Physics 11
Kinematics Retest 4 Solutions

	a.	b.	c.	d.
1.		X		
2.	X			
3.		X		
4.	X			
5.		X		
6.				X
7.		X		
8.	X			
9.		X		
10.				X
11.	X	X	X	X
12.				X
13.			X	
14.	X			
15.				X
16.			X	
17.			X	
18.		X		
19.		X		
20.			X	
21.		X		
22.				X
23.		X		
24.			X	
25.			X	

1. **Problem**
   True or false? When you throw a ball over to your friend, the ball's velocity is zero when it reaches its maximum height.

   a. True
   b. False

   Solution
   False. The vertical velocity is zero at the maximum height, but the horizontal velocity is not.

2. **Problem**
   True or false? The area under a velocity-time graph is the displacement.

   a. True
   b. False

   Solution
   True.

3. **Problem**
   True or false? If an object changes direction, then the line on its velocity-time graph must have a changing slope.

   a. True
   b. False

   Solution
   False. An object that changes direction is represented by a line that crosses the time axis in a velocity-time graph.

4. **Problem**
   True or false? If the velocity vector and the acceleration vector both point in the same direction, then the object must be speeding up.

   a. True
   b. False

   Solution
   True. Acceleration is the change in velocity over time. If acceleration is in the same direction as velocity, then the velocity vector must be increasing and the object must be speeding up.

5. **Problem**
   True or false? If an object is moving to the right, then its acceleration must also be to the right.

   a. True
   b. False

   Solution
   False. An object that is moving to the right at a constant speed has zero acceleration and an object that is moving right and slowing down has a leftward acceleration.

6. **Problem**
   An athlete throws a javelin at four different angles above the horizontal, each with the same speed: 30°, 40°, 60°, 80°. Which two throws cause the javelin to land the same distance away?

   a. 30° and 80°
   b. 40° and 80°
   c. 40° and 60°
   d. 30° and 60°

   Solution
   The range of a projectile can be calculated by first finding the time using the vertical velocity, and then using that time to find the horizontal range. The time can be found using the formula $v_{f,y} = v_{i,y} + at$ with $v_{f,y} = -v_{i,y}$

   $$t = \frac{-2v_{i,y}}{a} = \frac{-2v_i \sin(\theta)}{g}$$

   The horizontal velocity is constant, so the horizontal distance (i.e. range) is

   $$d = v_x t = v_i \cos(\theta)\left(\frac{-2v_i \sin(\theta)}{g}\right) = \frac{-2v_i^2 \cos(\theta)\sin(\theta)}{g}$$

   Therefore, the range would be the same for two angles θ_1 and θ_2 if $\cos(\theta_1)\sin(\theta_1) = \cos(\theta_2)\sin(\theta_2)$. Performing the calculations, we get:

   $$\cos(30^\circ)\sin(30^\circ) = 0.4330127$$
   $$\cos(40^\circ)\sin(40^\circ) = 0.4924039$$
   $$\cos(60^\circ)\sin(60^\circ) = 0.4330127$$
   $$\cos(80^\circ)\sin(80^\circ) = 0.1710101$$

   Since $\cos(\theta)\sin(\theta)$ are the same for 30° and 60°, the range is the same for these two angles.

   Alternatively, we can use the complementary angle identity ($\cos(\theta) = \sin(90^\circ - \theta)$) to get:

   $$\cos(\theta)\sin(\theta) = \sin(90^\circ - \theta)\sin(\theta)$$

   and notice that the expression would be equal for complementary angles (angles that sum to 90°). For example, if $\theta_1 = 30^\circ$ and $\theta_2 = 60^\circ$, then

   $$\sin(90^\circ - \theta_1)\sin(\theta_1) = \sin(90^\circ - 30^\circ)\sin(30^\circ) = \sin(60^\circ)\sin(30^\circ)$$
   $$\sin(90^\circ - \theta_2)\sin(\theta_2) = \sin(90^\circ - 60^\circ)\sin(60^\circ) = \sin(30^\circ)\sin(60^\circ)$$

7. **Problem**
   Suppose that several projectiles are launched. Which one will be in the air for the longest time?

   a. The one with the furthest horizontal range.
   b. The one with the greatest maximum height.
   c. The one with the greatest initial speed.
   d. None of the above.

