

# Physics 11

## Circular Motion and Gravity Test Solutions

1. a.  b.  c.  d.
2. a.  b.  c.  d.
3. a.  b.  c.  d.
4. a.  b.  c.  d.
5. a.  b.  c.  d.
6. a.  b.  c.  d.
7. a.  b.  c.  d.
8. a.  b.  c.  d.
9. a.  b.  c.  d.
10. a.  b.  c.  d.
11. a.  b.  c.  d.
12. a.  b.  c.  d.
13. a.  b.  c.  d.
14. a.  b.  c.  d.
15. a.  b.  c.  d.
16. a.  b.  c.  d.

**1. Problem**

What are the units of  $G$ , the universal gravitational constant?

- a.  $\text{kg m/s}^2$
- b.  $\text{N m}^2 \text{kg}^{-2}$
- c.  $G$  does not have any units
- d.  $\text{m s}^{-2}$

**Solution**

The universal gravitational constant is defined in Newton's law of universal gravitation:

$$F = \frac{GMm}{r^2}$$

The constant relates the combination of mass and distance units to force units. Therefore, it must have units of

$$\text{N m}^2 \text{kg}^{-2}$$

Or, since  $\text{N} = \text{kg m/s}^2$ , the units of  $G$  are also

$$\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$$

**2. Problem**

A 347-kg satellite is in a circular orbit at an altitude of 547 km around the earth. How much energy (kinetic and potential) would it release if it were to crash down onto the surface of the earth?

- a. 11.7 GJ
- b. 7.8 GJ
- c. 8.4 GJ
- d. 9.4 GJ

**Solution**

During the crash, the satellite would lose its kinetic energy and some of its potential energy.

$$\Delta E = GMm \left( \frac{1}{R} - \frac{1}{2(R+h)} \right) = 11.7 \text{ GJ}$$

**3. Problem**

What is the escape speed on Mars? The mass of Mars is  $6.39 \times 10^{23} \text{ kg}$  and its radius is  $3.39 \times 10^6 \text{ m}$ .

- a. 5.01 km/s
- b. 4.20 km/s
- c. 3.55 km/s
- d. 2.77 km/s

**Solution**

The escape speed is

$$v_{esc} = \sqrt{\frac{2GM}{r}} = 5.01 \text{ km/s}$$

**4. Problem**

An asteroid orbits the sun with a period of 3.44 years. What is the asteroid's average distance from the sun? Note: One astronomical unit (au) is the average distance from the earth to the sun.

- a. 1.94 au
- b. 1.39 au
- c. 1.69 au
- d. 2.28 au

**Solution**

Kepler's Third Law of Planetary Motion states that the square of the orbital period is proportional to the cube of the average distance

$$\left(\frac{T_{\text{asteroid}}}{T_{\text{earth}}}\right)^2 = \left(\frac{R_{\text{asteroid}}}{R_{\text{earth}}}\right)^3$$

We are given that  $T_{\text{asteroid}}/T_{\text{earth}} = 3.44$  and  $R_{\text{earth}} = 1$  au. Therefore,

$$R_{\text{asteroid}} = \sqrt[3]{(3.44)^2} \text{ au} = 2.28 \text{ au}$$

**5. Problem**

A rocket is launched straight up from earth's surface at a speed of 30 km/s. What is its speed when it is very far away from the earth? Consider only the effect of the earth's gravity in your answer. The mass of the earth is  $5.97 \times 10^{24}$  kg and its radius is  $6.38 \times 10^6$  m.

- a. 27.8 km/s
- b. 15.7 km/s
- c. 8.3 km/s
- d. 19.9 km/s

**Solution**

When the rocket is very far away, its potential energy is zero and it only has kinetic energy. This is equal to its initial energy.

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - \frac{GMm}{r}$$

Solving for  $v_f$  gives

$$v_f = \sqrt{v_i^2 - \frac{2GM}{r}} = 27.8 \text{ km/s}$$

**6. Problem**

A pilot makes an outside vertical loop (in which the path of the plane traces out the top half of a circle with centre of the loop is beneath the plane) of radius 4747 m. At the top of his loop, he is pushing down on his seat with only 1/5 of his normal weight. How fast is he going?

- a. 144 m/s
- b. 212 m/s
- c. 123 m/s
- d. 193 m/s

**Solution**

The centripetal force is

$$\frac{mv^2}{r} = mg(1 - 1/5)$$

Solving for  $v$  gives

$$v = \sqrt{gr(1 - 1/5)} = 193 \text{ m/s}$$

### 7. Problem

An asteroid orbits the sun at an average distance of 19.2 au. What is the period of the asteroid's orbit? Note: One astronomical unit (au) is the average distance from the earth to the sun.

