PHYS 410: Homework 1

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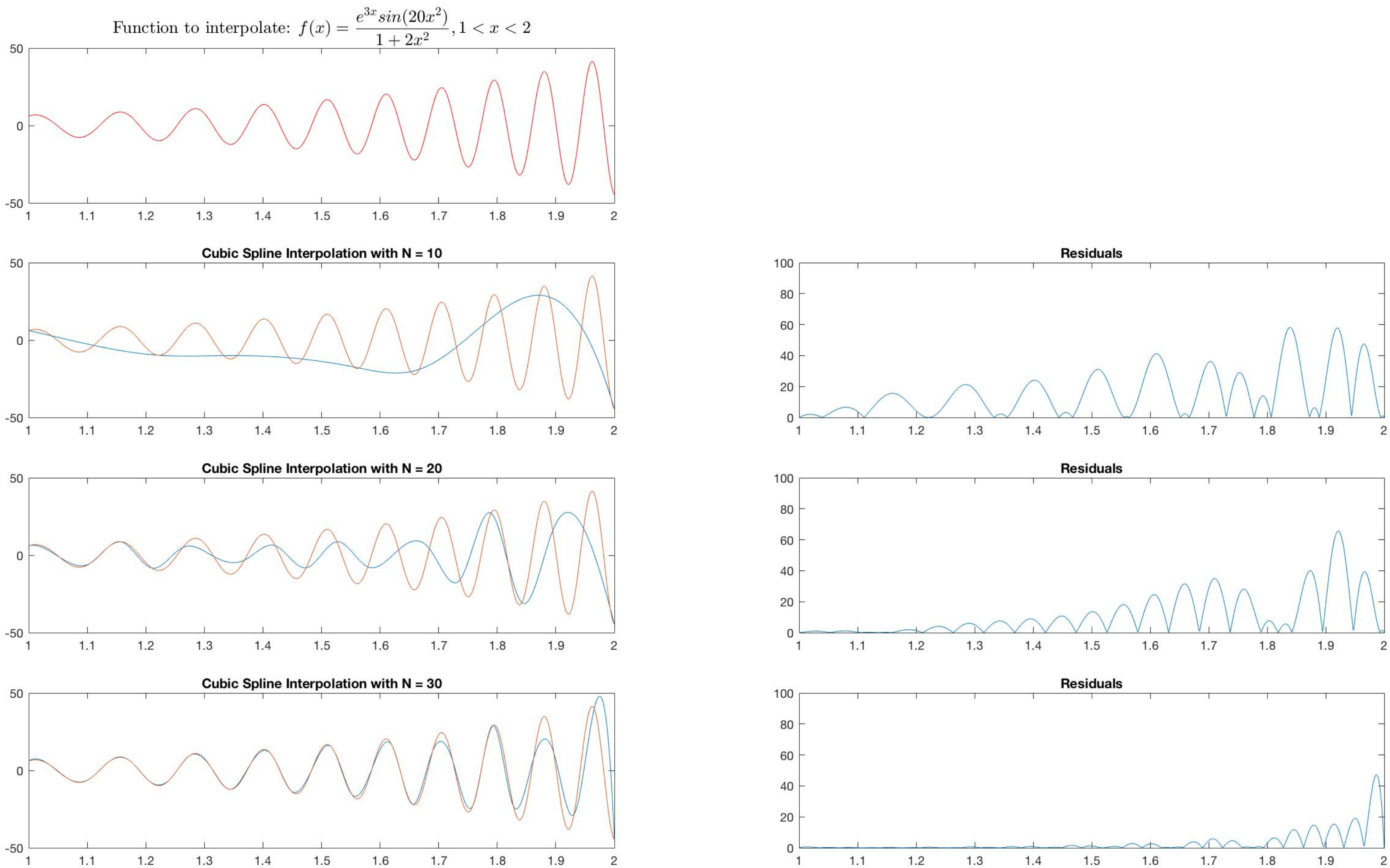
The MATLAB code is included in the code.zip file. After unzipping the folder, you can run the code by simply calling *interpolate* from the MATLAB command line (or by clicking the run button). The code generates three figures, reproduced below, that show the plots of the three interpolation techniques with $N = 10, 20, 30$ for each technique.

