Networks with Signed Edges

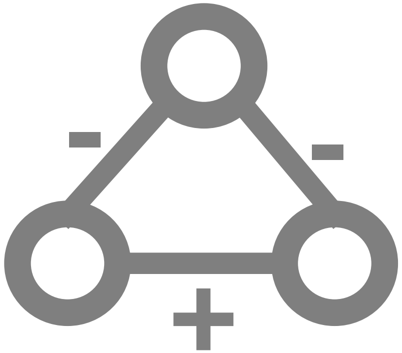
CS224W: Social and Information Network Analysis
Jure Leskovec, Stanford University
http://cs224w.stanford.edu

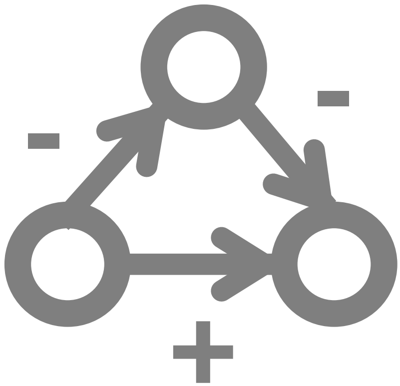
How the Class Fits Together

Observations	Models	Algorithms
Small diameter, Edge clustering	Erdös-Renyi model, Small-world model	Decentralized search
Patterns of signed edge creation	Structural balance, Theory of status	Models for predicting edge signs
Viral Marketing, Blogosphere, Memetracking	Independent cascade model, Game theoretic model	Influence maximization, Outbreak detection, LIM
Scale-Free	Preferential attachment, Copying model	PageRank, Hubs and authorities
Densification power law, Shrinking diameters	Microscopic model of evolving networks	Link prediction, Supervised random walks
Strength of weak ties, Core-periphery	Kronecker Graphs	Community detection: Girvan-Newman, Modularity

Signed Networks

- **Networks with positive and negative relationships**
- Our basic unit of investigation will be **signed triangles**
- First we talk about **undirected** networks then **directed**
- **Plan for today:**
  - **Model:** Consider two soc. theories of signed nets
  - **Data:** Reason about them in large online networks
  - **Application:** Predict if A and B are linked with + or -





Signed Networks

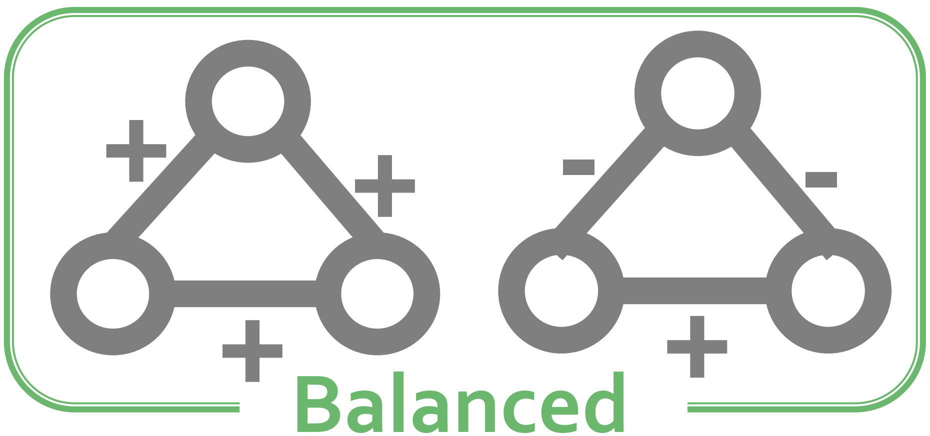
- Networks with positive and negative relationships

- Consider an **undirected complete graph**
- Label each edge as either:
  - **Positive:** friendship, trust, positive sentiment, …
  - **Negative:** enemy, distrust, negative sentiment, …

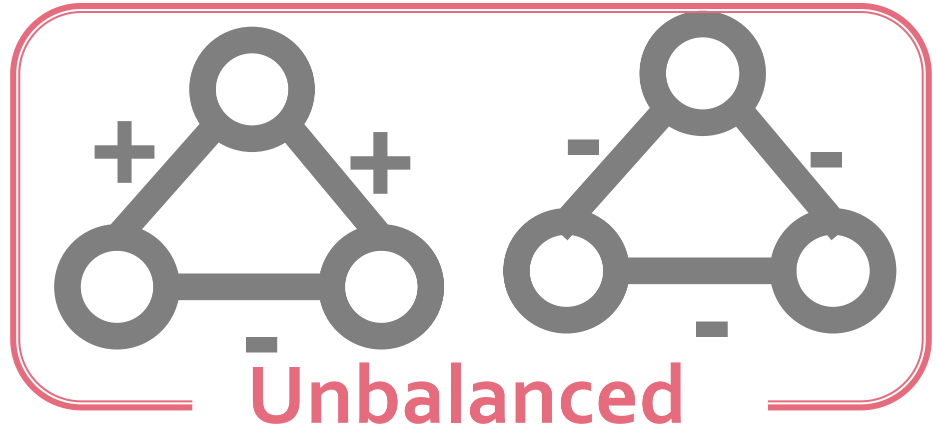
- Examine triples of connected nodes A, B, C

Theory of Structural Balance

- **Start with the intuition** **[Heider '46]:**
  - Friend of my friend is my friend
  - Enemy of enemy is my friend
  - Enemy of friend is my enemy
- Look at connected triples of nodes:



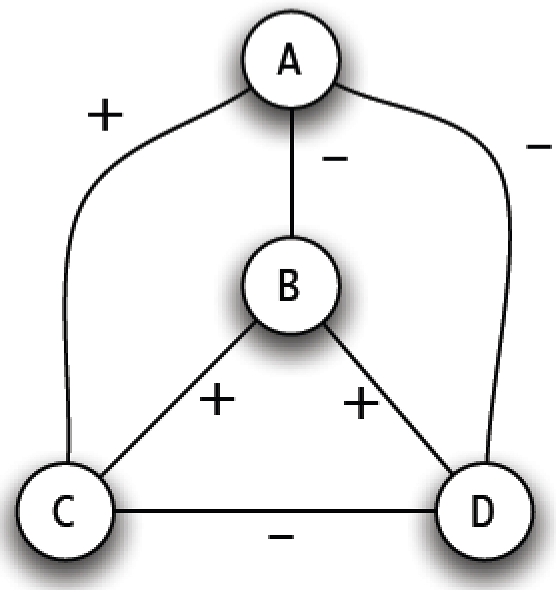
Consistent with "friend of a friend" or "enemy of the enemy" intuition



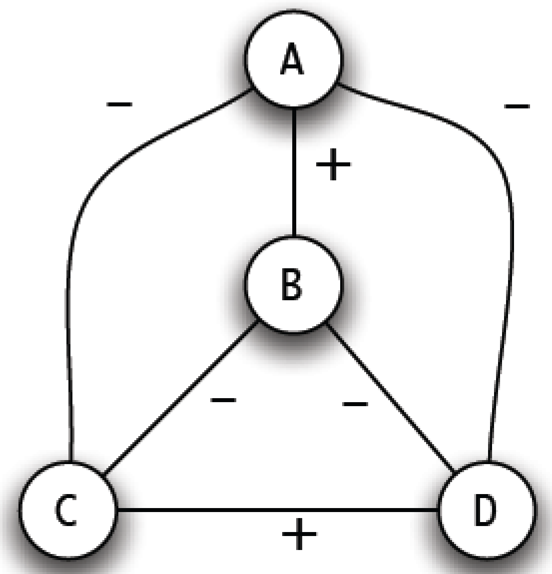
Inconsistent with the "friend of a friend" or "enemy of the enemy" intuition

Balanced/Unbalanced Networks

- Graph is **balanced** if every connected triple of nodes has:
  - All 3 edges labeled +, or
  - Exactly 1 edge labeled +



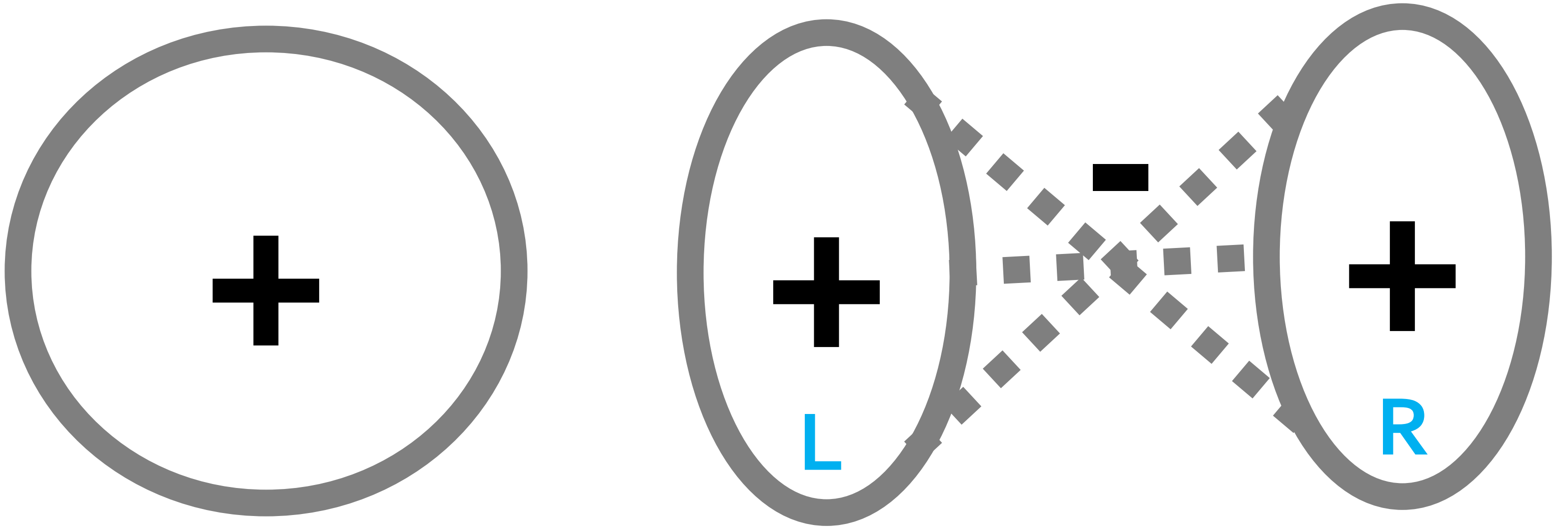
Unbalanced



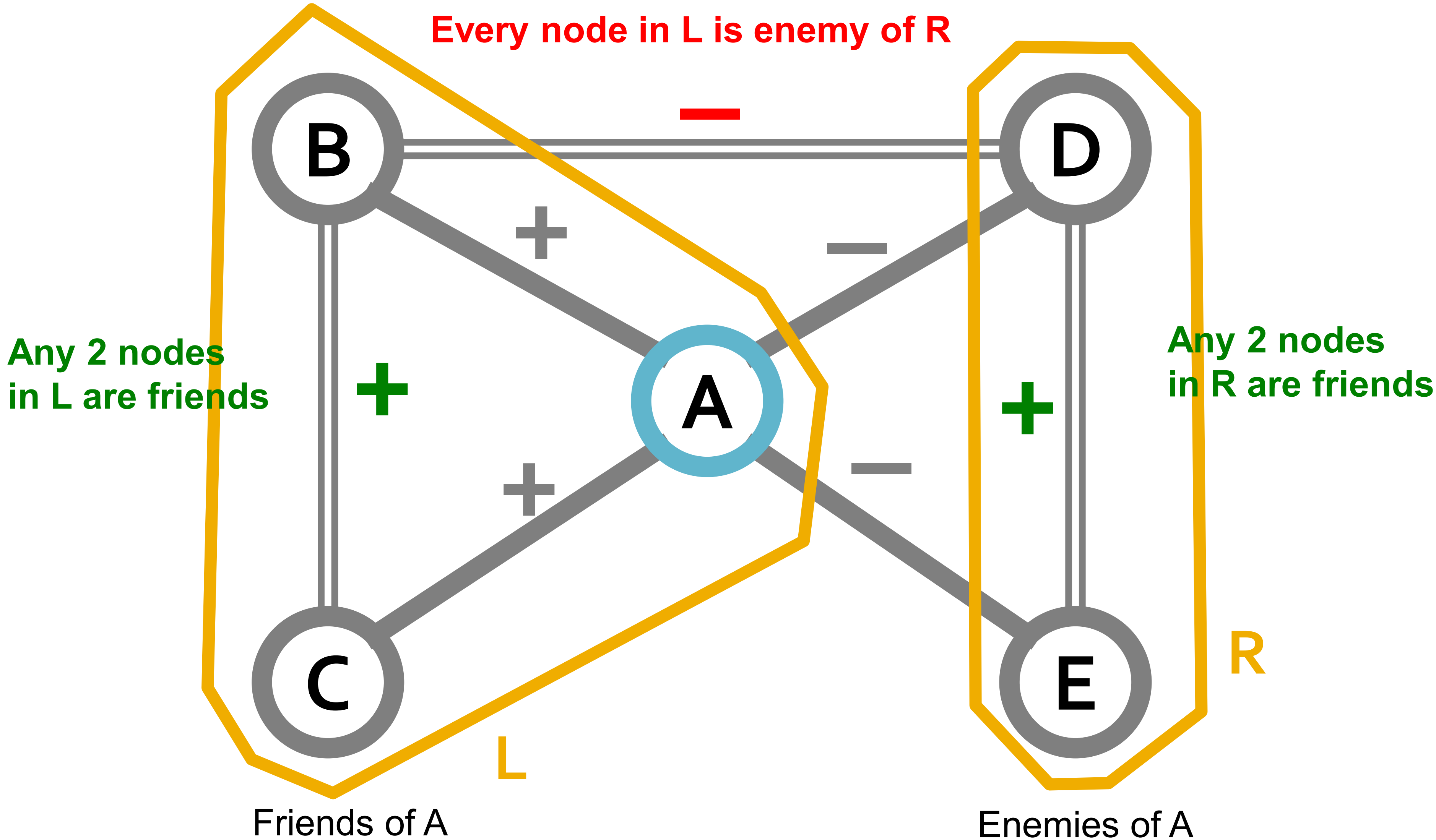
Balanced

Local Balance → Global Factions

- **Balance implies global coalitions** [Cartwright-Harary]
- If **all triangles are balanced**, then either:
  - The network contains only positive edges, or
  - Nodes can be split into 2 sets where negative edges only point between the sets