   Solution
   Since horizontal and vertical motions can be analyzed separately, only the vertical motion matters for the longest air time. The projectile with the greatest maximum height will stay up in the air for the longest time.

8. **Problem**
   A car traveling at speed v is able to stop in a distance d. Assuming the same constant acceleration, what distance does this car require to stop when it is traveling at speed $2v$?

   a. $4d$
   b. $2d$
   c. d
   d. $\sqrt{2}d$

   Solution
   Solve $v_f^2 = v_i^2 + 2ad$ for d and set $v_f = 0$ to get

   $$d = -\frac{v_i^2}{2a}$$

   Therefore, the braking distance is proportional to the square of the initial velocity. If the initial velocity is multiplied by 2, then the car would need 2^2 times the distance to stop.

9. **Problem**
   Ball 1 is dropped from the top of a building. One second later, ball 2 is dropped from the same building. If air resistance can be ignored, then as time progresses (and while the balls are still in free fall), the difference in their speeds

   a. increases.
   b. remains constant.
   c. decreases.
   d. cannot be determined from the given information.

   Solution
   Since both balls are accelerating at the same rate, the difference in their velocities will remain constant. Using the formula $v_f = v_i + at$, with $a = g$ and $v_i = 0$, we see that

   $$\Delta v = v_1 - v_2 = gt_1 - gt_2 = g(t_1 - t_2)$$

   and both $g = 9.8\,\mathrm{m/s^2}$ and $(t_1 - t_2) = 1$ s remain constant.

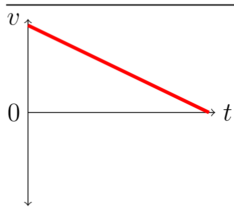
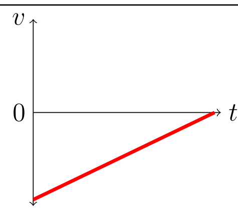
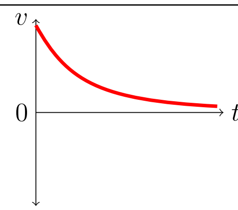
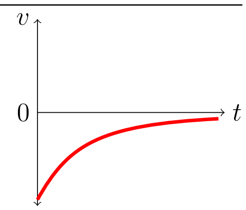
   Note that if air resistance is an important factor, then the difference would change until both balls are falling at their terminal velocity.

10. **Problem**
    A football is kicked with a velocity of 25 m/s at an angle of $45°$ above the horizontal. What is the vertical component of its acceleration as it travels along its trajectory? (Ignore air resistance.)

    a. $g\sin(45°)$ upward
    b. $g\sin(45°)$ downward
    c. g upward
    d. g downward

    Solution
    For projectile motion on Earth, the acceleration is always g downward.

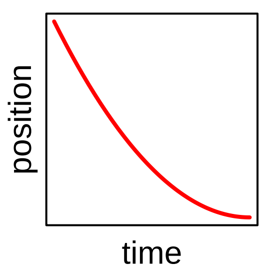
11. **Problem**
    Which velocity-time graphs represent the motion of an object that is slowing down? *Select all that apply.*

Graph A	Graph B	Graph C	Graph D
			

   a. Graph A
   b. Graph B
   c. Graph C
   d. Graph D

   Solution
   Slowing down is represented by a line approaching the time axis on a velocity-time graph.

12. **Problem**
    Which choice best matches the given position-time graph?

