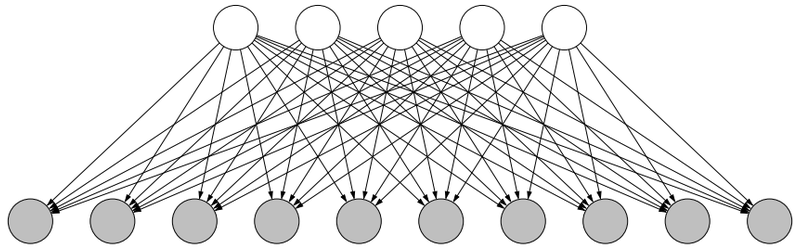
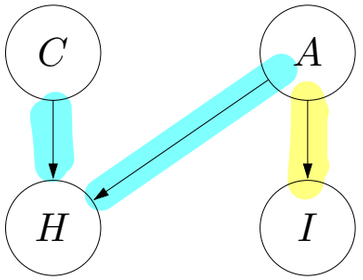
Bayesian networks: probabilistic inference



- In this module, I will talk about a strategy for performing probabilistic inference in general Bayesian networks.

Review: Bayesian network



Random variables:

cold C, allergies A, cough H, itchy eyes I

Joint distribution:

$$\mathbb{P}(C=c, A=a, H=h, I=i) = p(c)p(a)p(h \mid c, a)p(i \mid a)$$

Definition: Bayesian network

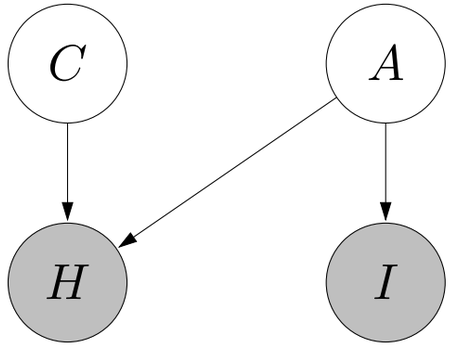
Let $X = (X_1, \ldots, X_n)$ be random variables.

A **Bayesian network** is a directed acyclic graph (DAG) that specifies a joint distribution over X as a product of local conditional distributions, one for each node:

$$\mathbb{P}(X_1 = x_1, \ldots, X_n = x_n) \stackrel{\text{def}}{=} \prod_{i=1}^{n} p(x_i \mid x_{\text{Parents}(i)})$$

- Recall that a Bayesian network is given by (i) a set of random variables, (ii) directed edges between those variables capturing qualitative dependencies, (iii) local conditional distributions of each variable given its parents which captures these dependencies quantitatively, and (iv) a joint distribution which is produced by multiplying all the local conditional distributions together.

Review: probabilistic inference



query

evidence

Question: $\mathbb{P}(C \mid H = 1, I = 1)$

given
1) evidence vars
2) query vars

Input

Bayesian network: $\mathbb{P}(X_1, \ldots, X_n)$

Evidence: $E = e$ where $E \subseteq X$ is subset of variables

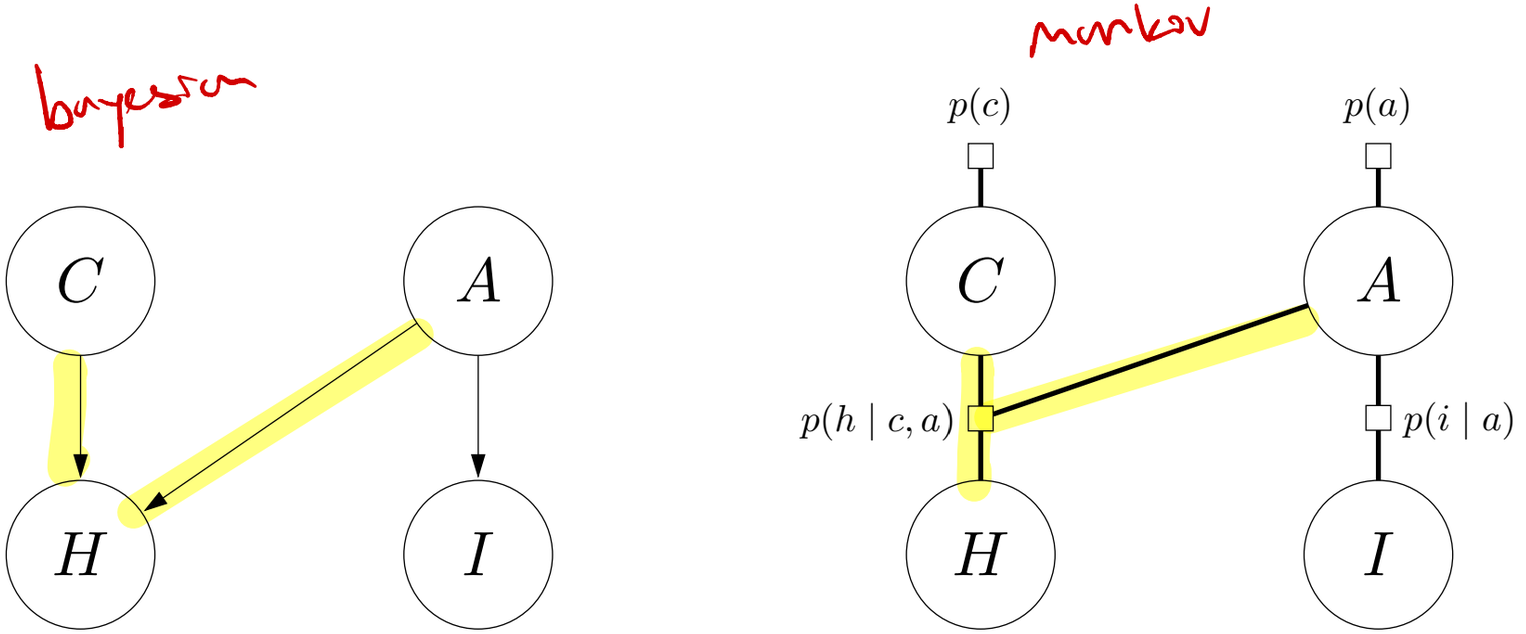
Query: $Q \subseteq X$ is subset of variables

compute
1) prob of query given evidence

Output

$\mathbb{P}(Q \mid E = e)$  $\mathbb{P}(Q = q \mid E = e)$ for all values q

Reduction to Markov networks



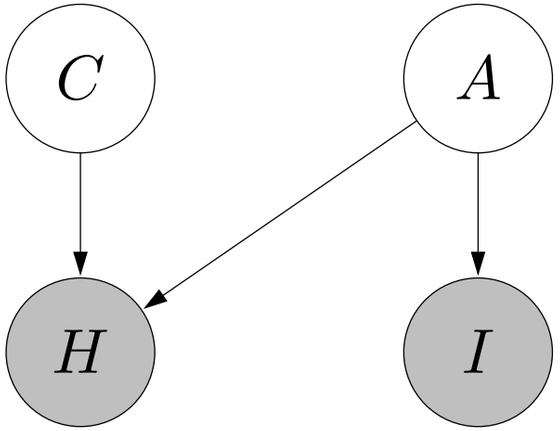
$$\mathbb{P}(C = c, A = a, H = h, I = i) = \frac{1}{Z} p(c)p(a)p(h \mid c, a)p(i \mid a)$$

