ASEN 3128 Homework 6

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I. Problem 1:

Using the non-dimensional data from table 6.1 in the textbook, along with data in table E.1 for case III (for a Boeing 747 airplane), the following dimensional derivatives were computed and are tabulated below. This case was for constant altitude flight.

Table 1. Dimensional Derivatives

	X [N]	**Z [N]**	**M [m · N]**
u $[\frac{m}{s}]$	$-2.209 * 10^3$	$-2.350 * 10^4$	$1.776 * 10^4$
w $[\frac{m}{s}]$	$4.486 * 10^3$	$-1.006 * 10^5$	$-1.742 * 10^5$
q $[\frac{m}{s}]$	0	$-5.0404 * 10^5$	$-1.695 * 10^7$
$\dot{w}[\frac{m}{s}]$	0	$1.891 * 10^3$	$-1.685 * 10^4$

II. Problem 2:

For the Linearized Longitudinal set, the following A Matrix was computed using MATLAB:

$$A = \begin{vmatrix} -0.0077 & 0.0155 & 0 & -9.81 \\ -0.0820 & -0.3509 & 265.47 & 0 \\ 4.265E-4 & -0.0037 & -0.477 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

III. Problem 3:

The eigenvalues and vectors for the linearized longitudinal set was calculated using the relation: $det(\lambda I - A) = 0$. Once the eigenvalues were calculated, they could be classified into their corresponding mode by finding the eigenvalues with the largest real part (magnitude wise). The short period response has a larger real eigenvalue due to the real parts function as a stabilizer. The larger the real part, the quicker the envelope of the oscillation will go to either zero or infinity. Since the real part was larger in magnitude and negative, the real part will drive the oscillations to zero as the e^{nt} part of $e^{nt} \cdot e^{j\omega t}$ dominates (where n is the real part of the eigenvalue $\lambda = n \pm i\omega$). The real part dominates since the imaginary part is just a sinusoidal oscillation, therefore, the real part drives this oscillation's amplitude to its end state. The eigenvectors, eigenvalues, dampening coefficient, and natural frequency were calculated for each mode using MATLAB and are provided below:

$$\lambda = n \pm i\omega, \quad \zeta = \frac{-n}{\omega_n}, \quad \omega_n = \sqrt{(\omega^2 + n^2} \tag{1}$$

A. Short Period Mode:

$$\lambda_{1,2} = -0.4142 \pm 0.9944i, \quad \zeta = 0.3845, \quad \omega_n = 1.077 \tag{2}$$

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$$Eigenvector_1 = \begin{vmatrix} 0.0155 + 0.0105i \\ 0.999 \\ -0.0002 + 0.0037i \\ 0.0033 - 0.0011i \end{vmatrix}$$

$$Eigenvector_2 = \begin{vmatrix} 0.0155 - 0.0105i \\ 0.999 \\ -0.0002 - 0.0037i \\ 0.0033 + 0.0011i \end{vmatrix}$$

B. Phugoid Mode:

$$\lambda_{1,2} = -0.0038 \pm 0.0620i, \quad \zeta = 0.0605, \quad \omega_n = 0.0622 \tag{3}$$

$$Eigenvector_1 = \begin{vmatrix} -0.998 \\ -0.0639 + 0.0072i \\ -3.933E - 4 - 5.515E - 6 \\ 0.0003 + 0.0063i \end{vmatrix}$$

$$Eigenvector_2 = \begin{vmatrix} -0.998 \\ -0.0639 - 0.0072i \\ -3.933E - 4 + 5.515E - 6 \\ 0.0003 - 0.0063i \end{vmatrix}$$

IV. Problem 4:

When approximating the short period mode using the reduced order model, the simplified set of equations reduces to :

$$\begin{vmatrix} \Delta\dot{q} \\ \Delta\dot{\theta} \end{vmatrix} = \begin{vmatrix} \frac{M_q}{I_y} & \frac{U_0 M_w}{I_y} \\ 1 & 0 \end{vmatrix} \begin{vmatrix} \Delta q \\ \Delta\theta \end{vmatrix}$$

Solving for the eigenvalue of this homogeneous set 2x2 state space model for pitch motion using the relation, $det(\lambda I - A) = 0$, where I is the identity matrix and A is from the model. The corresponding eigenvalues are:

$$\lambda_{1,2} = -0.1888 \pm 0.9973i \tag{4}$$

This shows a large discrepancy when comparing the real parts of the eigenvalue from the less simplified case. The real parts of the eigenvalue are the parts that ultimately have the control on the stability as they drive the envelope of the imaginary part. While the real parts were noticed to have a rather large discrepancy, the imaginary part was almost exactly the same from both models. This means that the corresponding period will also be very similar since the imaginary part of the eigenvalues control the period.

