HW #1

Jack Krebsbach

Sep 4

1. Let n be a positive integer that is not a perfect square. Prove that $\sqrt{n}$ is irrational.

2. Use the Principle of Mathematical Induction to prove:

$$1^2 + 3^2 + \cdots + (2n-1)^2 = \frac{4n^3 - n}{3} \quad \forall n \in \mathbb{N}$$

3. Let $n > 1$ be a positive integer and let $a_1, a_2, \ldots, a_n$ be real numbers. Prove that

$$\left| \sum_{k=1}^{n} a_k \right| \leq \sum_{k=1}^{n} |a_k| .$$

4. Exercise 1.2.8

5. Exercise 1.2.12

6. Grade the following proof: Prove that $\sqrt{3}$ is irrational. Does a similar argument work to show $\sqrt{6}$ is irrational?

Proof:To prove that $\sqrt{3}$ is irrational, we can use a proof by contradiction. The idea is to assume that $\sqrt{3}$ is rational, and then derive a contradiction.

Assumption: $\sqrt{3}$ is rational.

If $\sqrt{3}$ is rational, it can be expressed as a fraction $\frac{a}{b}$ in its simplest form, where a and b are integers with no common factors other than 1 (i.e., they are coprime), and b is not equal to 0 .

So, we have: $\sqrt{3} = \frac{a}{b}$

Squaring both sides of the equation: $3 = \frac{a^2}{b^2}$

Rearranging the equation: $a^2 = 3 \cdot b^2$

From this equation, we can see that a^2 is a multiple of 3 , which implies that a must also be a multiple of 3. Let's represent a as $3k$ where k is an integer:

$$(3k)^2 = 3 \cdot b^2$$
$$9k^2 = 3 \cdot b^2$$
$$3k^2 = b^2$$

Now, b^2 is also a multiple of 3 , which means b must be a multiple of 3 . However, this contradicts our initial assumption that a and b have no common factors other than 1 (i.e., they are coprime). This contradiction arises from assuming that $\sqrt{3}$ is rational.

Hence, our initial assumption that $\sqrt{3}$ is rational must be false, which means that $\sqrt{3}$ is irrational. Now, to address whether a similar argument works for $\sqrt{6}$:

No, a similar argument does not work for $\sqrt{6}$. The proof for the irrationality of $\sqrt{3}$ relies on the fact that it leads to a contradiction with respect to the common factors of a and b. However, for $\sqrt{6}$, this argument does not apply directly because the factors of 6 are not the same as the factors of 3. In fact, $\sqrt{6}$ is not irrational; it is a rational number. It can be expressed as the fraction $\frac{2\sqrt{3}}{3}$, which is a ratio of two integers.