[11.12] I first work a simple case of this problem. It has all the ingredients needed for the general case but it is much easier to understand and it then makes it much easier to do the general case.

Simple case:

Let $\boldsymbol{a} = a_1\eta_1 + a_2\eta_2 + a_3\eta_3$, $\boldsymbol{c} = c_1\eta_1 + c_2\eta_2 + c_3\eta_3$, and λ a scalar.

We wish to show $(\boldsymbol{a}+\lambda\boldsymbol{c})\wedge\boldsymbol{c} = \boldsymbol{a}\wedge\boldsymbol{c}$

Recall that $\eta_1\wedge\eta_1 = 0 = \eta_2\wedge\eta_2 = \eta_3\wedge\eta_3$.

So,
$$\begin{aligned}
\boldsymbol{a}\wedge\boldsymbol{c} &= \left[a_1c_1\ \eta_1\wedge\eta_1 + a_2c_2\ \eta_2\wedge\eta_2 + a_3c_3\ \eta_3\wedge\eta_3\right] \\
&\quad + \left[\left(a_1c_2\ \eta_1\wedge\eta_2 + a_2c_1\ \eta_2\wedge\eta_1\right) + \left(a_1c_3\ \eta_1\wedge\eta_3 + a_3c_1\ \eta_3\wedge\eta_1\right) + \left(a_2c_3\ \eta_2\wedge\eta_3 + a_3c_2\ \eta_3\wedge\eta_2\right)\right] \\
&= [0] + \left[\left(a_1c_2 - a_2c_1\right)\eta_1\wedge\eta_2 + \left(a_1c_3 - a_3c_1\right)\eta_1\wedge\eta_3 + \left(a_2c_3 - a_3c_2\right)\eta_2\wedge\eta_3\right] \\
&= \left(a_1c_2 - a_2c_1\right)\eta_1\wedge\eta_2 + \left(a_1c_3 - a_3c_1\right)\eta_1\wedge\eta_3 + \left(a_2c_3 - a_3c_2\right)\eta_2\wedge\eta_3
\end{aligned}$$

Replacing $\boldsymbol{a}$ by $\boldsymbol{a}+\lambda\boldsymbol{c}$ (and, also, a_1 by $a_1+\lambda c_1$, etc.) in this expression yields

$$\begin{aligned}
\left(\boldsymbol{a}+\lambda\boldsymbol{c}\right)\wedge\boldsymbol{c} &= \left[\left(a_1+\lambda c_1\right)c_2 - \left(a_2+\lambda c_2\right)c_1\right]\eta_1\wedge\eta_2 + \left[\left(a_1+\lambda c_1\right)c_3 - \left(a_3+\lambda c_3\right)c_1\right]\eta_1\wedge\eta_3 \\
&\quad + \left[\left(a_2+\lambda c_2\right)c_3 - \left(a_3+\lambda c_3\right)c_2\right]\eta_2\wedge\eta_3 \\
&= \left[\left(a_1c_2 - a_2c_1\right) + \lambda\left(c_1c_2 - c_2c_1\right)\right]\eta_1\wedge\eta_2 + \left[\left(a_1c_3 - a_3c_1\right) + \lambda\left(c_1c_3 - c_3c_1\right)\right]\eta_1\wedge\eta_3 \\
&\quad + \left[\left(a_2c_3 - a_3c_2\right) + \lambda\left(c_2c_3 - c_3c_2\right)\right]\eta_2\wedge\eta_3 \\
&= \left[\left(a_1c_2 - a_2c_1\right)\right]\eta_1\wedge\eta_2 + \left[\left(a_1c_3 - a_3c_1\right)\right]\eta_1\wedge\eta_3 + \left[\left(a_2c_3 - a_3c_2\right)\right]\eta_2\wedge\eta_3 \\
&= \boldsymbol{a}\wedge\boldsymbol{c}
\end{aligned}$$

General case:

Let
$$\begin{aligned}
\boldsymbol{a} &= a_1\eta_1 + \cdots + a_t\eta_t \\
\boldsymbol{b} &= b_1\eta_1 + \cdots + b_t\eta_t \\
&\ \ \vdots \\
\boldsymbol{d} &= d_1\eta_1 + \cdots + d_t\eta_t
\end{aligned}$$

Following DimBulb's approach, we wish to show $(\boldsymbol{a}+\lambda\boldsymbol{c})\wedge\boldsymbol{b}\wedge\cdots\wedge\boldsymbol{d} = \boldsymbol{a}\wedge\boldsymbol{b}\wedge\cdots\wedge\boldsymbol{d}$, where $\boldsymbol{c}$ is one of the vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\cdots$, $\boldsymbol{d}$

This is equivalent to showing $(\boldsymbol{a}+\lambda\boldsymbol{c})\wedge\boldsymbol{c}\wedge\boldsymbol{b}\wedge\cdots\wedge\boldsymbol{d} = \boldsymbol{a}\wedge\boldsymbol{c}\wedge\boldsymbol{b}\wedge\cdots\wedge\boldsymbol{d}$,

or to showing $[(\boldsymbol{a}+\lambda\boldsymbol{c})\wedge\boldsymbol{c}]\wedge[\boldsymbol{b}\wedge\cdots\wedge\boldsymbol{d}] = [\boldsymbol{a}\wedge\boldsymbol{c}]\wedge[\boldsymbol{b}\wedge\cdots\wedge\boldsymbol{d}]$.

Thus, we need but show that $(\boldsymbol{a}+\lambda\boldsymbol{c})\wedge\boldsymbol{c} = \boldsymbol{a}\wedge\boldsymbol{c}$, just as in the simple case.

We proceed as in the simple case, ignoring all terms $\eta_p \wedge \eta_p$ and grouping terms $\eta_p \wedge \eta_q$ and $\eta_q \wedge \eta_p$.

$$
\begin{aligned}
(\boldsymbol{a}+\lambda\boldsymbol{c})\wedge\boldsymbol{c} &= \left[(a_1+\lambda c_1)c_2-(a_2+\lambda c_2)c_1\right]\eta_1\wedge\eta_2+\cdots+\\
&\quad+\left[(a_p+\lambda c_p)c_q-(a_q+\lambda c_q)c_p\right]\eta_p\wedge\eta_q+\cdots+\\
&\quad+\left[(a_{t-1}+\lambda c_{t-1})c_t-(a_t+\lambda c_t)c_{t-1}\right]\eta_{t-1}\wedge\eta_t\\
&= \left[(a_1c_2-a_2c_1)\right]\eta_1\wedge\eta_2+\left[\lambda(c_1c_2-c_2c_1)\right]\eta_1\wedge\eta_2+\cdots\\
&\quad+\left[(a_pc_q-a_qc_p)\right]\eta_p\wedge\eta_q+\left[\lambda(c_pc_q-c_qc_p)\right]\eta_p\wedge\eta_q+\cdots\\
&\quad+\left[(a_{t-1}c_t-a_tc_{t-1})\right]\eta_{t-1}\wedge\eta_t+\left[\lambda(c_{t-1}c_t-c_tc_{t-1})\right]\eta_{t-1}\wedge\eta_t\\
&= \left[(a_1c_2-a_2c_1)\right]\eta_1\wedge\eta_2+\cdots\left[(a_pc_q-a_qc_p)\right]\eta_p\wedge\eta_q+\cdots\left[(a_{t-1}c_t-a_tc_{t-1})\right]\eta_{t-1}\wedge\eta_t\\
&= \boldsymbol{a}\wedge\boldsymbol{c}
\end{aligned}
$$

That concludes the proof.