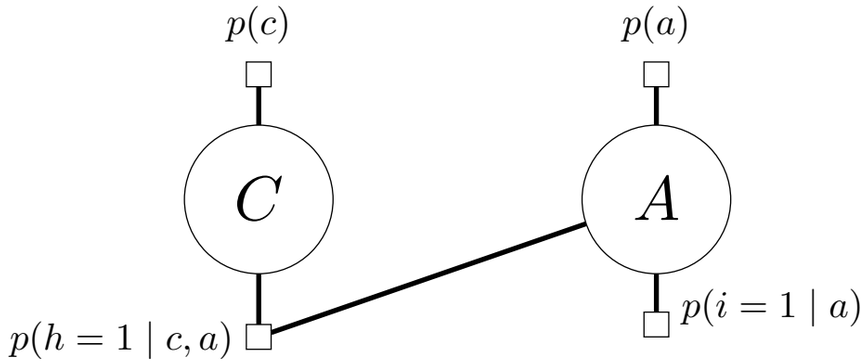
Bayesian network = Markov network with normalization constant $Z = 1$

Reminder: single factor that connects **all** parents!

- Our overarching strategy for performing inference in Bayesian networks is to convert them into Markov networks.
- Recall that the joint distribution is just the product of all the local conditional distributions. The local conditional distributions (e.g., $p(a \mid b, e)$) are all non-negative so they can be interpreted as simply factors in a factor graph.
- Recall that a Markov network defines the joint distribution as the product of all the factors divided by some normalization constant Z. But in this case, $Z = 1$ because the factors are local conditional distributions of a Bayesian network! Put it another way, Bayesian networks are just instances of Markov networks where the normalization constant $Z = 1$.
- It's important to remember that there is a single factor that connects all the parents. Don't let the directed graph in the Bayesian network deceive you into thinking that there are two factors, one per arrow, which is a common mistake.
- Now we can run any inference algorithm for Markov networks (e.g., Gibbs sampling) on this so-called Markov network and obtain quantities such as $\mathbb{P}(H = 1)$. But there is one important thing that's missing, which is the ability to condition on evidence...

Conditioning on evidence





Markov network:

$$\mathbb{P}(C = c, A = a \mid H = 1, I = 1) = \frac{1}{Z} p(c)p(a)p(h = 1 \mid c, a)p(i = 1 \mid a)$$

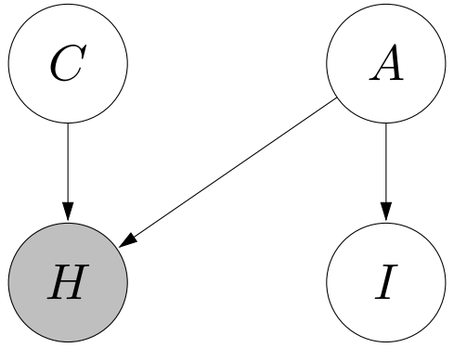
Bayesian network with evidence = Markov network with $Z = \mathbb{P}(H = 1, I = 1)$

Solution: run any inference algorithm for Markov networks (e.g., Gibbs sampling)!

[demo]

- Suppose we condition on evidence $H = 1$ and $I = 1$.
- We can define a new Markov network over the remaining variables (C and A) by simply plugging in the values to H and I. The normalization constant Z is the sum over all values of C and A, which is no longer 1, but rather the probability of the evidence $\mathbb{P}(H = 1, I = 1)$.
- To understand why this relationship holds, recall that the desired conditional probability is the joint probability over the marginal probability. The factors simply represent the joint probability, and thus the normalization constant must be the marginal probability.
- Now we can again run any inference algorithm for Markov networks (e.g., Gibbs sampling), and this allows us to do probabilistic inference in any Bayesian network.
- In the demo, we will run Gibbs sampling to compute $\mathbb{P}(C = 1 \mid H = 1, I = 1)$, and we see that it converges to the right answer (0.13).

Leveraging additional structure: unobserved leaves



Markov network:

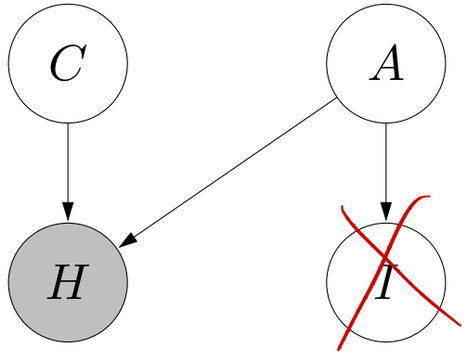
$$\mathbb{P}(C = c, A = a, I = i \mid H = 1) = \tfrac{1}{Z} p(c)p(a)p(h = 1 \mid c, a)p(i \mid a),$$

where $Z = \mathbb{P}(H = 1)$

Question: $\mathbb{P}(C = 1 \mid H = 1)$

Can we reduce the Markov network before running inference?

Leveraging additional structure: unobserved leaves



marginalize out unobserved leaves

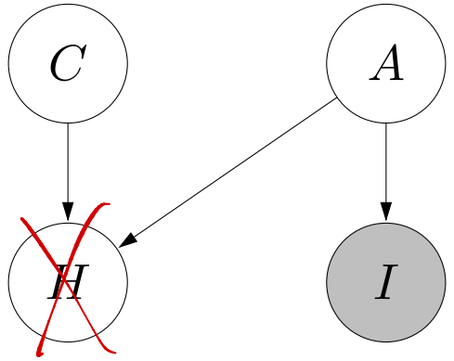
Markov network:

$$\begin{aligned}
\mathbb{P}(C = c, A = a \mid H = 1) &= \sum_i \mathbb{P}(C = c, A = a, I = i \mid H = 1) \\
&= \sum_i \frac{1}{Z} p(c)p(a)p(h = 1 \mid c, a)p(i \mid a) \\
&= \frac{1}{Z} p(c)p(a)p(h = 1 \mid c, a) \sum_i p(i \mid a) \\
&= \frac{1}{Z} p(c)p(a)p(h = 1 \mid c, a)
\end{aligned}$$

equal to 1

Throw away any unobserved leaves before running inference!

