

$$[10.8] \begin{cases} \bar{X} = x \\ Y = x + y \end{cases}$$

$$\frac{\partial x}{\partial \bar{X}} = 1; \quad y = Y - x = Y - \bar{X} \Rightarrow \frac{\partial y}{\partial \bar{X}} = -1$$

$$\frac{\partial x}{\partial Y} = \frac{\partial \bar{X}}{\partial Y} = 0 \text{ because } \bar{X} \text{ is held constant during partial with } Y$$

$$\frac{\partial y}{\partial Y} = \frac{\partial Y}{\partial Y} - \frac{\partial \bar{X}}{\partial Y} = 1 \quad [\text{Note: } x = Y - y \Rightarrow \frac{\partial x}{\partial Y} = \frac{\partial Y}{\partial Y} - \frac{\partial y}{\partial Y} = 1 - 1 = 0] \checkmark$$

$$\text{So, } \frac{\partial \Phi}{\partial \bar{X}} = \frac{\partial \Phi}{\partial x} \frac{\partial x}{\partial \bar{X}} + \frac{\partial \Phi}{\partial y} \frac{\partial y}{\partial \bar{X}} = \frac{\partial \Phi}{\partial x} - \frac{\partial \Phi}{\partial y} \Rightarrow \boxed{\frac{\partial}{\partial \bar{X}} = \frac{\partial}{\partial x} - \frac{\partial}{\partial y}}$$

$$\frac{\partial \Phi}{\partial Y} = \frac{\partial \Phi}{\partial x} \frac{\partial x}{\partial Y} + \frac{\partial \Phi}{\partial y} \frac{\partial y}{\partial Y} = 0 + \frac{\partial \Phi}{\partial y} \Rightarrow \boxed{\frac{\partial}{\partial Y} = \frac{\partial}{\partial y}}$$