[11.13] P is an element of grade p, and Q is an element of grade q.

Thus P, in general, is a sum of expressions with p n's wedged together

$$P = \sum_{i=1}^{n} a_i n_{i1} \wedge n_{i2} \wedge \cdots \wedge n_{ip}$$

$$= a_1 n_{11} \wedge n_{12} \wedge \cdots \wedge n_{1p} + a_2 n_{21} \wedge n_{22} \wedge \cdots \wedge n_{2p} + \cdots + a_n n_{n1} \wedge n_{n2} \wedge \cdots \wedge n_{np}$$

and similarly

$$Q = \sum_{j=1}^{m} b_j \, n_{n+j\,1} \wedge n_{n+j\,2} \wedge \cdots \wedge n_{n+j\,q}$$

$$= b_1 \, n_{n+1\,1} \wedge n_{n+1\,2} \wedge \cdots \wedge n_{n+1\,q} + b_2 \, n_{n+2\,1} \wedge n_{n+2\,2} \wedge \cdots \wedge n_{n+2\,q}$$

$$+ \cdots + b_m \, n_{n+m\,1} \wedge n_{n+m\,2} \wedge \cdots \wedge n_{n+m\,q}$$

(The reason for using "$n+j$" instead of just "j" is because, for example, the 1st expression for P is $a_1 n_{11} \wedge n_{12} \wedge \cdots \wedge n_{1p}$ but the 1st expression for Q isn't $b_1 n_{11} \wedge n_{12} \wedge \cdots \wedge n_{1q}$; rather Q must be free not to have to use n_{11} again. So I label the 1st expression in Q as $b_1 n_{n+1\,1} \wedge n_{n+1\,2} \wedge \cdots \wedge n_{n+1\,q}$.)

Example:

$p=2, n=3$: $P = a_1 n_1 \wedge n_2 + a_2 n_3 \wedge n_4 + a_3 n_5 \wedge n_6$

$$= \sum_{i=1}^{3} a_i n_{i1} \wedge n_{i2}, \text{ where } n_{11} = n_1, n_{12} = n_2, n_{21} = n_3, \text{ etc.}$$

$q=3, m=2$ $\quad Q = b_1 n_7 \wedge n_8 \wedge n_9 + b_2 n_{10} \wedge n_{11} \wedge n_{12}$

$$= \sum_{j=1}^{2} b_j n_{3+j\,1} \wedge n_{3+j\,2} \wedge n_{3+j\,3}$$

where $n_{41} = n_7$, $n_{42} = n_8$, $n_{43} = n_9$, $n_{51} = n_{10}$, etc.

$$P \wedge Q = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{[i} b_{j]} n_{i1} \wedge \cdots \wedge n_{ip} \wedge n_{n+j\,1} \wedge \cdots \wedge n_{n+j\,q}$$

Note that $a_{[i} b_{j]} = b_{[j} a_{i]}$. For example,

$$a_{[1} b_{2]} = \tfrac{1}{2}(a_1 b_2 - a_2 b_1) = \tfrac{1}{2}(b_2 a_1 - b_1 a_2) = b_{[2} a_{1]}.$$

[11.13, p.2] Observe that

$$a_{[i}b_{j]}\,\eta_{i1}\wedge\cdots\wedge\eta_{ip}\wedge\eta_{n+j1}\wedge\cdots\wedge\eta_{n+jq}$$

$$=(-1)^{p}a_{[i}b_{j]}\,\eta_{n+j1}\wedge\eta_{i1}\wedge\cdots\wedge\eta_{ip}\wedge\eta_{n+j2}\wedge\cdots\wedge\eta_{n+jq}$$

$$=(-1)^{2p}a_{[i}b_{j]}\,\eta_{n+j1}\wedge\eta_{n+j2}\wedge\eta_{i1}\wedge\cdots\wedge\eta_{ip}\wedge\eta_{n+j3}\wedge\cdots\wedge\eta_{n+jq}$$

$$=\cdots$$

$$=(-1)^{qp}a_{[i}b_{j]}\,\eta_{n+j1}\wedge\cdots\wedge\eta_{n+jq}\wedge\eta_{i1}\wedge\cdots\wedge\eta_{ip}$$

$$=(-1)^{pq}b_{[j}a_{i]}\,\eta_{n+j1}\wedge\cdots\wedge\eta_{n+jq}\wedge\eta_{i1}\wedge\cdots\eta_{ip}$$

So, $$P\wedge Q=\sum_{i=1}^{n}\sum_{j=1}^{m}a_{[i}b_{j]}\,\eta_{i1}\wedge\cdots\wedge\eta_{n+jq}$$

$$=(-1)^{pq}\sum_{j=1}^{m}\sum_{i=1}^{n}b_{[j}a_{i]}\,\eta_{n+j1}\wedge\cdots\wedge_{ip}$$

$$=(-1)^{pq}\,Q\wedge P$$

$$=\begin{cases}Q\wedge P & \text{if } p \text{ or } q \text{ or both are even}\\ -Q\wedge P & \text{if } p \text{ and } q \text{ are both odd}\end{cases}$$

[11.14] If p is odd, then by [11.13],

$$P\wedge P=-P\wedge P$$

$$\Rightarrow P\wedge P=0$$