PHYSICS 7B: Physics for Scientists and Engineers

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Contents

Chapter 0

Syllabus

0.1 Important Info

Syllabus schedule here

0.1.1 Contacts + Sections

Instructor OH:

1. Robert Birgeneau; robertjb@berkeley.edu; Tu 2-3pm

2. Dr.Chen/Giles-Donovan OH are Th 2-3pm.

Lecture 1: MWF 11-12p @ Physics North 1

Head GSI: Yucehn Tang; ytang5@berkeley.edu; begin emails with [7B Lec.1]

2 Sections: DIS + LAB; Labs not every week and may be replaced by DIS (check syllabus)

0.1.2 Materials

1. 7B Workbook by Hedeman (on bCourses)

2. Mastering Physics (Purchase access code)

3. Physics for scientists and engineers 5th edition by Giancoli (may come w/ mastering physics purchase)

0.1.3 Assignments

Homework will be made available Friday 11:59 and due next Friday 11:59.

Must complete all 4 Labs - Sep 29; Oct 27; 11/17; 12/1.

Midterms - Pimentel 1 - 9/25 8-10pm + 10/30 8-10pm.

Final - TBD.

Chapter 17

Temperature, Thermal Expansion, and the Ideal Gas Law

17.1 Atomic Theory of Matter

Definition 1 (Unified Atomic Mass Units (u))**.** We define ^{12}C to have exactly 12.0000 *unified atomic mass units* (u) such that $1\,\text{u} = 1.6605 \times 10^{-27}\,\text{kg}$.

Definition 2 (Elements, Compounds, Atoms, Molecules)**.** *Elements* are substances that cannot be broken down into simpler substances by chemical means, *compounds* are substances made up of elements that can be broken down, *atoms* are the smallest pieces of an element, and *molecules* are the smallest pieces of compounds made up of atoms.

Proposition 1. To convert between Celsius and Fahrenheit, use:

$$T(^\circ\text{C}) = \frac{5}{9}[T(^\circ\text{F}) - 32] \quad T(^\circ\text{F}) = \frac{9}{5}[T(^\circ\text{C}) + 32]$$

Remark. Different materials do not expand precisely linearly over a wide temerpature rage. Thus, we standardize with the *constant-volume gas thermometer*. This thermometer consists of a hollow rigid bulb with a low-pressure gas connected by a thin tube to a mercury manometer. If the height of the mercury tube is adjusted to ensure the gas maintains a constant volume, the new height reached by the mercury is the temperature.

Definition 3 (Freezing/Boiling Point)**.** The *freezing point* of a substance is defined as that temperature at which the solid and liquid phases coexist

in equilibirum s.t. there is no net liquid or solid changing into the other one. The *boiling point* is defined analogously for liquid and gas. These temperatures vary with pressure so pressure is specified usually at $1\,\mathrm{atm}$.

Definition 4 (Thermal Equilibrium)**.** Two objects are defined to be in *thermal equilibrium* is, when placed in contact, no net energy flows from to the other, and their temperatures don't change.

Remark. When two systems are in thermal equilibrium, their temperatures are (by definition) equal and no net energy is exchanged.

Definition 5 (0th Law of Thermodynamics)**.** Specifically experiments indicate that **if two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other**.

Proposition 2 (Thermal Expansion)**.** The thermal expansion in length, area, and volume of a material at a fixed pressure due to change in temperature ΔT is approximately given by:

$$\Delta l = \alpha l_0 \Delta T \quad \Delta A = \gamma A_0 \Delta T \quad \Delta V = \beta V_0 \Delta T$$

where α, γ, β denote the material's *coefficient of linear expansion* with unit $(\mathrm{C}^\circ)^{-1}$ and l_0, A_0, V_0 denote its initial length, area, and volume.

Proof. At a certain temperature, a thin rod of length l_0 at temperature T_0 is heated by ΔT to a temperature T. This causes it to expand by Δl to a length l, or $l(T)$, that is dependent on the original length l_0, the temperature change ΔT, and its coefficient of linear expansion α. Thus, $\Delta l = \alpha l_0 \Delta T$ so $l = l_0(1 + \alpha \Delta T)$. For a thin plate with area A_0, this becomes $A = A_0 + \Delta A = A_0(1 + \alpha \Delta T)^2 = A_0 + 2\alpha A_0 \Delta T + \alpha^2 A_0 \Delta T^2$. Thermal expansion for volume is similarly $V = V_0 + \Delta V = V_0(1 + \alpha \Delta T)^3 = V_0 + 3\alpha V_0 \Delta T + 3\alpha^2 V_0 \Delta T^2 + \alpha^3 V_0 \Delta T^3$.

Because α and (usually)ΔT are extremely small, we say $\Delta A \simeq 2\alpha A_0 \Delta T$ and $\Delta V \simeq 3\alpha V_0 \Delta T$ implying the coefficient for area $\gamma \simeq 2\alpha$ and the coefficient for volume $\beta \simeq 3\alpha$. $\qquad\square$

Note. If water at $0\,\mathrm{°C}$ is heated, its volume *decreases* until it reaches $4\,\mathrm{°C}$ when it behaves normally and expands as the temperature increases.

This implies that above $4\,\mathrm{°C}$ the surface water in a lake/river in contact with cold air sinks because it is denser bringing in warmer water from below causing convection to bring the whole body to the same temperature. As water cools below $4\,\mathrm{°C}$, however, its volume expands so its density decreases meaning the the surface water turning to ice is less dense than the water below so a layer of ice forms on top. The ice acts as an insulator, allowing life to exist under ice.

Definition 6 (Thermal Stresses). If 2 ends of a material are rigidly fixed, temperature change can cause compressive or tensile stresses called *thermal stresses*.

If the beam tries to expand by $\Delta\ell$ while rigid braces exert a force to hold the beam in place, compressing OR expanding it, the force required is $\Delta\ell = \frac{1}{E}\frac{F}{A}\ell_0$ where E is Young's modulus for the material. Substituting the thermal expansion equation gives the stress to be $\frac{F}{A} = \alpha E \Delta T$.

Definition 7 (State Variables). Quantities detectable by instruments are called *state variables*. For a gas in a container, they are pressure P, volume V, temperature T, and quantity of gas – mass m or equivalently *moles*.

Definition 8 (Mole). One *mole*, abbreviated mol, is defined to be the number of atoms in exactly $12\,\mathrm{g}$ of ^{12}C. This number N_A is called *Avogadro's number* and equal to 6.022×10^{23}.

Definition 9 (Equilibrium States). When the state variables of a system are not chaning in time and equal throughout the system.

Definition 10 (Absolute Zero). Absolute zero of temperature is $-273.15\,°\mathrm{C}$ or 0 K on the Kelvin scale s.t. $T(K) = T(°C) + 273.15$.

Proposition 3 (Gas Laws). For a fixed quantity of gas, the following 'laws' are valid (as long as the pressure and density are not too great and the gas is not too close to condensating):

1. (Boyle's Law) [At constant temperature], $P_1V_1 = P_2V_2$ and $P \propto \frac{1}{V}$.

2. (Charles's Law) [At constant pressure], $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ s.t. $V \propto T$.

3. (Gay-Lussac's Law) [At constant volume], $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ s.t. $P \propto T$.

Together these imply $PV \propto T$ so $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$.

Definition 11 ((Equation of State for an) Ideal Gas Law). Of course, once mass is left unfixed, experiments show $PV \propto mT$. Specifically,

$$PV = nRT$$

where n is the number of moles of gas and R is the universal gas constant found to be roughly $8.314\,\frac{\mathrm{J}}{\mathrm{mol.K}} = 0.0821\,\frac{\mathrm{L.atm}}{\mathrm{mol.K}} = 1.99\,\frac{\mathrm{calories}}{\mathrm{mol\,K}}$.

Remark (Important Remarks for Ideal Gas Law). Always give T in kelvins and P as absolute, rather than gauge, pressure. The equation is less 'ideal'

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for gases at high pressure/density or near the boiling/condensation point. Recall $1\,\mathrm{L} = 1 \times 10^{-3}\,\mathrm{m}^3$.

Definition 12 (Standard Temperature and Pressure)**.** Standard temperature and pressure, abbreviated STP, implies $T = 273.15K (= 0\,^\circ\,\mathrm{C})$ and $P = 1.00\,\mathrm{atm} = 1.013 \times 10^5\,\mathrm{Nm}^2 = 101.3\,\mathrm{kPa}$.

Note. $1.00\,\mathrm{mol}$ of gas at STP has $V = 22.4\,\mathrm{L}$.

Remark (Avogadro's Hypothesis)**.** Amedeo Avogadro proposed equal volumes of gas at the same pressure and temperature contain equal numbers of molecules. In other words, he proposed R is the same for all gases.

Definition 13 (Boltzmann constant)**.** Because N_A is constant, we can write $PV = \frac{N}{N_A}RT$ where N is the total number of molecules in a gas. Setting the *Boltzmann constant* $k = R/N_A = 1.38 \times 10^{-23}\,\mathrm{J/K}$ gives $PV = NkT$.

Definition 14 (Ideal Gas Temperature Scale)**.** The *ideal gas temperature scale* is based on the property of an ideal gas that pressure is directly proportional to the absolute temperature. Real gases approach this ideal at very low density. This scale takes points $P = 0$ at $T = 0$ K and the *triple point* of water when its solid, liquid, and gas states coexist in equilibrium at $P = 4.58\,\mathrm{torr}$ and $T = 0.01\,^\circ\mathrm{C} = 372.16$ K.

Definition 15 (Absolute/Kelvin Temperature)**.** The absolute temperature T of a substance is determind by putting that substance in good thermal contact with a constant-volume gas thermometer s.t., at constant volume, $T = (273.16\,\mathrm{K}) \lim P_{tp} \to 0(\frac{P}{P_{tp}})$.

Here, P_{tp} denotes the pressure of the gas in the rigid bulb of the thermometer when placed in water at triple point and P is the pressure in the thermometer when it's in contact with the substance determining T.

Chapter 18

Kinetic Theory of Gases

18.1 The Ideal Gas Law and the Molecular Interpreation of Temperature

> **Definition 16** (Kinetic Theory)**.** The analysis of matter in terms of atoms in continuous random motion is called *kinetic theory*.

> **Proposition 4** (Postulates of Kinetic Theory)**.** Under these conditions describing an 'ideal gas', real gases follow the ideal gas law quite closely.
>
> 1. There are a large number, N, of molecules, each of mass m, moving in random directions with a variety of speeds.
>
> 2. The molecules are, on average, far apart from one another. I.e. their average separation is much greater than their diameter.
>
> 3. The molecules obey the laws of classical mechanics and only interact when they collide s.t. the potential energy relating to attractive forces between them is much weaker than the kinetic energy.
>
> 4. Collisions with another molecule or the wall of the vessel are perfectly elastic and of very short duration relative to time between collisions.

> **Proposition 5** (Temperature to Average Kinetic Energy of Molecules)**.** The average translational kinetic energy of molecules in random motion in an ideal gas is directly proportional to the absolute temperature of the gas. In other words, $\overline{K} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$.

> **Proof.** Take a box of length ℓ and ends of area A filled with an ideal gas. Focus on a single molecule of mass m's collision with one wall. Newton's 2nd and 3rd laws tell us a force $F = \frac{dp}{dt}$ is exerted on the molecule. Because collisions are elastic, its velocity v_x is equal in magnitude so $\Delta p = 2mv_x$. If the molecule takes time Δt to travel 2ℓ to the other wall and back,

$F = \frac{\Delta(mv)}{\Delta t} = \frac{2mv_x}{2\ell/v_x} = \frac{mv_x{}^2}{\ell}$. Recall that although the particle may collide with the tops and sides of the container, its x component doesn't change and neither does its momentum.

We can average the force on a wall due to all the N molecules in the box by $F = \frac{m}{\ell}(v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2)$. Averaging, $\overline{v_x^2} = \frac{v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2}{N}$ s.t. $F = \frac{m}{\ell}N\overline{v_x^2}$. Because $v^2 = v_x^2 + v_y^2 + v_z^2 \simeq 3v_x^2$, we let $F = \frac{m}{l}\frac{N\overline{v^2}}{3}$. Thus, the pressure on the wall is $P = \frac{F}{A} = \frac{1}{3}\frac{Nm\overline{v^2}}{A\ell} = \frac{1}{3}\frac{Nm\overline{v^2}}{V} \Rightarrow PV = \frac{2}{3}N(\frac{1}{2}m\overline{v^2})$. From the ideal gas law $PV = NkT$, such that $\frac{3}{2}kT = \frac{1}{2}(m\overline{v^2}) = \overline{KE}$. $\square$

Definition 17 (Thermal Motion)**.** This definition explains the relationship between temperature as a measure of motion of molecules such that random motion of a gas is sometimes called *thermal motion.*

Definition 18 (Root-Mean-Square Speed v_{rms})**.** To calculate how fast molecules molecules move on average in an ideal gas, we can derive the *root-mean-square speed* $v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$.