When comparing the oscillation period of the Phugoid mode from problem 2, to the oscillation period given by the Lanchester approximation, it can be seen that the values were much different. The oscillation period calculated using the linearized model from problem 2 was calculated using the imaginary part of the eigenvalue, ω, using the following relation: $T = \frac{2\pi}{\omega}$. The oscillation period calculated using the Lanchester approximation, utilizes conservation of energy to get the relation: $T = \pi\sqrt{2}\frac{U_0}{g}$, where U_0 is the trim state for the the x component of velocity in body frame coordinates. These relations give the following results:

$$T = 101.28[s], \quad T_{Lanchester} = 120.23[s] \tag{5}$$

V. Problem 5:

The linearized longitudinal set of state variables, Δu, Δw, Δq, and $\Delta\theta$ were solved for using MATLAB's ODE45 differential equation solver. The set was solved over two different periods of time to analyze the two different modes. First, over a long period of time to analyze the Phugoid mode response and then over a short period of time to see the short phase response of the system. To check that the trim state is an equilibrium state, we check the system about zero perturbations first. This resulted in constant state variables as the trim states will not change in equilibrium. These trim states are plotted first for each mode below. The system is an approximation for a Boeing 747 flying at 40,000 feet with individual perturbations of $\Delta u = 10[\frac{m}{s}]$, $\Delta w = 10[\frac{m}{s}]$, $\Delta q = 0.1[\frac{rad}{s}]$, and $\Delta\theta = 0.1[rad]$, to see how the system will react. The following plots show the reactions about each of these perturbations.