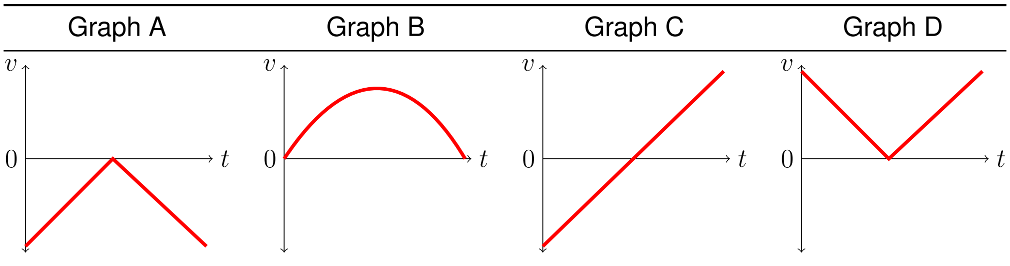


   a. moving to the right and speeding up.
   b. moving to the right and slowing down.
   c. moving to the left and speeding up.
   d. moving to the left and slowing down.

   Solution
   The object is moving to the right if its position is increasing and moving to the left if its position is decreasing. The object is speeding up if the tangent line is becoming more vertical and slowing down if the tangent line is becoming more horizontal.

   This graph describes an object that is moving to the left and slowing down.

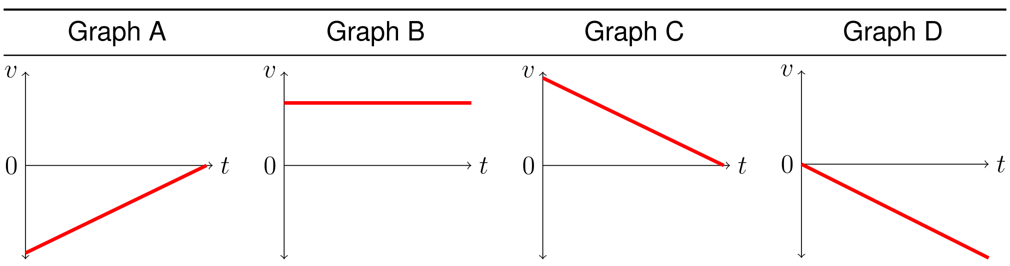
13. **Problem**
    Which velocity-time graph represents the motion of an object that changes its direction?



a. Graph A
b. Graph B
c. Graph C
d. Graph D

Solution
Changing direction on a velocity-time graph is represented by a line that crosses the time axis.

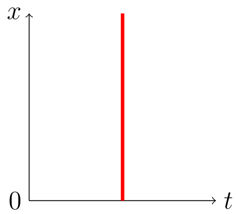
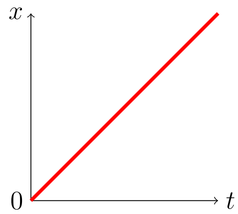
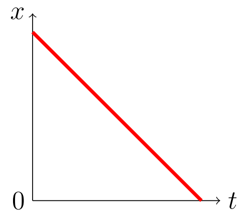
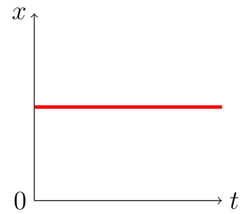
14. **Problem**
Which velocity-time graph represents motion with constant positive acceleration?



a. Graph A
b. Graph B
c. Graph C
d. Graph D

Solution
Acceleration is the slope of the velocity-time graph. Therefore, the correct answer is the graph with the positive slope. Note that for positive acceleration it does not matter if the velocity is always negative as long as the slope is positive.

15. **Problem**
Which position-time graph represents an object at rest?

Graph A	Graph B	Graph C	Graph D
			

a. Graph A
b. Graph B
c. Graph C
d. Graph D

Solution
An object at rest has zero velocity so the slope of its position-time graph must have a slope of zero (horizontal line).