Remark. It's noteworthy that the average speed $\overline{v} \neq v_{rms}$ necessarily. In fact, for an ideal gas, they differ by about 8%.

Note. $\overline{KE} = \frac{3}{2}kT$ tells us as $T \to 0$, $\overline{KE} \to 0$. However, modern quantum theory tells us this is not true and kinetic energy appraoches a small nonzero minimum.

18.2   Distribution of Molecular Speeds

Definition 19 (Maxwell Distribution of Speeds)**.** In 1859, James Maxwell worked out a formula for the most probable distribution of speeds in a gas with N molecules, that is

$$f(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}},$$

where $f(v)dv$ represents the number of molecules that have speeds between v and $v + dv$ such that $\int_0^\infty f(v)dv = N$.

Definition 20 (Activation Energy)**.** Two molecules may chemically react only if their kinetic energy is great enough to partially penetrate each other. This minimum energy is called *activation energy.* The rate of a chemical reaction is proportional to the number of molecules with energy greater than E_A such that rates increase rapidly with increased temperature.

Example (Determining $\bar{v}$ and$\vec{v_p}$). To find the average speed $\bar{v}$, we must integrate over the product of v and the number $f(v)dv$ which have speed v and divide by N the number of molecules. Thus, $\bar{v} = \frac{\int_0^\infty vf(v)dv}{N} = 4\pi(\frac{m}{2\pi kT})^{\frac{3}{2}} \int_0^\infty v^3 e^{-\frac{1}{2}\frac{mv^2}{kT}} dv = 4\pi(\frac{m}{2\pi kT})^{\frac{3}{2}} (\frac{2k^2T^2}{m}) = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$.

To find the most probable speed, we need simply find when the slope is 0 such that $\frac{df(v)}{dv} = 4\pi N(\frac{m}{2\pi kT})^{\frac{3}{2}}(2ve^{-\frac{mv^2}{2kT}} - \frac{2mv^3}{2kT}e^{-\frac{mv^2}{2kT}}) = 0$. Solving for v gives $v_p = \sqrt{\frac{2kT}{m}}$.

Note. In summary,

$$v_p \approx 1.41\sqrt{\frac{kT}{m}}, \quad \bar{v} \approx 1.60\sqrt{\frac{kT}{m}}, \quad v_{rms} \approx 1.73\sqrt{\frac{kT}{m}}.$$

18.3   Real Gases and Changes of Phase

Remark. At high pressure, the volume of a real gas is *less* than that predicted by the ideal gas law.

Explanation. This is because ideal gases assume the potential energy originating from attractive forces between molecules to be negligible relative to their kinetic energy. This energy pulls molecules closer together so the volume decreases. At lower temperatures, these forces result in *liquefaction* or *condensation*.

18.4   Vapor Pressure and Humidity

Definition 21 (Critical Point/Temperature). At a certain *critical temperature*, a gas will change to liquid phase if suffieicent pressure is applied. This is also called the *critical point*.

Definition 22 (Gas and Vapor). A substance in a gaseous state *below* its critical temperature is called a *vapor*; above the critical temperature it is called a *gas*.

Remark (Phase Diagram). A *PT Diagram* is often called a *phase diagram* because it compares the different phases of a substance.

Definition 23 (Sublimation). *Sublimation* refers to the process wherein, at low pressures, a solid changes directly into the vapor phases *without* passing through the liquid phase.

18.5   Partial Pressure and Humidity

Remark (Cooling Process). Moleculesi n a liquid roughly follow the Maxwell distribution. Molecules of high speeds in a liquid – i.e. E_A – may leave the liquid temporarily but be pulled back by attractive forces. Only those of highest speed *evaporate*, ultimately decreasing the average speed such that the absolute temperature is less. Kinetic theory, then, predicts evaporation is a *cooling process*.

Definition 24 (Saturated Vapor Pressure). At equilibrium, an equal number of molecules enter the vapor above a liquid and enter the liquid. At this point, the pressure of the vapor is said to be *saturated*.

Remark. The concentration of particular molecules in the gas phase above the liquid will not affect the saturated vapor pressure, but will lengthen the amount of time to reach equilibrium due to collisions.

Remark. Increased temperature increases the saturated vapor pressure of a liquid until it equals the external pressure and *boiling* occurs. Bubbles which indicate a change from liquid to the gas phase are crushed if vapor pressure inside the bubbles is less than the external pressure.

Definition 25 (Partial Pressure). *Partial pressure* is the pressure each gas would exert it it alone were present in a mixture. The *relative humidity* is defined to be the ratio of the partial pressure of water vapor to the saturated vapor pressure of water at a given temperature. If the partial pressureo f water exceeds the saturated vapor pressure, the air is *supersaturated*. This excess water may appear as dew or clouds.

Definition 26 (Dew Point). The *dew point* is when water is cooled s.t. the saturated vapor pressure of water equals its partial pressure.

Note. The pressure P of the amosphere as a function altitude y above sea level is, where P_0 is 1.00 atm:

$$P = P_0 e^{-(\rho_0 g/P_0)y} = P_0 e^{-(1.25\times 10^{-4} m^{-1})}$$

18.6   van der Waals Equation of State

Theorem 1 (van der Waals Equation of State). Given gas-dependent constants a and b,

$$(P + \frac{a}{(V/n)^2})(\frac{V}{n} - b) = RT.$$

Proof. To make the ideal gas law more accurate, we now resolve (1) the finite size of molecules in comparison to one another and the container and (2) forces between molecules may be greater than the size of molecules.

Let the molecules in a gas behave like hard spheres of radius r such that the volume molecules can move around in is less than the volume V of the container because the distance between molecules never shrinks below $2r$. Where b is the unavailable volume per mole of gas, replace V in $PV = nRT$ by $(V - nb)$. This relation, $P(\frac{V}{n} - b) = RT$ is called the *Clausius equation of state* and predicts ideal gases have less pressure than real gases. Next, molecules at the edge of the gas leaving towards a wall are slowed by a net attractive force pulling them back in, thus exerting less force and pressure on the wall. We say this pressure is proportionally reduced by the density squared, or $(n/V)^2$, for constant a s.t. $P = \frac{RT}{(V/n)-b} - \frac{a}{(V/n)^2}$ or

$$(P + \frac{a}{(V/n)^2})(\frac{V}{n} - b) = RT.$$

Note that at low densities, van der Waals reduces to the ideal gas law. $\qquad \square$

18.7   Mean Free Path

Definition 27 (Mean Free Path)**.** We define the *mean free path*, ℓ_M to be the average distance a molecule travels between collisions such that

$$\ell_M = \frac{1}{4\pi\sqrt{2}r^2(N/V)}.$$

Proof. Suppose the molecules of a gas are hard spheres of radius r. Let the path of one molecule with mean speed $\overline{v}$ be a cylinder of radius $2r$ such that if another molecule's center lies in the cylinder a collision will occur. We can assume for now the other molecules are not moving and the concentration of molecules is N/V. Then, $V_{cylinder} * N/V$ is the number of collisions that will occur. Over a time Δt, the molecule travels $\overline{v}\Delta t$ so the number of collisions is expected to be $\pi(2r)^2\overline{v}\Delta t(N/V)$. Thus, the average distance between collisions is

$$\ell_M = \frac{\overline{v}\Delta t}{\pi(2r)^2\overline{v}\Delta t(N/V)} = \frac{1}{4\pi r^2(N/V)}.$$

If the other molecules are moving, the number of collisions in Δt actually depends on the *relative* speed $v_{rel} \approx \sqrt{2}\overline{v}$ of the colliding molecules so

$$\ell_M = \frac{1}{4\sqrt{2}\pi r^2(N/V)}.$$

This of course loses meaning at low densities. $\qquad \square$

18.8   Diffusion

Definition 28 (Diffusion). In general, the *diffusing* substance moves from a region where its concentration is high to a region where its concentration is low.

Definition 29 (Fick's Law). Consider a tube of cross-sectional area A containg molecules of increasingly smaller concentration left-to-right. Take a small middle section of tube of length Δx with section 1 on the left and 2 on the right. 1 has more molecules causing greater pressure such that more will strike the boundary into the middle section than from 2. Let J be the rate of diffusion in numbehr of molecules/moles/kg per second. This is proportional, with diffusion constant D to the *concentration gradient* or difference in concentration per unit distance $\frac{C_1 - C_2}{\Delta x}$. Thus, $J = DA\frac{C_1 - C_2}{\Delta x} = DA\frac{dC}{dx}$. This is *Fick's Law.*

Chapter 19

Heat and the First Law of Thermodynamics

19.1   Heat as Energy Transfer

Remark. An 18-th century model pictured heatflow as movement of a fluid substance called *caloric* which could never be detected. Today, heat, like work, represents a transfer of energy.

Definition 30 (Calorie). A *calorie* (cal) is defined as the amount of heat necessary to raise the temperature of 1 gram of water by 1 Celsius degree. The more often used *kilocalorie* (kcal), or **C***alorie* is 1000 calories. In the UK, heat is measured through *British thermal units* (Btu) defined as the heat needed to raise the temperature of 1 lb of water by 1 F°. Gas companies use the *therm* defined to be 1×10^5 Btu.

Note (Mechanical Equivalent of Heat). The mechanical equivalent oheat is known as $4.186 \, \text{J} = 1 \, \text{cal}$. And, $1 \, \text{Btu} = 0.252 \, \text{kcal} = 1056 \, \text{J}$.

Definition 31 (Heat). *Heat* is energy transfered from one object to another because of a difference in temperature.

Definition 32 (Internal/Thermal Energy). The sum total of all the neergy of the molecules in an object is called its *internal/thermal energy*. e.g. 2 objects of equal temperature have more internal temperature than just 1.

For monatomic (one atom/molecule) gas, the internal energy

$$E_{int} = N(\frac{1}{2}\bar{v^2}) = \frac{3}{2}NkT = \frac{3}{2}nRT.$$

If the gas molecules contain more than one atom, the rotaional and vibrational energy of the molecules must also be taken into account.

Remark. Internal energy of real gases, as opposed to ideal gases, also depends on pressure and volume (due to atomic potential energy). Similarly, the internal energy of liquids and solids includes electric potential energy associated with chemical bonds.

19.2   Specific Heat

Definition 33 (Specific Heat). An amount of heat Q put into an object of mass m results, depending on its *specific heat* c, in a temperature change of ΔT such that
$$Q = mc\Delta T.$$

Definition 34 (Closed/Open/Isolated Systems). A *closed system* is one for which no mass enters or leaves, but energy may be exchanged with the environment. Mass may enter or leave in an *open system*. If no energy AND no mass passes its boundaries, a system is *isolated*.

Remark (Energy Conservation for Heat Transfer). Within an isolated system, we can write the energy conservation equation for heat transfer as $\Sigma Q = 0$.

Definition 35 (Calorimetry). *Calorimetry* is a technique which quantitatively measures heat exchange using a *calorimeter* that must be well insulated.

Definition 36 (Bomb Calorimeter). A *bomb calorimeter* measures the thermal energy released when a substance burns to determine their Calorie content. In this calorimeter, a carefully weighed sample is placed with an excess of oxygen in the "bomb" that is then placed in the water of a calorimeter with a fine wire passing into the bomb that ignites the mixture.

19.3   Latent Heat

Definition 37 (Change of Phase). A certain amount of energy is required to change a material's phase.

Definition 38 (Heat of Fusion/Vaporization). The heat required to change $1.0\,\mathrm{kg}$ of a substance from the solid to liquid state is called the *heat of fusion*, denoted by L_F. The heat required for the change from liquid to the vapor phase is called the *heat of vaporization* L_V. These are also called the *latent heats* of substances.

The heat involved in a change of phase is written as $Q = mL$ where m is the mass of the substance.

Remark. These latent heats also refer to the amount of heat *released* by a substance when it changes from gas to liquid or liquid to solid.

Remark. At the melting point, the latent heat of fusion does not increase the average kinetic energy (and thus the temperature) of the molecules in a solid, but instead overcomes the potential energy from the forces between molecules. This is the same for vaporization. However, vaporization requires a greater average distance between molecules such that the heat of vaporization is far greater than the heat of fusion for a given substance.

