# Orbit Determination of the Asteroid 2001 BY60

Team 3 - Moon Knights

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#### **Abstract**

The objective of this project was to determine the orbital elements (6 numbers that pinpoint the exact location and orientation of an astronomical body) of the Mars-crossing asteroid 2001 BY60, update information on the NASA Jet Propulsion Lab (JPL) Horizons database, and predict the asteroid's future trajectory. Over the course of five weeks, my team used a 20" telescope to collect five sets of 15 frames and selected three light frames from each to process. Then, using reference stars near the asteroid, we processed these 15 images and determined the asteroids Right Ascension and Declination (celestial coordinates). From this, we coded Method of Gauss programs to determine the asteroids position and velocity of the asteroid for the five sets of three frames. The results were then verified with Method of Laplace programs. Using these position and velocity vectors, the orbital elements of the asteroid were calculated. We compared the calculated orbital elements with JPLs values for 2001 BY60 and found that JPLs values were all within the margin of error. This means that the results can confirm JPLs values and it is improbable that the orbit of the asteroid has been affected by other passing bodies since the last time JPL Horizons updated its data. The results also indicated that 2001 BY60 is stable and likely will not collide with other celestial bodies in the future.

#### 1. Introduction

4.6 billion years ago, our Solar System would have been unrecognizable. In place of the Sun and its planets was a solar nebula, a wispy cloud of gas and dust, which eventually collapsed onto itself and formed a flat spinning disk. 99.8% of the material consolidated to form the Sun. Other larger clumps left over formed planets and moons. The chunks of debris left over from this cosmic construction site are the asteroids and the comets that orbit the Sun. [1]

Although asteroids and comets both formed at around the same time, they can be distinguished based on composition. Since asteroids were formed in the hot, inner part of the nebula [2] [3], where it was too warm for ice to remain solid, they are made of metal and rock. Comets, on the other hand, were formed in the cooler, outer part of the nebula and are therefore composed of ice as well as metal and rock.

Most asteroids orbiting the Sun are located in the main asteroid belt, between Mars and Jupiter. Other asteroids, Trojans, are in the same orbit around the sun as Jupiter [4]. Comets, on the other hand, are found farther away from the Sun in the Kuiper Belt (past Neptunes orbit), and the Oort Cloud (the edge of the Solar system) [5].

Near Earth Asteroids (NEAs) are asteroids whose orbits can bring them near Earth. Astronomers classify asteroids with a perihelion distance of less that 1.3 AU as NEAs. The ma-

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jority of NEAs originate from the main asteroid belt and are pushed towards the sun by collisions with asteroids or the gravitational effect of Jupiter. Mars Crossing Asteroids (MCAs) are asteroids whose orbits cross that of Mars.

Because of the proximity of NEAs and MCAs with the Earth and the possibility of one striking Earth, it is important to keep track of their orbit and future paths. If an asteroid is gravitationally nudged by another object, it could be redirected into a path that intersects Earth. If we know about an Earth-bound asteroid soon enough, we can take measures to reduce damage and casualties, or even deflect it.

History has highlighted the importance of the early detection of asteroids. Asteroids like the 15 kilometer monstrosity that wiped out the dinosaurs are extremely rare, so the smaller NEAs actually pose more of a threat. In February, 2013, a 20 meter meteor broke over Chelyabinsk, Russia and caused more than 1,500 casualties. Just earlier this year on June 2nd, an asteroid penetrated the atmosphere and disintegrated just 50 kilometers above South Africa. Scientists only saw it coming a day in advance. If this had been a larger asteroid, there would not have been sufficient time to deflect it. [6]

As of now, astronomers have mapped out 8,000 near-Earth asteroids that are at least 140 meters wide, big enough to wipe out an entire state. However, as there are about 25,000 such space rocks around Earth, we need to step up our asteroid-hunting game and find the other two-thirds as soon as possible.[7] The government has recognized this need and has taken action to speed up asteroid mapping. In the 1990s, Congress directed NASA to find 90 percent of the NEOs that are at least 0.6 miles (1 kilometer) in diameter. Furthermore, in 2005, NASA was instructed to spot 90 percent of all NEOs 460 feet and larger by the end of 2020. SSP strives to contribute to the effort of asteroid cataloging, so each group chose an asteroid and tracked it for about a month, hoping to verify NASA's data.

