[12.7] Show that $\phi_{[rs\ldots u]} = \phi_{rs\ldots u}$ where there are p indices *r*, *s*, … , *u*.

Proof: There are p! permutations of *r*, *s*, … , *u*, half of them even and half odd. Let $\mathcal{P}^+$ be the set of even permutations, $\mathcal{P}^-$ the set of odd permutations, and $\mathcal{P}$ the set of all permutations. Since ϕ, by definition, is antisymmetric in *r*, *s*, …, *u,*

$$\phi_{\pi(r)\pi(s)\ldots\pi(u)} = \left\{ \begin{array}{cc} \phi_{rs\ldots u} & \text{if } \pi \in \mathcal{P}^+ \\ -\phi_{rs\ldots u} & \text{if } \pi \in \mathcal{P}^- \end{array} \right\} = sign(\pi)\,\phi_{rs\ldots u} \qquad (1)$$

So,

$$\begin{aligned}\phi_{[rs\ldots u]} &\overset{defn}{=} \frac{1}{p!}\sum_{\pi\in\mathcal{P}} sign(\pi)\ \phi_{\pi(r)\pi(s)\ldots\pi(u)} \overset{(1)}{=} \frac{1}{p!}\sum_{\pi\in\mathcal{P}} sign(\pi)\left[sign(\pi)\,\phi_{rs\ldots u}\right] \\ &= \frac{1}{p!}\sum_{\pi\in\mathcal{P}} \phi_{rs\ldots u} = \frac{1}{p!}\left(p!\,\phi_{rs\ldots u}\right) \\ &= \phi_{rs\ldots u}\end{aligned}$$

Alternatively, we could solve this as

$$\begin{aligned}\phi_{[rs\ldots u]} &\overset{defn}{=} \frac{1}{p!}\sum_{\pi\in\mathcal{P}} sign(\pi)\ \phi_{\pi(r)\pi(s)\ldots\pi(u)} \overset{(a)}{=} \frac{1}{p!}\left(\sum_{\pi\in\mathcal{P}^+} \phi_{\pi(r)\pi(s)\ldots\pi(u)} + \sum_{\pi\in\mathcal{P}^-} -\phi_{\pi(r)\pi(s)\ldots\pi(u)}\right) \\ &\overset{(1)}{=} \frac{1}{p!}\left(\frac{p!}{2}\phi_{rs\ldots u} + \frac{p!}{2}(-)\left(-\phi_{rs\ldots u}\right)\right) \\ &= \phi_{rs\ldots u}.\end{aligned}$$

(a) *sign* (π) = 1 for $\pi \in \mathcal{P}^+$ and *sign* (π) = -1 for $\pi \in \mathcal{P}^-$