

# Physics 7B: Physics for Scientists and Engineers

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August 31, 2023

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# Chapter 0

## Syllabus

### 0.1 Important Info

Syllabus schedule [here](#)

#### 0.1.1 Contacts + Sections

Instructor OH:

1. Robert Birgeneau; robertjb@berkeley.edu; Tu 2-3pm
2. Dr.Chen/Giles-Donovan OH are Th 2-3pm.

Lecture 1: MWF 11-12p @ Physics North 1

Head GSI: Yucehn Tang; ytang5@berkeley.edu; begin emails with [7B Lec.1]

2 Sections: DIS + LAB; Labs not every week and may be replaced by DIS (check syllabus)

#### 0.1.2 Materials

1. 7B Workbook by Hedeman (on bCourses)
2. Mastering Physics (Purchase access code)
3. Physics for scientists and engineers 5th edition by Giancoli (may come w/ mastering physics purchase)

#### 0.1.3 Assignments

Homework will be made available Friday 11:59 and due next Friday 11:59.

Must complete all 4 Labs - Sep 29; Oct 27; 11/17; 12/1.

Midterms - Pimentel 1 - 9/25 8-10pm + 10/30 8-10pm.

Final - TBD.

## Chapter 17

# Temperature, Thermal Expansion, and the Ideal Gas Law

### 17.1 Atomic Theory of Matter

**Definition 1** (Unified Atomic Mass Units ( $u$ )). We define  $^{12}\text{C}$  to have exactly 12.0000 *unified atomic mass units* ( $u$ ) such that  $1\text{ u} = 1.6605 \times 10^{-27}\text{ kg}$ .

**Definition 2** (Elements, Compounds, Atoms, Molecules). *Elements* are substances that cannot be broken down into simpler substances by chemical means, *compounds* are substances made up of elements that can be broken down, *atoms* are the smallest pieces of an element, and *molecules* are the smallest pieces of compounds made up of atoms.

**Proposition 1.** To convert between Celsius and Fahrenheit, use:

$$T(^{\circ}\text{C}) = \frac{5}{9}[T(^{\circ}\text{F}) - 32] \quad T(^{\circ}\text{F}) = \frac{9}{5}[T(^{\circ}\text{C}) + 32]$$

**Remark.** Different materials do not expand precisely linearly over a wide temperature range. Thus, we standardize with the *constant-volume gas thermometer*. This thermometer consists of a hollow rigid bulb with a low-pressure gas connected by a thin tube to a mercury manometer. If the height of the mercury tube is adjusted to ensure the gas maintains a constant volume, the new height reached by the mercury is the temperature.

**Definition 3** (Freezing/Boiling Point). The *freezing point* of a substance is defined as that temperature at which the solid and liquid phases coexist

in equilibrium s.t. there is no net liquid or solid changing into the other one. The *boiling point* is defined analogously for liquid and gas. These temperatures vary with pressure so pressure is specified usually at 1 atm.

**Definition 4 (Thermal Equilibrium).** Two objects are defined to be in *thermal equilibrium* is, when placed in contact, no net energy flows from to the other, and their temperatures don't change.

**Remark.** When two systems are in thermal equilibrium, their temperatures are (by definition) equal and no net energy is exchanged.

**Definition 5 (0th Law of Thermodynamics).** Specifically experiments indicate that **if two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.**

**Proposition 2 (Thermal Expansion).** The thermal expansion in length, area, and volume of a material at a fixed pressure due to change in temperature  $\Delta T$  is approximately given by:

$$\Delta l = \alpha l_0 \Delta T \quad \Delta A = \gamma A_0 \Delta T \quad \Delta V = \beta V_0 \Delta T$$

where  $\alpha, \gamma, \beta$  denote the material's *coefficient of linear expansion* with unit  $(^\circ\text{C})^{-1}$  and  $l_0, A_0, V_0$  denote its initial length, area, and volume.

**Proof.** At a certain temperature, a thin rod of length  $l_0$  at temperature  $T_0$  is heated by  $\Delta T$  to a temperature  $T$ . This causes it to expand by  $\Delta l$  to a length  $l$ , or  $l(T)$ , that is dependent on the original length  $l_0$ , the temperature change  $\Delta T$ , and its coefficient of linear expansion  $\alpha$ . Thus,  $\Delta l = \alpha l_0 \Delta T$  so  $l = l_0(1 + \alpha \Delta T)$ . For a thin plate with area  $A_0$ , this becomes  $A = A_0 + \Delta A = A_0(1 + \alpha \Delta T)^2 = A_0 + 2\alpha A_0 \Delta T + \alpha^2 A_0 \Delta T^2$ . Thermal expansion for volume is similarly  $V = V_0 + \Delta V = V_0(1 + \alpha \Delta T)^3 = V_0 + 3\alpha V_0 \Delta T + 3\alpha^2 V_0 \Delta T^2 + \alpha^3 V_0 \Delta T^3$ .

