Solve Exponential and Logarithmic Equations

Ex 1: Solve each exponential equation. When necessary, round to the nearest thousandth.

A. $2^{x+5} = 2^{2x-3}$

$x+5 = 2x-3$

$8 = x$ ✓

$2^{13} = 2^{13}$

B. $7^{3x+4} = 49^{2x+1}$

$7^{3x+4} = (7^2)^{2x+1}$

$3x+4 = 2(2x+1)$

$3x+4 = 4x+2$

$x = 2$

$7^{10} = 49^5$ ✓

C. $27^{4x-1} = 9^{3x+8}$

$(3^3)^{4x-1} = (3^2)^{3x+8}$

$12x-3 = 6x+16$

$6x = 19$

$x = 19/6$ ✓

D. $10^{3x-10} = \left(\frac{1}{100}\right)^{6x-1}$

$10^{3x-10} = (10^{-2})^{6x-1}$

$3x-10 = -12x+2$

$15x = 12$

$x = 12/15 = 4/5$ ✓

E. $8^x = 20$

$\ln 8^x = \ln 20$

$x \ln 8 = \ln 20$

$x = \frac{\ln 20}{\ln 8}$

$x \approx 1.441$ ✓

F. $7^{6x} = 12$

$\ln 7^{6x} = \ln 12$

$6x \ln 7 = \ln 12$

$\frac{6x \ln 7}{6 \ln 7} = \frac{\ln 12}{6 \ln 7}$

$x = \frac{\ln 12}{6 \ln 7} \approx .213$ ✓

G. $4e^{3x} = 1$

$e^{3x} = \frac{1}{4}$

$\ln e^{3x} = \ln \frac{1}{4}$

$\frac{3x \ln e}{3} = \frac{\ln \frac{1}{4}}{3}$

$x \approx -.462$ ✓

H. $7^{2x-3} - 4 = 14$

$7^{2x-3} = 18$

$\ln 7^{2x-3} = \ln 18$

$(2x-3)\frac{\ln 7}{\ln 7} = \frac{\ln 18}{\ln 7}$

$2x-3 \approx 1.485$

$x \approx 2.243$ ✓

I. $\frac{3(2^{x+6})}{3} = \frac{17}{3}$

$2^{x+6} = \frac{17}{3}$

$\ln 2^{x+6} = \ln \frac{17}{3}$

$\frac{(x+6)\ln 2}{\ln 2} = \frac{\ln \frac{17}{3}}{\ln 2}$

$x \approx -3.479$

J. $5^{x+1} = 3^{2x-1}$

$\ln 5^{x+1} = \ln 3^{2x-1}$

$\frac{(x+1)\ln 5}{\ln 5} = \frac{(2x-1)\ln 3}{\ln 5}$

$x+1 = .683..(2x-1)$

$x+1 = 1.365x - .683$

$\frac{1.683}{.365} = \frac{.365x}{.365}$

$x+1 = 2\frac{\ln 3}{\ln 5}x - \frac{\ln 3}{\ln 5}$

$1 + \frac{\ln 3}{\ln 5} = 2\frac{\ln 3}{\ln 5}x - x$

$x\left(2\frac{\ln 3}{\ln 5} - 1\right)$

$x \approx 4.607$

Ex 2: Solve the exponential equation by factoring.

$2^{2x} - 12 \cdot 2^x + 32 = 0$

let $u = 2^x$

$(2^x)^2 - 12(2^x) + 32 = 0$

$u^2 - 12u + 32 = 0$

$(u-8)(u-4) = 0$

$u = 8 \quad u = 4$

$2^x = 8 \quad 2^x = 4$

$x = 3$

$x = 2$

Ex 3: You deposit $700 in an account that pays 2.5% annual interest. HOw long does it take the balance to reach the following amounts?

A. $1000 when interest is compounded quarterly

$\frac{1000}{700} = \frac{700}{700}\left(1 + \frac{.025}{4}\right)^{4t}$

$\frac{10}{7} = 1.00625^{4t}$

$\frac{\ln\left(\frac{10}{7}\right)}{4\ln 1.00625} = \frac{4t\ln(1.00625)}{4\ln 1.00625}$

$t \approx 14.3 \text{ yrs}$

B. $1500 when interest is compounded yearly

$1500 = 700(1 + .025)^t$

$\frac{15}{7} = 1.025^t$

$\frac{\ln \frac{15}{7}}{\ln 1.025} = \frac{t \ln 1.025}{\ln 1.025}$

$t \approx 30.9 \text{ yrs}$

C. $2000 when interest is compounded continuously

$\frac{2000}{700} = \frac{700e^{.025t}}{700}$

$\frac{20}{7} = e^{.025t}$

$\frac{\ln \frac{20}{7}}{.025} = \frac{.025 \ln e\, t}{.025}$

$t \approx 42 \text{ years}$