Physics 12

Solutions to Physics 12 Statics Unit Test

	a.	b.	c.	d.
1.			X	
2.	X			
3.				X
4.	X			
5.			X	
6.				X
7.		X		
8.			X	
9.	X			
10.		X		
11.				X
12.	X			
13.	X			
14.	X			
15.	X			
16.		X		
17.	X			
18.		X		
19.		X		
20.			X	

1. **Problem**
   What conditions are necessary for a body to be in static equilibrium? Note that $\sum \vec{F}$ is the net force on the body and $\sum \vec{\tau}$ is the net torque on the body.

   a. $\sum \vec{F} = 0$
   b. $\sum \vec{\tau} = 0$
   c. $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$
   d. $\sum \vec{F} = 0$ or $\sum \vec{\tau} = 0$ (but not both)

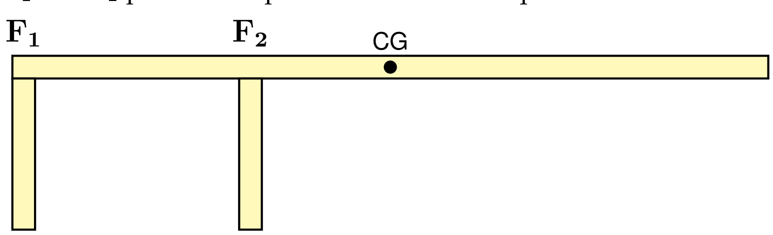
   Solution
   A body is in static equilibrium if the net force on the body is zero ($\sum \vec{F} = 0$) and the net torque on the body is zero ($\sum \vec{\tau} = 0$).

2. **Problem**
   A book weighs 4 N at the surface of the Earth. When held at rest on top of your head, the net force on the book is

   a. 0 N
   b. 4 N
   c. −4 N
   d. 9.8 N

   Solution
   Since the book is at rest, its acceleration is zero and the net force acting on it must be zero.

3. **Problem**
   A cantilever is held in static equilibrium by two vertical supports as shown in the figure. The beam is fastened to the supports with screws so that each support could apply an upward or downward force. The centre of gravity (CG) of the beam is to the right of the second support. In which direction must $\mathbf{F_1}$ and $\mathbf{F_2}$ point to keep the beam in static equilibrium?

   a. $\mathbf{F_1}$ and $\mathbf{F_2}$ both point upward
   b. $\mathbf{F_1}$ and $\mathbf{F_2}$ both point downward
   c. $\mathbf{F_1}$ points upward while $\mathbf{F_2}$ points downward
   d. $\mathbf{F_1}$ points downward while $\mathbf{F_2}$ points upward

   Solution
   The net force and the net torque (around any axis) on the beam must be zero in order for the beam to be in static equilibrium.

   In order to oppose the downward force of gravity on the beam, the vector sum of $\mathbf{F_1}$ and $\mathbf{F_2}$ must point upward. Then, considering the torque about the centre of gravity of the beam, $\mathbf{F_1}$ and $\mathbf{F_2}$ must point in opposite directions and $\mathbf{F_2}$ must be greater than $\mathbf{F_1}$ for the torques to cancel out (since $\mathbf{F_2}$ is closer to the axis). Therefore, $\mathbf{F_2}$ must point up and $\mathbf{F_1}$ must point down.

4. **Problem**
   A heavy child and a lightweight child are balanced on a massless seesaw. If both children move forward so that they are at half of their original distance from the pivot, what will happen to the seesaw?

   a. The seesaw will still be balanced.
   b. The side the lightweight child is sitting on will tilt downward.
   c. It is impossible to determine without knowing the masses and distances.
   d. The side the heavy child is sitting on will tilt downward.

   Solution
   Since the seesaw is initially balanced, we must have

   $$\tau_{net} = m_1 d_1 - m_2 d_2 = 0$$

   Halving both distances will result in

   $$\tau_{net} = m_1(d_1/2) - m_2(d_2/2) = \frac{1}{2}(m_1 d_1 - m_2 d_2) = \frac{1}{2}(0) = 0$$

   Therefore, the seesaw will remain balanced.