Because  $\alpha$  and (usually)  $\Delta T$  are extremely small, we say  $\Delta A \simeq 2\alpha A_0 \Delta T$  and  $\Delta V \simeq 3\alpha V_0 \Delta T$  implying the coefficient for area  $\gamma \simeq 2\alpha$  and the coefficient for volume  $\beta \simeq 3\alpha$ .  $\square$

**Note.** If water at  $0^\circ\text{C}$  is heated, its volume *decreases* until it reaches  $4^\circ\text{C}$  when it behaves normally and expands as the temperature increases.

This implies that above  $4^\circ\text{C}$  the surface water in a lake/river in contact with cold air sinks because it is denser bringing in warmer water from below causing convection to bring the whole body to the same temperature. As water cools below  $4^\circ\text{C}$ , however, its volume expands so its density decreases meaning the the surface water turning to ice is less dense than the water below so a layer of ice forms on top. The ice acts as an insulator, allowing life to exist under ice.

**Definition 6 (Thermal Stresses).** If 2 ends of a material are rigidly fixed, temperature change can cause compressive or tensile stresses called *thermal stresses*.

If the beam tries to expand by  $\Delta\ell$  while rigid braces exert a force to hold the beam in place, compressing OR expanding it, the force required is  $\Delta\ell = \frac{1}{E} \frac{F}{A} \ell_0$  where  $E$  is Young's modulus for the material. Substituting the thermal expansion equation gives the stress to be  $\frac{F}{A} = \alpha E \Delta T$ .

**Definition 7 (State Variables).** Quantities detectable by instruments are called *state variables*. For a gas in a container, they are pressure  $P$ , volume  $V$ , temperature  $T$ , and quantity of gas – mass  $m$  or equivalently *moles*.

**Definition 8 (Mole).** One *mole*, abbreviated mol, is defined to be the number of atoms in exactly 12 g of  $^{12}\text{C}$ . This number  $N_A$  is called *Avogadro's number* and equal to  $6.022 \times 10^{23}$ .

**Definition 9 (Equilibrium States).** When the state variables of a system are not changing in time and equal throughout the system.

**Definition 10 (Absolute Zero).** Absolute zero of temperature is  $-273.15^\circ\text{C}$  or 0 K on the Kelvin scale s.t.  $T(\text{K}) = T(^{\circ}\text{C}) + 273.15$ .

**Proposition 3 (Gas Laws).** For a fixed quantity of gas, the following 'laws' are valid (as long as the pressure and density are not too great and the gas is not too close to condensing):

1. (Boyle's Law) [At constant temperature],  $P_1 V_1 = P_2 V_2$  and  $P \propto \frac{1}{V}$ .
2. (Charles's Law) [At constant pressure],  $\frac{V_1}{T_1} = \frac{V_2}{T_2}$  s.t.  $V \propto T$ .
3. (Gay-Lussac's Law) [At constant volume],  $\frac{P_1}{T_1} = \frac{P_2}{T_2}$  s.t.  $P \propto T$ .

Together these imply  $PV \propto T$  so  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ .

**Definition 11 ((Equation of State for an) Ideal Gas Law).** Of course, once mass is left unfixed, experiments show  $PV \propto mT$ . Specifically,

$$PV = nRT$$

where  $n$  is the number of moles of gas and  $R$  is the universal gas constant found to be roughly  $8.314 \frac{\text{J}}{\text{mol}\cdot\text{K}} = 0.0821 \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}} = 1.99 \frac{\text{calories}}{\text{mol}\cdot\text{K}}$ .

**Remark (Important Remarks for Ideal Gas Law).** Always give  $T$  in kelvins and  $P$  as absolute, rather than gauge, pressure. The equation is less 'ideal'

for gases at high pressure/density or near the boiling/condensation point. Recall  $1 \text{ L} = 1 \times 10^{-3} \text{ m}^3$ .

**Definition 12 (Standard Temperature and Pressure).** Standard temperature and pressure, abbreviated STP, implies  $T = 273.15 \text{ K} (= 0^\circ \text{ C})$  and  $P = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Nm}^2 = 101.3 \text{ kPa}$ .

**Note.** 1.00 mol of gas at STP has  $V = 22.4 \text{ L}$ .

**Remark (Avogadro's Hypothesis).** Amedeo Avogadro proposed equal volumes of gas at the same pressure and temperature contain equal numbers of molecules. In other words, he proposed  $R$  is the same for all gases.

