Jack Hong
Partner: Walter Wasserman
September 25, 2014

PHYS 219: Experiment 3 – AC and Transient Response of a Resonant LCR Circuit

1. **<u>Transient response</u>**
   The circuit was set up as drawn.

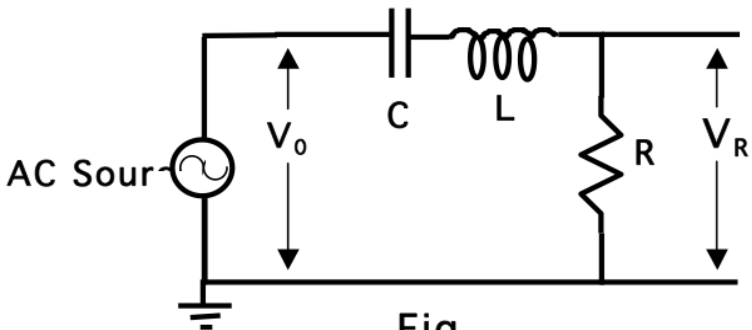


Fig.
using the component values L = 20 mH, C = 0.001 μF and R = 1000 Ω.

Components:

- Capacitor: 0.001uF ± 20% = 1e-9F ± 2e-10F
- Resistor: 1000Ω ± 5% = 1000Ω ± 50Ω
- Inductor: 4 * (5mH ± 1%) = 4 * (5mH ± 0.05mH) = 20mH ± sqrt(4 * .05^2)
  = 20mH ± 0.1mH = 2e-2H ± 1e-4H
- Wave generator (Rigol DG1012):
  - square waves
  - 5Hz
  - low: 0V; high: 5.000V
- Oscilloscope:
  - time divisions: 20.00us
  - Voltage division: 200mV

Measuring the frequency

Expected frequency based on the printed values:

frequency = 1/(2*pi*sqrt(LC)) = 1/(2 * pi * sqrt(20e-3H * 0.001e-6F))
= 3.558E+04Hz

Uncertainty calculations:

$$f_0(L,C)=\frac{1}{2\pi LC}$$

$$df=\left|\frac{\partial f}{\partial L}dL+\frac{\partial f}{\partial C}dC\right|$$

$$df=\left|-\frac{dL}{4\pi L^{3/2}\sqrt{C}}-\frac{dC}{4\pi C^{3/2}\sqrt{L}}\right|$$

$$\Delta f=\left|-\frac{\Delta L}{4\pi L^{3/2}\sqrt{C}}-\frac{\Delta C}{4\pi C^{3/2}\sqrt{L}}\right|$$

$$\Delta f=\left|-\frac{1\times 10^{-4}H}{4\pi(2\times 10^{-2}H)^{3/2}\sqrt{1\times 10^{-9}F}}-\frac{2\times 10^{-10}F}{4\pi(1\times 10^{-9})^{3/2}\sqrt{2\times 10^{-2}H}}\right|$$

$$\Delta f=\sqrt{(-88.97031793)^2+(-3558.812717)^2}$$

$$\Delta f=3559.924672Hz$$

The frequency based on the printed values of the components is ***3.558E+04Hz ± 3.560E+03Hz****.*

We measured the frequency of the oscillations by first measuring the period. The period was obtained by using the cursors to measure the time difference between the first peak and the last (6^{th}) peak and dividing that time by 5 (since that is 5 periods). We then took 1/T to find the frequency.

Time at first peak: 3.200us ± 0.4us
Time at 6^{th} peak: 147.2us ± 2us

The peaks were hard to measure because the voltage kept bouncing back and forth between two measurements.

The uncertainty in the 6^{th} peak is higher because it is less well defined than the first peak. We had to zoom in to find the peak.

Period times five = 147.2us-3.2us = 144us ± 2.0396us
Period = 144/5 = 28.8us ± 0.40792us

uncertainties for the measurements were added in quadrature, then divided by five to yield the uncertainty of one period.

Frequency = 1/28.8e-6 = ***3.472e+04Hz ± 491.8Hz***

the uncertainty is (28.8e-6)^-2 * 0.40792e-6 = 491.8Hz

t test between expected value and measured value:

t = abs(3.472e+04 – 3.558e+04)/sqrt(491.8^2 + 3560^2) = 0.239

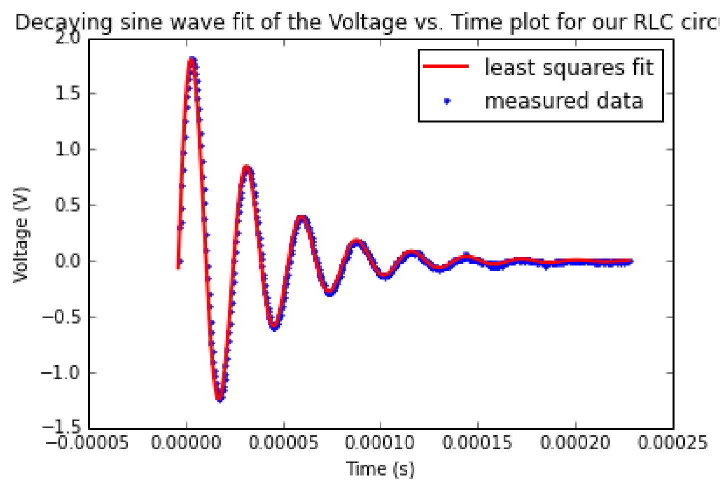
The t value is less than 1, which indicates that the measured frequency and calculated frequency agree very well, which is good.

