Physics 12
Special Relativity Solutions

	a.	b.	c.	d.
1.			X	
2.		X		
3.	X			
4.	X			
5.	X			
6.			X	
7.	X			
8.			X	
9.				X
10.		X		
11.				X
12.				X
13.	X			
14.		X		
15.				X
16.		X		
17.		X		
18.		X		
19.				X
20.			X	

1. **Problem**
   Calculate the Lorentz factor when $v = 0.870c$.

   a. 1.78
   b. 1.53
   c. 2.03
   d. 2.89

   Solution
   The Lorentz factor is

   $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.87^2}} = 2.03$$

2. **Problem**
   If you were to travel to a star 59.0 light-years from Earth at a speed of 1.100×10^8 m/s, what would you measure this distance to be?

   a. 54.3 ly
   b. 54.9 ly
   c. 63.4 ly
   d. 46.5 ly

   Solution
   The distance is contracted in your frame, so it would be shorter than when the Earth and star are at rest.

   The Lorentz factor is

   $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1.075$$

   Therefore, the contracted distance to the star is

   $$L = \frac{L_0}{\gamma} = \frac{59\,\text{ly}}{1.0748615} = 54.9\,\text{ly}$$

3. **Problem**
   Why did Michelson and Morley orient light beams at right angles to each other?

   a. To obtain an interference pattern that would indicate how much the speed of light differs when moving in different directions.
   b. To observe the wave-particle duality of light.
   c. To observe the scattering of photons at 90 degrees that could be analyzed to see if light is an electromagnetic wave.
   d. To obtain a diffraction pattern that would indicate if the speed of light is constant in all frames of reference regardless of their motion.

   Solution
   The two light beams form an interferometer. When the light beams recombine, they make an interference pattern that would indicate if the speed of light is different in each direction.

4. **Problem**
   Sitting in a stationary car, you observe a fast-moving train to be shorter than its rest length. An observer on the train observes your car to be

   a. shorter than its rest length
   b. longer than its rest length
   c. the same as its rest length
   d. not enough information to determine

   Solution
   In the train's reference frame, you are moving, so your car is length contracted to be shorter than its rest length.

5. **Problem**
   A clock moving at $v = 0.810c$ passes your clock when both clocks read $t = 0$. When your clock reads $t = 49.0\,\text{s}$, what does the moving clock read?

   a. 28.7 s
   b. 83.6 s
   c. 36.7 s
   d. 4.2 s

   Solution
   Time is dilated in your frame, so the clock in your frame reads a greater time interval than the moving clock.

   The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.81^2}} = 1.705$$

   The moving clock reads a proper time of

$$\Delta t_0 = \frac{\Delta t_0}{\gamma} = \frac{49\,\text{s}}{1.7052337} = 28.7\,\text{s}$$

6. **Problem**
   A rod passes by you at a speed of $0.380c$. You measure its length to be 64.0 m. How long would it be at rest?

   a. 73.1 m
   b. 119.7 m
   c. 69.2 m
   d. 59.2 m

   Solution
   The length of the rod is contracted in your frame, so it would be longer when at rest.

   The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.38^2}} = 1.081$$

Therefore, the proper length of the rod is

$$L_0 = \gamma L = (1.0810969)(64\,\text{m}) = 69.2\,\text{m}$$

7. **Problem**
   What was the purpose of the Michelson-Morley experiment?

   a. To measure the Earth's motion relative to the ether.
   b. To make a precise measurement of the speed of light.
   c. To establish that the Earth is the one true reference frame.
   d. To verify that light is an electromagnetic wave.

   Solution
   The purpose of the Michelson-Morley experiment was to establish the existence of the ether and to measure the Earth's motion relative to the ether.

8. **Problem**
   Which of the following is the correct expression for the Lorentz factor?

   a. $(1 + v^2/c^2)^{-1/2}$
   b. $(1 - v^2/c^2)^{1/2}$
   c. $(1 - v^2/c^2)^{-1/2}$
   d. $(1 + v^2/c^2)^{1/2}$

   Solution
   The Lorentz factor is defined to be

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = (1 - v^2/c^2)^{-1/2}$$

9. **Problem**
   Length contraction occurs

   a. only when the object is not moving.
   b. only when the object is approaching the speed of light.
   c. perpendicular to the direction of motion (transverse lengths).
   d. parallel to the direction of motion (longitudinal lengths).

   Solution
   Length contraction occurs for any object moving relative to an observer in the direction of motion (longitudinal lengths).

