PHYS 229: Experiment 1 — Expansion Coefficients of Copper and Invar Bars Through Laser Interferometry

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1 Introduction:

When objects are heated, the atoms in the object gain kinetic energy and causes the object to expand. Conversely, as objects cool, they shrink. The relationship between the length and the temperature of a metal bar is described by the equation: $\frac{\Delta L}{L_0} = \alpha \Delta T$. Where:

ΔL is the change in length

L_0 is the original length

α is the expansion coefficient

ΔT is the change in temperature

The goal of this experiment is to find the expansion coefficient for copper and invar (a nickel-iron alloy) bars. Rearranging the above equation and taking Δ to be infinitesimal, we find that the expansion coefficient is:

$$\alpha = \frac{1}{L}\frac{dL}{dT} \tag{1}$$

So we must find a way to measure how the length, L, changes as the temperature, T changes. (i.e. find $\frac{dL}{dT}$). However, the change in length of the bar is quite small, so we need a very precise instrument to make the measurement. For this purpose, we use the Michelson-Morley interferometer. See figure 1 for the setup.

The beam splitter in the middle of the instrument splits the laser into two beams, each of which bounces off their respective mirrors and recombines before hitting the detector. If the distance to each mirror is the same or off by λ, then the two light waves combine constructively, but if the distance is off by $\lambda/2$, then the light waves combine destructively, forming an interference pattern.

A metal bar is attached securely to mirror A (see figure 1), causing the mirror to move as the bar expands and contracts. The two laser beams will then recombine in and out of phase periodically as the bar moves mirror A.

Using a detector, we can measure how the length of the bar changes with great accuracy by measuring when a fringe appears (since a fringe appears every $\frac{\lambda}{2}$). The number of fringes is related to ΔL by:

$$\Delta L = \frac{N}{2}\lambda \tag{2}$$

and the length of the bar can be calculated by:

$$\begin{aligned} L &= L_0 + \Delta L && (3) \\ &= L_0 + \frac{N}{2}\lambda && (4) \end{aligned}$$

Using equation 3, and a copper-constantan thermocouple to measure the temperature of the bar, we can plot how the length changes with temperature. The slope of this length vs. temperature plot is then $\frac{dL}{dT}$. Dividing this slope by the original length will give α, the expansion coefficient.

2 Procedure:

We first set up the Michelson-Morley interferometer according to figure 2. We used a He-Ne laser with wave-length of 632.8nm, a photodetector that measures the laser light in electric potential, a red light filter in front of the detector to filter out most of the

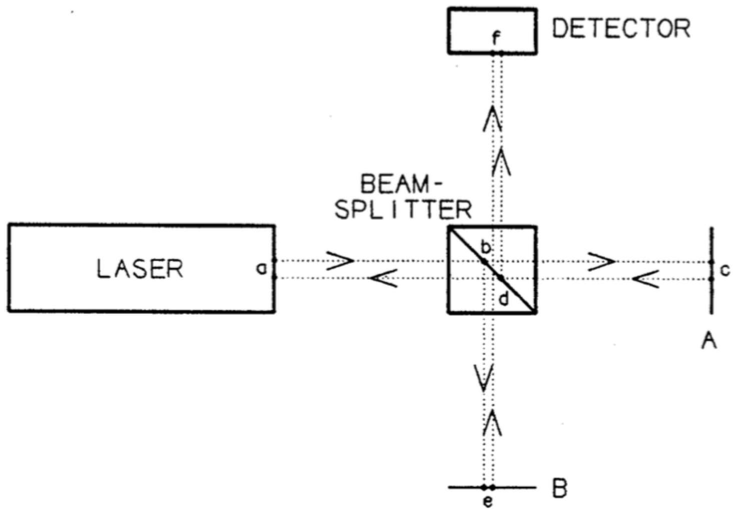

Figure 1: Laser interferometer set up used by Michelson and Morley in 1887

ambient light and only allow red light (the colour of the laser), a diffuser to allow us to see the diffraction pattern, copper-constantan thermocouples to measure temperature (placed inside the metal bar through a small hole), and finally two glass marbles to hold the bar inside the clamp.

It took us a long time (over an hour) to calibrate the laser. We first had to make sure the laser bounces off mirror A at 180° and bounces straight back into the laser, then we aligned the reflected beams over the detector so that a clear, circular diffraction pattern can be seen. This last step proved to be difficult, but we accomplished it with some perseverance and trial and error.

With the laser aligned, a heat gun (really a glorified blow dryer) is used to heat the bar up to about 50°C and data was measured by connecting the photodetector and thermocouples to a laptop computer and using LoggerLite to record the data.

We made measurements while the bar was *cooling* because the heat gun caused vibrations, which disrupted the photodetector. As well, we believe that the bar cools at a more uniform rate when cooling than heating because the heat gun is pointed at a more localized part of the bar while heating, but heat can disperse more uniformly when cooling.

