Two-Component Bivariate Signal Decomposition Based on Time-Frequency Analysis

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***Abstract*—A time-frequency analysis based approach for the decomposition of bivariate signals is presented. In particular, the well-known problem of two components overlapping in the time-frequency plane while having non-linear instantaneous frequencies is considered. The bivariate form of data leads to a significant modification of the Wigner distribution cross-terms. Therefore, the eigenvalue decomposition of Wigner distribution based signal autocorrelation matrix produces two significant eigenvalues instead of one in the common Wigner distribution. It is shown that the two corresponding eigenvectors can be linearly combined in order to produce fully separated signal components. The unknown coefficients are found by minimizing the time-frequency concentration measure of these particular eigenvectors linear combination. The presented approach is illustrated on the decomposition of a fast-varying real-valued signal with small instantaneous frequencies, so that its positive and negative frequency parts are so close that they degrade the analytical signal representation.**

***Index Terms*—bivariate signals; concentration measure; digital signal processing; time-frequency signal analysis**

I. Introduction

Time-frequency (TF) signal analysis is especially useful for signals with a time-varying spectral content [1]– [25], which therefore cannot be analyzed using the conventional Fourier analysis. A large number of time-frequency representations (TFR) has been proposed for the processing and characterization of univariate signals with a time-varying spectral content [6], [8], [9], with instantaneous frequency (IF) estimation being the central problem in this research field [7].

Multichannel signals, a form of multivariate data, arise routinely through recent sensor technology developments (for example, 3D inertial body sensors or 3D anemometers) [9], however, the processing of such signals is an ongoing research challenge [9]– [12], [17]– [21]. Recently developed concepts of modulated bivariate and multivariate data oscillations have opened the way to exploit multichannel signal interdependences, especially in time-frequency signal analysis [10], [11] [12]. In particular, the recently introduced concept of joint IF aims to characterize the multichannel data [10] and is defined as a weighted average of IFs in all individual channels, with the aim to capture their combined frequency characteristics. This concept stems from the multivariate oscillation model, which assumes one common oscillation that fits best all individual channel oscillations.

The multivariate signals IF estimation has been studied within the synchrosqueezed transform context in [9] as well as in the context of wavelet transform [10]. Empirical mode decomposition of multivariate data has also been recently considered [17]- [24].

The term multicomponent signals refers to signals that can described as a linear combination of independent signal components [1], [6]. In many applications, it is important to analyze every signal component independently, and therefore, decomposition of multi-component signals has attracted a significant research attention [1]– [6], [13], [14]. It has been shown that the decomposition of multi-component univariate signals can be performed using the S-method, under the condition that the components are not overlapping in the time-frequency plane [1]. However, in the case of overlapped components, the convenient condition of mutual orthogonality is violated, method presented in [1] cannot be applied and such cases are still subject of much research. For some specific cases, under the restricting assumptions of linearly or sinusoidally modulated components, such decomposition is possible [13]–[16]. However, in general, the univariate signals with components overlapped in the time-frequency plane cannot be decomposed. For multivariate signals, even with the multi-component signal decompositions using the EMD, this is possible only in cases of non-overlapped components.

The eigenvector decomposition of the univariate Wigner distribution (WD) autocorrelation matrix leads to one non-zero eigenvalue. As the S-method [1] of multicomponent signal having non-overlapped components can be expressed as the sum of WDs of individual components, this fact was used in the method presented in [1]. Decomposition of of overlapped non-linear components is still a challenging topic. In this paper, we demonstrate the possibility of the decomposition of two components in overlapped bivariate two-component signals, where [1] cannot be applied. Namely, in this case the autocorrelation matrix of the bivariate WD has exactly two significant eigenvalues. Each corresponding eigenvalue contains a linear combination of individual signal components. The idea is to apply the concentration measures on a time-frequency representation of eigenvector linear combination, in order to find coefficients producing the best possible concentration, corresponding to individual components.

The paper is organized as follows. Basic theory regarding Wigner distribution of bivariate signals and instantaneous frequency are presented Section II. The WD of two-component bivariate signals is analyzed in Section III. In Section IV we present the basic theory leading to the decomposition of bivariate multi-component signals. The theory is illustrated on a numerical example with real bivariate signal in Section V.