5. **Problem**
   A heavy seesaw is balanced with no one sitting on it. Then, a lightweight child sits on one side and a heavy child sits on the other side. They sit at different distances from the pivot so that the seesaw remains balanced. Now, if both children move forward so that they are at half of their original distance from the pivot, what will happen to the seesaw?

   a. It is impossible to determine without knowing the masses and distances.
   b. The side the lightweight child sitting on will tilt downward.
   c. The seesaw will still be balanced.
   d. The side the heavy child is sitting on will tilt downward.

   Solution
   Since the torque on the seesaw is balanced, we must have

   $$\tau_{net} = (m_1 d_1 + \tau_s) - (m_2 d_2 + \tau_s) = m_1 d_1 - m_2 d_2 = 0$$

   where τ_s is the torque from weight of the seesaw, m_1 and d_1 are the mass and distance of the lighter child, and m_2 an d_2 are the mass and distance of the heavier child.

   Note that since the seesaw is balanced when no one is sitting on it, the two torques from either side of the seesaw cancel out when calculating the net torque.

   Halving both distances will result in

   $$\tau_{net} = (m_1(d_1/2) + \tau_s) - (m_2(d_2/2) + \tau_s) = \frac{1}{2}(m_1 d_1 - m_2 d_2) = 0$$

   Therefore, the seesaw will still be balanced.

6. **Problem**
A heavy seesaw is out of balance. A lightweight child sits on the end that is tilted downward, and a heavy child sits on the other side so that the seesaw now balances. If both children then move forward so that they are at half of their original distance from the pivot, what will happen to the seesaw?

   a. It is impossible to determine without knowing the masses and distances.
   b. The seesaw will still be balanced.
   c. The side the heavy child is sitting on will now tilt downward.
   d. The side the lightweight child sitting on will once again tilt downward.

   Solution
   Since the heavy child sits on the side that initially had less torque (so that it was tilted up), he must be providing more torque than the lightweight child to balance the seesaw. Then, if both children move to half their original distance, the torque they provide are also halved. Since the heavier child provided more torque, the side with the heavier child will have a greater reduction in torque (the reduction is half of a greater amount). The side the lightweight child is sitting on will therefore have a greater torque tilting that side downward.

   For example, if the left side of the seesaw is initially tilted downward and

   - the left side of the seesaw applies $3\,\mathrm{Nm}$ of torque.
   - the right side of the seesaw applies $-1\,\mathrm{Nm}$ of torque.
   - the lightweight child sits on the left and applies $8\,\mathrm{Nm}$ of torque.
   - the heavy child sits on the right side and applies $-10\,\mathrm{Nm}$ of torque.

   Then the net torque when the seesaw is balanced is:

$$\tau_{net} = (3+8) - (1+10) = 0$$

   In the calculations above, we used the convention that counterclockwise rotation is positive.

   When the children move to half their original distance, the torque they provide will also be halved

   - the lightweight child sits at half the original distance and now applies $4\,\mathrm{Nm}$ of torque
   - the heavy child sits at half the original distance and now applies $-5\,\mathrm{Nm}$ of torque

   The net torque is now:

$$\tau_{net} = (3+4)\mathrm{Nm} - (1+5)\mathrm{Nm} = 1\,\mathrm{Nm}$$

   Since the net torque is positive, the seesaw is rotating counterclockwise. In other words, the left side (where the lightweight child is sitting) will once again tilt downward.

7. **Problem**
A person weighing $778\,\mathrm{N}$ stands with one foot on each of two bathroom scales. Which statement is correct about this situation?

   a. Each scale should read $389\,\mathrm{N}$
   b. The sum of the two scale readings should be $778\,\mathrm{N}$
   c. Each scale should read $778\,\mathrm{N}$
   d. None of the other statements are true.

   Solution
   Since the person is at rest, the net force on the person must be zero. The normal force on the person from the two scales must cancel out the person's weight. Therefore, the sum of the two scale readings should be equal to the person's weight.

