Physics 11
Circular Motion and Gravity Test Solutions

	a.	b.	c.	d.
1.	X			
2.			X	
3.			X	
4.			X	
5.		X		
6.		X		
7.	X			
8.				X
9.				X
10.				X
11.			X	
12.	X			
13.	X			
14.	X			
15.		X		
16.				X

1. **Problem**
   What are the units of G, the universal gravitational constant?

   a. $\mathrm{m^3\,kg^{-1}\,s^{-2}}$
   b. $\mathrm{kg\,m/s^2}$
   c. $\mathrm{m\,s^{-2}}$
   d. $\mathrm{kg\,m/s}$

   Solution
   The universal gravitational constant is defined in Newton's law of universal gravitation:

   $$F = \frac{GMm}{r^2}$$

   The constant relates the combination of mass and distance units to force units. Therefore, it must have units of

   $$\mathrm{N\,m^2\,kg^{-2}}$$

   Or, since $\mathrm{N} = \mathrm{kg\,m/s^2}$, the units of G are also

   $$\mathrm{m^3\,kg^{-1}\,s^{-2}}$$

2. **Problem**
   A 438-kg satellite is in a circular orbit at an altitude of 615 km around the earth. How much energy (kinetic and potential) would it release if it were to crash down onto the surface of the earth?

   a. 18.6 GJ
   b. 16.7 GJ
   c. 14.9 GJ
   d. 21.8 GJ

   Solution
   During the crash, the satellite would lose its kinetic energy and some of its potential energy.

   $$\Delta E = GMm\left(\frac{1}{R} - \frac{1}{2(R+h)}\right) = 14.9 \text{ GJ}$$

3. **Problem**
   What is the escape speed on Mars? The mass of Mars is 6.39×10^{23} kg and its radius is 3.39×10^{6} m.

   a. 3.55 km/s
   b. 2.59 km/s
   c. 5.01 km/s
   d. 4.13 km/s

   Solution
   The escape speed is

   $$v_{esc} = \sqrt{\frac{2GM}{r}} = 5.01 \text{ km/s}$$

4. **Problem**
   An asteroid orbits the sun with a period of 2.36 years. What is the asteroid's average distance from the sun? Note: One astronomical unit (au) is the average distance from the earth to the sun.

   a. 1.95 au
   b. 1.29 au
   c. 1.77 au
   d. 1.59 au

   Solution
   Kepler's Third Law of Planetary Motion states that the square of the orbital period is proportional to the cube of the average distance

   $$\left(\frac{T_{asteroid}}{T_{earth}}\right)^2 = \left(\frac{R_{asteroid}}{R_{earth}}\right)^3$$

   We are given that $T_{asteroid}/T_{earth} = 2.36$ and $R_{earth} = 1$ au. Therefore,

   $$R_{asteroid} = \sqrt[3]{(2.36)^2} \text{ au} = 1.77 \text{ au}$$

5. **Problem**
   A rocket is launched straight up from earth's surface at a speed of 22 km/s. What is its speed when it is very far away from the earth? Consider only the effect of the earth's gravity in your answer. The mass of the earth is 5.97×10^{24} kg and its radius is 6.38×10^6 m.

   a. 20.8 km/s
   b. 18.9 km/s
   c. 19.4 km/s
   d. 20.1 km/s

   Solution
   When the rocket is very far away, its potential energy is zero and it only has kinetic energy. This is equal to its initial energy.

   $$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - \frac{GMm}{r}$$

   Solving for v_f gives

   $$v_f = \sqrt{v_i^2 - \frac{2GM}{r}} = 19.0 \text{ km/s}$$

6. **Problem**
   A pilot makes an outside vertical loop (in which the path of the plane traces out the top half of a circle with centre of the loop is beneath the plane) of radius 4888 m. At the top of his loop, he is pushing down on his seat with only $1/3$ of his normal weight. How fast is he going?

   a. 246 m/s
   b. 179 m/s
   c. 198 m/s
   d. 267 m/s

   Solution
   The centripetal force is

   $$\frac{mv^2}{r} = mg(1 - 1/3)$$

Solving for v gives

$$v = \sqrt{gr(1 - 1/3)} = 179 \text{ m/s}$$

7. **Problem**
An asteroid orbits the sun at an average distance of 19.8 au. What is the period of the asteroid's orbit? Note: One astronomical unit (au) is the average distance from the earth to the sun.