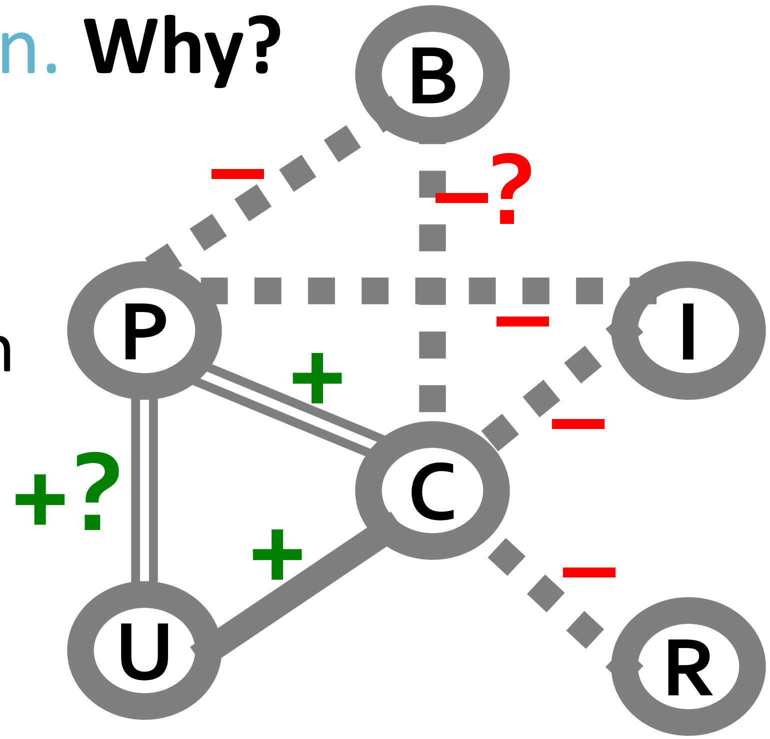


Analysis of Balance

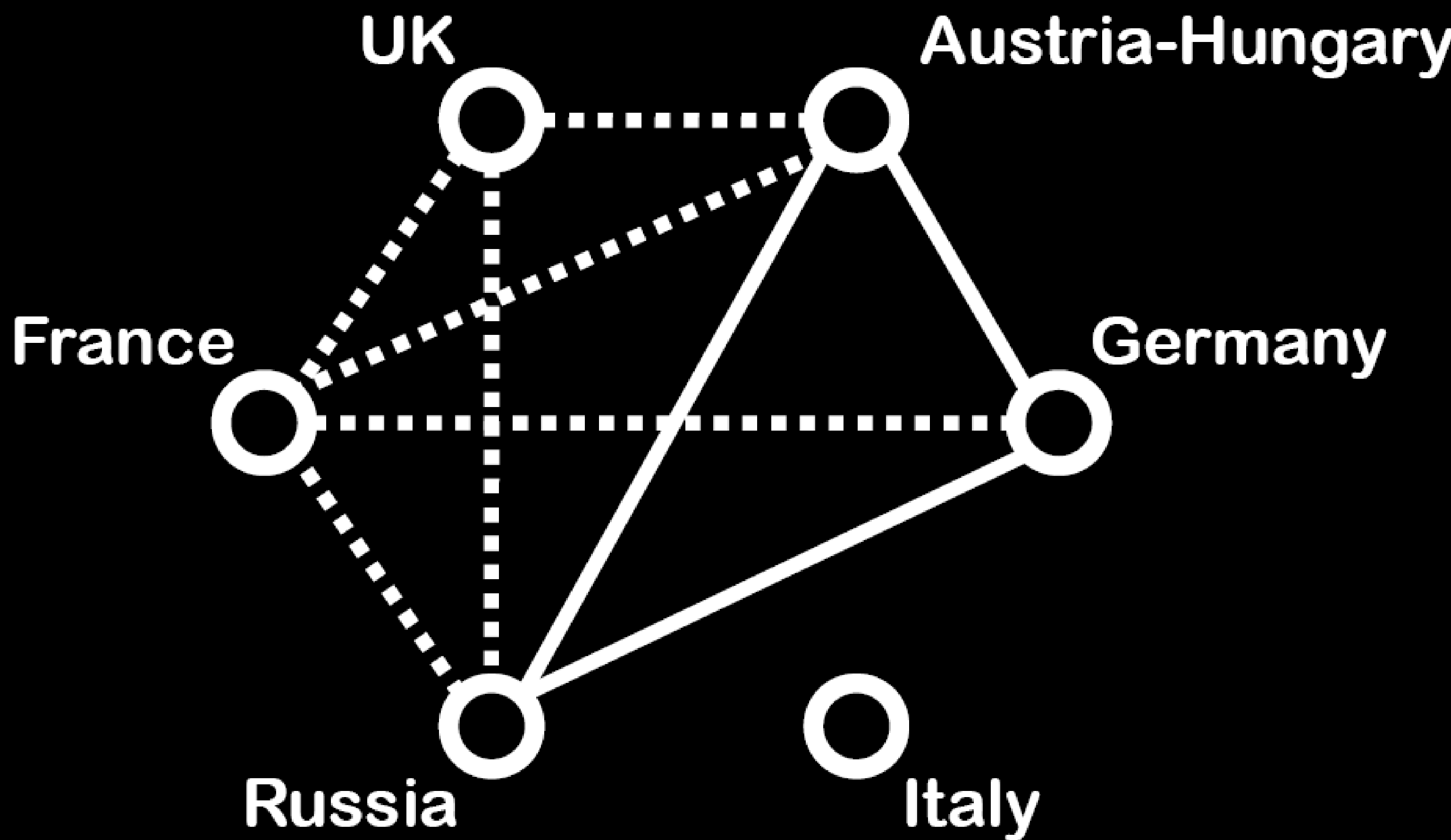


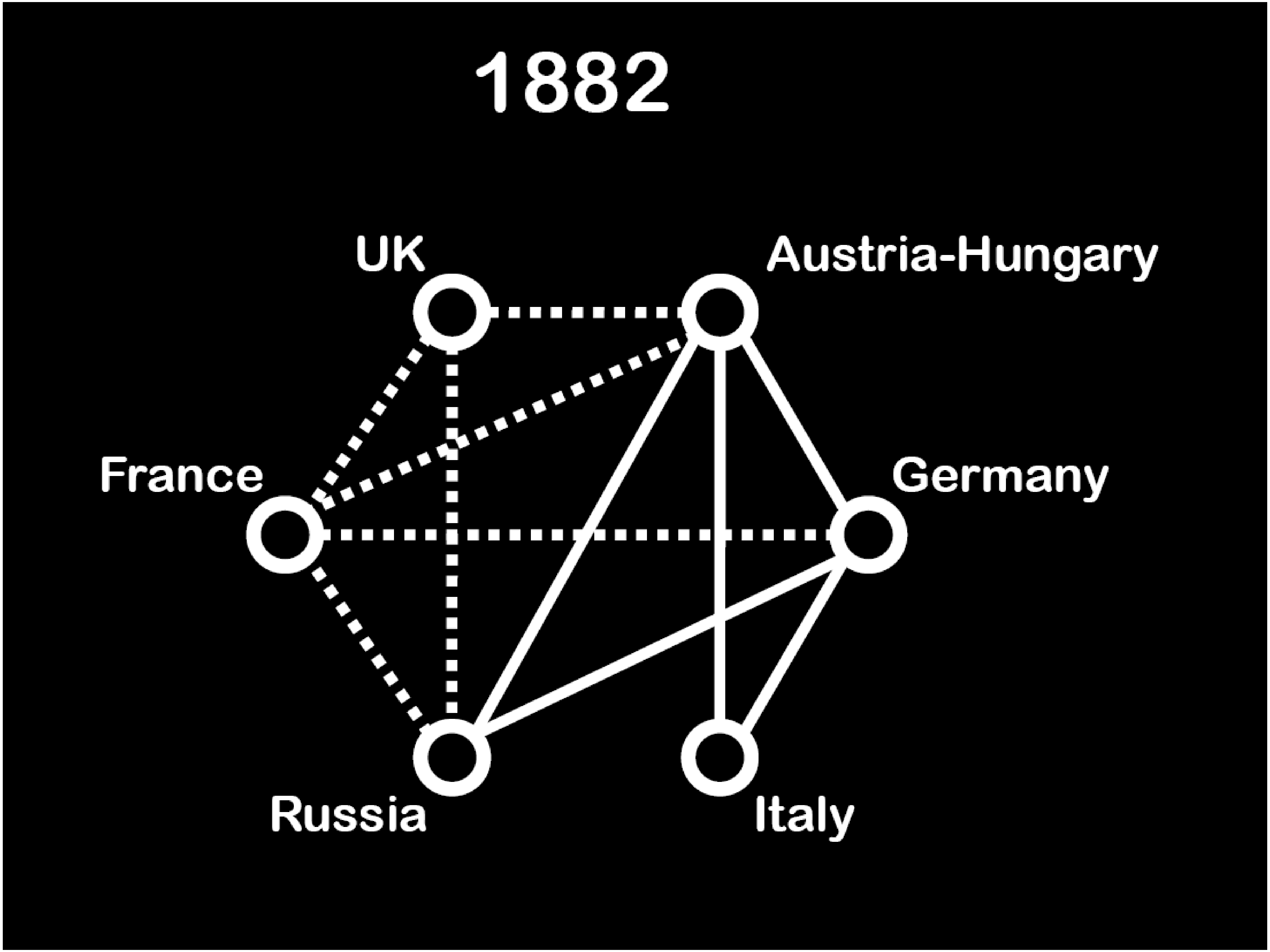
Example: International Relations

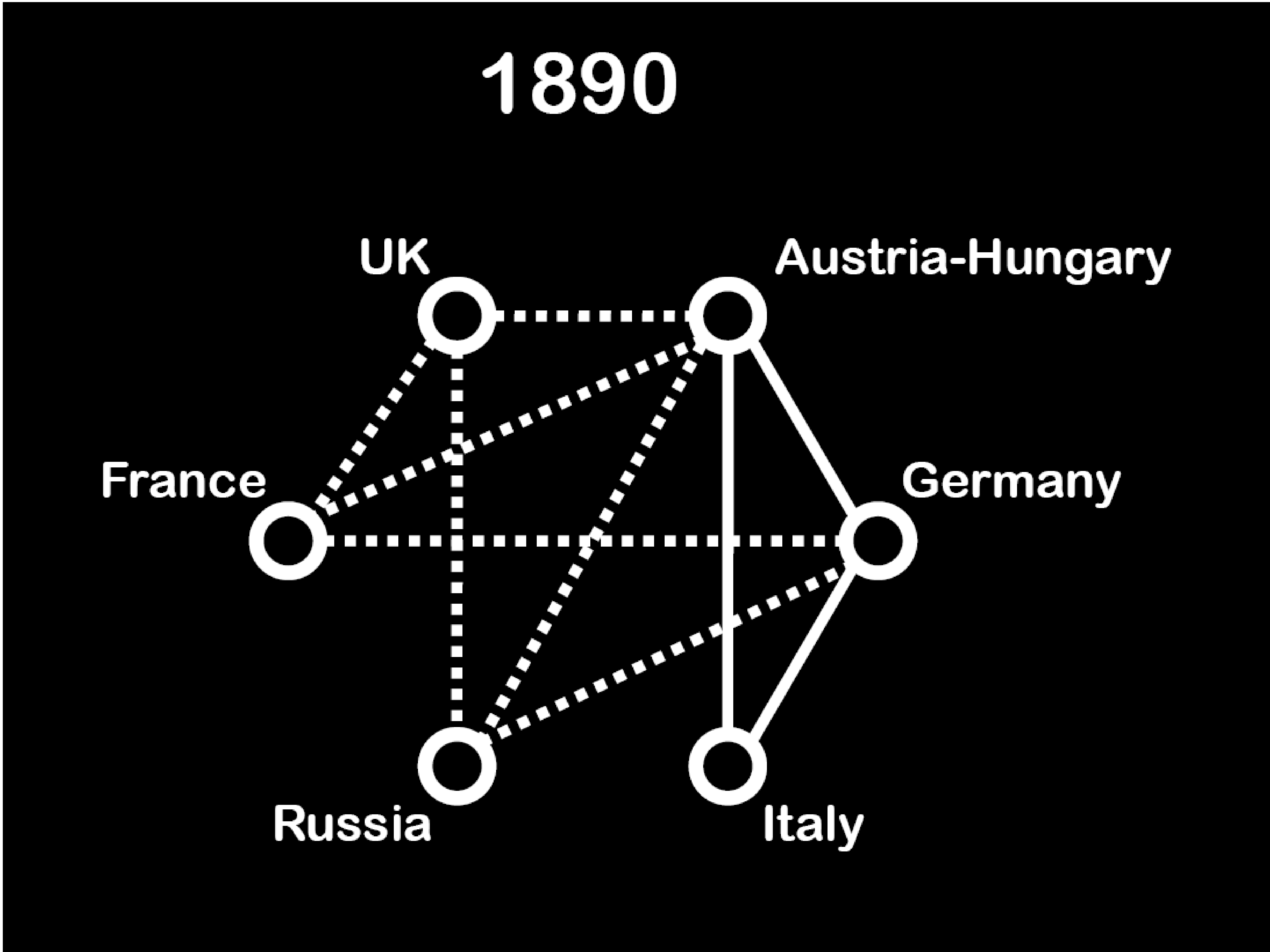
- **International relations:**
  - Positive edge: alliance
  - Negative edge: animosity
- Separation of Bangladesh from Pakistan in 1971: US supports Pakistan. **Why?**
  - USSR was enemy of China
  - China was enemy of India
  - India was enemy of Pakistan
  - US was friendly with China
  - China vetoed Bangladesh from U.N.