Leveraging additional structure: independence



Markov network:

$$\begin{aligned}
\mathbb{P}(C = c \mid I = 1) &= \textstyle\sum_{a,h} \mathbb{P}(C = c, A = a, H = h \mid I = 1) \\
&= \textstyle\sum_{a,h} \frac{1}{Z} p(c)p(a)p(h \mid c, a)p(i = 1 \mid a) \\
&= \textstyle\sum_{a} \frac{1}{Z} p(c)p(a)p(i = 1 \mid a) \\
&= p(c) \textstyle\sum_{a} \frac{1}{Z} p(a)p(i = 1 \mid a) \\
&= p(c)
\end{aligned}$$

LOTP w/ joint

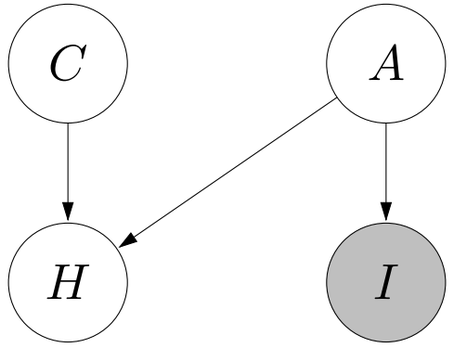
joint → local conditional

marginalize H bc unobserved leaf

independent of C

Throw away any disconnected components before running inference!

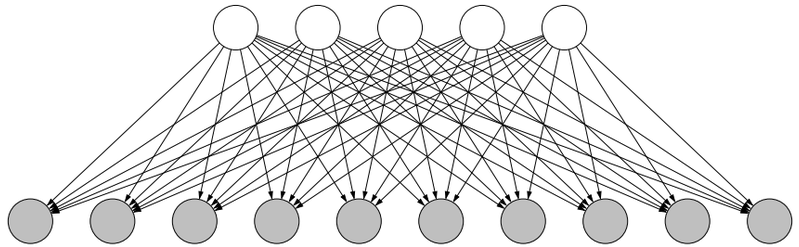
Summary



- Condition on evidence (e.g., $I = 1$)
- Throw away unobserved leaves (e.g., H)
- Throw away disconnected components (e.g., A and I)
- Define Markov network out of remaining factors
- Run your favorite inference algorithm (e.g., manual, Gibbs sampling)

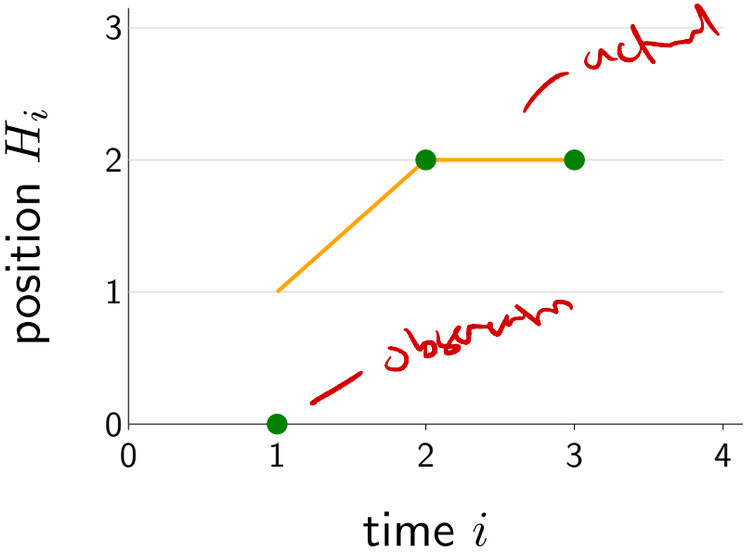
optimizations

Bayesian networks: forward-backward



- In this module, I will introduce the forward-backward algorithm for performing efficient and exact inference in Hidden Markov models, an important special case of Bayesian networks.

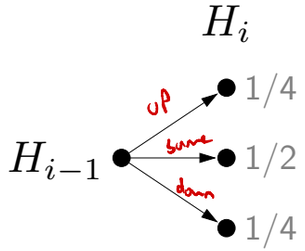
Hidden Markov models for object tracking



start

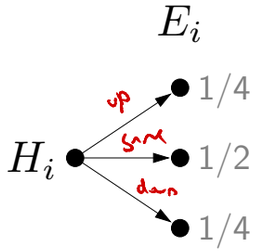
h_1	$p(h_1)$
0	1/3
1	1/3
2	1/3

transition

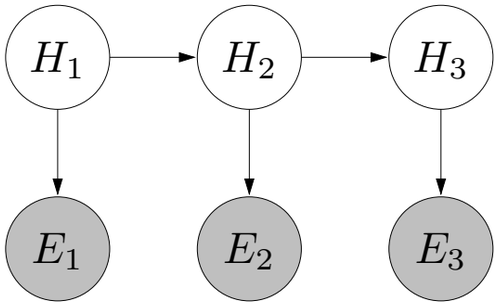


h_i	$p(h_i \mid h_{i-1})$
$h_{i-1}-1$	1/4
h_{i-1}	1/2
$h_{i-1}+1$	1/4

emission

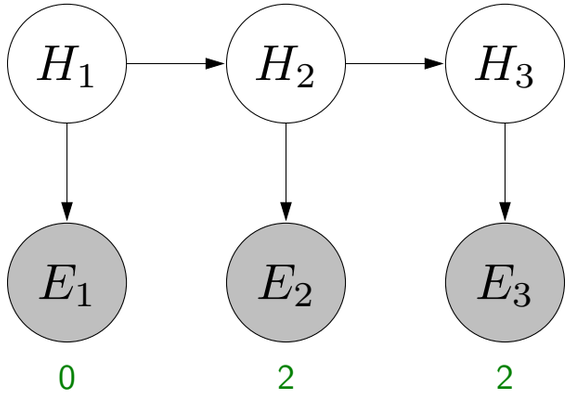


e_i	$p(e_i \mid h_i)$
h_i-1	1/4
h_i	1/2
h_i+1	1/4



$$\mathbb{P}(H = h, E = e) = \underbrace{p(h_1)}_{\text{start}} \prod_{i=2}^{n} \underbrace{p(h_i \mid h_{i-1})}_{\text{transition}} \prod_{i=1}^{n} \underbrace{p(e_i \mid h_i)}_{\text{emission}}$$

