LibKet: A Software Framework for Quantum-Accelerated Scientific Computing

# Hands-on Introduction to Quantum Computing with LibKet

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**Matthias Möller[1] and Carmen G. Almudever[2]**

[1]Delft University of Technology (m.moller@tudeflt.nl)

[2]Technical University of Valencia (cargara2@disca.upv.es)

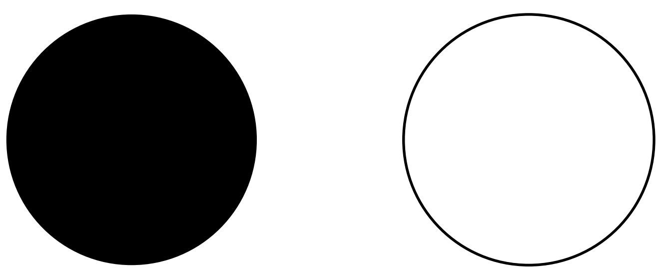
# What is different in QC?

## Basic unit of information

**Bit**

Exclusive state

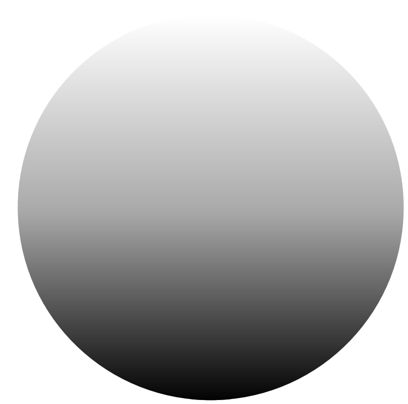
0 1



**Quantum bit (Qubit)**

$|0\rangle$ and $|1\rangle$

Superposition



$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$$

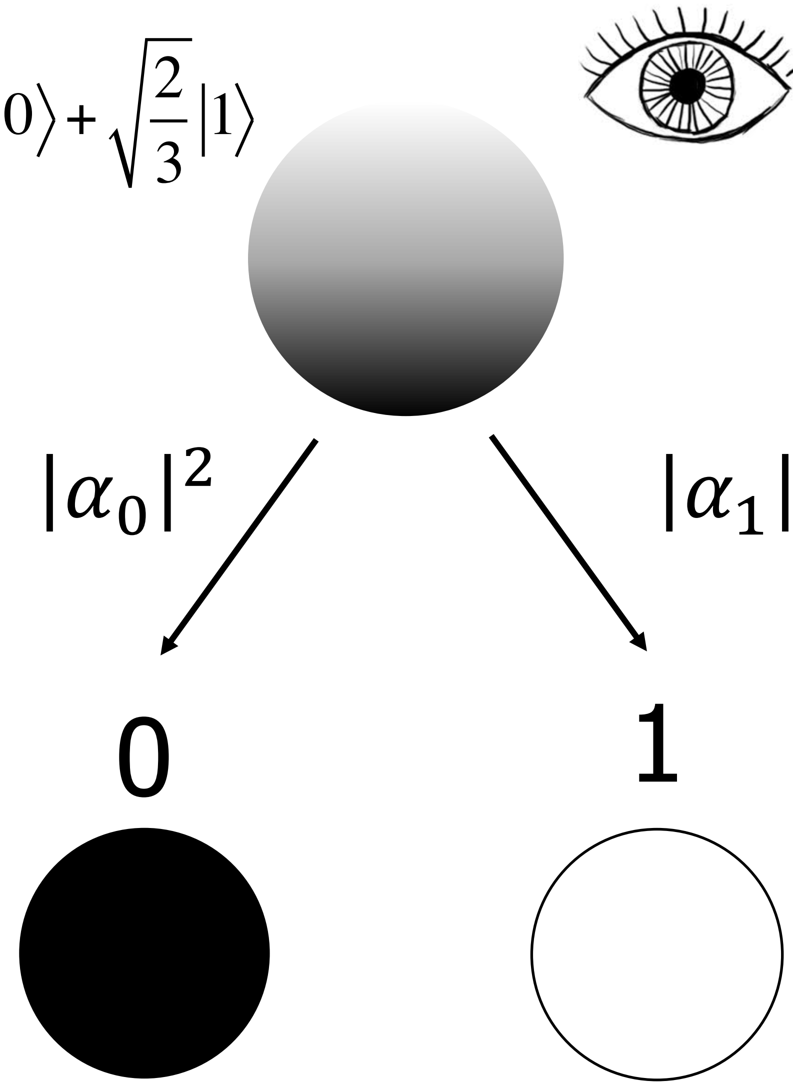
# What is different in QC?

## Reading out information

Memory

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 |

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$$




$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

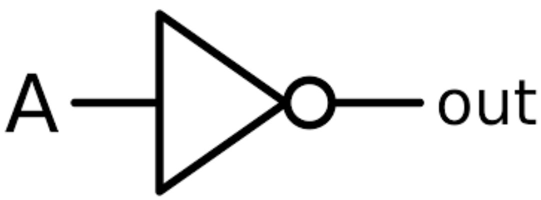
Probabilistic process

# What is different in QC?

## Operations

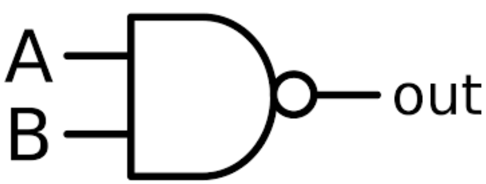
### Classical gates

**NOT**




| A | out |
|---|---|
| 0 | 1 |
| 1 | 0 |

**NAND**

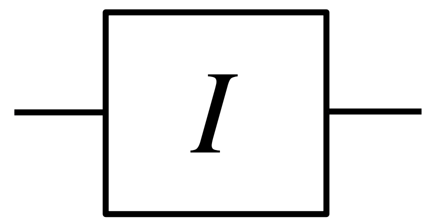



| A | B | out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

- Logical operation
- Truth tables
- Most of them only run forward

### Quantum gates

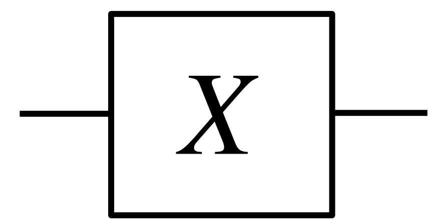
**Identity**




$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|0\rangle \rightarrow |0\rangle$$
$$|1\rangle \rightarrow |1\rangle$$

**Pauli X**




$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

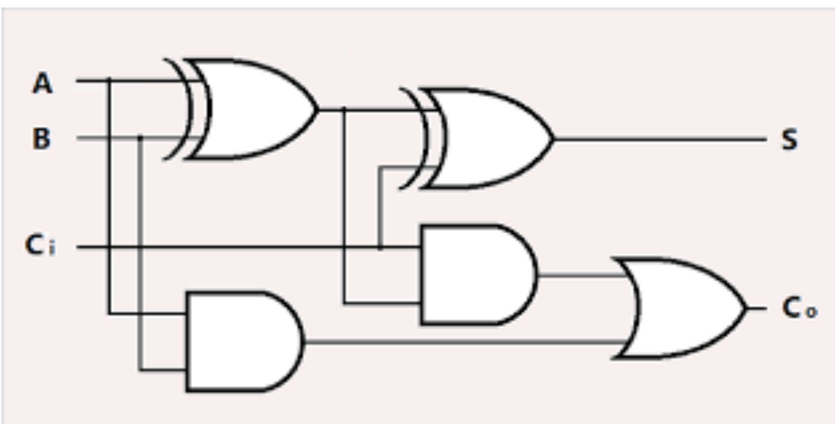
$$|0\rangle \rightarrow |1\rangle$$
$$|1\rangle \rightarrow |0\rangle$$

- Unitary operation
- Unitary matrix
- All quantum gates are reversible

# What is different in QC?

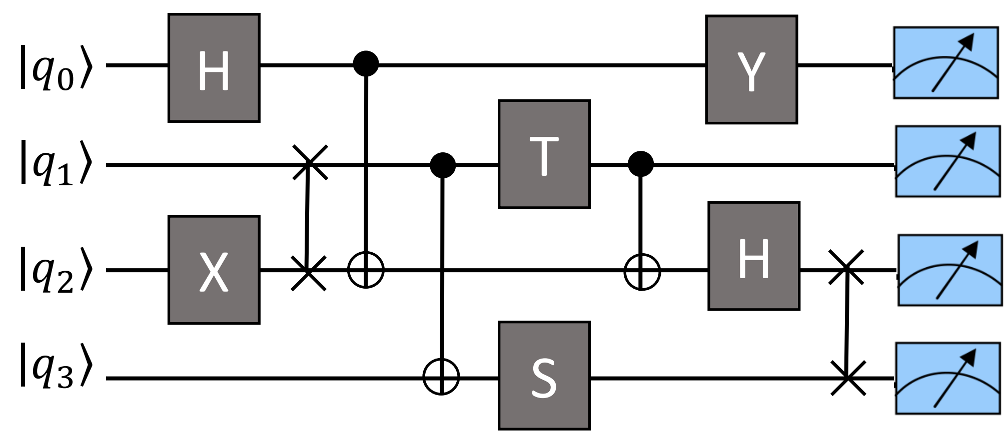
## Computation

### Classical circuits




### Quantum circuits

Circuit model of computation




- Single-qubit gates: H, X, T, S, Y
- Two-qubit gates: CNOT and SWAP

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# What is different in QC?

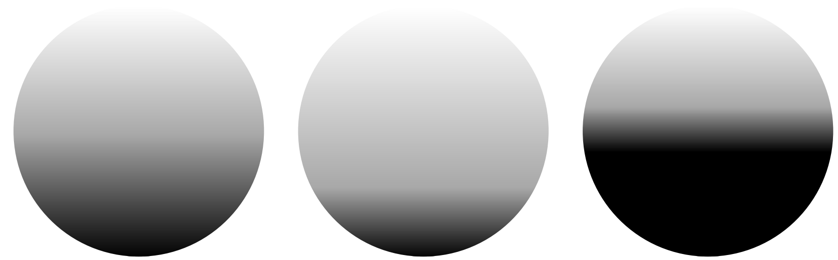
3 bits

000 **or** 001 **or** 010 **or** 011 **or**

100 **or** 101 **or** 110 **or** 111

- n bits hold 1 value: from 0 to $2^n$-1

3 qubits



$$\alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \alpha_{010}|010\rangle + \alpha_{011}|011\rangle$$
$$+\alpha_{100}|100\rangle + \alpha_{101}|101\rangle + \alpha_{110}|110\rangle + \alpha_{111}|111\rangle$$

000 **and** 001 **and** 010 **and** 011 **and** 100 **and** 101 **and** 110 **and** 111

- n qubits can hold $2^n$ values (50 qubits, $2^{50}$ complex amplitudes)
- All states (amplitudes) can be manipulated at the same time

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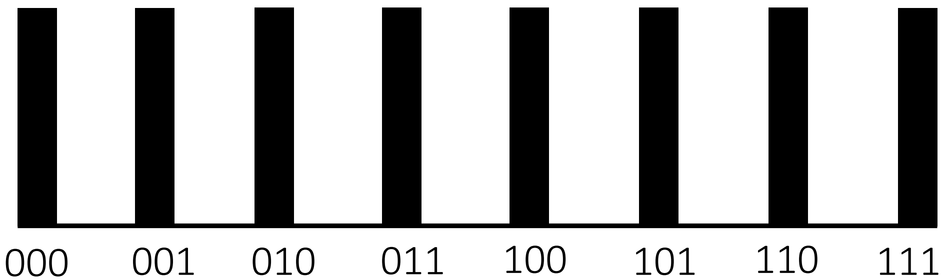
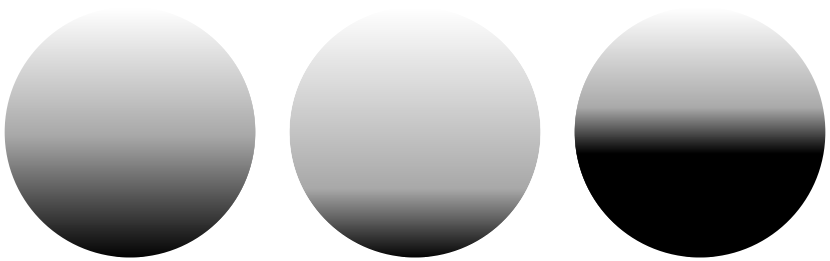
# What is different in QC?