We chose the Mars-crossing asteroid 2001 BY60, and used Gausss method for orbital determination to convert three roughly equally-spaced observations of the asteroids RA and Dec to position and velocity vectors for our asteroid. From there, we calculated the six orbital elements necessary to describe the orbit of any body: a and e (which describe the size and shape of the ellipse);  $\Omega$ , i, and  $\omega$  (which describe the orientation of the ellipse with respect to the orbital and reference planes); T (time of perihelion passage); and M (which describes the asteroid's position at a given time) [8].

### 2. Observations and image processing

#### 2.1. Methods

We took 3 sets of 5 pictures of our asteroid every other night (weather permitting) on the West Telescope at the Sommers-Bausch Observatory (254°44'15.0" E, 40°00'14.5"N, Altitude 1656.6 m) in Boulder, Colorado. The telescope was a PlaneWave CDK20 reflecting telescope, with a 20 inch mirror, and the pictures were taken by a STF-8300 CCD camera.

Our observations are listed in table 1.

Table 1: Journal of Observations

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1 Dark our test picture. It was fairly cloudy and		1 Dark	our test picture. It was fairly cloudy and		
the picture quality was average. There was			the picture quality was average. There was		
no wind or moon.					
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Using Maxim DL Pro 6, we processed our images. This allowed us to find the x-coordinates, y-coordinates, and magnitudes of 6-12 nearby reference stars, as well as those of our asteroid. We chose reference stars that were far away from the center of the image to ensure that our linear regression best approximated the curvature of the image. We also made sure there were no other stars close enough to the reference stars to affect the centroid determination. We then pulled up a catalog of the reference stars RA and Dec values in DS9, and ran Least Squares Plate Reduction (LSPR) on the data to calculate our asteroids RA and Dec.

## 2.2. Results

The Julian dates of our observations and observed RAs and Decs of our asteroid are listed in table 2

Table 2: I	RA and	Decs of	Asteroid
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Date/Time of Observa-	Julian Date	RA of Asteroid	Dec of Asteroid
tion (UTC)			
7/7/2018 5:38:17.940	2458306.735	17h 56m 55.67s	28°22' 20.2"
7/7/2018 5:58:12.320	2458306.749	17h 56m 55.51s	28°21' 50.1"
7/7/2018 6:18:55.920	2458306.763	17h 56m 55.40s	28°21' 28.60"
7/9/2018 5:27:31.100	2458308.727	17h 56m 16.77s	27°21' 42.5"
7/9/2018 5:51:17.190	2458308.744	17h 56m 16.40s	28°21' 42.5"
7/9/2018 6:08:20.250	2458308.756	17h 56m 16.13s	27°20' 49.1"
7/11/2018 5:06:47.829	2458310.713	17h 55m 43.80s	26°17' 52.0''
7/11/2018 5:24:27.309	2458310.725	17h 55m 43.55s	26°17' 27.6''
7/11/2018 5:44:26.800	2458310.739	17h 55m 43.27s	26°17' 0.1"
7/13/2018 5:00:24.899	2458312.709	17h 55m 17.17s	25°10′ 36.0′′
7/13/2018 5:21:56.000	2458312.724	17h 55m 16.93s	25°10′ 5.1″
7/13/2018 5:41:17.420	2458312.737	17h 55m 16.70s	25°09' 37.3"
7/19/2018 5:01:3.339	2458318.709	17h 54m 41.87s	21°31' 34.7"
7/19/2018 5:23:19.66	2458318.725	17h 54m 41.80s	21°30′ 59.1"
7/19/2018 5:41:34.439	2458318.737	17h 54m 41.75s	21°30′ 29.9″