- a. 84.1 years
- b. 122.6 years
- c. 110.9 years
- d. 97.9 years

#### Solution

Kepler's Third Law of Planetary Motion states that the square of the orbital period is proportional to the cube of the average distance

$$\left(\frac{T_{\text{asteroid}}}{T_{\text{earth}}}\right)^2 = \left(\frac{R_{\text{asteroid}}}{R_{\text{earth}}}\right)^3$$

We are given that  $R_{\text{asteroid}}/R_{\text{earth}} = 19.2$  and  $T_{\text{earth}} = 1$  year. Therefore,

$$T_{\text{asteroid}} = \sqrt{(19.2)^3} \text{ years} = 84.1 \text{ years}$$

### 8. Problem

What is the radius of the geosynchronous orbit around Jupiter? The mass of Jupiter is  $1.898 \times 10^{27}$  kg. A "day" on Jupiter lasts 9 hours and 55 minutes.

- a.  $235 \times 10^3$  km
- b.  $160 \times 10^3$  km
- c.  $200 \times 10^3$  km
- d.  $104 \times 10^3$  km

#### Solution

The radius of a circular geosynchronous orbit is given by

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = 160000 \text{ km}$$

### 9. Problem

An object is moving in a circle with a constant speed. Its acceleration is

- a. tangent to the circle.
- b. directed towards the centre of the circle.
- c. in the same direction as its velocity.
- d. opposite the direction of its velocity.

#### Solution

An object in uniform circular motion has a centripetal acceleration that always points toward the centre of the circle.

**10. Problem**

An object is moving in a circle with a constant speed. Its velocity is

- a. opposite the direction of its acceleration.
- b. directed towards the centre of the circle.
- c. tangent to the circle.
- d. directed away from the centre of the circle.

**Solution**

An object in uniform circular motion has a velocity that is always tangent to the circle.

**11. Problem**

A pilot, of mass  $m$ , executes a vertical dive, then follows a semi-circular arc until the plane is going straight up. Just as the plane is at its lowest point, the force that the pilot's seat exerts on him is

- a. less than  $mg$  and pointing up
- b. less than  $mg$  and pointing down
- c. more than  $mg$  and pointing up
- d. more than  $mg$  and pointing down

**Solution**

The centripetal force points towards the centre of the circle, which would be upward. This force must oppose and overcome the pilot's weight, so it is more than  $mg$ .

**12. Problem**

The maximum speed around a level circular curve is 41.0 km/h. What is the maximum speed around a curve with 3 times the radius? Assume all other factors remain unchanged.

- a. 43.6 km/h
- b. 36.0 km/h
- c. 123.0 km/h
- d. 71.0 km/h

**Solution**

For uniform circular motion, the centripetal force is given by

$$F_c = \frac{mv^2}{r}$$

If  $r \rightarrow 3r$ , then to maintain the same centripetal force (since the friction is assumed to be the same) we must have  $v \rightarrow \sqrt{3}v = 71.0$  km/h.

**13. Problem**

A rock, of mass  $m$ , is attached to a string and whirled in a vertical circle of radius  $r$ . At the top of the circle, the tension in the string is 8 times the rock's weight. The rock's speed at this point is given by

- a.  $8\sqrt{gr}$
- b.  $\sqrt{7gr}$
- c.  $\sqrt{8gr}$
- d.  $\sqrt{9gr}$

**Solution**

The net force at the top of the circle is the tension in the string plus the weight of the rock. This net

force is the centripetal force for circular motion.

$$\frac{mv^2}{r} = F_{tension} + mg$$

We are also given in the problem that  $F_{tension} = 8mg$ . Putting this into the equation above and solving for  $v$  gives

$$v = \sqrt{9gr}$$

#### 14. Problem

Consider a satellite moving in a circular orbit of radius  $r$  around a planet of mass  $M$ . Which expression gives the satellite's orbital speed?

- a.  $\sqrt{Gr}$
- b.  $\sqrt{2GM/r^2}$
- c.  $GM/r^2$
- d.  $\sqrt{GM/r}$

#### Solution

The centripetal force is the gravitational force for a circular orbit.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

Solving for  $v$  gives

$$v = \sqrt{GM/r}$$

#### 15. Problem

Halley's comet is in a highly elliptical orbit around the sun. Its speed

- a. is constant.
- b. increases as it nears the sun.
- c. decreases as it nears the sun.
- d. is zero when it reaches its furthest point from the sun.

#### Solution

Orbiting objects get faster when they get closer to the sun (Kepler's second law).

#### 16. Problem

A planet has 9 times the earth's mass and 8 times the earth's radius. If an object weighs 100 N on earth, how much would it weigh on this planet?

- a. 39.6 N
- b. 112.5 N
- c. 61.5 N
- d. 14.1 N

#### Solution

The gravitational field strength on the surface of a planet is

$$g = \frac{GM}{r^2}$$

So the gravitational field strength on the planet is

$$g_{planet} = \frac{G(9M_{earth})}{(8r_{earth})^2} = \frac{9}{64}g_{earth}$$

So the 100 N object would weigh

$$(100N) \frac{9}{64} = 14.1N$$