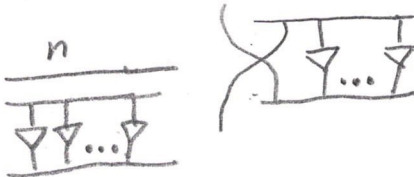
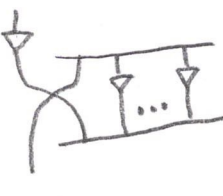


[13.19] Show that $T^{-1} = \frac{n}{\text{diagram}}$

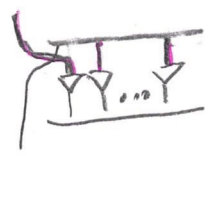


Proof: $TT^{-1} = \left[\begin{smallmatrix} \downarrow \\ \downarrow \end{smallmatrix} \right]^{-1} = \frac{n}{\text{diagram}}$

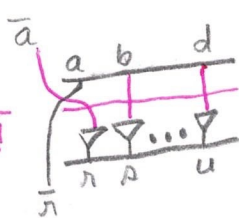


Note: $= n! \det(\tau)$ by [13.21].
Just a number

$= \frac{n}{\text{diagram}}$



$= \frac{n}{\text{diagram}} \cdot \frac{1}{n!}$



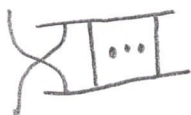
(See Note 1.)

$= \frac{n}{\text{diagram}} \cdot \frac{1}{n!}$



since $\text{diagram} = \frac{|| \dots ||}{|| \dots ||}$

$= \frac{n}{n!} \text{diagram}$



$= \frac{n(n-1)!}{n!} +$

$= 1$

$= I$

$= I \checkmark$

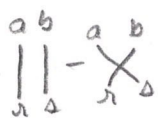
since $\text{diagram} = (n-p)! \text{diagram}$



since $1 = \delta_{[n]}^a (*) \delta_n^a = 1$ just

To see (*), compare this to

$H = 2 \delta_{[n]}^{a,b} = 2 \left(\frac{1}{2} \right) [\delta_n^a \delta_n^b - \delta_n^a \delta_n^b] = \text{diagram} - \text{diagram}$



$$T^{-1}T = \left[\begin{array}{c} \downarrow \\ \downarrow \\ \downarrow \end{array} \right]^{-1} = \frac{n}{\begin{array}{c} \text{---} \\ \downarrow \downarrow \dots \downarrow \end{array}} \begin{array}{c} \text{---} \\ \downarrow \end{array} \dots \begin{array}{c} \text{---} \\ \downarrow \end{array} = \frac{n}{\begin{array}{c} \text{---} \\ \downarrow \downarrow \dots \downarrow \end{array}} \begin{array}{c} \text{---} \\ \downarrow \downarrow \dots \downarrow \end{array}$$

$$= \frac{n}{\begin{array}{c} \text{---} \\ \downarrow \downarrow \dots \downarrow \end{array}} \frac{1}{n!} \begin{array}{c} \downarrow \downarrow \dots \downarrow \\ \vdots \\ \downarrow \downarrow \dots \downarrow \end{array} = \frac{n}{n!} \begin{array}{c} \downarrow \downarrow \dots \downarrow \\ \vdots \\ \downarrow \downarrow \dots \downarrow \end{array} \begin{array}{c} \text{---} \\ \downarrow \downarrow \dots \downarrow \end{array}$$

$$= \frac{n(n-1)!}{n!} \mathbb{I} = \mathbb{I}$$

Note 1: (bud)

$$\begin{array}{c} \bar{a} \\ \downarrow \\ \begin{array}{c} a \ b \ d \\ \vdots \\ \downarrow \downarrow \dots \downarrow \\ n \ \Delta \ u \end{array} \end{array} = n! \delta_{\bar{n}}^a \varepsilon_{ab\dots d} \in^{n\Delta\dots u} T_{\bar{n}}^{\bar{a}} T_{\Delta}^b \dots T_u^d$$

$$= \frac{n!}{n!} \delta_{\bar{n}}^a \varepsilon_{ab\dots d} \in^{n\Delta\dots u} \sum_{\pi \in \mathcal{P}_{ab\dots u}} \text{sign}(\pi) T_{\bar{n}}^{\pi(\bar{a})} T_{\Delta}^{\pi(b)} \dots T_u^{\pi(d)}$$

$$= \delta_{\bar{n}}^a \in^{n\Delta\dots u} \sum_{\pi} \varepsilon_{\pi^{-1}(a) \pi^{-1}(b) \dots \pi^{-1}(d)} \text{sign}(\pi) T_{\bar{n}}^{\bar{a}} T_{\Delta}^b \dots T_u^d$$

(E) (antisym)

$$= \delta_{\bar{n}}^a \in^{n\Delta\dots u} \sum_{\pi} \text{sign}(\pi) \varepsilon_{ab\dots d} \text{sign}(\pi) T_{\bar{n}}^{\bar{a}} T_{\Delta}^b \dots T_u^d$$

$$= n! \delta_{\bar{n}}^a \varepsilon_{ab\dots d} \in^{n\Delta\dots u} T_{\bar{n}}^{\bar{a}} T_{\Delta}^b \dots T_u^d$$

$$= n! \begin{array}{c} \bar{a} \\ \downarrow \\ \begin{array}{c} a \ b \ d \\ \vdots \\ \downarrow \downarrow \dots \downarrow \\ n \ \Delta \ u \end{array} \end{array}$$

rename $x \mapsto \pi^{-1}(x)$ for $x = \bar{a}, b, \dots, d$. Thus, e.g., $\pi(b) \mapsto b$ and $b \mapsto \pi^{-1}(b)$