II. BACKGROUND THEORY

The bivariate signal of the form

$$\mathbf{x}(t) = \begin{bmatrix} a_1(t)e^{j\phi_1(t)} \\ a_2(t)e^{j\phi_2(t)} \end{bmatrix} \tag{1}$$

can be obtained by measuring the signal $x(t)$ by e.g. two sensors. It is assumed that each signal component modifies amplitude and phase of the measured signal.

The Wigner distribution of the bivariate signal $\mathbf{x}(t)$ has the following form

$$WD(\omega,t) = \int_{-\infty}^{\infty} \mathbf{x}^H(t-\frac{\tau}{2})\mathbf{x}(t+\frac{\tau}{2})e^{-j\omega\tau}d\tau$$
$$= \sum_i \int_{-\infty}^{\infty} a_i^*(t-\frac{\tau}{2})a_i(t+\frac{\tau}{2})e^{j(\phi_i(t+\frac{\tau}{2})-\phi_i(t-\frac{\tau}{2})}e^{-j\omega\tau}d\tau$$

where $\mathbf{x}^H(t)$ is the Hermitian transpose of vector $\mathbf{x}(t)$ and $i = 1,\ 2$.

Bivariate signals can be characterized by the joint instantaneous frequency. Namely, starting from the center of mass in the frequency direction of bivariate signal Wigner distribution

$$\langle\omega(t)\rangle = \frac{\int_{-\infty}^{\infty}\omega WD(\omega,t)d\omega}{\int_{-\infty}^{\infty}WD(\omega,t)d\omega} \tag{2}$$

or

$$\langle\omega(t)\rangle = \frac{1}{2j}\frac{[\mathbf{x}^H(t)\mathbf{x}'(t) - \mathbf{x}'^H(t)\mathbf{x}(t)]}{\mathbf{x}^H(t)\mathbf{x}(t)} \tag{3}$$

we easily obtain

$$\langle\omega(t)\rangle = \frac{\phi_1'(t)a_1^2(t) + \phi_2'(t)a_2^2(t)}{a_1^2(t)+a_2^2(t)}. \tag{4}$$

Consider a monocomponent bivariate signal, for which the components from different channels change in amplitude and phase as $a_i(t)\exp(j\phi_i(t)) = \alpha_i x(t)\exp(j\varphi_i)$. If signal $x(t)$ has the form $x(t) = A(t)\exp(j\psi(t)$ then $d\phi_i(t)/dt = d\psi(t)/dt$ and

$$\langle\omega(t)\rangle = \psi'(t).$$

III. TWO-COMPONENT BIVARIATE SIGNALS

Consider a two-component bivariate signal, defined as

$$\mathbf{x}(t) = \mathbf{x}_1(t) + \mathbf{x}_2(t)$$
$$= \begin{bmatrix} \alpha_{11}(t)x_1(t)e^{j\varphi_{11}} \\ \alpha_{12}(t)x_1(t)e^{j\varphi_{12}} \end{bmatrix} + \begin{bmatrix} \alpha_{21}(t)x_2(t)e^{j\varphi_{21}} \\ \alpha_{22}(t)x_2(t)e^{j\varphi_{22}} \end{bmatrix} \tag{5}$$

for which the components of the form $x_1(t) = A_1(t)e^{j\psi_1(t)}$ and $x_2(t) = A_2(t)e^{j\psi_2(t)}$ exhibit slow-varying amplitude changes compared to phase-changes, i.e. $|d\alpha_{ij}(t)/dt| \ll |d\psi_i(t)/dt|$ so that the amplitudes may be considered as constant within the analyzed time interval, $\alpha_{ij}(t) \sim \alpha_{ij}$.

The WD of the bivariate two-component signal in (5) is given by

$$WD(\omega,t) = \int_{-\infty}^{\infty} \mathbf{x}^H(t-\frac{\tau}{2})\mathbf{x}(t+\frac{\tau}{2})e^{-j\omega\tau}d\tau$$
$$= WD_a(t,\omega) + WD_c(t,\omega) \tag{6}$$

where $WD_a(t,\omega)$ represents the auto-terms

$$WD_a(\omega,t) = \int_{-\infty}^{\infty}[\alpha_{11}^2+\alpha_{12}^2]x_1^*(t-\frac{\tau}{2})x_1(t+\frac{\tau}{2})e^{-j\omega\tau}d\tau$$
$$+\int_{-\infty}^{\infty}[\alpha_{21}^2+\alpha_{22}^2]x_2^*(t-\frac{\tau}{2})x_2(t+\frac{\tau}{2})e^{-j\omega\tau}d\tau$$