19.4   The First Law of Thermodynamics

Definition 39 (The First Law of Thermodynamics)**.** Because heat is defined to be the transfer of energy due to a difference in temperature, let work be the transfer of energy NOT due to a temperature difference such that, as a general statement of the *law of conservation of energy* $\Delta E_{int} = Q - W$.

Note (Sign Conventions)**.** Because Q is the net heat *added* to the system while W is the net work done *by* the system, we say the following:

1. Heat added is $+$

2. Heat lost is -

3. Work on system is -

4. Work by system is $+$

Remark. A full summary of the first law of thermodynamics would include kinetic energy K and potential energy U such that

$$\Delta K + \Delta U + \Delta E_{int} = Q - W.$$

Definition 40 (Idealized Processes)**.** An idealized process carried out at:

1. constant temperature $(\Delta T = 0)$ is called *isothermal*.

2. constant heat $(Q = 0)$ is called *adiabatic*. (This is the case for well-insulated systems or very rapid processes.)

3. constant pressure $(\Delta P = 0)$ is called *isobaric*.

4. constant volume $(\Delta V = 0)$ is called *isovolumetric* (or isochoric).

> **Definition 41** (Isotherms). Curves on a PV diagram at different temperatures are called *isotherms*.

> **Definition 42** (Quasistatically). A process done slowly enough such that a series of equilibrium states are essentially maintained is called *quasistatically*.

> **Definition 43** (Heat Reservoir). A body whose mass is so large that, ideally, its temperature does not change significantly when heat is exchanged, is called a *heat reservoir*.

Example. Suppose there is a gas of pressure P inside a cylindrical container of area A and length ℓ fitted with a movable piston.

If our system is just the gas, as it exerts a force $F = PA$ on the piston, the work done by the gas is $dW = \mathbf{F} \cdot d\ell = PAd\ell = PdV$ where $d\mathbf{l}$ is an infinitesimal displacement of the piston.

If the gas was *compressed* so $d\ell$ would point into the gas, its volume would decrease and $dV < 0$. Thus, the work done by the gas would be negative s.t. positive work is being done *on* the gas. For a finite change in volume from V_A to V_B, the work W done by the gas will be $W = \int dW = \int_{V_A}^{V_B} PdV$.

For an quasistatic isothermal expansion of an ideal gas, $P = nRT/V$ for constant T tells us $W = nRT \ln \frac{V_B}{V_A}$. This is the area under the curve between points A and B on a PV diagram.

We can replicate this same change of state from A to B for an ideal gas first isovolumetrically then isobarically. For an ideal gas undergoing an isovolumetric process where we lower the pressure from $P_A \to P_B$, we simply reduce the temperature by letting heat flow out ($dV = 0 \Rightarrow W = 0$ so no work is done). For an ideal gas undergoing an isobaric process taking the volume from $V_A \to V_B$, P is constant at P_B so $W = P_B \Delta V = P_B(V_B - V_A) = \frac{nRT_B}{V_B}(V_B - V_A) = nRT_B(1 - \frac{V_A}{V_B})$. Thus, the total work from A to B in this case was $\frac{nRT_B}{V_B}(V_B - V_A)$ which is quantitatively different from $nRT \ln \frac{V_B}{V_A}$.

> **Definition 44** (Free Expansion). *Free expansion* is a type of adiabatic process when a gas is allowed to expand in volume without doing any work. Picture 2 well-insulated containers connected by a valve such taht one container has gas while the other is empty. When the valve is open, no heat flows or work is done because no objects are moved so $\Delta E_{int} = Q = W = 0$. E_{int} only depends on T so $\Delta T = 0$ as well.

19.5   Molar Specific Heats for Gases

Remark. Specific heats easily apply to solids and liquids. Gases depend much more on how the thermodynamic process is carried out however. Specific heats are given at constant volume c_V and pressure c_P.

Definition 45 (Molar Specific Heats). Molar specific heats C_V and C_P denote the heat required to raise 1 mol of gas by 1 C° at constant volume and pressure respectively. Thus,

$$Q = nC_V\Delta T \qquad\qquad \text{[volume constant]}$$
$$Q = nC_P\Delta T \qquad\qquad \text{[pressure constant]}$$

where $C_V = Mc_V$ and $C_P = Mc_P$ where M is the molecular mass of the gas (g/mol).

Remark. Imagine an ideal gas is slowly heated by ΔT, first at constant volume, then constant pressure. In the constant volume process, no work is done since $\Delta V = 0$. Thus all the heat Q_V added goes towards increasing internal energy such that $Q_V = \Delta E_{int}$.

However, in the constant pressure process, work *is* done so the added heat Q_P increases the internal energy and does the work $W = P\Delta V_P$ so $\Delta E_{int} = Q_P - P\Delta V_P$.

Because ΔE_{int} is equal in both processes ($\Delta T_V = \Delta T_P$), $Q_P - Q_V = P\Delta V \Rightarrow nC_P\Delta T - nC_V\Delta T = P(\frac{nR\Delta T}{P})$ from the ideal gas law. This implies $C_P - C_V = R$ which is very accurate to what's obtained experimentally.

Remark. Now, a process done at constant volume on a *monatomic* gas does no work so $nC_V\Delta T = nC_V(T-0) = Q_V = \Delta E_{int} = N(\frac{1}{2}m\overline{v^2}) = \frac{3}{2}nRT$. So $C_V = \frac{3}{2}R$.

Definition 46 (Degrees of Freedom). Here, we denote *degrees of freedom* to mean the number of independent way molecules can possess energy.

Example. For instance, a diatomic molecule has 5 total degrees of freedom. 3 from translational energy (x,y,z) plus 2 from rotaional kinetic energy (not 3 because the axis along the line of the 2 molecules has such small inertia it is negligible).

Definition 47 (Principle of Quipartition of Energy). Energy is shared equally among the active degrees of freedom and each active degree of a molecule has on average an energy equal to $\frac{1}{2}kT$.

Note. This makes sense as diatomic molecules have energy $\frac{5}{2}nRT$ about 5/3 times monatomic molecules without degrees of freedom. Hence their C_V being 5/3 as much. Yet, at extreme low and high temperatures, this diverges such that at low temps, molecules only have translational kinetic energy (3/2). And at high temps, molecules also have vibration as 2 degree of freedom (as if from a spring) such that it has kinetic and potential energy (7/2).

Note. For solids, we maintain this same spring idea such that molecules have potential and kinetic energy to do with vibration in the x, y, z directions implying 6 degrees of freedom.

19.6   Adiabatic Expansion of a Gas

Remark. Take the first law of thermodynamics in an adiabatic process ($\Delta Q = 0$) for an ideal gas such that $dE_{int} = dQ - dW = -dW = -PdV$. For an ideal gas, $\Delta E_{int} = nC_V\Delta T$ tells us $nC_v dT = -PdV$. Taking the differential of the ideal gas law gives $PdV + VdP = nRdT$ so $nC_V(\frac{PdV+VdP}{nR}) + PdV = 0$. Rearranging and $C_V + R = C_P$ gives $C_P PdV + C_V VdP = 0$ or $\frac{C_P}{C_V}PdV + VdP = 0$. Defining $\gamma = \frac{C_P}{C_V}$ and integrating finally gives $\ln(P) + \gamma \ln(V) = \text{constant}$ which simplifies to

$$PV^\gamma = \text{constant}.$$

It's important to note the ideal gas law holds for an adiabatic expansion however PV is clearly not constant implying T must be nonconstant.

19.7   Heat Transfer

Definition 48 (Conduction). Heat *conduction* can be visualized via molecular collisions such that faster molecules at a heated end collide with slower-moving neighbors and transfer kinetic energy. In metals, collisions of free electrons are mainly responsible for conduction.

Remark (Thermal Conductivity). Take a uniform cylinder of cross-sectional area A and length ℓ such that ends with temperatures T_1 and T_2 with heat flow Q over a time interval t gives $\frac{Q}{t} = kA\frac{T_1-T_2}{\ell}$ where k is the *thermal conductivity* constant characteristic of the material. The rate of heat flow (J/s) is directly proportional to $(T_1 - T_2)/\ell$.

Definition 49 (Thermal Gradient). When k or A cannot be considered constant, we instead take the limit of an infinitesimally thin slab of thickness dx such that our equation becomes $\frac{dQ}{dt} = -kA\frac{dT}{dx}$ where dT/dx is the *temperaute gradient*. Here, the negative sign denotes that heat flow is in direction opposite to the temperature gradient.

Definition 50 (Conductors/Insulators). Substances for which k is very large conduct heat rapidly and are called good thermal *conductors*. When k is small, the substances are called good thermal *insulators*.

Definition 51 (Thermal Resistance). The insulating properties of bulding materials are usually specified by R-values or *thermal resistance*, defined as $R = \frac{\ell}{k}$ for a thickness ℓ of a material. Larger R values imply better insulation.

Definition 52 (Convection). Though liquids and gases are poor conductors of heat, *Convection* is the rapid process whereby heat flows via the bulk movement of molecules over larger distances.

Definition 53 (Forced/Natural Convection). For example, *Forced convection* occurs through a fan blowing air while *natural convection* occurs through hot air rising. Hot air or water (in convection currents) rise from buoyancy because heat causes them to expand, decreasing their relative density.

Definition 54 (Radiation). Radiation is heat transferred over empty space through EM waves. IR wavelengths are mainly responsible for heating the Earth.

Definition 55 (Stefan-Boltzmann equation). The rate at which energy leaves a radiating object is $\frac{Q}{t} = \varepsilon \sigma A T^4$ where σ is the *Stefan-Boltzmann constant* with value 5.67×10^{-8} W/m²·K⁴. The factor ε, or the *emissivity*, falls between 0 and 1 and is characteristic of the surface of the radiating material.

Remark. A good absorber is *also* a good emitter. Black and dark objects are good emitters with $\varepsilon \approx 1$.

Example. Take an object of emissivity ε and area A at temperature T_1 surrounded by an environment at temperature T_2. Because both objects radiate energy to each other so the object absorbs energy proportional to T_2^4, the *net* rate of radiant heat flow from the object is given by $\frac{Q}{t} = \varepsilon \sigma A (T_1^4 - T_2^4)$.

Definition 56 (Solar Constant). About 1350 J of energy from the Sun strike Earth's atmospher per second per square meter at right angles to the Sun's rays. This number 1350 W/m² is called the solar constant. The atmosphere may absorb as much as 70% of this energy such that an object of emissivity ε with area A facing the Sun absorbs at a rate about $\frac{Q}{t} = (1000 \text{ W/m}^2)\varepsilon A \cos(\theta)$ where θ is the angle between the Sun's rays and a line perpendicular to the area A. Seasons result from this "effective

area" $\cos(\theta)$ capture.

Definition 57 (Thermography). Diagnostic *thermography* uses a thermograph to scan the body and measure the intensity of infrared radiation to detect areas of high metabolic activity.

Chapter 20

Second Law of Thermodynamics

20.1 Introduction

Definition 58 (Second Law of Thermodynamics - Clausius Statement). Heat can flow spontaneously from a hot object to a cold object; heat will not flow spontaneously from a cold object to a hot object.

Remark. Energy could be conserved in a broken cup putting itself back together as it rises, but this is not observed to happen.

20.2 Heat Engines

Definition 59 (Heat Engine). A *heat engine* is any device that changes thermal energy into mechanical work.

Note. Sign convention *only* for heat engines: $Q_H, Q_L, W > 0$. where Q_H heat is input at a high temperature T_H into a system that then transforms it partly into work W and partly as exhausted as heat Q_L at a lower temperature T_L. By conservation of energy, $Q_H = W + Q_L$.

Remark. Mechanical energy can only be obtained from thermal energy when heat is allowed to flow from a high to low temperature. If the temperature were constant throughout, pressure on the intake and outtake would be equal such that the work done by and on the gas would be equal so net work would be done.

Definition 60 (Operating Temperatures). The high and low temperatures, T_H, T_L are the *operating temperatures* of the engine.

Definition 61 (Working Substance)**.** The material, usually steam, that is heated and cooled is the *working substance*.

Definition 62 (Efficiency e of any Heat Engine)**.** The efficiency, e, of any heat engine can be defined as the ratio of work done, W to the heat input at the high termperature, Q_H, over a complete cycle: $e \coloneqq \frac{W}{Q_H}$. Equivalently, This is $e = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$.