**Definition 13 (Boltzmann constant).** Because  $N_A$  is constant, we can write  $PV = \frac{N}{N_A}RT$  where  $N$  is the total number of molecules in a gas. Setting the *Boltzmann constant*  $k = R/N_A = 1.38 \times 10^{-23} \text{ J/K}$  gives  $PV = NkT$ .

**Definition 14 (Ideal Gas Temperature Scale).** The *ideal gas temperature scale* is based on the property of an ideal gas that pressure is directly proportional to the absolute temperature. Real gases approach this ideal at very low density. This scale takes points  $P = 0$  at  $T = 0 \text{ K}$  and the *triple point* of water when its solid, liquid, and gas states coexist in equilibrium at  $P = 4.58 \text{ torr}$  and  $T = 0.01^\circ \text{ C} = 372.16 \text{ K}$ .

**Definition 15 (Absolute/Kelvin Temperature).** The absolute temperature  $T$  of a substance is determined by putting that substance in good thermal contact with a constant-volume gas thermometer s.t., at constant volume,  $T = (273.16 \text{ K}) \lim_{P \rightarrow 0} P_{tp} \rightarrow 0 \left( \frac{P}{P_{tp}} \right)$ .

Here,  $P_{tp}$  denotes the pressure of the gas in the rigid bulb of the thermometer when placed in water at triple point and  $P$  is the pressure in the thermometer when it's in contact with the substance determining  $T$ .

## Chapter 18

# Kinetic Theory of Gases

### 18.1 The Ideal Gas Law and the Molecular Interpretation of Temperature

**Definition 16 (Kinetic Theory).** The analysis of matter in terms of atoms in continuous random motion is called *kinetic theory*.

**Proposition 4 (Postulates of Kinetic Theory).** Under these conditions describing an 'ideal gas', real gases follow the ideal gas law quite closely.

1. There are a large number,  $N$ , of molecules, each of mass  $m$ , moving in random directions with a variety of speeds.
2. The molecules are, on average, far apart from one another. I.e. their average separation is much greater than their diameter.
3. The molecules obey the laws of classical mechanics and only interact when they collide s.t. the potential energy relating to attractive forces between them is much weaker than the kinetic energy.
4. Collisions with another molecule or the wall of the vessel are perfectly elastic and of very short duration relative to time between collisions.

**Proposition 5 (Temperature to Average Kinetic Energy of Molecules).** The average translational kinetic energy of molecules in random motion in an ideal gas is directly proportional to the absolute temperature of the gas. In other words,  $\overline{K} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$ .

**Proof.** Take a box of length  $\ell$  and ends of area  $A$  filled with an ideal gas. Focus on a single molecule of mass  $m$ 's collision with one wall. Newton's 2nd and 3rd laws tell us a force  $F = \frac{dp}{dt}$  is exerted on the molecule. Because collisions are elastic, its velocity  $v_x$  is equal in magnitude so  $\Delta p = 2mv_x$ . If the molecule takes time  $\Delta t$  to travel  $2\ell$  to the other wall and back,



$F = \frac{\Delta(mv)}{\Delta t} = \frac{2mv_x}{2\ell/v_x} = \frac{mv_x^2}{\ell}$ . Recall that although the particle may collide with the tops and sides of the container, its  $x$  component doesn't change and neither does its momentum.

We can average the force on a wall due to all the  $N$  molecules in the box by  $F = \frac{m}{\ell}(v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2)$ . Averaging,  $\overline{v_x^2} = \frac{v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2}{N}$  s.t.  $F = \frac{m}{\ell}N\overline{v_x^2}$ . Because  $v^2 = v_x^2 + v_y^2 + v_z^2 \simeq 3v_x^2$ , we let  $F = \frac{m}{\ell}\frac{N\overline{v^2}}{3}$ . Thus, the pressure on the wall is  $P = \frac{F}{A} = \frac{1}{3}\frac{Nm\overline{v^2}}{A\ell} = \frac{1}{3}\frac{Nm\overline{v^2}}{V} \Rightarrow PV = \frac{2}{3}N(\frac{1}{2}m\overline{v^2})$ . From the ideal gas law  $PV = NkT$ , such that  $\frac{3}{2}kT = \frac{1}{2}(m\overline{v^2}) = \overline{KE}$ .  $\square$

**Definition 17 (Thermal Motion).** This definition explains the relationship between temperature as a measure of motion of molecules such that random motion of a gas is sometimes called *thermal motion*.

