R:17 P:14 C:10 99%

PHYS 410: Project 2
Monte Carlo Simulation of the 2D Ising Model

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The code generalized to 2D can be found in *ising2D.m*. The grid is of size $N \times N$. Each trial is now indexed with an x and a y coordinate.

The script used to generate all the plots and analyses can be found in *ising2D_analysis.m*.

The initial energy is calculated by doing two circshifts – one shift to the right and one down; this generates the products of all the nearest neighbour pairs (not including diagonal neighbours). The left-right neighbours and up-down neighbours are then summed up on line 17. The change in energy is just (line 34):

$$dE = 2J \times grid(x, y)[left + right + up + down]$$

The magnetization is still just the sum of all the elements in the grid, and the change in magnetization is also still just:

$$dM = 2 \times grid(x, y)$$

The four quantities are determined as follows:

$$\begin{aligned} M &= mean(Mlist) \\ E &= mean(Elist) \\ C_v &= var(Elist)/T^2 \\ X &= var(Mlist)/T \end{aligned}$$

1 Thermalization

From the plot of Energy vs Time in Figure 1, we can approximate the thermalization times by just looking at the approximate point where the fluctuations level off.

Approximate thermalization times:

- For $N = 20$: $t_{therm} \approx 0.2 \times 10^4$
- For $N = 40$: $t_{therm} \approx 0.8 \times 10^4$

We can conclude that thermalization time increases with N.

2 Observables of Ising Model

- Energy increases as temperature increases.
- Magnetization decreases as temperature increases.
- Heat capacity peaks around the Curie temperature of 2.3
- Magnetic susceptibility peaks around the Curie temperature of 2.3