whereas $WD_c(t,\omega)$ represents the cross-terms

$$WD_c(\omega,t) = \int_{-\infty}^{\infty}[\alpha_{11}\alpha_{21}x_1^*(t-\frac{\tau}{2})x_2(t+\frac{\tau}{2})e^{j(\varphi_{21}-\varphi_{11})}$$
$$+\alpha_{11}\alpha_{21}x_2^*(t-\frac{\tau}{2})x_1(t+\frac{\tau}{2})e^{j(\varphi_{11}-\varphi_{21})}$$
$$+\alpha_{12}\alpha_{22}x_1^*(t-\frac{\tau}{2})x_2(t+\frac{\tau}{2})e^{j(\varphi_{22}-\varphi_{21})}$$
$$+\alpha_{12}\alpha_{22}x_2^*(t-\frac{\tau}{2})x_1(t+\frac{\tau}{2})e^{j(\varphi_{12}-\varphi_{22})}]e^{-j\omega\tau}d\tau$$

Observe that phase shifts do not cancel out in the cross-terms $WD_c(t,\omega)$. The auto-terms are summed to be in phase, whereas the cross-term is formed as an off-phase summation, which leads to a significant change in these terms. Namely, for the univariate WD of a two-component signal, the undesirable cross-term is formed as the Fourier transform of $x_1^*(t-\frac{\tau}{2})x_2(t+\frac{\tau}{2})+x_2^*(t-\frac{\tau}{2})x_1(t+\frac{\tau}{2})$ having an oscillatory nature. However, for the bivariate WD, same phase shifted terms are summed at the same location in the time-frequency plane, but with different phases, thus averaging out.

IV. SIGNAL DECOMPOSITION

The inverse Wigner distribution has the following form

$$\mathbf{x}^H(t-\frac{\tau}{2})\mathbf{x}(t+\frac{\tau}{2}) = \frac{1}{2\pi}\int_{-\infty}^{\infty}WD(\omega,t)e^{j\omega\tau}d\omega. \tag{7}$$

After introducing the substitutions $t_1 = t+\tau/2$ and $t_2 = t-\tau/2$ we obtain

$$\mathbf{x}^H(t_2)\mathbf{x}(t_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty}WD\left(\frac{t_1+t_2}{2},\omega\right)e^{j\omega(t_1-t_2)}d\omega.$$

Assuming a proper discretization of the time and angular frequency axes, $t_1 = n_1\Delta t,\ t_2 = n_2\Delta t$ and $\omega = k\Delta\omega$ we obtain

$$\mathbf{x}^H(n_2)\mathbf{x}(n_1) =$$
$$= \tfrac{1}{K+1}\sum_{k=-K/2}^{K/2}WD\left(\tfrac{n_1+n_2}{2},k\right)e^{j\frac{\pi}{K+1}k(n_1-n_2)}. \tag{8}$$

Let us introduce the notation

$$R(n_1,n_2) = \mathbf{x}^H(n_2)\mathbf{x}(n_1). \tag{9}$$

Under the assumption that the cross-terms can be neglected, the inversion of WD for two-component bivariate signals produces a matrix with the elements of the form

$$\begin{aligned} R(n_1,n_2) = {} & [\alpha_{11}^2+\alpha_{12}^2]x_1(n_1)x_1^*(n_2) \\ & + [\alpha_{12}^2+\alpha_{22}^2]x_2(n_1)x_2^*(n_2). \end{aligned} \tag{10}$$

The eigenvalue decomposition of a square matrix $\mathbf{R}$ of dimensions $K \times K$ is given by

$$\mathbf{R} = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^T = \sum_{p=1}^{K} \lambda_p \mathbf{q}_p(n)\mathbf{q}_p^H(n), \tag{11}$$

where λ_p are eigenvalues and $\mathbf{q}_p(n)$ are eigenvectors of $\mathbf{R}$. Note that the eigenvectors $\mathbf{q}_p(n)$ are orthonormal.

