[12.5] Show (i) that dx^2 has components <0, 1, 0, … , 0> and (ii) represents the tangent hyperplane elements to x^2 = constant.

(i)

$$dx^2 = \boldsymbol{\alpha} = \sum_{i=1}^{n} \alpha_i dx^i \quad \Rightarrow \quad \alpha_i = \begin{cases} 1 & \text{if } i = 2 \\ 0 & \text{Otherwise} \end{cases}$$

Using component notation, $dx^2 = \langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle = \langle 0, 1, 0, \ldots, 0 \rangle$ ✔

(ii)

From [12.4], if $\Phi : \mathcal{M} \to \Re$ is a scalar field, then $d\Phi$ is a vector orthogonal to the (n-1) dimensional hyperplane where Φ is constant. So, define

$$\Phi : \mathcal{M} \to \Re : \Phi(x^1, x^2, \ldots, x^n) = x^2.$$

Thus, $dx^2 = d\Phi(x^1, x^2, \ldots, x^n)$ is orthogonal to the (n-1) dimensional hyperplane (i.e., the tangent hyperplane) at $(0, x^2, 0, \ldots, 0)$ where $\Phi = x^2$ is constant.