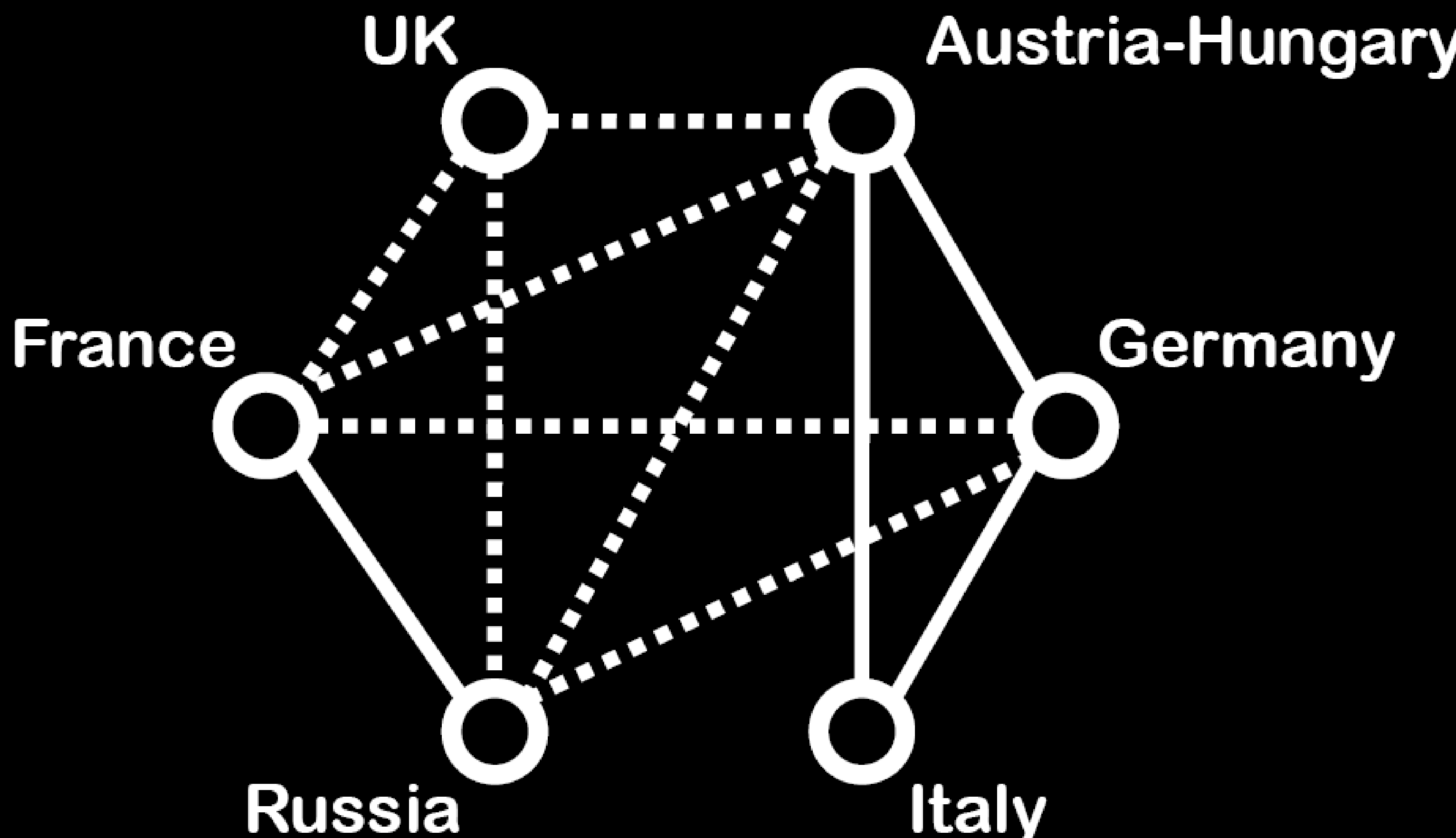
1872-1881

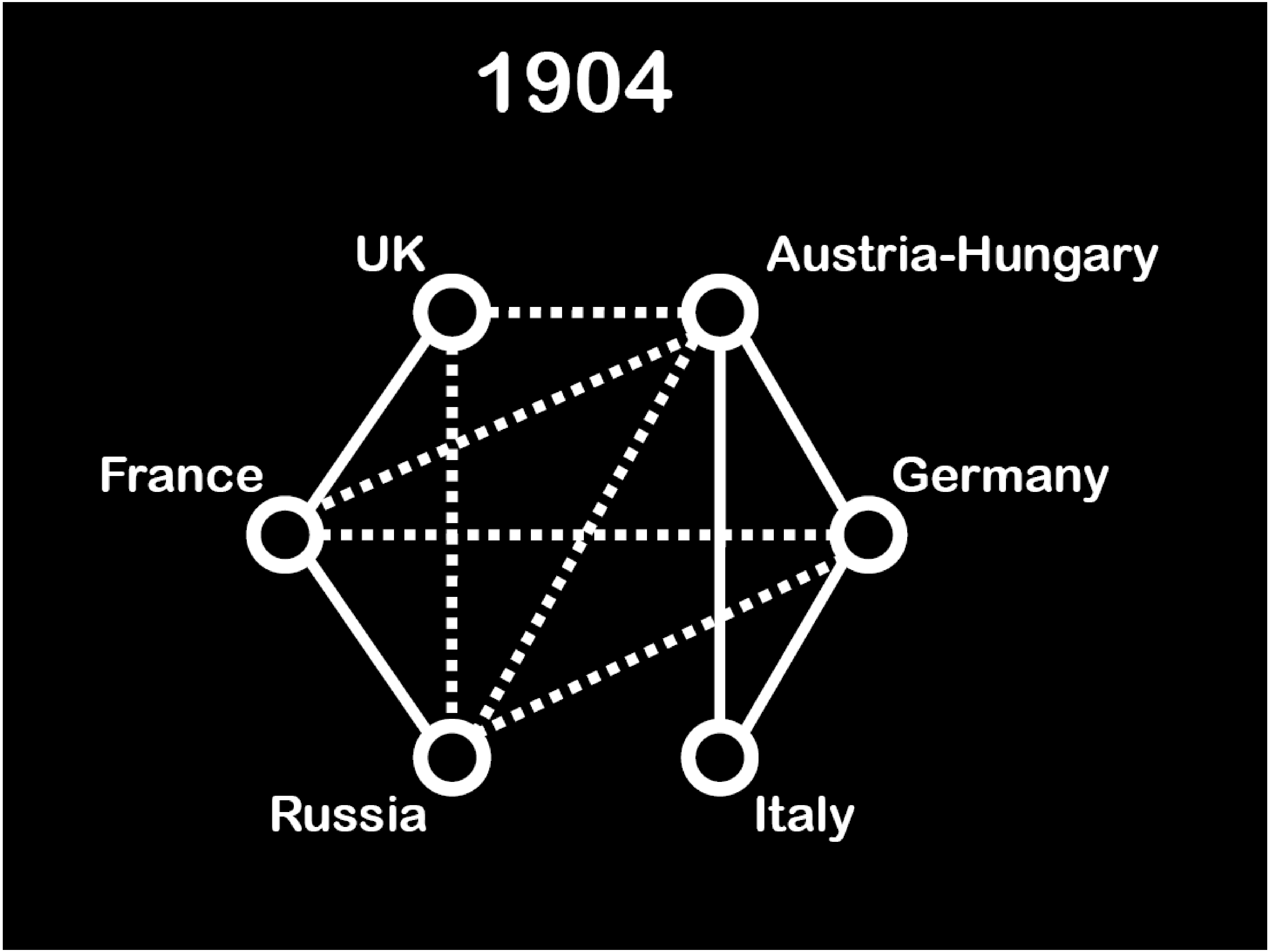


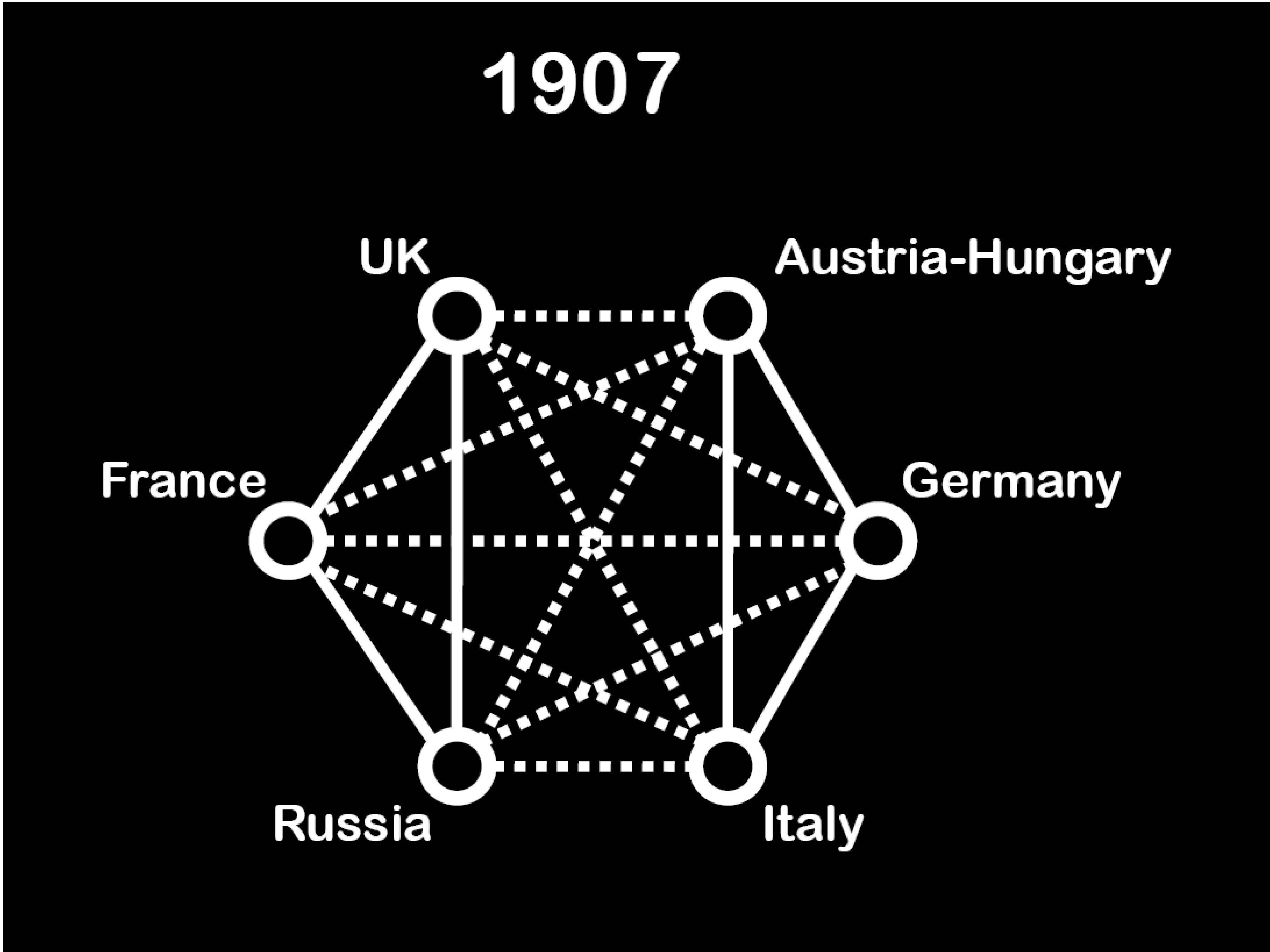

1882
UK
Austria-Hungary
France
Germany
Russia
Italy


1890
UK
Austria-Hungary
France
Germany
Russia
Italy

1891-1894

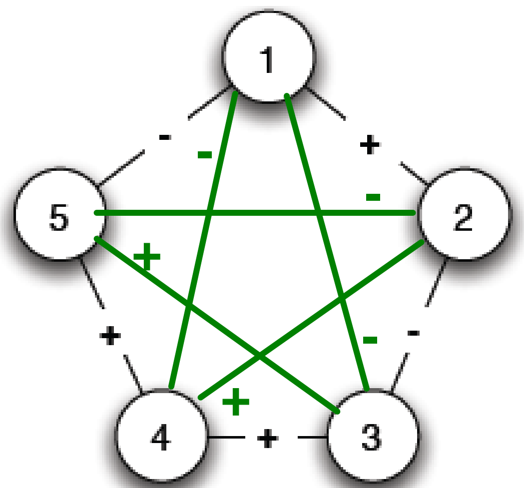



1904
UK
Austria-Hungary
France
Germany
Russia
Italy


1907
UK
Austria-Hungary
France
Germany
Russia
Italy

Balance in General Networks

- So far we talked about complete graphs



Balanced?