# 2001 BY60 is shown in Fig. 1.

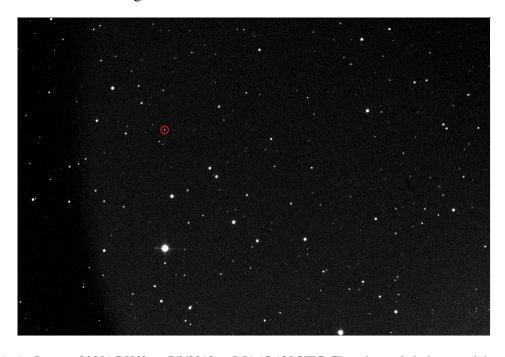


Figure 1: An Image of 2001 BY60 on 7/9/2018 at 5:51:17.190 UTC. There is a red circle around the asteroid.

#### 3. Orbit determination

#### 3.1. Methods

To determine the orbital elements of our asteroid, we used the Method of Gauss to calculate the position and velocity vectors for the second of three observations, given the RA and Dec values for all 3 observations. We first converted our observations to Earth-asteroid unit vectors in Cartesian coordinates. The conservation of angular momentum ensures that all three position vectors and all three velocity vectors are in the same plane, so the second position and velocity vectors can be expressed as linear combinations of the first and third. This allowed us to set up a system of linear equations to express the vectors in terms of each other and several time-dependent constants. However, to solve for the time-dependent constants, we needed the orbital elements, so we approximated the constants using a 2nd order Taylor series. With this approximation, we generated a rough initial guess for our position and velocity, and then iteratively improved the guesses using higher-order Taylor approximations. We wrote a Python program to run this process continuously until we converged on accurate position and velocity vectors.

We also used the Method of Laplace to check our results from the Method of Gauss. The Laplacian method uses a truncated Taylor series to express the position vector as a function of its derivative and second derivative, which gives us a system of vector equations we can solve for position and velocity. However, the Laplacian method is known to be less accurate than the Gaussian method for asteroids closer to the sun, so we did not run our entire OD using Laplaces method. We have included a comparison of the position and velocity vectors generated by Gausss method and Laplaces method for completeness.

After determining position and velocity, we used the conservation of angular momentum and properties of ellipses to calculate the orbital elements (a, e,  $\Omega$ , i,  $\omega$ , and M) given the position and velocity. The semi-major axis (a) is the average of the perihelion distance and aphelion distance. The eccentricity (e) is a number between 0 and 1 specifying how elliptical the orbit is. The longitude of the ascending node ( $\Omega$ ) specifies the rotation of the orbit in the reference plane. The inclination (i) is the angle between the reference plane and the orbital plane. The argument of perihelion ( $\omega$ ) is the angle of rotation of the orbit in its own orbital plane. The time of perihelion passage (T) is the time where the asteroid is closest to the Sun in its orbit. Finally, the time-dependent mean anomaly (M) is the hypothetical angle between the perihelion and what the asteroids current position would be if the orbit was circular and had the same orbital period as the actual orbit [8]. We wrote a program that performs this process on any given set of three observations.

With our 15 processed images, we were able to run our OD program on 5 different trios of images, generating 5 sets of orbital elements. We used the 5 sets to calculate the standard deviation and uncertainty of our values.

We also wrote an ephemeris generation program that takes in the orbital elements of an asteroid and outputs its expected RA and Dec on any given day. To check our process for self-consistency, we plugged our calculated orbital elements in into the ephemeris generation program. We then calculated the percent difference between the RA and Decs generated from our actual observations and the RA and Decs from the ephemeris generation. [9]

## 3.2. Results

We list our calculated orbital elements in table 3.

