Rounding error:

From lecture: round off error $\sim \frac{eps}{h}$

Every function evaluation introduces rounding error

$$REr = \frac{3}{4}\left(\frac{2\,eps}{h}\right) + \frac{3}{20}\left(\frac{2\,eps}{h}\right) + \frac{1}{60}\left(\frac{2\,eps}{h}\right)$$

$$= \frac{eps}{h}\left(\frac{3}{2} + \frac{3}{10} + \frac{1}{30}\right)$$

$$= \frac{11}{6}\frac{eps}{h}$$

$$\text{Total Error}_{\text{(upper bound)}} = \frac{2181.29}{140}h^6 + \frac{11}{6}\frac{eps}{h}$$

$$\frac{d(TE)}{dh} = \frac{2181.29}{140}(6h^5) - \frac{11\,eps}{6h^2} = 0$$

USING MATLAB fzero function to find the root closest to 0:

$$h_{optimal} = 0.00331016232216 9$$