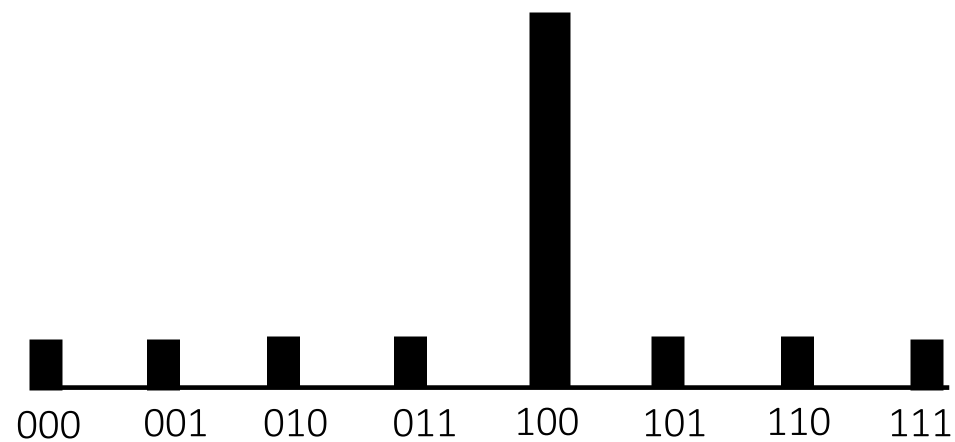
Superposition and entanglement

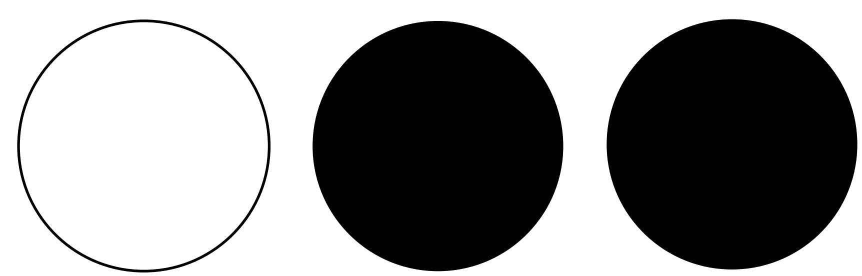


$$\alpha_{000}|000\rangle+\alpha_{001}|001\rangle+\alpha_{010}|010\rangle+\alpha_{011}|011\rangle$$
$$+\alpha_{100}|100\rangle+\alpha_{101}|101\rangle+\alpha_{110}|110\rangle+\alpha_{111}|111\rangle$$

Interference



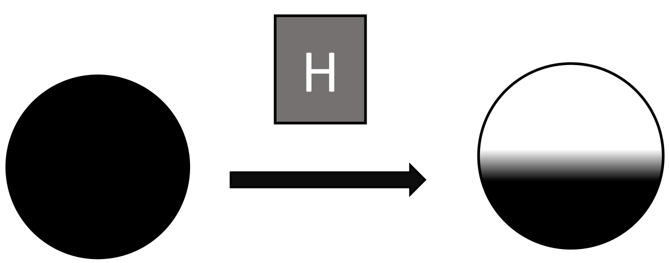



- Probabilistic computation
- Result is a binary value

# What is different in QC?

**In-memory computing**

- The qubits hold the information in a form of a quantum state which is modified by applying an operation on them.

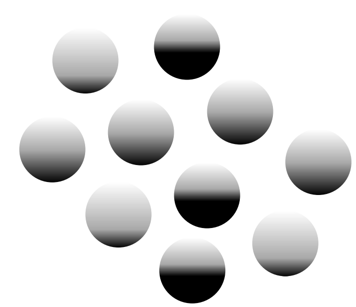



$|\psi\rangle = |0\rangle$ $\quad$ $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

**Qubits and gates are error prone**

- Qubits have short coherence time
- Imperfect operations
  - Gate error rates: $10^{-2}$-$10^{-3}$

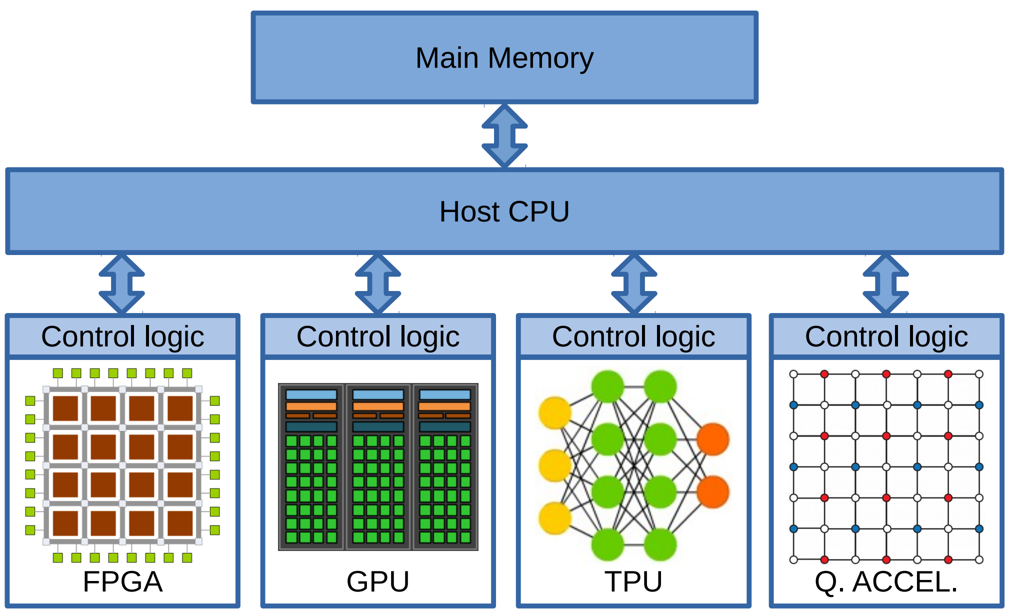
Quantum information needs to be protected

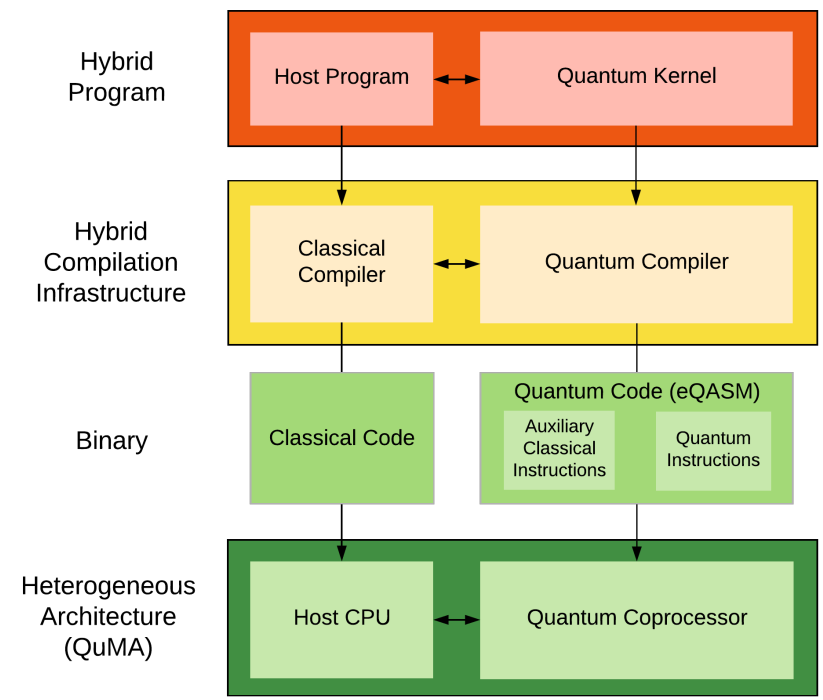


Quantum error correction

# A quantum computer is not (is)

- It is not a replacement for classical computers
- It is an in-memory-computing device
- It is a co-processor in a (heterogeneous) multi-core architecture




X. Fu et. al, "eQASM: An Executable Quantum Instruction Set Architecture", *IEEE International Symposium on High Performance Computer Architecture (HPCA), 2019*.

Riesebos, L., et al. "Quantum Accelerated Computer Architectures." *2019 IEEE International Symposium on Circuits and Systems (ISCAS)*. IEEE, 2019.

# Quantum Computation is based on Linear Algebra

# Reading out quantum information

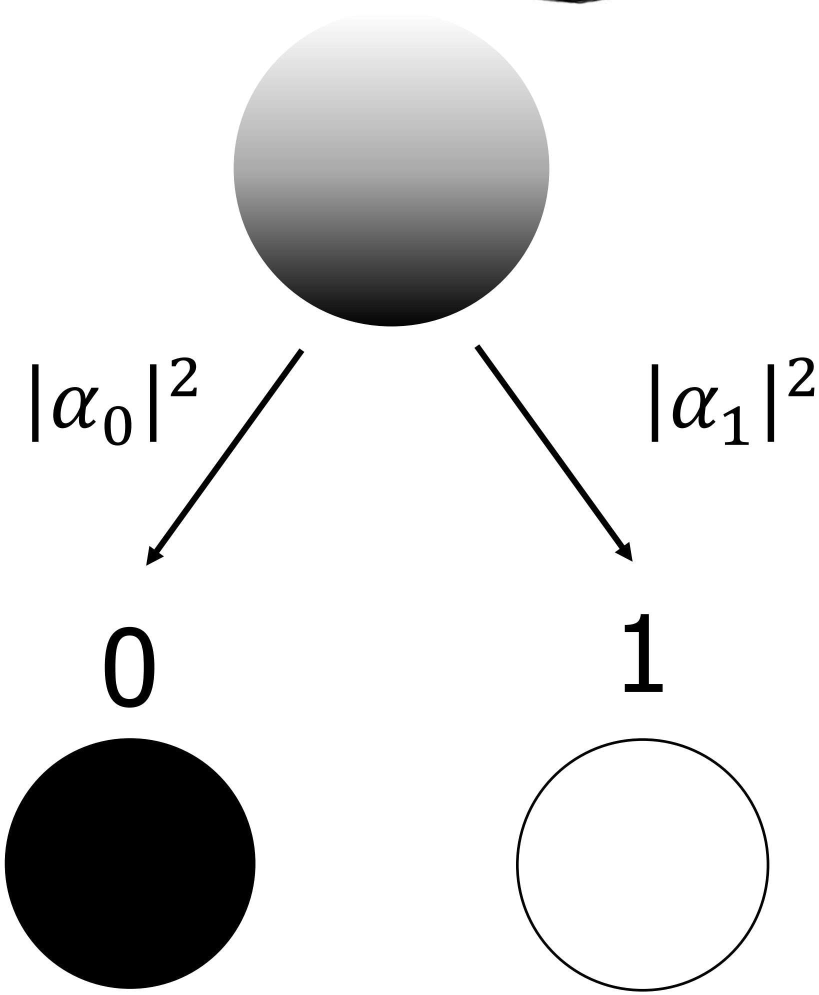
## Measuring a qubit (quantum state)

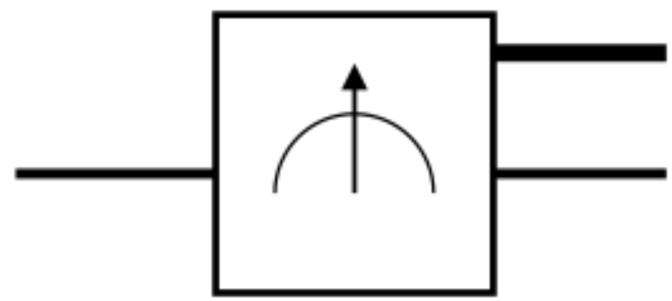
$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

$$\alpha_0, \alpha_1 \in \mathbb{C}$$

$$|\alpha_i|^2$$

is the probability of finding the qubit in state $|i\rangle$ when we measure it (in the computational basis)



$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

Probabilistic process
Binary value
Projective measurement

# Measuring a qubit

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

| | $|\alpha_0|^2$ Prob. result 0 | $|\alpha_1|^2$ Prob. result 1 |
|---|---|---|
| $\frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$ | $\left\|\frac{1}{\sqrt{2}}\right\|^2 = \frac{1}{2}$ | $\left\|\frac{1}{\sqrt{2}}\right\|^2 = \frac{1}{2}$ |
| $\frac{1}{2}\|0\rangle + \frac{\sqrt{3}}{2}\|1\rangle$ | $\left\|\frac{1}{2}\right\|^2 = \frac{1}{4}$ | $\left\|\frac{\sqrt{3}}{2}\right\|^2 = \frac{3}{4}$ |
| $\frac{1}{2}\|0\rangle + \frac{i\sqrt{3}}{2}\|1\rangle$ | $\left\|\frac{1}{2}\right\|^2 = \frac{1}{4}$ | $\left\|\frac{i\sqrt{3}}{2}\right\|^2 = \frac{3}{4}$ |
| $\frac{1}{2}\|0\rangle + \frac{1}{2}\|1\rangle$ | $\left\|\frac{1}{2}\right\|^2 = \frac{1}{4}$ | $\left\|\frac{1}{2}\right\|^2 = \frac{1}{4}$ |

This is not a quantum state

$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

# A quantum state is a vector

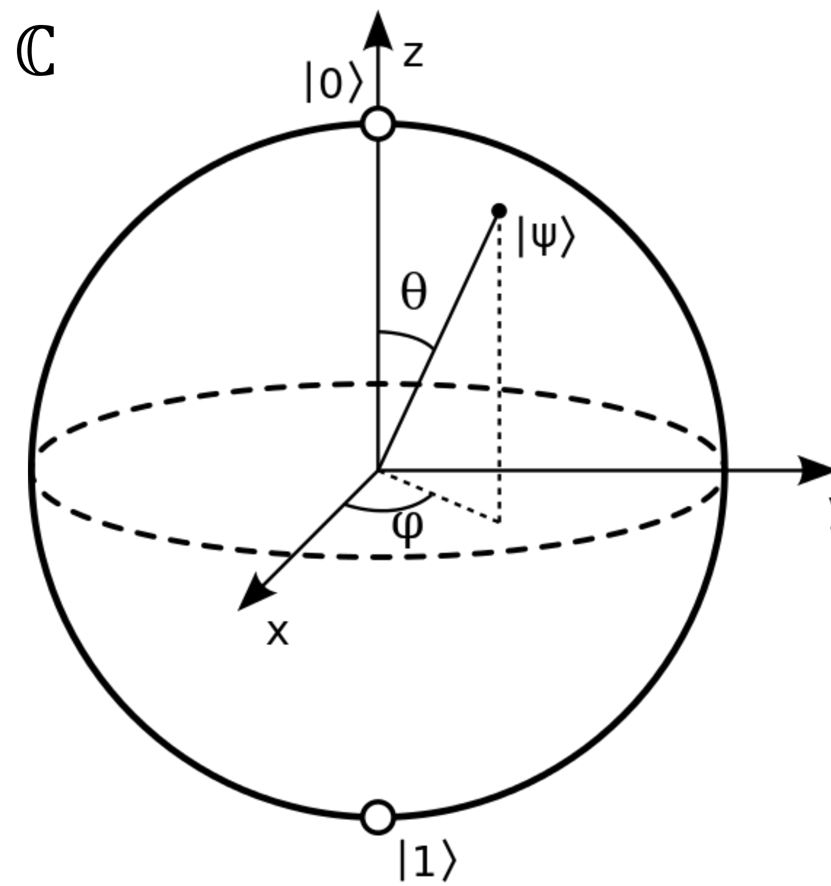
$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\boxed{|\psi\rangle = \alpha_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}}$$