**Definition 18 (Root-Mean-Square Speed  $v_{rms}$ ).** To calculate how fast molecules move on average in an ideal gas, we can derive the *root-mean-square speed*  $v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$ .

**Remark.** It's noteworthy that the average speed  $\bar{v} \neq v_{rms}$  necessarily. In fact, for an ideal gas, they differ by about 8%.

**Note.**  $\overline{KE} = \frac{3}{2}kT$  tells us as  $T \rightarrow 0$ ,  $\overline{KE} \rightarrow 0$ . However, modern quantum theory tells us this is not true and kinetic energy approaches a small nonzero minimum.

## 18.2 Distribution of Molecular Speeds

**Definition 19 (Maxwell Distribution of Speeds).** In 1859, James Maxwell worked out a formula for the most probable distribution of speeds in a gas with  $N$  molecules, that is

$$f(v) = 4\pi N \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} v^2 e^{-\frac{1}{2} \frac{mv^2}{kT}},$$

where  $f(v)dv$  represents the number of molecules that have speeds between  $v$  and  $v + dv$  such that  $\int_0^\infty f(v)dv = N$ .

**Definition 20 (Activation Energy).** Two molecules may chemically react only if their kinetic energy is great enough to partially penetrate each other. This minimum energy is called *activation energy*. The rate of a chemical reaction is proportional to the number of molecules with energy greater than  $E_A$  such that rates increase rapidly with increased temperature.

**Example (Determining  $\bar{v}$  and  $v_p$ ).** To find the average speed  $\bar{v}$ , we must integrate over the product of  $v$  and the number  $f(v)dv$  which have speed  $v$  and divide by  $N$  the number of molecules. Thus,  $\bar{v} = \frac{\int_0^\infty v f(v) dv}{N} = 4\pi \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \int_0^\infty v^3 e^{-\frac{1}{2} \frac{mv^2}{kT}} dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \left(\frac{2k^2 T^2}{m}\right) = \sqrt{\frac{8}{\pi}} \frac{kT}{m}$ .

To find the most probable speed, we need simply find when the slope is 0 such that  $\frac{df(v)}{dv} = 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \left(2ve^{-\frac{mv^2}{2kT}} - \frac{2mv^3}{2kT} e^{-\frac{mv^2}{2kT}}\right) = 0$ . Solving for  $v$  gives  $v_p = \sqrt{\frac{2kT}{m}}$ .

**Note.** In summary,

$$v_p \approx 1.41 \sqrt{\frac{kT}{m}}, \quad \bar{v} \approx 1.60 \sqrt{\frac{kT}{m}}, \quad v_{rms} \approx 1.73 \sqrt{\frac{kT}{m}}.$$

## 18.3 Real Gases and Changes of Phase

**Remark.** At high pressure, the volume of a real gas is *less* than that predicted by the ideal gas law.

**Explanation.** This is because ideal gases assume the potential energy originating from attractive forces between molecules to be negligible relative to their kinetic energy. This energy pulls molecules closer together so the volume decreases. At lower temperatures, these forces result in *liquefaction* or *condensation*.

## 18.4 Vapor Pressure and Humidity

**Definition 21 (Critical Point/Temperature).** At a certain *critical temperature*, a gas will change to liquid phase if sufficient pressure is applied. This is also called the *critical point*.

**Definition 22 (Gas and Vapor).** A substance in a gaseous state *below* its critical temperature is called a *vapor*; above the critical temperature it is called a *gas*.

**Remark (Phase Diagram).** A *PT Diagram* is often called a *phase diagram* because it compares the different phases of a substance.

**Definition 23 (Sublimation).** *Sublimation* refers to the process wherein, at low pressures, a solid changes directly into the vapor phases *without* passing through the liquid phase.

## 18.5 Partial Pressure and Humidity

**Remark (Cooling Process).** Molecules in a liquid roughly follow the Maxwell distribution. Molecules of high speeds in a liquid – i.e.  $E_A$  – may leave the liquid temporarily but be pulled back by attractive forces. Only those of highest speed *evaporate*, ultimately decreasing the average speed such that the absolute temperature is less. Kinetic theory, then, predicts evaporation is a *cooling process*.

**Definition 24 (Saturated Vapor Pressure).** At equilibrium, an equal number of molecules enter the vapor above a liquid and enter the liquid. At this point, the pressure of the vapor is said to be *saturated*.

**Remark.** The concentration of particular molecules in the gas phase above the liquid will not affect the saturated vapor pressure, but will lengthen the amount of time to reach equilibrium due to collisions.

**Remark.** Increased temperature increases the saturated vapor pressure of a liquid until it equals the external pressure and *boiling* occurs. Bubbles which indicate a change from liquid to the gas phase are crushed if vapor pressure inside the bubbles is less than the external pressure.

