Final Exam
Physics 410
December 19, 2015

- Do all problems, read them all first and budget your time accordingly.
- Write on blank pages provided, clearly label the problem and section you are doing. Write your name on each sheet of paper separately.
- No books or notes, no computers or wireless internet devices. Calculators are fine, but should not be necessary. Needless to say- no collaboration of any kind.
- You have two hours, good luck!

Problem 1

- Define polynomial interpolation. Show that the Lagrange polynomial

$$P(x) = \sum_{j=1}^{k} f(x_j) L_j(x) \qquad\qquad L_j(x) = \prod_{i \neq j} \frac{x - x_i}{x_j - x_i} \tag{0.1}$$

  provides such an interpolation, where x_j is a grid of values of the independent variable x and $f(x_j)$ are the values of the interpolated function f on the grid.

- To evaluate the derivative $f'(x_0)$, construct an equally spaced grid around the point x_0, including the points x_0 and $x_0 \pm jh$ where $j = 1, 2$ and h is a constant. Write the resulting Lagrange polynomial $P(x)$; evaluate the derivative $P'(x_0)$ to find a centered five-point formula for the derivative.

Problem 2

In the following we use the centered formulas to approximate the first and second derivatives of the function $f(x)$, evaluated at the point x.

$$\begin{aligned} f'(x) &= \frac{f(x+h) - f(x-h)}{2h} \\ f''(x) &= \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} \end{aligned}$$

where h is a constant.

- Show the above are good approximations to the respective derivatives, using a Taylor expansion. Estimate the error in those formulas.
- For a grid of five equally spaced points, with periodic boundary conditions, write a differentiation matrix for the first and second derivatives, using the above formulas.
- Use the above expressions to discretize the differential equation

  $$f''(x) + 5xf'(x) = 0 \tag{0.2}$$

  on the same five point grid, with periodic boundary conditions.

Problem 3

- Explain the idea of importance sampling, and exemplify it in the context of Monte Carlo integration.
- Write a pseudo-code (i.e. a series of steps for the computer to follow, written in plain language) for a Monte Carlo simulation of the one-dimensional Ising model using the Metropolis-Hastings algorithm. Explain the main ideas behind the algorithm, including generating configurations, thermalization and importance sampling.