HW1:

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Sample Question [0 points]

Sa Generate data of size 12 from a normal distribution with mean=0 and var=1 and store it in a vector named A. Print the data so that it is visible in the pdf file that is generated.

Hint: Use the function rnorm() available in R

Solution Sa

```
A=rnorm(12,0,1)
A
```

```
##  [1] -1.03756412 -0.64657807  0.38682939  0.35029975 -0.08893851 -0.33990580
##  [7]  0.62219403 -1.46735333 -0.72151962  0.78173131  0.07245817  0.41382664
```

Problem 1 [10 points]

Write a function that takes in two parameters n and q ($n > q + 1$) and outputs the plot of the time series $MA(q)$ given by $v_t = w_t + w_{t-1} + ... + w_{t-q}$, $t = q + 1, ...n, w_t \sim N(0, 1)$. Make sure that the function code and the plot are printed out.

The title of the graph should indicate the value of q. for exampe, it should "moving Average 10" for MA(10)

Problem 2 [10 +5 points]

a) Write a function that takes in two parameters n and p and outputs the plot of n points from $AR(p)$ given by $Y_t = a_1 Y_{t-1} + a_2 Y_{t-2} + ... + a_p Y_{t-p} + w_t, w_t \sim N(0, 1), Y_0, Y_{-1}... = 0, a_i = 1/i, t = p + 1, ...n.$ Make sure that the function code and the plot are printed out.
b) Use the filter function from R to check your function from a) is correct. (the plots from the output of filter function and your function should match). Display both these plots

Problem 3 [3 points]

Let x_t be a random walk with drift δ.

We know that $x_t = \delta t + \sum_{j=1}^{t} w_j$. Use this result to generate a random walk with $\delta = 0.1$.

Problem 4 [2+2+1 points]

Generate $n = 100$ observations from $x_t = -0.9x_{t-2} + w_t, w_t \sim N(0, 1)$. Let $v_t = (x_t + x_{t-1} + x_{t-2} + x_{t-3})/4$. Take initial points to be 0. (Do NOT use the filter function)

a) Plot x_t
b) Plot v_t
c) Now plot x_t as a line and superimpose v_t as a dashed line.

Problem 5 [3 points]

Show that the autocovariance function can be written as

$\gamma(s, t) = E[(x_t - \mu_t)(x_s - \mu_s)] = E(x_s x_t) - \mu_s \mu_t.$

Problem 6 [2+2+2+1+1 points]

Let

$w_t \sim wn(0, \sigma^2), y_t = w_t + a_1 w_{t-1} + a_2 w_{t-2}$

Show that

a) $\gamma_y(t, t) = (1 + a_1^2 + a_2^2)\sigma^2$

b) $\gamma_y(t, t-1) = \gamma_t(t, t+1) = (a_1 + a_1 a_2)\sigma^2$

c) $\gamma_y(t, t-2) = \gamma_t(t, t+2) = (a_2 \sigma^2)$

d) $\gamma_y(t, s) = 0, |s - t| \geq 3$

e) Summarize the above results in to an autocovariance function

Problem 7 [5 points]

a) Construct a function that accepts two inputs 1) time series data X and 2) positive lag value h, and outputs the ACF at lag h. Compare the output of your function with that from the inbuilt acf() function on the speech data (library astsa, see textbook example 1.3) set at h=0:10.

Problem 8 [2 points]

If X and Y are dependent but $var(X) = var(Y)$, find $cov(X + Y, X - Y)$

Problem 9 [1+2+3 points]

Consider the time series

$$x_t = \beta_1 + \beta_2 t + w_t,$$

where β_1 and β_2 are known constants and w_t is a white noise process with variance σ^2

a) Determine whether x_t is stationary

b) Show that $y_t = x_t - x_{t-1}$ is stationary

c) Show that the mean of

$$v_t = \frac{1}{2q+1} \sum_{j=-q}^{j=q} x_{t-j}$$

is $\beta_1 + \beta_2 t$

Problem 10 [1+2+3 points]

Suppose we would like to predict a single stationary series x_t with mean 0 and autocorrelation function $\rho(h)$ at some point in the future $t + l$, for $l > 0$.

a) If we predict using only x_t and some scalar multiplier A, show that the mean square prediction error

$$MSE(A) = E[(x_{t+l} - Ax_t)^2]$$

is minimized at the value $A = \rho(l)$

b) show that minimum mean square prediction error is

$$MSE(A) = \gamma(0)[1 - \rho^2(l)]$$

c) Show that if $x_{t+l} = Ax_t$, then $\rho(l) = 1$, if $A > 0$ and $\rho(l) = -1$, if $A < 0$

Problem 11 [3 points]

Show that the autocovariance function is symmetric i.e $\gamma(h) = \gamma(-h)$