While recording data, we noticed that the interferometer is *very* sensitive. Small vibrations, slight bumps against the table, and even laughing can cause unwanted jags to appear in the plot. As a result, we had to retake data several times, making sure that we remained quiet and away from the table.

We recorded the temperature, potential, and time as the copper bar cooled from about 50°C to about 38°C. This process took approximately 20 minutes and we collected 2689 data points.

Similarly for the invar bar, we recorded the temperature, potential, and time as it cooled from 53°C to 37°C. This process took around 15 minutes and we collected 1970 data points.

Because of the amount of data collected, the raw data will not be reproduced in a table in this report. However, they will be plotted and represented visually.

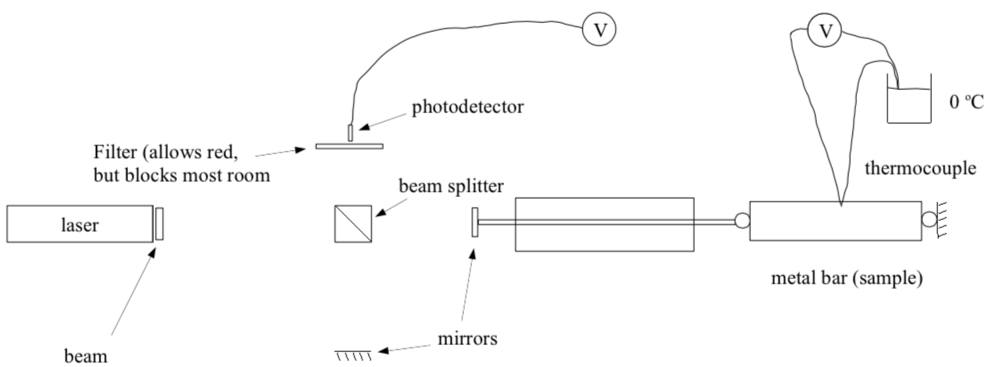

Figure 2: A more detailed figure of the interferometer set up used in this experiment.

3 Results:

The potential vs time plot is shown on LoggerLite while we were collecting data. This looked like a sine wave with bright fringes represented by peaks and dark fringes by troughs. The raw data for copper is shown in figure 3. This revealed that the voltage shifted during our data collection process. This was attributed to inconsistencies in the laser itself. We determined that this does not affect our analysis because we only care about the fringe count and the fringes are well defined throughout the plot.

Similarly for the invar bar, the raw data is shown in figure 4. The laser remained more or less steady this time, but note that the fringes became more spreadout as the cooling rate decreases.

We used an analog caliper to measure both the copper and the invar bars. Both bars were measured to have an original length of 92.3mm $\pm$ 0.05mm.

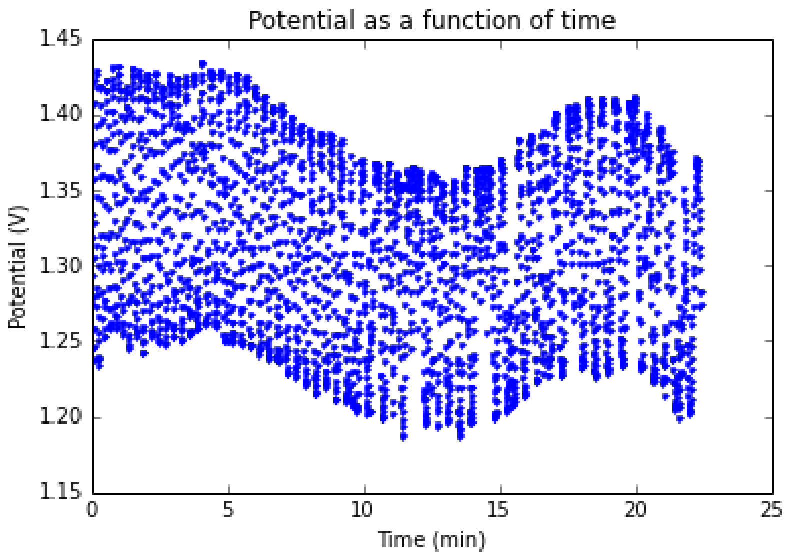

Figure 3: Data collected for copper. It is not important that the potential shifted during our measurement because we only care about the fringe count and the wave is well shaped throughout the plot.