Definition 63 (Carnot Engine)**.** An ideal engine, or *Carnot engine*, consists of 4 processes done in a cycle, 2 adiabatic and 2 isothermal done reversibly.

The isothermal processes, where heats Q_H and Q_L are transferred assume constant temperatures T_H and T_L such that the system is in contact with idealized *heat reservoirs* which are large enough to not fluctuate in temperature.

Definition 64 (Reversible Processes)**.** Processes done *reversibly* are considered to be carried out infinitely slowly so that the process is a series of equilibrium states so the whole process could be reversed with no change in the magnitude of work done or heat exchanged.

Remark. *Real processes* are always *irreversible*.

Remark. Suppose a Carnot engine uses an ideal gas. In the first isothermal process ab, $W_{ab} = nRT_H \ln \frac{V_b}{V_a}$. This is isothermal so E_{int} does not change and thus the heat, Q_H added equals the work done by the gas. So

$$Q_H = nRT_H \ln \frac{V_b}{V_a}.$$

Similarly, the heat lost in the isothermal process cd is

$$Q_L = nRT_L \ln \frac{V_c}{V_d}.$$

The paths bc, da are adiabatic so $P_b V_b^{\gamma} = P_c V_c^{\gamma}$ and $P_d V_d^{\gamma} = P_a V_a^{\gamma}$ where $\gamma = \frac{C_P}{C_V}$. From the ideal gas law, $\frac{P_b V_b}{T_H} = \frac{P_c V_c}{T_L}$ and $\frac{P_d V_d}{T_L} = \frac{P_a V_a}{T_H}$. Together, this tells us $T_H V_b^{\gamma-1} = T_L V_c^{\gamma-1}$ and $T_L V_d^{\gamma-1} = T_H V_a^{\gamma-1}$. This implies $(\frac{V_b}{V_a})^{\gamma-1} = (\frac{V_c}{V_d})^{\gamma-1}$ so $\frac{V_b}{V_a} = \frac{V_c}{V_d}$. Substituting this into our original equations show

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \qquad \text{[Carnot Cycle]}$$

so

$$e_{ideal} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}.$$

Theorem 2 (Carnot's Theorem). For any ideal reversible engine with fixed input and exhaust temperatures T_H and T_L, the fundamental upper limit to the efficiency of any heat engine is $e = 1 - \frac{T_L}{T_H}$. In other words,

> All reversible engines operating between the same two constant temperatures T_H and T_L have the same efficiency. Any irreversible engine operating between the same two fixed temperatures will have an efficiency less than this.

Note. Real engines that are well designed reach 60 to 80% of the Carnot efficiency.

Definition 65 (Second Law of Thermodynamics - Kelvin-Planck Statement). No device is possible whose sole effect is to transform a given amount of heat completely into work.

Definition 66 (Otto Cycle). The operation of an automobile internal combustion engine can be approximated by a reversible cycle called the *Otto Cycle* wherein the input and exhaust tempeartures are *not* constant. Paths ab, cd are adiabatic while bc, da are isovolumetric.

Remark. For an ideal gas as working substance, the efficiency of an Otto cycle engine is:

$$e = 1 - (\frac{V_a}{V_b})^{1-\gamma},$$

where V_a/V_b is the *compression ratio*.

Explanation. Because the exchange takes place at constant volume,

$$Q_H = nC_V(T_c - T_b) \quad \text{and} \quad Q_L = nC_V(T_d - T_a).$$

Thus,

$$e = 1 - \frac{Q_L}{Q_H} = 1 - \left[\frac{T_d - T_a}{T_c - T_b}\right].$$

From the adibatic processes, $P_a V_a^\gamma = P_b V_b^\gamma$ and $P_c V_c^\gamma = P_d V_d^\gamma$. From ideal gas law then, $T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}$ and $T_c V_c^{\gamma-1} = T_d V_d^{\gamma-1}$. This gives efficiency

$$e = 1 - \left[\frac{(V_b/V_a)^{\gamma-1}(T_c - T_b)}{T_c - T_b}\right] = 1 - (\frac{V_b}{V_a})^{\gamma-1} = 1 - (\frac{V_a}{V_b})^{1-\gamma}.$$

20.3 Refrigerators, Air Conditioners, and Heat Pumps

Definition 67 (Perfect Refrigerator)**.** A *perfect refrigerator* is one in which no work is required to take heat from the low-temperature region to the high-temperature region. This is impossible

Remark. Refrigerators, air conditioners, and heat pumps do the reverse of a heat engine, transferring heat *out* of a cool environment int oa warm environment. Each does work W to remove heat Q_L from *inside* and exhaust heat Q_H *outside*. This is usually done by an electric motor which compresses a fluid.

Definition 68 (The Coefficient of Performance (COP))**.** The *coefficient of performance (COP)* of a refrigerator is defined as the ratio between the heat Q_L removed and the work W done so COP $= \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$. For an ideal refrigerator, COP$_\text{ideal} = \frac{T_H}{T_H - T_L}$.

Definition 69 (Heat Pump)**.** A *heat pump* does work W to take heat Q_L from outside to deliver heat Q_H inside. Thus its COP is $\frac{Q_H}{W}$. Typical heat pumps have COP ≈ 2.5 to 3.

Remark (SEER Rating)**.** Cooling devices typically have SEER (Seasonal Energy Efficiency Ratio) ratings defined as $\frac{\text{(heat removed in Btu)}}{\text{(electrical input in watt-hours)}}$.

Definition 70 (Entropy)**.** When an amount of heat Q is added to a system by a reversible process at constant temperature (K), the change in entropy S of the system is given by
$$\Delta S = \frac{Q}{T}.$$
If the temperature is not constant, $dS = \frac{dQ}{T}$ so

$$\Delta S = \int_a^b dS = \int_a^b \frac{dQ}{dT} \quad \text{[reversible process]}.$$

Definition 71 (State Variable)**.** Entropy depends only on the states a and b of the system. Thus, it is a state variable.

Remark (Important Remark)**.** The change in entropy is path-independent.

Remark. Any reversible cycle can be approximated as a series of Carnot cycles. More cycles leads to better approxumations. For each cycle, we can write $\Sigma \frac{Q}{T} = 0$ so that over the limit of infinitely many Carnot cycles,

we get, as an integral around a closed path:

$$\oint \frac{dQ}{T} = 0 \quad \text{[reversible paths]}.$$

Thus entropy for an irreversible can be calculated for a reversible process between the same two states.

Note. One form of the *third law of thermodynamics* states that as $T \to 0$, $S \to 0$.

20.4 Entropy and the Second Law of Thermodynamics

Remark. Given an isolated system of two objects at temperatures T_H and T_L, the objects eventually come to some intermediate tempearture T_M so that the heat Q_H lost equals $-Q$ and the heat Q_L gained by the colder object equal Q so the total change is $\Delta S = \Delta S_H + \Delta S_L = -\frac{Q}{T_{HM}} + \frac{Q}{T_{LM}}$ where T_{HM}, T_{LM} are the intermediate temperatures. $T_{HM} > T_{LM}$ at all times so $\Delta S > 0$. So the *total change* is positive. (While a reversible (ideal, impossible) process has $\Delta S = 0$.)

Definition 72 (General Statement - Second Law of Thermodynamics)**.** The entropy of an isolated system never increases. It either stays constant (reversible) or increases (irreversible processes).

The total entropy of any system plus that of its environment increases as a result of any natural process: $\Delta S = \Delta S_{sys} + \Delta S_{env} > 0$.

We can also say natural processes tend to move toward a state of greater disorder. Hence, entropy is called *time's arrow*.

20.5 Order to Disorder

Note. The entropy of a system can be considered a measure of the disorder of the system.

Example. A stone falling is ordered macroscopically because each molecule is moving at the same velocity in the same direction. However, its fall turns into thermal energy which is more random and disordered.

20.6 Unavailability of Energy; Heat Death

Remark. An important aspect of the 2nd Law of Thermodynamics is that in any natural processes, some energy becomes unavailable to do useful

work. As time goes on, energy is never lost, but is instead *degraded.*

Definition 73 (Heat Death)**.** The *heat death* of the universe is the prediction that all energy of the universe will degrade to thermal energy.

20.7   Statistical Interpretation of Entropy

Definition 74 (Micro/Macrostate)**.** The *microstate* of a system describes the position and velocity of every particle (or molecule) while the *macrostate* of a system describes the properties of a system like temperature, pressure, number of moles, etc.

Remark. A basic assumption behind the statistical approach is that each microstate is equally probable. And a great number of microstates can equal the same macrostate.

Remark. The most probable state is the one with the greatest entropy or greatest disorder and randomness. In other words, it corresponds to the most *microstates.*

Definition 75 (Thermodynamic Probability)**.** In consistency with the Clausius statement that $dS = dQ/dT$, we can say the entropy of a system in a (macro)state is $S = k \ln \mathscr{W}$ where $\mathscr{W}$ is the number of corresponding microstates, or the *thermodynamic probaility*, of the occurence of some state.

Definition 76 (Third Law of Thermodynamics)**.** It is not possible to reach absolute zero in any finite number of processes.

Remark. From $\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$, we can assign the value of the triple point of water $T_{tp} = 273.16K$ to the thermodynamic scale such taht $T = (273.16K) * (\frac{Q}{Q_{tp}})$ where Q, Q_{tp} are the magnitudes of the heats exchanged by a Carnot engine with such reservoirs. Thus, the thermodynamic scale is identical to the ideal gas scale as long as the ideal gas is valid.

Note. Because T_L of an engine is thus never 0, a 100% efficiency engine such that $e = 1 - \frac{T_L}{T_H} = 1$ is not possible.

Note. Mechanical energy is transformed to electric energy with a turbine and generator.

Definition 77 (Thermal Pollution)**.** The heat Q_L output from every heat engine is referred to as *thermal pollution* because the environment must absorb it.

Chapter 21

Electric Charge and Electric Field

21.1 Static Electricity, Electric Charge and Conservation

Definition 78 (Law of Conservation of Electric Charge). The net amount of electric charge produced in any process is 0.

Definition 79 (Free/Conduction Electrons). In a good (metal) conductor, some *free/conduction electrons* are bound very loosely and can move about freely inside.

When a positively charged object is brought close, these electrons move quickly toward it. And swiftly away from a negatively charged object.

There are fewer free electrons in a semiconductor, and almost none in an insulator.

21.2 Induced Charge

Definition 80 (Charging by Conduction). Take a positively charged object A and neutral object B. If A touches B, electrons from B will pass to A such that when A leaves, B will have a net positive charge. This is *charging by conduction.*

Definition 81 (Charging by Induction). If a positively charged object simply approaches a neutral metal rod but doesn't touch it, a charge is *induced* at either end.

Definition 82 (Grounded). Because the Earth is so large and can conduct, it easily accepts and gives up electrons causing any object connected to it to be *grounded*.

Definition 83 (Electroscope). An *electroscope* is a device used to detect charge. A knob on the electroscope charges 1 or 2 gold leaves which are insulated with glass. The leaves separate when a positive object is brought close as electrons in the leaves go to the charge making the leaves positive so they repel each other.

If instead, the knob is charged by touching/conduction, the whole apparatus acquires a net charge which separates the leaves.

Note the electroscope doesn't tell you the sign of the charge unless you charged it first by conduction.

Modern sensitive electroscopes are called electrometers.

21.3   Coulomb's Law

Definition 84 (Coulomb's Law). The force between 2 charged particles is

$$F = -k\frac{Q_1 Q_2}{r^2} \quad \text{where}$$

$$k = 8.988 \times 10^9 \, \text{Nm}^2/\text{C}^2 \quad \text{and C is the coulomb.}$$

In vector form, with unit vector $\hat{r_{21}}$, we can write this as

$$\vec{F_{12}} = k\frac{Q_1 Q_2}{r_{21}^2}\hat{r_{21}}.$$

Definition 85 (Elementary Charge). The smallest charge observed in nature, the elementary charge, is denoted as e and equal to 1.6022×10^{-19} C.

Note this is positive so the charge on the electron is $-e$.

Definition 86 (Quantized). Charge cannot be created or destroyed or split into partial charges (except for quarks) so electric charge is *quantized* such that it exists only in discrete amounts like $1e, 2e, 3e$, etc.

Remark (Permittivity of Free Space). The constant k in Coulomb's Law is actually often written in terms of the *permittivity of free space* where $k = \frac{1}{4\pi\varepsilon_0}$ and $\varepsilon_0 = 8.85 \times 10^{-12} \, \text{C}^2/\text{Nm}^2$.

