ASEN 3128 Homework 1

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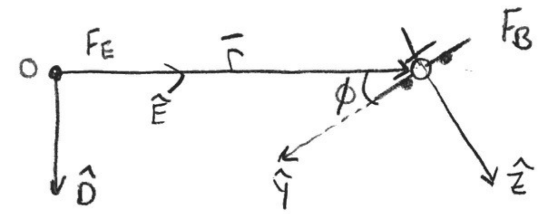
A. Questions 1-8

Jack Lambert | ASEN 3128 | Section 012 | HW #1



$F_E = 100 (m/s)$

$\bar{r} = 1 km$

$F_E = \begin{bmatrix} \hat{N} \\ \hat{E} \\ \hat{D} \end{bmatrix}$ $F_B = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}$

1.) $\bar{r} = 1 \hat{E} [km] = 1 \hat{y}_E = -\cos\phi \hat{y} + \sin\phi \hat{z}$

$\bar{r}_E = 1\hat{E} = 1 \hat{y}_E = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} km$

$\bar{r}_B = -\cos\phi \hat{y} + \sin\phi \hat{z} = \begin{bmatrix} 0 \\ -\cos\phi \\ \sin\phi \end{bmatrix}$

- $\bar{r}$ is the position vector of the air plane
- $\bar{r}_E$ is an array of numbers representing the vector, $\bar{r}$, in an inertial frame
- $\bar{r}_B$ is an array of numbers representing the vector, $\bar{r}$, in a body frame.

2.) • $\bar{V}^E = \frac{d^E}{dt} \bar{r}$ • The rate of change of the vector $\bar{r}$ w/ respect to an inertial reference frame

$= \frac{d}{dt}(1 \hat{N})$ or $\frac{d}{dt}(-\cos\phi \hat{y} + \sin\phi \hat{z})$

- $\bar{V}^E_E \rightarrow$ The rate of change of $\bar{r}$ as viewed from an inertial frame, in interial coordinates.

$= -100 \hat{N} \left(\frac{m}{s}\right) = \begin{bmatrix} -100 \\ 0 \\ 0 \end{bmatrix} (m/s) = \begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{z}_E \end{bmatrix}$

- $\bar{V}^E_B \rightarrow$ The rate of change of $\bar{r}$ as viewed from an inertial frame in body coordinates

$= 100 \hat{x} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} U^E \\ V^E \\ W^E \end{bmatrix}$

3.) • $\bar{\omega}^{EB} \longrightarrow$ the angular velocity of the body as seen by the inertial frame

$$= \frac{\bar{V}^E}{\bar{r}} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

• $\bar{\omega}_E^{EB} \rightarrow$ the angular velocity of the body as seen by the inertial frame - in inertial coordinates

$$= \left(\frac{\bar{V}^E}{\bar{r}}\right)_E = \boxed{0.1\,\hat{D}\left(\frac{rad}{s}\right)} = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}\left(\frac{rad}{s}\right)$$

• $\bar{\omega}_B^{EB} \longrightarrow$ the angular velocity of the body as seen by the inertial frame - in body coordinates

$$= \left(\frac{\bar{V}^E}{\bar{r}}\right)_B = -0.1\sin\phi\,\hat{y} + 0.1\cos\phi\,\hat{z} = \begin{bmatrix} 0 \\ 0.1\sin\phi \\ 0.1\cos\phi \end{bmatrix}\left(\frac{rad}{s}\right)$$

4.). $\frac{d^B}{dt}\bar{r} \longrightarrow$ the rate of change in $\bar{r}$ as seen from a body frame (velocity of body)

$$= \bar{V}^B = \begin{bmatrix} u \\ v \\ w \end{bmatrix}(m/s) = \frac{d^E}{dt}\bar{r} - \bar{\omega}^{EB} \times \bar{r}$$

• $\left(\frac{d^B}{dt}\bar{r}\right)_E \longrightarrow$ the velocity of the body as seen from the body in body coordinates

$$= \begin{bmatrix} \dot{x}_E^B \\ \dot{y}_E^B \\ \dot{z}_E^B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}(m/s)$$

• $\left(\frac{d^B}{dt}\bar{r}\right)_B \longrightarrow$ the velocity of the body as seen from the body in body coordinates

$$\frac{d^E}{dt}\bar{r}_c = \bar{V}^E$$

$$= \begin{bmatrix} u^B \\ v^B \\ w^B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}(m/s) \qquad \begin{bmatrix} u^E \\ v^E \\ w^E \end{bmatrix} = \bar{V}^E_B \qquad \bar{V}_E = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

