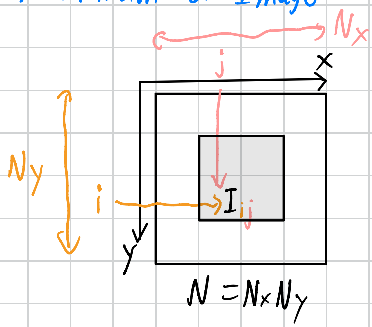
Quantum Hadamard Edge detection (QHED)

① Definition of edge
② Classical edge detection
③ QHED

① Definition

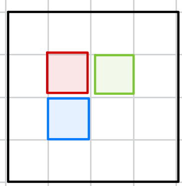
(i) Definition of Image



- For B&W image, each pixel is represented by I_{ij}

  Usually, $I_{ij} \in [0, 255]$ (black ⟷ white)

  $256 = 2^8$, known as 8-bit colour
  ⇒ representing 256 type of colour



- For RGB image each pixel is represented by vector $\vec{v}$

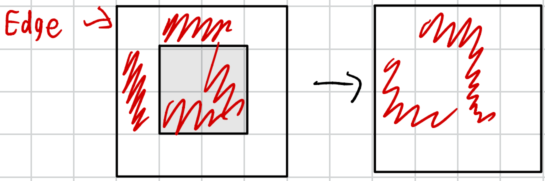
  $$\vec{v} = \begin{pmatrix} R_{ij} \\ G_{ij} \\ B_{ij} \end{pmatrix}$$

  $R_{ij}, G_{ij}, B_{ij} \in [0, 255]$

We can convert RGB → B&W using the following function

$$I_{ij} = 0.299R + 0.587G + 0.114B$$

(ii) Definition of Edge



We say pixel (i,j) is edge iff

$$|I_{i+1\,j} - I_{ij}| \geq I_{threshold}$$

OR

$$|I_{i\,j+1} - I_{ij}| \geq I_{threshold}$$

Where $I_{threshold}$ is a hyper parameter for tunning

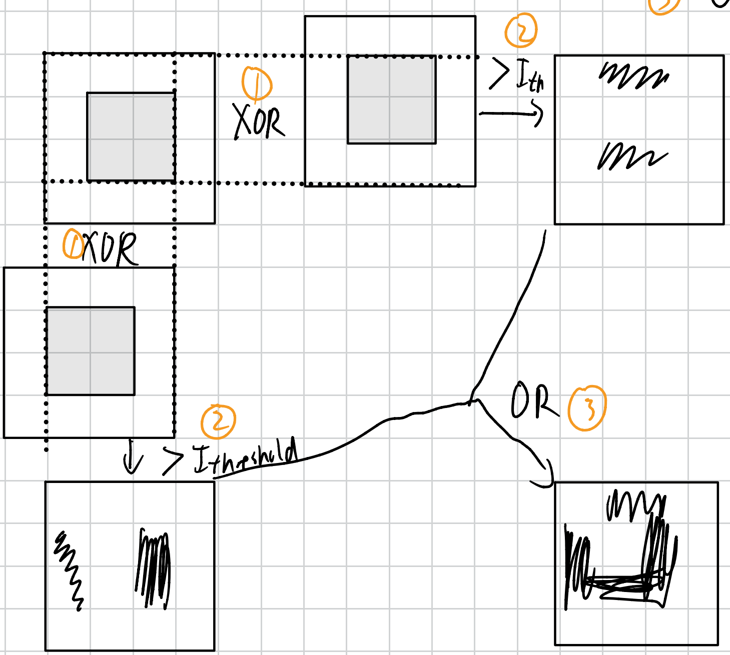
② Classical edge detection

Edge can be detected by (XOR gate on 2'complement) performing

① element wise absolute difference in vertical and horizontal direction

② Elementwise comparision

③ ORing them



Problem of classical approach :

Let $n_x = \lceil \log_2 N_x \rceil$

$n_y = \lceil \log_2 N_y \rceil$

Then $\# FLOP = \underset{O(N_x N_y)}{XOR} + \underset{O(N_x N_y)}{>0} + \underset{O(N_x N_y)}{OR}$