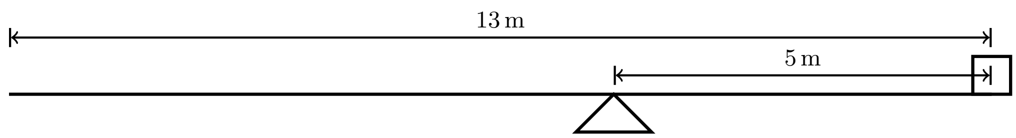
8. **Problem**
A boy and a girl are balanced on a massless seesaw. The boy's mass is 41 kg and the girl's mass is 29 kg. If the boy is sitting 1.9 m from the pivot, how far from the pivot must the girl be sitting on the other side of the seesaw?

   a. 1.8 m
   b. 1.5 m
   c. 2.7 m
   d. 1.7 m

Solution
Since the torque on the seesaw is balanced, we must have $m_b d_b = m_g d_g$ where m_b is the mass of the boy, d_b is the distance of the boy from the pivot, and m_g and d_g are corresponding quantities for the girl. Solving for d_g gives:

$$d_g = \left(\frac{m_b}{m_g}\right) d_b = \left(\frac{41\,\mathrm{kg}}{29\,\mathrm{kg}}\right) 1.9\,\mathrm{m} = 2.7\,\mathrm{m}$$

9. **Problem**
A lever is 13 m long. The distance from the fulcrum to the load to be lifted is 5 m. If a rock weighing 8000 N is to be lifted, how much force must be exerted on the lever?



   a. 5000 N
   b. 4690 N
   c. 4920 N
   d. 2810 N

Solution
The worker must generate a torque equal to the torque from the rock.

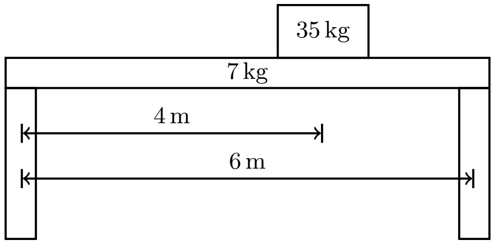
$$F_{worder} d_{worker} = F_{load} d_{load}$$

Where $d_{worker} = 13\,\mathrm{m} - 5\,\mathrm{m} = 8\,\mathrm{m}$. Therefore, solving for F_{worker}:

$$F_{worker} = \frac{d_{load}}{d_{worker}} F_{load} = \left(\frac{5}{8}\right) 8000\,\mathrm{N} = 5000\,\mathrm{N}$$

10. **Problem**
A uniform 7 kg beam, 6 m long, supports a 35 kg box. The beam is supported by two vertical columns

and the centre of gravity of the box is $4\,\mathrm{m}$ from the left column. Calculate the force on the right column.



a. $1920\,\mathrm{N}$
b. $263\,\mathrm{N}$
c. $120\,\mathrm{N}$
d. $790\,\mathrm{N}$

Solution
Let $m = 7\,\mathrm{kg}$, $M = 35\,\mathrm{kg}$, $d_1 = 4\,\mathrm{m}$, and $d_2 = 6\,\mathrm{m}$. Also, let F_1 be the force on the left support and F_2 be the force on the right support.

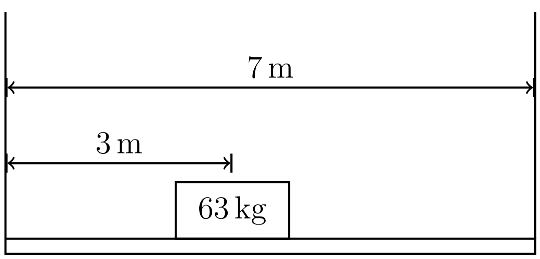
The equation for the net torque about the axis passing through the left end of the beam is

$$\tau_{net} = d_2 F_2 - \frac{d_2}{2} mg - d_1 M g = 0$$

Solving for F_2:

$$F_2 = g\left(\frac{1}{2}m + \frac{d_1}{d_2}M\right) = (9.8\,\mathrm{N/kg})\left(\frac{1}{2}(7\,\mathrm{kg}) + \frac{4\,\mathrm{m}}{6\,\mathrm{m}}(35\,\mathrm{kg})\right) = 263\,\mathrm{N}$$

11. **Problem**
A scaffold of negligible mass is hanging horizontally from wires on each end. The scaffold is $7\,\mathrm{m}$ long. A $63\,\mathrm{kg}$ box sits $3\,\mathrm{m}$ from the left end. What is the tension in the left wire?



a. $239\,\mathrm{N}$
b. $42\,700\,\mathrm{N}$
c. $157\,000\,\mathrm{N}$
d. $353\,\mathrm{N}$