Remark. These laws only apply for objects whose size are much smaller

than the distance between them. Ideally, like point charges. For finite-sized objects, charge may not be distributed uniformly and r values may not be clear.

Definition 87 (Electrostatic/Coulomb Force)**.** The *electrostatic* or *Coulomb force* is when one leectrically charged particle exerts a force on a second one. When charges are all at rest, this is the study of *electrostatics*.

Definition 88 (Principle of Superposition)**.** With several charges, the net force is the vector sum of the forces due to each charge. This sum relies on the pricniple of superposition and is based on experiment.

21.4   Electric Field

Definition 89 (Electric Field)**.** The electric field $\vec{E}$ defined at any point in space is defined as $\vec{E} = \frac{\vec{F}}{q}$ as $q \to 0$. Here q is a test charge whose impact on the rest of the field is negligible.

E ahs magnitude $k\frac{Q}{r^2}$.

Note. The superposition principle also applies to electric fields.

Remark. We can split a charge distribution into infinitesimal charges dQ which will act on a tiny point charge at a distance r such that $dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}$ which shows that $\vec{E} = \int d\vec{E}$.

Example. A thin ring of radius a holds a uniform total charge of $+Q$. Its center is x away from a point P. Thus, any point charge dQ along the ring is $\sqrt{a^2 + x^2}$ away from point P. Clearly, any electric field force perpendicular to the line of symmetry through the center of the ring will cancel out due to the ring's symmetry. So, the net force, and net field then, is only along and away from the line of symmetry. Thus, $dE = k\frac{dQ}{a^2+x^2}\cos(\theta) = k\frac{dQ}{a^2+x^2}\frac{x}{\sqrt{a^2+x^2}}$. Suppose a segment $d\ell$ of the ring has point charge dQ with distribution λ. Thus $E = \int dE = \frac{\lambda}{4\pi\varepsilon_0}\frac{x}{(x^2+a^2)^{\frac{3}{2}}}\int_0^{2\pi a} d\ell = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x(2\pi a)}{(x^2+a^2)^{\frac{3}{2}}}$.
This comes out to just

$$\frac{1}{4\pi\varepsilon}\frac{Qx}{(x^2+a^2)^{\frac{3}{2}}}.$$

This makes sense as $x \gg a$, the field comes out to just the formula for if the ring were a point charge.

Example. Take a very long straight wire of uniformly distributed positive charge with charge per length λ. Take a point P a distance x from its

midpoint. Clearly, $dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dy}{x^2+y^2}$ where the wire lies along the y axis. Again, the y-component of the field will cancel from both sides so only $\cos(\theta)$ will be summed giving $E = E_x = \int dE\cos(\theta) = \frac{\lambda}{4\pi\varepsilon_0}\int\frac{\cos\theta dy}{x^2+y^2}$. Note that we can write $y = x\tan\theta$ so $dy = xd\theta/\cos^2\theta$ giving $\frac{1}{x^2+y^2} = \frac{\cos^2\theta}{x^2}$ so

$$E = \frac{\lambda}{4\pi\varepsilon_0 x}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}$$

.

Example. Take a thin circular disk of radius R with a uniformly distributed charge per unit area σ. Take a point P z above the disk's center. Everything not in the z-axis cancels out so $dE = \frac{1}{4\pi\varepsilon_0}\frac{zdQ}{(z^2+r^2)^{\frac{3}{2}}}$ for each ring of radius r from the 1st example. Clearly, the ring has area $(dr)(2\pi r)$ so $dQ = \sigma 2\pi rdr$ giving $dE = \frac{z\sigma rdr}{2\varepsilon_0(z^2+r^2)^{\frac{3}{2}}}$. Now, integrating over all the rings from 0 to R gives

$$E = \frac{z\sigma}{2\varepsilon_0}\int_0^R\frac{rdr}{(z^2+r^2)^{\frac{3}{2}}} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{\frac{1}{2}}}\right].$$

As $z \ll R$, $E = \frac{\sigma}{2\varepsilon_0}$ which approximates a point over an infinite plane of charge.

21.5   Field Lines

Definition 90 (Lines of Force). Electric field lines or *lines of force* are drawn to indicate the direction of force due to a given field on a positive test charge.

Lines starting on a positive charge terminate radially at negative charges.

Remark. The closer together lines are, the stronger the electric field is in that region. In fact, fields lines can be drawn so the number of lines crossing unit area perpendicular to $\vec{E}$ is proportional to the magnitude of the electric field.

Definition 91 (Electric Dipole). An electric field created by 2 equal charges of opposite sign is an *electric dipole*.

Remark. In the central reguon between two closely spaced, oppositely charged, parallel plates, the electric field has the same magnitude at all points $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$.

Note. We can equivalently make gravitational field diagrams.

21.6   Electric Fields and Conductors

Remark. The electric field inside a conductor is 0 in the static situation.

Any net charge on a conductor distributes itself on the surface.

Remark. Imagein a neutral spherical metal shell with a positive charge fixed inside. There will be an induced charge on the inside of the shell surface. Electric fields from the positive charge will thus end at the shell inside. An induced charge will emerge at the surface of the shell given off new field lines radially. No field exists within the conducting shell itself.

Remark. The electric field is always perpendicular to the surface outside of a conductor.

However, the electric field outside a nonconductor does not necessarily make an angle of 90° to the surface.

Remark. Obviously, the force on a particle due to a field is $\vec{F} = q\vec{E}$.

21.7   Electric Dipoles

Definition 92 (Dipole Moment). Given an electric dipole formed by 2 equal charges Q of opposite sign separated by a distance ℓ, the quantity $p = Q\ell$ is called the *dipole moment*. $\vec{p}$ can be considered as a vector pointing from the negative to positive charge.

Definition 93 (Polar Molecules). Molecules with dipole moments are called *polar molecules*.

Remark. If a dipole of moment $p = Q\ell$ is placed in a uniform electric field $\vec{E}$ will result in a torque $\tau = QE\frac{\ell}{2}\sin\theta + QE\frac{\ell}{2}\sin\theta = pE\sin\theta$. That is,

$$\vec{\tau} = \vec{p} \times \vec{E}$$

will be exerted until the dipole is parralel to the field.

Thus, the work done on the dipole by the electric field to change the angle from θ_1 to θ_2 is $W = \int_{\theta_1}^{\theta_2} \tau d\theta$. In this case, $\tau = -pE\sin\theta$ because its direction is opposite to the direction of increasing θ. This gives $W = pE(\cos\theta_2 - cos\theta_1)$. If $U = 0$ when $\vec{p} \perp \vec{E}$, $U = -W = -pE\cos\theta = -\vec{p}\cdot\vec{E}$.

Definition 94 (Hydrogen Bond). When H^+ is involved, the weak bond it can make with a nearby negative charge is relatively strong and referred to as a *hydrogen bond*.

Chapter 22

Gauss's Law

22.1 Electric Flux

Definition 95 (Electric Flux). *Electric flux* is defined as the electric field passing through a given area.

Remark. For a uniform electric field $\vec{E}$ passing through an area AA, the electric flux is defined as $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos\theta$.

Remark. The total, or net, flux through a closed surface is given by $\Phi_E = \oint \vec{E} \cdot d\vec{A}$. A closed surface is one which fully encloses a volume.

Remark. We define *arbitrarily* that the direction of $d\vec{A}$ is *outward* from the enclosed volume. Thus, the flux entering the enclosed volume is negative and flux leaving is positive.

Definition 96 (Gauss's Law). $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ where ε_0 is the permittivity of free space from Coulomb's Law.

Note. This is valid for any such surface.

Remark. We can derive Coulomb's Law from this. Take an imaginary sphere around a point charge, $+Q$, such that $\frac{+Q}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A} = \oint E dA = E(4\pi r^2)$ so $E = \frac{Q}{4\pi\varepsilon_0 r^2}$.

Example. Take a thin spherical shell of radius r_0 which possesses a total net charge Q. Suppose our imaginary gaussian surface is a sphere of radius r,

$r > r_0$ concentric with our charged shell such that $\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q}{\varepsilon_0}$ so $E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$ $[r > r_0]$. Now looking inside the shell with an imaginary sphere of radius $r < r_0$, the electric field is again symmetric and perpendicular to the walls of the shell but with no enclosed charge so $\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = 0 \Rightarrow E = 0$.

Note. For a large plane nonconductor, $E = \sigma/2\varepsilon_0$ while for a conductor, it's $E = \sigma/\varepsilon_0$ because a field goes out both ends of a nonconductor but out only the surface of a conductor.

Chapter 23

Electric Potential

23.1 Electric Potential Energy

Definition 97 ((Electric) Potential)**.** *Electric potential*, or simply potential, is the electric potential energy per unit charge. It is denoted by the symbol V. Thus, a tiny positive test charge q in an electric field will have poential energy U_a at some arbitrary point a (relative to 0 potential energy) such that $V_a = \frac{U_a}{q}$.

Remark (Potential Difference)**.** Note that only differences in potential energy are measurable and hence physically meaningful. We call this quantity the *potential difference* so $V_{ba} = \Delta V = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}$.

Note. Volts and Voltage Electric potential is measured in *volts*, or $1\,\text{J/C}$. The potential difference is the *voltage*.

23.2 Electric Potential and Electric Field

Remark. Recall the difference in potential energy is given by $U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{\ell}$ where $d\vec{\ell}$ is an infitesimal displacement. Because $\vec{E} = \vec{F}/q$, we can rewrite this as $V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$.

Example. Take a charged conducting sphere of radius r_0. At a distance $r > r_0$ from the center, $E = k\frac{Q}{r^2}$ and points radially outward so $V_b - V_a$ for points r_a and r_b away from the center, with $d\ell = dr$ give $-\int_{r_a}^{r_b} \vec{E} \cdot d\vec{\ell} = -Qk\int_{r_a}^{r_b} \frac{dr}{r^2} = Qk(\frac{1}{r_b} - \frac{1}{r_a})$. If we assign the potential to be 0 at $r = \infty$, then $V = kQ/r$. At $r = r_0$, this just becomes $V = Qk/r_0$. And for within the conductor, $E = 0$ so between any point on the surface and inside the conductor gives 0 change so $V = Qk/r_0$.

Note. This equation $V = kQ/r$ is known as the *Coulomb potential* in regards to a point charge Q and $V = 0$ at $r = \infty$.

Definition 98 (Breakdown). When high voltages are present, air ionizes due to the high electric fields and any free electrons in the air can be accelerated to knock electrons out of O_2 and N_2 so a *breakdown* of air occurs.

Note. When high voltages are present, a glow known as *corona discharge* may be seen around sharp points. Lightning rods, with sharp tips, are intended to ionize the surrounding air.

23.3   Equipotential Lines and Surfaces

Definition 99 (Equipotential Lines/Surfaces). Electric potential can be represented by drawing *equitpotential lines* or, in 3 dimension, *equipotential surfaces*. That is, the potential difference between any two points on the surface is 0 so no work is required.

Note. An equipotential surface must be *perpendicular* to the electric field at any point.

Remark (Gradient of V). $dV = -\vec{E} \cdot d\vec{\ell} = -E_\ell d\ell$ so $E_\ell = -\frac{dV}{d\ell}$ where E_ℓ is the component of the electric field in the direction of the infitiesimal displacement $d\vec{\ell}$. This quantity is also known as the *gradient* of V so $\vec{E} = -\vec{\nabla}V = -(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z})V$. If $\vec{E}$ is written as a function of x, y, and z, then we let

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}.$$

Definition 100 (Electron volt eV). One electron volt, or eV, is defined as the energy aquired by moving a particle carrying a charge e^- througha potential difference of 1 V. This gives $(1.6022 \times 10^{-19}\,\text{C})(1.00\,\text{V}) = 1.6022 \times 10^{-19}\,\text{J}$.

Definition 101 (Supply/Signal Voltage). Batteries and wall sockets provide a steady *supply voltage*. On the other hand, a *signal voltage* carries information.

Definition 102 (Quanitzation Error). The difference between the original continuous analog signal and its digital approximation is called the *quantization error*, or loss. Reducing this loss requires bit depth or resolution (the number of bits for the voltage of each sample) and sampling rate (the

number of times per second the analog voltage is measured). This process is executed by a digital-to-analog converter, or DAC. External distortion is called *noise*. Thermal noise refers to the random mnotion of electrons.

Definition 103 (Oscilloscope). An *oscilloscope* is a device for amplifying, measuring, and visually displaying an electric signal as a function of time. For instance, an *electrocardiogram* (ECG or EKG) records the potential changes for a person's heart.

