

[2.5] Given that stereographic projection from the upper half sphere to the equatorial plane (a) preserves angles and (b) sends circles to circles, show that vertical semicircles map to circles that intersect the equatorial circle orthogonally

We start with a vertical circle on the sphere; that is, the intersection of a vertical plane with the upper hemisphere of the Riemann sphere. By (b), its stereographic projection in the equatorial plane is a circle. By (a), the 2 angles the projected circle makes with the equatorial circle are the same as the 2 angles that the vertical circle on the sphere makes with the equatorial circle. But a vertical circle is perpendicular to the equatorial plane. Therefore the angles it makes with the equatorial circle are 90° . Thus the projected circle makes right angles with the equatorial circle.