$= O(N_x N_y)$

$= O(2^{n_x + n_y})$

$\Rightarrow$ expotential time

② QHED :

If we neglect the time for encoding image & measurement

The runtime of QHED $O(2^n) \rightarrow O(1)$

(i) Quantum probability image encoding (QPIE)

we can represent intensity as probability

$C_{ij} = \dfrac{I_{ij}}{\sum_{ij} I_{ij}^2}$ (normalisation)

Each pixel $=$ each basis vector

For simplicity, assume $N_x, N_y =$ even,

binary $|b_0 \cdots b_{N-1}\rangle$

0000	0001	0010	0011
0100	0101	0110	0111
1000	1001	1010	1011
1100	1101	1110	1111

$\rightarrow$

hex $|i\rangle$

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Using row-major order, we can unroll image into 1D-array

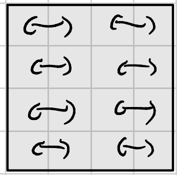
$C_{ij} \rightarrow C_i$

$|Img\rangle = \sum_i C_i |i\rangle$ (State preparation)

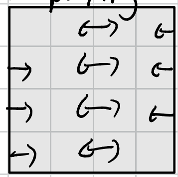
- It canbe proven that arbitary state can be prepared using only CNOT & $R(\theta, \phi)$
- It canbe proven that such encoding have runtime $O(n^2)$

(ii) QHED:

Define even & odd pairing as even pairing
odd & even pairing as odd pairing



even pairing



odd pairing

Goal: Find their absolute difference of even & odd pairing

If we can do this, then we acheive horizontal scanning ✓

Vertical scanning = horizontal scan(image^T)

Method ①

- For even-pairing:

0000	0001	0010	0011
0100	0101	0110	0111
1000	1001	1010	1011
1100	1101	1110	1111

One notice that only least significant bit is different

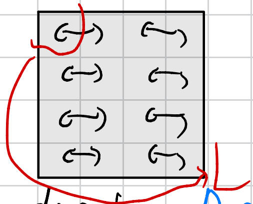
By the magic of H-gate

$$I_{N-1} \otimes H_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & & & \\ 1 & -1 & & & \\ & & 1 & 1 & \\ & & 1 & -1 & \\ & & & & \ddots \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} c_0 \\ \vdots \\ c_{N-1} \end{pmatrix}}_{|Img\rangle} = \begin{pmatrix} c_0+c_1 \\ c_0-c_1 \\ c_2+c_3 \\ c_2-c_3 \\ \vdots \end{pmatrix}$$

✓ Difference in $O(1)$

- For odd-pairing: We just need to append first pixel to the last



This can be done by Decrement gate

$$\begin{pmatrix} & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ 1 & & & \cdots & \end{pmatrix}\begin{pmatrix} c_0 \\ \vdots \\ c_{N-1} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_0 \end{pmatrix}$$

Then repeat last procedure

Method ② Auxilary qubit

One realize even pairing & Odd pairing can be done at the same time by adding 1 extra qubit

$|Img\rangle \otimes |0\rangle$

And Apply H-gate to it

$$|Img\rangle \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{pmatrix} c_0 \\ c_0 \\ c_1 \\ c_1 \\ \vdots \\ c_{N-1} \\ c_{N-1} \end{pmatrix}$$

Apply Decrement gate Then H gate

$$I_{N+1} \otimes H_0 \begin{pmatrix} c_0 \\ c_1 \\ c_1 \\ c_2 \\ \vdots \\ c_{N-1} \\ c_{N-1} \\ c_0 \end{pmatrix} = \begin{pmatrix} c_0 + c_1 \\ c_0 - c_1 \\ c_1 + c_2 \\ c_1 - c_2 \\ \vdots \\ c_{N-1} + c_0 \\ c_{N-1} - c_0 \end{pmatrix}$$

✓ Diffrency in $O(1)$