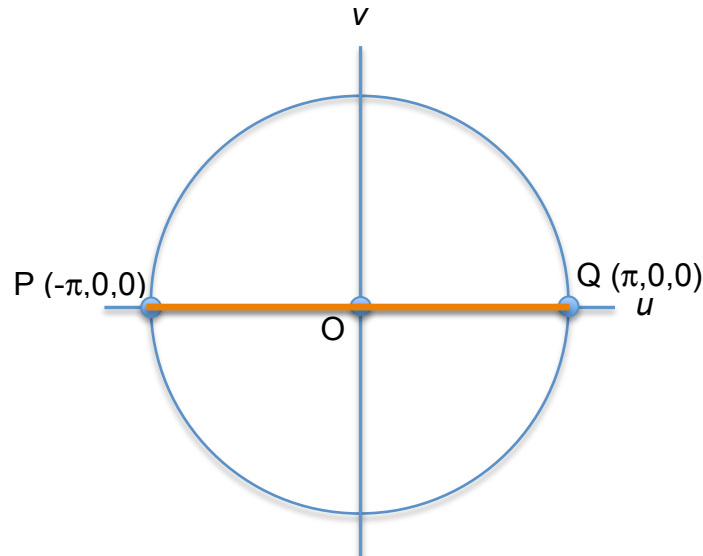
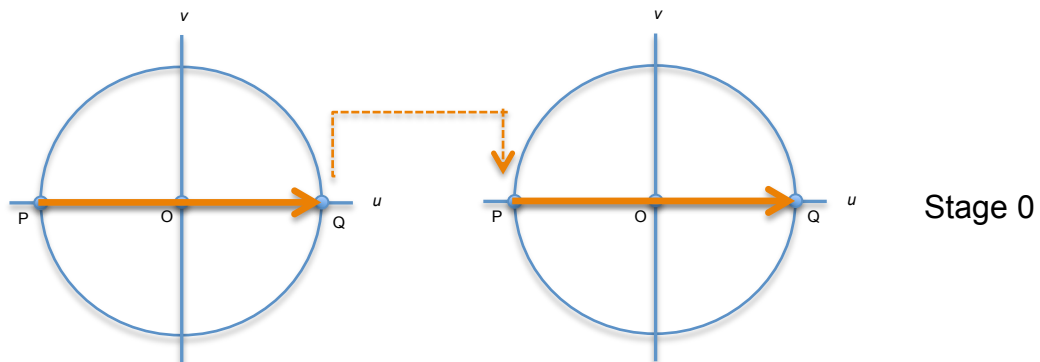


[12.2] In problem [12.17] we showed that a single 2π rotation in configuration space is not homotopic to a point. In this problem we are asked to extend this result by showing that a 4π rotation *is* homotopic to a point.

As in problem [12.17], let the Configuration Space $\mathcal{R} = \{ (u, v, w) \}$, a 3-ball of radius π where each vector (u, v, w) represents a rotation in \mathfrak{R}^3 .



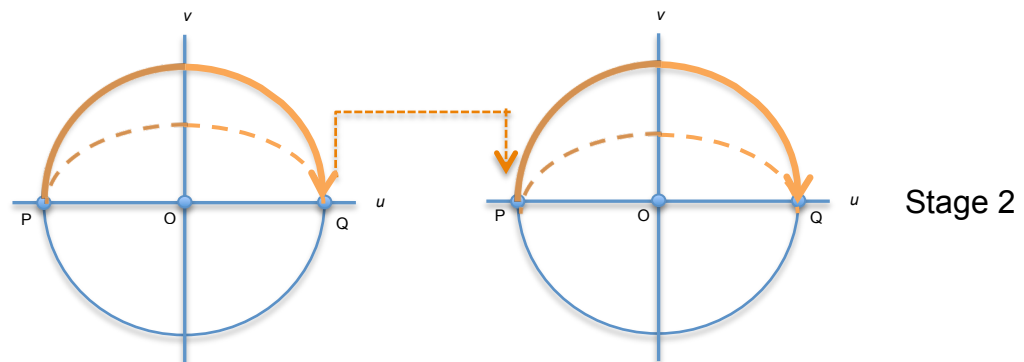
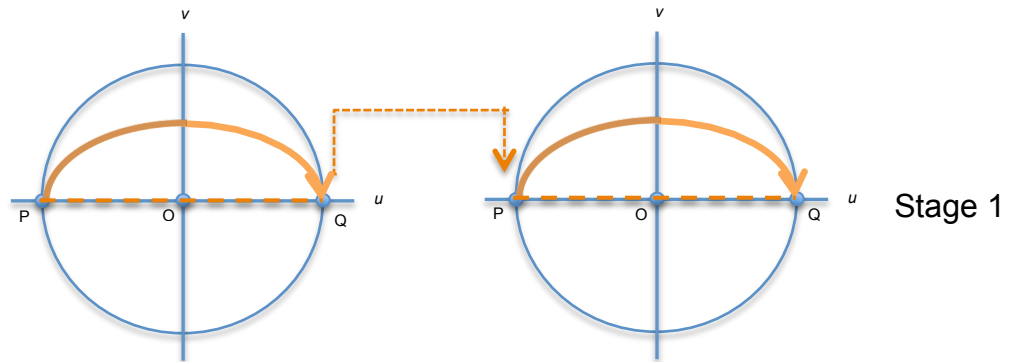
The figure represents the uv -cross section of the 3-ball \mathcal{R} . O is the origin $(0, 0, 0)$. The line segment \overline{PQ} is a loop because P is identified with Q . The 4π rotation is represented below by a trajectory that consists of traversing \overline{PQ} twice. I demonstrate the homotopy mapping by using a sequence of diagrams that shows a progressive continuous deformation of the 4π rotation to a point P .



