Page 91, Problem 1

Proof. Say $f\colon C \to C$ is a map that is NOT homeomorphic to the identity where C is the unit circle. Suppose also $f(x) \neq -x$ for all $x \in C$. Thus, no antipodal points exist (taking points as (x,y) with additive inverse (-x, -y)) so no straight line between the identity and $f(x)$ pass through the origin. Therefore, via example 2 on page 89, we can define the homotopy from f to i $F\colon C \times I \to C$ as

$$F(x,t) = \frac{(1-t)f(x) + tx}{||(1-t)f(x) + tx||}.$$

Thus, $f(x) \underset{F}{\cong} i$ which is a contradiction so there exists some point $x \in C$ so $f(x) = -x$. □

Page 95, Problem 13

Proof. Say $f\colon C \to C$ is a map that is NOT homeomorphic to the identity where C is the unit circle. Suppose also $f(x) \neq -x$ for all $x \in C$. Thus, no antipodal points exist (taking points as (x,y) with additive inverse (-x, -y)) so no straight line between the identity and $f(x)$ pass through the origin. Therefore, via example 2 on page 89, we can define the homotopy from f to i $F\colon C \times I \to C$ as

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