SOFR Curve Construction

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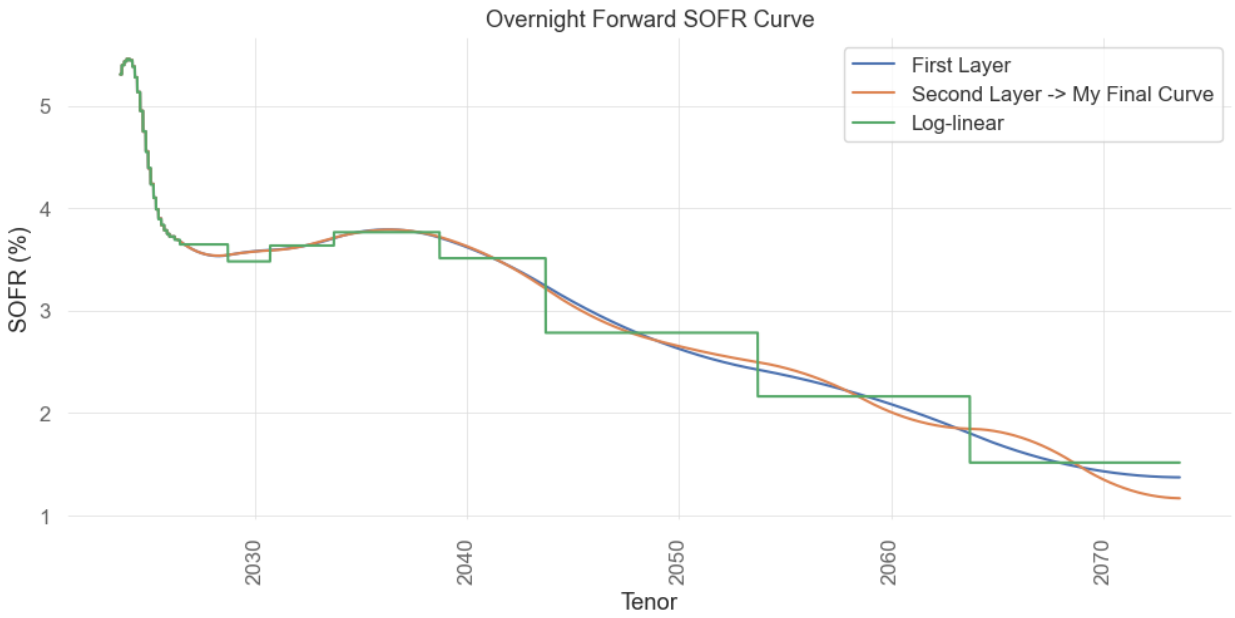
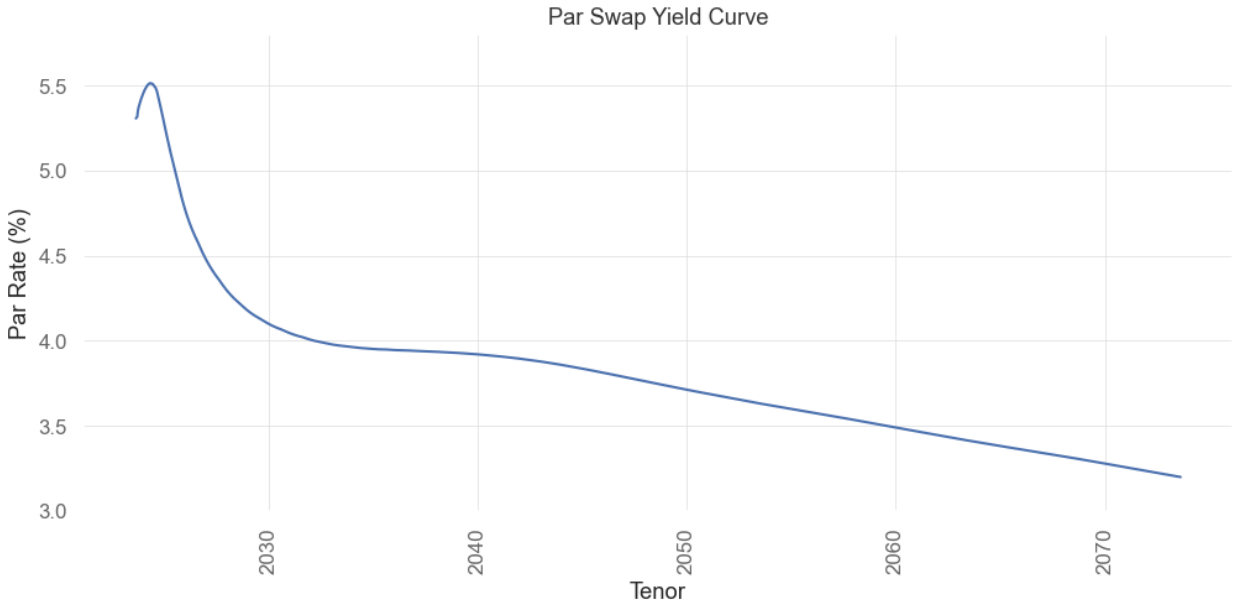
I. SOFR Curve Construction