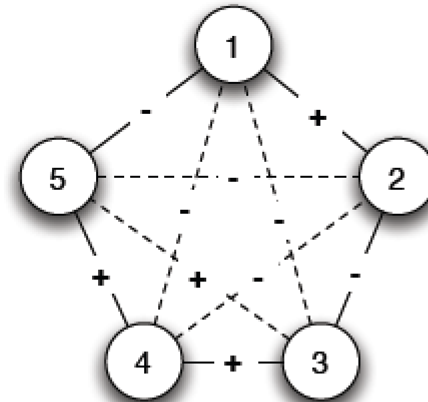
Def 1: Local view

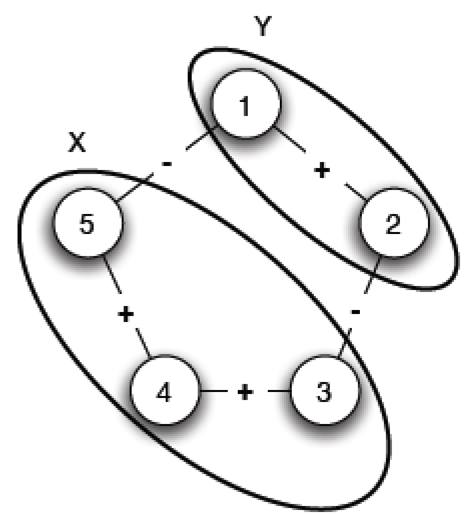
Fill in the missing edges to achieve balance

Def 2: Global view

Divide the graph into two coalitions

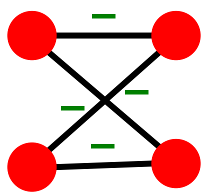
The 2 definitions are equivalent!



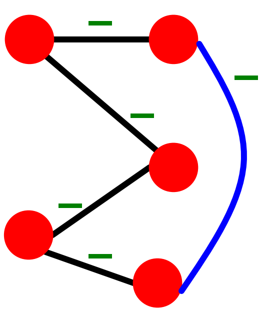


Is a Signed Network Balanced?

- Graph is **balanced** if and only if it contains **no cycle with an odd number of negative** edges
- **How to compute this?**
  - Find connected components on + edges
    - If we find a component of nodes on +edges that contains a –edge ⇒ **Unbalanced**
  - For each component create a super-node
  - Connect components A and B if there is a negative edge between the members
  - Assign super-nodes to sides using BFS

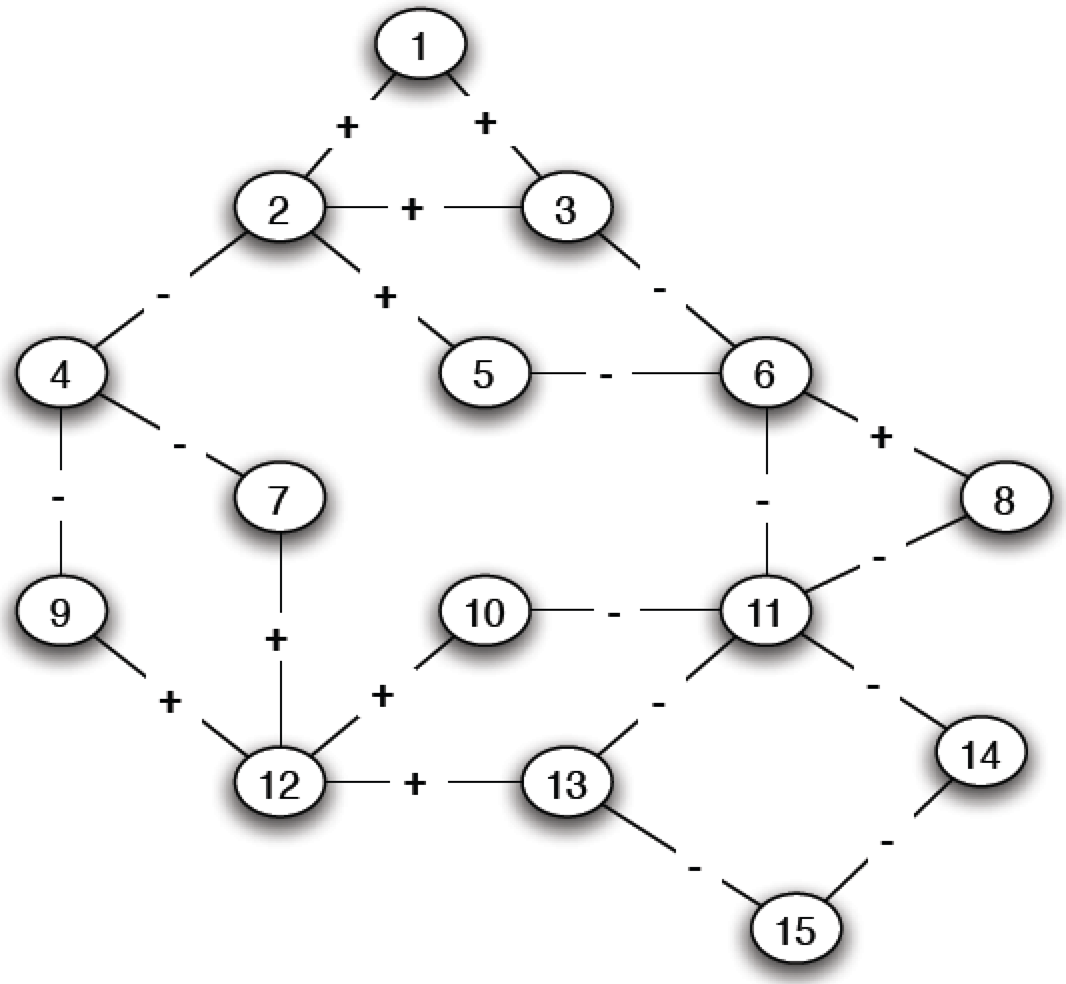


Even length cycle

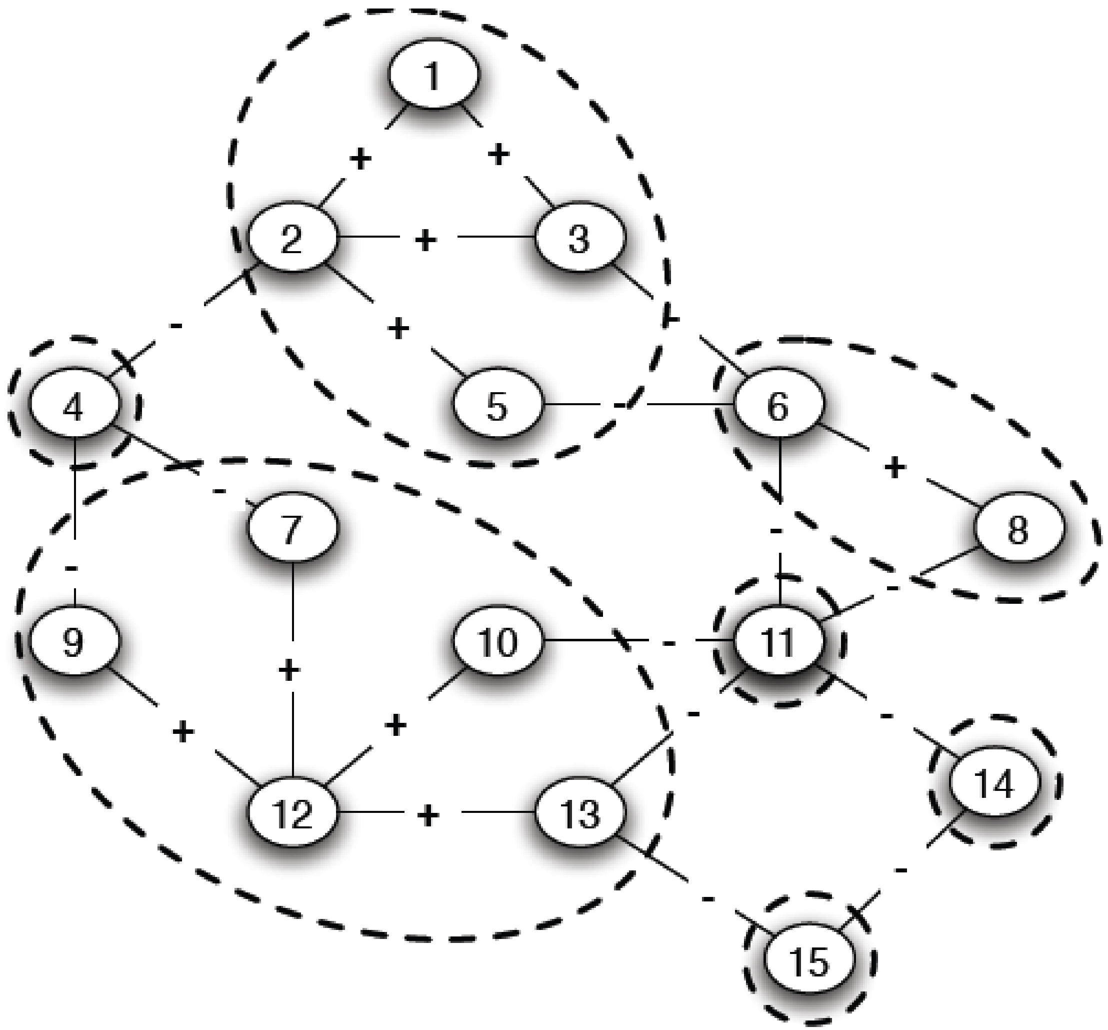


Odd length cycle

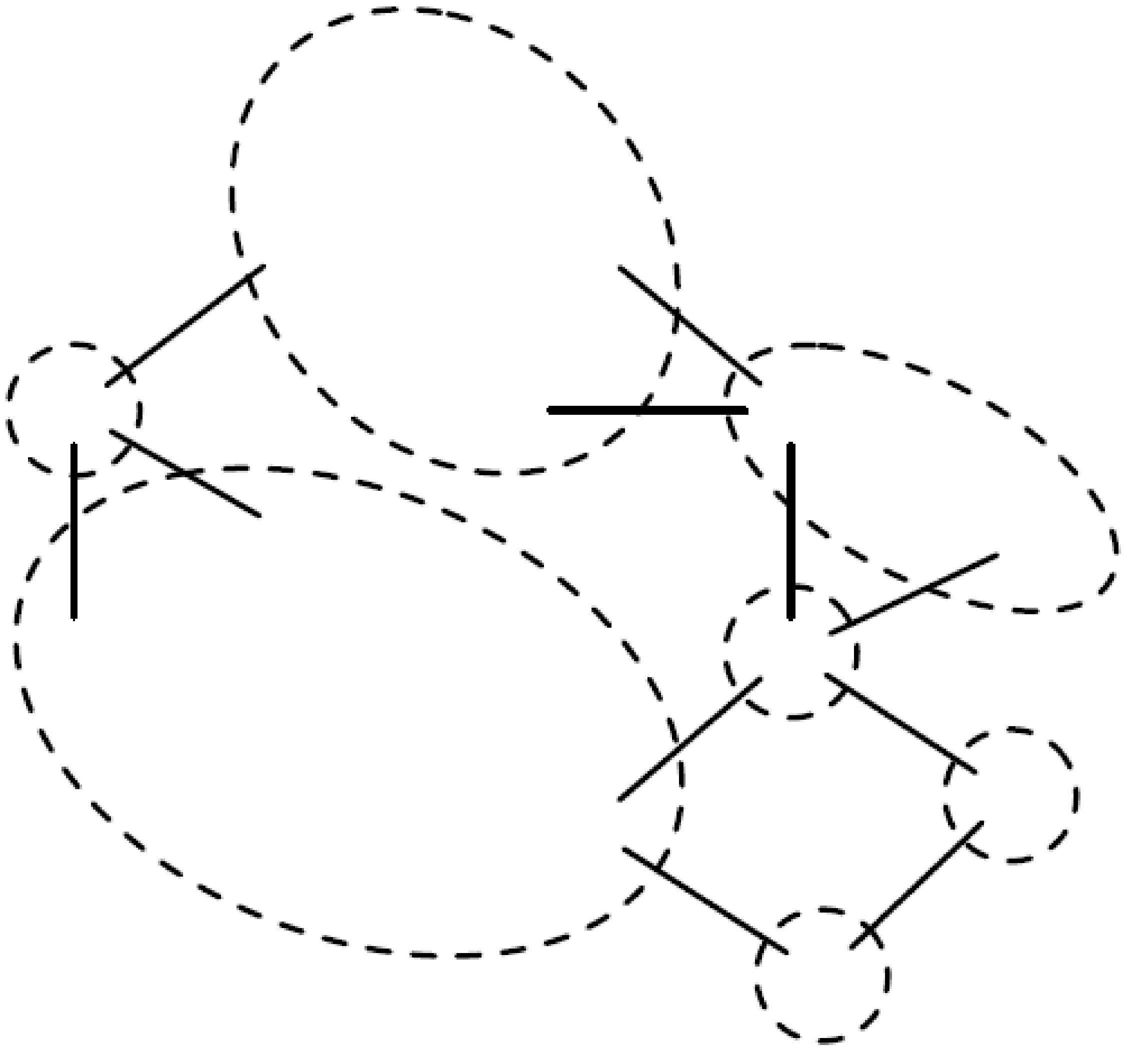
Signed Graph: Is it Balanced?