For a two-component signal, in a noiseless case, the elements of this matrix are

$$R(n_1,n_2) = \lambda_1 q_1(n_1)q_1^*(n_2) + \lambda_2 q_2(n_1)q_2^*(n_2). \tag{12}$$

Note that although the eigenvectors are mutually orthogonal, the overlapped signal components are not orthogonal. Therefore, both eigenvectors $\mathbf{q}_1$ and $\mathbf{q}_2$ contain a linear combination of signal components, i.e.

$$\mathbf{q}_1 = \gamma_{11}\mathbf{x}_1 + \gamma_{12}\mathbf{x}_2, \tag{13}$$

$$\mathbf{q}_2 = \gamma_{21}\mathbf{x}_1 + \gamma_{22}\mathbf{x}_2, \tag{14}$$

with unknown coefficients $\gamma_{i,j}$. This means that each component can be expressed as a linear combination of eigenvectors $\mathbf{q}_1$ and $\mathbf{q}_2$:

$$\mathbf{x}_1 = \nu_{11}\mathbf{q}_1 + \nu_{12}\mathbf{q}_2, \tag{15}$$

$$\mathbf{x}_2 = \nu_{21}\mathbf{q}_1 + \nu_{22}\mathbf{q}_2. \tag{16}$$

Notice that individual signal components are better concentrated (more sparse) in the time-frequency plane than their linear combinations contained within eigenvectors $\mathbf{q}_1$ and $\mathbf{q}_2$. Therefore, it is natural to search for the unknown coefficients $\nu_{ij},\ i,j = 1,2$ which produce the best possible individual component concentrations (sparsities). This search procedure can be described as follows.

First, the signal

$$\mathbf{y} = \nu_{11}\mathbf{q}_1 + \nu_{12}\mathbf{q}_2, \tag{17}$$

is formed, then we fix $\nu_{11} = 1$ and vary the real and imaginary part of ν_{12} until the best possible concentration of (17) is found. To this end, the concentration (sparsity) measure

$$\mathcal{M}[TFR_{\mathbf{y}}(n,k)] = \sum_n \sum_k |TFR_{\mathbf{y}}(n,k)| \tag{18}$$

is exploited, where the underlying $TFR_{\mathbf{y}}(n,k)$ is calculated for the normalized signal $\mathbf{y}/\|\mathbf{y}\|_2$ and can be any time-frequency distribution, including the spectrogram, S-method with narrow frequency window, WD etc. As these representations are quadratic, the measure (18) corresponds to ℓ_1-norm, which has recently been intensively used as the signal sparsity measure.

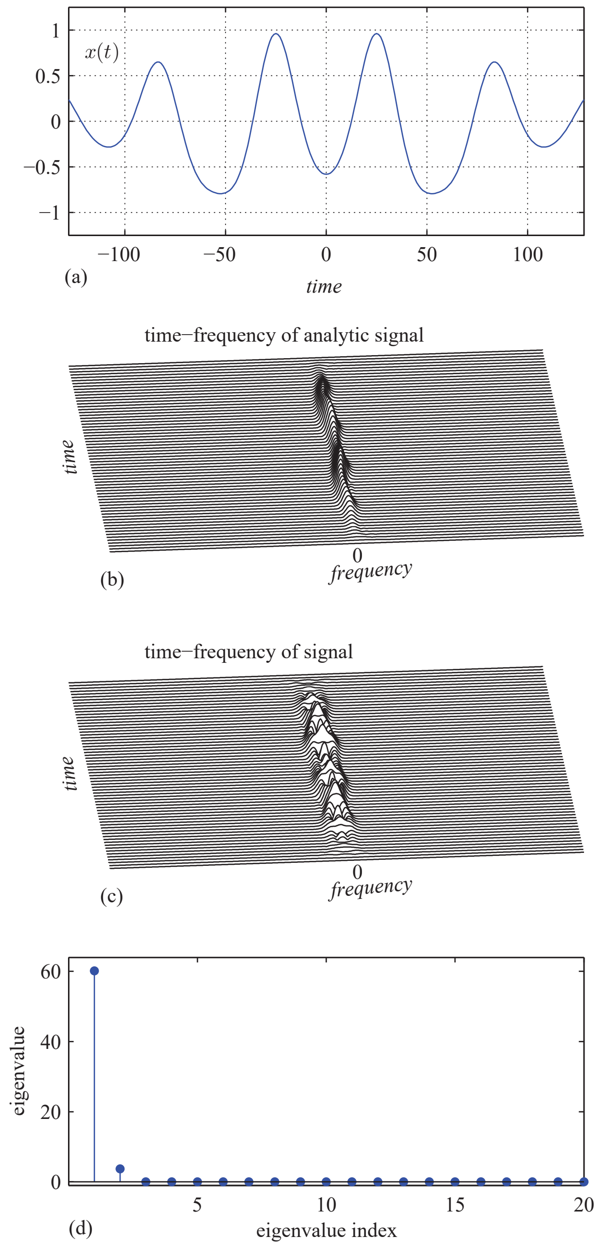


Fig. 1. The bivariate real signal under consideration. (a) time domain waveform; (b) PWD of the corresponding analytic signal; (c) PWD of the original signal; (d) eigenvalues of the autocorrelation matrix $\mathbf{R}$.