I have built the SOFR curve as of closing 8/25/2023. The accuracy to Bloomberg benchmark is always within 0.02bp. The library is written in Python, using datasets from Bloomberg.

See below the overnight forward curve and the par swap yield curve.





1. Assumptions

My focus in on the overnight forward curve backboned upon SOFR Index, which can easily transfer to par swap rate curve. The initial date is set to be 8/25/2023, settlement is 8/27/2023.

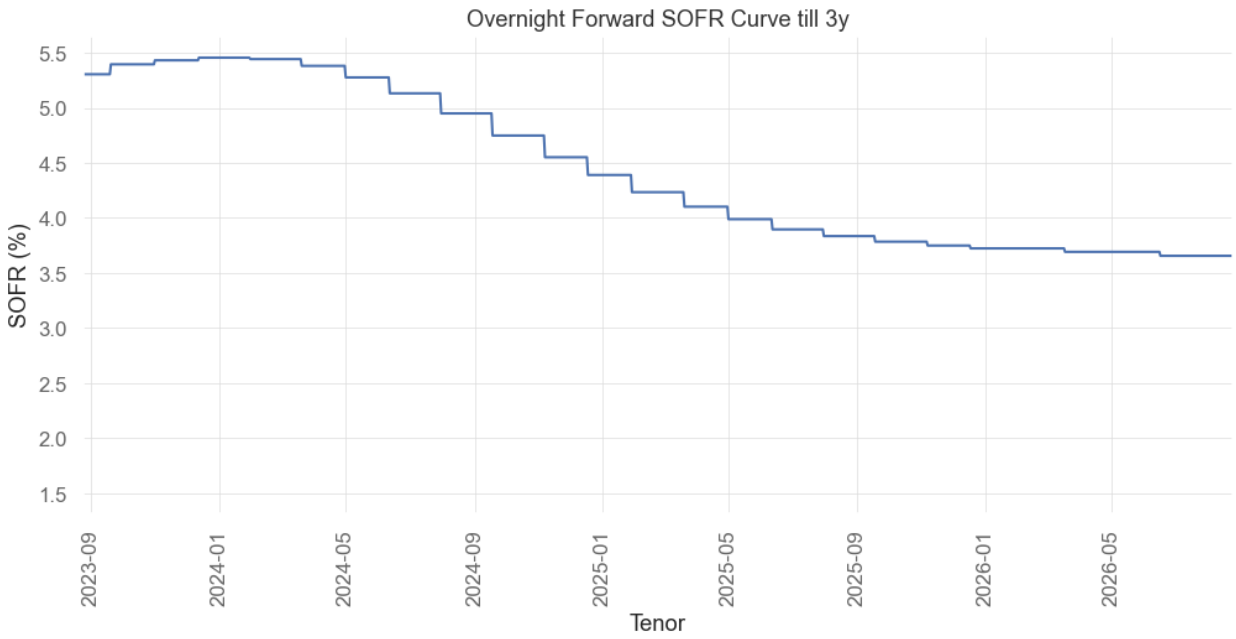
- Using the SR3 futures in the front end (<3yrs), adjusted by convexity.
- Using Swap rates for 3+ tenor, first layer is 5y, 7y, 10y, 15y, 20y, 30y, 40y and 50y. and implemented the skewness adjustment for 4y, 6y, 8y, 9y, 12y, 25y, 35y and 45y, which are the second layer. In practice, I would implement 3 layers:
    - First layer: Cash yield on (5y, 10y, 30y) + corresponding swap spreads
    - Second layer: 4y, 7y, 15y, 20y
    - Third layer: 6y, 8y, 9y, 11y, 12y, 25y, 40y, 50y
- Using Log-linear interpolation on DF for front end, and log-cubic spline interpolation for 3+ tenor.
- The curve is solved by auto-differentiation under Dual number. Levenberg-Marquardt method is used to blend both the gradient descent and Gauss-Newton method.
- No turns, no holidays and no business day convention for simplicity.
- Knots are placed at strategic points as IMM dates, rather than at the maturity of par tenor. instruments. This is to avoid large exchange basis and carry fluctuation when market advanced.