# Representation on the Bloch Sphere

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \qquad \alpha_0, \alpha_1 \in \mathbb{C}$$




- $|\psi\rangle = \alpha_0|0\rangle + e^{i\varphi}\alpha_1|1\rangle$

  $\alpha_0, \alpha_1 \in \mathbb{R}$

- $\sqrt{\alpha_0^2 + \alpha_1^2} = 1 \qquad (\sqrt{sin^2 x + cos^2\, x = 1})$

  $\alpha_0 = cos\frac{\theta}{2} \qquad \alpha_1 = sin\frac{\theta}{2}$

  $|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi} sin\frac{\theta}{2}|1\rangle$

  $\theta, \varphi \in \mathbb{R}$

$\theta$ : polar angle $\in [0, \pi]$

$\varphi$ : azimutal angle $\in [0, 2\pi]$

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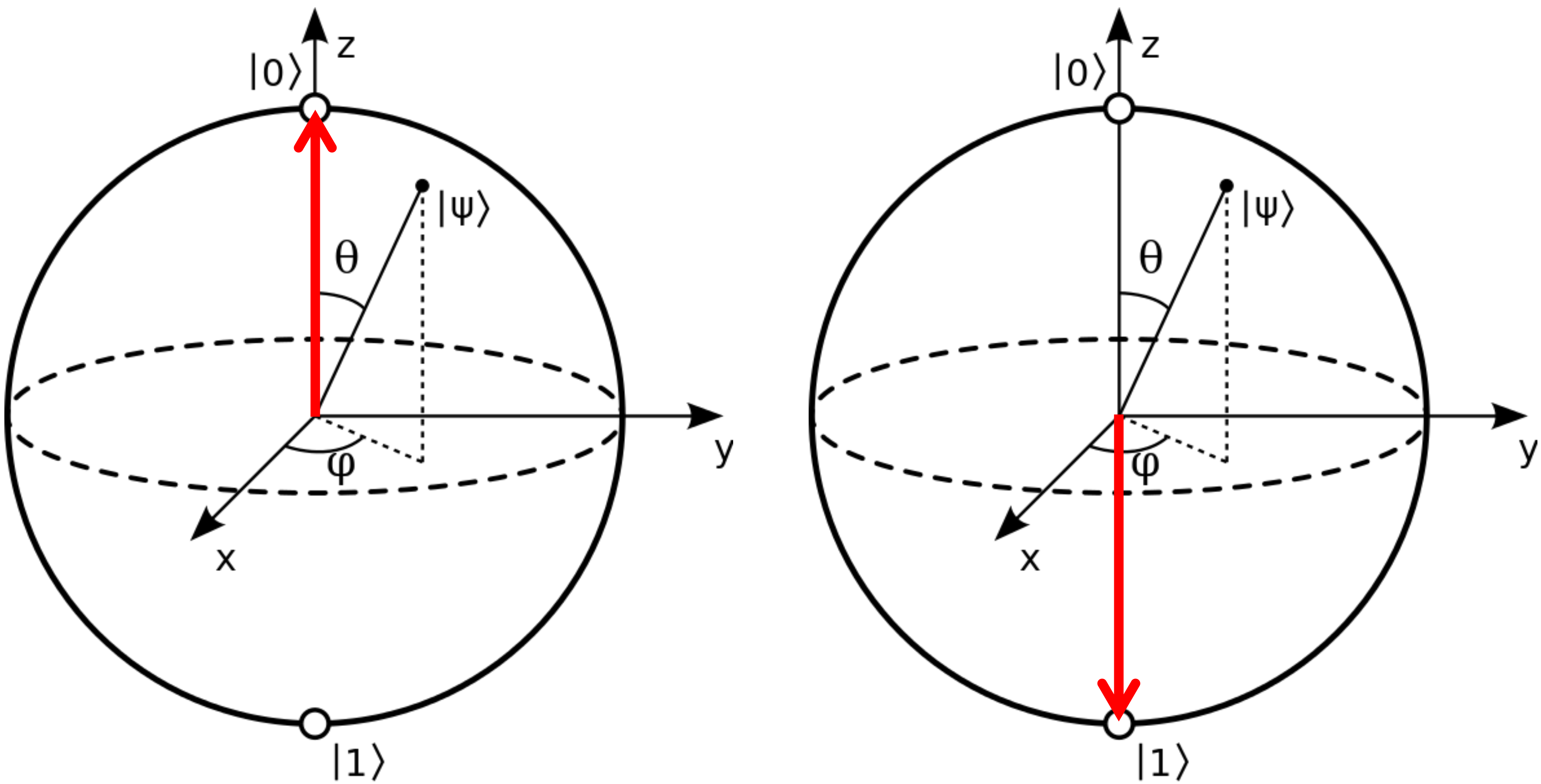
https://qiskit.org/textbook/ch-states/representing-qubit-states.html#bloch-sphere

# Representation on the Bloch Sphere

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# Single-qubit gates

**Identity**

$$I$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|0\rangle \rightarrow |0\rangle$$
$$|1\rangle \rightarrow |1\rangle$$

**Pauli X**

$$X$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|0\rangle \rightarrow |1\rangle$$
$$|1\rangle \rightarrow |0\rangle$$

**Pauli Z**

$$Z$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|0\rangle \rightarrow |0\rangle$$
$$|1\rangle \rightarrow -|1\rangle$$

**Pauli Y**

$$Y$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$|0\rangle \rightarrow \mathrm{i}|1\rangle$$
$$|1\rangle \rightarrow -\mathrm{i}|0\rangle$$

**Hadamard**

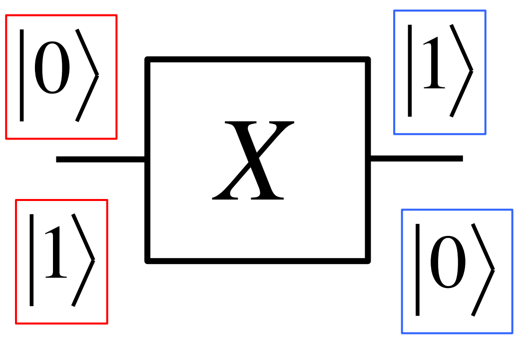
$$H$$

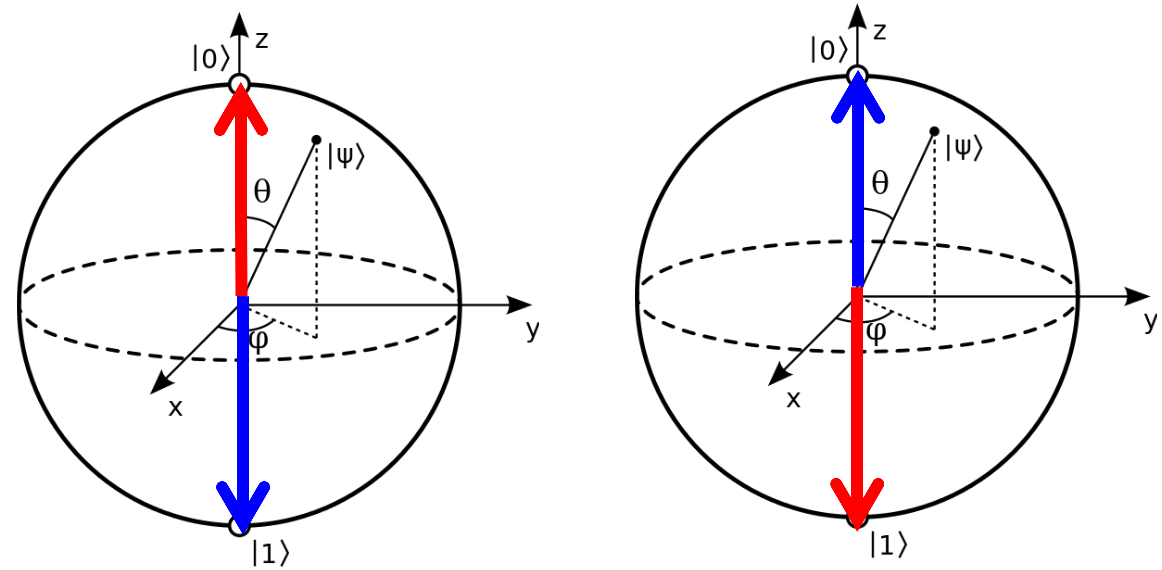
$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

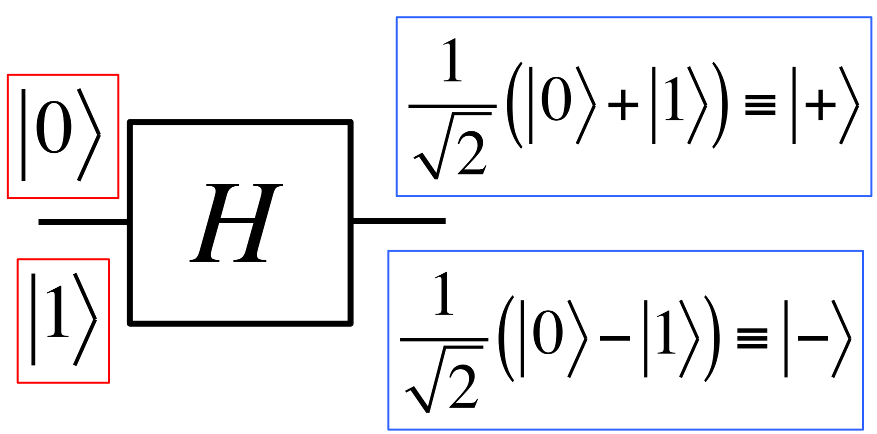
$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

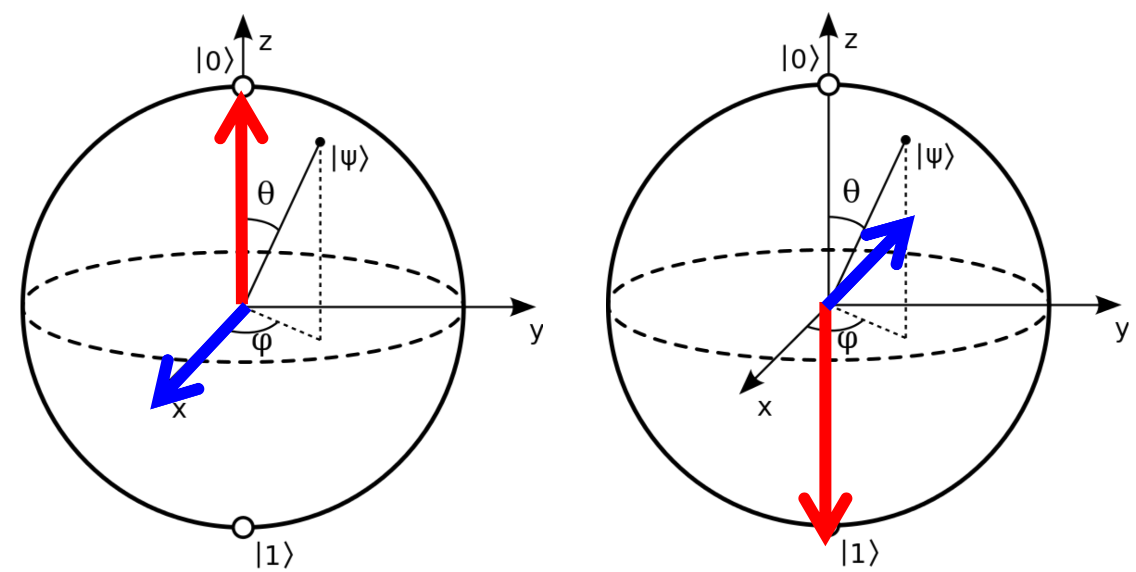
$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