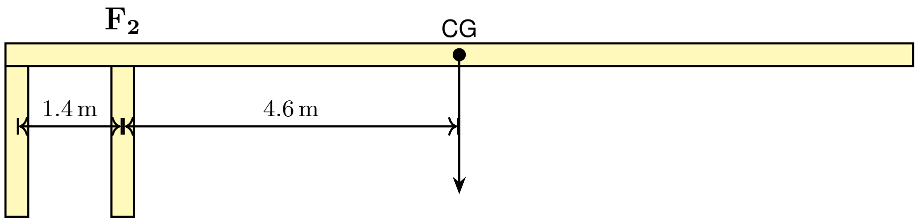
Solution
To avoid needing to know the tension in the right wire, we can calculate the torque about the rotation axis going through the right side of the scaffold. Let T_1 be the tension in the left wire, $d_B = 4\,\mathrm{m}$, and $d_L = 7\,\mathrm{m}$. Then,

$$T_1 d_L = mgd_B$$

Solving for T_1:

$$T_1 = \left(\frac{d_B}{d_L}\right) mg = \left(\frac{4}{7}\right)(63\,\text{kg})(9.8\,\text{N/kg}) = 353\,\text{N}$$

12. **Problem**
A cantilever beam is held in static equilibrium by two vertical supports separated by 1.4 m. The beam's mass is 63 kg and its centre of gravity (CG) is 4.6 m from the second support. What is the magnitude of the force applied by the second support, $\mathbf{F_2}$?



a. 2600 N
b. 690 N
c. 260 N
d. 1400 N

Solution
The net force on the beam must be zero. Assuming that F_1 points down and F_2 points up:

$$-F_1 + F_2 - mg = 0$$

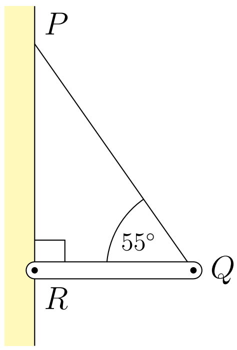
The net torque about the centre of gravity (CG) must be zero. Let $d_1 = 6\,\text{m}$ be the distance from the CG to the first support and $d_2 = 4.6\,\text{m}$ be the distance from the CG to the second support:

$$d_1F_1 - d_2F_2 = 0$$

Solving the above system of linear equations for F_2 (by substitution or elimination) gives:

$$F_2 = \left(\frac{d_1}{d_1 - d_2}\right) mg = \left(\frac{6}{6 - 4.6}\right)(63\,\text{kg})(9.8\,\text{N/kg}) = 2600\,\text{N}$$

13. **Problem**
A 7.0 m long uniform beam of mass 4.0 kg, QR, is mounted by a hinge on a wall and held in a horizontal position by wire PQ, forming a 55° angle at point Q as shown in the figure. What is the magnitude of the tension in wire PQ?



a. 24 N
b. 48 N
c. 2.4 N
d. 7.5 N

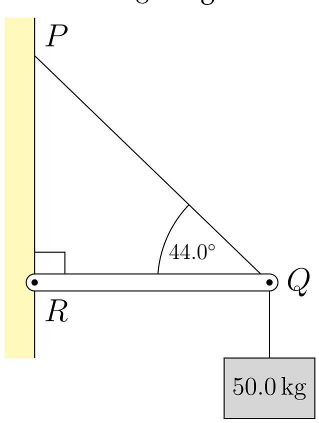
Solution
The net torque about the hinge point R must be zero because the beam is in static equilibrium. The force of gravity on the beam and the tension in the wire are the two forces applying a torque to the beam. Let l be the length of the beam, θ be the angle at point Q, and T be the tension in wire.

$$\tau_{net} = lT\sin(\theta) - \frac{lmg}{2} = 0$$

Solving for T, we get

$$T = \frac{mg}{2\sin(\theta)} = \frac{(4.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{2\sin(55°)} = 24\,\mathrm{N}$$

14. **Problem**
A 9.0 m long uniform beam of mass 4.00 kg, QR, is mounted by a hinge on a wall and held in a horizontal position by wire PQ, forming a 44.0° angle at point Q as shown in the figure. A load of mass 50.0 kg hangs vertically down from point Q. What is the magnitude of the tension in wire PQ?



a. 734 N
b. 1000 N
c. 73.4 N
d. 328 N

Solution
The net torque about the hinge point R must be zero because the beam is in static equilibrium. The

