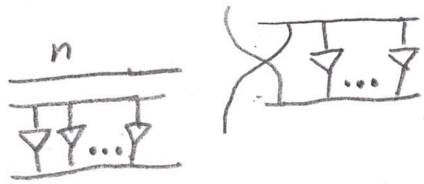
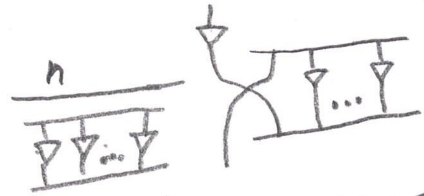
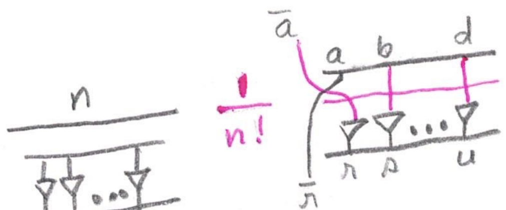
[13.19] Show that $T^{-1} =$ 

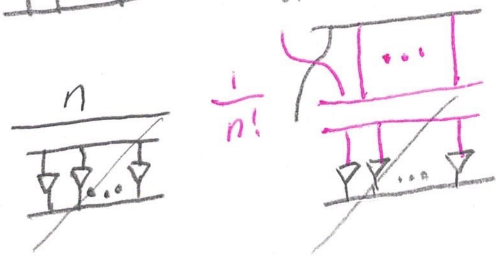
Proof: $TT^{-1} =$  $=$ 

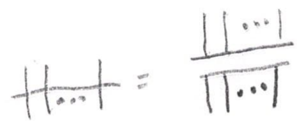
Note: $= n! \det(T)$ by [13.21].
Just a number

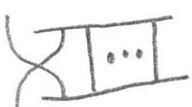
$=$ 

$=$ 

See Note 1.

$=$ 

since 

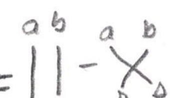
$= \frac{n}{n!}$ 

since  $= (n-p)!$ 

$= \frac{\cancel{n}(n-1)!}{\cancel{n!}}$ |

since | $= \delta^a_{[r]} \overset{(*)}{=} \delta^a_r =$ | just

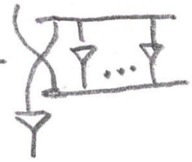
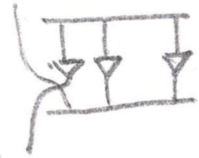
To see (*), compare this to

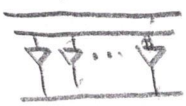
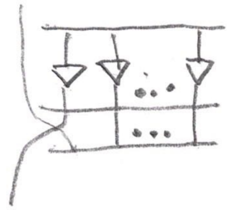
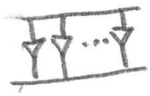
 $= 2\delta^a_{[r}\delta^b_{s]} = 2\left(\tfrac{1}{2}\right)\left[\delta^a_r\delta^b_s - \delta^a_s\delta^b_r\right] =$ 

$=$ |

$= I$

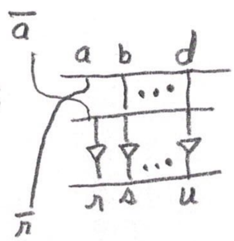
$= I$ ✓

$T^{-1}T =$  $= \dfrac{n}{}$   $= \dfrac{n}{}$  

$= \dfrac{n}{}$  $\dfrac{1}{n!}$  $= \dfrac{n}{n!}$  

$= \dfrac{n(n-1)!}{n!}$  $= | = I$

Note 1:
(bud)



$= n!\, \delta^{a}_{\bar{r}}\, \varepsilon_{ab\cdots d}\, \epsilon^{rs\cdots u}\, T^{[\bar{a}}{}_{r} T^{b}{}_{s} \cdots T^{d]}{}_{u}$

$= \dfrac{n!}{n!} \delta^{a}_{\bar{r}}\, \varepsilon_{ab\cdots d}\, \epsilon^{rs\cdots u} \sum_{\pi \in P_{\bar{a}b\cdots u}} \mathrm{sign}(\pi)\, T^{\pi(\bar{a})}{}_{r}\, T^{\pi(b)}{}_{s} \cdots T^{\pi(d)}{}_{u}$

$= \delta^{a}_{\bar{r}}\, \epsilon^{rs\cdots u} \sum_{\pi} \varepsilon_{\pi^{-1}(a)\pi^{-1}(b)\cdots\pi^{-1}(d)}\, \mathrm{sign}(\pi)\, T^{\bar{a}}{}_{r} T^{b}{}_{s} \cdots T^{d}{}_{u}$

rename $x \mapsto \pi^{-1}(x)$ for $x = \bar{a}, b, \ldots, d$. Thus, e.g., $\pi(b) \mapsto b$ and $b \mapsto \pi^{-1}(b)$

$\underset{(\text{antisym})}{\overset{(\varepsilon)}{=}} \delta^{a}_{\bar{r}}\, \epsilon^{rs\cdots u} \sum_{\pi} \mathrm{sign}(\pi)\, \varepsilon_{ab\cdots d}\, \mathrm{sign}(\pi)\, T^{\bar{a}}{}_{r} T^{b}{}_{s} \cdots T^{d}{}_{u}$

$= n!\, \delta^{a}_{\bar{r}}\, \varepsilon_{ab\cdots d}\, \epsilon^{rs\cdots u}\, T^{\bar{a}}{}_{r} T^{b}{}_{s} \cdots T^{d}{}_{u}$

$= n!$