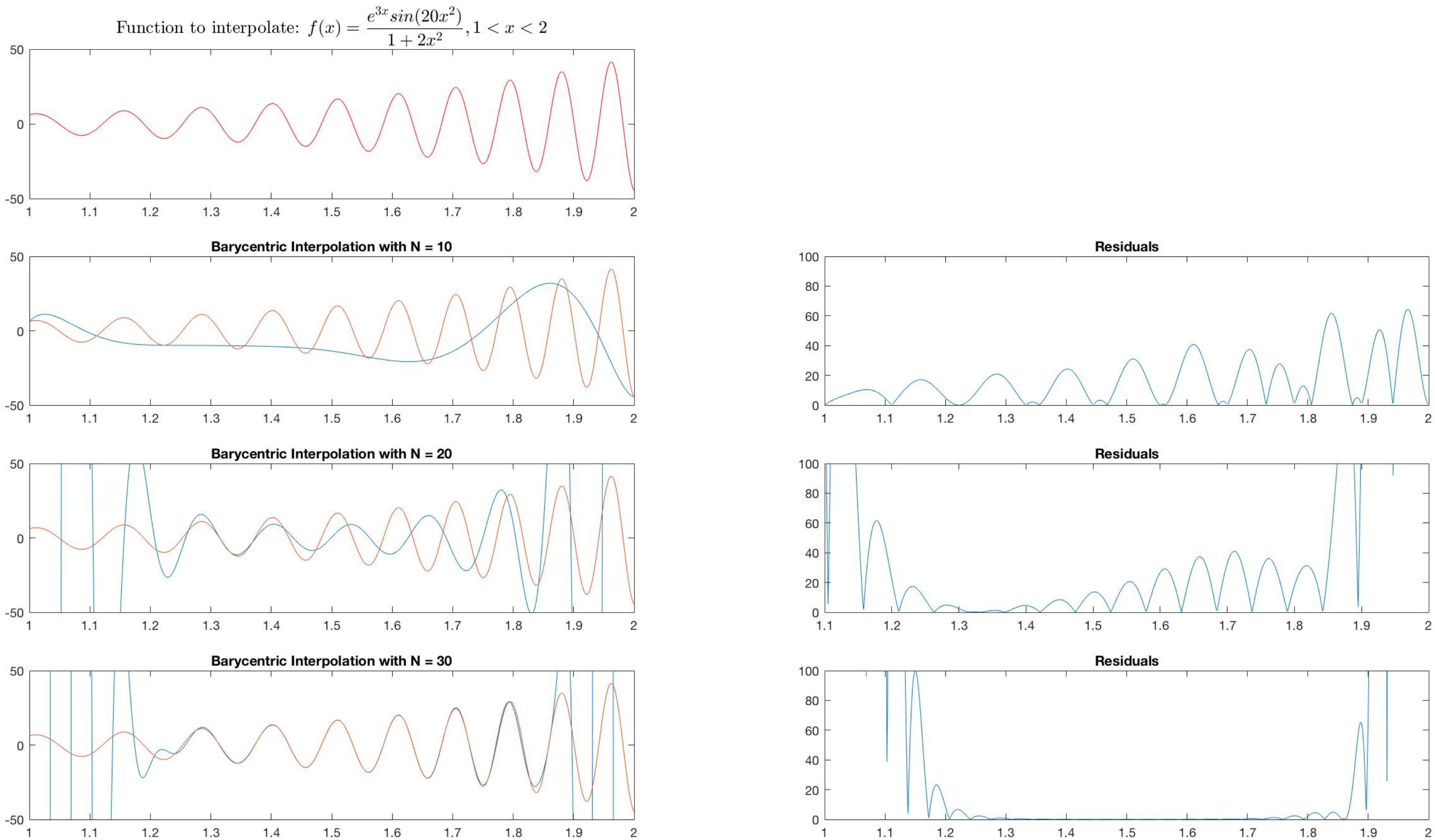
The variable user parameters are located at the top of the code file. You can change which interpolation techniques to use, as well as change whether or not a plot of the function should be overlayed on top of the interpolant functions, and whether or not to plot the residuals.