10. **Problem**
A car moving at $v = 0.661c$ turns on its headlights. In the car's reference frame, what distance does the light cover in $7.98 \times 10^{-8}\,\mathrm{s}$?

a. $213.0\,\mathrm{m}$
b. $23.9\,\mathrm{m}$
c. $65.9\,\mathrm{m}$
d. $179.0\,\mathrm{m}$

Solution
The speed of light in any frame is $3.00 \times 10^{8}\,\mathrm{m/s}$. Therefore the distance the light traveled is

$$d = (3.00 \times 10^{8}\,\mathrm{m/s})(7.98 \times 10^{-8}\,\mathrm{s}) = 23.9\,\mathrm{m}$$

11. **Problem**
Why was it once believed that light must travel through a medium called the *ether* and could not propagate across empty space?

a. Light shows the phenomenon of diffraction and interference.
b. Maxwell's theory of electromagnetism implies this.
c. The speed of light is the maximum possible speed.
d. All other known waves need a medium to travel through.

Solution
All other waves, such as sound and ocean waves, need a medium to travel through (e.g. air or water). Light, which is an electromagnetic wave according to Maxwell's theory, is the only wave that can propagate through empty space.

12. **Problem**
If Michelson and Morley had observed the interference pattern shift in their interferometer, what would that have indicated?

a. The speed of light is boosted in the direction of Earth's motion.
b. The speed of light is the same in all frames of reference.
c. The speed of light changes upon reflection from a surface.
d. The speed of light depends on the motion relative to the ether.

Solution
If Michelson and Morley had observed the interference pattern shift, then they would conclude that the speed of light depends on the motion relative to the ether, and that the Earth is moving relative to the ether. They did not observe this and their null result was key to the rejection of the ether hypothesis.

13. **Problem**
According to the postulates of special relativity, the speed of light in a vacuum

a. is constant for all observers regardless of their motion.
b. is constant only in the rest frame of the ether.
c. depends on the speed of the observer.
d. depends on the speed of the light source.

Solution
The speed of light is constant for all observers regardless of their motion.

14. **Problem**
An astronaut goes on a long space voyage near the speed of light. When he returns home, how will his age compare to the age of his twin who stayed on Earth?

   a. This is a paradox in special relativity that does not have a clear answer.
   b. The astronaut will be younger than his twin because of time dilation.
   c. Both will be the same age because each can claim that it was the other who was moving.
   d. The astronaut will be older than his twin because of time dilation.

Solution
Time dilation means that moving clocks run slow. The astronaut is moving and so his clock runs slow. Therefore, the astronaut will be younger than his twin. Note that since the astronaut must turn around to travel back to the Earth, he cannot analyze the situation with the time dilation formula that only applies for an observer in a single inertial reference frame.

15. **Problem**
What best describes the Lorentz factor in the nonrelativistic limit?

   a. $\gamma \approx c$
   b. $\gamma \approx 0$
   c. $\gamma \to \infty$
   d. $\gamma \approx 1$

Solution
The Lorentz factor is defined to be

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

In the nonrelativisitc limit, $v \ll c$ so $v^2/c^2 \approx 0$ and

$$\gamma \approx \frac{1}{\sqrt{1-0}} = 1$$

16. **Problem**
In your spaceship, you see an alien spaceship moving at $0.68c$. Considering the effects of time dilation and length contraction, the aliens would see your spaceship moving

   a. faster than $0.68c$
   b. at $0.68c$
   c. slower than $0.68c$
   d. not enough information to determine

Solution
Relative speeds are not affected under Lorentz transformations. The aliens would see your spaceship moving at $0.68c$ (but in the opposite direction).

17. **Problem**
Which statement accurately describes the relativity of simultaneity?
   a. Only events at the same location can be simultaneous.
   b. Events simultaneous in one frame may not be simultaneous in another.
   c. Simultaneity is absolute.
   d. All observers in inertial reference frames agree on which events are simultaneous.

   Solution
   The relativity of simultaneity says that events that are simultaneous in one inertial reference frame may not be simultaneous in another inertial reference frame.

18. **Problem**
Suppose you decide to travel to a star 60.0 light-years away in the reference frame of the Earth. How fast would you have to travel so that the distance would be only 50.0 light years?
   a. $0.438c$
   b. $0.553c$
   c. $0.794c$
   d. $0.646c$

   Solution
   The Lorentz factor is

$$\gamma = \frac{L_0}{L} = 1.200$$

   We solve for v in terms of γ

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{1.2^2}} = 0.553$$

19. **Problem**
A clock moving at $v = 0.370c$ passes your clock when both clocks read $t = 0$. When the moving clock reads $t = 52.0\,\mathrm{s}$, what do the clocks in your frame read?
   a. 48.3 s
   b. 19.2 s
   c. 161.0 s
   d. 56.0 s

   Solution
   Time is dilated in your frame, so the clocks in your frame would read a greater time interval.

   The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.37^2}} = 1.076$$

   The clocks in your frame read

$$\Delta t = \gamma \Delta t_0 = (1.0763895)(52\,\mathrm{s}) = 56.0\,\mathrm{s}$$

20. **Problem**
    Time dilation means that

    a. time flies when you're having fun.
    b. moving clocks run faster than clocks at rest.
    c. moving clocks run slower than clocks at rest.
    d. moving clocks run at the same rate as clocks at rest.

    Solution
    Time dilation is the effect that moving clocks run slower than clocks at rest.