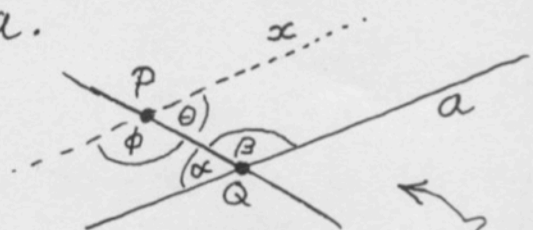
Solutions to Exercises in The Road to Reality

by Roger Penrose

Note: many readers may find different ways of solving these problems. The solutions given below merely represent a reasonably concise set of solutions that come to mind, and other possibilities, provided they are logically sound, which may occur to the reader may be just as acceptable.

[2.1] ☺ Let P be a point and a be a line, in a given plane, where P does not lie on a. Let Q be some point on a. Join P to Q and consider some other line x, in the plane, through P.



Let the angles $\theta, \phi, \alpha, \beta$ be as in the figure.

Now $\theta + \phi = 2$ right angles $= 180° = \pi$.
and $\beta + \alpha = 2$ right angles $= \pi$.

Adding these two equations: $(\theta+\beta) + (\phi+\alpha) = 2\pi$ (=360°)

Now if $\theta+\beta < \pi$, then (by Euclid's 5th) x and a meet (on the right). If $\theta+\beta > \pi$, then by equation above $\phi+\alpha < \pi$, so x and a meet (on the left). So the only possibility for x and a to be parallel is $\theta+\beta=\pi$. This gives a unique parallel to a through P ∴ Playfair