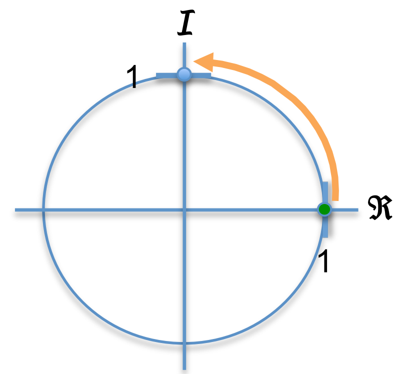
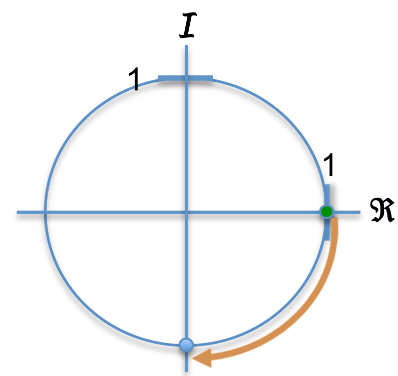
[9.7] Derive the expression

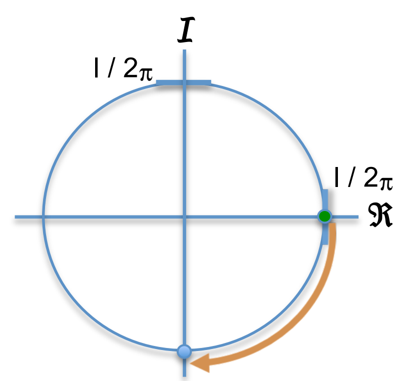
$$z = \frac{il}{2\pi}\left(e^{-i\chi} - 1\right) \quad \text{for } 0 \le \chi < 2\pi$$



1. $z = e^{i\chi}$ for $0 \le \chi < 2\pi$
   - Start with standard unit circle
     - $re^{i\theta}$ is equation of circle of radius r
     - Thus $e^{i\theta}$ is equation of unit circle
   - Center: Origin = (0, 0)
   - Starting point: z = 1 on real axis
   - Trajectory is Counter-clockwise (CCW) as χ goes from 0 to 2π



2. $z = e^{-i\chi}$ for $0 \le \chi < 2\pi$
   - Reverse the direction
   - Center: Origin = (0,0)
   - Starting point: z = 1
   - Clockwise (CW) trajectory

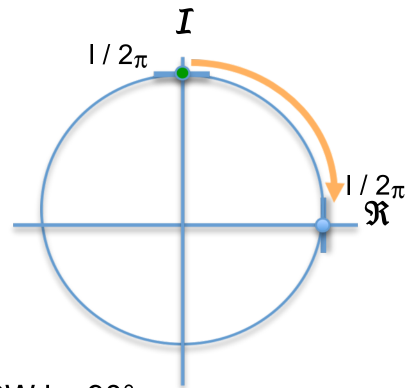


3. $z = \frac{l}{2\pi} e^{-i\chi}$ for $0 \le \chi < 2\pi$
   - Expand radius
   - Multiply by $\frac{l}{2\pi}$
   - Starting point: z = $\frac{l}{2\pi}$

4. $z = \frac{i\,l}{2\pi} e^{-i\chi}$ for $0 \le \chi < 2\pi$

- Rotate circle CCW by 90°
- New starting point at z = l i / 2π on Imaginary axis
- $e^{i\theta}$ is equation of unit circle

  and i is at 90° on unit circle $\Rightarrow$ $i = e^{\frac{\pi}{2} i}$
- Equation is multiplied by i
- $e^{\frac{\pi}{2} i}$ shows points on circle are rotated CCW by 90°



5. $z = \frac{i\,l}{2\pi}\left(e^{-i\chi} - 1\right)$ for $0 \le \chi < 2\pi$

- Translate circle downward by l / 2π
- Simply subtract $\frac{l}{2\pi} i$:

  $$z = \frac{i\,l}{2\pi} e^{-i\chi} - \frac{i\,l}{2\pi} = \frac{i\,l}{2\pi}\left(e^{-i\chi} - 1\right)$$

- New center: z = $-\frac{l}{2\pi} i$
- New starting point: Origin
- Trajectory: CW