WS11: Smoothed Particle Hydrodynamics

Jackie Villadsen

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1 Test 7: Uniform Shear

Figures 1 through 1 show the evolution of flow in a tube due to intrinsic shear as the particles try to reorganize themselves into a regular grid due to kernel smoothing (see second section for explanation of this process). Initially (Figure 1), the tube contains particles with well-ordered motions: all particles in the center of the tube move in the negative direction with the same velocity, and a velocity gradient leads outward to the edges, where all the particles move in the positive direction with the same velocity. However, after a few time steps, the particles have moved out of their initial well-ordered grid, and the effect of the smoothing kernels is to attempt to restore the particles to a regular grid. This leads to the particles being pushed and pulled around attempting to form a grid, which is always disrupted by further motion. This leads to a spread in the velocities (Figure 1). As this process continues, some of the random motions along the length of the tube are converted into random transverse motions, resulting in a reduction in the total energy in the ordered flow along the tube, and so a reduction in the amplitude of the velocities (Figure 1).

2 Test 3.1: 2D Shock Tube

In hydrodynamic simulations, if the particles are in a regular (lattice) configuration, then the error on the computed gradient will be much lower than if the particles are in a purely random configuration. Thus, in SPH it is desirable to have a way for the particles to "remesh" themselves onto a regular grid, even if they are initially in a random configuration (which is more reflective of their physical state). This remeshing can be introduced artificially, or it can be achieved by using the Hamiltonian formulation to calculate the forces between particles. The Hamiltonian formulation introduces a derivative of the kernel (particle smoothing) function, which is always negative, so the net result is an effective repulsive force between particles. These forces cancel out only when the particles are on a regular grid, with a hexagonal grid being the lowest "energy" state.

Figure 2 shows this process of remeshing occurring after a shock front. The shock front randomizes the particle motions, and it takes quite a large distance

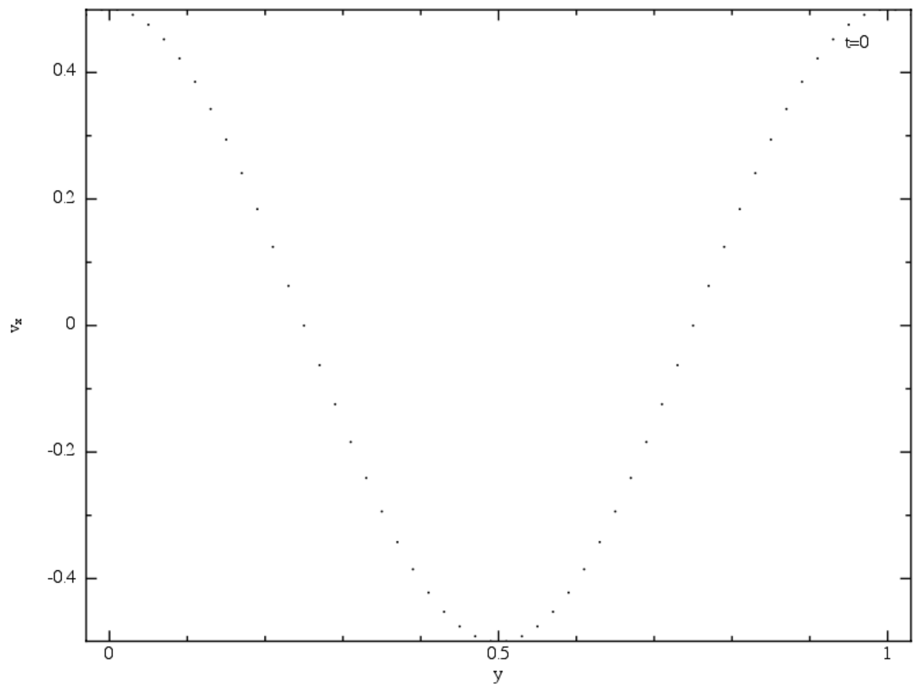

Figure 1: Cross-section of initial velocity in a tube, where the velocities are in opposite directions in the center of the tube and on the sides.

for the particles to get pushed into a regular lattice. The result is that there is "noise" in the velocity immediately after the shock, which smooths out later. This noise is not physical but rather a result of the remeshing process.

Figure 2 shows the same simulation repeated with the quintic spline kernel, which is smoother than the cubic spline kernel (which has discontinuous 3rd derivatives) and extends to a larger radius, and so remeshes more quickly.