Inference questions



Question (**filtering**):

$$\mathbb{P}(H_2 \mid E_1 = 0, E_2 = 2)$$

position at step 2 given observations

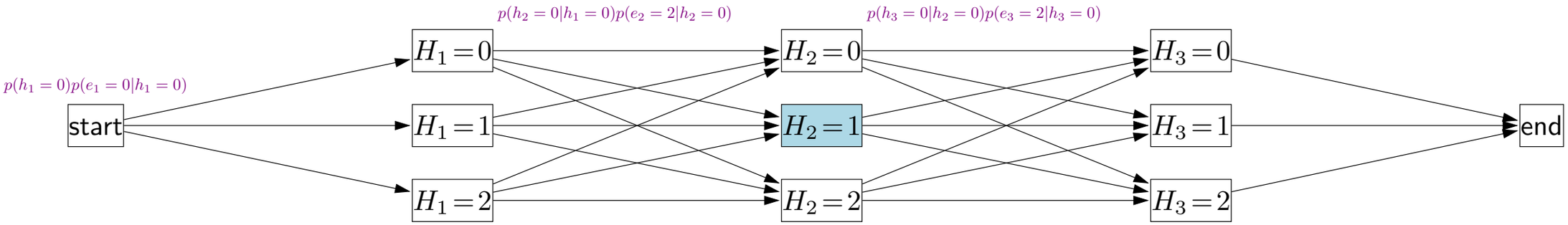
Question (**smoothing**):

$$\mathbb{P}(H_2 \mid E_1 = 0, E_2 = 2, E_3 = 2)$$

Note: filtering is a special case of smoothing if marginalize unobserved leaves

- **Filtering** asks for the distribution of some hidden variable H_i conditioned on only the evidence up until that point. This is useful when you're doing real-time object tracking, and you can't see the future.
- **Smoothing** asks for the distribution of some hidden variable H_i conditioned on all the evidence, including the future. This is useful when you have collected all the data and want to retrospectively go and figure out what H_i was.

Lattice representation

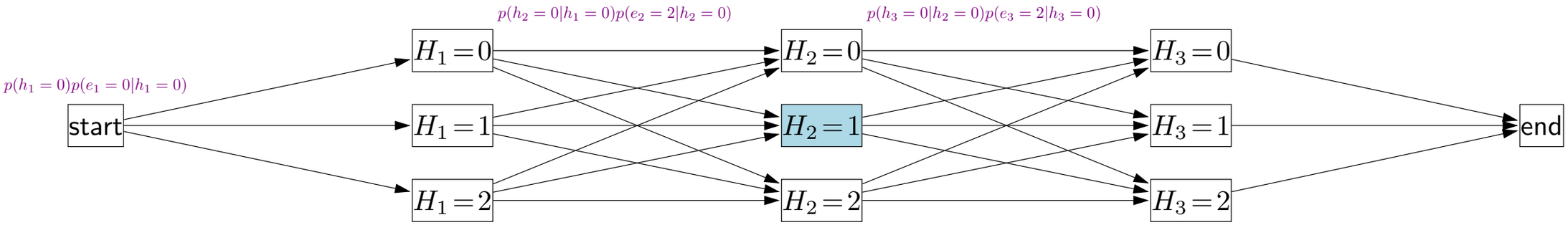


- Edge $\boxed{\text{start}} \Rightarrow \boxed{H_1 = h_1}$ has weight $p(h_1)p(e_1 \mid h_1)$
- Edge $\boxed{H_{i-1} = h_{i-1}} \Rightarrow \boxed{H_i = h_i}$ has weight $p(h_i \mid h_{i-1})p(e_i \mid h_i)$
- Each path from $\boxed{\text{start}}$ to $\boxed{\text{end}}$ is an assignment with weight equal to the product of edge weights

Key: $\mathbb{P}(H_i = h_i \mid E = e)$ is the weighted fraction of paths through $\boxed{H_i = h_i}$

- The forward-backward algorithm is based on a form of dynamic programming.
- To develop this, we consider a **lattice representation** of HMMs. Consider a directed graph (not to be confused with the HMM) with a start node, an end node, and a node for each assignment of a value to a variable $H_i = v$. The nodes are arranged in a lattice, where each column corresponds to one variable H_i and each row corresponds to a particular value v. Each path from the start to the end corresponds exactly to a complete assignment to the nodes.
- Each edge has a weight (a single number) determined by the local conditional probabilities (more generally, the factors in a factor graph). For each edge into $\boxed{H_i = h_i}$, we multiply by the transition probability into h_i and emission probability $p(e_i \mid h_i)$.
- This defines a weight for each path (assignment) in the graph equal to the joint probability $P(H = h, E = e)$.
- Note that the lattice contains $O(n|\text{Domain}|)$ nodes and $O(n|\text{Domain}|^2)$ edges, where n is the number of variables and $|\text{Domain}|$ is the number of values in the domain of each variable (3 in our example).
- Now comes the key point. Recall we want to compute a smoothing question $\mathbb{P}(H_i = h_i \mid E = e)$. This quantity is simply the weighted fraction of paths that pass through $\boxed{H_i = h_i}$. This is just a way of visualizing the definition of the smoothing question.
- There are an exponential number of paths, so it's intractable to enumerate all of them. But we can use dynamic programming...

Forward and backward messages



Forward: $F_i(h_i) = \sum_{h_{i-1}} F_{i-1}(h_{i-1})\text{Weight}(\boxed{H_{i-1} = h_{i-1}}, \boxed{H_i = h_i})$

sum of weights of paths from $\boxed{\text{start}}$ to $\boxed{H_i = h_i}$

Backward: $B_i(h_i) = \sum_{h_{i+1}} B_{i+1}(h_{i+1})\text{Weight}(\boxed{H_i = h_i}, \boxed{H_{i+1} = h_{i+1}})$

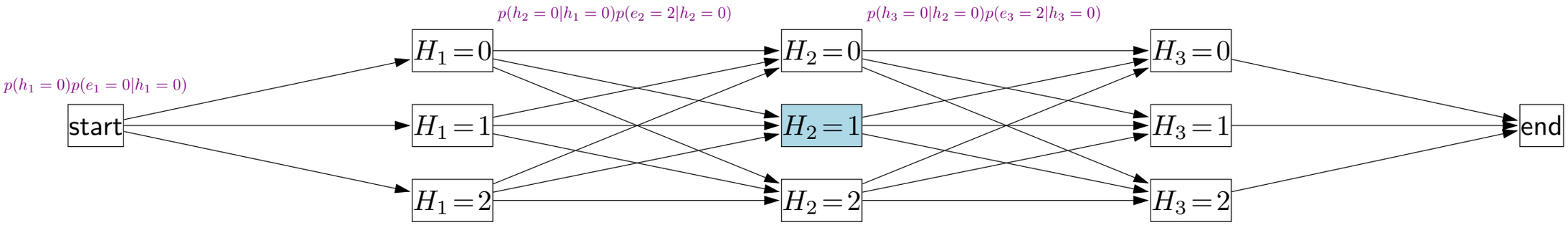
sum of weights of paths from $\boxed{H_i = h_i}$ to $\boxed{\text{end}}$

Define $S_i(h_i) = F_i(h_i)B_i(h_i)$:

sum of weights of paths from $\boxed{\text{start}}$ to $\boxed{\text{end}}$ through $\boxed{H_i = h_i}$

- First, define the forward message $F_i(v)$ to be the sum of the weights over all paths from the start node to $\boxed{H_i = v}$. This can be defined recursively: any path that goes $\boxed{H_i = h_i}$ will have to go through some $\boxed{H_{i-1} = h_{i-1}}$, so we can sum over all possible values of h_{i-1}.
- Analogously, let the backward message $B_i(v)$ be the sum of the weights over all paths from $\boxed{H_i = v}$ to the end node.
- Finally, define $S_i(v)$ to be the sum of the weights over all paths from the start node to the end node that pass through the intermediate node $\boxed{X_i = v}$. This quantity is just the product of the weights of paths going into $\boxed{H_i = h_i}$ $(F_i(h_i))$ and those leaving it $(B_i(h_i))$.
- This is analogous to factoring: $(a + b)(c + d) = ab + ad + bc + bd$.
- Note: $F_1(h_1) = p(h_1)p(e_1 = 0 \mid h_1)$ and $B_n(h_n) = 1$ are base cases, which don't require the recurrence.