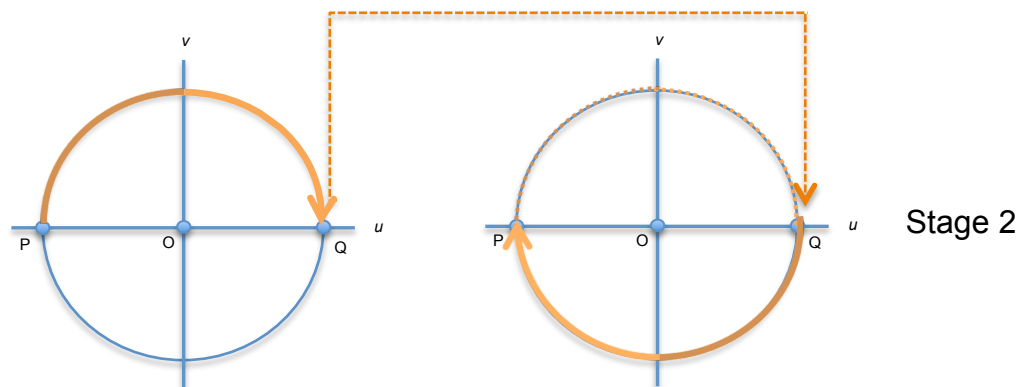
The requirements for each diagram are:

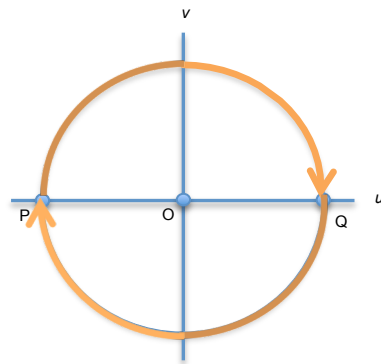
1. Must clearly indicate a continuous transformation from the prior diagram
2. Initial and final points of trajectory must be fixed at $P (=Q)$

Also note that the point P is traversed 3 times during a 4 rotation rather than only twice as was the case for the 2 rotation. Thus, this time we are free to move the middle point P during the deformation process. This is the key to why homotopy works here but not in the prior problem [2.12].



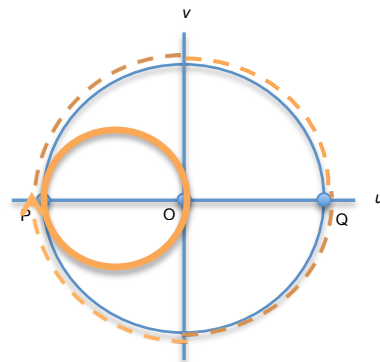
The next two figures repeat Stage 2, modifying it into a more convenient representation. The first figure is valid because antipodal points are identified.





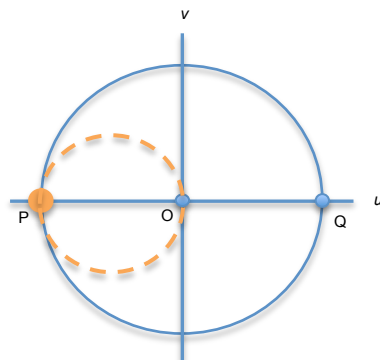
Stage 2

Below is where we free the middle occurrence of P (actually, Q).



Stage 3

Finally, the trajectory is contracted continuously to a point.



Final Stage