A. Phugoid Mode:

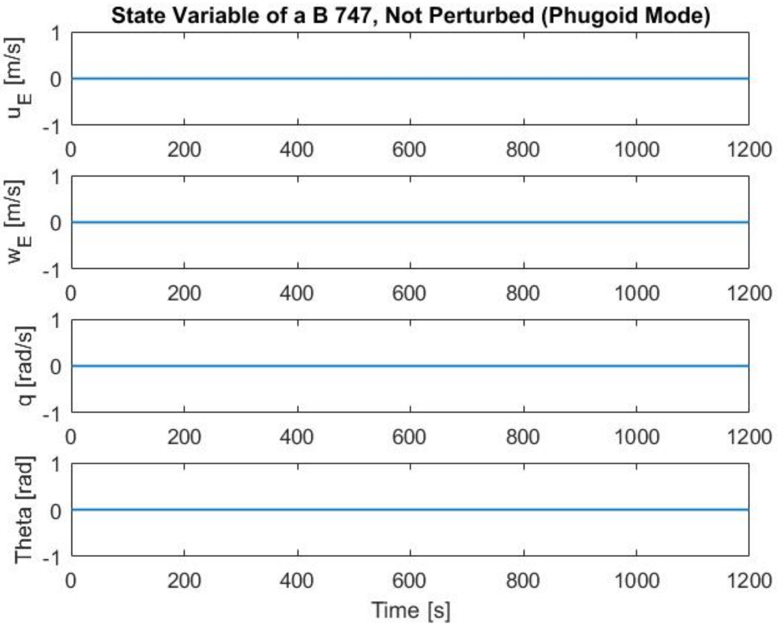

Figure 1. No Perturbations

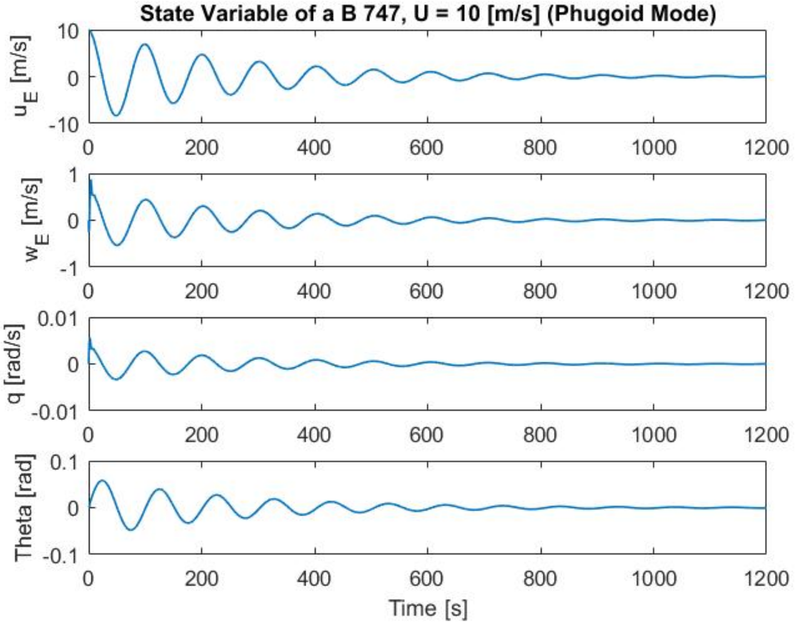

Figure 2. $\Delta u = 10[\frac{m}{s}]$

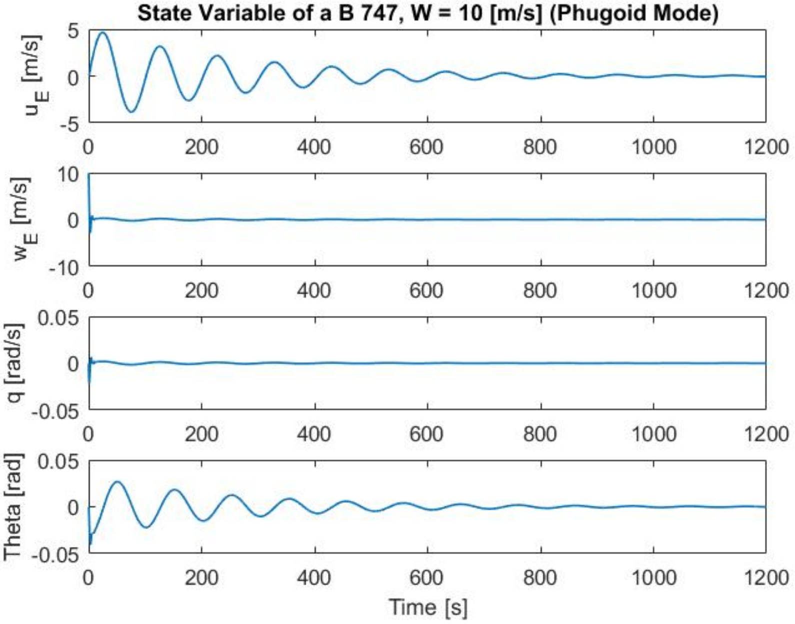

Figure 3. $\Delta w = 10[\frac{m}{s}]$

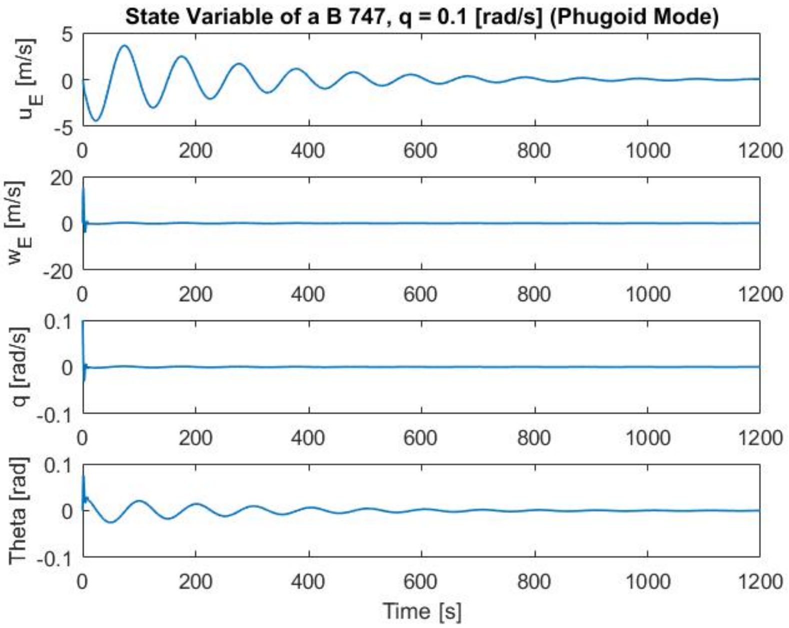

Figure 4. $\Delta q = 0.1[\frac{rad}{s}]$

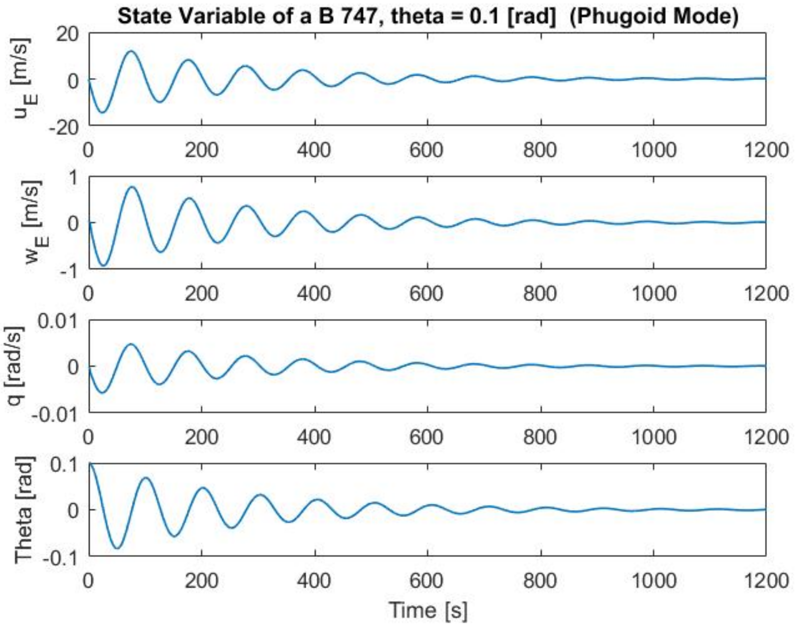

Figure 5. $\Delta \theta = 0.1[rad]$

B. Short Period Mode

