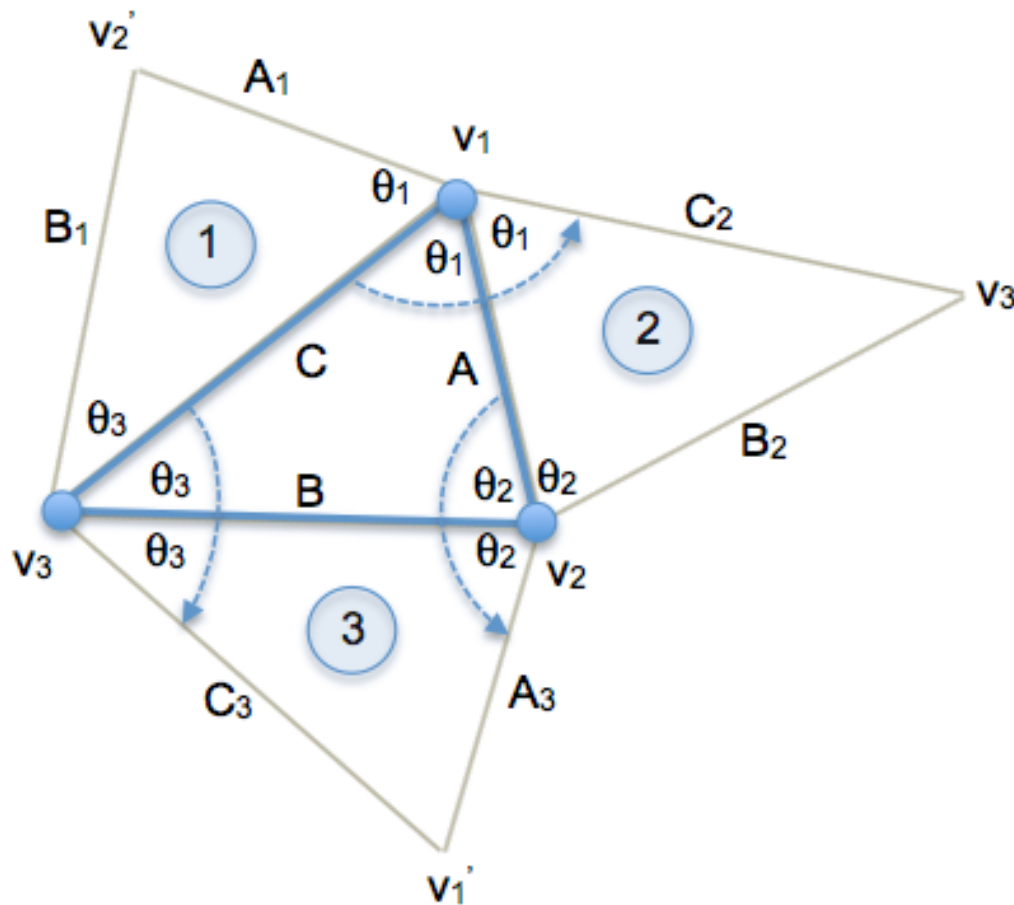


[11.5] I believe deant provided a correct solution to this problem, but he made some mistakes in explaining it. For example, in his figure he claimed that his side A gets rotated into his side C, and that is clearly not correct because those sides in his figure are definitely not the same size. I am making a pass at clarifying his explanation.



Consider 2 rotations about different axes. Let vertices  $v_1$  and  $v_2$  be the two axes of rotation with angles of rotation  $2\theta_1$  and  $2\theta_2$ , respectively, in the directions shown in the figure. By halving the angles, we create the blue triangle  $\langle v_1 v_2 v_3 \rangle$  with sides A, B, and C. Next we create triangles 1, 2, and 3 as shown by reflecting this triangle about sides C, A, and B, respectively.

It is easy to see that  $2\theta_1$  rotates triangle 1 into triangle 2, sliding side  $A_1$  into A,  $B_1$  into  $B_2$ , and C into  $C_2$ . (Sides  $A_1$  to A and  $C_1$  to C are easy to visualize, and since that fixes all 3 vertices, then slide  $B_2$  must also match up.) Similarly  $2\theta_2$  rotates triangle 2 into triangle 3 with the respective sides matching up. Thus the composition rotates triangle 1 into triangle 3. Finally, we also see that  $2\theta_3$  rotates triangle 1 into triangle 3, and thus  $v_3$  is the resultant axis of rotation and  $2\theta_3$  is the rotation amount.