[12.5] Show (i) that dx^2 has components <0, 1, 0, … , 0> and (ii) represents the tangent hyperplane elements to x^2 = constant.

(i)

$$dx^2 = \boldsymbol{\alpha} = \sum_{i=1}^{n} \alpha_i dx^i \quad \Rightarrow \quad \alpha_i = \begin{cases} 1 & \text{if } i = 2 \\ 0 & \text{Otherwise} \end{cases}$$

Using component notation, $dx^2 = < \alpha_1, \alpha_2, \ldots, \alpha_n > = < 0, 1, 0, \ldots, 0 >$ ✔

(ii)

Let $\Phi = \boldsymbol{\alpha}$. Then $d\Phi = d\boldsymbol{\alpha}$ = <0, 1, 0, … , 0>. This implies that Φ is constant on the (n-1)-dimensional hyperplane $x^1 \times x^3 \times x^4 \times \bullet\bullet\bullet \times x^n$. That is, the hyperplane is tangential to the surface of constant Φ. ✔