3} + 5$

$= -1 + 3 - 1 + 5$

$= 6$ Rpta.

11: Completo + ordenado decreciente

$P_{(x)} = x^{n+4} + \ldots + x^{a-1} + x^{a-2} + x^{a-3}$

$a-3 = 0 \sim a = 3$

$x^{n+4} + \ldots + x^{2} + x + 1$

$\boxed{NT = GA+1}$

$14 = n+4+1$

$14 = n+5$

$9 = n$

$a+n = 3+9 = 12$

Hombres:

14: Polinomio ident. nulo

$P_{(x)} = (a-5)x^2 + (b-3)x + c - 6$

$P_{(3)} = P_{(200)} = P_{(-999)} = 0$

$M = 2a + b + 2c$

$a - 5 = 0 \rightarrow a = 5$

$b - 3 = 0 \rightarrow b = 3$

$c - 6 = 0 \rightarrow c = 6$

$M = 10 + 3 + 12 = 25$

Mujeres:

21: $GA = 6006$

$M = a^6 b^{24} c^{60} d^{120} \ldots$

$6 + 24 + 60 + 120 + \ldots = GA$

$\underbrace{1.2.3 + 2.3.4 + 3.4.5 + \ldots}$ $= GA$

$\frac{n(n+1)(n+2)(n+3)}{4} = 6006$

$n = 11$

K

Hallar: $y^{z^n} = 2^{2^1} = 4$ Rpta.

DANI: → Ident. nulo.

09: Anula para más de un valor.

$P_{(x)} = z^{y^n} + x\{x[y^{z^2} - 16(1 + \frac{1}{8x})] + y - \frac{4}{x}\}$

$P_{(x)} = z^{y^n} + x^2[y^{z^2} - 16(1 + \frac{1}{8x})] + xy - 4$

$P_{(x)} = z^{y^n} + x^2 y^{z^2} - 16x^2(1 + \frac{1}{8x}) + xy - 4$

$P_{(x)} = z^{y^n} + x^2 y^{z^2} - 16x^2 - 2x + xy - 4$

$P_{(x)} = (\underbrace{y^{z^2} - 16}_{0})x^2 + (\underbrace{y - 2}_{0})x + (\underbrace{z^{y^n} - 4}_{0})$

$y = 2$ | $2^{z^2} = 2^4$, $z^2 = 4$, $z = 2$ | $2^{2^n} = 2^2$, $n = 1$

07:

$P_{(x)} = ($

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