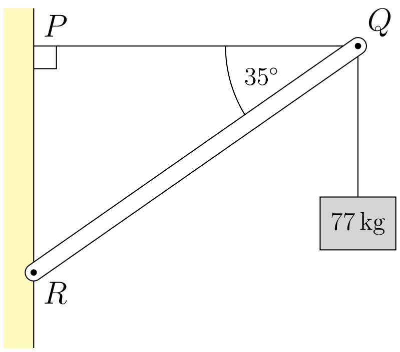
weight of the beam, the tension in the wire, and the hanging load each apply a torque to the beam. The force of gravity on a uniform beam can be modeled as a force, $F_g = mg$, acting on the centre of the beam. Let l be the length of the beam, θ be the angle at point Q, T be the tension in wire, m be the mass of the beam, and M be the mass of the hanging load.

$$\tau_{net} = lT\sin(\theta) - \frac{lmg}{2} - lMg = 0$$

Solving for T, we get

$$T = \frac{g}{\sin(\theta)}\left(\frac{m}{2} + M\right) = \frac{9.80\,\mathrm{N/kg}}{\sin(44.0°)}\left(\frac{4.00\,\mathrm{kg}}{2} + 50.0\,\mathrm{kg}\right) = 734\,\mathrm{N}$$

15. **Problem**
A uniform beam QR, 1.0 m long with negligible mass, is mounted by a hinge on a wall and held in position by a horizontal wire PQ as shown in the figure. The beam supports a load of mass 77 kg hanging vertically down from point Q. What is the magnitude of the force on the hinge at point R?



a. 1320 N
b. 11 900 N
c. 207 N
d. 4310 N

Solution
Since the mass of the beam is negligible, only two forces apply a torque on the beam about the hinge point R: the tension in the horizontal wire, T_{PQ}, and the force from the hanging load, mg.

To calculate the torque from these two forces, we need to calculate the component of the force that is perpendicular to the beam. For the horizontal wire, this means multiplying by $\sin(35°)$ while for the vertical wire, this means multiplying by $\cos(35°)$.

The net torque must be zero since the beam is in static equilibrium. Let r be the length of the beam. Then, the equation for the net torque is:

$$\tau_{net} = rT_{PQ}\sin(35°) - rmg\cos(35°) = 0$$

The factors of r cancel out (so the length of 2.0 m was not needed). Solving for T_{PQ}, we get

$$T_{PQ} = \frac{mg\cos(\theta)}{\sin(\theta)}$$

To keep the beam in static equilibrium, the net force must also be zero. This means that the force on the hinge must be equal to the sum of the forces applied by the tension in the wire and the hanging mass. Since the two forces are perpendicular, we can use the Pythagorean theorem to calculate the magnitude of their sum.

$$F_{hinge} = \sqrt{(mg)^2 + (mg)^2\frac{\cos^2(\theta)}{\sin^2(\theta)}} = mg\sqrt{1 + \frac{\cos^2(\theta)}{\sin^2(\theta)}} = mg\sqrt{\frac{\sin^2(\theta)}{\sin^2(\theta)} + \frac{\cos^2(\theta)}{\sin^2(\theta)}}$$

Using the Pythagorean identity ($\sin^2\theta + \cos^2\theta = 1$), we can simplify further:

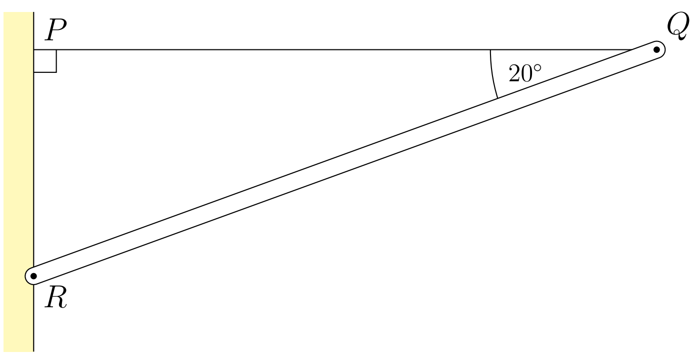
$$F_{hinge} = mg\sqrt{\frac{\sin^2(\theta) + \cos^2(\theta)}{\sin^2(\theta)}} = mg\sqrt{\frac{1}{\sin^2(\theta)}} = \frac{mg}{\sin(\theta)}$$