Note: frequencies were measured with the oscilloscope triggering with the decaying sine wave. This resulted in fuzzier, doubled waves on the oscilloscope. From this point onward, we are triggering on the up slope of the square wave, making the lines much cleaner.

Measuring the time constant:

The plan: We will use the oscilloscope to measure voltage and the time. Then, we will use Python to perform a least squares fit of a decaying sine wave to our data. Python calculated the time constant as one of its parameters.

The following plot was produced using Python. This is the equation used to fit the data:
V(t) = amplitude * e ^ (-t/tau) * sin(2 * pi * frequency * t + phi) + y_offset



Python calculated a time constant (tau) of: 3.740E-05 +/- 7.403E-06 seconds.

The time constant using tau = RC (and the printed values):

R = 1000Ω ± 5%; C = 1E-09F ± 20%

RC = 1E-06s ± 20.62% (adding the relative uncertainties in quadrature)

RC = 1E-06s ± 2.062E-07s

t-test between measured time constant and the time constant from the printed values:

$$t = \frac{|3.740E\text{-}05s - 1E\text{-}06s|}{\sqrt{(7.403E\text{-}06s)^2 + (2.062E\text{-}07s)^2}}$$

$$t = 4.915$$

This indicates that the measured time constant may not agree with the calculated time constant by using the equation tau = RC. This may indicate that the time constant is not RC in RLC circuits like it is in RC circuits. Possibly, the formula will work if we use impedance instead of resistance.

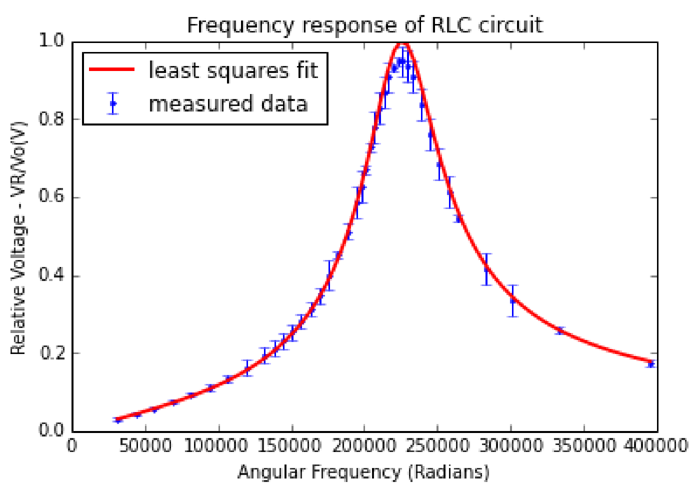
2. **Driven Harmonic Oscillator**

The setup: everything was the same as part one, except the wave generator is now producing sinusoidal waves instead of square waves and the high voltage is now set to 10V, with the low voltage still at 0V.

The plan: We will use the cursors of the oscilloscope in tracker mode to measure the voltage across the resistor, the voltage across the generator (because it may fluctuate) and the phase shift between the peaks of the two waves for several different frequencies. We incremented the frequency by 5 kHz at a time to start, then slowed to to 1 kHz and 0.5 kHz intervals as we neared the peak because that is an important part of the graph and we wanted better resolution at those points. We will then use Python to plot the data and use a least squares fit to fit a Lorentzian curve to our data. We will be plotting VR/Vo vs Angular frequency in radians.

We obtained to following fit by fitting the following equation:

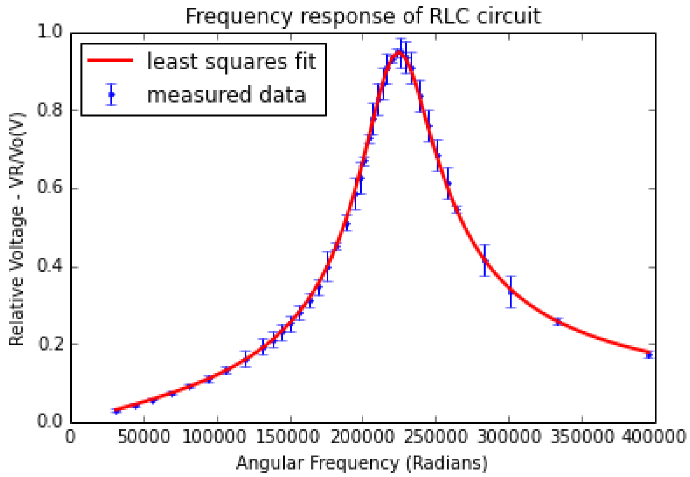
$$\frac{V_R}{V_0} = \frac{1}{\sqrt{\left(1+\frac{\omega}{\gamma}-\frac{\omega_0^{\,2}}{\omega\gamma}\right)^2}}$$



The resulting fit was not so good, so we added a scaling factor to the beginning like so:

$$\frac{V_R}{V_0} = k\frac{1}{\sqrt{\left(1+\frac{\omega}{\gamma}-\frac{\omega_0^{\,2}}{\omega\gamma}\right)^2}}$$

Using the latter fit, we got a much better result:



Python calculated:

resfreq = 2.248e+05 +/- 4.200e+02 radians

= 3.5778E+04Hz ± 66.845Hz

bandwidth (gamma) = 5.142e+04 +/- 7.792e+02 radians/s

We needed to add a scaling constant for the fit to work properly. This may be because there was extra resistance from the components, especially the inductors because they are really long lengths of wire. This resistance would need to be added in the equation somewhere, so we thought we would just stick a constant in front and see what happens. This resulted in a much better fit, so we just left it at that.