

TRANSFORMACIONES TRIGONOMETRICAS

■ Identidades de la Suma o diferencia a Producto (A > B)

$$1) \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$2) \sin A - \sin B = 2 \sin\left(\frac{A-B}{2}\right) \cos\left(\frac{A+B}{2}\right)$$

$$3) \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$4) \cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

■ Identidades de Producto a Suma o diferencia (x > y)

$$1) 2 \sin x \cos y = \sin(x+y) + \sin(x-y)$$

$$2) 2 \sin y \cos x = \sin(x+y) - \sin(x-y)$$

$$3) 2 \cos x \cos y = \cos(x+y) + \cos(x-y)$$

$$4) 2 \sin x \sin y = \cos(x-y) - \cos(x+y)$$

1 Réduire:

$$K = \frac{\sin 8x + \sin 2x}{\sin 10x + \sin 4x}$$

$$K = \frac{2 \sin\left(\frac{8x+2x}{2}\right) \cos\left(\frac{8x-2x}{2}\right)}{2 \sin\left(\frac{10x+4x}{2}\right) \cos\left(\frac{10x-4x}{2}\right)}$$

$$K = \frac{\cancel{\sin 5x} \cancel{\cos 3x}}{\cancel{\sin 7x} \cancel{\cos 3x}} = \frac{\sin 5x}{\sin 7x} \#$$

2 Simplifier:

$$K = \frac{\cos x - \cos 5x}{\sin 5x - \sin x}$$

$$K = \frac{2 \sin\left(\frac{5x+x}{2}\right) \sin\left(\frac{5x-x}{2}\right)}{2 \sin\left(\frac{5x-x}{2}\right) \cos\left(\frac{5x+x}{2}\right)}$$

$$K = \frac{\sin 3x}{\cos 3x} = \tan 3x \#$$

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④ Reducir:

$$K = \frac{\sin 2A + \sin 2B}{\sin C} \cdot A+B+C=180^\circ$$

$$K = \frac{2 \sin\left(\frac{2A+2B}{2}\right) \cos\left(\frac{2A-2B}{2}\right)}{\sin C}$$

$$K = \frac{2 \sin(A+B) \cos(A-B)}{\sin C}$$

$$K = 2 \cos(A-B)$$

$$* Si: \alpha + \theta = 180$$

$$\sin \alpha = + \sin \theta$$

$$\cos \alpha = - \cos \theta$$

$$\tan \alpha = - \tan \theta$$

$$\cot \alpha = - \cot \theta$$

$$\sec \alpha = - \sec \theta$$

$$\csc \alpha = + \csc \theta$$

17 Reducir:

$$\frac{\frac{\sqrt{2}}{2} \sin x + \cos y \cdot \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} \sin y + \cos x \cdot \frac{\sqrt{2}}{2}}$$

$$\frac{2 \sin x \cos 45^\circ + 2 \sin 45^\circ \cos y}{2 \sin y \cos 45^\circ + 2 \sin 45^\circ \cos x}$$

$$= \frac{\sin(x+45^\circ) + \sin(x-45^\circ) + \sin(y+45^\circ) + \sin(45^\circ-y)}{\sin(y+45^\circ) + \sin(y-45^\circ) + \sin(45^\circ+x) + \sin(45^\circ-x)}$$

18 Reducir: $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\frac{\sqrt{3} \cos 20^\circ - \sin 40^\circ}{\cos 10^\circ}$$

$$\frac{2 \sin 60^\circ \cos 20^\circ - \sin 40^\circ}{\cos 10^\circ}$$

$$\frac{\sin(60^\circ+20^\circ) + \sin(60^\circ-20^\circ) - \sin 40^\circ}{\cos 10^\circ}$$

$$\frac{\sin 80^\circ}{\sin 80^\circ} = 1$$

12 Simplificar:

$$C = \frac{\sin 54^\circ + \sin 6^\circ}{\cos 24^\circ}$$

$$C = \frac{2 \sin 30^\circ \cos 24^\circ}{\cancel{\cos 24^\circ}}$$

$$C = 2 \sin 30^\circ = 2 \left(\frac{1}{2} \right)$$

$$C = 1 \quad \#$$

4 Reducir:

$$\frac{\sin 2x + \sin 4x + \sin 6x}{\cos 2x + \cos 4x + \cos 6x}$$

$$C = \frac{(2 \sin 4x \cos 2x) + \sin 4x}{(2 \cos 4x \cos 2x) + \cos 4x}$$

$$C = \frac{\sin 4x (2 \cos 2x + 1)}{\cos 4x (2 \cos 2x + 1)} = \tan 4x \quad \#$$

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26) Calcule: K ; $\theta = \frac{\pi}{17}$

* $\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$

* $\cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B$

$$K = \frac{\cos^2 60^\circ - \sin^2 30^\circ}{\cos 110^\circ + \cos 50^\circ}$$

$17\theta = \pi$

$17\theta = 180^\circ$

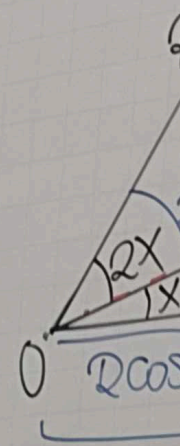
$$K = \frac{\cos(60+30) \cdot \cos(60-30)}{2 \cos 80^\circ \cos 30^\circ}$$

* Si: $\alpha + \theta = 180^\circ$

$\Rightarrow \cos \alpha = -\cos \theta$

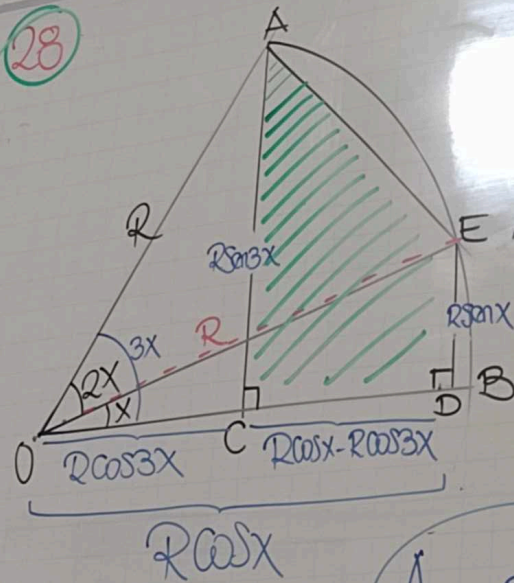
$$K = \frac{\cos 90^\circ \cos 30^\circ}{2 \cos 80^\circ \cos 30^\circ}$$

$$K = \frac{-\cos 80^\circ}{2 \cos 80^\circ} = -\frac{1}{2}$$



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$\vec{A} \cdot \vec{B}$
 $\vec{A} \cdot \vec{B}$



$$A_T = \left(\frac{B+b}{2} \right) h$$

$$A_T = \left(\frac{R \sin 3x + R \sin x}{2} \right) (R \cos x - R \cos 3x)$$

$$A_T = \frac{R^2}{2} \frac{(\sin 3x + \sin x)(\cos x - \cos 3x)}{\sin 2x \cos x \sin 2x \sin x}$$

$$A_T = R^2 \sin^3 2x$$