Evaluate Logarithms and Graph Logarithmic Functions

KEY CONCEPT *For Your Notebook*

Definition of Logarithm with Base b

Let b and y be positive numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

Ex 1: Rewrite each logarithm in exponential form.

Logarithmic Form	Exponential Form
A. $\log_5 625 = 4$	$5^4 = 625$
B. $\log_2 \frac{1}{16} = -4$	$2^{-4} = \frac{1}{16}$
C. $\log_{64} 4 = \frac{1}{3}$	$64^{1/3} = 4$

Ex 2: Solve for the unknown in the logarithmic equation.

A. $\log_9 x = 2$

$9^2 = x$

$x = 81$

B. $\log_5 125 = y$

$5^y = 125$

$y = 3$

Ex 3: Use a calculator to evaluate each logarithm. Round to the nearest thousandth.

log is $\log_{10}$

A. $\log 15.125$ ≈ 1.180

B. $\ln 2.371$ $\approx .863$

C. $\ln e^2$ 2

Ex 4: Evaluate each logarithmic expression without a calculator.

A. $\log_7 49 = 2$ $7^2=49$

B. $\log_3 27 = 3$ $3^3=27$

C. $\log_6 \sqrt{6} = 1/2$ $6^{1/2}=\sqrt{6}$

D. $\log_3 \frac{1}{9} = -2$ $3^{-2}=\frac{1}{9}$

E. $\log_{81} 9 = 1/2$ $81^{1/2}=9$

F. $\log_{11} 11 = 1$ $11^1=11$

G. $\log_6 1 = 0$ $6^0=1$

H. $\log_4 4^6 = 6$ $4^x=46$

I. $\log_8 32 = 5/3$ $8^x=32$ $\sqrt[3]{8}=2^5=32$ $(2^3)^x=32$ $8^{5/3}=32$

Properties of Logarithms

★ $\log_b 1 =$ 0

★ $\log_b b =$ 1

★ $\log_b b^x =$ x

★ $b^{\log_b x} =$ x

Ex 5: Simplify the expression.

A. $\log_6 36^x = 2x$ $6^? = 36^x$ $6^? = (6^2)^x$ $?=2x$

B. $7^{\log_7 x} = x$

C. $10^{\log 4} = 4$ $10^{\log_{10} 4}$

Ex 6: Biologists have found that an alligator's length l (in inches) and weight w (in pounds) are related by the function $l = 27.1 \ln w^{\frac{1}{100}} - 32.8$. Use a graphing calculator to estimate the weight of an alligator that is 10 feet long.

~~$w \approx 4.85$~~

$10 = 27.1 \ln w^{\frac{1}{100}} - 32.8$

$\frac{42.8}{27.1} = \frac{27.1 \ln w^{\frac{1}{100}}}{27.1}$

$1.579 = \ln w^{\frac{1}{100}}$

$e^{1.579} \approx w^{\frac{1}{100}}$

~~$w \approx 4.85$~~

$w \approx 485$ lb