[9.4] Characterize the holomorphic mappings of the Riemann Sphere to itself that do not preserve the north or south poles but which map the north and south hemisphere into themselves

I don't have a handle on a full characterization. In fact, I can only think of a single class of mappings that satisfy this statement. Namely, pick any point on the equatorial circle and map the entire Riemann Sphere to that point. Call the point z_o. Then the function $f(z) = z_o$ for every z in the Riemann sphere is clear holomorphic, does not preserve either pole, and maps the north and south hemispheres into themselves.

I can imagine there may be such holomorphic functions that, say, map the north pole N to an interior point A of the northern hemisphere. Such a function might freeze the equatorial point and simply stretch points on "the North Pole side" of A and squeeze points "beyond" A towards the equatorial sphere. But, I couldn't come up with any such function and my gut feel is that it is not possible. If it were possible, then a 2nd function could rotate the equatorial circle and the composite of the two holomorphic functions would still be holomorphic and satisfy the required condition.

Also, towards the end of Chapter 9, Penrose states (without proof) that the only holomorphic functions defined on the entire Riemann Sphere are constant functions. That further suggests to me that the constant functions, where the constant in on the equatorial circle, are the only ones that meet the required condition.

This is pretty weak but no one has posted a stab at this problem so I thought I would post these thoughts. Maybe someone can build on it.