# Single-qubit gates

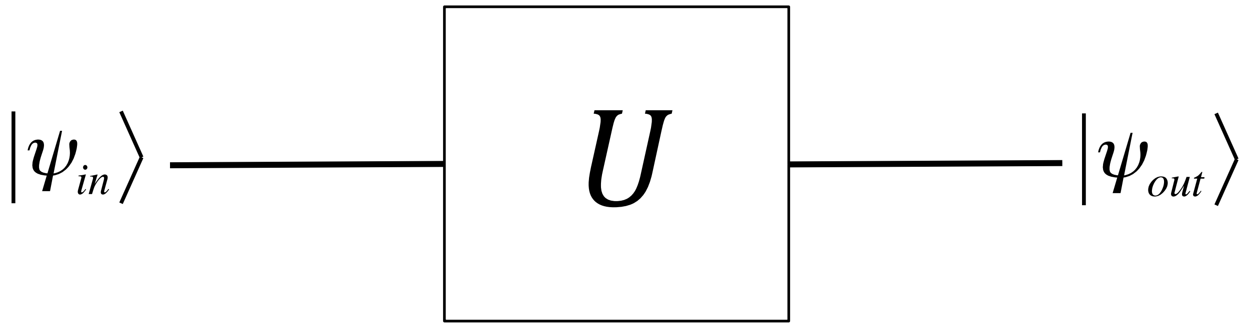




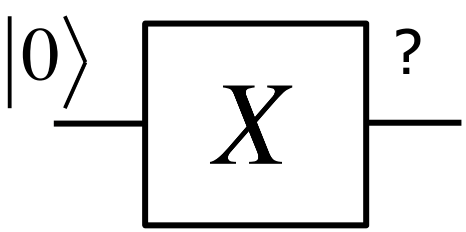
# Single-qubit gates




$$|\psi_{out}\rangle = U|\psi_{in}\rangle$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$




$$|\psi_{out}\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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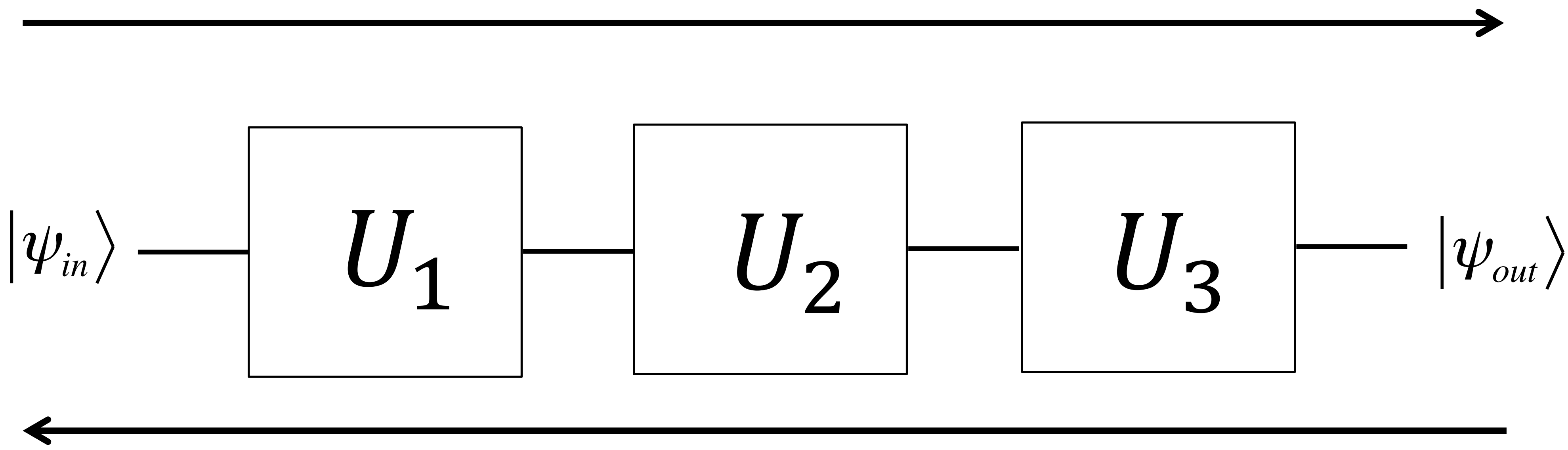
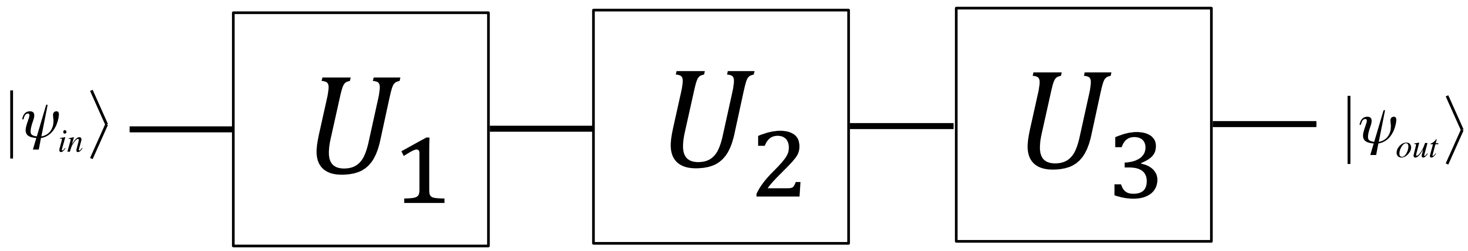
# Quantum circuit (single qubit)

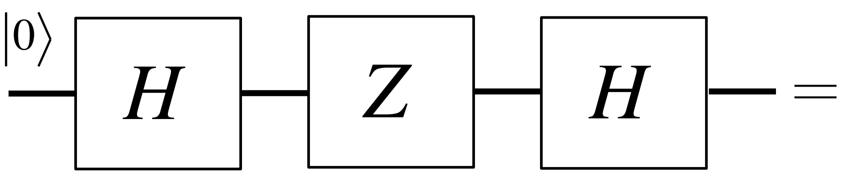


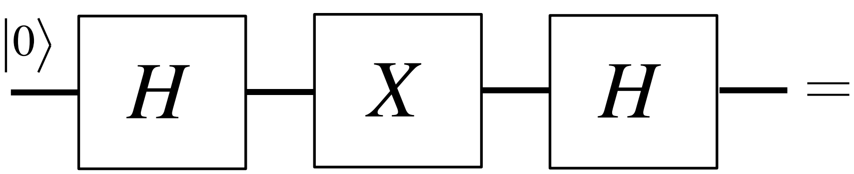

$$|\psi_{out}\rangle = U_3 U_2 U_1 |\psi_{in}\rangle$$

# Quantum circuit (single qubit )




$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



# Multi-qubit state

$$|\psi_0\rangle = \alpha_0|0\rangle + \beta_0|1\rangle \qquad |\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$|\psi\rangle = |\psi_0\rangle \otimes |\psi_1\rangle$$

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle = \begin{pmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{pmatrix}$$

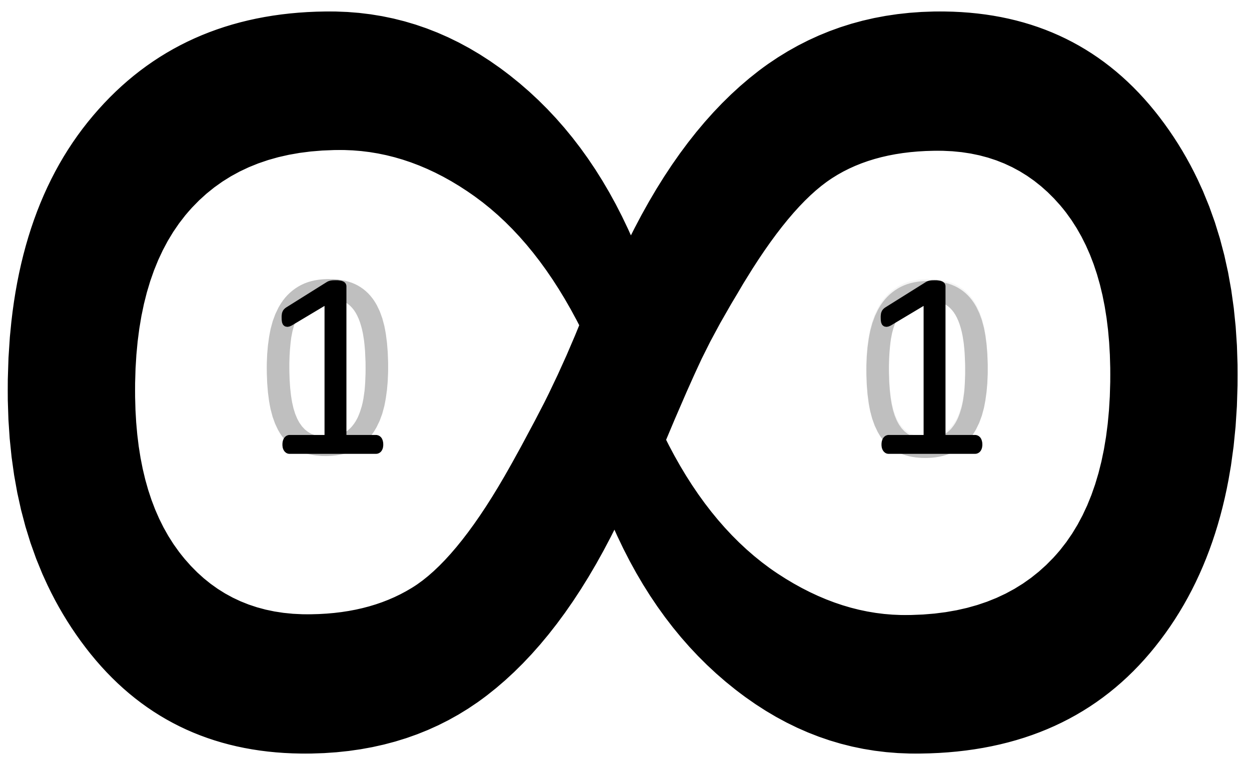
**Tensor product**

$$|A\rangle \otimes |B\rangle = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix}$$

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

# Multi-qubit state

**Entanglement**




Two qubits in a superposition are correlated with one another

# Multi-qubit state

## Entanglement

Quantum entanglement means that multiple particles are linked together in a way such that the measurement of one particle's quantum state determines the possible quantum states of the other particles.

This connection isn't depending on the location of the particles in space. Even if you separate entangled particles by billions of kms, changing one particle will induce a change in the other.

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

https://www.youtube.com/watch?v=CC_XES4xQD4

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# Multi-qubit state

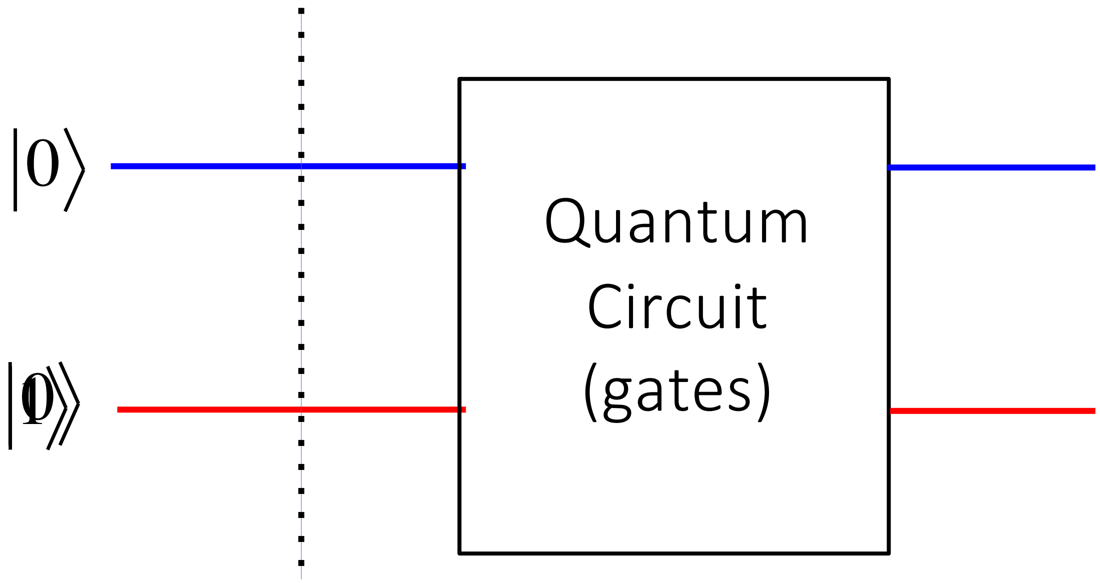
Two qubits are *entangled* when their joint states cannot possibly be separated into a product of individual qubit states

$$|\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle \qquad \text{vs.} \qquad |\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle + |\varphi'\rangle \otimes |\psi'\rangle$$

$$\frac{1}{\sqrt{2}} \left(|10\rangle + |11\rangle\right) = |1\rangle \otimes \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle\right)$$ (not entangled)

$$\frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}} |0\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}} (|1\rangle \otimes |1\rangle)$$ (entangled)

# Quantum circuit (multiple qubits)



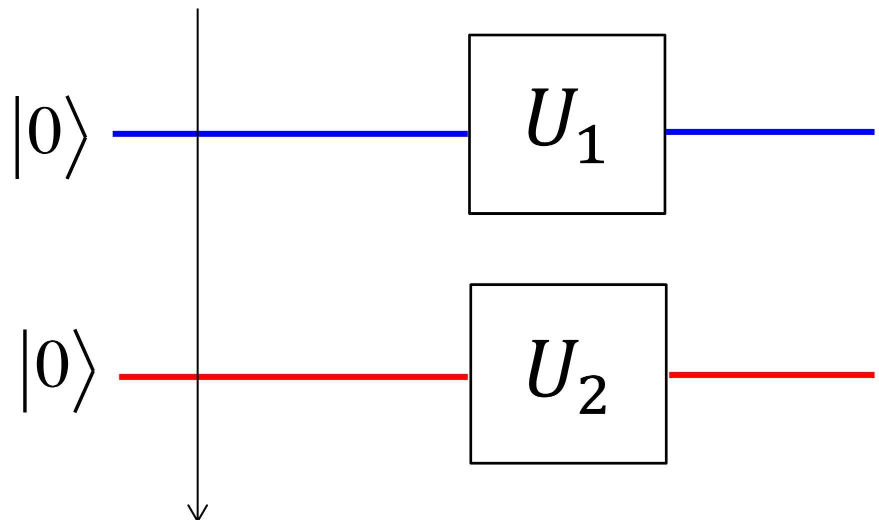

$$|01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$|0\rangle \otimes |0\rangle = |0\rangle|0\rangle = |00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