16. **Problem**
A particle initially moving with a velocity of 2 m/s in the x-direction experiences a constant acceleration of 1 m/s^2 in the x-direction and –2 m/s^2 in the y-direction. What are the velocity components of the particle after 4 s?

a. $v_x = 4\,\mathrm{m/s}, v_y = -8\,\mathrm{m/s}$
b. $v_x = -6\,\mathrm{m/s}, v_y = 4\,\mathrm{m/s}$
c. $v_x = 6\,\mathrm{m/s}, v_y = -8\,\mathrm{m/s}$
d. $v_x = 3\,\mathrm{m/s}, v_y = -2\,\mathrm{m/s}$

Solution
The velocities can be calculated separately.

$$v_x = 2\,\mathrm{m/s} + (1\,\mathrm{m/s^2})(4\,\mathrm{s}) = 6\,\mathrm{m/s}$$
$$v_y = 0\,\mathrm{m/s} + (-2\,\mathrm{m/s^2})(4\,\mathrm{s}) = 8\,\mathrm{m/s}$$

17. **Problem**
A car travels 20 km at 31 km/h and 261 km at 106 km/h. What is the average speed for this trip?

a. 101 km/h
b. 96 km/h
c. 90 km/h
d. 106 km/h

Solution
The average speed is the total distance divided by the total time.

$$v_{avg} = \frac{d_{total}}{t_{total}} = \frac{d_1 + d_2}{d_1/v_1 + d_2/v_2} = \frac{20\,\mathrm{km} + 261\,\mathrm{km}}{\frac{20\,\mathrm{km}}{31\,\mathrm{km/h}} + \frac{261\,\mathrm{km}}{106\,\mathrm{km/h}}} = 90\,\mathrm{km/h}$$

18. **Problem**
A car accelerates from 47 km/h to 81 km/h, at an average rate of 6 m/s^2. How much time does it take to complete this speed increase?

    a. 0.62 s
    b. 1.57 s
    c. 14.2 s
    d. 5.67 s

    Solution
    First, convert the speeds from km/h to m/s.

$$v_i = 47\,\mathrm{km/h} \times \frac{1\,\mathrm{h}}{3600\,\mathrm{s}} \times \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} = 13.1\,\mathrm{m/s}$$
$$v_f = 81\,\mathrm{km/h} \times \frac{1\,\mathrm{h}}{3600\,\mathrm{s}} \times \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} = 22.5\,\mathrm{m/s}$$

    Then, use the formula for constant acceleration motion.

$$t = \frac{v_f - v_i}{a} = \frac{22.5\,\mathrm{m/s} - 13.1\,\mathrm{m/s}}{6\,\mathrm{m/s^2}} = 1.57\,\mathrm{s}$$

19. **Problem**
An F1 car accelerates from 0 to 60 miles per hour in $2.53\,\mathrm{s}$. What is the acceleration of the car in SI units? $(1\,\mathrm{mile} = 1609.34\,\mathrm{m})$

    a. 19.2 m/s^2
    b. 10.6 m/s^2
    c. 22.6 m/s^2
    d. 18.2 m/s^2

    Solution
    First, convert 60 mph to m/s.

$$60\,\mathrm{mph} \times \frac{1609.34\,\mathrm{m}}{1\,\mathrm{mi}} \times \frac{1\,\mathrm{h}}{3600\,\mathrm{s}} = 26.8223\,\mathrm{m/s}$$

    Then, divide by the time.

$$a = \frac{v_f - v_i}{t} = \frac{26.8223\,\mathrm{m/s} - 0}{2.53\,\mathrm{s}} = 10.6\,\mathrm{m/s^2}$$

20. **Problem**
Suppose an object travels at a constant velocity of $27.0\,\mathrm{km/h}$. What distance would it travel in $23.0\,\mathrm{minutes}$?

    a. 621 km
    b. 210 km
    c. 10.4 km
    d. 452 km

    Solution
    Use the formula for constant velocity motion making sure to convert to the proper units.

$$d = vt = (27.0\,\mathrm{km/h})(23.0\,\mathrm{min})\left(\frac{1\,\mathrm{h}}{60\,\mathrm{min}}\right) = 10.4\,\mathrm{km}$$

