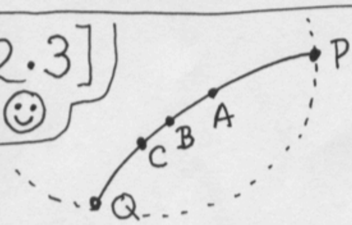
[2.2] It's all a matter of units. If we rescale the length-unit so that a previously measured distance x now comes out as λx, then a previously measured area Δ will now come out as $\lambda^2\Delta$. Thus, if we start with a length-unit giving $C=1$ (so the Lambert triangle-area formula is simply $\pi-(\alpha+\beta+\gamma)=\Delta$), then the choice $\lambda=\sqrt{C}$ will give us the more general-looking Lambert formula $\pi-(\alpha+\beta+\gamma)=C\Delta$. Accordingly, if we start with the length-unit for which the length (hyperbolic) from A to B is given by $\log\frac{QA\cdot PB}{PA\cdot QB}$, then with the rescaled length-unit, this formula gives us $\lambda\times$ the hyperbolic distance from A to B, i.e. it is $\frac{1}{\lambda}\times\log\frac{QA\cdot PB}{PA\cdot QB}$ that gives us this distance, i.e. $C^{-1/2}\log\frac{QA\cdot PB}{PA\cdot QB}$.

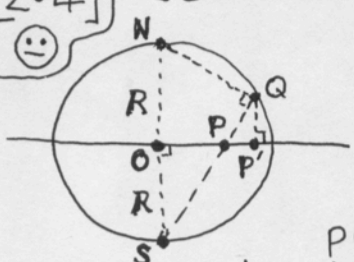
[2.3]



$$\log\frac{QA\cdot PB}{QB\cdot PA}+\log\frac{QB\cdot PC}{QC\cdot PB}=\log\frac{QA\cdot \cancel{PB}\cdot \cancel{QB}\cdot PC}{\cancel{QB}\cdot PA\cdot QC\cdot \cancel{PB}}$$

$$=\log\frac{QA\cdot PC}{PA\cdot QC}$$ (cancelling PB and QB).

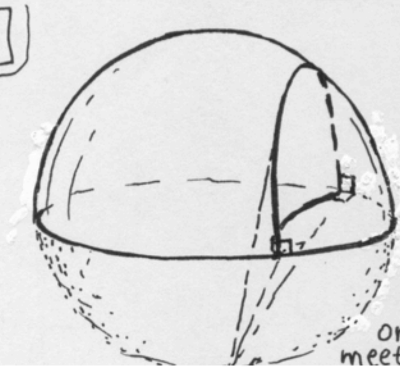
[2.4] Use Beltrami's geometry of Fig. 2.17, and consider the vertical plane through the sphere's centre O and the point P of the conformal representation and the corresponding point P′ of the projective representation (so this plane also contains the north pole N and the south pole S). Q is the corresponding point of the hemispherical representation. We have SO=ON=R (=radius)



Now $\frac{2R}{SQ}=\frac{SP}{R}$ (Δs SQN & SOP similar) and $\frac{OP'}{OP}=\frac{SQ}{SP}$ (vertical projections)

But $SP^2=R^2+OP^2$ (Pythagoras). Hence $\frac{OP'}{OP}=\frac{2R^2}{R^2+OP^2}$, as was required.

[2.5]



To get Beltrami's hemispheric repn. from the conformal one, in the equatorial plane, we project stereographically from the south pole S. We are allowed to know that this projection is conformal (sending circles to circles), so the conformal repn in the equatorial plane, with hyp. str. lines as circles meeting bounding ⊙ at rt. angles going to conformal one on northern hemisph. with. hyp. str. lines as ⊙s meeting equator at rt. angles, i.e. vertical semi-circles.