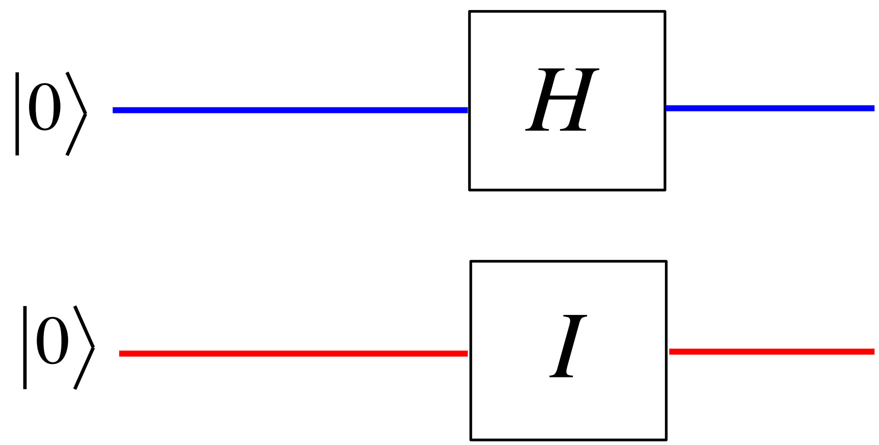
# Quantum circuit (multiple qubits)




$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|\psi_{out}\rangle = U_1 \otimes U_2 |\psi_{in}\rangle$$

# Quantum circuit (multiple qubits)

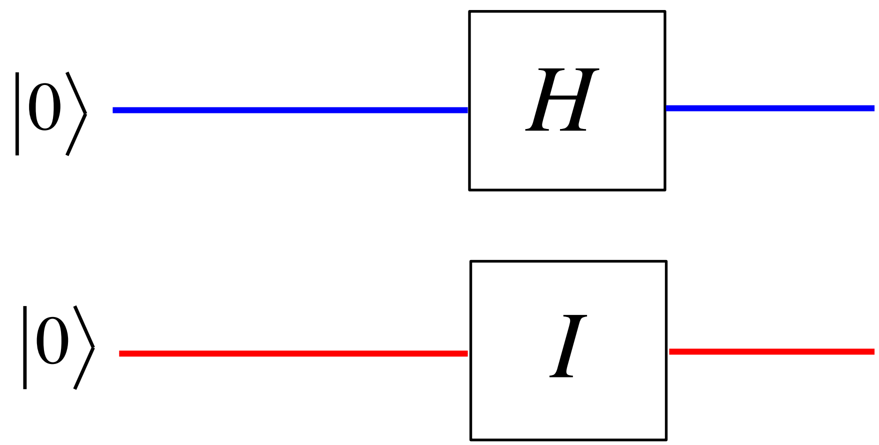



$$|\psi_{out}\rangle = H \otimes I |\psi_{in}\rangle$$

$$\hat{U}_1 \otimes \hat{U}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$|A\rangle \otimes |B\rangle = \begin{bmatrix} A_{11}B & A_{12}B \\ A_{21}B & A_{22}B \end{bmatrix}$$

# Quantum circuit (multiple qubits)




$$|\psi_{out}\rangle = H \otimes I |\psi_{in}\rangle$$

$$|\psi_{out}\rangle =$$

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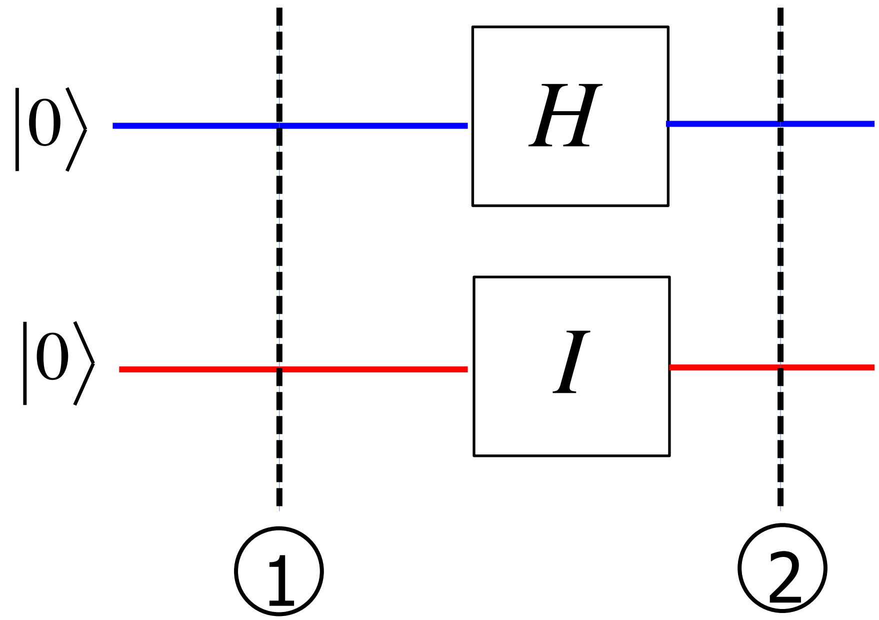
# Quantum circuit (multiple qubits)



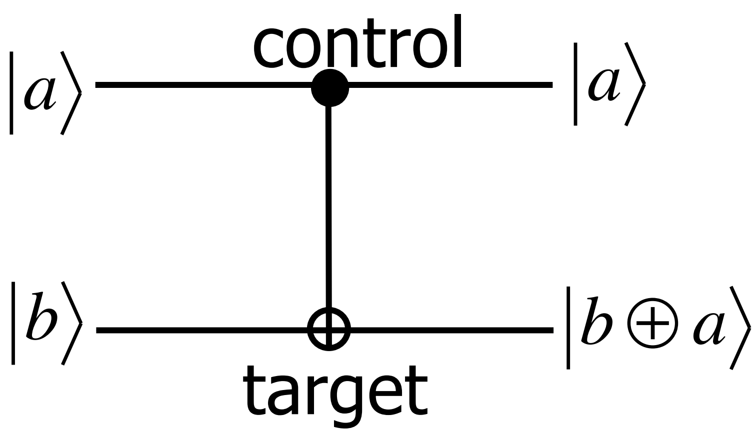

$$|\psi_{out}\rangle = H \otimes I|\psi_{in}\rangle$$

1 $|00\rangle$

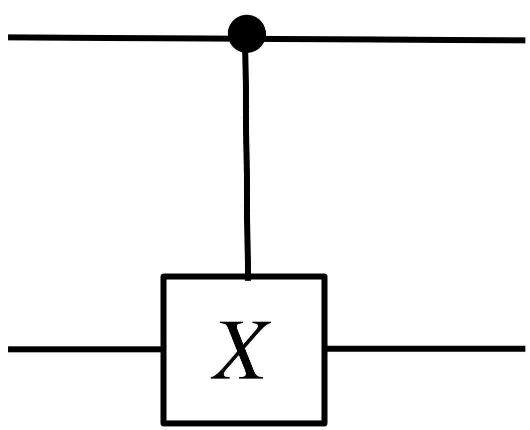
2 $\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$

# Multi-qubit gates

## CNOT gate




$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



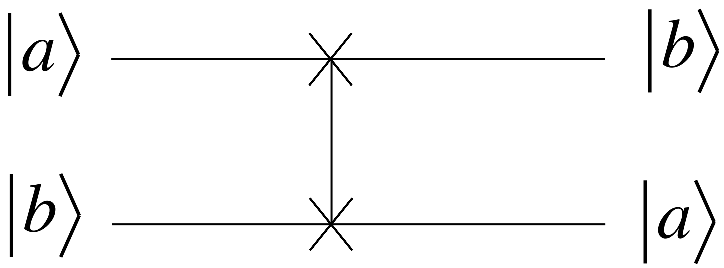

## XOR gate

| a | b | out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$$a \cdot \overline{b} + \overline{a} \cdot b$$

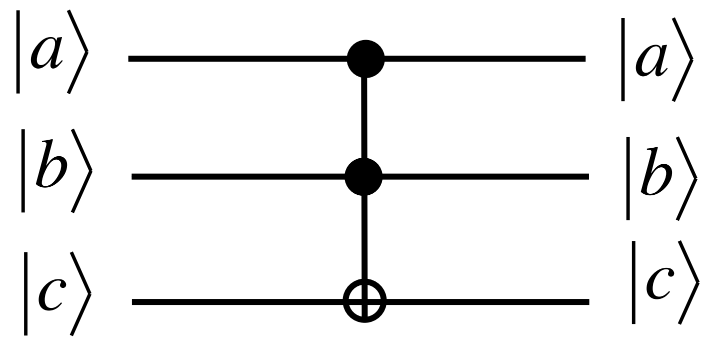
# Multi-qubit gates

**SWAP gate**




$$
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

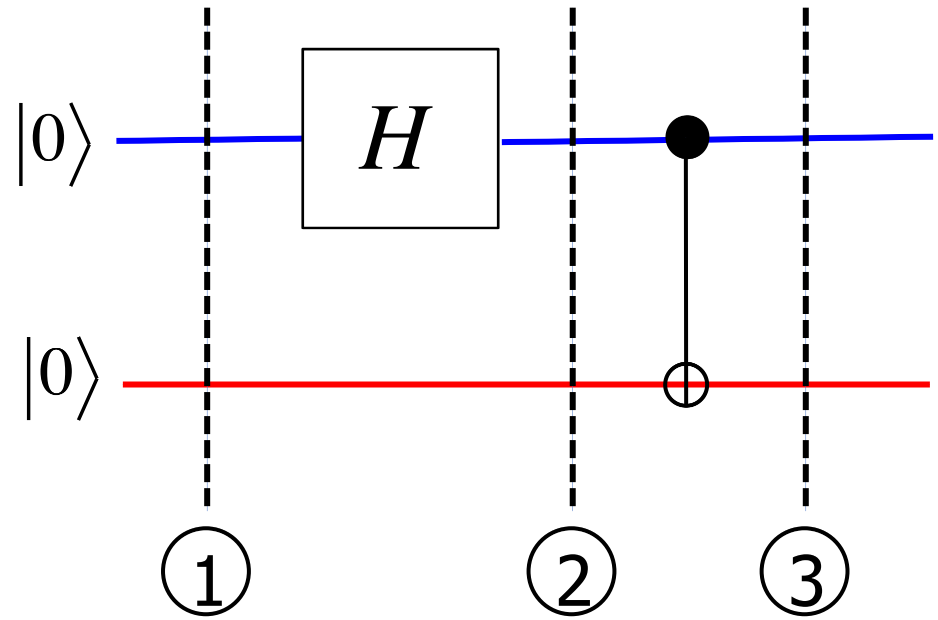
**Toffoli gate**




$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
$$

| Input | | | Output | | |
|---|---|---|---|---|---|
| 0 | 0 | **0** | 0 | 0 | **0** |
| 0 | 0 | **1** | 0 | 0 | **1** |
| 0 | 1 | **0** | 0 | 1 | **0** |
| 0 | 1 | **1** | 0 | 1 | **1** |
| 1 | 0 | **0** | 1 | 0 | **0** |
| 1 | 0 | **1** | 1 | 0 | **1** |
| 1 | 1 | **0** | 1 | 1 | **1** |
| 1 | 1 | **1** | 1 | 1 | **0** |

# Quantum circuit (multiple qubits)




1 $|00\rangle$

2 $\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right)|0\rangle=\frac{1}{\sqrt{2}}\left(|00\rangle+|10\rangle\right)$

3 $\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right)$

# Bell states

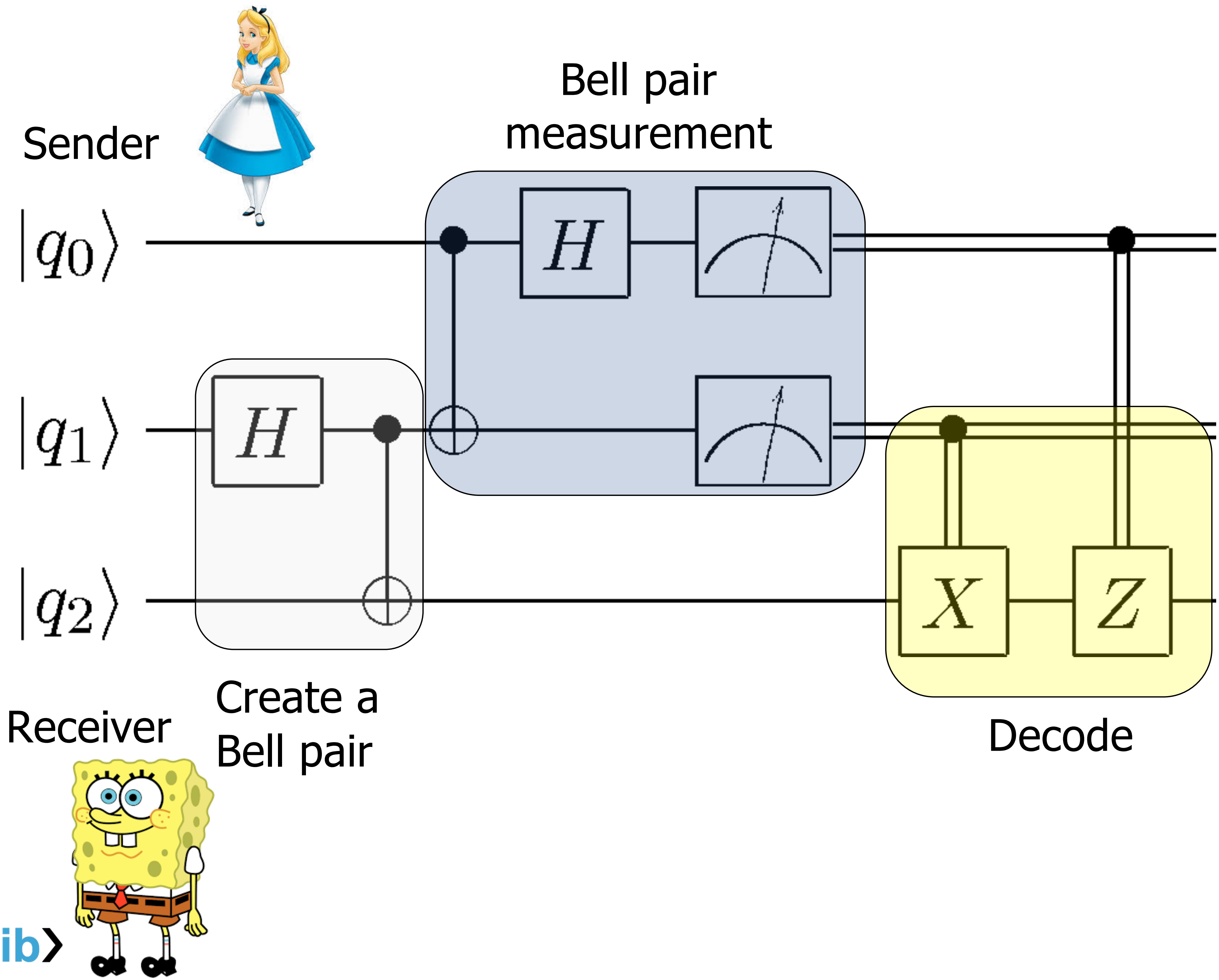
$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