5.) $\left(\frac{\partial^E}{\partial t}\bar{V}^E\right)_E \rightarrow$ time rate of change seen from an inertial frame of a velocity of a body seen in an inertial frame in inertial coordinates

$$=\left(\frac{\partial^B}{\partial t}V^E\right)_E + \bar{\omega}_E^{EB}\times\bar{V}_E^E$$

$$= \begin{bmatrix} \ddot{x}_E \\ \ddot{y}_E \\ \ddot{z}_E \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \\ 0 \end{bmatrix} m/s^2$$

$\left(\frac{\partial^B}{\partial t}\bar{V}^E\right)_B \rightarrow$ time rate of change seen from a body frame of a velocity seen in an inertial frame in body coordinates

$$= \frac{\partial}{\partial t}\bar{V}_B^E = \begin{bmatrix} \dot{u}^E \\ \dot{v}^E \\ \dot{w}^E \end{bmatrix} = \left(\frac{\partial^E}{\partial t}\bar{V}^E\right)_B - \left(\bar{\omega}_B^{EB}\times\bar{V}_B^E\right)$$

6.) $\frac{\partial^E}{\partial t}\bar{V}^E = \boxed{\frac{\partial^B}{\partial t}V^E + \bar{\omega}^{EB}\times\bar{V}^E} = \bar{\omega}_B^{EB}\times\bar{V}_B^E = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0.1\sin\phi & 0.1\cos\phi \\ 100 & 0 & 0 \end{bmatrix}$

$$\left(\frac{\partial^E}{\partial t}\bar{V}^E\right)_E = \left(\bar{\omega}_E^{EB}\times V_E^E\right) = \begin{bmatrix} \hat{N} & \hat{E} & \hat{D} \\ 0 & 0 & 0.1 \\ -100 & 0 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ -10 \\ 0 \end{bmatrix}(m/s)}$$

$$= \boxed{\begin{bmatrix} 0 \\ 10\cos\phi \\ -10\sin\phi \end{bmatrix} m/s}$$

7.) • $\vec{f}$ – External force exerted on an object

• $\bar{f}_E$ External force exerted on an object in inertial coordinates

$$= m\left(\frac{\partial^E}{\partial t}\bar{V}^E\right)_E = m\left(\frac{\partial^B}{\partial t}\bar{V}^E + \bar{\omega}^{EB}\times\bar{V}^E\right)_E = m\begin{bmatrix} 0 \\ -10 \\ 0 \end{bmatrix}$$

• $\bar{f}_B$ External force exerted on an object in body coordinates

$$= m\left(\frac{\partial^E}{\partial t}\bar{V}^E\right)_B = m\left(\frac{\partial^B}{\partial t}\bar{V}^E + \omega^{EB}\times\bar{V}^E\right)_B$$

$$= m\begin{bmatrix} 0 \\ 10\cos\phi \\ -10\sin\phi \end{bmatrix}$$

8) $\overline{W}_E = \begin{bmatrix} 10 \\ 20 \\ -5 \end{bmatrix}$ (m/s) Find - $\overline{V}_E$, $\overline{V}_B$

$$\overline{V}^E = \overline{V} + \overline{W} \longrightarrow \overline{V} = \overline{V}^E - \overline{W}$$