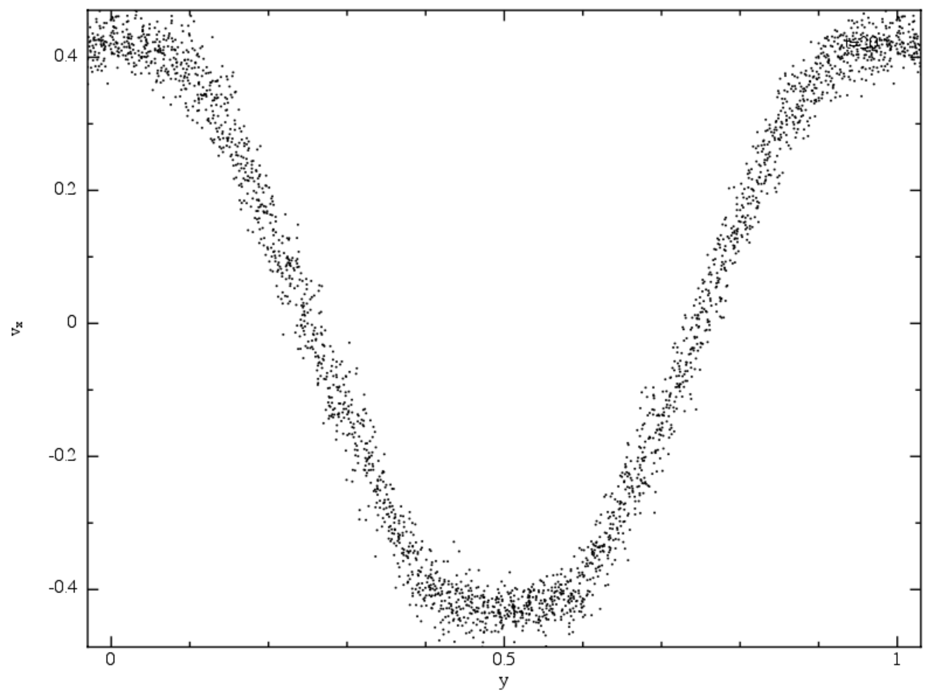

Figure 2: Cross-section of velocity in the same tube, once shear has started to cause a spread in the velocities of the particles.

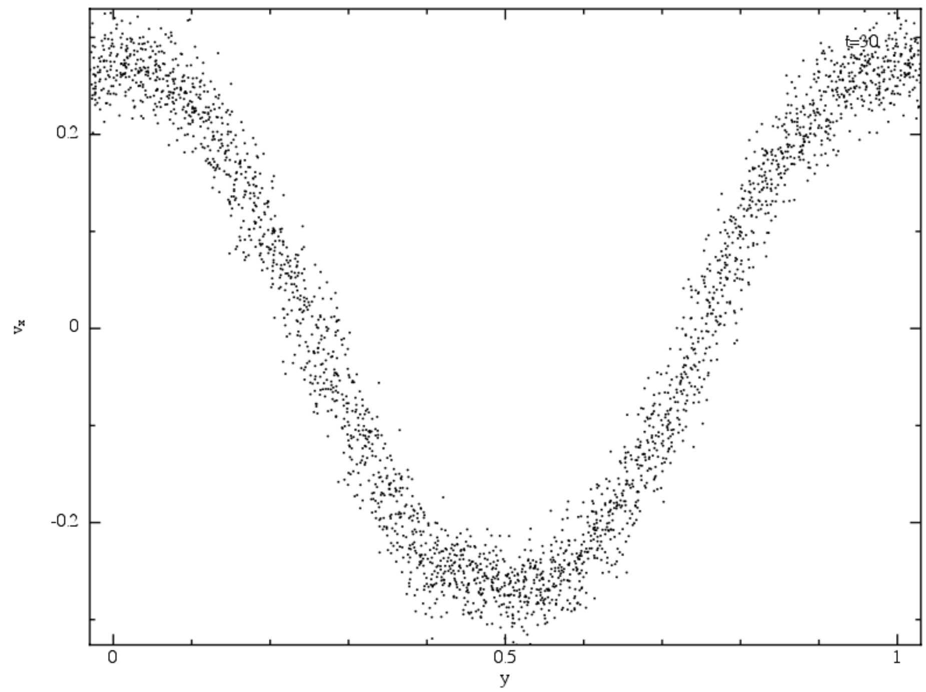

Figure 3: Cross-section of velocity in the same tube, after the effects of shear have been felt long enough that the energy in ordered velocity has been partially dissipated, reducing the amplitude of the maximum velocity.

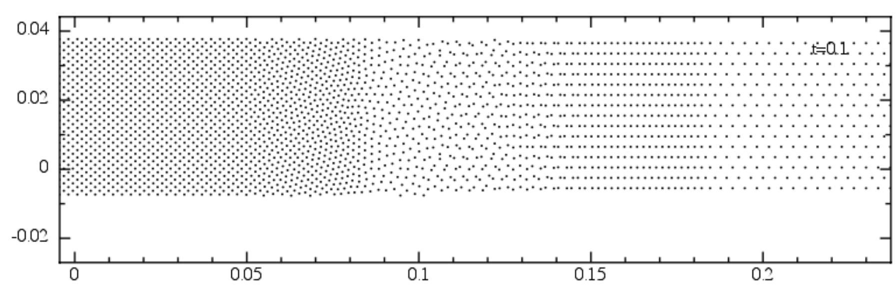

Figure 4: 2D Sod shock tube using cubic spline kernel.

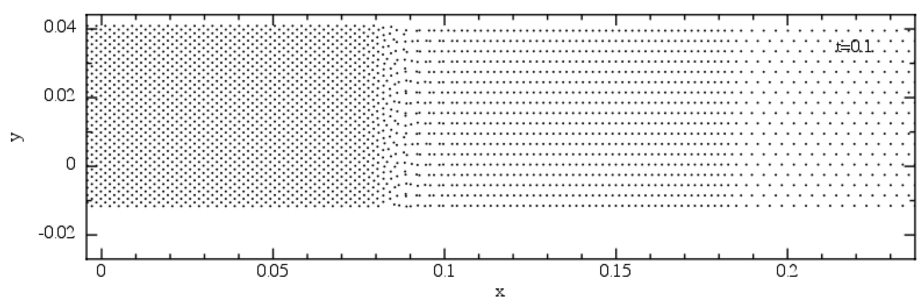

Figure 5: 2D Sod shock tube using quintic spline kernel.