Solve Exponential and Logarithmic Equations

Ex 1: Solve each logarithmic equation. **Check for extraneous solutions.**

A. $\log_5(5x+9) = \log_5 6x$

$5x+9=6x$

$x=9$ ✓

B. $\log_6(3x-10) = \log_6(14-5x)$

NO SOLUTIONS

$3x-10=14-5x$

$8x=24$

$x=3$ extraneous

$\log_6(-1)$ can't log a neg #!

Ex 2: Solve each logarithmic equation. Round solutions to the nearest thousandth when necessary. **Check for extraneous solutions.**

A. $\log_4 x = -1$

$4^{-1}=x$

$x=.25$

B. $\frac{1}{3}\log_5 12x = 2$

$\log_5 12x = 6$

$5^6 = 12x$

$\log_5 \sqrt[3]{12x} = 2$

$5^2 = \sqrt[3]{12x}$ SAME

$(25)^3 = 12x$

C. $\log_2(x-4) = 6$

$2^6 = x-4$

$x=2^6+4$

$x=64+4$

$x=68$

D. $4\ln(-x)+3=21$

$4\ln(-x)=18$

$\ln(-x)=\frac{9}{2}$

$e^{9/2}=-x$

$x=-e^{9/2}$

E. $\log_4(-x)+\log_4(x+10)=2$

$\log_4[(-x)(x+10)]=2$

$16=-x^2-10x$

$x^2+10x+16=0$

$(x+2)(x+8)$

$x=-2,-8$

F. $\log_6 3x + \log_6(x-1) = 3$

$\log_6[3x(x-1)]=3$

$6^3 = 3x^2-3x$

$3x^2-3x-216=0$

$x^2-x-72=0$

$(x-9)(x+8)=0$

$x=9, \cancel{-8}$

$36 \times 6 = 216$

$3\overline{)216}$

Newton's Law of Cooling

An object that is hotter than its surroundings will cool off, and an object that is cooler than its surroundings will warm up. Newton's Law of Cooling states that the difference between an object and its surroundings decreases exponentially as a function of time according to the formula:

$$T(t) = T_s + (T_0 - T_s)\,e^{-kt}$$

T_0 initial temp of object t time

T_s temp of surroundings $-k$ ~~constant of~~ rate of decrease in the temp. difference

Ex 3: When a container of milk is taken out of the refrigerator, its temperature is 40°F. An hour later, its temperature is 50°F. Assume that the temperature of the air is a constant 70°F.

A. Write the function for the temperature of this container of milk as a function of time t.

$T(t) = 70 + (40-70)e^{-k(t)}$

$50 = 70 + (-30)e^{-k}$

$-20 = (-30)e^{-k}$

$\ln\frac{2}{3} = \ln e^{-k}$

$k = -\ln\frac{2}{3}$

$k \approx -.405$

$T(t) = 70 + -30e^{.405t}$

B. What is the temperature of the milk after 2 hours?

$T(2) = 70 + (-30)^{+.405(2)}$

$T(2) \approx 57°F$

C. After how many hours is the temperature 65°F?

$65 = 70 + -30^{.405t}$

$-5 = -30e^{.405t}$

$\ln .16\overline{6} = \ln e^{.405t}$

$\frac{\ln .166}{.405} = \frac{.405t}{.405}$

$t \approx 4.4$ hours

Ex 4: Detective Jamison is called to the scene of a crime where a dead body has just been found. She arrives on the scene at 10:23 pm and begins her investigation. Immediately, the temperature of the body is taken and found to be 80°F. The detective checks the programmable thermostat and finds that the room has been kept at 68°F for the past 3 days. An hour later, the victim's temperature is taken again and found to be 78.5°F. Assuming that the victim's body temperature was normal (98.6°F) prior to death, when did the victim die?

$T_0 = 80°F$

$T(t) = 68 + (80-68)e^{-k(t)}$

$78.5 = 68 + 12e^{-k}$

$10.5 = 12e^{-k}$

$\ln .875 = \ln e^{-k}$

$k = -\ln .875$

$k \approx .1335$

$98.6 = 68 + 12e^{-.1335t}$

$2.55 = e^{-.1335t}$

$\frac{\ln 2.55}{-.1335} = \frac{-.1335t}{-.1335}$

$t \approx 7$ hours ago