Putting everything together



$$\mathbb{P}(H_i = h_i \mid E = e) = \frac{S_i(h_i)}{\sum_v S_i(v)}$$

Algorithm: forward-backward algorithm

Compute $F_1, F_2, \ldots, F_n$

Compute $B_n, B_{n-1}, \ldots, B_1$

Compute S_i for each i and normalize

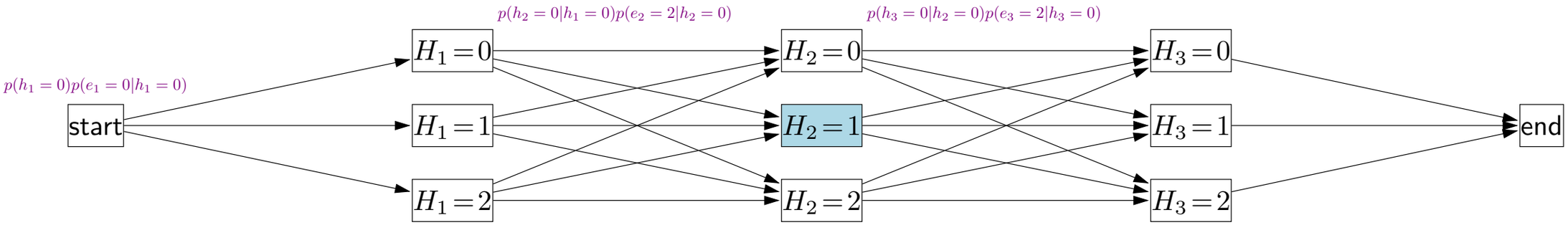
Running time: $O(n|\text{Domain}|^2)$

└ domain of a variable H_i

[demo]

- Now the smoothing question $\mathbb{P}(H_i = h_i \mid E = e)$ is just equal to the normalized version of S_i.
- The algorithm is thus as follows: for each node $\boxed{H_i = h_i}$, we compute three numbers: $F_i(h_i), B_i(h_i), S_i(h_i)$. First, we sweep forward to compute all the F_i's recursively. At the same time, we sweep backward to compute all the B_i's recursively. Then we compute S_i by pointwise multiplication.
- The running time of the algorithm is $O(n|\text{Domain}|^2)$, which is the number of edges in the lattice.
- In the demo, we are running the variable elimination algorithm, which is a generalization of the forward-backward algorithm for arbitrary Markov networks. As you step through the algorithm, you can see that the algorithm first computes a forward message F_2 and then a backward message B_2, and then it multiplies everything together and normalizes to produce $\mathbb{P}(H_2 \mid E_1 = 0, E_2 = 2, E_3 = 2)$. The names and details don't match up exactly, so you don't need to look too closely.
- Implementation note: we technically can normalize S_i to get $\mathbb{P}(H_i \mid E = e)$ at the very end but it's useful to normalize F_i and B_i at each step to avoid underflow. In addition, normalization of the forward messages yields $\mathbb{P}(H_i = v \mid E_1 = e_1, \ldots, E_i = e_i)$ which are exactly the filtering queries!

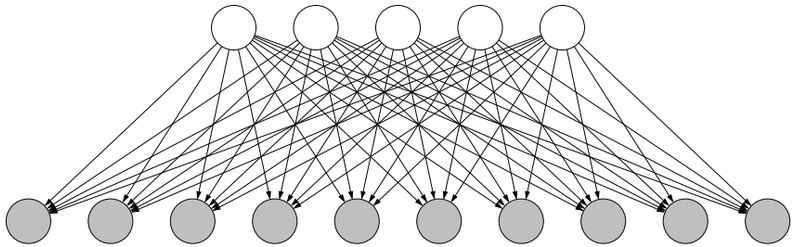
Summary



- Lattice representation: paths are assignments
- Dynamic programming: compute sums efficiently
- Forward-backward algorithm: compute all smoothing questions, share intermediate computations

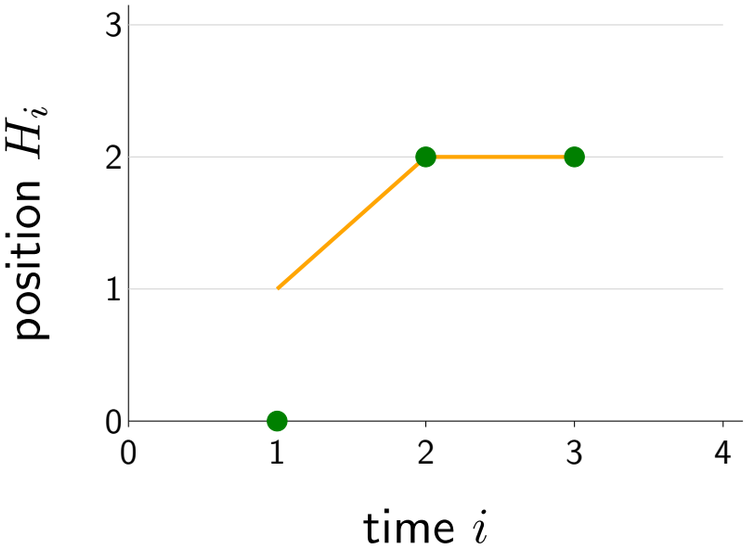
- In summary, we have presented the forward-backward algorithm for probabilistic inference in HMMs, in particular smoothing queries.
- The algorithm is based on the lattice representation in which each path is an assignment, and the weight of path is the joint probability.
- Smoothing is just then asking for the weighted fraction of paths that pass through a given node.
- Dynamic programming can be used to compute this quantity efficiently.
- This is formalized using the forward-backward algorithm, which consists of two sets of recurrences.
- Note that the forward-backward algorithm gives you the answer to all the smoothing questions ($\mathbb{P}(H_i = h_i \mid E = e)$ for all i), because the intermediate computations are all shared.

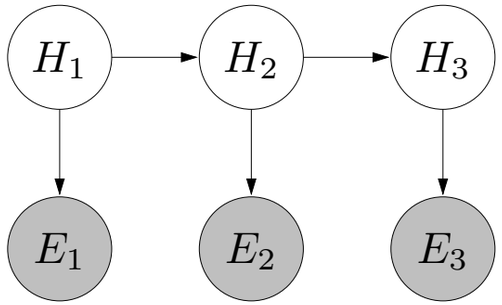
Bayesian networks: particle filtering



- In this module, I will present the particle filtering algorithm for performing approximate inference in Hidden Markov models which is useful when the size of the domain of the variables is large.

