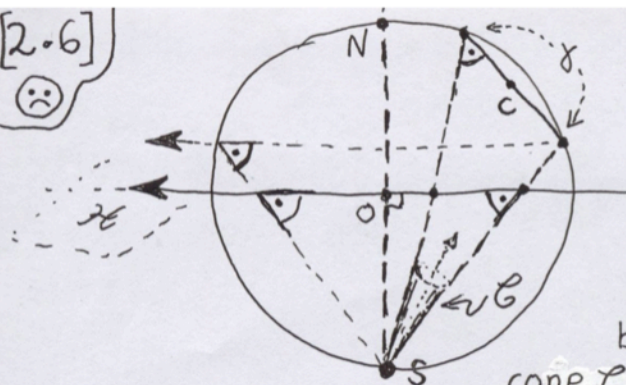


[2.6]



Take the plane through the vertical diameter SN and the centre C of some chosen circle γ on the sphere.

The angles marked \triangle are all equal (the lines with \leftarrow being parallel), so the (elliptical) cone C joining S to γ is intersected

by two planes (namely the plane of γ) and the horizontal equatorial plane H) at opposite slopes to the symmetry axis of the cone (marked with arrow \nearrow). Hence, these intersections are similar (the elliptical cone being symmetrical under reflection in the axis \nearrow), so γ being a circle is equivalent to the cone C 's intersection with H being a circle (the limiting case, when γ passes through S giving a straight line in the plane). Thus, stereographic projection sends circles to circles, and if we allow our circles to become infinitesimal we see that the map is conformal (Fig. 8.4).

[2.7]

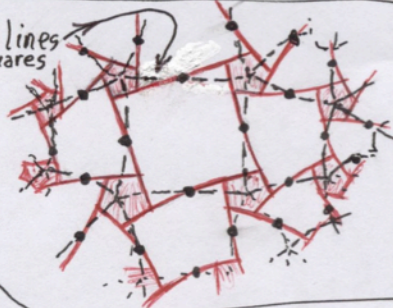


See Fig. 2.19a; black broken lines give original squares

modify thus:

(I'm not sure what this is good for!)

red lines



Rotate each line of original square of Fig. 2.19a through the same angle, about centre of side of square (marked point) to get new (red) square.

[2.8]



Area of mēlon of skin is $\frac{\alpha}{2\pi}$ of the total

From \nearrow and we have

$$2\alpha R^2 + 2\beta R^2 + 2\gamma R^2 = 2\pi R^2 + 2\Delta$$

$$\therefore \Delta = (\alpha + \beta + \gamma - \pi)R^2$$

Since triply counted $\therefore 2$ extra Δ s

$$\text{This area is } \frac{\alpha}{2\pi} \times 4\pi R^2 = 2\alpha R^2$$

redraw

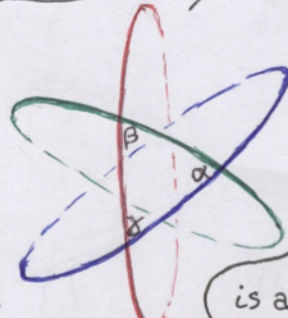


Fig. 2.20

Area is one-half the total sphere area $4\pi R^2$ (since the remainder is also the same shape and size. Let the area of the triangle in the middle be Δ).

