Page 91, Problem 1

Proof. Say $f\colon C \to C$ is a map that is NOT homeomorphic to the identity where C is the unit circle. Suppose also $f(x) \neq -x$ for all $x \in C$. Thus, no antipodal points exist (taking points as (x,y) with additive inverse (-x, -y)) so no straight line between the identity and $f(x)$ passes through the origin. Therefore, via example 2 on page 89, we can define the homotopy from f to i where $F\colon C \times I \to C$ as

$$F(x,t) = \frac{(1-t)f(x) + tx}{||(1-t)f(x) + tx||}.$$

Thus, $f(x) \underset{F}{\cong} i$ which is a contradiction so there exists some point $x \in C$ so $f(x) = -x$. □

Page 95, Problem 13

Proof. Diagrams and more detailed proof attached. For written explanation: F shows that α, β are homotopies to themselves. From this, I use F to construct two new homotopies from α, β to themselves called P, Q. These essentially squash β, α into e respectively and drag α around the square. Multiplying these maps PQ gives a homotopy from the concatanted loops $\alpha.\beta$ to the product in G of $\alpha(t).\beta(t)$. We can reverse this product to get QP for a homotopy from $\alpha.\beta$ to $\beta(t).\alpha(t)$ instead. Linking these together relative to $\{0,1\}$, $\langle\beta.\alpha\rangle = \langle\alpha.\beta\rangle$ which can be split up to show $\pi_1(G)$ is abelian for any 2 loops in G. □