Positive Connected Components

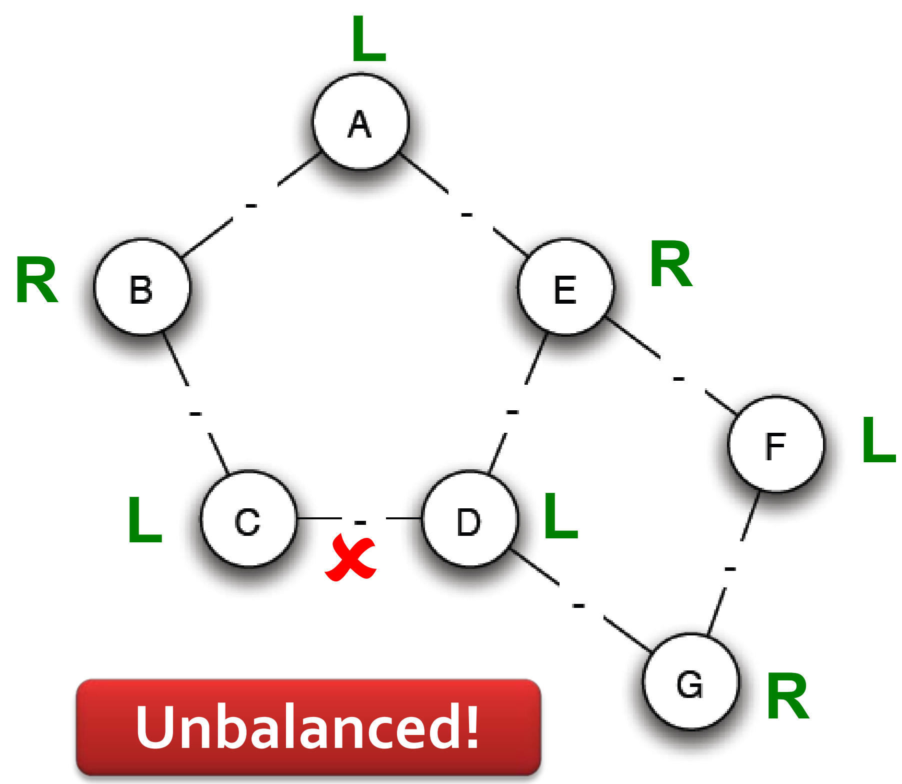


Reduced Graph on Super-Nodes



BFS on Reduced Graph

- Using BFS assign each node a **side**
- Graph is **unbalanced** if any two super-nodes are assigned the **same side**

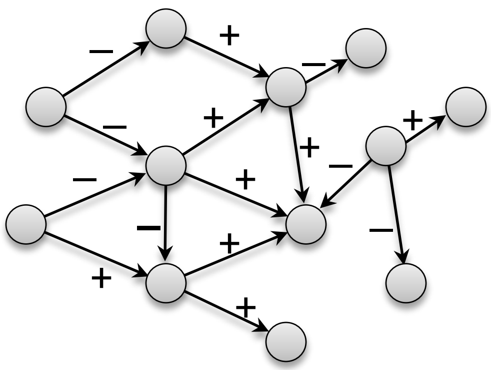


Exploring Real Data

[CHI '10]

Real Large Signed Networks

- Each link A→B is **explicitly** tagged with a sign:
  - Epinions: Trust/Distrust
    - Does A trust B's product reviews?
      (only positive links are visible)
  - Wikipedia: Support/Oppose
    - Does A support B to become Wikipedia administrator?
  - Slashdot: Friend/Foe
    - Does A like B's comments?
  - **Other examples:**
    - Online multiplayer games



	Epinions	Slashdot	Wikipedia
Nodes	119,217	82,144	7,118
Edges	841,200	549,202	103,747
+ edges	85.0%	77.4%	78.7%
– edges	15.0%	22.6%	21.2%

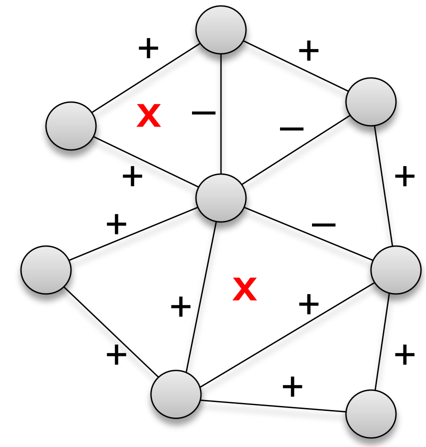
Balance in Our Network Data

- **Does structural balance hold?**
  - Compare frequencies of signed triads in real and "shuffled" data

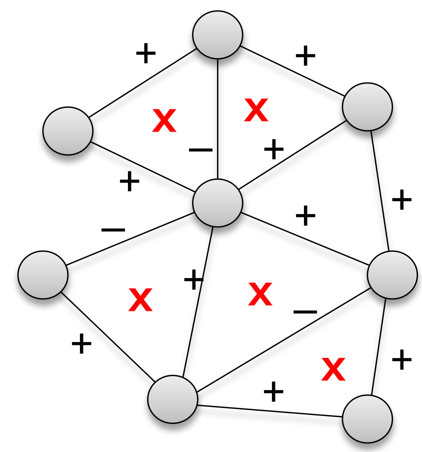
	Triad	Epinions P(T)	Epinions $P_0(T)$	Wikipedia P(T)	Wikipedia $P_0(T)$	Balance
Balanced	+ + +	0.87	0.62	0.70	0.49	✓
Balanced	- - +	0.07	0.05	0.21	0.10	✓
Unbalanced	+ + -	0.05	0.32	0.08	0.49	✓
Unbalanced	- - -	0.007	0.003	0.011	0.010	✗

P(T) fraction of a triads

$P_0(T)$ triad fraction if the signs would be random



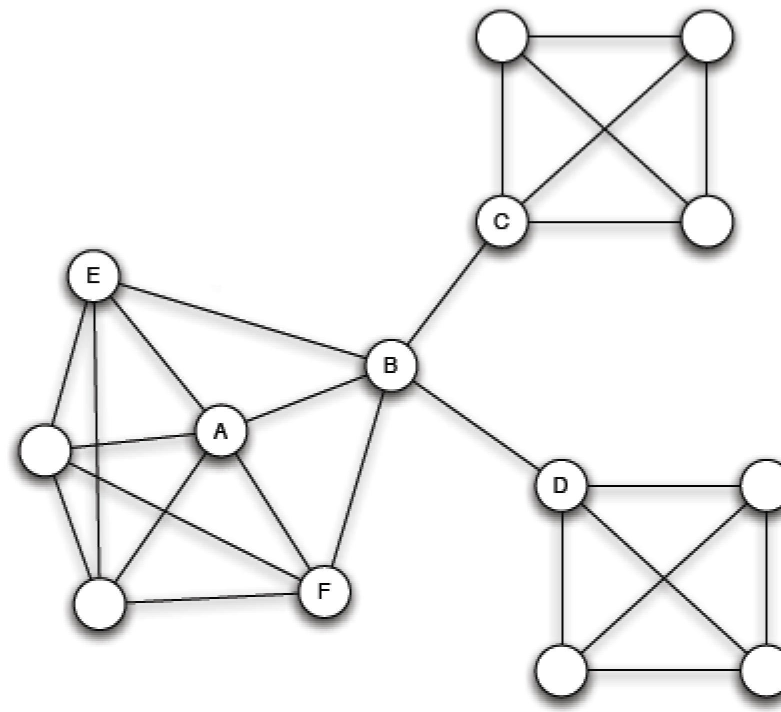
Real data



Shuffled data

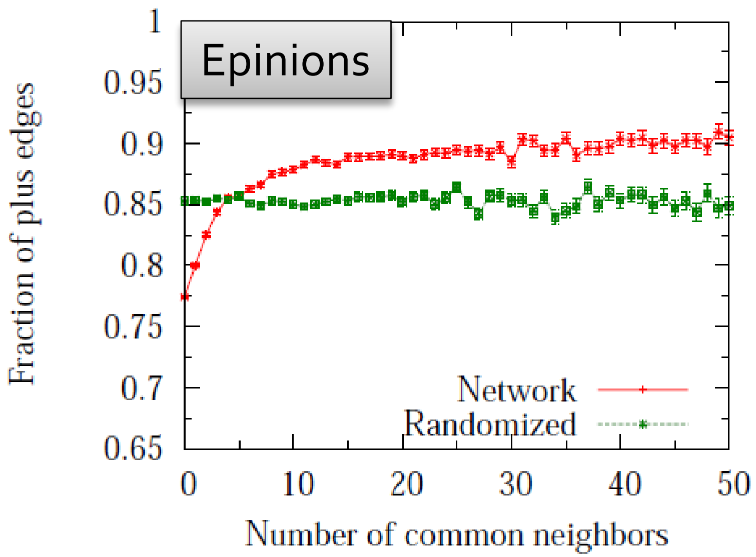
Global Structure of Signed Nets

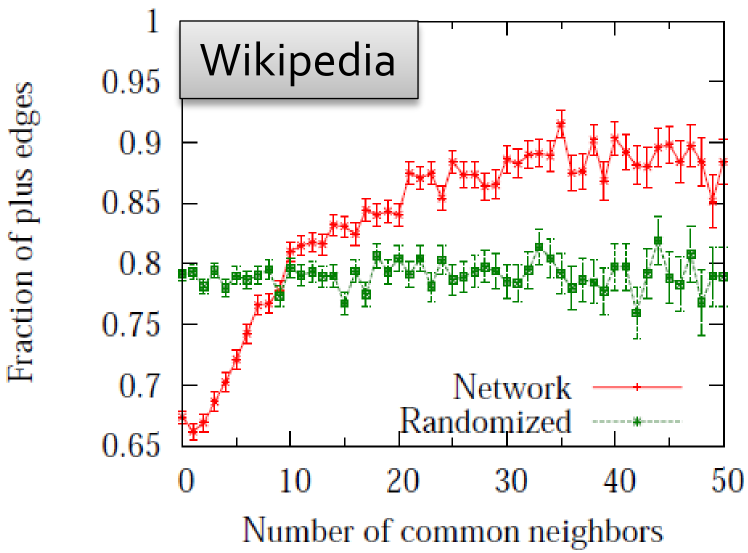
- Intuitive picture of social network in terms of densely linked clusters
- **How does structure interact with links?**
- **Embeddedness of link (A,B):** Number of shared neighbors



Global Factions: Embeddedness

- **Embeddedness of ties:**
  - Positive ties tend to be **more** embedded
- Positive ties tend to be more clumped together
  - Public display of signs (votes) in Wikipedia further attenuates this

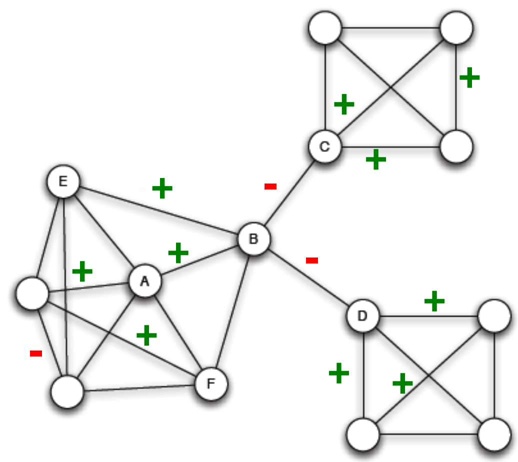




Global Structure of Signed Nets

	Size		Clustering		Component	
	Nodes	Edges	Real	Rnd	Real	Rnd
Epinions: −	119,090	123,602	0.012	0.022	0.308	0.334
Epinions: +	119,090	717,027	0.093	0.077	0.815	0.870
Slashdot: −	82,144	124,130	0.005	0.010	0.423	0.524
Slashdot: +	82,144	425,072	0.025	0.022	0.906	0.909
Wikipedia: −	7,115	21,984	0.028	0.031	0.583	0.612
Wikipedia: +	7,115	81,705	0.130	0.103	0.870	0.918

- **Clustering:**
  - +net: More clustering than baseline
  - –net: Less clustering than baseline
- **Size of max. component:**
  - +/–net: Smaller than the baseline

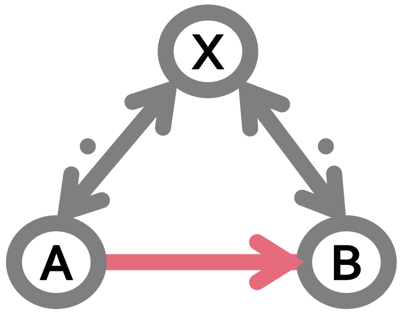


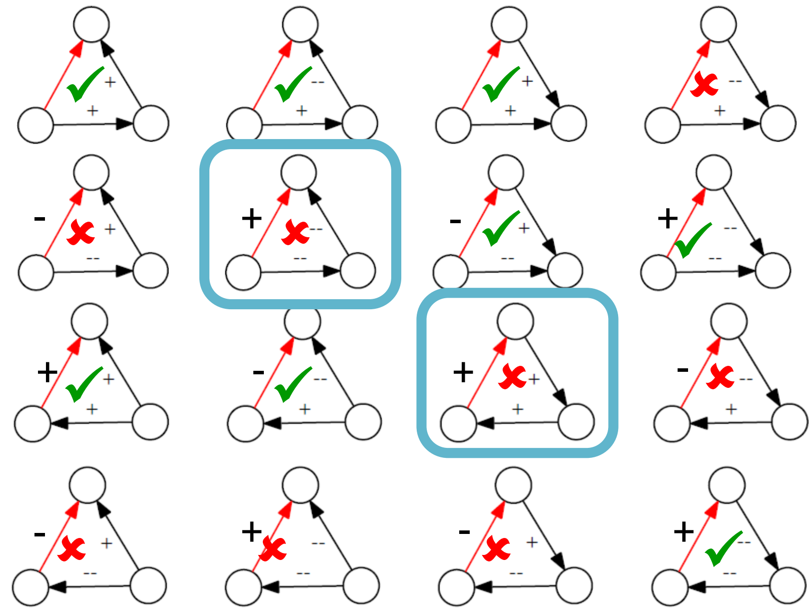
[CHI '10]