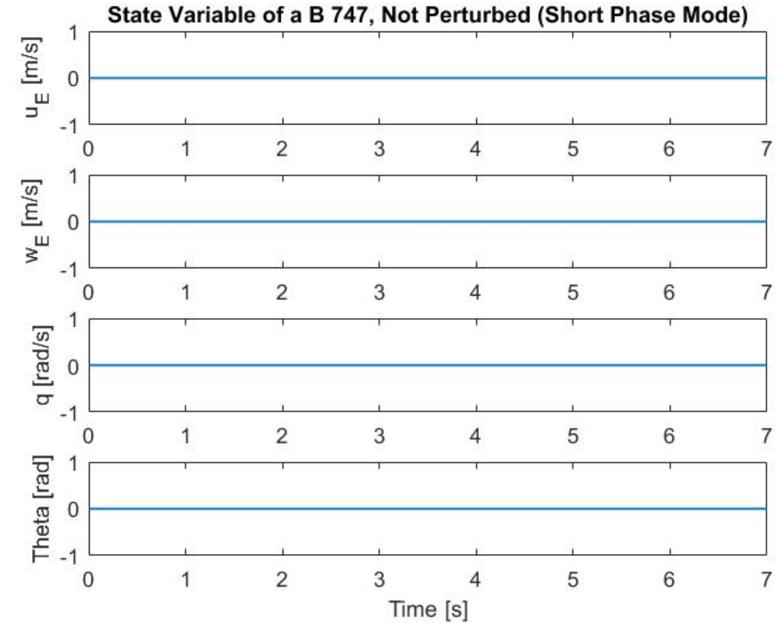

Figure 6. No Perturbations

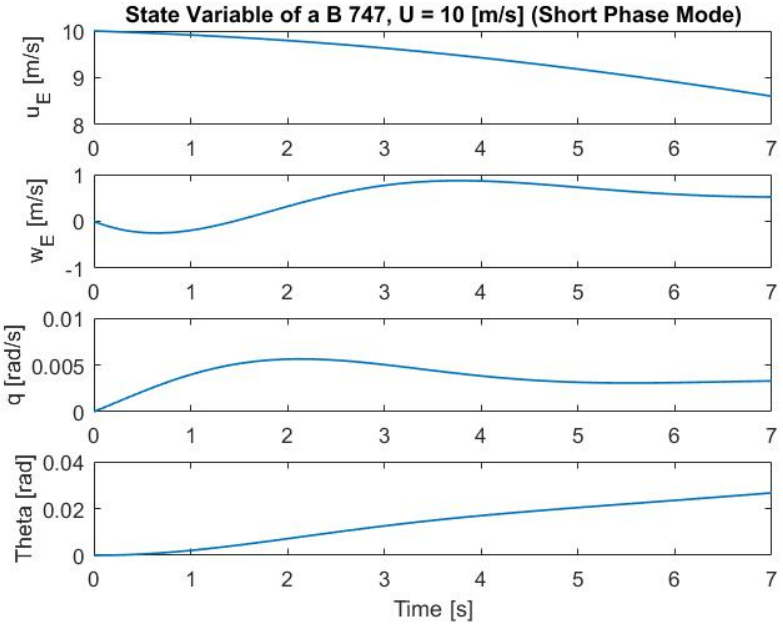

Figure 7. $\Delta u = 10[\frac{m}{s}]$

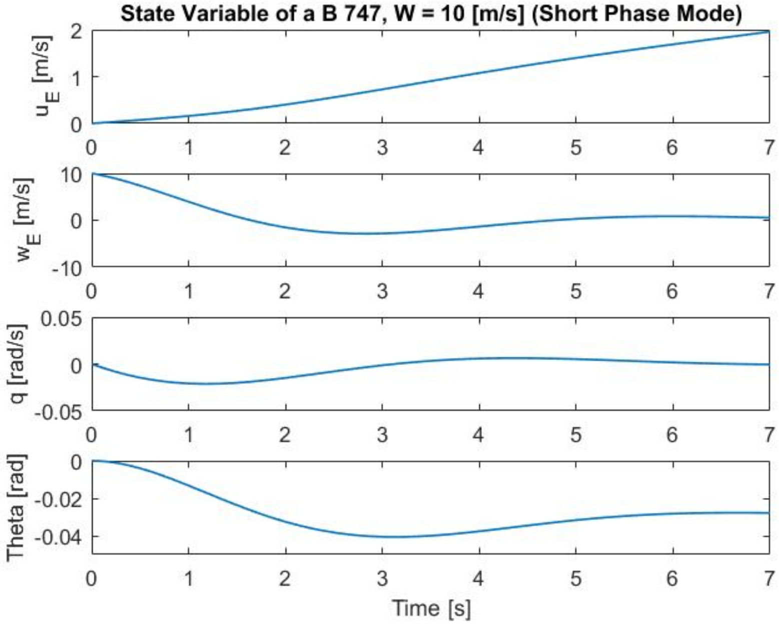

Figure 8. $\Delta w = 10[\frac{m}{s}]$

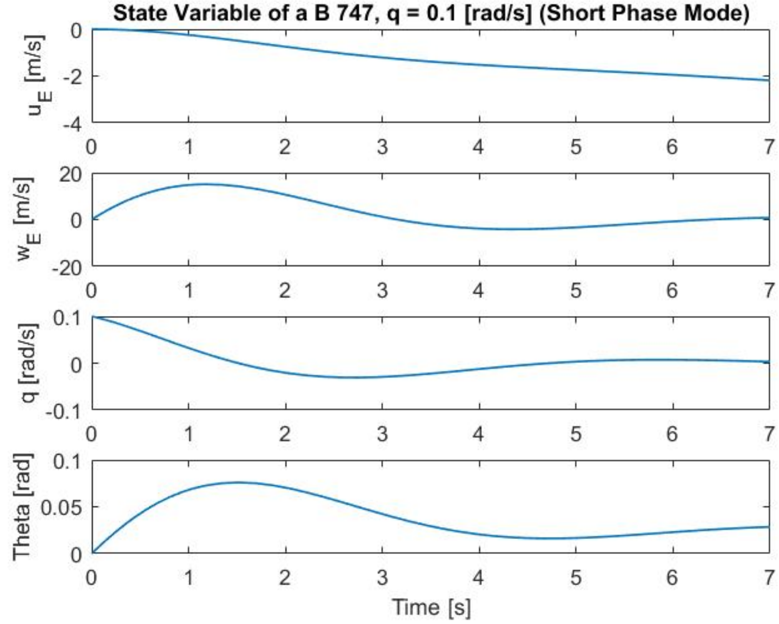

Figure 9. $\Delta q = 0.1[\frac{rad}{s}]$

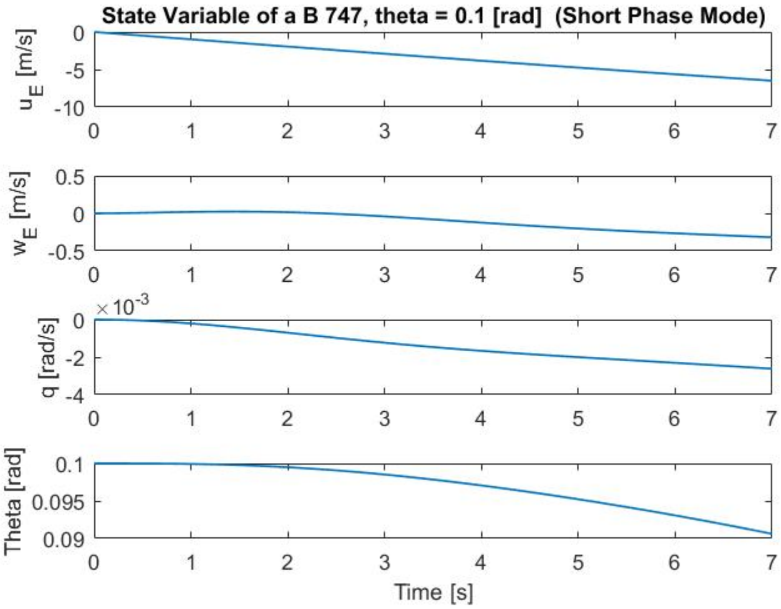

Figure 10. $\Delta\theta = 0.1[rad]$

After analyzing the plots it can be seen that the short phase mode is more pronounced for perturbations in $Deltaw^E$ and Δq, where the phugoid mode is nearly none existent for the state variables $Deltaw^E$ andΔq and is largely diminished for $\Delta\theta$. Perturbations in Δu^E and θ had more pronounced phugoid modes as they system oscillates about trim for a much longer period of time. The short phase mode was still present for the state variables $Deltaw^E$ andΔq in the case for a perturbation about Δu^E.

