



Any incident ray  $O_2R_1$ , striking the cone surface  $O_3D$  at point  $R_1$ , is reflected as  $R_1G_1$ , reaching the ground at point  $G_1$ ; Needs to calculate the length of  $O_1G_1$ :

$$O_1 G_i = O_1 F_i + F_i G_i \quad (1)$$

$$O_1 F_i = O_i R_i = O_3 J_i + J_i K_i = O_2 O_3 \tan \theta + J_i K_i = O_2 O_3 \tan \theta + R_i K_i \tan \theta \quad (2)$$

$$R_i K_i = O_i O_3 = \mathbf{O}_i \mathbf{R}_i \tan \beta \quad (3)$$

$$(3) \text{ substituting into (2) gives: } O_1 R_i = O_2 O_3 \tan \theta + R_i K_i \tan \theta = O_2 O_3 \tan \theta + O_i R_i \tan \beta \tan \theta \quad (4)$$

$O_2O_3$  is known,  $O_iR_i$  can be determined from equation (4):  $O_iR_i = \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta}$  (5)

From (3) and (5),  $R_i K_i = O_i R_i \tan \beta = \left( \frac{O_2 O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) \tan \beta = \frac{O_2 O_3 \tan \theta \tan \beta}{1 - \tan \beta \tan \theta}$  (6)

From  $\Delta RFG_i$  we have  $F_i G_i = R_i F_i \tan(90 - 2\beta - \theta) = (R_i K_i + K_i F_i) \tan(90 - 2\beta - \theta) = (O_2 O_3 \tan \theta \tan \beta / (1 - \tan \beta \tan \theta) + O_2 O_3 + O_2 O_1) \cot(2\beta + \theta)$

$$\mathbf{O}_1\mathbf{G}_i = O_1F_i + F_iG_i = O_iR_i + F_iG_i = \left( \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) + \left( \frac{O_2O_3 \tan \theta \tan \beta}{1 - \tan \beta \tan \theta} + O_2O_3 + O_2O_1 \right) \cot(2\beta + \theta) =$$

$$O_2 O_3 \frac{\tan \theta (1 + \tan \beta \cot(2\beta + \theta))}{1 - \tan \beta \tan \theta} + (O_2 O_3 + O_2 O_1) \cot(2\beta + \theta) = O_2 O_3 \frac{\tan \theta (1 + \tan \beta \cot(2\beta + \theta))}{1 - \tan \beta \tan \theta} + (O_3 O_1) \cot(2\beta + \theta) \quad (8)$$

From equation (8), it can be seen that  $O_1O_2$  represents the height of the robot,  $O_2O_3$  represents the height of the vertex of the panoramic conical mirror relative to the installation point,  $O_1O_3$  represents the height of the vertex of the conical mirror, and  $\beta$  is the complementary angle of the cone angle of the conical mirror, all of which are known quantities.  $\theta$  is the variable.



Along the horizontal y-direction (perpendicular to the optical axis), an obstacle located at  $G_i$  with the width  $G_iG_i'$  forms a circular image; the circle size corresponds to the radius  $O_iR_i$  associated with the incident angle  $\theta$  in the previous figure. The image of the obstacle of width  $G_iG_i'$  is  $O_iO_i'$ , whose magnitude is computed as follows:

The radial incident angle for the obstacle of width  $G_iG_i'$  is  $\phi_i$ , which is computed as follows:

$$\phi_i = \sin^{-1} \left( \frac{G_iG_i'}{O_iR_i} \right) = \sin^{-1} \left( \frac{G_iG_i'}{\left( O_2O_3 \tan \theta / 1 - \tan \beta \tan \theta \right)} \right) \quad (9)$$

$$O_iO_i' = O_iM_i' \tan \Delta O_iM_i'O_i' = O_iM_i' \tan 2\phi_i = \left( \frac{O_iR_i}{2} / \cos \phi_i \right) \tan 2\phi_i = \left( \left( \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) / 2 \cos \phi_i \right) \tan 2\phi_i = \frac{\left( \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) \tan 2\phi_i}{2 \cos \phi_i} \quad (10)$$

$$O_iO_i' = \frac{\left( \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) \tan 2\phi_i}{2 \cos \phi_i} = \frac{\left( \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) \sin 2\phi_i}{2 \cos \phi_i \cos 2\phi_i} = \frac{\left( \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) 2 \sin \phi_i \cos \phi_i}{2 \cos \phi_i (1 - 2(\sin \phi_i)^2)} = \frac{\left( \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) \sin \phi_i}{(1 - 2(\sin \phi_i)^2)} \quad (11)$$

Substitute (9) into (11) gives

$$O_iO_i' = \frac{\left( \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) \sin \phi_i}{(1 - 2(\sin \phi_i)^2)} = \frac{\left( \frac{O_2O_3 \tan \theta}{1 - \tan \beta \tan \theta} \right) \left[ \frac{G_iG_i'}{\left( O_2O_3 \tan \theta / 1 - \tan \beta \tan \theta \right)} \right]}{\left( 1 - 2 \left( \frac{G_iG_i'}{\left( O_2O_3 \tan \theta / 1 - \tan \beta \tan \theta \right)} \right)^2 \right)} = \frac{G_iG_i'}{1 - 2 \left( \frac{G_iG_i'}{O_2O_3} \right) \left( \frac{1 - \tan \beta \tan \theta}{\tan \theta} \right)^2} \quad (12)$$

From equation (12),  $O_2O_3$  represents the height of the vertex of the panoramic conical mirror relative to its installation point,  $G_iG_i'$  represents the horizontal width of the obstacle,  $\beta$  is the complementary angle of the cone angle of the conical mirror; all of these are known quantities.  $\theta$  is the variable. Thus for any direction  $\theta$ , the conical image width of the obstacle with horizontal width  $G_iG_i'$  is  $O_iO_i'$ .