Plugging in the numbers from the problem, we get:

$$F_{hinge} = \frac{(77\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{\sin(35°)} = 1320\,\mathrm{N}$$

16. **Problem**

A 5.0 m long uniform beam of mass 15 kg (QR) is mounted by a hinge on a wall and held in position by a horizontal wire (PQ), forming a 20° angle at point Q as shown in the figure. What is the tension in the horizontal wire PQ?



a. 172 N
b. 202 N
c. 2020 N
d. 198 N

Solution

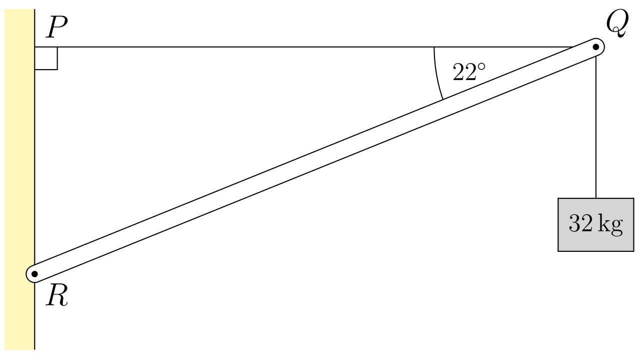
The force of gravity on a uniform beam can be modeled as a force, $F_g = mg$, acting on the centre of the beam. The net torque about the hinge point R must be zero because the beam is in static equilibrium. The weight of the beam and the tension in the wire are the two forces applying a torque to the beam. Let l be the length of the beam, θ be the angle at point Q, and T be the tension in wire.

$$\tau_{net} = lT\sin(\theta) - \frac{1}{2}lmg\cos(\theta) = 0$$

Solving for T, we get

$$T = \frac{mg\cos(\theta)}{2\sin(\theta)} = \frac{(15\,\mathrm{kg})(9.8\,\mathrm{m/s^2})\cos(20°)}{2\sin(20°)} = 202\,\mathrm{N}$$

17. **Problem**
A uniform beam QR, 2.0 m long with negligible mass, is mounted by a hinge on a wall and held in position by a horizontal wire PQ as shown in the figure. The beam supports a load of mass 32 kg hanging vertically down from point Q. What is the tension in the horizontal wire PQ?



a. 776 N
b. 418 N
c. 243 N
d. 182 N

Solution
Since the mass of the beam is negligible, only two forces apply a torque on the beam about the hinge point R: the tension in the horizontal wire, T_{PQ}, and the force from the hanging load, mg.

To calculate the torque from these two forces, we need to calculate the component of the force that is perpendicular to the beam. For the horizontal wire, this means multiplying by $\sin(22°)$ while for the vertical wire, this means multiplying by $\cos(22°)$.

The net torque must be zero since the beam is in static equilibrium. Let r be the length of the beam. Then, the equation for the net torque is:

$$\tau_{net} = rT_{PQ}\sin(22°) - rmg\cos(22°) = 0$$

The factors of r cancel out (so the length of 2.0 m was not needed). Solving for T_{PQ}, we get

$$T_{PQ} = \frac{mg\cos\theta}{\sin\theta} = \frac{(32\,\text{kg})(9.8\,\text{N/kg})\cos(22°)}{\sin(22°)} = 776\,\text{N}$$

18. **Problem**
Two scales are separated by 7.0 m, and a plank of mass 6.0 kg is placed between them. Each scale is observed to read 3.0 kg. A rock is placed somewhere on the plank, after which the left scale reads 50.0 kg and the right scale reads 60.0 kg. How far from the left scale was the rock placed?

a. 1.6 m
b. 3.84 m
c. 2.65 m
d. 2.77 m

Solution
The first condition for static equilibrium is that the net force on the plank must be zero. This implies that the mass of the rock is the sum of the two scale readings minus the plank's mass:

$$m_{rock} = 50\,\text{kg} + 60\,\text{kg} - 6\,\text{kg} = 104\,\text{kg}$$

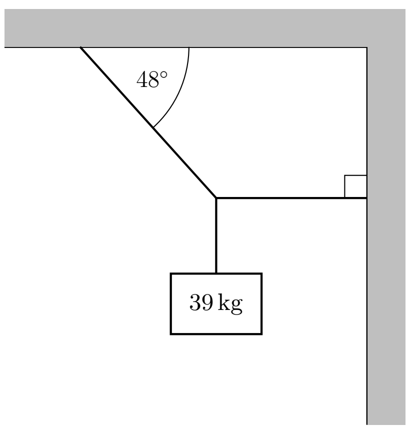
The second condition for static equilibrium is that the net torque on the plank must be zero. You can choose any axis to calculate the torque. Let's arbitrarily choose the left scale as the pivot. Also, let s be the separation between the two scales and x be the distance of the rock from the left scale. Then,