C. Appendix A - MATLAB Code

1. Script that Answers Questions 1-4

```
%% Author: Jack Lambert
% ASEN 3128
% Homework 6
% Problems 1-4
% Last Edited: 3/11/2018
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
close all;
%% Problem 1 - B747 Dimensional Derivatives at 40,000 ft
% Nondimensional Derivatives
% Table 6.1 -
Cx = [-.108, .2193, 0, 0];
Cz = [-.106, -4.92, -5.921, 5.896];
Cm = [.1043, -1.023, -23.92, -6.314];

% Table E.1 B747 Case 3
Alt = 40000*(0.3048); % Altitude [ft] -> [m]
[T, a, P, rho] = atmosisa(Alt); % Standard Atmosphere Properties at Alt.
Vel = 871*(0.3048);% Velocity [ft/s] -> [m/s]
u0 = Vel; % Initial Velocity in x-coord - Stability Axis Frame
W = 6.366*10^5*4.44822; % Weight [lb]->[N]
Ix_PA = 1.82*10^7*1.35581795; % Moment of Interia x-PA [slug ft^2]-> [kg m^2]
Iy_PA = 3.31*10^7*1.35581795; % Moment of Interia y-PA [slug ft^2]-> [kg m^2]
Iz_PA = 4.97*10^7*1.35581795; % Moment of Interia z-PA [slug ft^2]-> [kg m^2]
Izx_PA = 9.70*10^5*1.35581795; % Moment of Interia zx-PA [slug ft^2]-> [kg m^2]
zeta = -2.4; % Angle between Stability Axis and PA [degrees]
```

```
I = [Ix_PA, 0,-Izx_PA;...
    0, Iy_PA,0;...
    -Izx_PA, 0, Iz_PA]; % Inertia Matrix in PA
Q_PA_SA = [cosd(zeta), 0, -sind(zeta);...
    0, 1, 0;...
    sind(zeta), 0, cosd(zeta)]; % Transformation Matrix [PA-SA]
I_SA = Q_PA_SA * I * Q_PA_SA'; % MOI in Stability axis Frame
Ix = I_SA(1,1); % Moment of Interia x-SA [kg m^2]
Iy = I_SA(2,2); % Moment of Interia y-SA [kg m^2]
Iz = I_SA(3,3); % Moment of Interia z-SA [kg m^2]
Izx = (1/2)*(Ix-Iz)*sind(2*zeta)+Izx_PA*...
    (sind(zeta)^2-cosd(zeta)^2); % Moment of Interia zx-SA [kg m^2]

CD = .043; % Coefficient of Drag
theta0 = 0; % Initial Pitch Angle [deg]
cbar = 27.31*(0.3048); % Mean Chord Length [ft]->[m]
S = 5500*(0.3048)^2; % Surface Area [ft^2]->[m^2]
Cw0 = W/(.5*rho*S*u0^2);
g = 9.81; % Gravity Constant [m/s^2]
m = W/g; % Mass of Plane [kg]

% Function that Computes Dimensional Derivatives from Non-Dimenional derivatives
[X, Z, M ] = NonDimLong(rho,u0,S,W,theta0,Cx,Cz,Cm,cbar);
T = table(X',Z',M');
T.Properties.VariableNames = {'X' 'Z' 'M'};
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Problem 2 - A Matrix for Linearized Longitudinal Dynamics
[A,theta0,u0] = A_Matrix(); % Function that Computes the A Matrix
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Problem 3 - Short Period and Phugoid Modes
[eigVec,eigVal] = eig(A);

modes = diag(eigVal);
max_real = max(abs(real(modes)));
% Short Mode has Larger Real Part
 j = 1;
 k = 1;
for i = 1:length(modes)
    if abs(real(modes(i))) == max_real
        SP_Mode(j) = modes(i); % Short Period Mode
        SP_vec(:,j) = eigVec(:,i); % Short Period Eigen Vec
        j = j+1;
    else
        Phu_Mode(k) = modes(i); % Phugoid Mode
        Phu_vec(k,:) = eigVec(:,i); % Short Period Eigen Vec
        k = k+1;

    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Natural Frequency and Dampeing Ratio

% Phugoid Mode

Wn_PM = ( real(Phu_Mode(1))^2+imag(Phu_Mode(1))^(2) )^(1/2); % Natural Frequency
zeta_PM = -real(Phu_Mode(1))/Wn_PM; % Dampening Coefficient
Period_PM = (2*pi) / imag(Phu_Mode(1)); % Period

% Short Period Mode

Wn_SP = ( real(SP_Mode(1))^2+imag(SP_Mode(1))^(2) )^(1/2); % Natural Frequency
zeta_SP = -real(SP_Mode(1))/Wn_SP; % Dampening Coefficient
Period_SP = 1 / imag(SP_Mode(1)); % Period
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Problem 4

% Linearized Short Period Approximation
A_Lin = [M(3)/Iy u0*M(2)/Iy; 1 0]; % State Variable Matrix
```

```
[eigVec_Lin,eigVal_Lin] = eig(A_Lin);
Mode_SP_Lin = diag(eigVal_Lin); % EigenValues

% Lanchester Approximation for Phugoid Period

Period_PM_Lan = pi*(2)^(1/2)*(u0/g);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

2. Non-Dimensional Coefficients to Dimensional Coefficients Function

```
function [X, Z, M ] = NonDimLong(rho,u0,S,W,theta0,Cx,Cz,Cm,cbar)

% Computing the Nondimesnional Inital Weight Derivative
Cw0 = W/ ((1/2)*rho*S*u0^2);

% X
Xu = rho*u0*S*Cw0*sind(theta0) + .5*rho*u0*S*Cx(1);
Xw = .5*rho*u0*S*Cx(2);
Xq = .25*rho*u0*cbar*S*Cx(3);
Xwdot = .25*rho*cbar*S*Cx(4);

X = [Xu, Xw, Xq, Xwdot]';

% Z
Zu = -rho*u0*S*Cw0*cosd(theta0) + .5*rho*u0*S*Cz(1);
Zw = .5*rho*u0*S*Cz(2);
Zq = .25*rho*u0*cbar*S*Cz(3);
Zwdot = .25*rho*cbar*S*Cz(4);

Z = [Zu, Zw, Zq, Zwdot]';

% M
Mu = .5*rho*u0*cbar*S*Cm(1);
Mw = .5*rho*u0*cbar*S*Cm(2);
Mq = .25*rho*u0*(cbar^2)*S*Cm(3);
Mwdot = .25*rho*(cbar^2)*S*Cm(4);