We wrote a Python script to analyze the data by counting the fringes. This program counts the number of times the data "switches direction", which is defined as when a THRESHOLD number (9) points are going in the opposite direction. This threshold is used to prevent small jolts or variations in the graph to count as fringes. We tested the function on small sub samples of our data (where we also manually counted the fringes) and determined that it accurately counts the fringes to within +/- 1 fringe. (This might be because of the way it handles the beginning and the end.) The program then converts the fringe count to length using equation 2 and outputs a csv file that contains data for the length at various temperatures. Note that units of micrometers was used so that Python did not deal with numbers outside of its range. The code used to analyze the data is reproduced below:

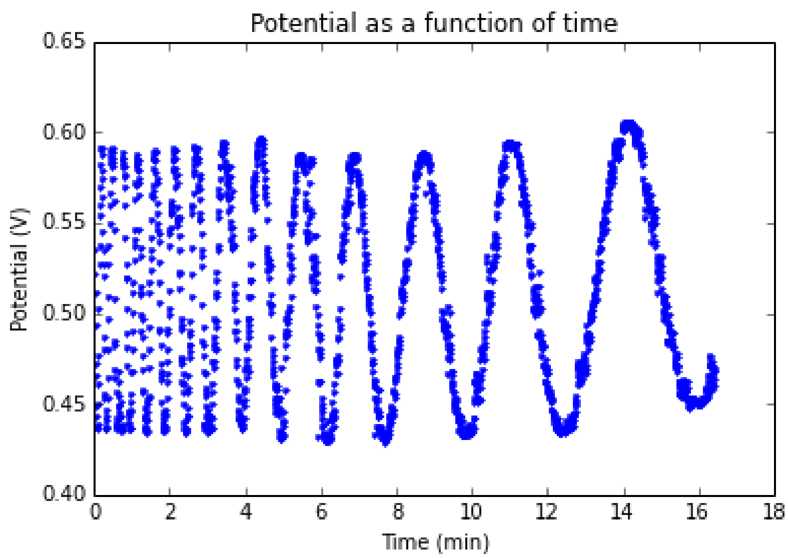

Figure 4: Data collected for invar

```
   # -*- coding: utf-8 -*-
"""
Created on Fri Jan 16 16:07:12 2015

@author: jackhong

IMPORTANT: DELETE the file generated by the program if you rerun it.
          Otherwise, it will add to the existing file.
"""
import numpy as np
import csv

# loads data
data = np.loadtxt('exp1_copper_data.csv', delimiter=',', comments='#')
temp = data[:,1]
potential = data[:,2]

# CONSTANT
THRESHOLD = 9
LENGTH_O = 150000  # original length of the bar in micrometers
WAVE_LENGTH = 0.6328 # wave length of He-Ne laser in micrometers

# VARIABLES -- All GLOBAL
# direction is one of:
# - "up"
# - "down"
direction = "down"
fringe_count = 0     # increments after each switch
error_counter = 0    # for making sure the wave is actually decreasing and not
    just bumped

# converts fringe_count to bar length
def fringes_to_length(n):
    return LENGTH_O - (n / 2) * WAVE_LENGTH

# writes the given data point to a new csv file
def write_data(temp,n):
    length = fringes_to_length(n)
    with open('length_v_temp_copper.csv', 'ab') as csvfile:
        data_writer = csv.writer(csvfile)
        data_writer.writerow([temp,length])

# switches direction, increments fringe_count, reset error_counter, writes data
def switch_direction(d,i):
    global direction
```

```
    global fringe_count
    global error_counter

    if d == "up":
        direction = "down"
    else:
        direction = "up"

    fringe_count += 1
    error_counter = 0
    write_data(temp[i],fringe_count)

enumPot = enumerate(potential)
next(enumPot)

def main():
    global error_counter
    for i, val in enumPot:
        if direction == "up":
            if val > potential[i-1]:
                pass
            else:
                error_counter += 1
                if error_counter > THRESHOLD:
                    switch_direction(direction,i)
        else:
            if val < potential[i-1]:
                pass
            else:
                error_counter += 1
                if error_counter > THRESHOLD:
                    switch_direction(direction,i)

# runs the count_fringes program when called in Python console.
# Other wise do nothing (other than define things)
if __name__ == "__main__":
    main()
```

Now we can plot the length vs the temperature. (see figures 5 and 6 We find that $\frac{dL}{dT}$ for copper is 3.427μm/K $\pm$ $.02722\mu$m/K and that $\frac{dL}{dT}$ for invar is 3.220μm/K $\pm$ 0.01993μm/K. Using equation 1 we find:

$$\alpha_{copper} = \frac{1}{92300\mu m} 3.427\mu m/K = 3.71E-05/K \pm 2.95E-6/K \tag{5}$$

$$\alpha_{invar} = \frac{1}{92300\mu m} 3.220\mu m/K = 3.49E-05 \pm 2.20E-6/K \tag{6}$$

Uncertainties are calculated using standard uncertainty propagation techniques, taking into account the uncertainty in the slope and the uncertainty in the original length. For uncertainty in the slope, we took the value given to us by the Python curve fit function, and for the length, the uncertainty is how accurately we could read the analog caliper. Since this is a division of two quantities with uncertainties we simply added the relative uncertainties in quadrature.