$$\longrightarrow \overline{V}_B = \overline{V}_B^E - \overline{W}_B$$

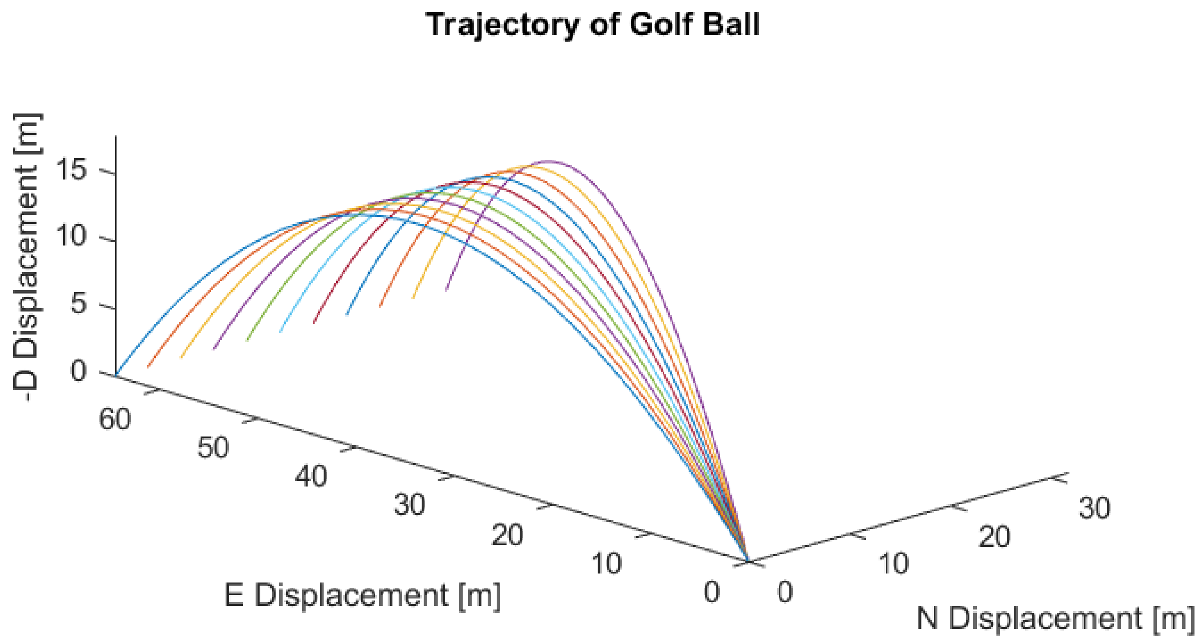
$$\overline{W}_B = \begin{bmatrix} -10 \\ -20\cos\phi + 5\sin\phi \\ -5\cos\phi + 20\sin\phi \end{bmatrix}$$

$$\overline{V}_B = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -10 \\ -20\cos\phi + 5\sin\phi \\ -5\cos\phi + 20\sin\phi \end{bmatrix}$$

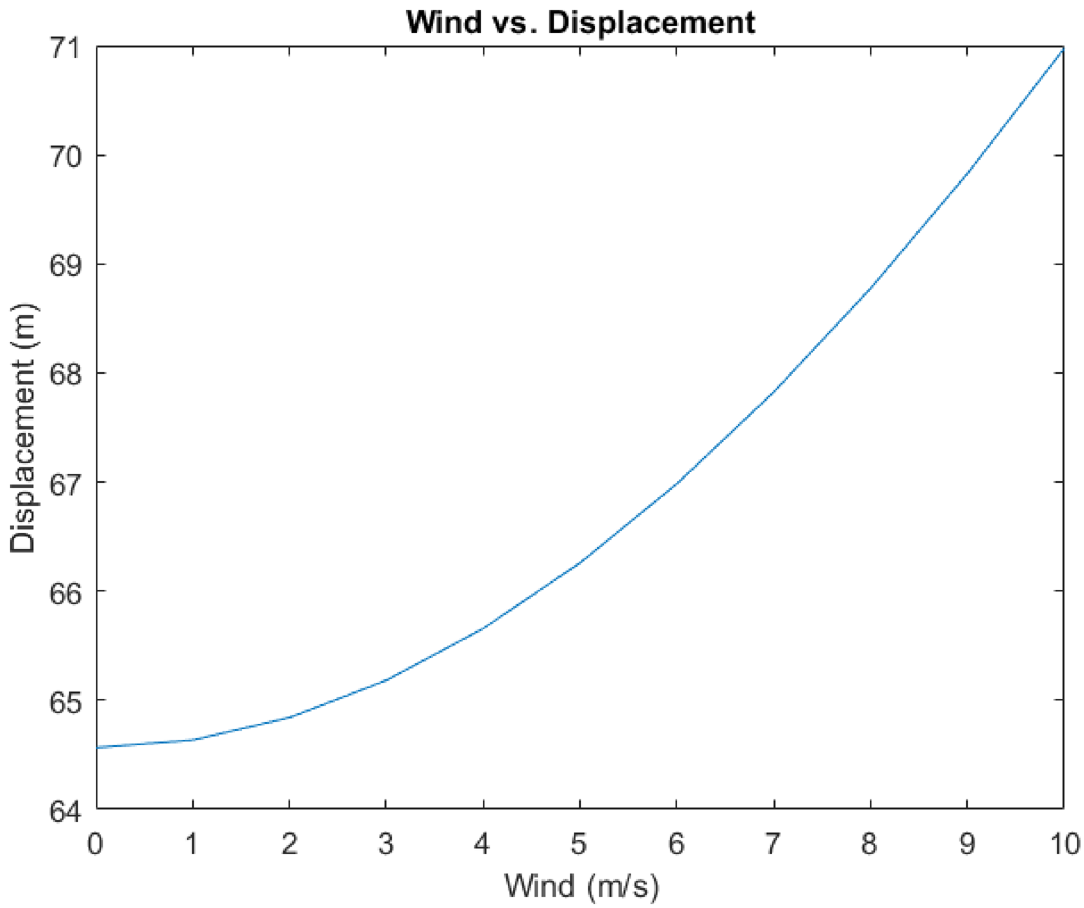
$$\boxed{= 90\hat{x} + (20\cos\phi - 5\sin\phi)\hat{y} + (5\cos\phi - 20\sin\phi)\hat{z}}$$

