Level Sets

$$f(x,y) = x^3y^2 - \sqrt{x} \cdot \ln y$$

Level sets of f: $\{(x,y) \in \mathbb{R}^2 : x^3y^2 - \sqrt{x}\ln y = z\}$

e.g. @ 2, $\{(x,y) \in \mathbb{R}^2 : \underline{x^3y^2 - \sqrt{x}\ln y} = \underline{2}\}$

function goes here

chosen level set goes here

Ch. 8

$f(x,y) = \sin(x-3y) = c \quad , \; |c| \leq 1$

$x - 3y = \arcsin(c) + \underline{2\pi n}$ ← it repeats

$y = \frac{1}{3}x - \frac{1}{3}(\arcsin(c) + 2\pi n)$

↳ $n \in \mathbb{Z}$

$y = mx + b$

it's parallel lines w/ different y-intercepts

Contour Plots

Magnitude: closeness of contours

↳ close = higher magnitude

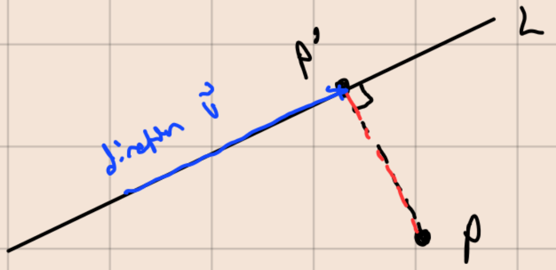
↳ far = lower magnitude

Ex: line $L = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and point $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$p + t\vec{v}$

What point on L is closest to p?

↳ can't just project directly bc L is not on origin



$p - p'$ is orthogonal to direction of line L

any point on L is represented by this point

$$p - p' \cdot \vec{v} = 0$$

$$\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} t \\ 1+t \\ 1+t \end{pmatrix} \right] \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$$

↳ $\begin{pmatrix} 1-t \\ -1-t \\ -1-t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = -3t - 1 = 0$

$t = -1/3$

$$p' = \begin{pmatrix} -1/3 \\ 2/3 \\ 2/3 \end{pmatrix}$$

* Look at fourier formula

Linear Regression

$(1,-1)$, $(0,2)$, $(0,3)$, $(4,2)$

$$x = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 4 \end{pmatrix}, \quad y = \begin{pmatrix} -1 \\ 2 \\ 3 \\ 2 \end{pmatrix}, \quad \bar{x} = \frac{5}{4}, \quad \bar{y} = \frac{3}{2}$$

$$y = mx + b = \begin{pmatrix} mx_1 + b \\ mx_2 + b \\ mx_n + b \end{pmatrix} = m\,\vec{x} + b\,\vec{1}$$

Line of best fit is $\text{Proj}_{\text{span}\{x,1\}}\, y$

① Make orthogonal basis

keep $\vec{1}$, find $\hat{x} = x - \text{Proj}_{\vec{1}}\, x = x - \frac{x \cdot 1}{1 \cdot 1} 1$

$$\hat{x} = x - \bar{x}\, 1$$

$$\hat{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 5/4 \\ 5/4 \\ 5/4 \\ 5/4 \end{pmatrix}$$

☆ look at orthogonal projection theorem (6.smth)

$$\hat{x} = \begin{pmatrix} -1/4 \\ -5/4 \\ -5/4 \\ 11/4 \end{pmatrix}$$

$$\text{Proj}_{\text{span}\{x,1\}}\, y = \text{Proj}_{\text{span}\{\hat{x},1\}}\, y$$

$$= \text{Proj}_{\hat{x}}\, y + \text{Proj}_{1}\, y = \frac{y \cdot \hat{x}}{\hat{x} \cdot \hat{x}} \hat{x} + \frac{y \cdot 1}{1 \cdot 1} 1$$

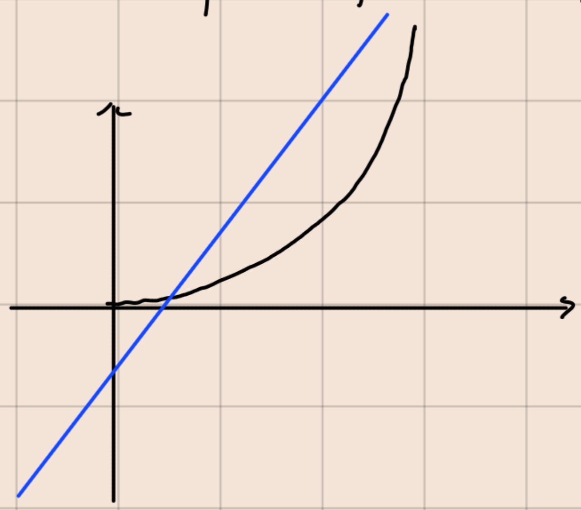
$$= -\frac{2}{43} \hat{x} + \frac{3}{2} 1 = -\frac{2}{43}\left(x - \frac{5}{4} 1\right) + \frac{3}{2} 1$$

$$= -\frac{2}{43} x + \frac{67}{43} 1$$

$$y = -2/43\, x + 67/43$$

Linear Regression

Data: $y = x^2$, $x \in \{0, 1, 2, \dots, 5\}$



step 1: move into a higher dimension space & create X and Y vectors

$$X = (0, 1, 2, 3, 4, 5)$$
$$Y = (0, 1, 4, 9, 16, 25)$$

step 2: make $\hat{X}$, the mean adjusted value of X

$$\hat{X} = X - \text{Proj}_{\mathbb{1}} X$$

← shifting X so that the average is zero. Therefore, average of $\hat{X}$ is zero

$$\hat{X} = X - \text{avg}(X) * \mathbb{1}$$
$$= X - (\text{avg}(X), \text{avg}(X), \text{avg}(X), \dots)$$

$$\text{avg}(X) = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \frac{X \cdot \mathbb{1}}{\mathbb{1} \cdot \mathbb{1}}$$

Step 3: project Y onto $\text{span}(\hat{X}, \mathbb{1})$

$\hat{X}$ and $\mathbb{1}$ are orthogonal, so their span is a orthogonal basis

$$\text{Proj}_{\text{span}(\hat{x}, 1)} Y = \text{Proj}_{\hat{x}} Y + \text{Proj}_1 Y$$

$$= \frac{Y \cdot \hat{X}}{\hat{X} \cdot \hat{X}} \hat{X} + \frac{Y \cdot 1}{1 \cdot 1} 1$$

$$= 5 \hat{X} + \frac{55}{6} 1$$

$$= 5 \left(X - \bar{x} 1 \right) + \frac{55}{6} 1$$

$$= 5X - \frac{75}{6} 1 + \frac{55}{6} 1$$

$$= 5X - \frac{20}{6} 1 = 5X - \frac{10}{3} 1$$

$$y = 5x - \frac{10}{3}$$