[11.10] Let $a = a_1\eta_1 + a_2\eta_2$ and $b = b_1\eta_1 + b_2\eta_2$.

The wedge product is defined as follows:

$\eta_1\wedge\eta_1 \equiv 0 \equiv \eta_2\wedge\eta_2$ $\eta_1\wedge\eta_2$ and $\eta_2\wedge\eta_1$ represent fundamental entities that are not further simplified (just as i, j, and k represent fundamental quaternion entities). They are related by $\eta_1\wedge\eta_2 = -\eta_2\wedge\eta_1$.

$a\wedge b$ is defined as

$$\begin{aligned} a\wedge b &\equiv a_1b_1\eta_1\wedge\eta_1 + a_1b_2\eta_1\wedge\eta_2 + a_2b_1\eta_2\wedge\eta_1 + a_2b_2\eta_2\wedge\eta_2 \\ &= a_1b_2\eta_1\wedge\eta_2 + a_2b_1\eta_2\wedge\eta_1 \\ &= (a_1b_2 - a_2b_1)\eta_1\wedge\eta_2 \end{aligned}$$

Finally, $a_{[p}b_{q]}$ is defined as $a_{[p}b_{q]} = \frac{1}{2}(a_pb_q - a_qb_p)$ for $p, q = 1, 2$.

In this problem I believe Penrose is asking us to verify that

$$a\wedge b = a_{[1}b_{1]}\eta_1\wedge\eta_1 + a_{[1}b_{2]}\eta_1\wedge\eta_2 + a_{[2}b_{1]}\eta_2\wedge\eta_1 + a_{[2}b_{2]}\eta_2\wedge\eta_2$$

Note that

$$\begin{aligned} &a_{[1}b_{1]}\cancelto{0}{\eta_1\wedge\eta_1} + a_{[1}b_{2]}\eta_1\wedge\eta_2 + a_{[2}b_{1]}\eta_2\wedge\eta_1 + a_{[2}b_{2]}\cancelto{0}{\eta_2\wedge\eta_2} \\ &= \tfrac{1}{2}(a_1b_2 - a_2b_1)\eta_1\wedge\eta_2 + \tfrac{1}{2}(a_2b_1 - a_1b_2)\eta_2\wedge\eta_1 \\ &= \tfrac{1}{2}(a_1b_2 - a_2b_1)\eta_1\wedge\eta_2 + \tfrac{1}{2}(a_1b_2 - a_2b_1)\eta_1\wedge\eta_2 \\ &= (a_1b_2 - a_2b_1)\eta_1\wedge\eta_2 \\ &= a\wedge b \quad \text{per definition above} \end{aligned}$$

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