2. Market Instruments

Interpolation	Node Dates		Market Instruments	
Log-linear	Initial Date	8/25/2023	Partial Jun'23 Futures	SFRM3
	FOMC Meetings	9/20/2023	12 IMM Futures	SFRU3
		11/1/2023		SFRZ3
		12/13/2023		SFRH4
	Provisonal FOMC Meetings	1/31/2024		SFRM4
		3/20/2024		SFRU4
		5/1/2024		SFRZ4
		6/12/2024		SFRH5
		7/31/2024		SFRM5
		9/18/2024		SFRU5
		11/7/2024		SFRZ5
		12/18/2024		SFRH6
	Estimated FOMC Meetings	1/29/2025		SFRM6
		3/20/2025	Par IRS	4Y
		5/1/2025		5Y
		6/12/2025		7Y
		7/31/2025		10Y
		9/18/2025		15Y
		11/7/2025		20Y
		12/18/2025		30Y
Logic-cubic	Strategic IMM Dates	3/18/2026		40Y
		6/17/2026		50Y
	Long term IMM dates	9/15/2027		
		9/20/2028		
		9/18/2030		
		9/21/2033		
		9/15/2038		
		9/16/2043		
		9/17/2053		
		9/19/2063		
		9/20/2073		

3. Front End Curve

Here we zoomed in to have a better view on the front end curve. For up to 3y, we have



The nodes are placed at FOMC meeting dates.

Note that the future convexity adjustment is based on the following 2 papers.

- Hull. 2002. Options, Future and Other Derivatives p. 566.
- Piterbarg and Renedo. 2006. Eurodollar Futures Convexity Adjustments in Stochastic Volalitiy Model. 2006

The mean reversion speed is set to be 0.03 and rate vol is set to be 140bps. The closing for SFRM5 is 96.075, so if you look at *SFRM5C 96.0000 COMB Comdty,* this serial option has implied volatility at 37.188% which at current rates (3.925%) is around 140bps.

Start Date	Days	Ticker	Last	Implied Forward
08/30/2023	21	SFRM3	94.6847	5.3153
09/20/2023	91	SFRU3	94.545	5.4543
12/20/2023	91	SFRZ3	94.51	5.4868
03/20/2024	91	SFRH4	94.645	5.3482
06/19/2024	91	SFRM4	94.925	5.0634
09/18/2024	91	SFRU4	95.295	4.6874
12/18/2024	91	SFRZ4	95.645	4.3304
03/19/2025	91	SFRH5	95.91	4.0572
06/18/2025	91	SFRM5	96.075	3.883
09/17/2025	91	SFRU5	96.16	3.7877
12/17/2025	91	SFRZ5	96.195	3.7414
03/18/2026	91	SFRH6	96.215	3.7092
06/17/2026	91	SFRM6	96.235	3.6759

4. 3y+ Curve Residual control

In the daily trading activities, we need to mark level 2 and level 3 residuals according to broker's screen. I have also implemented this in my curve, which gives me control to skew the curve on 4y, 6y, 8y, 9y, 12y, 25y, 35y and 45y. this is implemented by having a layered curve set.

Input Instruments	First Layer Rate (bps)	Second Layer Skew (bps)
4y		*0*
5y	421.5	
6y		*0*
7y	406.3	
8y		*-0.05*
9y		*-0.05*
10y	396.75	
12y		*-0.02*
15y	393.125	
20y	386.525	
25y		*-0.5*
30y	362.7	
35y		*0.4*
40y	340.845	
45y		*0.7*
50y	319.735	

By implement the skew, our final curve (i.e., the 2nd layer below) have less than 0.02bps difference to Bloomberg's closing yield.