[11.11] Recall that $2^n = \sum_{k=0}^{n} \binom{n}{k}$ where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

So $\binom{n}{0} = 1$, $\binom{n}{1} = n$, $\binom{n}{2} = \frac{1}{2!}n(n-1)$, $\binom{n}{3} = \frac{1}{3!}n(n-1)(n-2) = \frac{1}{6}n(n-1)(n-2)$,
$\binom{n}{4} = \frac{n!}{(4!)(n-4)!} = \frac{n(n-1)(n-2)(n-3)}{4!} = \frac{1}{24}n(n-1)(n-2)(n-3)$

Note that if you start with ordered subscripts $pqrs$, as in $a_p b_q c_r d_s$, that any simple swap, like $pqsr$, reverses sign; i.e. you get $-a_p b_q c_s d_r$. Similarly for a, b, c, d. Thus

$$a\wedge b\wedge c\wedge d = -a\wedge b\wedge d\wedge c = -a\wedge c\wedge b\wedge d = a\wedge c\wedge d\wedge b = a\wedge d\wedge b\wedge c = -a\wedge d\wedge c\wedge b$$
$$= -b\wedge a\wedge c\wedge d = b\wedge a\wedge d\wedge c = b\wedge c\wedge a\wedge d = -b\wedge c\wedge d\wedge a = -b\wedge d\wedge a\wedge c = b\wedge d\wedge c\wedge a$$
$$= c\wedge a\wedge b\wedge d = -c\wedge a\wedge d\wedge b = -c\wedge b\wedge a\wedge d = c\wedge b\wedge d\wedge a = c\wedge d\wedge a\wedge b = -c\wedge d\wedge b\wedge a$$
$$= -d\wedge a\wedge b\wedge c = d\wedge a\wedge c\wedge b = d\wedge b\wedge a\wedge c = -d\wedge b\wedge c\wedge a = -d\wedge c\wedge a\wedge b = d\wedge c\wedge b\wedge a$$

and for the coefficient of $\eta_p\eta_q\eta_r\eta_s$, where $q \neq p$, $r \neq p$ or q, and $s \neq p, q,$ or r:

$$a_{[p}b_q c_r d_{s]}$$
$$= \frac{1}{24}(a_pb_qc_rd_s - a_pb_qc_sd_r - a_pb_rc_qd_s + a_pb_rc_sd_q + a_pb_sc_qd_r - a_pb_sc_rd_q$$
$$- a_qb_pc_rd_s + a_qb_pc_sd_r + a_qb_rc_pd_s - a_qb_rc_sd_p - a_qb_sc_pd_r + a_qb_sc_rd_p$$
$$+ a_rb_pc_qd_s - a_rb_pc_sd_q - a_rb_qc_pd_s + a_rb_qc_sd_p + a_rb_sc_pd_q - a_rb_sc_qd_p$$
$$- a_sb_pc_qd_r + a_sb_pc_rd_q + a_sb_qc_pd_r - a_sb_qc_rd_p - a_sb_rc_pd_q + a_sb_rc_qd_p)$$

and, if, for example

$$a = a_1\eta_1 + a_2\eta_2 + a_3\eta_3 + a_4\eta_4$$
$$b = b_1\eta_1 + b_2\eta_2 + b_3\eta_3 + b_4\eta_4$$
$$c = c_1\eta_1 + c_2\eta_2 + c_3\eta_3 + c_4\eta_4$$
$$d = d_1\eta_1 + d_2\eta_2 + d_3\eta_3 + d_4\eta_4$$

then

$$a\wedge b\wedge c\wedge d = \sum_{p=1}^{4}\sum_{q=1}^{4}\sum_{r=1}^{4}\sum_{s=1}^{4} a_{[p}b_q c_r d_{s]}\,\eta_p\wedge\eta_q\wedge\eta_r\wedge\eta_s$$

(Note: Terms are zero unless $p, q, r,$ and s are distinct.)

In general, if

$$a = a_1\eta_{11} + a_2\eta_{12} + a_3\eta_3 + a_4\eta_{14} = \sum_{p=1}^{4} a_p \eta_{1p}$$

$$b = b_1\eta_{21} + b_2\eta_{22} + b_3\eta_{23} + b_4\eta_{24} = \sum_{q=1}^{4} b_q \eta_{2q}$$

$$c = c_1\eta_{31} + c_2\eta_{32} + c_3\eta_{33} + c_4\eta_{34} = \sum_{r=1}^{4} c_r \eta_{3r}$$

$$d = d_1\eta_{41} + d_2\eta_{42} + d_3\eta_{43} + d_4\eta_{44} = \sum_{s=1}^{4} d_s \eta_{4s}$$

(where possibly some $\eta_{ij} = \eta_{kl}$ when $(i,j) \neq (k,l)$);

$$a \wedge b \wedge c \wedge d = \sum_{p=1}^{4} \sum_{q=1}^{4} \sum_{r=1}^{4} \sum_{s=1}^{4} a_{[p} b_q c_r d_{s]}\, \eta_{1p} \wedge \eta_{2q} \wedge \eta_{3r} \wedge \eta_{4s}$$