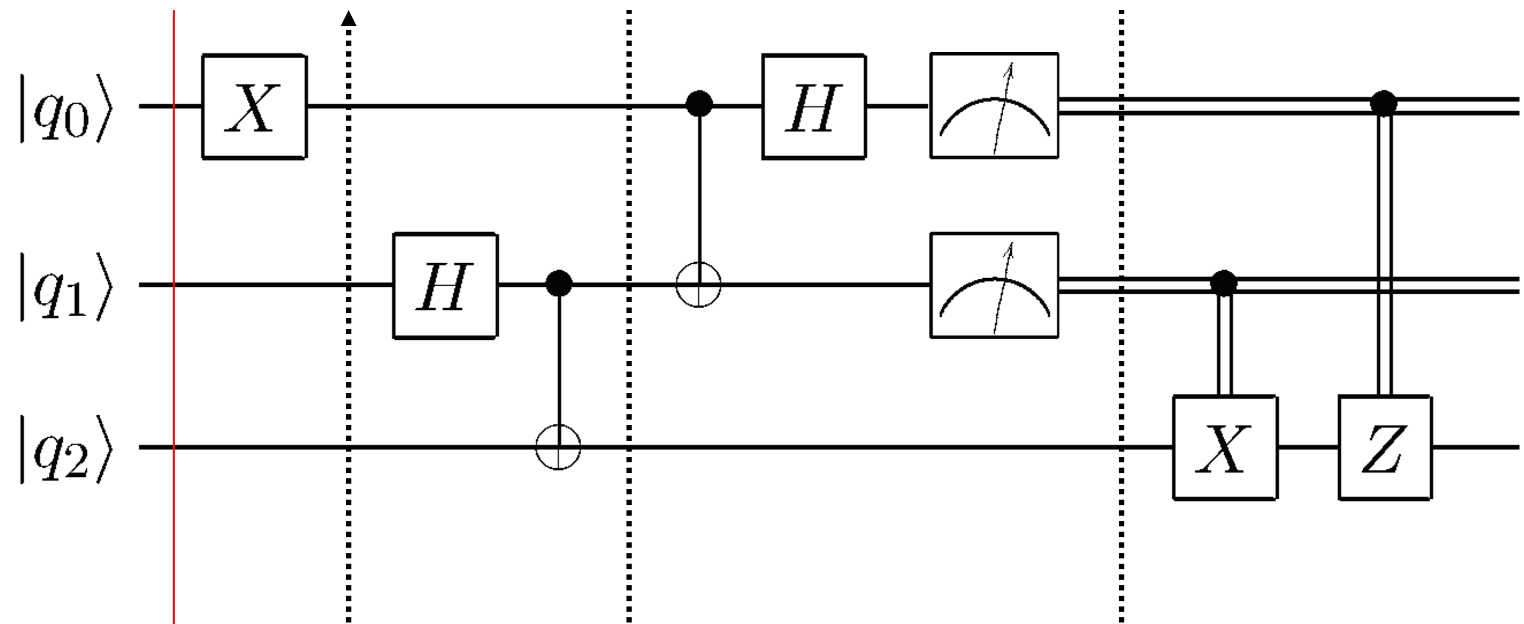
$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

# Quantum teleportation



# Quantum teleportation




$$|q_0\rangle|q_1\rangle|q_2\rangle = |q_0 q_1 q_2\rangle = |000\rangle$$

**QASM-like code**

```
X q0
H q1
CNOT q1,q2
CNOT q0,q1
H q0
```

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# Quantum teleportation

```
X q0
H q1
CNOT q1,q2
CNOT q0,q1
H q0
```

$$|q_0\rangle|q_1\rangle|q_2\rangle = |q_0 q_1 q_2\rangle = |000\rangle$$

**X $q_0$** → $|100\rangle$

**H $q_1$** → $|1\rangle\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right)|0\rangle = \frac{1}{\sqrt{2}}\left(|100\rangle+|110\rangle\right)$

**CNOT $q_1$,$q_2$** → $\frac{1}{\sqrt{2}}\left(|100\rangle+|111\rangle\right)$

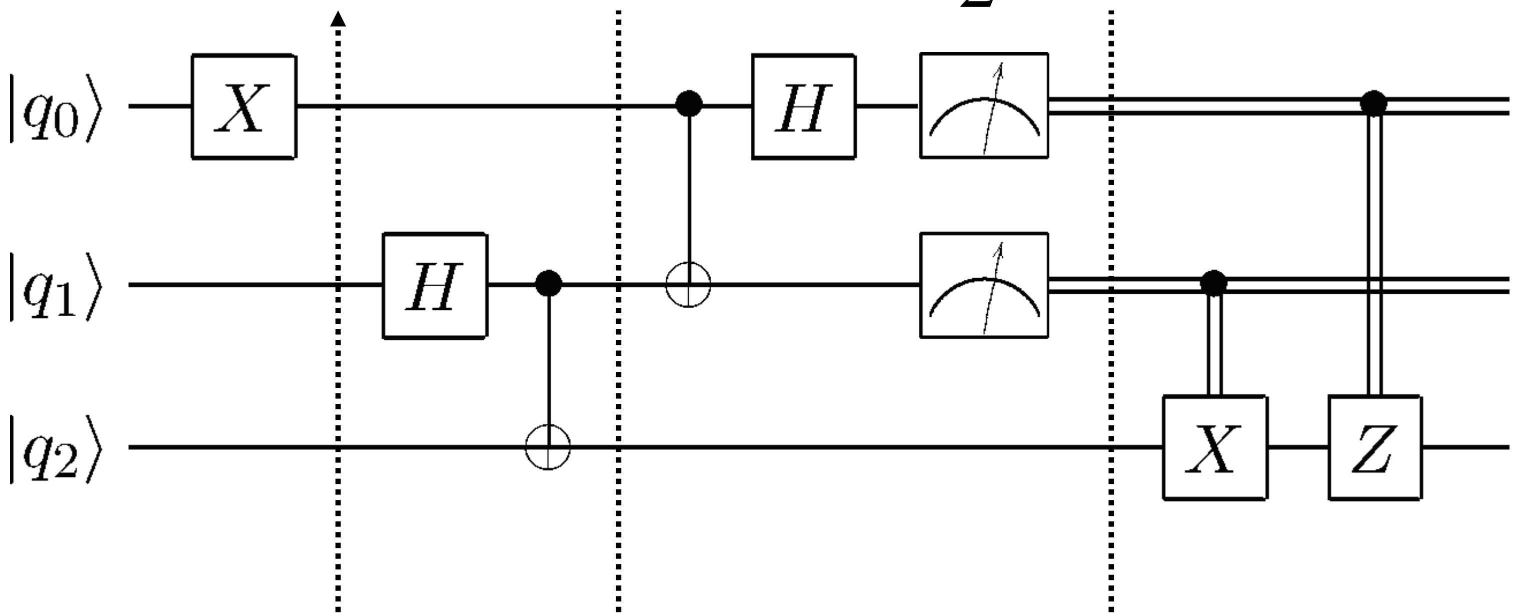
**CNOT $q_0$,$q_1$** → $\frac{1}{\sqrt{2}}\left(|110\rangle+|101\rangle\right)$

**H $q_0$** → $\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|0\rangle-|1\rangle\right)|10\rangle+\frac{1}{\sqrt{2}}\left(|0\rangle-|1\rangle\right)|01\rangle\right]=$

$$=\frac{1}{2}\left(|010\rangle-|110\rangle+|001\rangle-|101\rangle\right)$$

# Quantum teleportation

$$\frac{1}{2}\left(|010\rangle - |110\rangle + |001\rangle - |101\rangle\right)$$




- Measure 00 → $|q_2\rangle = |1\rangle$ → no correction
- Measure 01 → $|q_2\rangle = |0\rangle$ → bit-flip q2 → $|q_2\rangle = |1\rangle$
- Measure 10 → $|q_2\rangle = -|1\rangle$ → phase-flip q2 → $|q_2\rangle = |1\rangle$
- Measure 11 → $|q_2\rangle = -|0\rangle$ → bit-flip and phase-flip → $|q_2\rangle = |1\rangle$

# Quantum error correction (QEC)

## Qubits are very fragile!

- Extremely short coherence time: ~ us – s
- Gate error rates: ~$10^{-1}$ -$10^{-3}$ vs. CMOS $10^{-20}$-$10^{-15}$

1. Bit flip error

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2. Phase flip error

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle$$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

3. Bit and Phase flip error

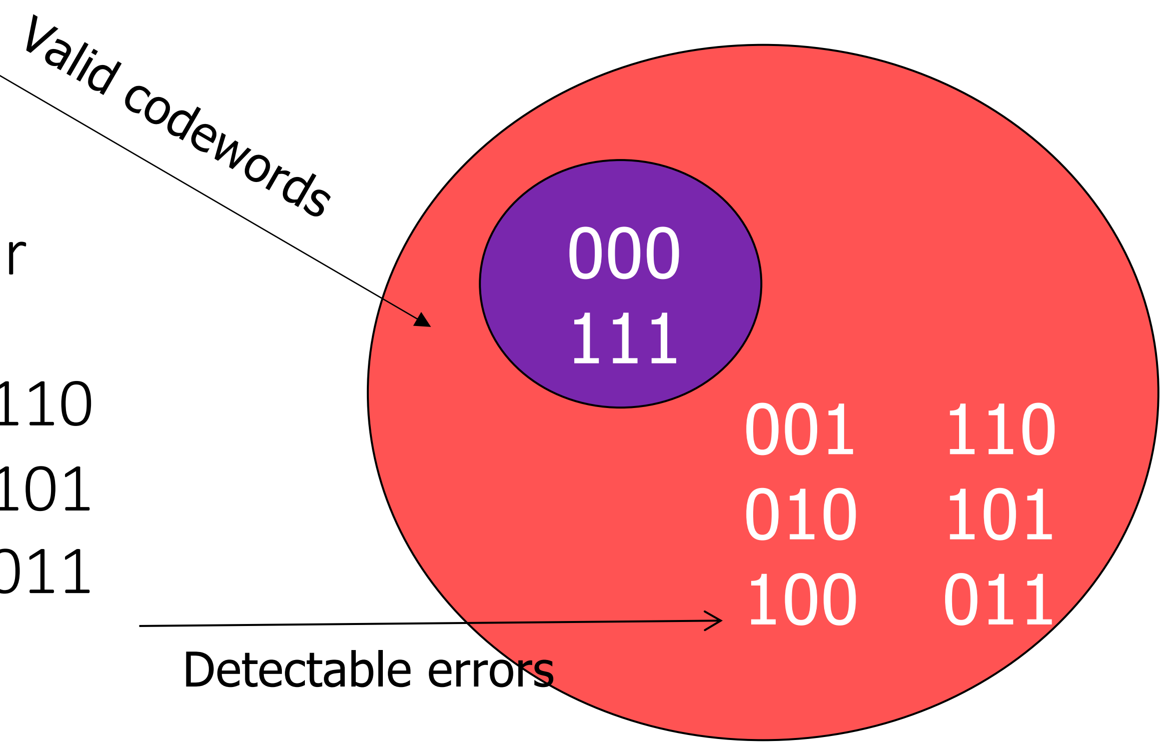
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle - \beta|0\rangle$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

# Quantum error correction (QEC)

Classical error correction encoding: 3 bit repetition code

- 0 is encoded as 000
- 1 is encoded as 111




- Assuming only 1 bit error

$$000 \rightarrow \left\{ \begin{array}{l} 001 \\ 010 \\ 100 \end{array} \right. \qquad 111 \rightarrow \left\{ \begin{array}{l} 110 \\ 101 \\ 011 \end{array} \right.$$

- Decoding: Majority voting
  - E.g. 000→001 ⟶ bit-flip bit 3

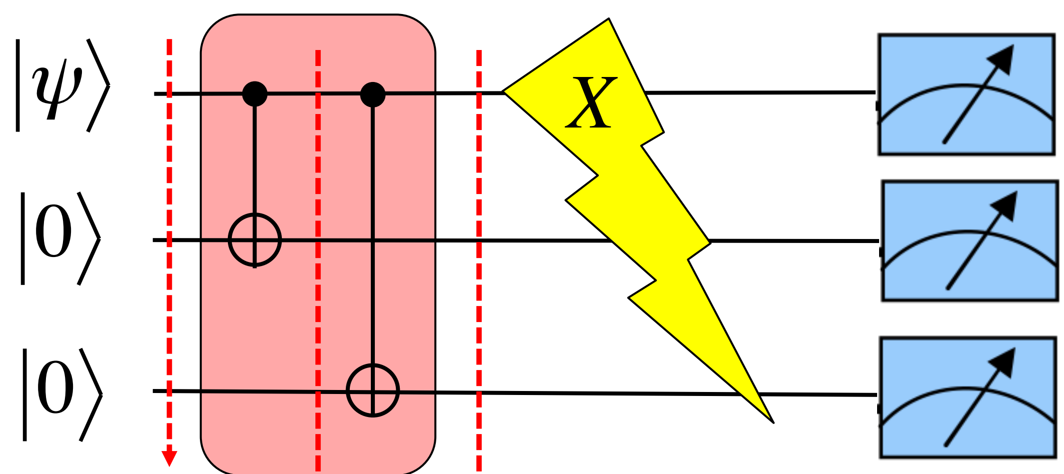
# Quantum error correction (QEC)