B. Problem 9:

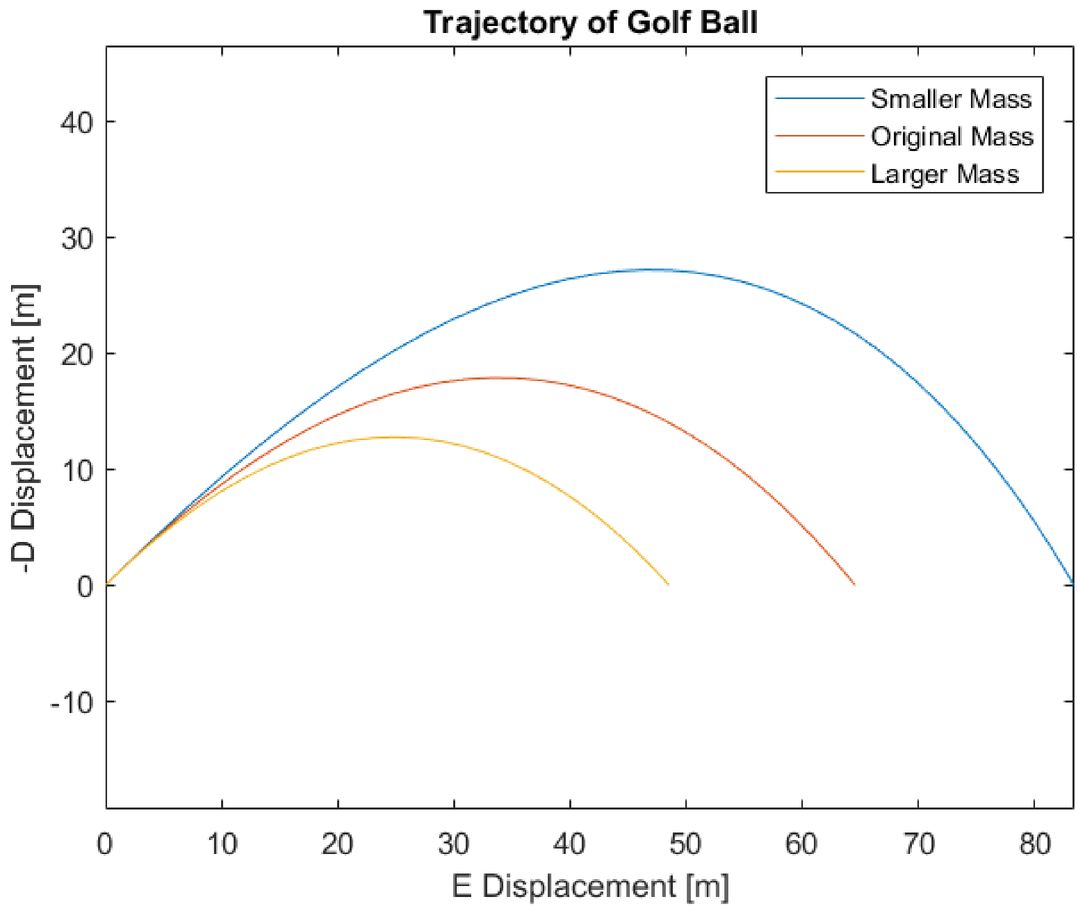
The first task we to develop a simulation for a golf ball that could account for variations of wind. Using MATLAB to vary the wind from 0 to 10 meters a second in the North direction, the following trajectory was found:



The next task was to find the variation in landing location to the amount of wind in the North direction. The landing location is characterized by the total horizontal displacement relative to the golf balls origin and was computed over the increments of wind from 0 to 10 meters a second. The plot of total horizontal distance achieved version wind speed was then plotted as such:

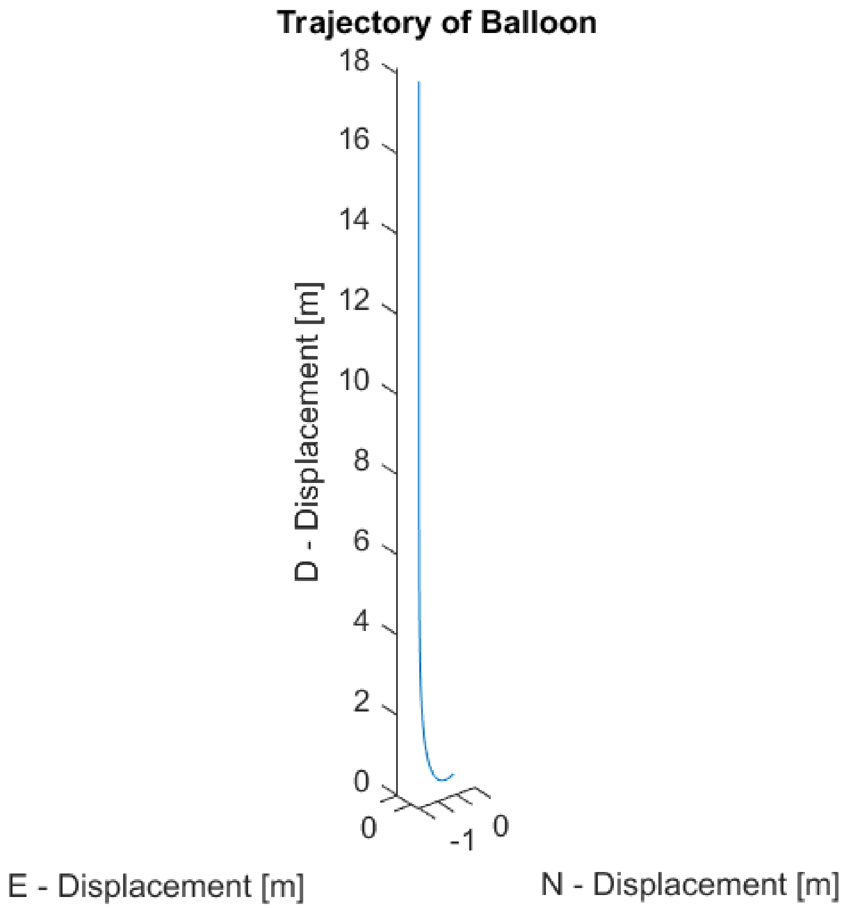


Lastly, we were tasked with finding the distance traveled by the golf ball if the ball was constrained by the total amount of kinetic energy and the mass were to be variable. It was found that the ball traveled further when the mass of the ball were lighter than if it were its original mass or heavier. The following plots display this behavior for the masses of 0.25, 0.5, and 0.75 kilograms:



C. Problem 10:

For this problem we had to find the trajectory of a balloon in variable wind conditions, particularly a balloon filled with 1 m^3 of helium with wind prevailing at 4 $\frac{m}{s}$ north and 2 $\frac{m}{s}$ east. Plot for the balloons trajectory are as follows:



Lastly, we had to find the relationship between wind speed and required helium volume to maintain an accent angle of 45 degrees. This was found by iterating through varying volumes of at high resolution and taking the first point at which the angle of the balloon crosses 45°. This would need to be done for each unique wind condition.

