Physics 410 – Project 3

November 9, 2016

This project discusses a driven damped pendulum motion, exemplifying chaotic motion.

The angle of the pendulum is θ, and it moves under the influence of gravity, friction, and potentially an external periodic force. The angular velocity of the pendulum is $v = l\frac{d\theta}{dt}$ and the angular acceleration is $\alpha = l\frac{d^2\theta}{dt^2}$. The equation obeyed by $\theta(t)$ is then:

$$m\frac{d^2\theta}{dt^2} + \nu\frac{d\theta}{dt} + mg\sin(\theta) = A\sin(\omega t) \tag{1}$$

Henceforth we set the length of the pendulum to be $l = 1$ meter, the mass $m = 1$ kg, and $g = 1\,\frac{m}{sec^2}$. To solve the second order equation you'd need to put it in a standard form, as pair of first order equations.

- Using the provided code (wither RK4 or RFK), study the unforced ($A = 0$) motion for ten cycles, for $\nu = 1, 5, 10$. Choose as initial conditions $\theta = 0.2$ and $v = 0$. Make plots of θ as function of time (recalling that θ it is a periodic variable), as well as phase portraits of θ versus v. Comment on the resulting motion, which case corresponds to damped, under-damped or over-damped motion?

- Now we switch on the driving force, with the same constants and initial values as above, and $\nu = \frac{1}{2}$. Set $\omega = \frac{2}{3}$ and $A = 0.5, 1.2$. In both cases study the motion for at least 300 periods, making plots of $\theta(t)$ and phase portraits. Comments on features of the resulting motion.

- You might need to choose your parameters (step size in RK4, or tolerance and minimum step size in RFK) appropriately. Comment and explain the results for different amplitudes. Comment on the choices involved.

- For the same parameters, except for $A = 1.35, 1.44$ and 1.465, run the simulation for at least 300 periods and plot $\theta(t)$. Comment on your results.

- Further analyze the motion by making phase space plots using only points for which $\omega t = 2\pi n$ for integer n. This is an example of s Poincare section. Comment on your results.