The first set of coefficients is obtained as the solution of

$$\min_{\nu_{12}} \mathcal{M}\left\{TFR_{\mathbf{y}}(n,k)\right\} \qquad \text{subject to } \nu_{11} = 1 \tag{19}$$

where a direct search can be applied. The linear combination in (17) with the so obtained coefficients ν_{11} and ν_{12} produces the first signal component.

Upon replacing the eigenvector $\mathbf{q}_1$ with the detected component, $\hat{\mathbf{q}}_1$, if $\nu_{12} \neq 0$ then the orthogonal projection of the detected component is removed from eigenvector $\mathbf{q}_2$, as

$$\hat{\mathbf{q}}_2 = \frac{1}{\sqrt{1-\mathbf{q}_1^H\mathbf{q}_2}}(\mathbf{q}_2 - \mathbf{q}_1^H\mathbf{q}_2\mathbf{q}_1), \tag{20}$$

to ensure that it is not detected again.

Subsequently, the same procedure is repeated for the calculation of the second set of coefficients. In other words, the second component is obtained as the linear combination of eigenvectors with coefficients being the solution of:

$$\min_{\nu_{21}} \mathcal{M}\left\{TFR_{\mathbf{y}}(n,k)\right\} \qquad \text{subject to } \nu_{22}=1, \tag{21}$$

where the TFR is calculated for the new linear combination

$$\mathbf{y}=\nu_{21}\hat{\mathbf{q}}_1+\nu_{22}\hat{\mathbf{q}}_2 \tag{22}$$

with the normalization $\mathbf{y}/\|\mathbf{y}\|_2$, where the first eigenvector is equal to the previously detected signal component, whereas the component's orthogonal projection is removed from the second eigenvector. It is important to note that both (19) and (21) are easily found using a direct search over the unknown parameter values.

V. Numerical example

Consider a real bivariate signal $\mathbf{x}(t) = [x_1(t)\ x_2(t)]^T$, where the signal from channel i has the form

$$\begin{aligned} x_i(t) &= e^{-(t/128)^2}\cos\left(\cos((300/64)\pi t/256)+\varphi_i\right) \\ &= 0.5e^{-(t/128)^2}\left[e^{j(\cos((300/64)\pi t/256)+\varphi_i)}+\right. \\ &\qquad \left.+\, e^{-j(\cos((300/64)\pi t/256)+\varphi_i)}\right] \\ &= x_{1i}(t)+x_{2i}(t),\ i=1,\ 2, \end{aligned} \tag{23}$$

for $-128 \le t \le 128$. The phases $\varphi_1 \neq \varphi_2$ are drawn from a uniform distribution, from the interval $[0,\ 2\pi]$. The signal in time domain is shown in Fig. 1 (a). Since the considered signal is real-valued, two symmetric components $x_{1i}(t)$ and $x_{2i}(t),\ i=1,\ 2$ exist in the time-frequency plane.

The pseudo-WD (PWD) of the considered signal is shown in Fig. 1 (c), for the first channel $i=1$. Observe that the two components $x_{1i}(t)$ and $x_{2i}(t)$ overlap in the TF plane. A common approach in the TF analysis of real-valued signals is to calculate their Hilbert transform prior to the calculation of the TFR.

However, as shown in Fig. 1 (b), since the components are very close (and close to zero frequency), the analytic signal does not provide a meaningful TF representation. Since these components are also nonlinear, none of the known techniques can be applied for their separation in order to, for example, estimate the instantaneous frequency of the considered signal. Moreover, due to the non-orthogonality of the overlapped components, the S-method based decomposition [1] cannot be applied in a straight-forward manner either.

Since the analyzed signal is bivariate, its WD has significantly reduced cross-terms. The WD autocorrelation matrix contains exactly two non-zero eigenvalues, as shown in Fig. 1 (d). The two corresponding eigenvectors contain linear combinations of two signal components, and their PWDs are shown in Fig. 2 (a) and (b). Using the proposed decomposition method, both components have been successfully extracted, as shown in Fig. 3 (a) and (b).