Evolving Directed Networks

- **New setting:**

  Links are directed and created over time





16 signed directed triads

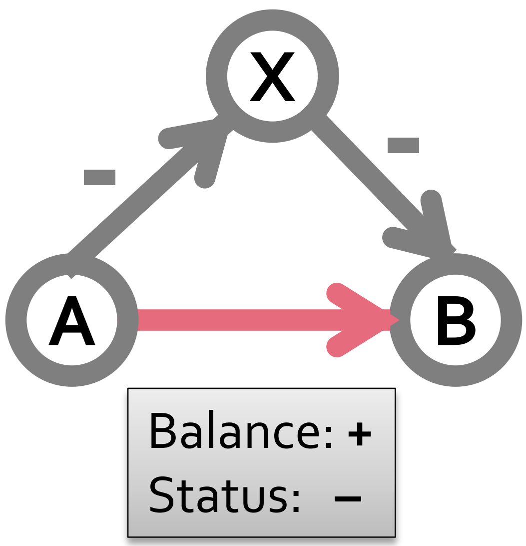
(in directed networks people traditionally applied balance by ignoring edge directions)

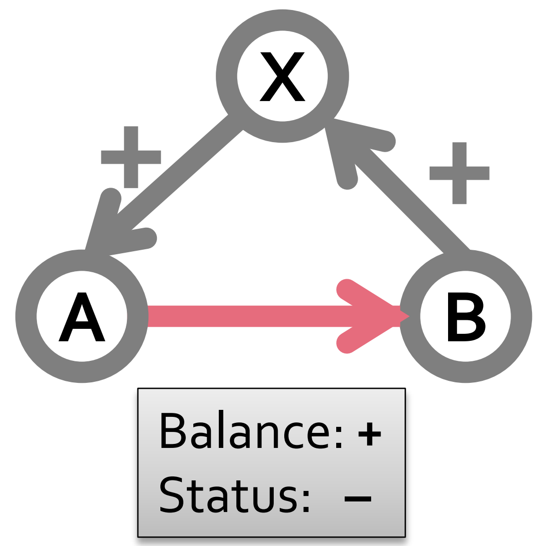
- **How many △ are now explained by balance?**
  - **Only half** (8 out of 16)
- **Is there a better explanation?** Yes. **Status.**

Alternate Theory: Status

- **Status** in a network [Davis-Leinhardt ’68]
  - $A \xrightarrow{+} B$ :: B has **higher** status than A
  - $A \xrightarrow{-} B$ :: B has **lower** status than A
    - (Note the notion of status is now implicit)
  - **Apply this principle transitively over paths**
    - Can replace each $A \xrightarrow{-} B$ with $A \xleftarrow{+} B$
    - Obtain an all-positive network with same status interpretation

Status vs. Balance



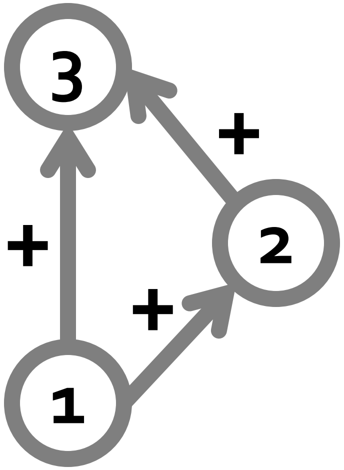


Status and balance give different predictions!

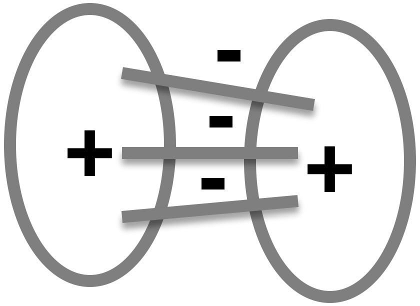
Status vs. Balance

At a global level:

- **Status ⇒ Hierarchy**
  - All-positive directed network should be (approximately) **acyclic**

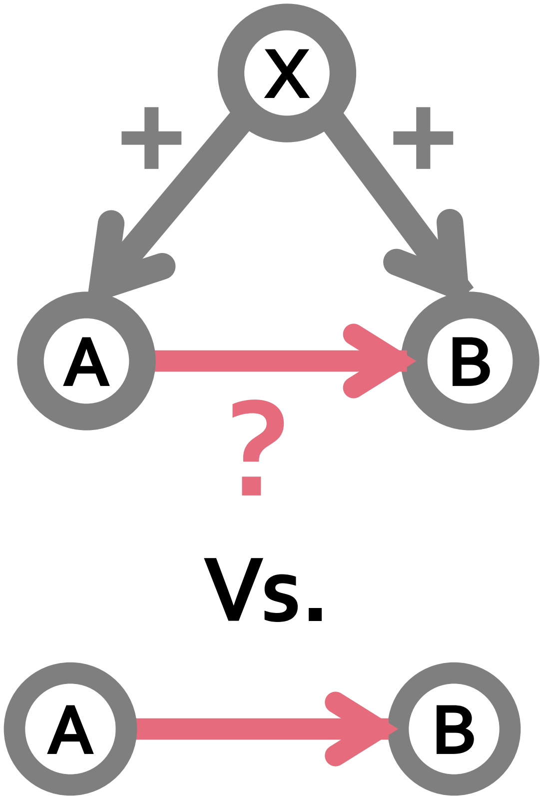


- **Balance ⇒ Coalitions**
  - Balance ignores directions and implies that subgraph of negative edges should be (approximately) **bipartite**



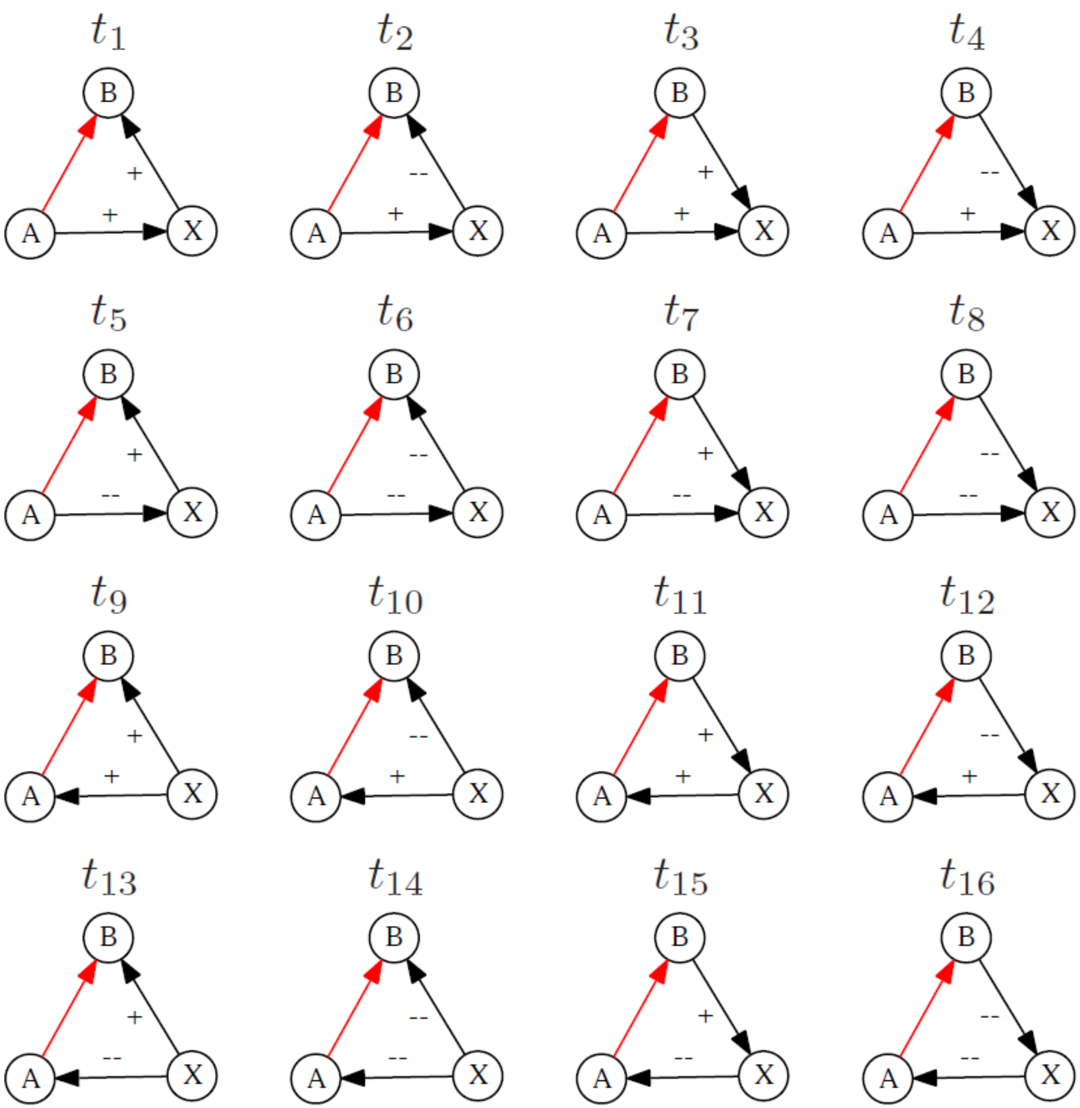
Theory of Status

- Edges are **directed and created over time**
  - X has links to A and B
  - Now, A links to B (triad A-B-X)
  - **How does sign of A⟶B depend signs from/to X?**
    $P(A \xrightarrow{+} B \mid X)$ vs. $P(A \xrightarrow{+} B)$
- **We need to formalize:**
  - **1) Links are embedded in triads:** Triads provide **context for signs**
  - **2)** Users are **heterogeneous** in their linking behavior



1) Context: 16 Types

- **Link A⟶B appears in context X: A⟶B | X**
- **16 possible contexts:**



2) Heterogeneity in linking behavior

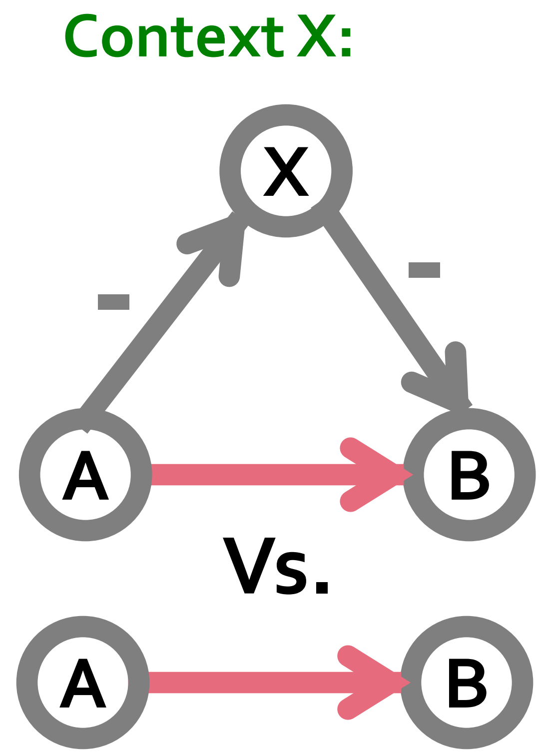
- **Users differ in frac. of + links they give/receive**
- **For a user U:**
  - **Generative baseline:** Frac. of + given by U
  - **Receptive baseline:** Frac. of + received by U

Basic question:

- How do **different link contexts** cause users to **deviate from their baselines**?
  - Link contexts as modifiers on a person's predicted behavior
  - **Surprise**: How much behavior of A/B **deviates** from his/her **baseline** when A/B is in **context X**

Computing Surprise

- **Surprise:** How much behavior of user **deviates** from **baseline** in **context X**
  - **Baseline:** For every user A_i :
    $p_g(A_i)$… **generative baseline** of A_i
    - Fraction of times A_i gives a plus
  - **Context:** $(A_1, B_1 | X_1),…, (A_n, B_n | X_n)$
    … all instances of triad context X
    - (A_i, B_i, X_i) … an instance where when user A_i links to user B_i the triad of type X is created.
    - Say k of those triads closed with a plus
      - k out of n times: $A_i \xrightarrow{+} B_i$