## 3 qubit bit-flip code: single bit-flip errors

Physical qubit    Logical qubit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$

Encoding (entanglement)    Detection




**Measurement destroys quantum information**

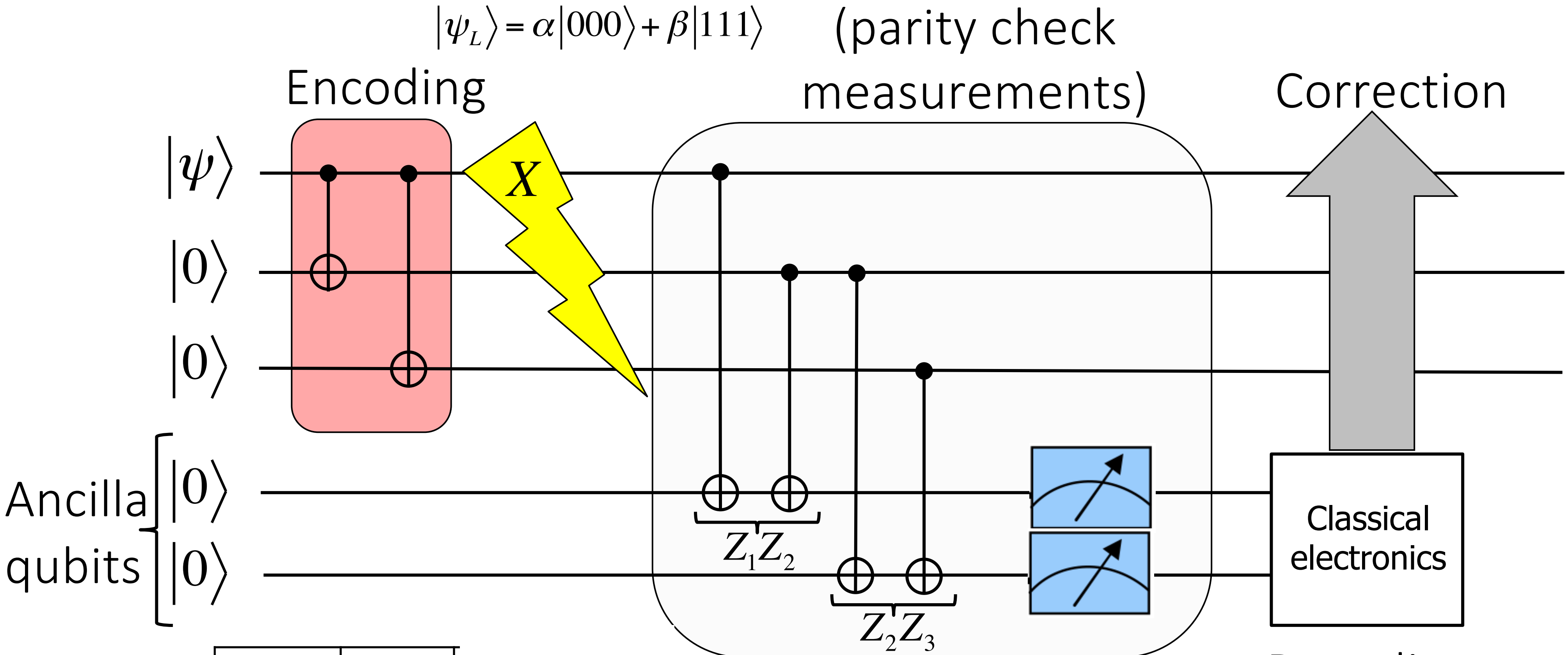
$$|\psi\rangle = \alpha|000\rangle + \beta|100\rangle$$

$$|\psi\rangle = \alpha|000\rangle + \beta|110\rangle$$

$$|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$$

# Quantum error correction (QEC)

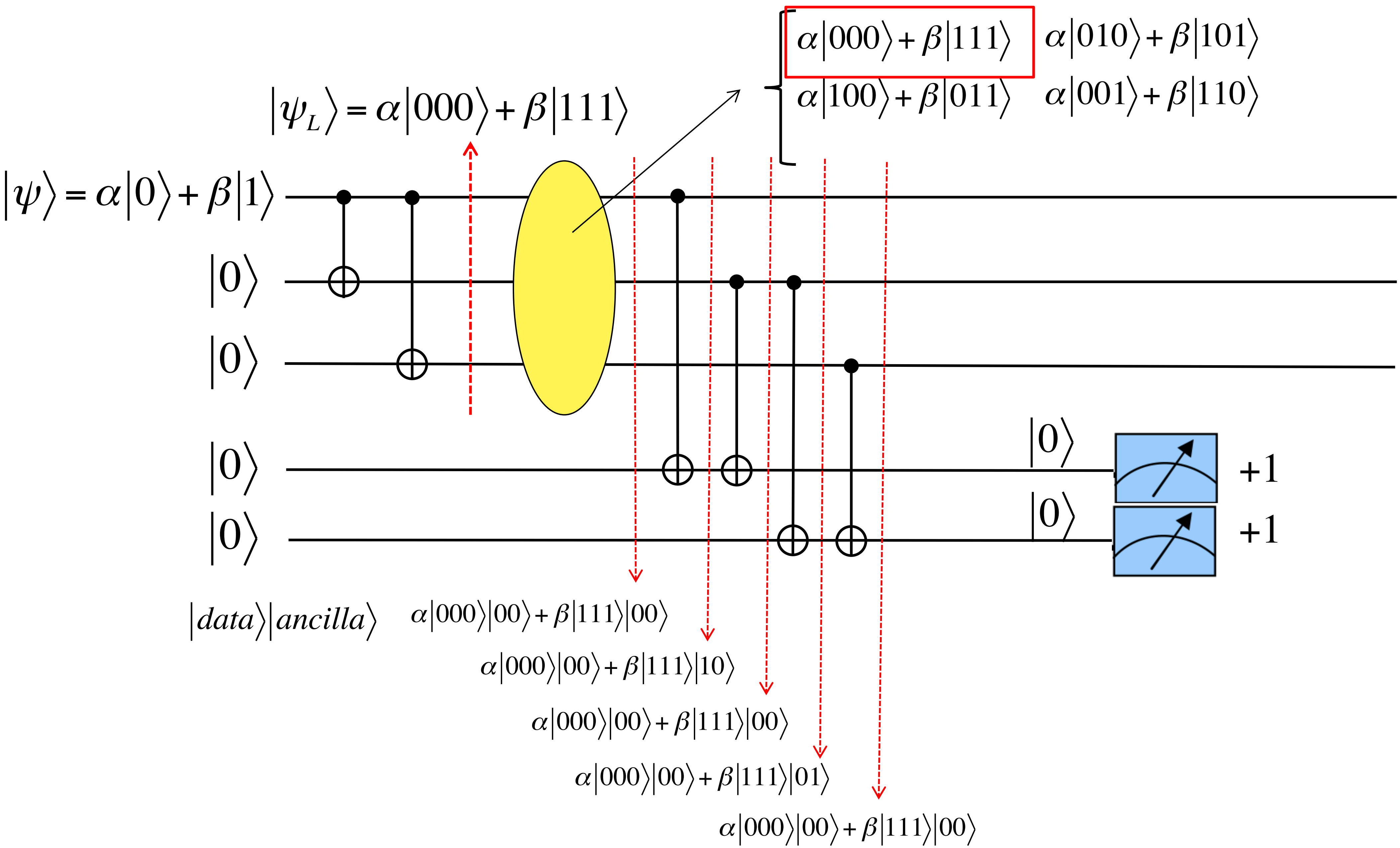
3 qubit bit-flip code



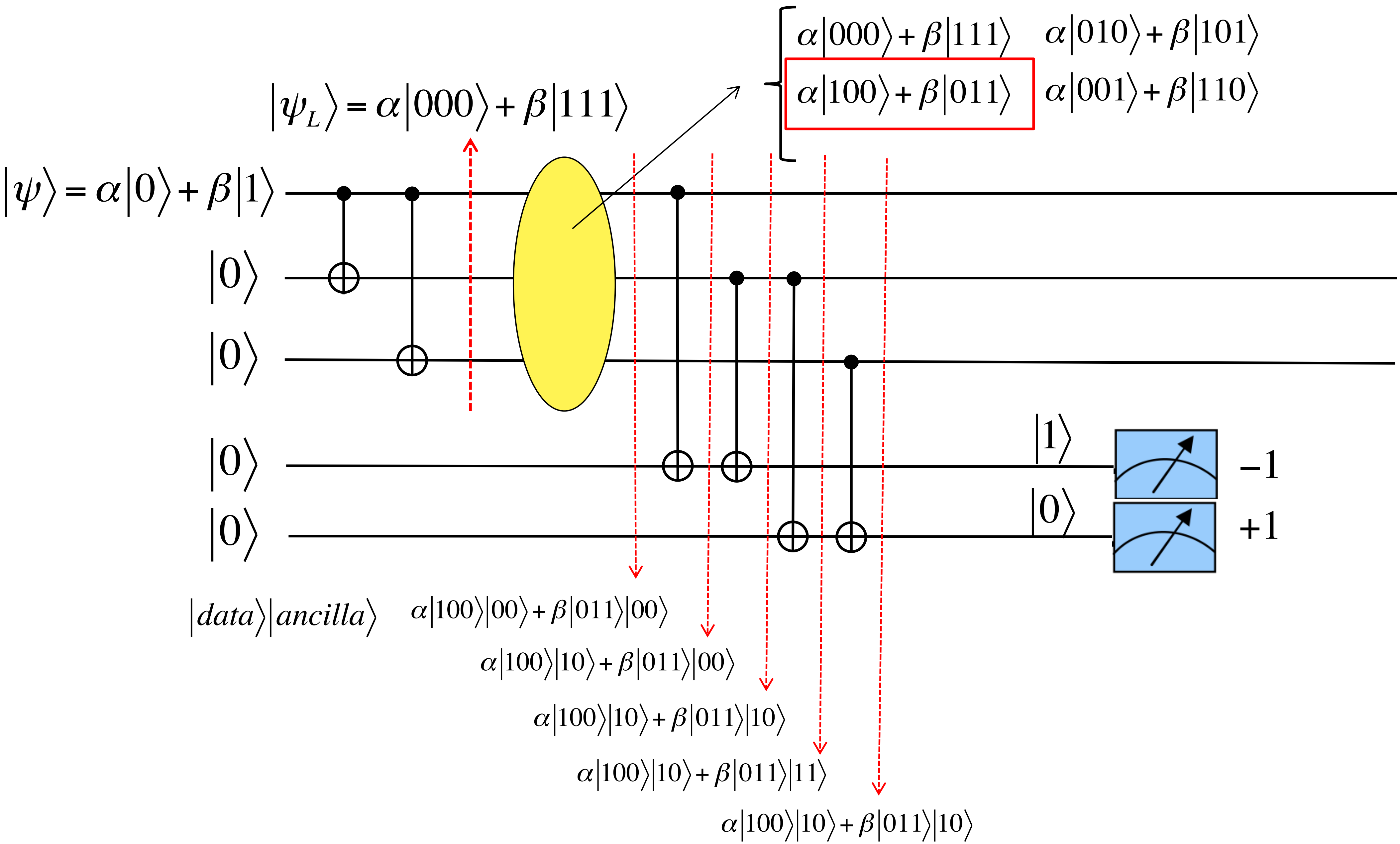

| $Z_1Z_2$ | $Z_2Z_3$ |
|---|---|
| 0(+1) | 0(+1) |
| 0(+1) | 1(-1) |
| 1(-1) | 0(+1) |
| 1(-1) | 1(-1) |

Error syndromes

# Single bit-flip error occurs…



# Single bit-flip error occurs…

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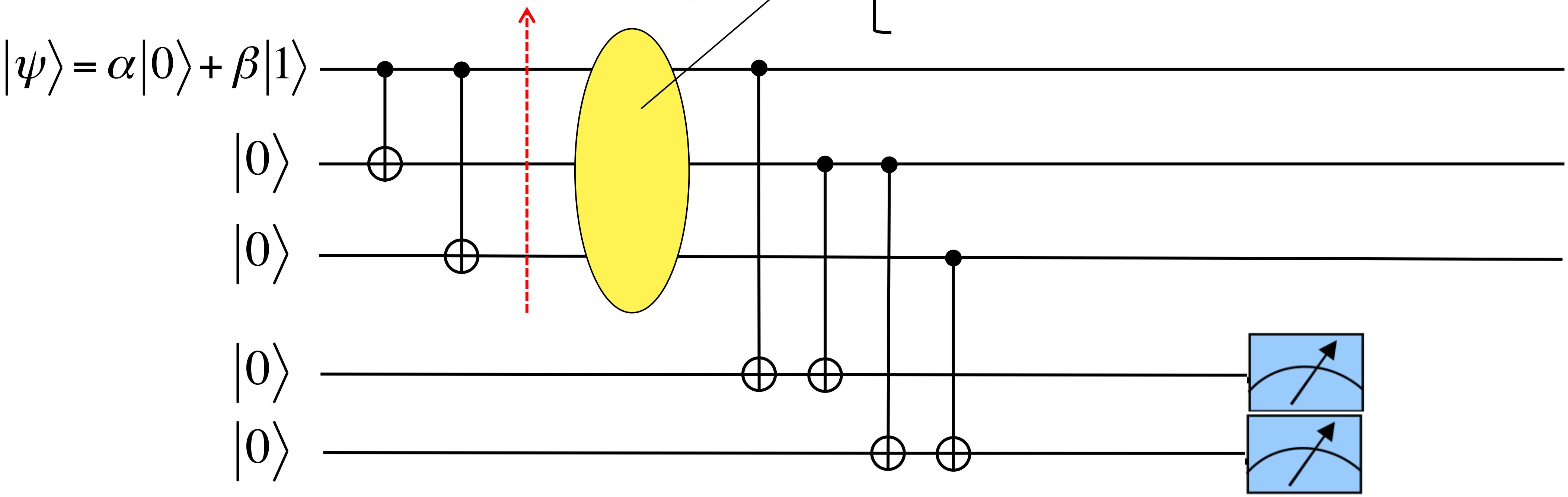
# Single bit-flip error occurs…