The instantaneous frequency (IF) estimation is a common problem in the TF signal analysis. The exact IF of the considered signal is shown in Fig. 4 (black line). For real signals, it is usual to calculate the analytic form based on the Hilbert transform in order to perform the TF-based IF estimation. However, in the case of the considered signal, the estimation based on the analytic signal WD (Fig. 1 (b)) is not accurate, as can be seen in Fig. 4 (red line). Namely, the IF estimation based on the standard TFR-maxima approach does not appropriately track the IF variations, as they are lost in the corresponding TFR due to significant overlapping of the components and the fact that amplitude and phase variations are of the same order.

On the other side, it can be observed that the nonnegative IF estimation based on the pseudo-WDs of two extracted components, shown by green and blue dots in Fig. 4, is accurate, up to the theoretically expected bias caused by the IF non-linearity, which can be further reduced using some well-known IF estimation techniques [6].

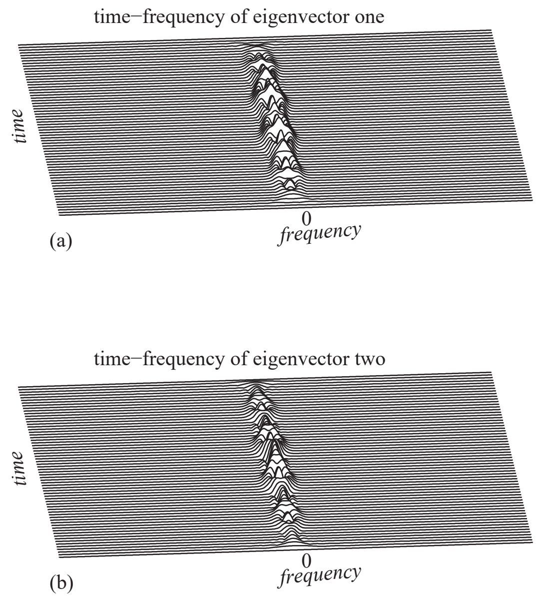
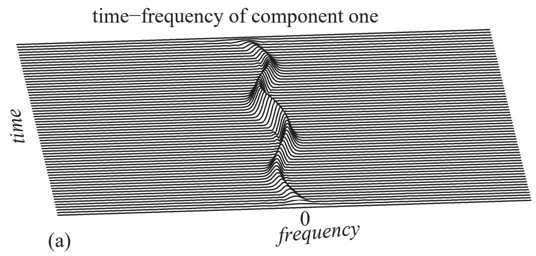


Fig. 2. Pseudo-WD of first two eigenvectors of matrix $\mathbf{R}$.

VI. Conclusion

We have considered the decomposition of two-component bivariate signals. The autocorrelation matrix of bivariate Wigner distribution is a subject of eigenvalue decomposition, leading to an eigenvector linear combination that produces separated signal components. The coefficients of this linear combination are found by minimizing the time-frequency concentration (sparsity) measure of the eigenvector linear combination. It can be concluded that two overlapped signal components can be decomposed in the bivariate case. The generalization of the concept to multivariate signals with an arbitrary number of components is a subject of our separate research paper.



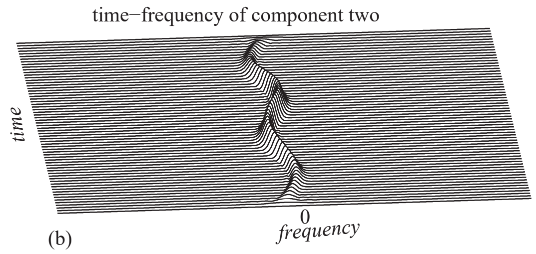


Fig. 3. Pseudo-WD of components extracted using the proposed approach.

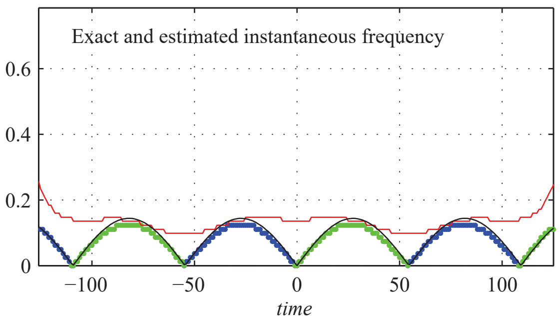


Fig. 4. Estimation of the IF: True IF (black line); The IF estimation using the analytic signal (red line); The nonnegative IF estimation based on components extracted using the proposed approach (green and blue line).

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