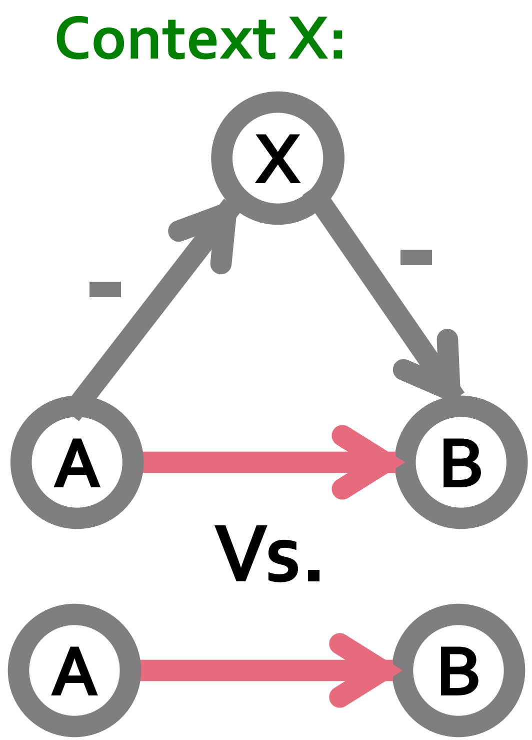


Computing Surprise

- **Surprise:** How much behavior of user **deviates** from **baseline** in **context X**
  - **Generative surprise of context *X*:**

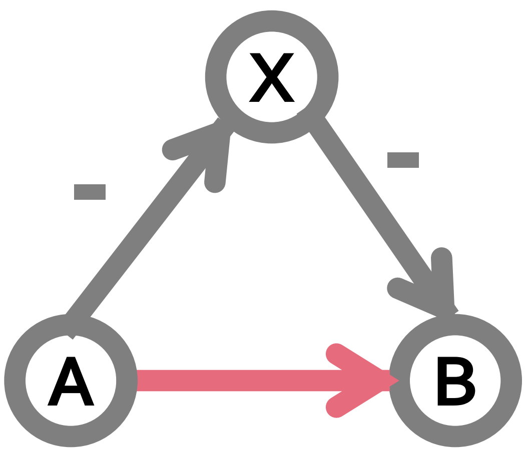
$$s_g(X) = \frac{k - \sum_{i=1}^{n} p_g(A_i)}{\sqrt{\sum_{i=1}^{n} p_g(A_i)(1 - p_g(A_i))}}$$

    - $p_g(A_i)$ … **generative baseline** of A_i
    - **Context X:** $(A_1, B_1 | X_1), \ldots, (A_n, B_n | X_n)$
    - *k* of instances of triad *X* closed with a plus edges
  - Receptive surprise is similar, just use $p_r(A_i)$

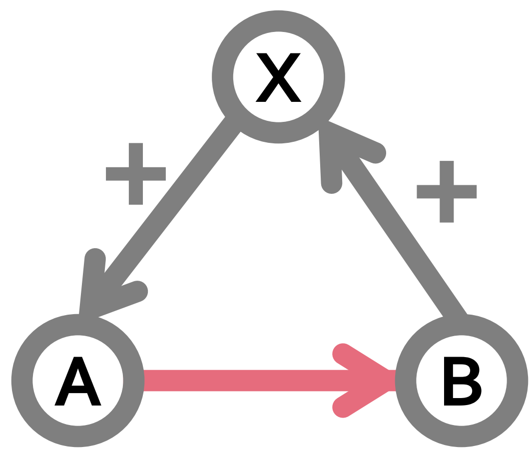


Status: Two Examples

- Assume status is at work
- What happens?



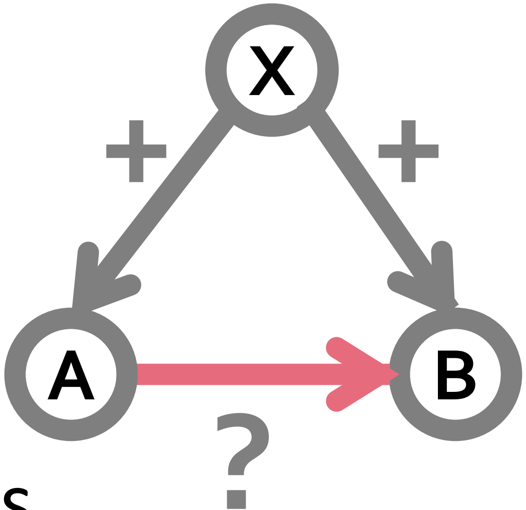
Gen. surprise of A: –
Rec. surprise of B: –



Gen. surprise of A: –
Rec. surprise of B: –

Joint Positive Endorsement

- X positively endorses A and B
- Now A links to B



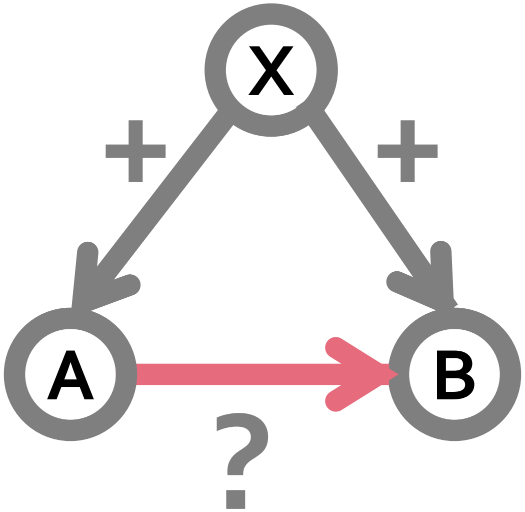
A puzzle:

- In our data we observe:
  Fraction of positive links deviates
  - **Above generative baseline of A: $S_g(X) > 0$**
  - Below receptive baseline of B: $S_r(X) < 0$

- **Why?**

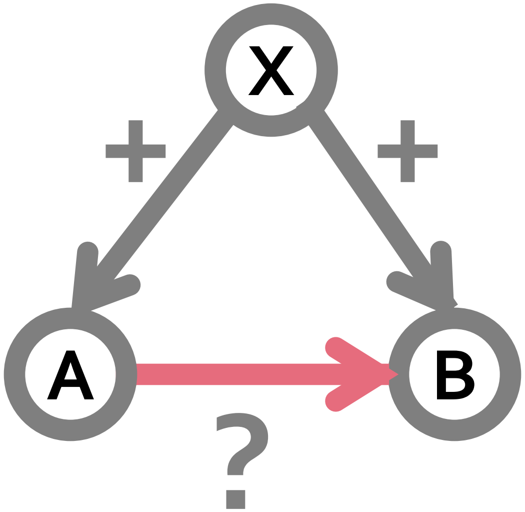
A Story: Soccer Team

- Ask every node: How does skill of B compare to yours?
  - Build a signed directed network
- We haven't asked A about B
- But we know that X thinks A and B are both better than him
- What can we infer about A's answer?



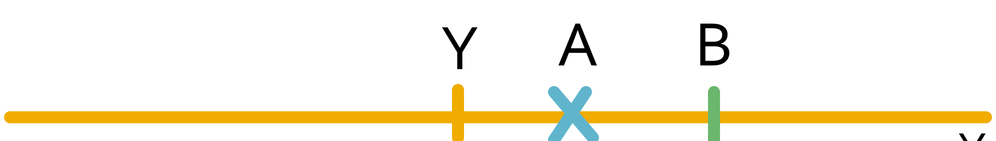
A Story: Soccer Team

- **A's viewpoint:**
  - Since B has positive evaluation, B is high status
  - Thus, evaluation A gives is **more likely to be positive** than the baseline



How does A evaluate B?

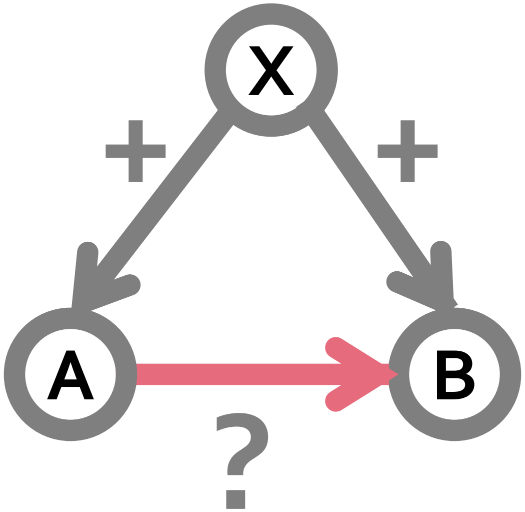
A is evaluating someone who is better than avg.
→ A is more positive than average



Y… average node

A Story: Soccer Team

- **B's viewpoint:**
  - Since A has positive evaluation, A is high status
  - Thus, evaluation B receives is **less likely to be positive** than the baseline



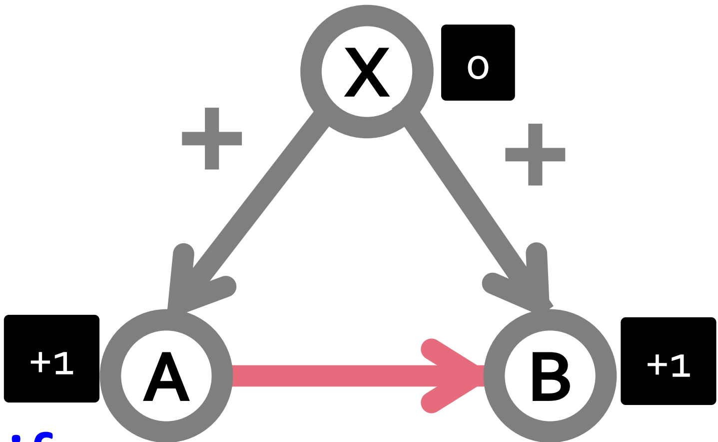
How is B evaluated by A?

B is evaluated by someone better than average.
→ They will be more negative to B than average

Y B A

Sign of A→B deviates in different directions depending on the viewpoint!

Consistency with Status



- **Determine node status:**
  - Assign *X* status 0
  - Based on signs and directions of edges set status of A and B
- Surprise is **status**-consistent, if:
  - Gen. surprise is status-consistent if it has **same** sign as status of B
  - Rec. surprise is status-consistent if it has the **opposite** sign from the status of A
- Surprise is **balance**-consistent, if:
  - If it completes a balanced triad

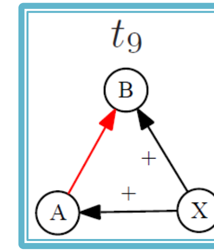
Status-consistent if:
Gen. surprise > 0
Rec. surprise < 0

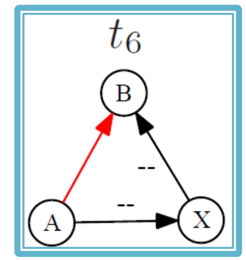
[CHI '10]

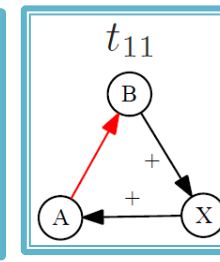
Status vs. Balance (Epinions)

- **Predictions:**

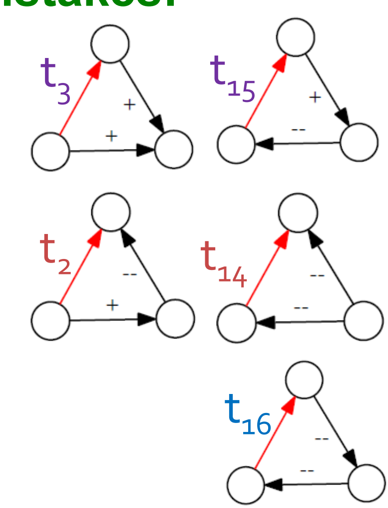
t_i	count	$P(+)$	$S_g(t_i)$	$S_r(t_i)$	B_g	B_r	S_g	S_r
t_1	178,051	0.97	95.9	197.8	✓	✓	✓	✓
t_2	45,797	0.54	-151.3	-229.9	✓	✓	✓	●
t_3	246,371	0.94	89.9	195.9	✓	✓	●	✓
t_4	25,384	0.89	1.8	44.9	○	○	✓	✓
t_5	45,925	0.30	18.1	-333.7	○	✓	✓	✓
t_6	11,215	0.23	-15.5	-193.6	○	○	✓	✓
t_7	36,184	0.14	-53.1	-357.3	✓	✓	✓	✓
t_8	61,519	0.63	124.1	-225.6	✓	○	✓	✓
t_9	338,238	0.82	207.0	-239.5	✓	○	✓	✓
t_{10}	27,089	0.20	-110.7	-449.6	✓	✓	✓	✓
t_{11}	35,093	0.53	-7.4	-260.1	○	○	✓	✓
t_{12}	20,933	0.71	17.2	-113.4	○	✓	✓	✓
t_{13}	14,305	0.79	23.5	24.0	○	○	✓	✓
t_{14}	30,235	0.69	-12.8	-53.6	○	○	✓	●
t_{15}	17,189	0.76	6.4	24.0	○	○	●	✓
t_{16}	4,133	0.77	11.9	-2.6	✓	○	✓	●
Number of correct predictions					8	7	14	13





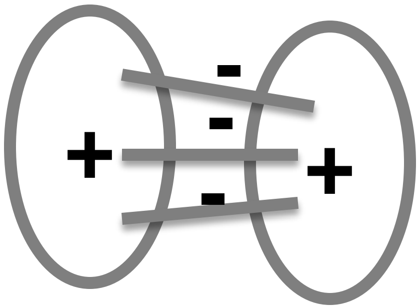


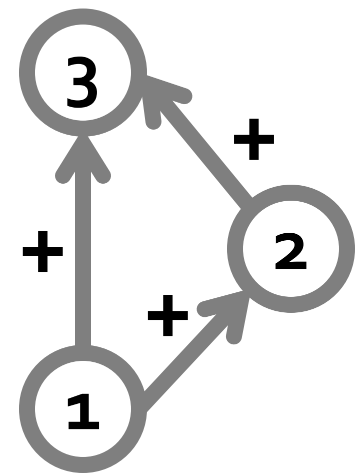
Mistakes:



Status vs. Balance

- At a global level:
  - Balance ⇒ Coalitions
  - Status ⇒ Hierarchy
- Observations:
  - No evidence for global balance beyond the random baselines
    - Real data is 80% consistent vs. 80% consistency under random baseline
  - Evidence for global status beyond the random baselines
    - Real data is 80% consistent, but 50% consistency under random baseline

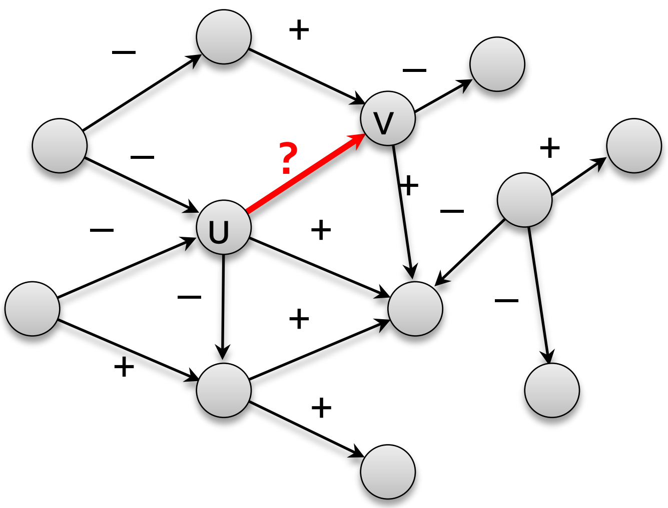




Predicting Edge Signs

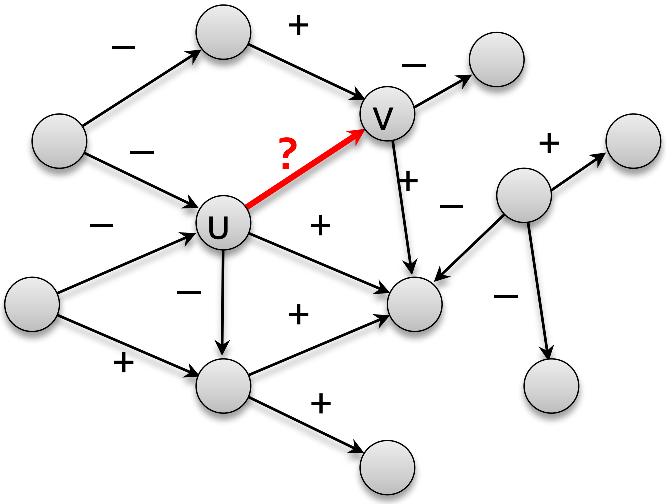
Edge sign prediction problem

- Given a network and signs on all but one edge, predict the missing sign



- Friend recommendation:
  - Predicting whether you know someone vs. Predicting what you think of them

Predicting Edge Signs

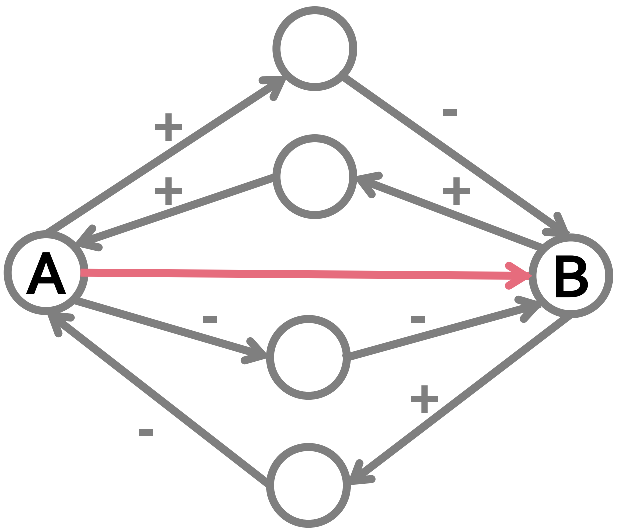


- **Problem Formulation:**
  - Predict sign of edge (u,v)
- Class label:
  - +1: positive edge
  - -1: negative edge
- Learning method:
  - Logistic regression

$$P(+|x) = \frac{1}{1 + e^{-(b_0 + \sum_i^n b_i x_i)}}$$

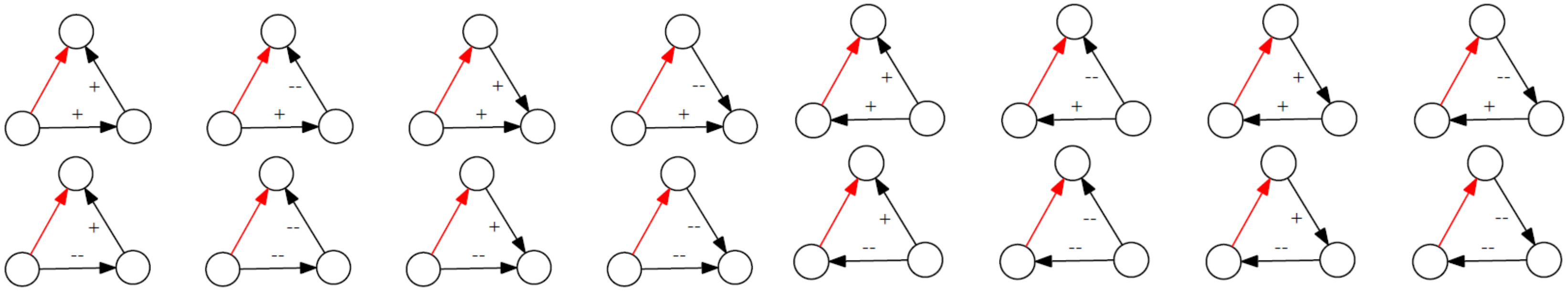
  - Each feature "votes" for/against a positive edge.
- Dataset:
  - Balanced: 50% +edges
- Evaluation:
  - Accuracy
- Features for learning:
  - Next slide

Features for Learning



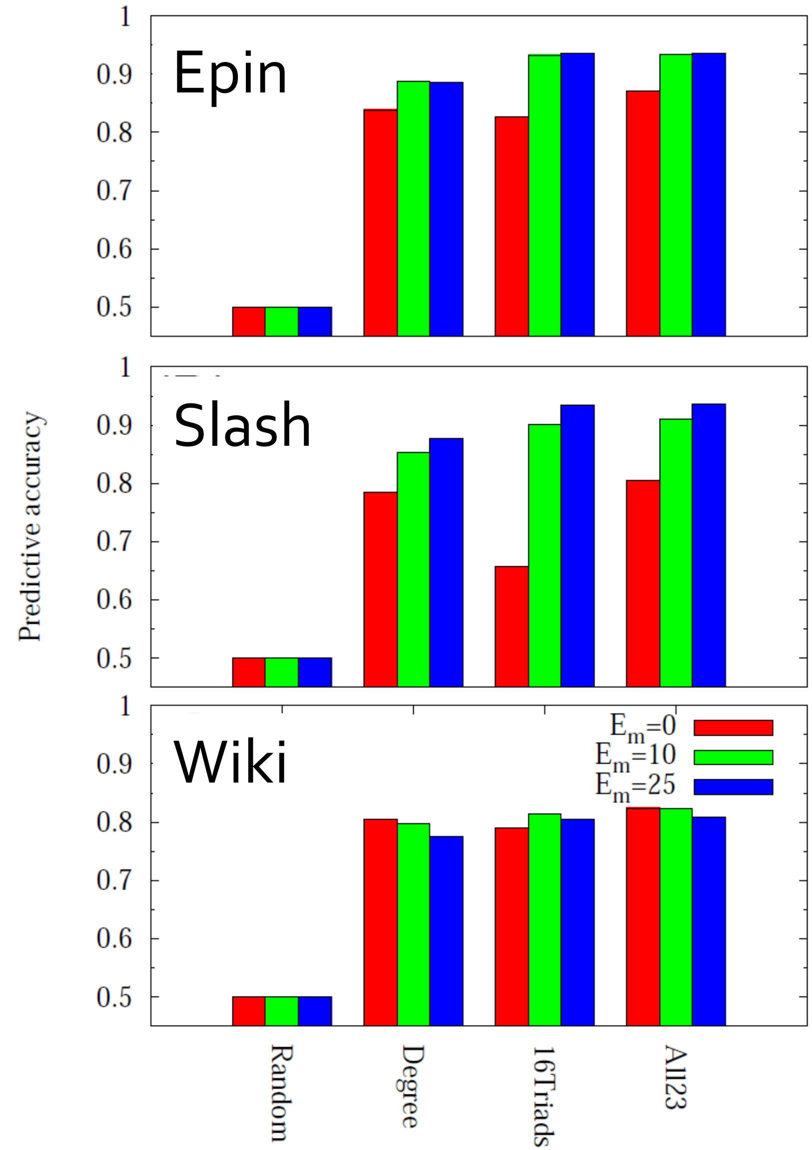
For each edge (A,B) create a set of features:

- **Triad counts of edge (A,B):**
  - In what types of triads does our red-edge participate in?



Edge Sign Prediction

- **Classification Accuracy:**
  - Epinions: 93.5%
  - Slashdot: 94.4%
  - Wikipedia: 81%
- **Signs can be modeled from local network structure alone!**
  - Trust propagation model of [Guha et al. '04] has 14% error on Epinions
- **Triad features perform less well for less embedded edges**
- Wikipedia is harder to model:
  - Votes are publicly visible



Balance and Status: Complete Model

Feature	Bal	Stat	**Epin**	**Slashd**	**Wikip**
const			-0.2	0.02	-0.2
● +→ ● +→ ●	1	1	**0.5**	**0.9**	**0.3**
● +→ ● -→ ●	-1	0	**-0.5**	**-0.9**	**-0.4**
● -→ ● +→ ●	-1	0	**-0.4**	**-1.1**	**-0.3**
● -→ ● -→ ●	1	-1	**-0.7**	**-0.6**	**-0.8**
○ +→ ○ ←+ ○	1	0	0.3	0.4	0.05
○ +→ ○ ←- ○	-1	1	-0.01	-0.1	-0.01
● -→ ● ←+ ●	-1	-1	**-0.9**	**-1.2**	**-0.2**
○ -→ ○ ←- ○	1	0	0.04	-0.07	-0.03
○ ←+ ○ +→ ○	1	0	0.08	0.4	0.1
● ←+ ● -→ ●	-1	-1	**-1.3**	**-1.1**	**-0.4**
○ ←- ○ +→ ○	-1	1	-0.1	-0.2	0.05
○ ←- ○ -→ ○	1	0	0.08	-0.02	-0.1
○ ←+ ○ ←+ ○	1	-1	-0.09	-0.09	-0.01
○ ←+ ○ ←- ○	-1	0	-0.05	-0.3	-0.02
○ ←- ○ ←+ ○	-1	0	-0.04	-0.3	0.05
○ ←- ○ ←- ○	1	1	-0.02	0.2	-0.2

Generalization

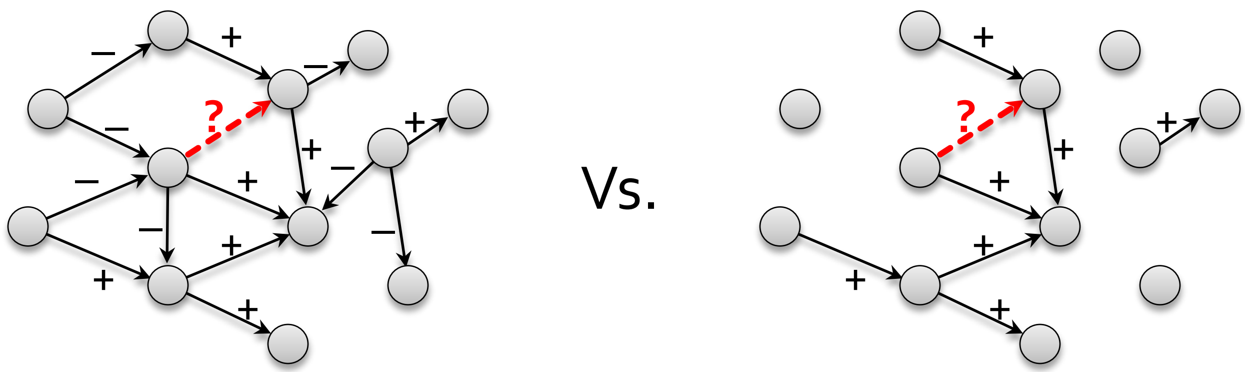
- Do people use these very different linking systems by obeying the same principles?
  - How generalizable are the results across the datasets?

All23	Epinions	Slashdot	Wikipedia
Epinions	0.9342	0.9289	0.7722
Slashdot	0.9249	0.9351	0.7717
Wikipedia	0.9272	0.9260	0.8021

- Nearly **perfect generalization** of the models even though networks come from very different applications!

Negative information helps?

- Suppose we are only interested in predicting whether there is a positive edge or no edge
- **Does knowing negative edges help?** **YES!**



Features	Epinions	Slashdot	Wikipedia
Positive edges	0.5612	0.5579	0.6983
Positive and negative edges	0.5911	0.5953	0.7114

Summary

- **Signed networks provide insight into how social computing systems are used:**
  - Status vs. Balance
  - Role of embeddedness and public display
  - More evidence that networks are globally organized based on status
- **Sign of relationship can be reliably predicted from the local network context**
  - ~90% accuracy sign of the edge
  - People use signed edges consistently regardless of particular application
    - Near perfect generalization of models across datasets