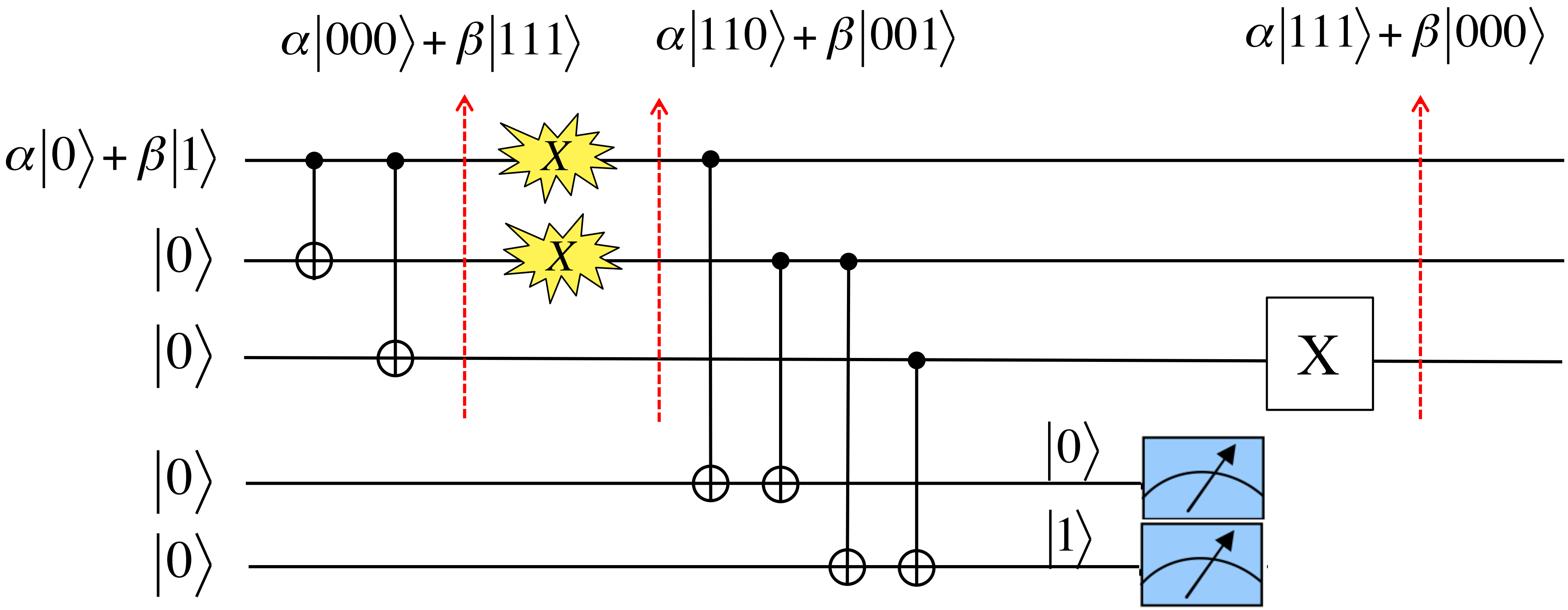
$$
\left\{
\begin{array}{ll}
\alpha|000\rangle+\beta|111\rangle & \alpha|010\rangle+\beta|101\rangle \\
\alpha|100\rangle+\beta|011\rangle & \alpha|001\rangle+\beta|110\rangle
\end{array}
\right.
$$

$$|\psi_L\rangle=\alpha|000\rangle+\beta|111\rangle$$



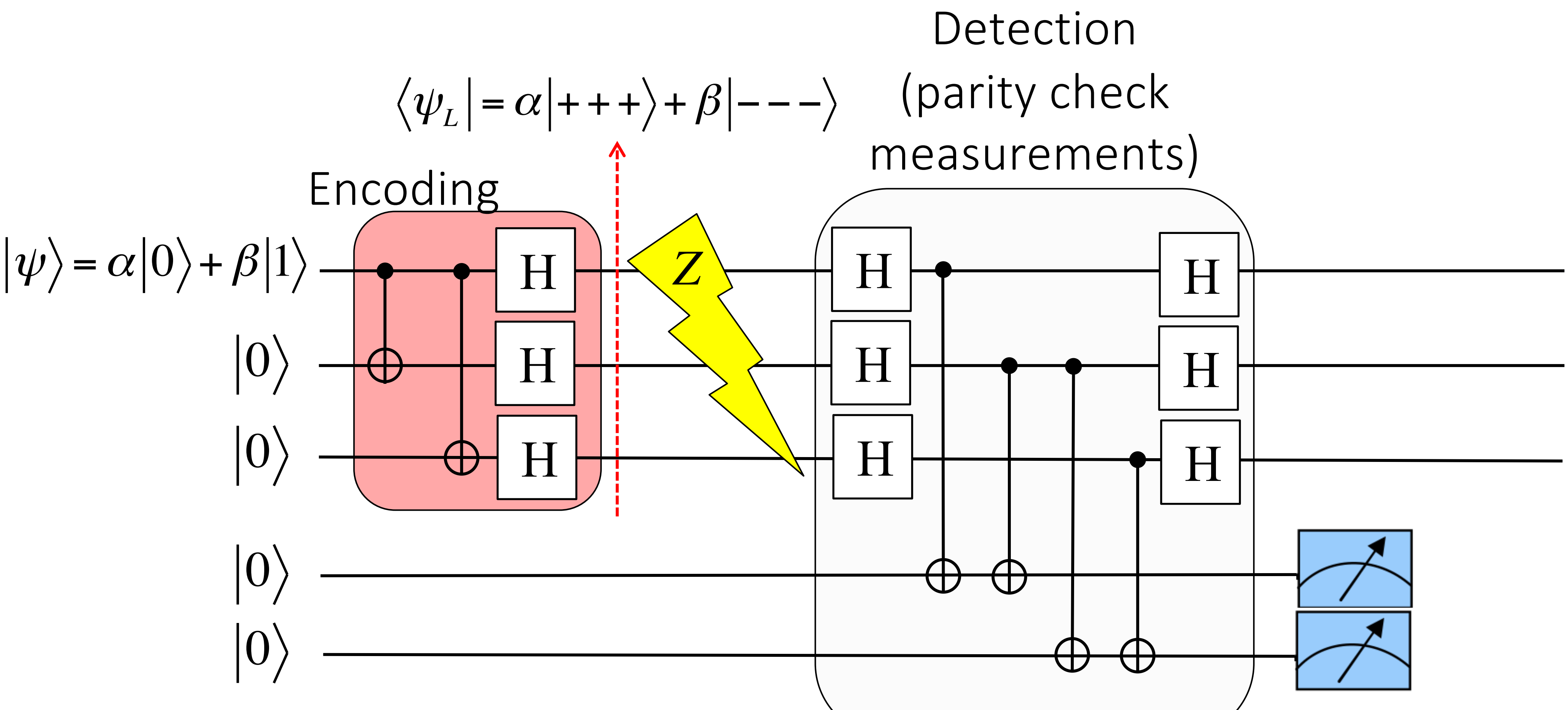

| Error location | Final state, $|data\rangle|ancilla\rangle$ |
|---|---|
| No Error | $\alpha|000\rangle|00\rangle+\beta|111\rangle|00\rangle$ |
| Qubit 1 | $\alpha|100\rangle|10\rangle+\beta|011\rangle|10\rangle$ |
| Qubit 2 | $\alpha|010\rangle|11\rangle+\beta|101\rangle|11\rangle$ |
| Qubit 3 | $\alpha|001\rangle|01\rangle+\beta|110\rangle|01\rangle$ |

# When two bit-flip errors occur…




| Error location | Final state, $|data\rangle|ancilla\rangle$ |
|---|---|
| No Error | $\alpha|000\rangle|00\rangle + \beta|111\rangle|00\rangle$ |
| Qubit 1 | $\alpha|100\rangle|10\rangle + \beta|011\rangle|10\rangle$ |
| Qubit 2 | $\alpha|010\rangle|11\rangle + \beta|101\rangle|11\rangle$ |
| Qubit 3 | $\alpha|001\rangle|01\rangle + \beta|110\rangle|01\rangle$ |

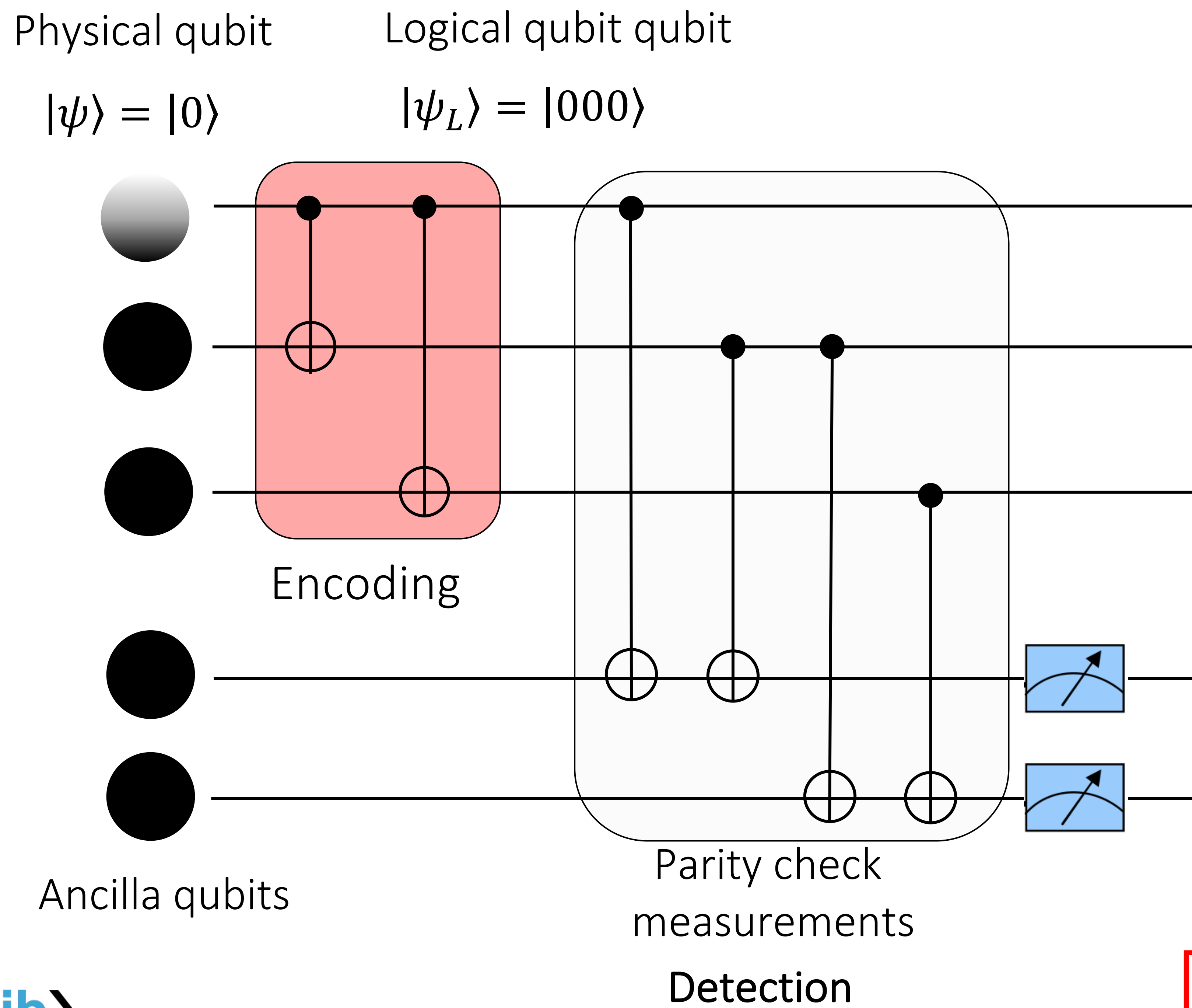
# 3-qubit phase-flip code: single phase-flip errors




| Error location | Final state, $\vert data\rangle\vert ancilla\rangle$ |
|---|---|
| No Error | $\alpha\vert{+}{+}{+}\rangle\vert 00\rangle+\beta\vert{-}{-}{-}\rangle\vert 00\rangle$ |
| Qubit 1 | $\alpha\vert{-}{+}{+}\rangle\vert 10\rangle+\beta\vert{+}{-}{-}\rangle\vert 10\rangle$ |
| Qubit 2 | $\alpha\vert{+}{-}{+}\rangle\vert 11\rangle+\beta\vert{-}{+}{-}\rangle\vert 11\rangle$ |
| Qubit 3 | $\alpha\vert{+}{+}{-}\rangle\vert 01\rangle+\beta\vert{-}{-}{+}\rangle\vert 01\rangle$ |

# Quantum error correction (QEC) - Exercise

**Encoding**

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