

Physics 410 - Homework 2

1. Write a code for a 7-point centered formula for differentiating a function at the point x^* , using evaluations of the function at x^* and $x^* \pm n * h$, for $n = 1, 2, 3$.
 - Derive the required weights using the idea of differentiating the Lagrange interpolation.
 - Derive the optimal values of h , which minimizes the combination of discretization and round off errors, and use it in your code.
 - Check your formula by differentiating the function $\sin(x^2)$ for $0 \leq x \leq 1$ and comparing to expectation.
2. Write a code for Romberg integration of a function $f(x)$ over an interval $[a, b]$, using the composite trapezoid formula (with error of order h^2 , where $h = b - a$) applied to sub-intervals of length $h2^{-m}$. Eliminate all errors to order h^6 . You might want to use the ideas in section 3.9 to organize the calculation.

Apply the code to calculate $\int_0^1 \sin(x) dx$, and compare the results to expectations.