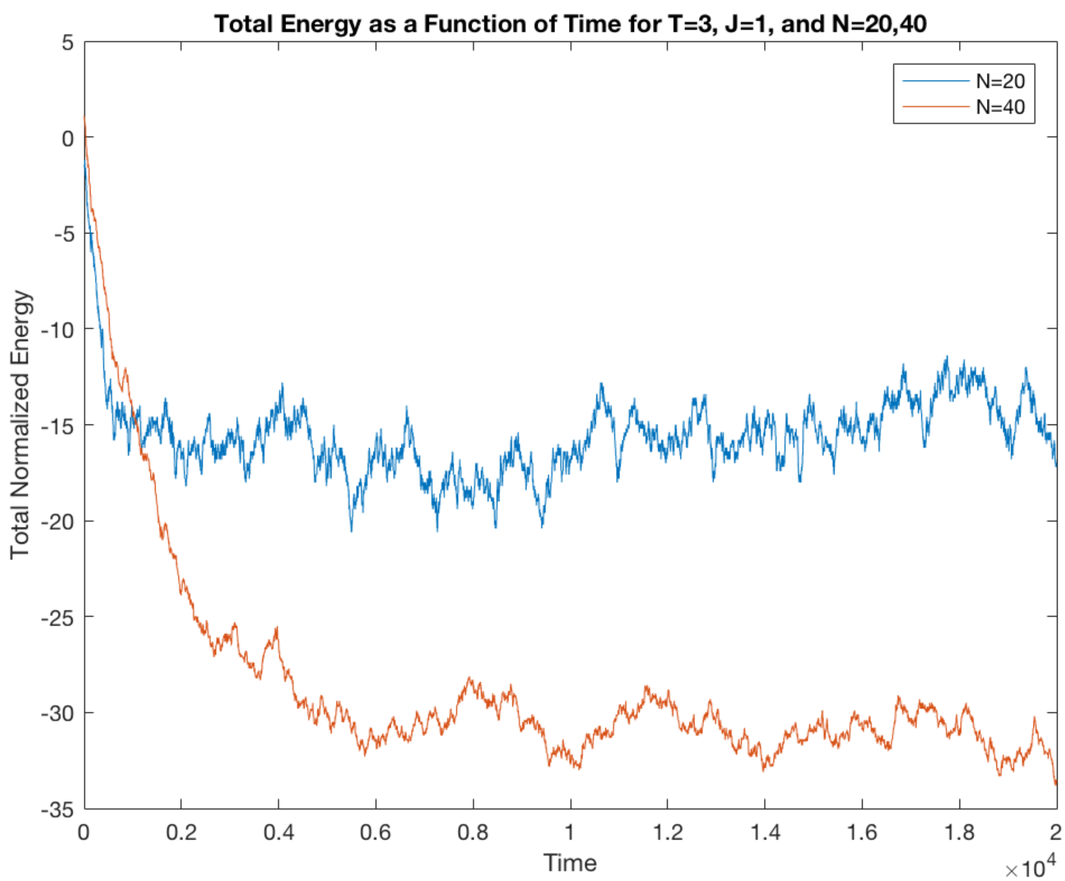

Figure 1: Plot of total energy as a function of time for T=3, J=1, and N=20,40. A rough eyeball estimate of thermalization time is 0.2×10^4 and 0.8×10^4 for N=20 and N=40 respectively.

