

[10.6] Given $f(x, y) = x^3 - y^3$ and $\begin{cases} X = x - y \\ Y = xy \end{cases}$

$$\text{So } \begin{cases} X^2 = x^2 - 2xy + y^2 \\ 2Y = 2xy \end{cases}$$

$$X^2 + 2Y = x^2 + y^2$$

$$\begin{aligned} \text{Also } (x^3 - y^3) &= (x - y)(x^2 + xy + y^2) \\ &= X(x^2 + y^2 + Y) \\ &= X(X^2 + 2Y + Y) \\ &= \boxed{X(X^2 + 3Y) = F(X, Y)} \end{aligned}$$