Review: Hidden Markov models for object tracking





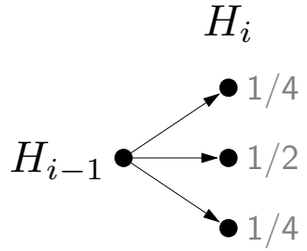
start

H_1

- 1/3
- 1/3
- 1/3

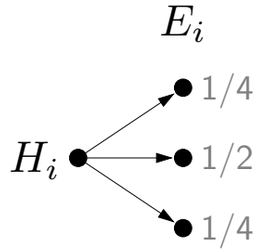
h_1	$p(h_1)$
0	1/3
1	1/3
2	1/3

transition



h_i	$p(h_i \mid h_{i-1})$
$h_{i-1}-1$	1/4
h_{i-1}	1/2
$h_{i-1}+1$	1/4

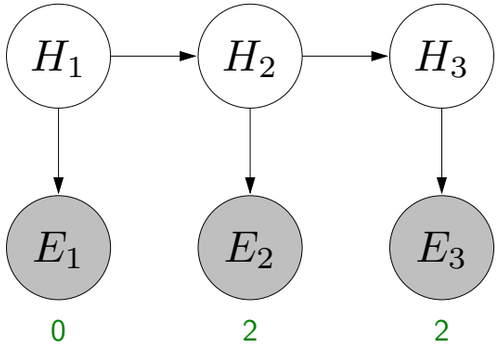
emission



e_i	$p(e_i \mid h_i)$
h_i-1	1/4
h_i	1/2
h_i+1	1/4

$$\mathbb{P}(H=h, E=e) = \underbrace{p(h_1)}_{\text{start}} \prod_{i=2}^{n} \underbrace{p(h_i \mid h_{i-1})}_{\text{transition}} \prod_{i=1}^{n} \underbrace{p(e_i \mid h_i)}_{\text{emission}}$$

Review: inference in Hidden Markov models

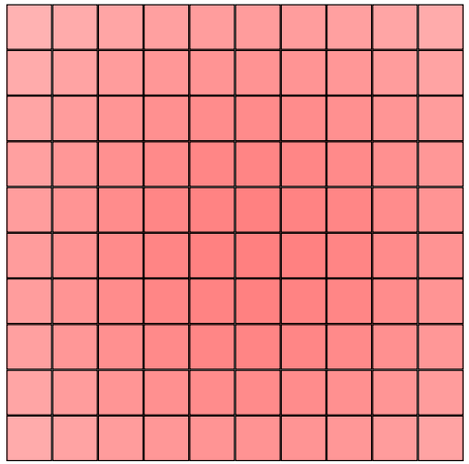


Filtering questions:

$$\mathbb{P}(H_1 \mid E_1 = 0)$$
$$\mathbb{P}(H_2 \mid E_1 = 0, E_2 = 2)$$
$$\mathbb{P}(H_3 \mid E_1 = 0, E_2 = 2, E_3 = 2)$$

p(current | observation so far)

Problem: many possible location values for H_i

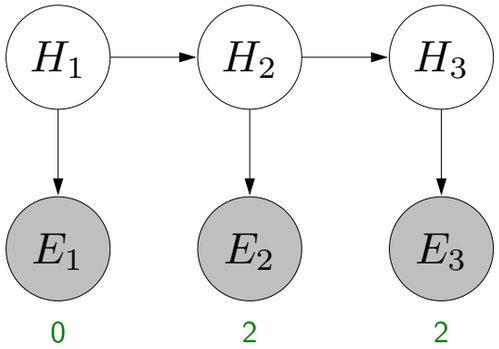


Forward-backward is too slow ($O(n|\text{Domain}|^2)$)...

- Recall that the two common types of inference questions we ask on HMMs are filtering and smoothing.
- Particle filtering, as the name might suggest, performs filtering, so let us focus on that. Filtering asks for the probability distribution over object location H_i at a current time step i given the past observations $E_1 = e_1, \ldots, E_i = e_i$.
- Last time, we saw that the forward-backward algorithm could already solve this. But it runs in $O(n|\text{Domain}|^2)$, where $|\text{Domain}|$ is the number of possible values (e.g., locations) that H_i can take on. On this example, $H_i \in \{0, 1, 2\}$ but for real applications, there could easily be hundreds of thousands of values, not to mention what happens if H_i is continuous. This could be a very large number, which makes the forward-backward algorithm very slow (even if it's not exponentially so).
- The motivation of particle filtering is to perform **approximate probabilistic inference**, and leverages the fact that most of the locations are very improbable given evidence.
- Particle filtering actually applies to general factor graphs, but we will present them for hidden Markov models for concreteness.

Beam search for HMMs

Idea: keep $\leq K$ partial assignments (**particles**)



Algorithm: beam search

Initialize $C \leftarrow [\{\}]$

For each $i = 1, \ldots, n$:

Extend:

$C' \leftarrow \{h \cup \{H_i : v\} : h \in C, v \in \text{Domain}_i\}$

Prune:

$C \leftarrow K$ particles of C' with highest weights

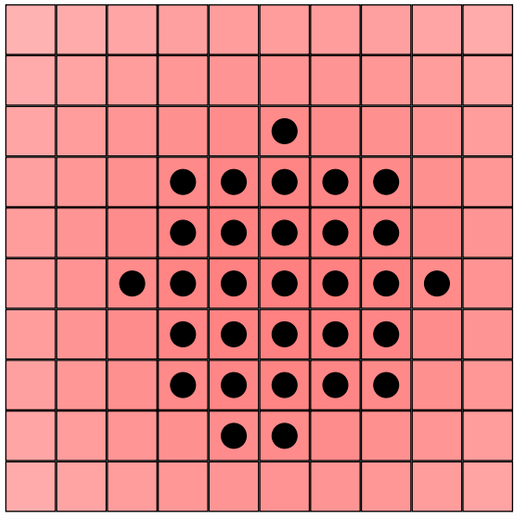
Normalize weights to get approximate $\hat{\mathbb{P}}(H_1, \ldots, H_n \mid E = e)$

Sum probabilities to get any approximate $\hat{\mathbb{P}}(H_i \mid E = e)$

[demo: `beamSearch({K:3})`]

- Our starting point for motivating particle filtering is beam search, an algorithm for finding an approximate maximum weight assignment in arbitrary constraint satisfaction problems (CSPs).
- Since HMMs are Bayesian networks, which are Markov networks, which have an underlying factor graph, we can simply apply beam search to HMMs (for now putting aside the goal of finding the maximum weight assignment).
- Recall that beam search maintains a list of candidate partial assignments to the first i variables. There are two phases. In the first phase, we **extend** all the existing candidates C to all possible assignments to H_i; this results in $K = |\text{Domain}|$ candidates C'. We then take the subset of K candidates with the highest weight, where the weight of a partial assignment is simply the product of all the factors (transitions, emissions) that can be computed on the partial assignment.
- In the demo, we start with partial assignments to H_1, whose weights are given by $p(h_1)p(e_1 = 0 \mid h_1)$. In the next step, we can multiply in factors $p(h_2 \mid h_1)p(e_2 = 2 \mid h_2)$, and so on.
- At the very end, we obtain $K = 3$ complete assignments, each with a weight (equal to the joint probability of the assignment and observations). We can normalize these weights to form an approximate distribution over all assignments (conditioned on the observations). From here, we can manually compute any marginal probabilities (e.g., $\mathbb{P}(H_3 = 2 \mid E = e)$) by summing the probabilities of assignments satisfying the given condition (e.g., $H_3 = 2$).