M = [Mu, Mw, Mq, Mwdot]';
```

3. Function that Computes the A Matrix

```
%% Author: Jack Lambert
% ASEN 3128
% Homework 6
% Purpose: To keep all constants in one function so they are not defined
% more than once and then Compute the A Matrix and provide Trim States
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [A,theta0,u0] = A_Matrix()
%% Airplane Parameters
% Nondimensional Derivatives
% Table 6.1 -
Cx = [-.108, .2193, 0, 0];
Cz = [-.106, -4.92, -5.921, 5.896];
Cm = [.1043, -1.023, -23.92, -6.314];

% Table E.1 B747 Case 3
Alt = 40000*(0.3048); % Altitude [ft] -> [m]
[T, a, P, rho] = atmosisa(Alt); % Standard Atmosphere Properties at Alt.
W = 6.366*10^5*4.44822; % Weight [lb]->[N]
Ix_PA = 1.82*10^7*1.35581795; % Moment of Interia x-PA [slug ft^2]-> [kg m^2]
Iy_PA = 3.31*10^7*1.35581795; % Moment of Interia y-PA [slug ft^2]-> [kg m^2]
Iz_PA = 4.97*10^7*1.35581795; % Moment of Interia z-PA [slug ft^2]-> [kg m^2]
Izx_PA = 9.70*10^5*1.35581795; % Moment of Interia zx-PA [slug ft^2]-> [kg m^2]
```

```
zeta = -2.4; % Angle between Stability Axis and PA [degrees]
I = [Ix_PA, 0,-Izx_PA;...
    0, Iy_PA,0;...
    -Izx_PA, 0, Iz_PA]; % Inertia Matrix in PA
Q_PA_SA = [cosd(zeta), 0, -sind(zeta);...
    0, 1, 0;...
    sind(zeta), 0, cosd(zeta)]; % Transformation Matrix [PA-SA]
I_SA = Q_PA_SA * I * Q_PA_SA'; % MOI in Stability axis Frame
Ix = I_SA(1,1); % Moment of Interia x-SA [kg m^2]
Iy = I_SA(2,2); % Moment of Interia y-SA [kg m^2]
Iz = I_SA(3,3); % Moment of Interia z-SA [kg m^2]
Izx = (1/2)*(Ix-Iz)*sind(2*zeta)+Izx_PA*...
    (sind(zeta)^2-cosd(zeta)^2); % Moment of Interia zx-SA [kg m^2]
CD = .043; % Coefficient of Drag
cbar = 27.31*(0.3048); % Mean Chord Length [ft]->[m]
S = 5500*(0.3048)^2; % Surface Area [ft^2]->[m^2]
g = 9.81; % Gravity Constant [m/s^2]
m = W/g; % Mass of Plane [kg]

%% Trim States
Vel = 871*(0.3048);% Velocity [ft/s] -> [m/s]
u0 = Vel; % Initial Velocity in x-coord - Stability Axis Frame (Trim State)
theta0 = 0; % Initial Pitch Angle [deg]
Cw0 = W/(.5*rho*S*u0^2);
%% Function that Computes Dimensional Derivatives from Non-Dimenional derivatives
[X, Z, M ] = NonDimLong(rho,u0,S,W,theta0,Cx,Cz,Cm,cbar);

%% State Variable Matrix A
deltaU_dot = [X(1)/m, X(2)/m, 0, -g*cosd(theta0)];
w_dot = [Z(1)/(m-Z(4)), Z(2)/(m-Z(4)), (Z(3)+m*u0)/(m-Z(4)), (-W*sind(theta0))/(m-Z(4))];
q_dot = [(1/Iy)*(M(1) + ((M(4)*Z(1))/(m-Z(4))) ),...
        (1/Iy)*(M(2) + ((M(4)*Z(2))/(m-Z(4))) ),...
        (1/Iy)*(M(3) + ((M(4)*(Z(3)+m*u0))/(m-Z(4))) ),...
        -((M(4)*W*sind(theta0))/(Iy*(m-Z(4))))];
deltaTheta_dot = [0, 0, 1, 0];

A = [deltaU_dot;w_dot;q_dot;deltaTheta_dot];