-1 normalize as 2D , not 1D

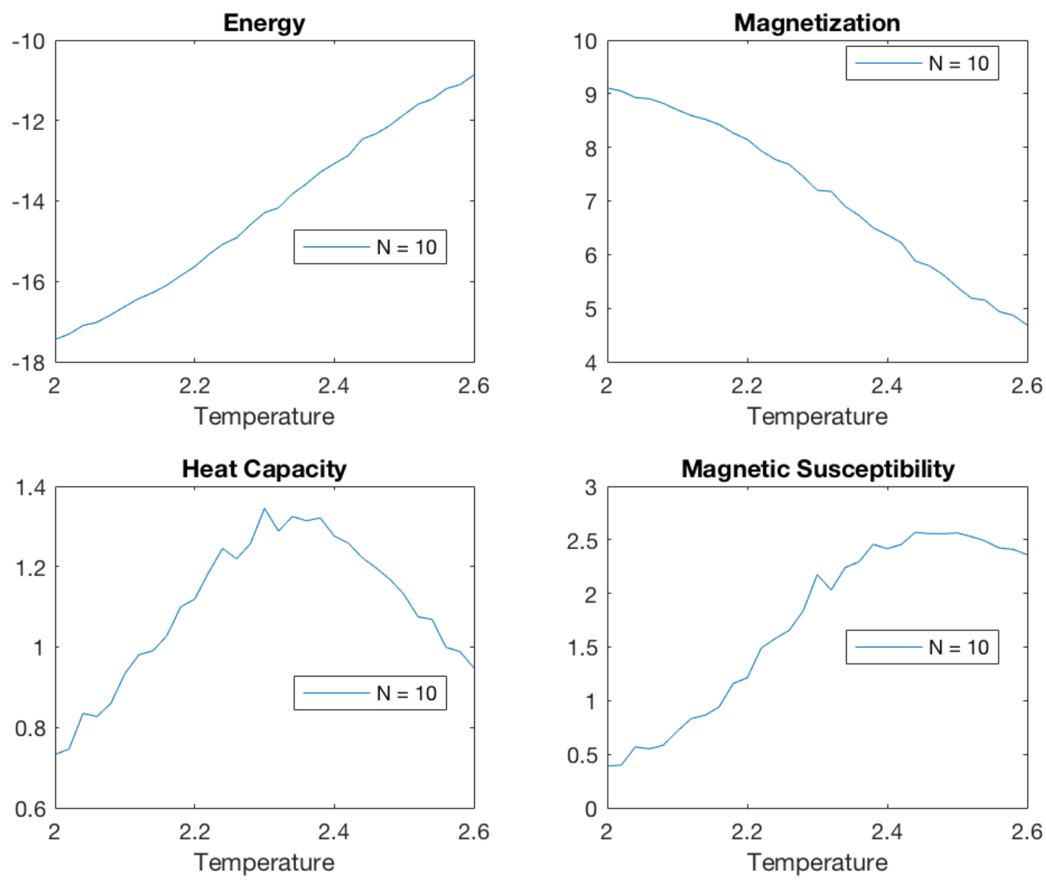

Figure 2: N=10; Number of steps = $5 \times 10^4 \times 10^2$

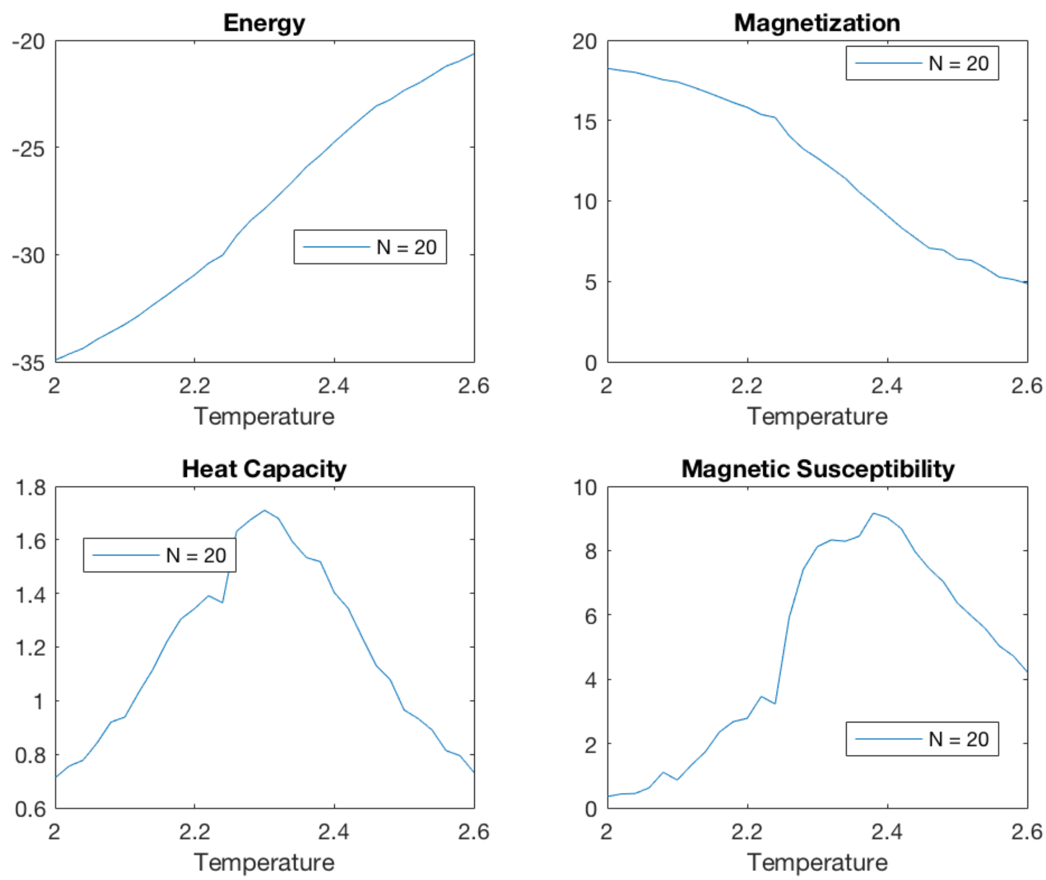

