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CS 430

FINAL PROJECT PSEUDOCODE

* Let L be a collection of lines added to the graph to separate points, either horizontal or vertical. In actual code this was done with a dictionary, with key k signifying a line with an x or y value of k, and value 0, 1, or 2:

    0: a horizontal line exists at y = k
    1: both a horizontal <u>and</u> vertical line exist at x = k and y = k
    2: a vertical line exists at x = k

This is done to achieve quick lookup and improve runtime

```
def LOCAL_OPTIMIZATION(P, n): # P is set of pts, n is number of pts
    for i in range(1, n):
        ADD_LINE(i+.5, hor.)          # O(1) operation
    notDone = True
    while notDone:
        notDone = False
        for line1 ∈ L:
            success = False
            REMOVE_LINE(line1)     # O(1)
            for line2 ∈ L:
                REMOVE_LINE(line2)     # O(1)
                for j in range(1, n):
                    if TRY_LINE(j+.5, vert.) or TRY_LINE(j+.5, hor.):
                        success = notDone = True
                        break
                if success == False:
                    ADD_LINE(line2)
                else: break
            if success == False:
                ADD_LINE(line1)
```

DESCRIPTION:

- Lines 1 and 2 are the initialization — the algorithm starts with a horizontal line in between each point (feasible solution)
- Lines 3-5: boolean notDone keeps track of whether any permanent changes were made to L. If there is, the optimization loop must run again in case a line is added to a region that has already been checked
- Lines 6-10: 2 lines are permuted and chosen to be removed
- Lines 11-14: first checks if adding a vertical line at j+.5 will result in another feasible solution, and if not, check if adding a horizontal line at the same value results in a feasible solution. If one is successful, the loop breaks and looks for a new combination. notDone and success both set to True
- Lines 15-19: if all combinations for a line have been exhausted and no success has been found, that line is restored and a new one will be removed.

METHODS:

- ADD_LINE(): adds or updates dictionary entry. $\underline{O(1)}$
- REMOVE_LINE(): deletes or updates (eg. $1 \to 0$). $\underline{O(1)}$
- TRY_LINE(): calls ADD_LINE ($O(1)$), then evaluates if all pts are isolated by dividing the graph into the regions created by the lines, and checking every x and y coordinate to count the number of points inside. If successful, return True, else REMOVE_LINE. Thus total runtime = $O(1) + \underline{O(n^2)} + O(1)$

every coordinate must be checked

note:
*this description only describes the logical structure of the program. Source code contains different methods but follows the structure

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FINAL PROJECT RUNTIME ANALYSIS

LINE BY LINE ANALYSIS

- 1-2: runs (n-1) times to init. $O(n)$.
- 4-19: worst case, the size of L is decreased by 1 every iteration. The lowest size L can be and still separate all pts is $2(\sqrt{n}-1)$, so $O(n)$.
- 6-19: this for loop runs approximately 1 iteration for every line in L (approximate because some are added during). $O(n)$
- 9-19: same as above. $O(n)$
- 11-14: runs (n-1) times at most. $O(n)$
- 12: as mentioned on previous page, the cost to test if all points are isolated is $O(n^2)$
- all other operations are $O(1)$

TOTAL THEORETICAL RUNTIME: $\cancel{O(n)+}\left(O(n)\cdot O(n)\cdot O(n)\cdot O(n)\cdot O(n^2)\right)$
$= O(n^6)$

EXPERIMENTAL RUNTIME

This estimate is much higher than the algorithm actually takes. Due to the random distribution of points, probability states it is highly unlikely that the while loop at 4 will run more than 2 or 3 times. Therefore the actual average case will be closer to $O(n^5)$, at least for small/medium n values.
Actual runtimes are listed below:

n	time (s)
5	0.0019
10	0.0268
20	0.4804
27	1.2072
30	1.4898
100	133.0074
150	1021.2179

results follow the model mx^b, where $m = 1.755\cdot10^{-8}$ and $b = 5.03$ with an R^2 value of 1.

This confirms the hypothesis that runtime is $O(n^5)$ for small/medium n, but remains $O(n^6)$ for $n \to \infty$.

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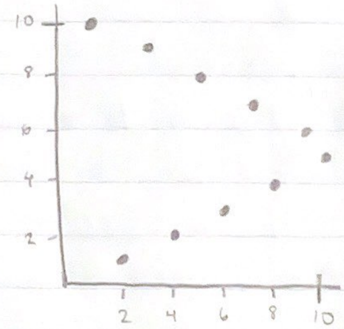
FINAL PROJECT SUBOPTIMAL SOLUTION

INSTANCE:



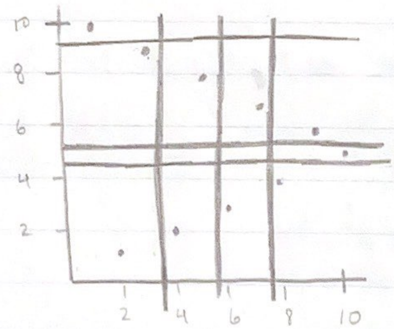
labeled as "instance00.txt" in project

ALGORITHM RUN:

1. start with all horizontal lines dividing between pts.
2. lines $y = 1.5$ and $y = 8.5$ are replaced with $x = 3.5$
3. lines $y = 2.5$ and $y = 7.5$ are replaced with $x = 5.5$
4. lines $y = 3.5$ and $y = 6.5$ are replaced with $x = 7.5$

Result:

6 lines total



OPTIMAL SOLUTION:

The intuitive solution would be to place a vertical divider after every 2 points, so that each region has 1 high point and 1 low point. Then, 1 horizontal line can be added to separate each of these pairs for a total of only 5 lines:

5 lines total

min possible, because 4 lines only creates 9 regions for 10 pts.