Chapter 24

Capacitance, Dielectrics, Electric Energy Storage

24.1 Capacitors

Definition 104 (Capacitor). A *capacitor, or sometimes a condenser,* is a device that can store electric charge and normally consists of two conducting objects. They store charge for later use and block surges of charge to protect circuits.

Note. Very tiny capcitors serve as memory for the bits of code in RAM.

Remark. A simple capacitor has a pair of parallel plates of area A separated by a distance d.

Definition 105 (Capacitance). The *capacitance, or constant of proportionality, C,* of the capacitor is measured in *farads (F)*. The amount of charge Q acquired by each plate, equal and opposite charges on each, acquires a potential difference V such that $Q = CV$. C is always postive.

Remark. Suppose the 2 conductor plates of a capacitor are separated by a vacuum of air and that the area $A \gg d$ so that the $\vec{E}$ is uniform and we ignore fringing at the edges. This implies that $E = \frac{Q}{\varepsilon_0 A}$. such that $V = \frac{Qd}{\varepsilon_0 A}$ so that

$$C = \frac{Q}{V} = \varepsilon_0 \frac{A}{d}. \quad \text{[parallel-plate capacitor]}$$

Note. Capacitors are superior to rechargable batteries because they can be recharged more than 10^5 times with no degradation.

> **Note.** The change in a capacitance results in an electric signal detected by a circuit.

> **Example.** A cylindirical capacitor consists of a small cylinder, or wire, or radius R_b surrounded by a coaxial cylindrical shell of inner radius R_a. Both wires are of length $\ell \gg R_a - R_b$. We charge the capacitor via a battery so the inner one has charge $+Q$, and the outer one $-Q$. The Electric field outside a long wire is directed radially outward and has magnitude $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$ where λ is the charge per unit length Q/ℓ. $V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell} = -\frac{Q}{2\pi\varepsilon_0\ell}\int_{R_a}^{R_b}\frac{dR}{R} = \frac{Q}{2\pi\varepsilon\ell}\ln\frac{R_a}{R_b}$. Thus, $C = \frac{2\pi\varepsilon_0\ell}{\ln(R_a/R_b)}$. Note that if the space between cylinders shrinks then $ln(R_a/R_b) = ln(1 + \Delta/R_b) \approx \Delta R/R_b$.

24.2   Capacitors in Series and Parallel

> **Definition 106** (Capacitors in Parallel). Capacitors which are not in sequence are in *parallel*. Because a battery of Voltage V is connected to points before and after them, the same potential difference exists across each of them. Each one acquires a charge given by $Q_i = C_i V$ so that $Q = \Sigma_i Q_i = \Sigma_i C_i V$. Thus, a single equivalent capacitor will have $C_{eq}V = \Sigma_i C_i V$ implies
> $$C_{eq} = \Sigma_i C_i. \quad \text{[parallel]}$$
> Thus, connecting capacitors in parallel increases the capacitance.

> **Definition 107** (Capcitors in Series). Capacitors can also be connected *in series*, or end to end. Thus, a charge $+Q$ flows from the battery to one plate of C_1, $-Q$ flows to a plate of C_n. The regions in between each capacitor are neutral so the net charge must be neutral so a positive charge cascades down each capacitor so it is the same. However, the total voltage increases across each capacitor so $V = \Sigma_i V_i$ so
> $$\frac{1}{C_{eq}} = \Sigma_i \frac{1}{C_i}. \quad \text{[series]}$$

24.3   Storage of Electric Energy

> **Remark.** A charged capacitor stores electric energy equal to the work done to charge it. The net effect of charging a capacitor is to remove charge from one plate and add it to the other. This takes time. And, the more charge already added makes it more difficult to move more charge because of the electric repulsion. The work needed to add a small amount of charge dQ against a potential difference V gives $dW = V\,dq$. Because $V = q/C$,

$W = \int_0^Q V\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}$. We can rewrite this also as

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV.$$

Definition 108 (Energy density u). Recall that the electric field $\vec{E}$ is nearly uniform between two plates and has potential difference $V = Ed$ such that $C = \frac{\varepsilon_0 A}{d}$. Thus $U = \frac{1}{2}\varepsilon_0 E^2 Ad$. This quantity Ad is the volume between the plates. When we divide both sides by Ad, we get $u =$ energy density $= \frac{\text{energy}}{\text{volume}} = \frac{1}{2}\varepsilon_0 E^2$.

Definition 109 (Dielectric). In most capacitors, there is an insulating sheet of material called a *dielectric* between the plates. This is useful because it does not break down as easily as air so higher voltages can be applied without charge passing the gap. Also, it allows the plates to be closer without touching increaseing capacitance. Lastly, it has been experimentally found that it can increase the capacitance by a factor K so that $C = KC_0$ where C_0 is the capacitance of a gap of air, or vacuum. $K > 1$ as we will see later.

Remark. We can also say $C = K\varepsilon_0\frac{A}{d}$.

Definition 110 (Permittivity). We can define a new quantity

$$\varepsilon = K\varepsilon_0$$

where ε is simply the *permittivity of a material*. So, $C = \varepsilon\frac{A}{d}$ and $u = \frac{1}{2}\varepsilon E^2$.

Remark. An electric field is also altered by a dielectric being inserted. The potential difference drops to $V = V_0/K$ such that $E_D = \frac{E_0}{K}$ and $E = E_D = \frac{V}{d} = \frac{V_0}{Kd}$.

The electric field is reduced because the dielectric has an induced charge opposite to each side of the capacitor. Thus, normal field lines may end on the dielectric meaning that the field within is less than in air. $E_D = E_0 - E_{ind} = \frac{E_0}{K}$ so $E_{ind} = E_0(1 - \frac{1}{K})$.

Definition 111 (Free Charge). Recall that $E_0 = \sigma/\varepsilon_0$ where $\sigma = Q/A$. Q is often the net charge on the conductor and called the *free charge*. Similarly, we define $E_{ind} = \sigma_{ind}/\varepsilon_0$ so $Q_{ind} = \sigma_{ind}A$ is often called the *bound charge* because it is on the insulator and not free to move. We can conclude with $\sigma_{ind} = \sigma(1 - \frac{1}{K})$ and $\mathbb{Q}_{ind} = Q(1 - \frac{1}{K})$.

Chapter 25

Electric Current and Resistance

25.1 Electric Cells and Batteries

Note. A potential difference is required to move charge in a conductor.

Remark. Batteries transform chemical energy into electrical energy.

Definition 112 (Electrodes/Electrolytes)**.** Two plates or rods made of dissimilar metals (one can be carbon) are *electrodes*. Electrodes are typically immersed in a solution or paste, such as a dilute acid, called an *electrolyte*.

The positive electrode is the *anode* while the negative is the *cathode*. A cell dies when an electrode is exhausted.

Definition 113 (Electric Cell/Battery)**.** A device of electrodes connected by an electrolyte is called an *electric cell*. Several connected cells form a battery.

Definition 114 (Terminal)**.** The part of each electrode outside the solution is called the *terminal*. Connections to wires and circuits are made here. Voltage depends on what the electrodes are nade of and their ability to be dissolved or give up electrons.

Remark. When a positive terminal of one cell is connected to the negative of another, they are in *series* and their voltages are simply added.

25.2 Electric Current

Definition 115 (Electric Circuit). An *electric circuit* is a continuous conducting path between an electrical device and the terminals of a battery. Any flow of charge from one terminal to another is *electric current*.

Remark. The average current, or net amount of charge passing through the wire's full cross section at any point in time is $\bar{I} = \frac{\Delta Q}{\Delta t}$ while its instantaneous current is $I = \frac{dQ}{dt}$. Current is measured in *amperes* where $1\,\mathrm{A} = 1\,\mathrm{C/s}$.

Definition 116 (Complete/Open Circuit). A continuous conducting path is a *complete circuit*. If a break, like a cut wire, exists in the circuit, no current flows and the circuit is *open*.

Definition 117 (Ground). In real circuits, wires are connected to a common conductor which provides continuity called the *ground*.

Remark (Conventional Current). When we refer to current direction, we mean the direction *positive charge* would flow. However, electron flow is negative positive flow. In fluids, both negatives and positives can move.

25.3   Ohm's Law

Definition 118 (Electrical Resistance). *Electrical Resistance* is the proportionality factor between the voltage V and the current I. Resistance uses units Ω, or ohms, such that $1.0\,\Omega$ is equal to $1.0\,\mathrm{V/A}$.

Definition 119 (Ohm's Law). Experiimentally, in metal conductors, R is a constant independent of V such that $V = IR$.

Definition 120 (Resistors). *Resistors* are components in electronic devices which are used to control the amount of current.

Definition 121 (Potential/Voltage Drop). A *potential/voltage drop* is a an electric potential decrease between two points in a circuit.

Definition 122 (Resistivity). Experimentally, it's been found that a wire's resistance is directly proportional to its length ℓ and inversely proportional to its cross-sectional area A such that $R = \rho\frac{\ell}{A}$ where ρ is the constant of proportionality, or *resistivity*, and depends on the material used.

Remark. The resistivity of a material generally incrases with temperature

as atoms are moving more rapidly and interfere with the flow of electrons such that $\rho_T = \rho_0[1+\alpha(T-T_0)]$ where ρ_0 is the resistivity at some reference temperature T_0 and α is the *temperature coefficient of resistivity*

25.4   Electric Power

Note. Motors transform electric energy into mechanical energy. High resistance appliances turn electric energy into thermal energy through collisions.

Definition 123 (Power). The power P is the rate of energy transformed dU per time dt however $dU = Vq$ so $P = \frac{dq}{dt}V = IV$. Power is measured in watts where $1\,\text{W} = 1\,\text{J/s}$.

Definition 124 (Fuses/Circuit Breakers). If current is large enough, wires will heat up and produce thermal energy at a rate equal to I^2R. Thicker wires have less resistance and can carry more current without becoming too hot. When a wire carries more current than is safe, it is "overloaded." *Fuses* or *circuit breakers* are installed to prevent this by opening the circuit at a certain value.

Definition 125 (Short Circuit). A *short circuit* implies 2 wires have touched that should not have because insulation has worn through.

25.5   Alternating Current

Definition 126 (Direct Current (DC)). *Direct current* is when a battery is connected to a circuit and the current moves steadily one direction.

Definition 127 (Alternating Current (AC)). On the other hand, electric generators adal. t power plants produce *alternating current* which reverses direction many times per second and is commonly sinusoidal. Almost all electricty provided in the world is AC.

Definition 128 (Peak Voltage). AC electric generator voltage is given as a function of time such that $V = V_0\sin(2\pi ft) = V_0\sin(\omega t)$ where $\omega = 2\pi f$ and V oscillates between $\pm V_0$, also known as the *peak voltage*.

This similarly gives $I = I_0\sin(\omega t)$ where I_0 is the *peak current*. From here, we get $P = I^2R = I_0^2 R\sin^2(\omega t)$ such that the power is always positive and has average value $\frac{1}{2}$ so $\bar{P} = \frac{1}{2}I_0^2 R. = \frac{1}{2}\frac{V_0^2}{R}$.

We can also get

$$I_{rms} = \sqrt{\bar{I^2}} = \frac{I_0}{\sqrt{2}} \quad V_{rms} = \sqrt{\bar{V^2}} = \frac{V_0}{\sqrt{2}}.$$

These are sometimes called the *effective values* and are useful because

$$\bar{P} = I_{rms}V_{rms} = I_{rms}^2 R = \frac{1}{2}\frac{V_0^2}{R} = \frac{V_{rms}^2}{R}.$$

25.6   Microscopic View of Electric Current

Remark. When a potential difference is applied to 2 ends of a wire of uniform cross section, the direction of the electric field $\vec{E}$ is parallel to the walls of the wire. (Because we are no longer dealing with the static case, $\vec{E}$ does no have to equal 0 so) charges are free to move and thus move under the action of the field.

Definition 129 (Current Density)**.** The wire carries a *current density* which points hwerever a positive charge would move at that point (i.e. it is generally the same as $\vec{E}$). $\vec{j} = I/A$ if the current density is uniform. If not, the general relation is $I = \int \vec{j} \cdot d\vec{A}$ where I is the current through the whole surface.

Definition 130 (Drift Speed)**.** Although electrons feel a force from the wire's electric field and begin to accelerate, they soon reach a steady average *drift speed* v_d. Collisions with atoms keep them from accelerating further.

Note. For clarity, electrons flow from the negative to positive terminal.