$$\frac{s}{2}m_{\mathtt{plank}} + xm_{\mathtt{rock}} - sm_{\mathtt{right}} = 0$$

Solving for x, we get:

$$x = \frac{s}{m_{\mathtt{rock}}}\left(m_{\mathtt{right}} - \frac{m_{\mathtt{plank}}}{2}\right) = \frac{7\,\mathrm{m}}{104\,\mathrm{kg}}\left(60\,\mathrm{kg} - \frac{6\,\mathrm{kg}}{2}\right) = 3.8\,\mathrm{m}$$

19. **Problem**
A box of mass 39 kg hangs down from three attached cords secured to the ceiling and wall as shown in the diagram. Find the maximum tension in any one of the three cords.



a. 272 N
b. 514 N
c. 659 N
d. 669 N

Solution
The vertical cord can only apply a vertical force and the horizontal cord can only apply a horizontal cord. However, the angled cord must oppose both the vertical force and the horizontal forces from the other two cords. Therefore, the maximum tension is found in the angled cord.

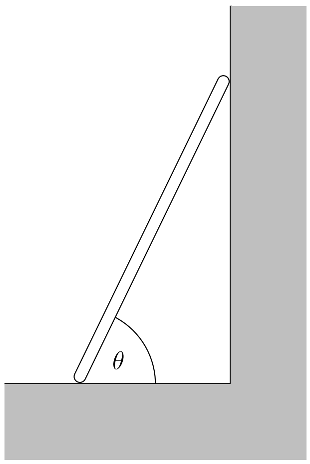
Let T be the tension in the angled cord. The vertical component of this tension, T_y, must be equal to the weight of the box, mg.

$$T_y = T\sin\theta = mg$$

Solving for T:

$$T = \frac{mg}{\sin\theta} = \frac{(39\,\mathrm{kg})(9.8\,\mathrm{N/kg})}{\sin(48^\circ)} = 514\,\mathrm{N}$$

20. **Problem**
A uniform ladder of mass $8\,\mathrm{kg}$ and length $8\,\mathrm{m}$ leans against a frictionless wall. Let θ be the angle that the ladder makes with the ground. If the coefficient of static friction between the ladder and the ground is 0.34, what is the minimum value of θ at which the ladder will not slip?



a. 48°
b. 46°
c. 56°
d. 38°

Solution
Let F_G be the force from the ground, F_f be the force of friction that the ground exerts on the bottom of the ladder, and $\mu = 0.34$ be the coefficient of static friction between the ladder and the ground.

The horizontal component of F_G comes from the friction between the ladder and the ground:

$$F_f = F_{G,x}$$

The vertical component of F_G is the normal force used to calculate the friction force. It is also the only force that balances the vertical gravitational force on the ladder:

$$F_f = \mu F_{normal} = \mu F_{G,y} = \mu m g$$

Therefore, combining the two equations, we have:

$$F_{G,x} = \mu m g$$

To find the values of $F_{G,y}$, we need to consider the net torque about the axis passing through the bottom of the ladder. Let the weight of the ladder be mg, the force from the wall be F_W, and the length of the ladder be d.

$$\tau_{net} = dF_W \sin\theta - \frac{d}{2} mg \cos\theta = 0$$

Solving for F_W, we get:

$$F_W = \frac{mg\cos\theta}{2\sin\theta}$$

Notice that as θ decreases, F_W increases and the corresponding friction force has to increase as well to keep the ladder from slipping. Since the only horizontal forces are from the ground and the wall, we must have

$$F_W = F_{G,x} = \mu m g = \frac{mg\cos\theta}{2\sin\theta}$$

Some algebra to rearrange the equation so that we can solve for θ:

$$\frac{1}{2\mu} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{1}{2\mu}\right) = \tan^{-1}\left(\frac{1}{0.68}\right) = 56^\circ$$