Table 3: Orbital Elements

	Set 1	Set 2	Set 3	Set 4	Set 5
Date/Times of Data	7/7/2018	7/7/2018	7/7/2018	7/9/2018	7/9/2018
Used	5:38:17.940	5:58:12.320	6:18:55.92	5:51:17.190	6:08:20.250
	7/13/2018	7/9/2018	7/11/2018	7/13/2018	7/11/2018
	5:21:56.000	5:27:31.100	5:06:47.829	5:41:17.420	5:24:27.309
	7/19/2018	7/11/2018	7/13/2018	7/19/2018	7/19/2018
	5:41:34.439	5:44:26.800	5:00:24.899	5:23:19.66	5:01:3.339
Semi-major Axis (a)	2.2468204914	2.1680371894	2.1500923130	2.2241752716	2.2200542453
(AU)					
Eccentricity (e)	0.3248828011	0.311365880	0.3042321139	0.3198555474	0.3189486676
Inclination (i) (de-	27.055010893	26.298425564	26.215170764	26.890801180	26.860403105
grees)					
Longitude of As-	147.47637254	147.40652889	148.28839374	147.73702277	147.78581358
cending Node $(\Omega)$					
(degrees)					
Argument of Perihe-	118.60544460	123.94248991	117.13949652	117.16622193	116.88277000
$lion (\Omega) (degrees)$					
Time of Perihelion	2458288.8765	2458297.1953	2458287.4856	2458286.8600	2458286.4608
Passage (T) (Julian					
Days)					
Mean Anomaly at	13.059154	11.20901455	14.38508906	13.85826084	14.01581164
2458333.5 JD (M)					
(Degrees)					

Table 4: Orbital Elements Compared with JPL Values

Orbital Element	Value	JPL Value	Percent Difference (%)
Semi-major Axis (a)	$2.20 \pm 0.04 \text{ AU}$	2.223446326 AU	0.98
Eccentricity (e)	$0.316 \pm 0.007$	0.320	1.21
Inclination (i)	$26.70 \pm 0.34^{\circ}$	26.9°	0.83
Longitude of Ascending	147.7 ± 0.3°	148°	0.00028
Node $(\Omega)$			
Argument of Perihelion ( $\omega$ )	119.0 ± 2.7°	117°	1.30
Time of Perihelion Passage	2458290 ± 4 JD	2458286.937 JD	0.000099
(T)			

Table 5: Self-Consistency Test

Dates	RA Percent Error (%)	Dec Percent Error (%)
7/7/2018 5:38:17.940	-7.97E-07	-5.74E-07
7/7/2018 5:58:12.320	-5.79E-9	3.09E-8
7/7/2018 6:18:55.920	-1.15E-07	3.65E-8
7/9/2018 5:27:31.100	5.98E-11	-9.36E-09
7/9/2018 5:51:17.190	-1.06E-07	-1.82E-08
7/9/2018 6:08:20.250	-4.69E-9	2.29E-08
7/11/2018 5:06:47.829	-1.07E-10	-9.52E-09
7/11/2018 5:24:27.309	5.84E-11	-9.59E-09
7/11/2018 5:44:26.800	2.37E-9	3.16E-08
7/13/2018 5:00:24.899	0.00E+00	3.55E-08
7/13/2018 5:21:56.000	0.00E+00	-9.99E-09
7/13/2018 5:41:17.420	0.00E+00	-9.92E-09
7/19/2018 5:01:3.339	0.00E+00	3.13E-06
7/19/2018 5:23:19.66	0.00E+00	1.04E-06
7/19/2018 5:41:34.439	0.00E+00	8.87E-07

Fig. 2 shows a plot of the orbit of 2001 BY60.

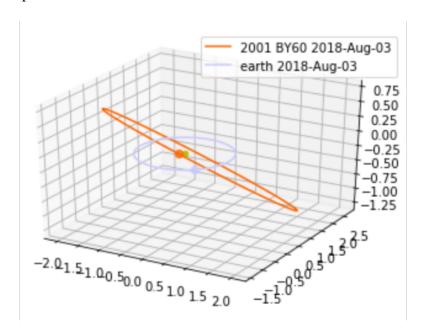


Figure 2: Plot of the orbit of 2001 BY60

We also used Laplace's method to find the position vector and velocity vector of 2001 BY60 on the same test dates that we used the Method of Gauss for. We compared these vectors with what we got from the Method of Gauss to check our values.