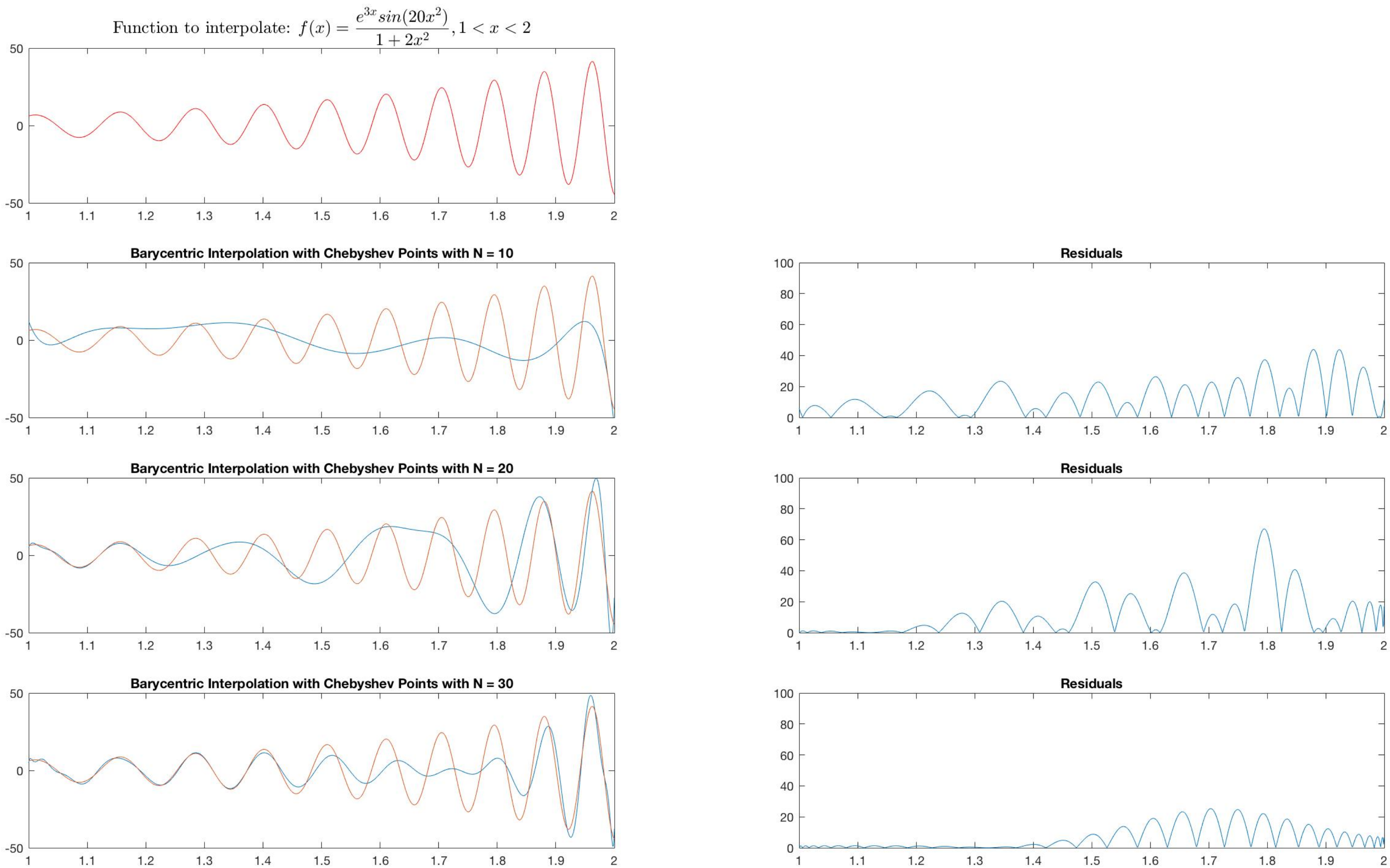
Comments on the interpolation results:

1. The first figure shows the cubic spline interpolation using the MATLAB *spline* function. As expected from a local method, the interpolation residuals are more or less even across the domain, with the residuals increasing slightly as the function increases. As expected, the residuals decrease as N increases.
2. The second figure shows the barycentric interpolation with evenly spaced points. It works very well, and is perhaps the best, in the middle of the desired domain where the residuals are virtually zero. However, as discussed in class, the end points of the domain swing wildly and have huge errors appear at around $x < 1.3$ and $x > 1.7$.
3. Finally, the third figure shows the barycentric interpolation with chebyshev points. Using chebyshev points appears to eliminate the wild swings at the ends of the domains. The quality of cubic spline interpolation and barycentric interpolation with chebyshev points look to be about the same quality.

In conclusion, the barycentric interpolation works best if you only care to interpolate the middle of the domain (of the known points) and not at all about the two extremes. In general, it is best to use either cubic spline, or barycentric interpolation with chebyshev points.


Function to interpolate: $f(x) = \frac{e^{3x} sin(20x^2)}{1+2x^2}, 1 < x < 2$
Cubic Spline Interpolation with N = 10
Residuals
Cubic Spline Interpolation with N = 20
Residuals
Cubic Spline Interpolation with N = 30
Residuals
50
0
-50
100
80
60
40
20
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2


Function to interpolate: $f(x) = \dfrac{e^{3x} sin(20x^2)}{1+2x^2}, 1 < x < 2$
Barycentric Interpolation with N = 10
Residuals
Barycentric Interpolation with N = 20
Residuals
Barycentric Interpolation with N = 30
Residuals
50
0
-50
100
80
60
40
20
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2



Function to interpolate: $f(x) = \dfrac{e^{3x} sin(20x^2)}{1+2x^2}, 1 < x < 2$
50
0
-50
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Barycentric Interpolation with Chebyshev Points with N = 10
Residuals
Barycentric Interpolation with Chebyshev Points with N = 20
Residuals
Barycentric Interpolation with Chebyshev Points with N = 30
Residuals
100
80
60
40
20