Beam search problems



Algorithm: beam search

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Prune:

$C \leftarrow K$ particles of C' with highest weights

issues

- Extend: slow because requires considering every possible value for H_i
- Prune: greedily taking best K doesn't provide diversity

Particle filtering solution (3 steps): **propose, weight, resample**

Step 1: propose

Old particles: $\approx \mathbb{P}(H_1, H_2 \mid E_1 = 0, E_2 = 2)$

$\{H_1 : 0, H_2 : 1\}$
$\{H_1 : 1, H_2 : 2\}$

Key idea: proposal distribution

For each old particle (h_1, h_2), sample $H_3 \sim p(h_3 \mid h_2)$.

h_i	$p(h_i \mid h_{i-1})$
$h_{i-1} - 1$	1/4
h_{i-1}	1/2
$h_{i-1} + 1$	1/4

New particles: $\approx \mathbb{P}(H_1, H_2, H_3 \mid E_1 = 0, E_2 = 2)$

$\{H_1 : 0, H_2 : 1, H_3 : 1\}$
$\{H_1 : 1, H_2 : 2, H_3 : 2\}$

↑ extend particles based on same evidence

↑ Random from distribution

Step 2: weight

Old particles: $\approx \mathbb{P}(H_1, H_2, H_3 \mid E_1 = 0, E_2 = 1)$

$\{H_1 : 0, H_2 : 1 : H_3 : 1\}$
$\{H_1 : 1, H_2 : 2 : H_3 : 2\}$

Key idea: weighting based on evidence

For each old particle (h_1, h_2, h_3), weight it by $p(e_3 = 2 \mid h_3)$.

prob of new evidence given new proposal

h_3	$p(e_3 = 2 \mid h_3)$
0	0
1	1/4
2	1/2

New particles: $\approx \mathbb{P}(H_1, H_2, H_3 \mid E_1 = 0, E_2 = 1, E_3 = 2)$

$\{H_1 : 0, H_2 : 1 : H_3 : 1\}$ (1/4)
$\{H_1 : 1, H_2 : 2 : H_3 : 2\}$ (1/2)

Step 3: resample

Old particles: $\approx \mathbb{P}(H_1, H_2, H_3 \mid E_1 = 0, E_2 = 2, E_3 = 2)$

$\{H_1 : 0, H_2 : 1 : H_3 : 1\}$ $(1/4) \Rightarrow 1/3$
$\{H_1 : 1, H_2 : 2 : H_3 : 2\}$ $(1/2) \Rightarrow 2/3$

normalize weights to make it new distribution

Key idea: resampling

Normalize weights and draw K samples to redistribute particles to more promising areas.

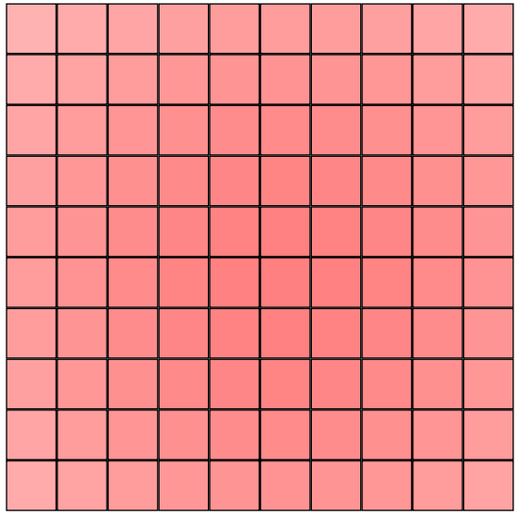
New particles: $\approx \mathbb{P}(H_1, H_2, H_3 \mid E_1 = 0, E_2 = 2, E_3 = 2)$

$\{H_1 : 1, H_2 : 2 : H_3 : 2\}$
$\{H_1 : 1, H_2 : 2 : H_3 : 2\}$

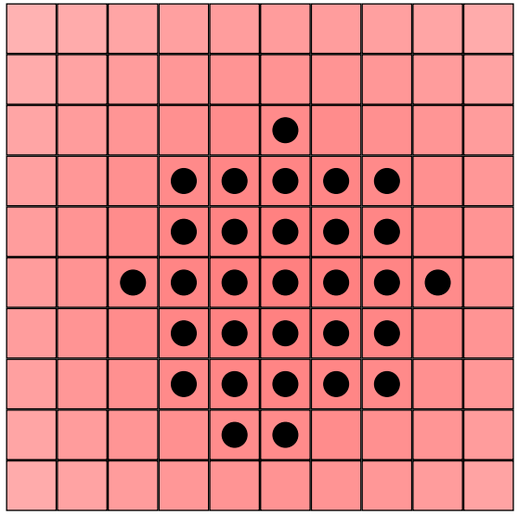
draw samples from new distribution

Why sampling?

distribution

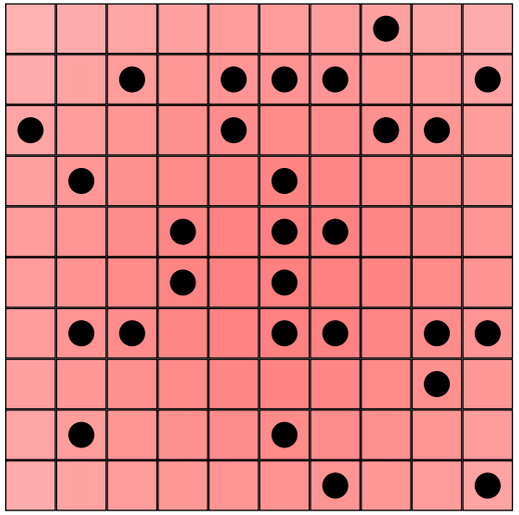


K with highest weight



not representative

K sampled from distribution



more representative

Sampling is especially important when there is high uncertainty!