**Definition 25 (Partial Pressure).** *Partial pressure* is the pressure each gas would exert if it alone were present in a mixture. The *relative humidity* is defined to be the ratio of the partial pressure of water vapor to the saturated vapor pressure of water at a given temperature. If the partial pressure of water exceeds the saturated vapor pressure, the air is *supersaturated*. This excess water may appear as dew or clouds.

**Definition 26 (Dew Point).** The *dew point* is when water is cooled s.t. the saturated vapor pressure of water equals its partial pressure.

**Note.** The pressure  $P$  of the atmosphere as a function altitude  $y$  above sea level is, where  $P_0$  is 1.00 atm:

$$P = P_0 e^{-(\rho_0 g / P_0)y} = P_0 e^{-(1.25 \times 10^{-4} \text{ m}^{-1})y}$$

## 18.6 van der Waals Equation of State

**Theorem 1 (van der Waals Equation of State).** Given gas-dependent constants  $a$  and  $b$ ,

$$\left(P + \frac{a}{(V/n)^2}\right)\left(\frac{V}{n} - b\right) = RT.$$

**Proof.** To make the ideal gas law more accurate, we now resolve (1) the finite size of molecules in comparison to one another and the container and (2) forces between molecules may be greater than the size of molecules.

Let the molecules in a gas behave like hard spheres of radius  $r$  such that the volume molecules can move around in is less than the volume  $V$  of the container because the distance between molecules never shrinks below  $2r$ . Where  $b$  is the unavailable volume per mole of gas, replace  $V$  in  $PV = nRT$  by  $(V - nb)$ . This relation,  $P(\frac{V}{n} - b) = RT$  is called the *Clausius equation of state* and predicts ideal gases have less pressure than real gases. Next, molecules at the edge of the gas leaving towards a wall are slowed by a net attractive force pulling them back in, thus exerting less force and pressure on the wall. We say this pressure is proportionally reduced by the density squared, or  $(n/V)^2$ , for constant  $a$  s.t.  $P = \frac{RT}{(V/n)-b} - \frac{a}{(V/n)^2}$  or

$$(P + \frac{a}{(V/n)^2})(\frac{V}{n} - b) = RT.$$

Note that at low densities, van der Waals reduces to the ideal gas law.  $\square$

## 18.7 Mean Free Path

**Definition 27 (Mean Free Path).** We define the *mean free path*,  $\ell_M$  to be the average distance a molecule travels between collisions such that

$$\ell_M = \frac{1}{4\pi\sqrt{2}r^2(N/V)}.$$

**Proof.** Suppose the molecules of a gas are hard spheres of radius  $r$ . Let the path of one molecule with mean speed  $\bar{v}$  be a cylinder of radius  $2r$  such that if another molecule's center lies in the cylinder a collision will occur. We can assume for now the other molecules are not moving and the concentration of molecules is  $N/V$ . Then,  $V_{cylinder} * N/V$  is the number of collisions that will occur. Over a time  $\Delta t$ , the molecule travels  $\bar{v}\Delta t$  so the number of collisions is expected to be  $\pi(2r)^2\bar{v}\Delta t(N/V)$ . Thus, the average distance between collisions is

$$\ell_M = \frac{\bar{v}\Delta t}{\pi(2r)^2\bar{v}\Delta t(N/V)} = \frac{1}{4\pi r^2(N/V)}.$$

If the other molecules are moving, the number of collisions in  $\Delta t$  actually depends on the *relative* speed  $v_{rel} \approx \sqrt{2}\bar{v}$  of the colliding molecules so

$$\ell_M = \frac{1}{4\sqrt{2}\pi r^2(N/V)}$$

. This of course loses meaning at low densities.  $\square$

## 18.8 Diffusion

**Definition 28 (Diffusion).** In general, the *diffusing* substance moves from a region where its concentration is high to a region where its concentration is low.

**Definition 29 (Fick's Law).** Consider a tube of cross-sectional area  $A$  containing molecules of increasingly smaller concentration left-to-right. Take a small middle section of tube of length  $\Delta x$  with section 1 on the left and 2 on the right. 1 has more molecules causing greater pressure such that more will strike the boundary into the middle section than from 2. Let  $J$  be the rate of diffusion in number of molecules/moles/kg per second. This is proportional, with diffusion constant  $D$  to the *concentration gradient* or difference in concentration per unit distance  $\frac{C_1 - C_2}{\Delta x}$ . Thus,  $J = DA \frac{C_1 - C_2}{\Delta x} = DA \frac{dC}{dx}$ . This is *Fick's Law*.