D. Appendix A - MATLAB Code

1. *Problem 9 Main*

```
global m cD A rhoAir g windMag windAng
m = 50/1000; % Mass [kg]
d = 3/100; % Diameter [m]
cD = 0.5;
A = pi*(d/2)^2; % Surface Area of golf ball [m^2]
rhoAir = 1.225; % Density of Air [ kg/m^3]
g = [0, 0, 9.81]; % Graitational acceleration [m/s^2]
windAng = 0; % Inital Wind Direction (Deg)
Velvec = [0 20 20]; % [m/s]
%% Intial Conditions
condition(1) = 0; % Displacemnt in N direction
condition(2) = 0; % Displacemnt in E direction [m/s]
condition(3) = 0; % Displacemnt in D direction [m/s]
condition(4) = Velvec(1); % Velocity in the N direction
condition(5) = Velvec(2); % Velocity in the E direction
condition(6) = Velvec(3); % Velocity in the -D direction
%% First part of Question 9
wind = (0:1:10); % Inital Wind Magnitude
for i = 1:length(wind)
    windMag = wind(i);
```

```
    [t,z] = ode45('Problem9',[0 5],condition);
    figure(1)
    plot3(z(:,1),z(:,2),z(:,3))
    hold on
    ydist(i) = max(z(:,2));
    xdist(i) = max(z(:,1));
    totaldist(i) = (ydist(i)^2+xdist(i)^2)^(1/2); % Gives total Displacemnt of ball
end
axis equal
title('Trajectory of Golf Ball')
xlabel('N Displacement [m]')
ylabel('E Displacement [m]')
zlabel('-D Displacement [m]')
hold off
% Plots the total distance traveled versus wind speed
figure(2)
plot(wind,totaldist)
title('Wind vs. Displacement')
xlabel('Wind (m/s)')
ylabel('Displacement (m)')
%% Second part of question 9
windMag = 0; % Inital Wind Magnitude (Reinitializing)
massvec = [25 50 75]/1000; % [kg]
KE = 1/2*m*norm(Velvec)^2;
tangent = Velvec./norm(Velvec);
for i = 1:3
    m = massvec(i);
    Vnew = sqrt((2*KE)/m).*tangent;
    condition(1) = 0; % Displacemnt in N direction
    condition(2) = 0; % Displacemnt in E direction [m/s]
    condition(3) = 0; % Displacemnt in D direction [m/s]
    condition(4) = Vnew(1); % Velocity in the N direction
    condition(5) = Vnew(2); % Velocity in the E direction
    condition(6) = Vnew(3); % Velocity in the -D direction

    [t,z] = ode45('Problem9',[0 5],condition);

    figure(3)
    plot(z(:,2),z(:,3))
    hold on
    ydistnew(i) = max(z(:,2));
    xdistnew(i) = max(z(:,1));
end
axis equal
title('Trajectory of Golf Ball')
xlabel('E Displacement [m]')
ylabel('-D Displacement [m]')
legend('Smaller Mass','Original Mass','Larger Mass')
```

2. *Problem 9 ODES*

```
function [dydt] = Problem9(t,y)
global m cD A rhoAir g windMag windAng
dx = y(1); % N - location
dy = y(2); % E - location
dz = y(3); % -D - location
Vx = y(4); % N - component of velocity
Vy = y(5); % E compenent of velocity
Vz = y(6); % -D component of velocity

%% Accounting for Wind
Vwx = -1*windMag*cosd(windAng); %Cross range wind [m/s]
Vwy = windMag*sind(windAng); %Down range wind [mph]
Vwz = 0;

vrel = [Vx-Vwx Vy-Vwy Vz-Vwz]; % Relative Wind
mag = sqrt(vrel(1)^2+vrel(2)^2+vrel(3)^2); % Magnitude of velocity rel to body
```

```
if mag == 0
    unitrep = [0 0 0];
else
    unitrep = vrel./mag; % Direction of drag
end

Drag = 0.5*rhoAir*mag^2*cD*A.*unitrep;
Accell = (-Drag - m.*g)./m;

% Kinematic Equations
dydt(1) = vrel(1);
dydt(2) = vrel(2);
dydt(3) = vrel(3);
dydt(4) = Accell(1);
dydt(5) = Accell(2);
dydt(6) = Accell(3);

if dz < 0.0 % stops the trajectory from continueing past ground level
    dydt = 0;
    for i = 1:6
        y(i) = 0;
    end
end
dydt = dydt';
```