- You might wonder why we are resampling, leaving the result of the algorithm up to chance.
- To see why resampling can be more favorable than beam search, consider the setting where we start with a set of particles on the left where the weights are given by the shade of red (darker is more weight). Notice that the weights are all quite similar (i.e., the distribution is close to the uniform distribution).
- Beam search chooses the K locations with the highest weight, which would clump all the particles near the mode. This is risky, because we have no support out farther from the center, where there is actually substantial probability.
- However, if we sample from the distribution which is proportional to the weights, then we can hedge our bets and get a more representative set of particles which cover the space more evenly.
- In cases where the original weights much more skewed towards a few particles, then taking the highest weight particles is fine and perhaps even slightly better than resampling.

Particle filtering

Algorithm: particle filtering

Initialize $C \leftarrow [\{\}]$

For each $i = 1, \ldots, n$:

Propose:

$C' \leftarrow \{h \cup \{H_i : h_i\} : h \in C, h_i \sim p(h_i \mid h_{i-1})\}$

Weight:

Compute weights $w(h) = p(e_i \mid h_i)$ for $h \in C'$

Resample:

$C \leftarrow K$ particles drawn independently from $\frac{w(h)}{\sum_{h' \in C} w(h')}$

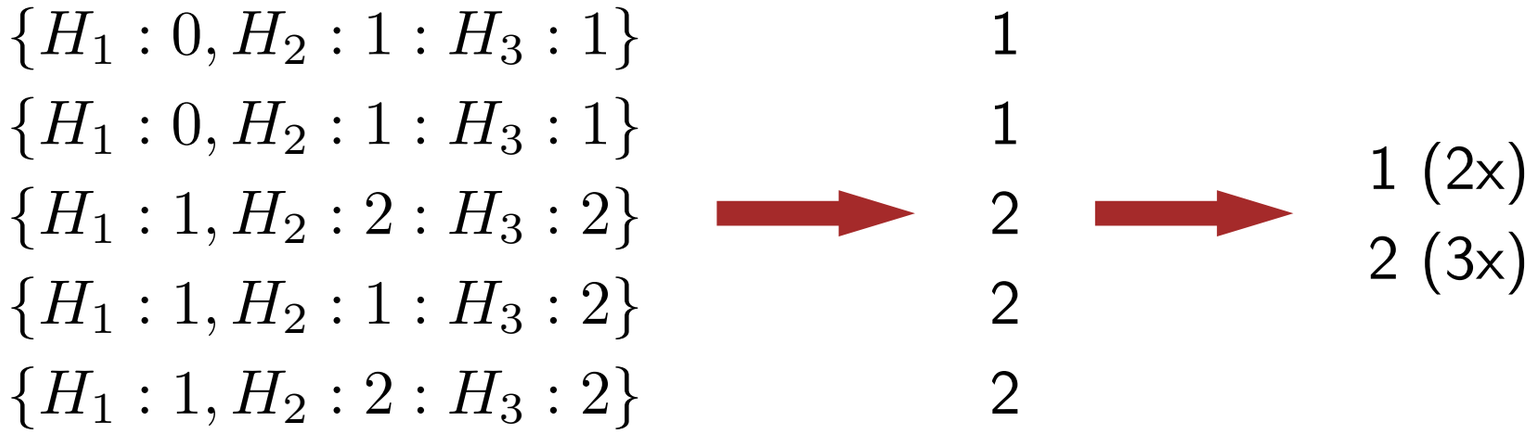
[demo: `particleFiltering({K:100})`]

- We now present the final particle filtering algorithm, which is structurally similar to beam search. We go through all the variables $H_1, \ldots, H_n$.
- For each candidate $h \in C$, we propose h_i according to the transition distribution $p(h_i \mid h_{i-1})$.
- We then weight this particle using the emission probability $w(h) = p(e_i \mid h_i)$.
- Finally, we normalize the weights $\{w(h) : h \in C\}$ and sample K particles independently from this distribution.
- In the demo, we can go through the extend (propose) and prune (weight + resample) steps, ending with a final set of full assignments, which can be used to approximate the filtering distribution $\mathbb{P}(H_3 \mid E = e)$.

Particle filtering: implementation

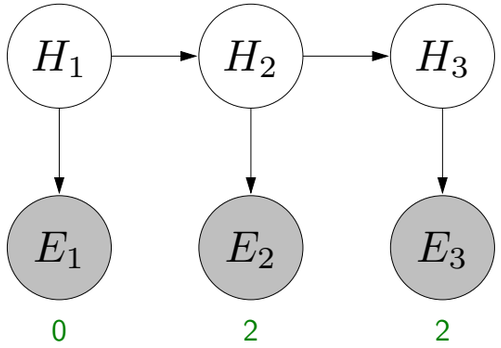
For filtering questions, can optimize:

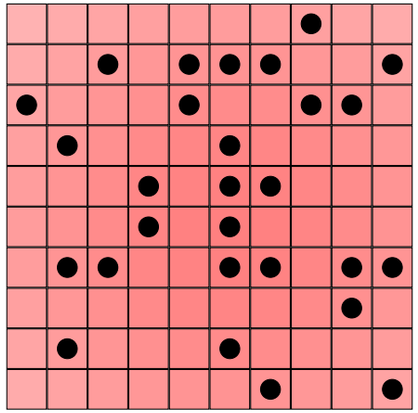
- Keep only value of last H_i for each particle
- Store count for each unique particle



- So far, we have presented a version of particle filtering where each particle at the end is a full assignment to all the variables. This allows us to approximately answer a variety of different questions based on the induced distribution.
- However, if we're only interested in filtering questions, then we can perform two optimizations.
- First, in tracking applications, we only care about the last location H_i, and future steps only depend on the value of H_i. Therefore, we often just store the value of H_i rather than the entire trajectory.
- Second, since we have discrete variables, many particles might have the same value of H_i, so we can just store the counts of each value rather than storing duplicate values.

Summary





$$\mathbb{P}(H_3 \mid E_1 = 0, E_2 = 2, E_3 = 2)$$

- Use particles to represent an approximate distribution

Propose (transitions) **Weight** (emissions) **Resample**

- Can scale to large number of locations (unlike forward-backward)
- Maintains better particle diversity (compared to beam search)

- In summary, we have presented particle filtering, an inference algorithm for HMMs that approximately computes filtering questions of the form: where is the object currently given all the past noisy sensor readings?
- Particle filtering represents distributions over hidden variables with a set of particles. To advance the particles to the next time step, it proposes new positions based on transition probabilities. It then weights these guesses based on evidence from the emission probabilities. Finally, it resamples from the normalized weights to redistribute the precious particle resources.
- Compared to the forward-backward algorithm, both beam search and particle filtering can scale up to a large number of locations (assuming most of them are unlikely). Unlike beam search, however, particle filtering uses randomness to ensure better diversity of the particles.
- Particle filtering is also called sequential Monte Carlo and there are many more sophisticated extensions that I'd encourage to learn about.