   a. 88.1 years
   b. 67.5 years
   c. 76.9 years
   d. 116.4 years

   Solution
   Kepler's Third Law of Planetary Motion states that the square of the orbital period is proportional to the cube of the average distance

$$\left(\frac{T_{asteroid}}{T_{earth}}\right)^2 = \left(\frac{R_{asteroid}}{R_{earth}}\right)^3$$

   We are given that $R_{asteroid}/R_{earth} = 19.8$ and $T_{earth} = 1$ year. Therefore,

$$R_{asteroid} = \sqrt{(19.8)^3} \text{ years} = 88.1 \text{ years}$$

8. **Problem**
What is the radius of the geosynchronous orbit around Jupiter? The mass of Jupiter is 1.898×10^{27} kg. A "day" on Jupiter lasts 9 hours and 55 minutes.

   a. 231×10^3 km
   b. 187×10^3 km
   c. 138×10^3 km
   d. 160×10^3 km

   Solution
   The radius of a circular geosynchronous orbit is given by

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = 160000 \text{ km}$$

9. **Problem**
An object is moving in a circle with a constant speed. Its acceleration is

   a. tangent to the circle.
   b. opposite the direction of its velocity.
   c. directed away from the centre of the circle.
   d. directed towards the centre of the circle.

   Solution
   An object in uniform circular motion has a centripetal acceleration that always points toward the centre of the circle.

10. **Problem**
    An object is moving in a circle with a constant speed. Its velocity is

    a. directed towards the centre of the circle.
    b. in the same direction as its acceleration.
    c. opposite the direction of its acceleration.
    d. tangent to the circle.

    Solution
    An object in uniform circular motion has a velocity that is always tangent to the circle.

11. **Problem**
    A pilot, of mass m, executes a vertical dive, then follows a semi-circular arc until the plane is going straight up. Just as the plane is at its lowest point, the force that the pilot's seat exerts on him is

    a. less than mg and pointing up
    b. less than mg and pointing down
    c. more than mg and pointing up
    d. more than mg and pointing down

    Solution
    The centripetal force points towards the centre of the circle, which would be upward. This force must oppose and overcome the pilot's weight, so it is more than mg.

12. **Problem**
    The maximum speed around a level circular curve is 79.0 km/h. What is the maximum speed around a curve with 5 times the radius? Assume all other factors remain unchanged.

    a. 176.7 km/h
    b. 64.0 km/h
    c. 375.8 km/h
    d. 35.3 km/h

    Solution
    For uniform circular motion, the centripetal force is given by

    $$F_c = \frac{mv^2}{r}$$

    If $r \to 5r$, then to maintain the same centripetal force (since the friction is assumed to be the same) we must have $v \to \sqrt{5}v = 176.6$ km/h.

13. **Problem**
    A rock, of mass m, is attached to a string and whirled in a vertical circle of radius r. At the top of the circle, the tension in the string is 3 times the rock's weight. The rock's speed at this point is given by

    a. $\sqrt{4gr}$
    b. $\sqrt{2gr}$
    c. $\sqrt{3gr}$
    d. $3\sqrt{gr}$

    Solution
    The net force at the top of the circle is the tension in the string plus the weight of the rock. This net

force is the centripetal force for circular motion.

$$\frac{mv^2}{r} = F_{tension} + mg$$

We are also given in the problem that $F_{tension} = 3mg$. Putting this into the equation above and solving for v gives

$$v = \sqrt{4gr}$$

14. **Problem**
Consider a satellite moving in a circular orbit of radius r around a planet of mass M. Which expression gives the satellite's orbital speed?

a. $\sqrt{GM/r}$
b. GM/r
c. $\sqrt{Gr}$
d. GM/r^2

Solution
The centripetal force is the gravitational force for a circular orbit.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

Solving for v gives

$$v = \sqrt{GM/r}$$

15. **Problem**
Halley's comet is in a highly elliptical orbit around the sun. Its speed

a. is constant.
b. increases as it nears the sun.
c. decreases as it nears the sun.
d. is zero when it reaches its furthest point from the sun.

Solution
Orbiting objects get faster when they get closer to the sun (Kepler's second law).

16. **Problem**
A planet has 6 times the earth's mass and 6 times the earth's radius. If an object weighs 100 N on earth, how much would it weigh on this planet?

a. 30.2 N
b. 26.6 N
c. 100.0 N
d. 16.7 N

Solution
The gravitational field strength on the surface of a planet is

$$g = \frac{GM}{r^2}$$

So the gravitational field strength on the planet is

$$g_{planet} = \frac{G(6M_{earth})}{(6r_{earth})^2} = \frac{6}{36}g_{earth}$$

So the 100 N object would weigh

$$(100N)\frac{6}{36} = 16.7N$$