Figure 3: N=20; Number of steps = $5 \times 10^4 \times 20^2$

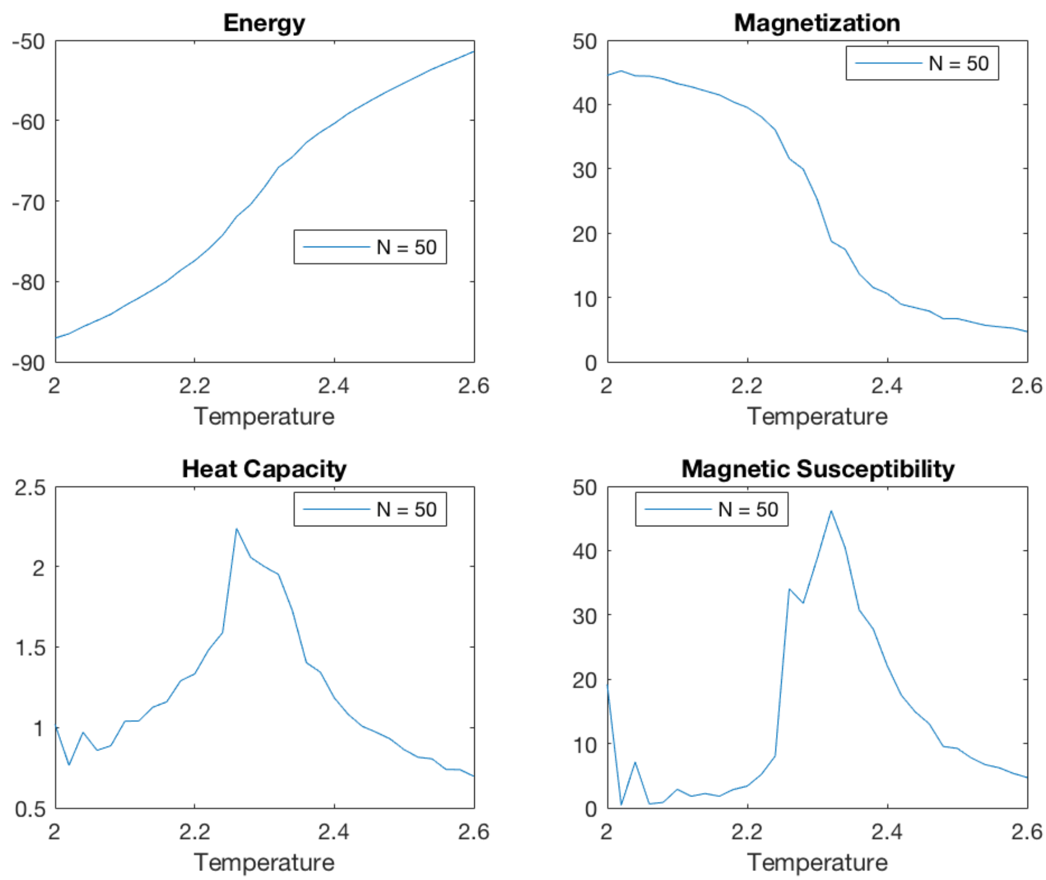

Figure 4: N=50; Number of steps = $5 \times 10^4 \times 50^2$

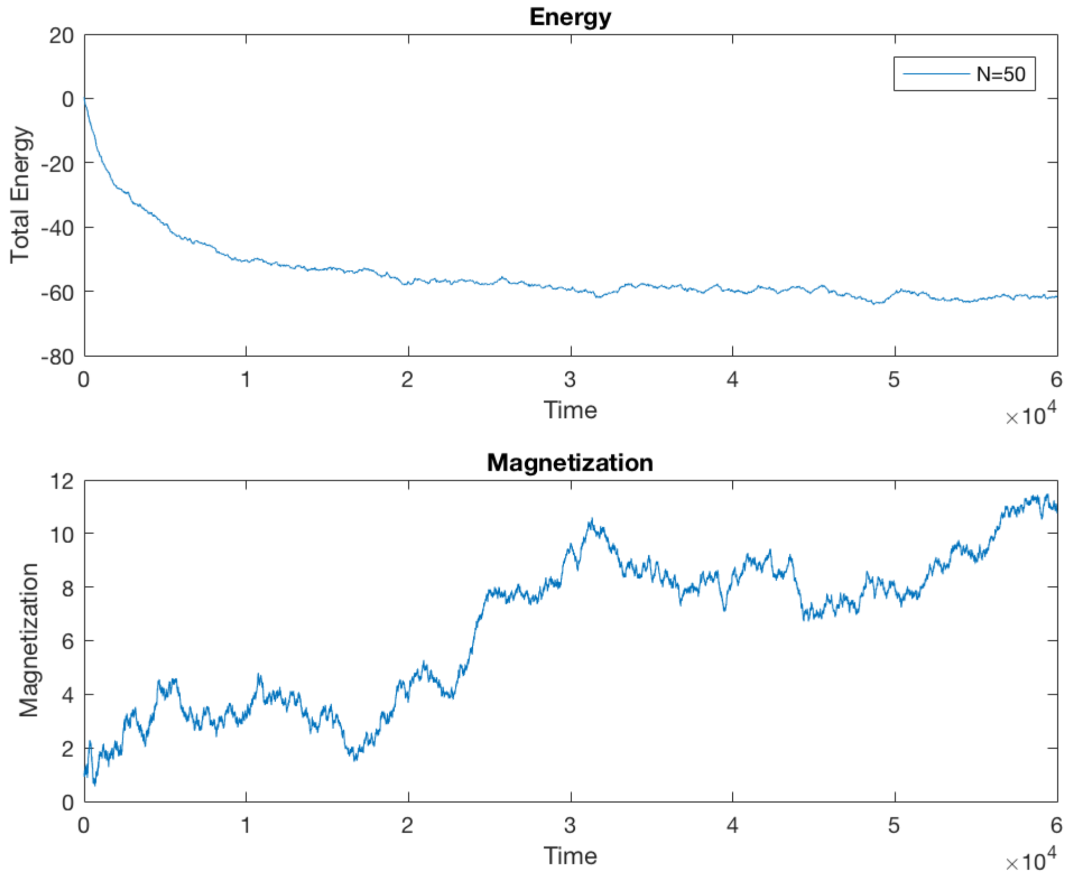

Figure 5: T=2.3; N=50;

