Classifying left ventricle shapes

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Abstract. Abstract

1 Introduction

1.1 subsection

subsubsection This is the caption of the figure displaying a white eagle and a white horse on a snow field

Since u_1 is a smooth function, we will have $\|hu_1\|_\infty \leq \eta$ for h small enough, and inequality (**??**) will hold, yielding thereby:

$$\psi(hu_1) \leq \frac{h^2}{2}\frac{1}{\lambda}\|u_1\|_2^2 + \frac{h^2}{2}\frac{1}{\delta'}\|u_1\|^2 \ . \tag{1}$$

If we choose δ' close enough to δ, the quantity $\left(\frac{1}{\lambda} + \frac{1}{\delta'}\right)$ will be negative, and we end up with

Proof. The spectrum of Λ is $\frac{2\pi}{T}\mathbb{Z} + a_\infty$. The largest negative eigenvalue λ is given by $\frac{2\pi}{T}k_o + a_\infty$, where

$$\frac{2\pi}{T}k_o + a_\infty < 0 \leq \frac{2\pi}{T}(k_o + 1) + a_\infty \ . \tag{2}$$

Hence:

$$k_o = E\left[-\frac{T}{2\pi}a_\infty\right] \ . \tag{3}$$

The condition $\gamma < -\lambda < \delta$ now becomes:

$$b_\infty - a_\infty < -\frac{2\pi}{T}k_o - a_\infty < \omega - a_\infty \tag{4}$$

which is precisely condition (**??**). $\qquad\square$

Lemma 1. *Assume that H is C^2 on $\mathbb{R}^{2n}\backslash\{0\}$ and that $H''(x)$ is non-degenerate for any $x \neq 0$. Then any local minimizer $\widetilde{x}$ of ψ has minimal period T.*

Proof. We know that $\widetilde{x}$, or $\widetilde{x} + \xi$ for some constant $\xi \in \mathbb{R}^{2n}$, is a T-periodic solution of the Hamiltonian system:

$$\dot{x} = JH'(x) . \tag{5}$$

There is no loss of generality in taking $\xi = 0$. So $\psi(x) \geq \psi(\widetilde{x})$ for all $\widetilde{x}$ in some neighbourhood of x in $W^{1,2}\left(\mathbb{R}/T\mathbb{Z};\mathbb{R}^{2n}\right)$.

But this index is precisely the index $i_T(\widetilde{x})$ of the T-periodic solution $\widetilde{x}$ over the interval $(0, T)$, as defined in Sect. 2.6. So

$$i_T(\widetilde{x}) = 0 . \tag{6}$$

Now if $\widetilde{x}$ has a lower period, T/k say, we would have, by Corollary 31:

$$i_T(\widetilde{x}) = i_{kT/k}(\widetilde{x}) \geq ki_{T/k}(\widetilde{x}) + k - 1 \geq k - 1 \geq 1 . \tag{7}$$

This would contradict (6), and thus cannot happen. □

Theorem 1 (Ghoussoub-Preiss). *Assume $H(t, x)$ is $(0, \varepsilon)$-subquadratic at infinity for all $\varepsilon > 0$, and T-periodic in t*

$$H(t, \cdot) \qquad \text{is convex} \quad \forall t \tag{8}$$

$$H(\cdot, x) \qquad \text{is } T-\text{periodic} \quad \forall x \tag{9}$$

$$H(t, x) \geq n\left(\|x\|\right) \qquad \text{with} \quad n(s)s^{-1} \to \infty \quad \text{as} \quad s \to \infty \tag{10}$$

$$\forall \varepsilon > 0 , \quad \exists c \; : \; H(t, x) \leq \frac{\varepsilon}{2}\|x\|^2 + c . \tag{11}$$

Assume also that H is C^2, and $H''(t, x)$ is positive definite everywhere. Then there is a sequence x_k, $k \in \mathbb{N}$, of kT-periodic solutions of the system

$$\dot{x} = JH'(t, x) \tag{12}$$

such that, for every $k \in \mathbb{N}$, there is some $p_o \in \mathbb{N}$ with:

$$p \geq p_o \Rightarrow x_{pk} \neq x_k . \tag{13}$$

□

References

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