end
```

4. Main Function

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Author: Jack Lambert
% Dale Lawrence
% Aircraft Dynmaics Homework 6
% Problem 5
% Purpose: Sets Initial COnditions for each Pertubation Case and Calls ODE45
% to plot the State Variables vs time
% Date Modefied: 2/12/18
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% ODE45 Variable Allocation
%                     X_E = z(1); % z-position, Inerital Frame
%                     Z_E = z(2); % z-position, Inerital Frame
%                     u_dot = z(3); % x-component of Velocity, Body Frame
%                     z_dot = z(4); % x-component of Velocity, Body Frame
%                     q_dot = z(6); % Angular Velocity about the y-axis [rad/s]
%                     theta_dot = z(5); % Pitch Angle
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Inital Conditions:
%                     i = 1 ----> 10 [m/s] - U
%                     i = 2 ----> 10 [m/s] - W
%                     i = 3 ----> 0.1 [rad/s] - q
%                     i = 4 ----> 0.1 [rad] - Pitch
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Initial Conditions
```

```
c1 = [0 0 0 0]; % xE: Location in Inertial Coordinates [m]
c2 = [0 0 0 0]; % zE: Location in Inertial Coordinates [m]
c3 = [10 0 0 0]; % Delta U: x-comp, BF Interial Velocity [m/s]
c4 = [0 10 0 0]; % Delta W: z-comp, BF Interial Velocity [m/s]
c5 = [0 0 0.1 0]; % Delta q: y-comp, BF Angular Velocity [rad/s]
c6 = [0 0 0 0.1]; % Delta Theta: Pitch Angle

for i = 1:4
    condition{i}= [c1(i) c2(i) c3(i) c4(i) c5(i) c6(i)];
end

%% State Variables vs. Time
t_Phugoid = [0 1200]; % Longer time to see Phugoid Mode
t_ShortP = [0 7]; % Shorter time to see Short Phase Mode

string = ["U = 10 [m/s]","W = 10 [m/s]","q = 0.1 [rad/s]",...
    "theta = 0.1 [rad] "]; % Title for Varying IC's
% Phugoid Response (Longer Time)
for i = 1:4
    % Calling ODE45
    [t,z] = ode45('Linearized_Longitudinal_Dynamics',t_Phugoid,condition{i});

    % Plotting Conditions
    figure
    % U_E vs time
    subplot(4,1,1)
    plot(t ,z(:,3),'Linewidth',1)
    tit = sprintf('%s %s %s','State Variable of a B 747,',string(i),'(Phugoid Mode)');
    title(tit)
    ylabel('u_E [m/s]')


    % W_E vs time
    subplot(4,1,2)
    plot(t ,z(:,4),'Linewidth',1)
    ylabel('w_E [m/s]')

    % q vs time
    subplot(4,1,3)
    plot(t ,z(:,5),'Linewidth',1)
    ylabel('q [rad/s]')

    % Theta vs time
    subplot(4,1,4)
    plot(t ,z(:,6),'Linewidth',1)
    ylabel('Theta [rad]')
    xlabel('Time [s]')

end

% Short Phase Response (shorter Time)
for i = 1:4
    % Calling ODE45
    [t,z] = ode45('Linearized_Longitudinal_Dynamics',t_ShortP,condition{i});

    % Plotting Conditions
    figure
    % U_E vs time
    subplot(4,1,1)
    plot(t ,z(:,3),'Linewidth',1)
    tit = sprintf('%s %s %s','State Variable of a B 747,',string(i),'(Short Phase Mode)');
    title(tit)
    ylabel('u_E [m/s]')


    % W_E vs time
    subplot(4,1,2)
    plot(t ,z(:,4),'Linewidth',1)
    ylabel('w_E [m/s]')
    % q vs time
    subplot(4,1,3)
    plot(t ,z(:,5),'Linewidth',1)
    ylabel('q [rad/s]')

    % Theta vs time
    subplot(4,1,4)
    plot(t ,z(:,6),'Linewidth',1)
    ylabel('Theta [rad]')
    xlabel('Time [s]')

end

%% Plotting Position
for i = 1:4
    [t,z] = ode45('Linearized_Longitudinal_Dynamics',t_Phugoid,condition{i});

    % xE vs zE
    figure
    plot(z(:,1) ,z(:,2),'Linewidth',1)
    tit = sprintf('%s %s %s','Position of a B747,',string(i));
    title(tit)
    xlabel('xE [m]')
    ylabel('zE [m]')
    axis equal
end
```

5. *Function ODE45 calls for the Linearized Longitudinal Set*

```
%% Author: Jack Lambert
% ASEN 3128
% Problem 5
% Purpose: Function for ODE45 to call to calculate the State variables xE,
% zE, u_dot, w_dot, q_dot, and theta_dot. This function uses the simplified
% assumptions for the Linearized Longitudinal Dynamics Set
% Last Edited: 3/11/2018
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [dzdt] = Linearized_Longitudinal_Dynamics(t,z)

X_E = z(1); % x-position, Ineritial Frame
Z_E = z(2); % z-position, Ineritial Frame
u_dot = z(3); % x-component of Velocity, Body Frame
w_dot = z(4); % z-component of Velocity, Body Frame
q_dot = z(5); % y-component of Angular Velocity, Body Frame
theta_dot = z(6); % Pitch Angle

%% State Variable Matrix for Linearized Longitudinal Set
[A,theta0,u0] = A_Matrix(); % A matrix function based on plane and parameters
State = [u_dot, w_dot, q_dot, theta_dot]'; % Couple State Variables in Long. Set
var = A*State; % % Couple State Variables in Long. Set
%% Solving for Inertial Position
dzdt(1) = u_dot*cosd(theta0) + w_dot*sind(theta0) - u0*theta_dot*sind(theta0); % xE
dzdt(2) = -u_dot*sind(theta0) + w_dot*cosd(theta0)-u0*theta_dot*cosd(theta0); % zE
%% Solving for State Variables in the Linearized Longitudinal Set
dzdt(3) = var(1); % uE
dzdt(4) = var(2); % wE
dzdt(5) = var(3); % q
dzdt(6) = var(4); % theta

dzdt = dzdt'; % Inverts for ODE45
end
```