Table 6: Position and Velocity Vectors from Gaussian and Laplacian Methods

	Set 1	Set 2	Set 3	Set 4	Set 5
Date/Time	7/13/2018	7/9/2018	7/11/2018	7/13/2018	7/11/2018
	5:21:56.000	5:27:31.100	5:06:47.829	5:41:17.420	5:24:27.309
Position Vec-	[0.34403936,	[0.28330940,	[0.3136791,	[0.34428529,	[0.31352121,
tor (Gaussian)	-1.4866935,	-1.4658349,	-1.4707788,	-1.4843302,	-1.4890556,
	-0.0903594]	-0.090911]	-0.0930926]	-0.0915062]	-0.0840925]
Velocity Vec-	[0.88748823,	[0.89623398,	[0.89195082,	[0.88746412,	[0.89067596,
tor (Gaussian)	0.16032759,	0.15857097,	0.14877881,	0.15660781,	0.14134238,
	-0.2234416]	-0.2179795]	-0.2111054]	-0.2201823]	-0.2204387]
Position Vec-	[0.34409789,	[0.28267121,	[0.31278469,	[0.34504937,	[0.31505520,
tor (Laplace)	-1.4838509,	-1.5051447,	-1.5187757,	-1.4472477,	-1.4068125,
	-0.0916954]	-0.0705650]	-0.0693693]	-0.1089283]	-0.1247306]
Velocity Vec-	[0.88727696,	[0.89328768,	[0.89413767,	[0.88277670,	[0.87758438,
tor (Laplace)	0.16167428,	0.16926882,	0.15174095,	0.16192386,	0.15625823,
	-0.2203355]	-0.2495024]	-0.2587181]	-0.1824077]	-0.1301732]
Percent	[0.01700982,	[0.22551550,	[0.28552761,	[0.22168481,	[0.48808525,
Difference	-0.1913861,	-2.6462508,	-3.2109708,	-2.5298640,	-5.6800312,
(Position) (%)	-1.4676660]	-25.199696]	-29.204776]	-17.384283]	-38.921043]
Percent	[0.02380828,	[0.32928424,	[0.2448753,	[0.52958006,	[1.48073000,
Difference	0.83645053,	6.52626558,	1.97134281,	3.33784772,	10.0240696,
(Velocity) (%)	-1.3998707]	-13.486232]	-20.268313]	-18.765788]	-51.490221]

#### 4. Discussion

All of our values for the orbital elements were close to JPLs values. Our largest percent difference was 1.3% (for the argument of perihelion), and JPLs values were within the margin of uncertainty for every single value. This lets us conclude that our data and methods were reasonably accurate and gives us confidence in JPLs values. When performing a self-consistency test on our data, our expected RA and Dec values matched our actual RA and Dec values to about one millionth of a percent, which gives us further confidence that our methods were sound.

Our process did have several sources of error, however. First of all, we used Newtons method (which is an approximation) to calculate the roots of several functions that were analytically unsolvable. We also used truncated Taylor series approximations in the Gaussian Method for orbital determination, and finding the RA and Dec of our asteroid from the pictures is an inexact process, with room for mistakes if the centroid of the stars was miscalculated because of background noise. Finally, rounding errors further added to our uncertainty. Improving the results could be possible with higher-order Taylor series approximations and non-linear higher-order regression on our images. Using data structures with higher numerical precision would also improve our accuracy.

We all enjoyed working on this project. It gave us an opportunity to try experimental science, and we were able to experience its intensity. Spending late nights taking pictures with the telescope as a team was our favorite part of the project. On the other hand, though processing the images and finding reference stars was significantly less enjoyable, it was equally important. Seeing

the tiny speck of moving light after animating our images filled us with a sense of pride and accomplishment. Writing the programs to determine the asteroids orbit was difficult, confusing, and sometimes frustrating, but again, the satisfaction we felt after plugging in our asteroid data and seeing the orbital elements pop out made it all worth it. Overall, our project went well and was enjoyable and educational. If we could redo the project, we would try to be more focused while observing, as we could have taken more pictures if we improved our efficiency on the observation deck.

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