Remark. If there are n free electrons, each of charge $-e$ per unit volume, then the total charge that passes through an area A in a time Δt is $\Delta Q = (nV)(-e) = -(nAv_d\Delta t)(e)$. This implies the wire's current is $-neAv_d$ so the current density is $j = -nev_d$ implying the direction of positive current flow is opposite to the drift speed of the electrons.

If there exists several types of ions including free electrons, each of density n_i and charge q_i and drift speed v_{di} then the net current density at any point is $j = \Sigma_i n_i q_i v_{di}$ or $I = \Sigma_i n_i q_i v_{di} A$.

Remark (Conductivity)**.** From $R = \rho\frac{\ell}{A}$, we can write $I = jA$ and $V = E\ell$ such that $E\ell = (jA)(\rho\frac{\ell}{A}) = j\rho\ell = \frac{1}{\rho}E = \sigma E$ where $\sigma = \frac{1}{\rho}$ is the *conductivity* for a metal conductor. We can write the microscopic statement of Ohm's law, then, as $\vec{j} = \sigma\vec{E} = \frac{1}{\rho}\vec{E}$.

25.7   Superconductivity

Definition 131 (Superconducting)**.** Metal and certain compounds of alloys which reach 0 resistivity at very low temperatures are called *superconduc-*

tors. They become superconducting at a certain *transition temperature* or *critical temperature* T_C which is generally within a few degrees of absolute zero.

Remark (Dipole Layer). The potential difference across the cell membrane, or *dipole layer*l results from neurons having a positive charge on the outer surface and negative on the inner surface. When a neuron is not transmitting, this is the *resting potential* normally stated as $V_{inside} - V_{outside}$. The voltage pulse is known as the *action potential*.

Chapter 26

DC Circuits

26.1 EMF and Terminal Voltage

Remark (Electromotive Force (emf)). To have current in an electric circuit, we need a device to transofrm one type of energy into electric. This is called a *source of electromotive force* or *emf*, denoted $\mathscr{E}$. The potential difference between the terminals of such a source, when no current flows to an external circuit, is called the emf.

Note. Voltage dims over time because chemical reactions cannot supply charge fast enough to maintain the full emf.

Definition 132 (Internal Resistance). A battery itself has some *internal resistance* r.

Definition 133 (Terminal Voltage). We measure the *terminal voltage* $V_{ab} = V_a - V_b$ between the two terminals of a battery. $V_a b = \mathscr{E}$.

26.2 Resistors in Series/Parallel

Remark (Resistors in Series). When resistors are connected in *series*, the voltage drops across each. However, due to conservation of charge, their current remains the same. Also, because of conservation of energy, the total voltage drop must equal the voltage of the battery. So, $V = \Sigma_i V_i \Rightarrow \frac{V}{I} = \Sigma_i \frac{V_i}{I}$ finally implying

$$R_{eq} = \Sigma_i R_i \quad \text{[series]}.$$

Remark (Resistors in Parallel). In parallel, however, the current diverges

across each pathway although each path encounters the same potential difference. Thus, $\frac{V}{R_{eq}} = I = \Sigma_i I_i = \Sigma_i \frac{V}{R_i}$. Dividing by V gives us

$$R_{eq} = (\Sigma_i \frac{1}{R_i})^{-1} \quad \text{[parallel]}.$$

Remark. We can perform a sanity check with $R = \rho \frac{l}{A}$ such that adding resistors in series adds length increasing the resistance while adding them in in parallel increases the area of resistance current flows through, reducing the overall resistance.

26.3 Kirchhoff's Rules

Remark. To make the process of solving for currents systematic, Kirchhoff developed 2 laws and a series of steps.

Definition 134 (Kirchhoff's First Rule/Junction Rule)**.** Using the conservation of electric charge, *at any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction.*

Definition 135 (Kirchhoff's Second Rule/Loop Rule)**.** Using the conservation of energy, *the sum of the changes in potential around any closed loop of a circuit must be zero.*

Remark (Kirchhoff's Process)**.** We can use the following steps to solve circuits, i.e. determine each current and direction along every pathway:

(Step 1) Label the current in each separate branch of the circuit using different subscripts and choose an arrow direction.

(Step 2) Apply Kirchhoff's junction rule to create sum of current equations.

(Step 3) Apply Kirchhoff's loop rule on voltages with careful reference to resistor and current and potential difference direction. Apply enough loops to solve for all unknowns.

26.4 Charging a Battery

Definition 136 (Battery Charger)**.** A *battery charger* works precisely by connecting batteries in reverse forcing charge through one terminal back into a battery.

Note. The negative (-) terminal is ground or "neutral" around 0 V and the "hot" terminal is positive (+). Always connect the ground cable last

and disconnect it first.

26.5   RC Circuits

Definition 137 (RC Circuits). Circuits which include capacitors and resistors are called *RC circuits*. Notably, its current varies over time.

Remark (Capacitor Charging). After a switch S is closed, current immediately flows through the circuit out from the negative terminal through the resistor R and accumulate on the upper plate of the capacitor, increasing its potential difference, reducing the current due to opposing voltage. Eventually, the voltage across the capacitor will equal the emf of the battery $\mathscr{E}$ at which point no further current flows and there is no potential difference across the resistor. From $V_C = Q/C$, $\mathscr{E} = IR + \frac{Q}{C} = R\frac{dQ}{dt} + \frac{1}{C}Q$. This gives $\frac{dQ}{C\mathscr{E} - Q} = \frac{dt}{RC}$ which we can integrate from $0 to Q$ and $0 to t$ such that $ln(1 - \frac{Q}{C\mathscr{E}}) = -\frac{t}{RC}$. Taking the exponential finally gives us $1 - \frac{Q}{C\mathscr{E}} = e^{-t/RC}$ so

$$Q = C\mathscr{E}(1 - e^{-t/RC}) = Q_0(1 - e^{-t/RC})$$

where $Q_0 = C\mathscr{E}$ represents the maximum charge on the capacitor (and notably *not* the Q at $t = 0$). Thus, the potential difference across the capacitor is also

$$V_C = Q/C = \mathscr{E}(1 - e^{-t/RC}).$$

Thus, it's evident that after a very long time, $V_C = \mathscr{E}$.

The quantity RC is called the *time constant τ of the circuit*. This represents the time required for the current to drop to $1/e \approx 0.37$ of its initial value. In fact, the current I through the circuit at any time t is finally equal to

$$I = \frac{dQ}{dt} = \frac{\mathscr{E}}{R}e^{-t/RC} = \frac{\mathscr{E}}{R}e^{-t/\tau}.$$

where $I_0 = \mathscr{E}/R$.

Remark (Capacitor Discharging). When the switch S is closed again, charge begins to flow through resistor R from one side of the capacitor toward the other until fully discharged. At any instant, the voltage across the capacitor equals that across the resistor so $\frac{Q}{C} = IR = -\frac{dQ}{dt}$ so $dQ/Q = -dt/RC$ so integrating and exponentiating gives

$$Q = Q_0 e^{-t/RC} \quad V_C = Q/C \quad \text{gives} \quad V_C = V_0 e^{-t/RC} \quad (V_0 = Q_0/C).$$

Thus, the charge decreases exponentially so $I = -\frac{dQ}{dt} = \frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC}$.

Definition 138 (Sawtooth Voltage). In some instances, like with a gas-filled tube with electrical breakdown, the voltage stored on a capacitor is discharged very rapidly after charging slowly. This produces a *sawtooth*

> *voltage.*

> **Note.** One danger is *leakage current* wherein current travels an unintended path.

26.6 Ammeters and Voltmeters

Definition 139 (Ammeter/Voltmeter)**.** An *ammeter* measures current while a *voltmeter* measures potential difference between points. Either one can be analog displaying by position on a scale or digital.

Definition 140 (Galvanometer)**.** Analog reading is done by a *galvanometer*. A galvanometer works via the force between a magnetic field and a current-carrying coil of wire. The deflection of the detecting needle is proportional to the current flowing through it.

Definition 141 (Full-scale Current Sensitivity)**.** The *full-scale current sensitivity* of a galvanometer I_m is the electric current needed to make the needle deflect full scale.

Definition 142 (Shunt Resistor)**.** An analog ammeter consists of a galvanometer in parallel with a resistor called a *shunt resistor R_{sh}*. Here "shunt" implies "in parallel." The resistance of the galvanometer coil is r. R_{sh} is normally very small so the ammeter has very small net resistance and most of the current passes through it rather than the galvanometer.

Similarly, an analog voltmeter consists of a galvanometer and a resistor R_{ser} connected in series. R_{ser} is usually large to generate a high internal resistance.

Definition 143 (Other Meters)**.** A *multimeter* can measure voltage, current, and resistance. It is thus sometimes called a VOM(Volt-Ohm-Meter).

An *ohmmeter* measures resistance and must contain a battery of known voltage connected in series to a resistor R_{ser} as well as an ammeter which has a shunt R_{sh}. Here, the needle deflection is inversely proportional to the resistance. This best not be used on very delicate devices because it sends a current through, itself.

> **Note.** Digital meters are used similarly but are constructed extremely differently. They use semiconductor devices.

Chapter 27

Magnetism

27.1 Magnets and Magnetic Fields

Definition 144 (Poles). *Poles* are where the magnetic effect is strongest. A freely suspended magnet that points towards north is the north pole and one that points south is called the south pole. Like poles repel and unlike attract. *True north* is different from geographic north pole. The angular difference is called *magnetic declination.*

Note. No *magnetic monopole* has ever been observed. Cutting produces new poles.

Definition 145 (Ferromagnetic). Materials which exhibit strong magnetic effects are *ferromagnetic.*

Remark. We can illustrate a magnetic field with field lines so the field direction is tangent to a field at any point and the number of lines per unit area is proportional to the strength of the field.

Magnetic field lines always form closed loops, unlike electric field lines.

Field lines fringe at edges of two wide poles.

Note (Angle of Dip). The *angle of dip* is the angle the Earth's magnetic field makes with the horizontal at any point.

27.2 Force on an Electric Current in a Magnetic Field

Definition 146 (Magnetic Field Force on Current). Given magnetic field $\vec{B}$,

the force on a wire carrying current I with current direction $\vec{\ell}$ is equal to

$$\vec{F} = I\vec{\ell} \times \vec{B}.$$

If $\vec{B}$ is not uniform, then we can more generally write $d\vec{F} = Id\vec{\ell} \times \vec{B}$ and integrate.

Note that $\otimes$ represents a field going into a page while $\odot$ represents a field coming out. (X marks the spot).

Note (Tesla). The SI unit for magnetic field B is the *tesla* T so 1 T $=$ 1 N/A. The *gauus* (G) is also used where $1\,\text{G}=1 \times 10^{-4}\,\text{T}$.

Remark. If N particles of charge q pass by a given point in time t and travel a distance ℓ this constitutes a current $I = Nq/t$ such that $\vec{\ell} = \vec{v}t$ where $\vec{v}$ is the velocity of each particle. Therefore the force on the N particles if $\vec{F} = I\vec{\ell} \times \vec{B} = (Nq/t)(\vec{v}t) \times \vec{B} = Nq\vec{v} \times \vec{B}$ so each particle experiences a force $\vec{F} = q\vec{v} \times \vec{B} = qvB\sin\theta$.

Remark (Cyclotron Frequency). The time T required for a particle of charge q moving with constant speed v to make one circular revolution is $T = 2\pi r/v = \frac{2\pi m}{qB}$. We call $f = \frac{1}{T} = \frac{qB}{2\pi m}$ the *cyclotron frequency*.

Note. The *aurora borealis* represents charged ions approaching the Earth from the sun and being increasingly affected by the magnetic field.

27.3   Lorentz Equation

Remark (Lorentz Equation). For a particle of charge q moving with velocity $\vec{v}$ in the presence of both a magnetic field $\vec{B}$ and electric field $\vec{E}$,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}).$$

This is the Lorentz equation.

27.4   Torque on Current Loop & Magnetic Dipole

Remark. Suppose there exists a rectangular loop of current I with sides a perpendicular to the field and sides b parallel to the field. The loop can spin about the axis lying inbetween and parallel to both sides of length a. Thus, the loop will experience net torque $2*a*I*B*b/2 = IabB = IAB$ where I is the area of the loop. For multiple loops, this becomes $\tau NIAB$. We call the quantity $\vec{\mu} = NI\vec{A}$ the *magnetic dipole moment*. Where $\vec{A}$

points in the direction determined by current and righthand rule. Thus,

$$\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}.$$

Remark. Defining $U = 0$ when the loop is perpendicular to the field gives $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$.