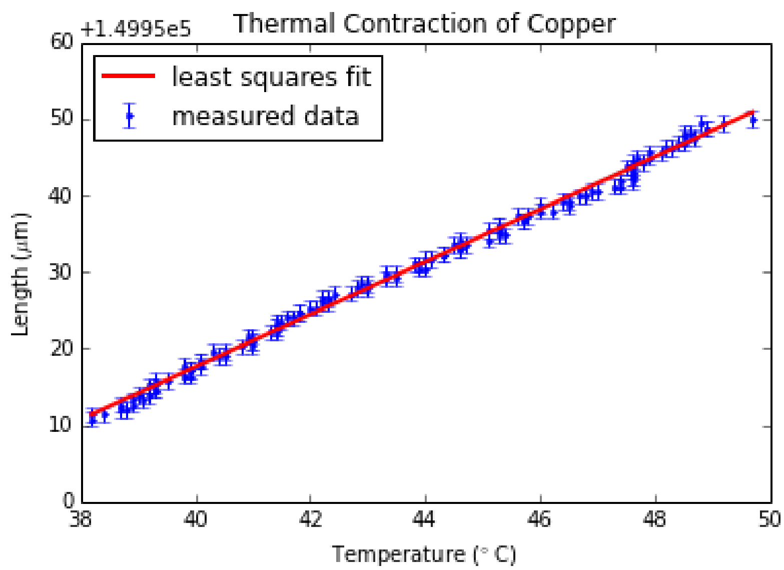

Figure 5: Plot of length vs temperature for copper. The slope is 3.427μm/K $\pm$ $.02722\mu$m/K.

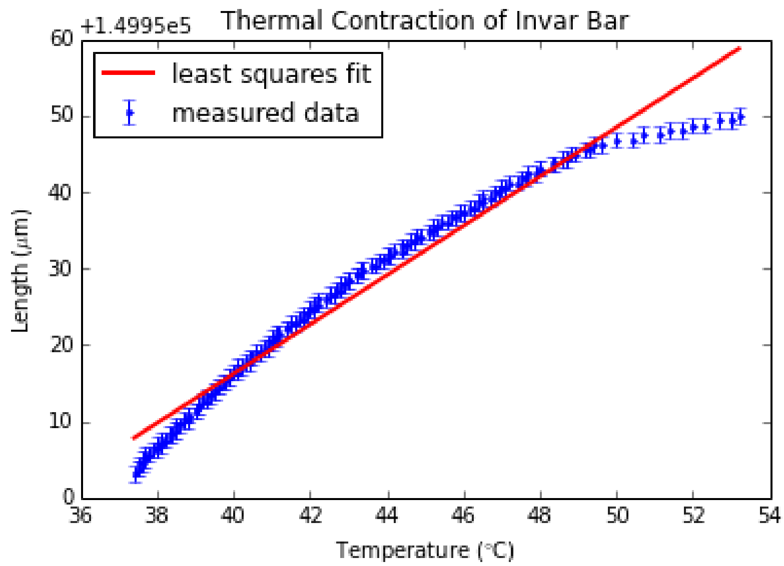

Figure 6: Plot of length vs temperature for invar. Although ths curve is clearly NOT linear, we fitted with a line as a quick apporximation, following the approach used with the copper bar. The slope is $3.220\mu\text{m/K} \pm 0.01993\mu\text{m/K}$.

4 Discussion:

The accepted values for the coefficient of expansion for copper is 1.67E-05/K for copper and 8E-07/K for invar. Which are not close at all to our measured values. (All of the accepted values in this report are taken from the lab instructions for PHYS 229: Expansion Coefficients Through Laser Interferometry). However, it is quite likely that the heat gun heated up the surrounding clamps and made them expand much more than the invar bar itself, which should have seen very very little expansion, resulting in the wrong expansion coefficients to be measured. If we assume that the invar bar did not expand significantly during the experiment, then the expansion coefficient of the surrounding clamps can be estimated to be the above calculated expansion coefficient for the invar bar. Taking this into account for the copper bar and estimating an original length of 5cm for the clamp, we found that $\frac{dL}{dT}$ for the clamp is 1.745 μm. We then adjusted $\frac{dL}{dT}$ for the copper bar by this amount and the new value for the expansion coefficient of copper is:

$$\alpha_{copper} = \frac{1}{92300\mu m}(3.427\mu m - 1.745\mu m) \tag{7}$$

$$= 1.8E - 05/K \tag{8}$$

Since there was a lot of estimating involved in reaching the above number, the uncertainty is large. However it is now within 7.78% of the accepted value.