21. **Problem**
A person throws a rock straight down from a bridge with an initial speed of 28.6 m/s. It falls 14.7 m to the water below. How much time does it take for the rock to hit the water?

    a. 0.65 s
    b. 0.48 s
    c. 0.37 s
    d. 0.66 s

    Solution
    Use the quadratic formula to solve the equation $h = \frac{1}{2}gt^2 + v_i t$ for t.

$$\frac{1}{2}gt^2 + v_i t - h = 0$$

$$a = \frac{1}{2}g = \frac{1}{2}(9.8\,\mathrm{m/s^2}) = 4.9\,\mathrm{m/s^2}, b = v_i = 28.6\,\mathrm{m/s}, c = -h = -14.7\,\mathrm{m}$$

$$t = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = 0.48\,\mathrm{s}$$

22. **Problem**
A ball tossed straight up returns to its starting point in 5.93 s. What was its initial speed? Ignore air resistance.

    a. 34.4 m/s
    b. 30.2 m/s
    c. 43.5 m/s
    d. 29.1 m/s

    Solution
    The final velocity is equal to the initial velocity, but in the opposite direction ($v_f = -v_i$). Substitute this into the equation $v_f = v_i + at$ and solve for vi:

$$-v_i = v_i + at$$

$$-2v_i = at$$

$$vi = -\frac{at}{2} = -\frac{(-9.8\,\mathrm{m/s^2})(5.93\,\mathrm{s})}{2} = 29.1\,\mathrm{m/s}$$

23. **Problem**
A golf ball is hit with an initial velocity of 69 m/s at an angle of 33° above the horizontal. What is its range (horizontal distance before hitting the ground)? Ignore air resistance and assume a flat golf course.

   a. 233 m
   b. 444 m
   c. 645 m
   d. 632 m

Solution
First, analyze the vertical motion to find the time it takes the ball to hit the ground using the formula $v_f = v_i + at$ with $v_f = -v_i$:

$$t = -\frac{2v_i}{a} = -\frac{2(69\,\mathrm{m/s})\sin(33^\circ)}{-9.8\,\mathrm{m/s^2}} = 7.669\,406\,8\,\mathrm{s}$$

Then, use the time to figure out how far the ball travels horizontally (recall that the horizontal velocity is constant).

$$d_x = v_x t = (69\,\mathrm{m/s})(\cos(33^\circ))(7.669\,406\,8\,\mathrm{s}) = 444\,\mathrm{m}$$

24. **Problem**
A person throws a rock horizontally, with an initial velocity of 12 m/s, from a bridge. It falls 7.04 m to the water below. How far does it travel horizontally before striking the water?

   a. 14.3 m
   b. 12.7 m
   c. 14.4 m
   d. 19.9 m

Solution
First analyze the vertical motion to find the time it takes the rock to hit the water using the formula $d = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(7.04\,\mathrm{m})}{9.8\,\mathrm{m/s^2}}} = 1.198\,638\,7\,\mathrm{s}$$

Then, use the time to figure out how far the rock travels horizontally (recall that the horizontal velocity is constant).

$$d_x = v_x t = (12\,\mathrm{m/s})(1.198\,638\,7\,\mathrm{s}) = 14.4\,\mathrm{m}$$

25. **Problem**
What is the maximum height reached by a ball thrown straight up with an initial velocity of 16.3 m/s? Assume that the ball is thrown on the surface of the Earth and that it undergoes constant acceleration due to gravity (ignore air resistance).

 a. 9.1 m
 b. 11.7 m
 c. 13.6 m
 d. 7 m

Solution
Use the formula $v_f^2 = v_i^2 + 2ad$ with $a = g = -9.8\,\mathrm{m/s^2}$ and $v_f = 0$ (the velocity of the ball at its maximum height is zero). Solve for d:

$$d = \frac{-v_i}{2a} = \frac{-16.3\,\mathrm{m/s}}{2(-9.8\,\mathrm{m/s^2})} = 13.6\,\mathrm{m}$$