Definition 147 (Galvanometer). A *galvanometer* consists of a coil of N loops of wire with an attached pointer suspended in the magnetic field of a permanent magnet. Thus, the torque on the loop is $\tau = NIAB\sin\theta$. This torque is opposed by a spring which has torque $\tau_s = k\phi$ where ϕ is the angle through which it is turned and θ is the angle of the loop relative to the field lines. This implies $\phi = \frac{NIAB\sin\theta}{k}$. To prevent the measured angle θ from depending on θ and instead only depending on I, a useful meter wraps a galvaonmeter coil around a cylindrical iron core to concentrate the field lines.

Definition 148 (Electric Motor). An *electric motor* converts electric energy into mechanical energy. A current-carrying coil of wire in a magnetic field must turn continuously in one direction and is mounted on an iron cylinder called the *rotor* or *armature*. This armature is mounted on a shaft or axle. Because a coil would only rotate past its equilibrium point and then rotate back the opposite way, a current switch is necessary to turn continuously. This can be achieved with a *dc motor* with the use of *commutators and stationary brushes*. On the other hand, an *ac motor* can work withour commutators as its current alternates independently.

Definition 149 (Cathode Rays). *Cathode rays* are accelerated via a high voltage and pass between a pair of parallel plates built into a cathode ray tube. When another voltage is applied to the plates, an electric field $\vec{E}$ is produced and a pair of coil produce magnetic field $\vec{B}$. Changing the amount of each field deflects the cathode rays up or down. The force on the rays is $F = evB$. Without an electric field, the rays are bent into a curved path such that $evB = m\frac{v^2}{r}$ so $e/m = v/Br$ which is also equal to $E/B^2 r$. These righthandside quantities can all be measured allowing e/m to be determined as *electrons*, or "carriers of electricity." A cathode ray tube applies this high voltage across a small volume of gas in a tube producing a glow at one end.

Remark (Oil-Drop Experiment). The *oil-drop experiment* of Robert A. Millikan yielded a precise value for the electron's charge. Tiny droplets of mineral oil carrying an electric charge were allowed to fall under gravity between two parallel plates. The electric field E between the plates was adjusted until the drop was suspended in midair implying $qE = mg$ so $q = mg/E$ which could be fully measured, giving us both the charge and mass of an electron.

Definition 150 (Quantized Charge)**.** Experimental results suggest charges
are all integral multiples of e implying electric charge is *quantized*, or exists
only in discrete amounts.

Remark (The Hall Effect)**.** Take a current-carrying wire of diameter d that's
held fixed in a magnetic field into the wire from the viewer's perspective.
As electrons move right, the magnetic field pushes the electrons downward
as $\vec{F_b} = -e\vec{v_d} \times \vec{B}$. This creates a greater density of electrons towards
the bottom which creates a potential difference that provides an electric
field $\vec{E_H}$ that exerts a force $e\vec{E_H}$ on the moving charges equal and opposite
to the magnetic force. This is called the *Hall effect* and the potential,
$\mathscr{E}_H = E_H d = v_d B d$ is called the *Hall emf.*

Definition 151 (Mass Spectrometer)**.** A *mass spectrometer* is a device used
to measure the masses of atoms. Ions can be produced by heating a sample
or through a current. Such ions of mass m and charge q will be shot
through a slit S_1 and enter a region S_2 before another slit where there are
perpendicular electric and magnetic fields. To pass through straight, the
net force must be 0 so $qE = qvB$ implying $v = E/B$. Thus, only ions
of this speed will emerge through slit S_2 after which a magnetic field B'
is applied forcing the ions to follow a circular path or radius r. The field
is varied until a detector is triggered or a detector simply measures the
diameter of the path of the ion given the known magnetic field. Either
way, $qvB' = mv^2/r$ so $m = \frac{qB'r}{v} = \frac{qBB'r}{E}$.

Remark (Isotopes)**.** Using a mass spectrometer on some materials and rul-
ing out the possibility of impurities implied there must exists different types
of atoms with different masses called *isotopes.*

Chapter 28

Sources of a Magneetic Field

28.1 Magnetic Field Due to a Straight Wire

> **Definition 152** (Permeability of Free Space)**.** The proportionality constant of magentic field near to a long straight wire is $B = \frac{\mu_0}{2\pi} \frac{I}{r}$ where $\mu_0 = 4\pi \times 10^{-7} \cdot \text{m/A}$ is the *permeability of free space*.

> **Remark.** Given two wires carrying parallel currents I_1, I_2 in the same direction, the force exerted by one on the other is $F_2 = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} \ell_2$ where d is the distance between the wires and ℓ_2 is the length of the second wire. For parallel currents the force is attractive. For antiparallel currents, it is repulsive.

> **Remark.** Operational Definitions Because μ_0 is defined precisely as $4\pi \times 10^{-7} \cdot \text{m/A}$, we can redefine one *ampere* to be the current flowing in each of two long parallel wires exactly $1\,\text{m}$ apart which results in a force of exactly $2 \times 10^{-7}\,\text{N}$ per meter length of each wire. From here, we can define a *coulomb* via $1\,\text{C} = 1\,\text{A} \cdot \text{s}$.
>
> We use *operational definitions* in this way for purposes of reproducibility and precise measurements.

> **Remark** (Ampèré's Law)**.** To generalize results from a long straight wire to current with a path of any size and shape, we simply sum the parallel magnetic field component of each infitesimal segment such that $\sigma B_\parallel \Delta \ell = \mu_0 I_{encl}$ so, as infitesimal length vector $\Delta \ell \to 0$, we get
>
> $$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encl}. \quad [\text{Ampéré's Law}]$$

> **Example.** Applying this law in the case of our standard long straight wire

carrying a current I. Given a circle of radius r symmetric around the wire, all points experience the same field $\vec{B}$ and B will be parallel to each segment so $\mu_0 I = \oint B d\ell = B(2\pi r)$ so $B = \frac{\mu_0 I}{2\pi r}$.

Note. Note that this doesn't capture the exact total magnetic field, just from one source, unlike Gauss's Law.

28.2   Mangetic Field of a Solenoid and Toroid

Definition 153 (Solenoid). A long coil of wire consisting of many loops, or simply turns, carrying a dc current is called a *solenoid*. Between any two loops, the fields due to each loop tend to cancel. However, towards the center, the fields add up to give a large and uniform field, particularly for a long solenoid with closely packed loops. The field outside a solenoid is very small compared to the field inside, except near the ends.

Remark. Take a rectangular path overlayed one side of the solenoid with current into the page towards the bottom and out of the page at the top such that the magnetic field inside is uniformly to the right and negigible outside. Breaking this path loop into its 4 sides, we can ignore the side outside and the sides perpendicular to the field leaving just the side inside such that $\oint \vec{B} \cdot d\vec{\ell} = B\ell$ for side length inside ℓ. Therefore, this loop captures the current of $n = N/\ell$ loops per unit length such that

$$B = \mu_0 n I \quad \text{[solenoid]}.$$

Definition 154 (Toroid). A *toroid* is a solenoid bent into the shape of a circle. Applying this result for solenoids with Ampèré's Law gives $B(2\pi r) = \mu_0 N I$ where r is the radius from the center of the toroid to the center of the solenoid gives $B = \frac{\mu_0 N I}{2\pi r}$. For the net curent enclosed by a path outside the whole toroid, the net current enclosed is 0 so $B = 0$. Similarly, for a path taken at a radius smaller than the inner edge of the toroid, $I_{encl} = 0 \Rightarrow B = 0$.

28.3   Biot-Savart Law

Remark. The magnetic equivalent to Coulomb's Law, according to Biot and Savart, a current I flowing in any path can be considered as many infitesimal current elements such that the magnetic field due to a current $d\vec{\ell}$ flowing anywhere in space is in fact equal to

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{\mathbf{r}}}{r^2} \quad \text{[Biot-Savart Law]}$$

where $\hat{\mathbf{r}} = \vec{r}/r$ and r is the unit vector of $\vec{r}$ from $d\vec{\ell}$ to the point measured.

Note. An important distinction between the Biot-Savart's and Ampèré's laws is that the vector $\vec{B}$ in Ampèré's considers the field not just due to the current enclosed while Biot-Savart does.

Remark (Magnetic Dipole Field). A current-carrying coil has a magnetic dipole moment $\mu = NIA$ where A is the area of the coil. The magnetic field *produced* by a magnetic dipole has magnetic, along the dipole axis, of $B = \frac{\mu_0 IR^2}{2(R^2+x^2)^{\frac{3}{2}}}$ where $\mu = IA = I\pi R^2$ for a single loop $N = 1$.

Note. We cannot apply the Biot-Savart law to a moving charge because the current is not constant at each point.

Definition 155 (Ferromagnetic). Materials are said to be *ferromagnetic* if no electric current is needed to make a material magnetic.

Definition 156 (Domains). Microscopic examination reveals that a piece of iron is made up of tiny regions, or *domains* less than 1mm in length or width which behave like a tiny magnet with a north and south pole. In an unmagnetized piece of iron, these domains are arranged randomly and cancel. But, the domains are preferentially aligned in a magnet. Therefore, a metal can be made magnetic by placing it in a strong magnetic field which rotates the domains.

Remark (Curie Temperature). An iron magnet may remain magnetized for a long time, but dropping it on the floor or striking it with a hammer can jar domains and destroy magnetism, as can heating it. Above a certain *Curie temperature*, the domains become too random and no magnet can be made.

Soft iron loses its magnetism readily when current is turned on or off. However, *hard* iron maintains its magnetism even with no external field.

Remark (Electron Spin). It is the magnetic field due to electron *spin* that is believed to produce ferromagnetism. It is also thought all magnetic fields are caused by electric currents implying all magnetic field lines form closed loops, unlike electric ones.

28.4   Applications of Electromagnets

Definition 157 (Electromagnet). If a piece of iron is placed inside a solenoid, the magnetic field is increased greatly because the domains of the iron are aligned by the field produced by the current turning the iron into a magnetic. This resulting field is hte sum of the iron's field as well which

can be hundreds or thousands of times larger. An iron-core solenoid is called a *electromagnet*.

A large field requires a large current producing a lot of waste heat, but superconducting material kept below the transition temperature can produce very high magnetic fields with no electric power needed to maintain it. However, energy is required to refrigerate the coils at low temperatures.

The new field $\vec{B} = \vec{B}_0 + \vec{B}_M$ where $\vec{B}_M$ is the field due to the added magnetic material can replace the constant μ_0 with a new *magnetic permeability* μ characteristic of the material inside the coil. Note this is separate from the magnetic dipole moment $\vec{\mu}$.

Remark. A solenoid can also be modififed to have a moveable rod of iron partially inserted which, when current passes through, works as a switch.

Definition 158 (Saturation)**.** At *saturation*, nearly all the domains of a metal are aligned. If the current making it magnetic is now applied in the other direction, the metal's field will approach 0 and then saturation in the other direction. If the current is ever turned off, some domains will stay where they were.

Definition 159 (Hysteresis)**.** After first reaching saturation and then undergoing a reverse process, the field will never reach 0 net field and 0 field caused by the current in the coil again. Instead, it will loop around this point, following a *hysteresis loop*. In such a cycle, much energy is transformed to thermal energy due to realigning domains. In fact, the energy dissipated is proportional to the area of this $B_0 v.B$ curve.

Definition 160 (Retentivity)**.** Materials which keep a magnetized iron core despite no current passing through coils have high *retentivity*. For a permanent magnet, these have the highest retention.

28.5   Para/Diamagnetism

Definition 161 (Para/Diamagnetic)**.** All materials are somewhat magnetic. They are either *paramagnetic* or *diagmagnetic* in which case $\mu > \mu_0$ or $\mu < \mu_0$ respectively. This ratio $K_m = \frac{\mu}{\mu_0}$ is called the *relative permeability*. Moreover, $\chi_m = K_m - 1$ is called *magnetic susceptibility*.

Remark. Paragmagnetism may occur in materials with a permanent magnetic dipole moment.

Conversely, diamagnetic materials ahave no permanent magnetic dipole moment, only an induced one. All materials are somewhat diamagnetic.

Definition 162 (Magnetization Vector). A useful quantity is the *magnetization vector* $\vec{M} = \frac{\vec{\mu}}{V}$ where $\vec{\mu}$ is the magnetic dipole moment of the sample and V is its volume. Curie's Law states $M = C\frac{B}{T}$ where C is a constant. M will approach some max as B is increased or T is decreased. This corresponds to complete alignment of all permanent magnetic dipoles.

Chapter 29

Electromagnetic Introduction and Faraday's Law

29.1 Induced EMF

> **Definition 163.**