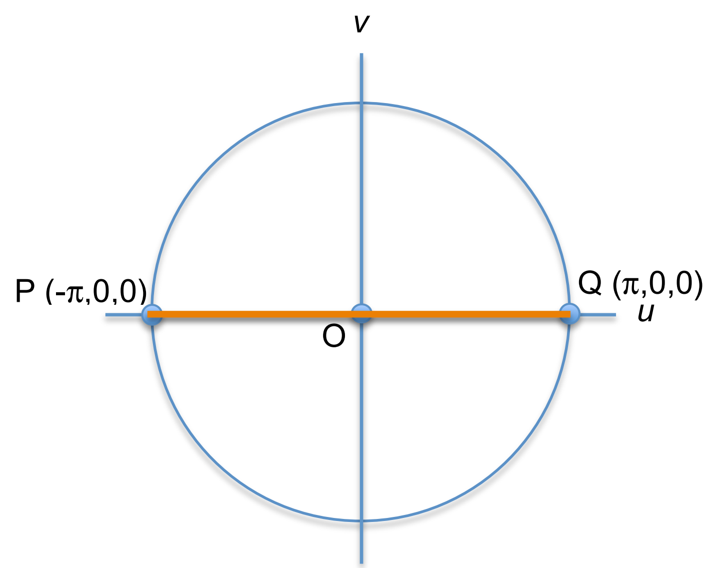
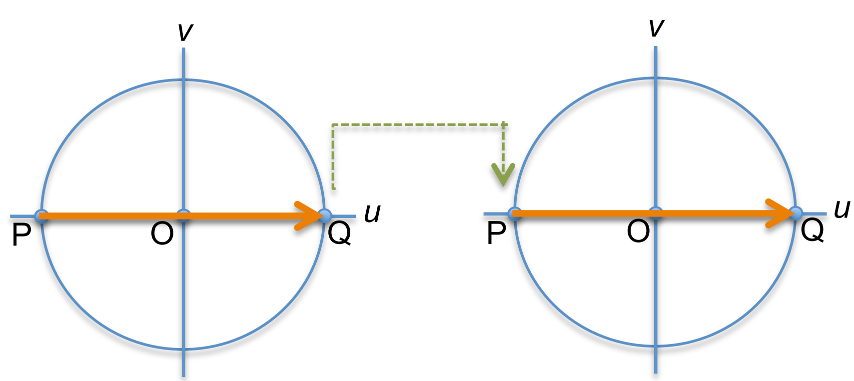
[12.2] In problem [12.17] we showed that a single 2π rotation in configuration space is not homotopic to a point. In this problem we are asked to extend this result by showing that a 4π rotation *is* homotopic to a point.

As in problem [12.17], let the Rotation Subspace $\mathcal{R} = \{ (u, v, w) \}$ be a 3-ball of radius π where each vector (u, v, w) represents a rotation in $\mathbb{R}^3$.



The figure represents the *uv*-cross section of the 3-ball $\mathcal{R}$. O is the origin (0, 0, 0). The line segment $\overline{PQ}$ is a loop because P is identified with Q. The 4π rotation is represented below by a trajectory that consists of traversing $\overline{PQ}$ twice. I demonstrate the homotopy mapping by using a sequence of diagrams that shows a progressive continuous deformation of the 4π rotation to a point P.



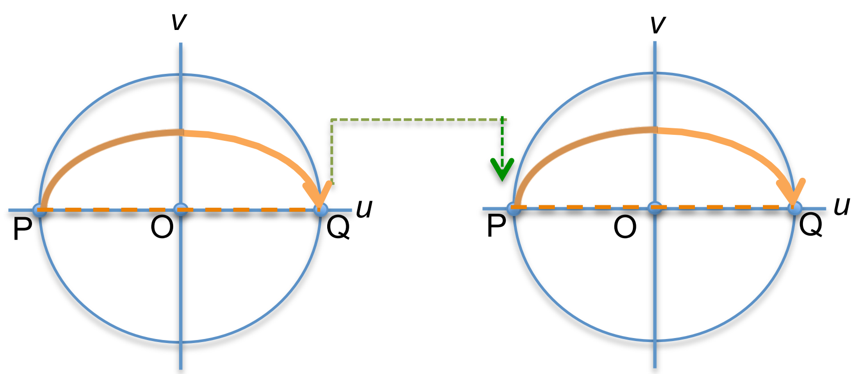
Stage 0

The requirements for each diagram are:

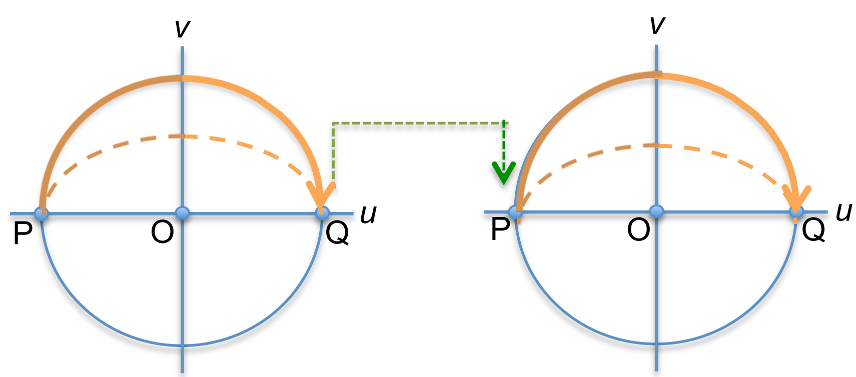
1. Must clearly indicate a continuous transformation from the prior diagram

   a. The rotation trajectory from the prior diagram will be indicated in dashed orange
   b. The new rotation trajectory is indicated in solid orange
2. Initial and final points of trajectory must be fixed at P (=Q)

Also note that the point P is traversed 3 times during a 4 rotation rather than only twice as was the case for the 2 rotation. Thus, this time we are free to move the middle point P during the deformation process. This is the key to why homotopy works here but not in the prior problem [2.12].

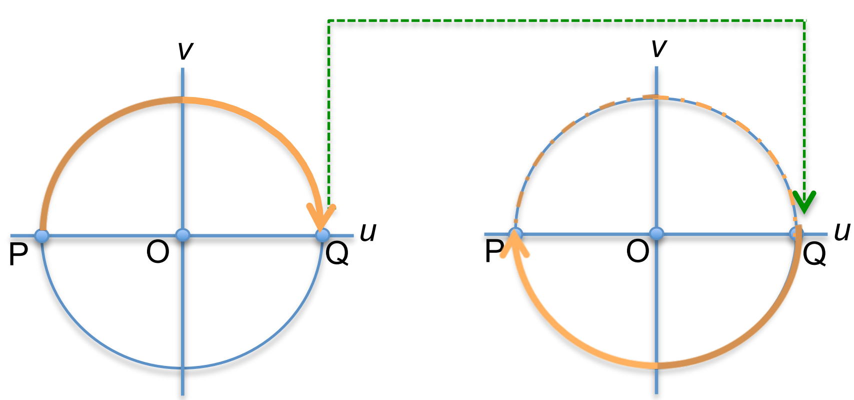


Stage 1

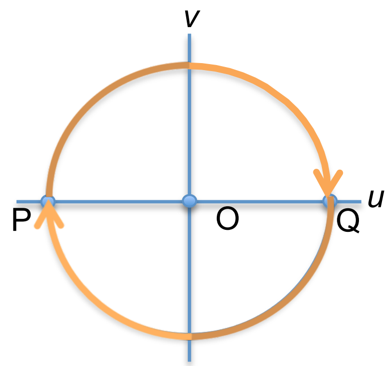


Stage 2

The next two figures repeat Stage 2, modifying it into a more convenient representation. The first figure is valid because antipodal points are identified.

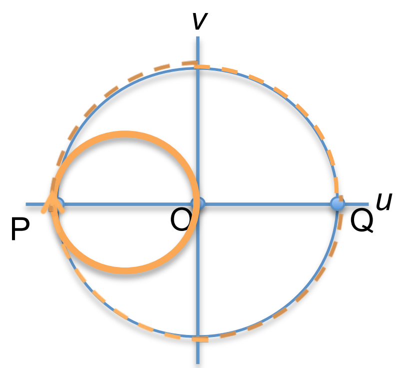


Stage 2



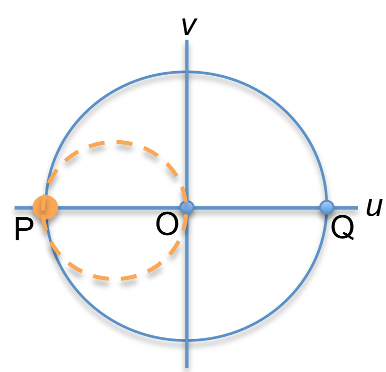
Stage 2

Below is where we free the middle occurrence of P (actually, Q).



Stage 3

Finally, the trajectory is contracted continuously to a point.



Final Stage