3 Curie Temperature

- The value of the Curie temperature is 2.3. This is where the plots of heat capacity and magnetic susceptibility peak. (Figure 6)
- Thermalization near the Curie temperature takes longer, and by looking at the Figure 5, we see that the thermalization time around the Curie temperature (T=2.3) is around 2×10^4. So I removed the first 2×10^4 points. I still used a total of $5 \times 10^4 \times 50^2$ trials as that seems to be large enough to get consistent values.
- The magnetic susceptibility is the observable that gives the clearest signature of the Curie temperature. Both the heat capacity and magnetic susceptibility give clear signatures of the Curie temperature; they both have a peak at the Curie temperature. However, note that the magnetic susceptibility's peak is much larger the heat capacity's peak.

  Energy and Magnetization appear to have an inflection point near the Curie temperature, but those are harder to discern than a peak.

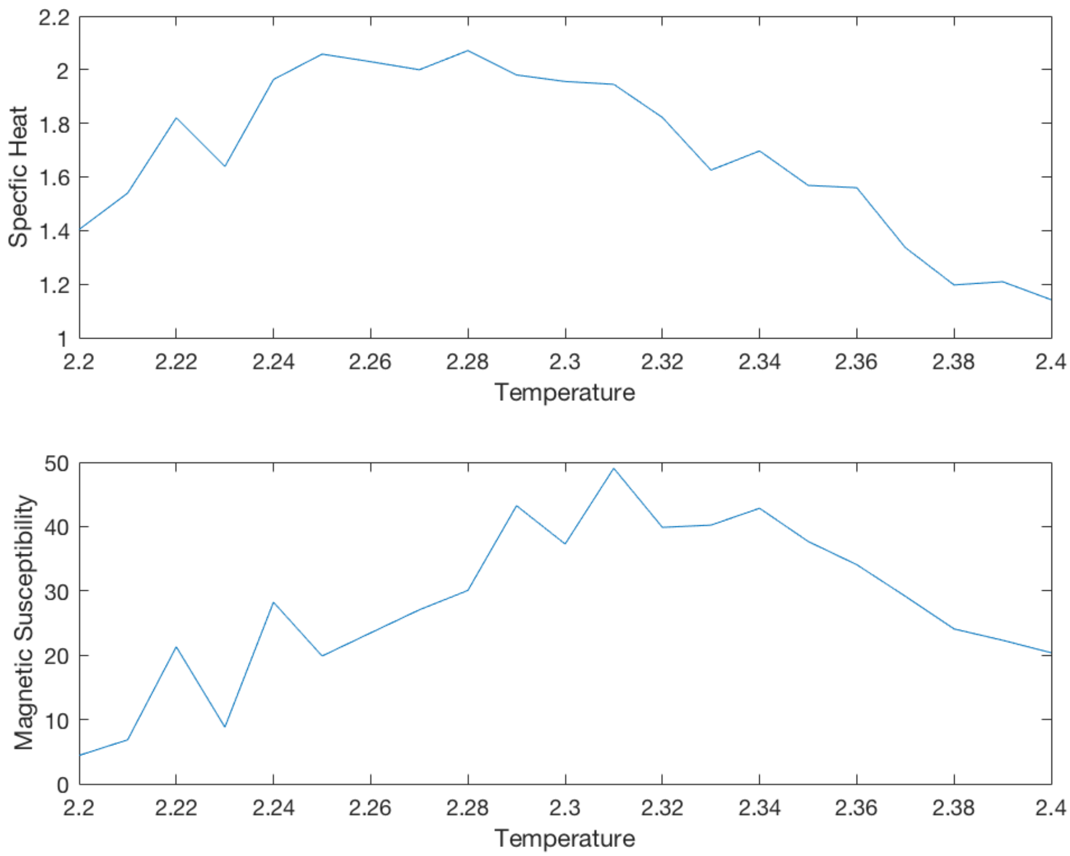

Figure 6: The peak of these plots is the Curie temperature. This is roughly around T=2.3