PSYC 529 Written Homework 1

Due January 20th, 2025

Name: _____

Use a pencil (not a pen). Write neatly in the space provided after each problem. If you need more space, use the back of the page. Answers to a few of the problems are available so you can check your work. Unless otherwise specified you may use a calculator or use R as a calculator.

1. Some of the following probability statements can be interpreted as frequency probabilities, while the rest can only be interpreted as belief (Bayesian) probabilities. Which are which? **Hint:** frequency probability can only describe outcomes of a *repeatable action*, such as flipping a coin.
   (a) (2 points) $P(\text{it rains on days in March}) = 0.30$
   (b) (2 points) $P(\text{it will rain on 3/13/2025}) = 0.25$
   (c) (2 points) $P(\text{the therapy's success rate is between 60\% and 85\%}) = 0.95$
   (d) (2 points) $P(\text{the therapy is successful}) = 0.73$ (i.e. the success rate is 73%)
   (e) (2 points) $P(\text{Cliff's football predictions are correct}) = 0.51$

2. Let's say you roll a fair six-sided die 200 times.
   (a) (5 points) About how many times out of 200 would you expect to roll a five, six, or seven?

   (b) (5 points) About how many times out of 200 would you expect to roll an even number or a five?

3. Dana Scully and Fox Mulder are FBI agents who investigate unusual cases (they are the main characters in the X-Files). Mulder believes in paranormal phenomena (e.g. psychic powers, ghosts, UFOs), but Scully is very skeptical. In their most recent case, Mulder and Scully are investigating the disappearance of a Yale graduate student. Someone suggests the possibility that the student was abducted by aliens.

   (a) (5 points) Which of the two agents do you think has a higher subjective (belief) probability in the alien theory, Mulder or Scully?

   (b) (5 points) One of Scully and Mulder's colleagues doesn't think the culprit is an alien, and is willing to bet $20 on it. Who is more likely to take that bet, Scully or Mulder? Explain your answer.

   (c) (5 points) The beliefs of Langly (one of Mulder's friends) in the alien theory can expressed as the following probability: $P(\text{it was aliens}) = 0.35$.

   For Langly, what is $P(\text{it was not aliens})$, i.e. the probability that the perpetrator is from Earth?

   (d) (5 points) While investigating the missing student's apartment, Scully finds a mysterious device that does not resemble any Earth technology. Should this new evidence increase or decrease Scully's $P(\text{it was aliens})$? Answer based on your intuition; we will learn the relevant math (Bayes' Rule) later.

4. A house plant has been destroyed by a cat! There are three suspects in this case of herbicide: Ana, Oskar, and Cliff. For the sake of simplicity, we will assume that exactly one cat is guilty and thus the other two cats are innocent. You haven't observed any evidence in the case yet, so all you have to go on is your assessment of each cat's moral character, expressed as a belief probability. Later we will learn how to use forensic evidence (e.g. fur left at the crime scene) to update our beliefs using Bayes' Rule. You know that Oskar is a scoundrel, so: $P(\text{Oskar is guilty}) = 0.7$. Ana is mostly well-behaved, so $P(\text{Ana is guilty}) = 0.1$.

   Compute the following belief probabilities (show your work):

   (a) (2½ points) $P(\text{Cliff is guilty})$

   (b) (2½ points) $P(\text{Oskar is innocent})$, i.e. not guilty

   (c) (2½ points) $P(\text{Ana is innocent})$

   (d) (2½ points) $P(\text{Oskar or Cliff is guilty})$

5. Here is a table with some numbers in it.

x
4
9
1
100

Compute the following, referring to this table (feel free to use a calculator, or to use R as a calculator):

(a) (5 points) $\sum_{i=1}^{4} x_i$

(b) (5 points) $\sum_{i=3}^{4} x_i$

(c) (5 points) $\sum_{i=1}^{4} \sqrt{x_i}$

6. Compute the following, referring to the table in the problem above:

(a) (5 points) $\prod_{i=1}^{4} x_i$

(b) (5 points) $\prod_{i=3}^{4} x_i$

(c) (5 points) $\prod_{i=1}^{4} \sqrt{x_i}$

7. Simplify the following expressions involving exponents. **DO NOT USE A CALCULATOR**: I do not care about getting a final numeric result (e.g. I don't care that $2^3 = 8$). Here are some examples of the kind of answers I want (the left hand side is the problem and the right hand side is the solution):

   - $8^2 \cdot 8 = 8^3$
   - $d^4 d^2 e^9 e = d^6 e^{10}$

   Expressions to simplify:

   (a) (2 points) $2 \cdot 2 \cdot 2 \cdot 2$

   (b) (2 points) $(0.3)^0 (0.7)^{1000}$

   (c) (2 points) $0.4 \cdot (0.4)^9$

   (d) (2 points) $2^3 \cdot 2^2$

   (e) (2 points) $12^2 \cdot 12 \cdot (100)^0$

   (f) (2 points) aa

   (g) (2 points) $aabccc$

   (h) (2 points) $z^4 z^5$

   (i) (2 points) $pppp(1-p)(1-p)$

   (j) (2 points) $p(1-p)pp(1-p)p$

Answers to Selected Problems

1-d) The evidence should increase Scully's P(it was aliens), i.e. Scully should believe in the alien theory at least a little bit more. Later we will learn about Bayes' Rule, which explains the relevant math.

2-a) $P(5 \text{ or } 6 \text{ or } 7) = P(5) + P(6) + P(7) = \frac{1}{6} + \frac{1}{6} + 0 = \frac{2}{6} = \frac{1}{3} \approx 0.33$ number of times $\approx 0.33 \cdot 200 = 66$

4-a) $P(\text{Cliff is guilty}) = 1 - P(\text{Oskar or Ana is guilty}) = 1 - P(\text{Oskar is guilty}) - P(\text{Ana is guilty}) = 1 - 0.7 - 0.1 = 0.2$

5-c) $\sum_{i=1}^{4} \sqrt{x_i} = 2 + 3 + 1 + 10 = 16$

7-b) $(0.3)^0(0.7)^{1000} = 1 \cdot (0.7)^{1000} = (0.7)^{1000}$

7-f) $aa = a^2$