3. Problem 10 Main

```
global Wind Wind2
%% Part 1
Problem10part1();
%% Part 2
Wind = 0:1:20;
t2 = 1;

for i=1:length(Wind)
   Wind2 = [Wind(i) 0 0];
   V45 = Problem10part2(t2,V45);
   t2 = t2+1;
end

figure(2)
hold on
plot(Wind,V45);
title('Angle of Balloon')
xlabel('Wind Velocity [m/s]')
ylabel('Volume of Helium')
axis equal
```

4. Problem 10 ODES

```
function [dydt] = Problem10(t,y)
global m cD A V rhoAir rhoHe g Wind
dx = y(1); % x - location
dy = y(2); % y - location
dz = y(3); % z - location
Vx = y(4); % x - component of velocity
Vy = y(5); % y - compenent of velocity
Vz = y(6); % z - component of velocity

%% Accounting for Wind
Vwx = Wind(1);
Vwy = Wind(2);
Vwz = Wind(3);
vrel = [Vx-Vwx Vy-Vwy Vz-Vwz]; % Relative Wind
mag = sqrt(vrel(1)^2+vrel(2)^2+vrel(3)^2); % Magnitude of velocity rel to body
```

```
if mag == 0
    unitrep = [0 0 0];
else
    unitrep = vrel./mag; % Direction of drag
end

Drag = -0.5*rhoAir*mag^2*cD*A.*unitrep;
Bouyancy =  V * (rhoAir - rhoHe) * g;
Accell = (Bouyancy + Drag - m.*g)./m;

% Kinematic Equations
dydt(1) = vrel(1);
dydt(2) = vrel(2);
dydt(3) = vrel(3);
dydt(4) = Accell(1);
dydt(5) = Accell(2);
dydt(6) = Accell(3);

if dz < 0.0 % stops the trajectory from continueing past ground level
    dydt = 0;
    for i = 1:6
        y(i) = 0;
    end
end
dydt = dydt';
```

5. Problem 10 Part 1

```
function Problem10part1()

global m cD A V rhoAir rhoHe g Wind
m = 0.5; % Mass of Balloon[kg]
cD = 0.5;
V = 1; % Volume of Helium  [m^3]
r = (V /((4/3)*pi))^(1/3);
A = pi*r^2;
rhoAir = 1.225; % Density of Air [kg/m^3]
rhoHe = 0.164; % Density of Helium [kg/m^3]
g = [0, 0, 9.81]; % Graitational acceleration [m/s^2]
Velvec = [0 0 0]; % [m/s]

%% Intial Conditions
condition(1) = 0; % Displacemnt in N direction
condition(2) = 0; % Displacemnt in E direction [m/s]
condition(3) = 0; % Displacemnt in D direction [m/s]
condition(4) = Velvec(1); % Velocity in the N direction
condition(5) = Velvec(2); % Velocity in the E direction
condition(6) = Velvec(3); % Velocity in the D direction

[t,z] = ode45('Problem10',[0 5],condition);
figure(1)
plot3(z(:,1),z(:,2),z(:,3))
axis equal
title('Trajectory of Balloon')
xlabel('N - Displacement [m]')
ylabel('E - Displacement [m]')
zlabel('D - Displacement [m]')
```

6. Problem 10 Part 2

```
function [v45] = Problem10part2(t2,v45)

global m cD A V rhoAir rhoHe g Wind Wind2
m = 0.5; % Mass of Balloon[kg]
cD = 0.5;
V = 1; % Volume of Helium  [m^3]
```

```matlab
r = (V /((4/3)*pi))^(1/3);
A = pi*r^2;
rhoAir = 1.225; % Density of Air [kg/m^3]
rhoHe = 0.164; % Density of Helium [kg/m^3]
g = [0, 0, 9.81]; % Graitational acceleration [m/s^2]
Velvec = [0 0 0]; % [m/s]
Wind = [4 2 0];
%% Intial Conditions
condition(1) = 0; % Displacemnt in N direction
condition(2) = 0; % Displacemnt in E direction [m/s]
condition(3) = 0; % Displacemnt in D direction [m/s]
condition(4) = Velvec(1); % Velocity in the N direction
condition(5) = Velvec(2); % Velocity in the E direction
condition(6) = Velvec(3); % Velocity in the D direction

while true
    [t,z] = ode45('Problem10part2',[0 10],condition); %does ODE45
    angle1 = -atand(z(end,1)/z(end,3));
    angle2 = atand(1);
    if angle1<angle2
        V45(j) = V;
        break
    else
        V = V+(V/50);
    end
end
```