Backpropagation

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Gradient Descent

Network parameters $\theta = \{w_1, w_2, \cdots, b_1, b_2, \cdots\}$

Starting Parameters $\theta^0 \longrightarrow \theta^1 \longrightarrow \theta^2 \longrightarrow \ldots\ldots$

$$\nabla L(\theta) = \begin{bmatrix} \partial L(\theta)/\partial w_1 \\ \partial L(\theta)/\partial w_2 \\ \vdots \\ \partial L(\theta)/\partial b_1 \\ \partial L(\theta)/\partial b_2 \\ \vdots \end{bmatrix}$$

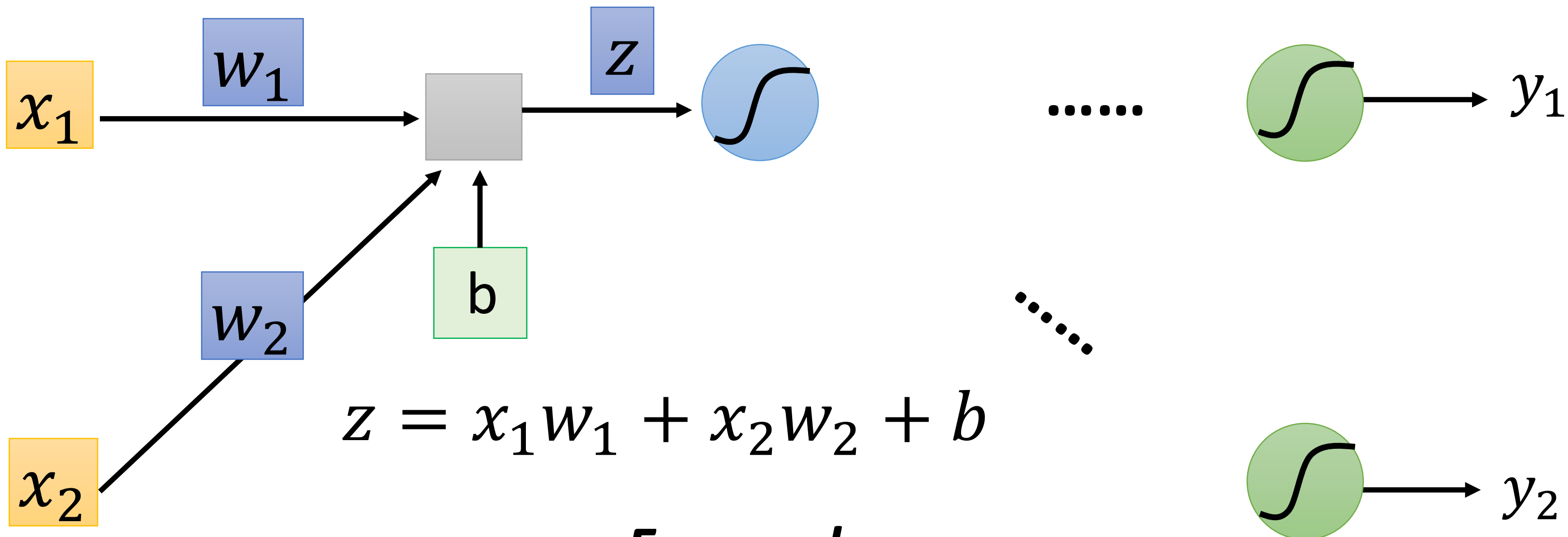
Compute $\nabla L(\theta^0)$ $\qquad \theta^1 = \theta^0 - \eta \nabla L(\theta^0)$

Compute $\nabla L(\theta^1)$ $\qquad \theta^2 = \theta^1 - \eta \nabla L(\theta^1)$

Millions of parameters ……

To compute the gradients efficiently, we use ***backpropagation***.

Backpropagation



$$z = x_1 w_1 + x_2 w_2 + b$$

Forward pass:

Compute $\partial z / \partial w$ for all parameters

Backward pass:

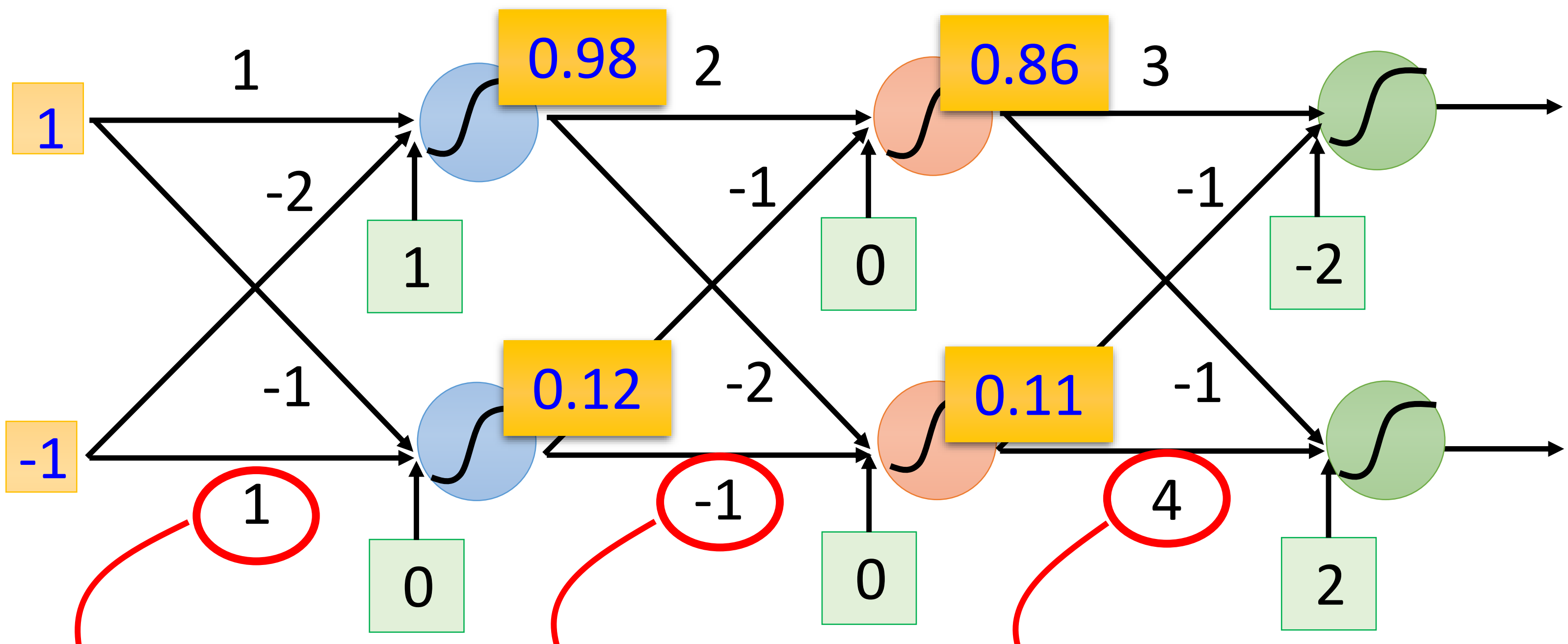
Compute $\partial l / \partial z$ for all activation function inputs z

$$\frac{\partial l}{\partial w} = ? \quad \frac{\partial z}{\partial w} \frac{\partial l}{\partial z}$$

(Chain rule)

Backpropagation – Forward pass

Compute $\partial z / \partial w$ for all parameters

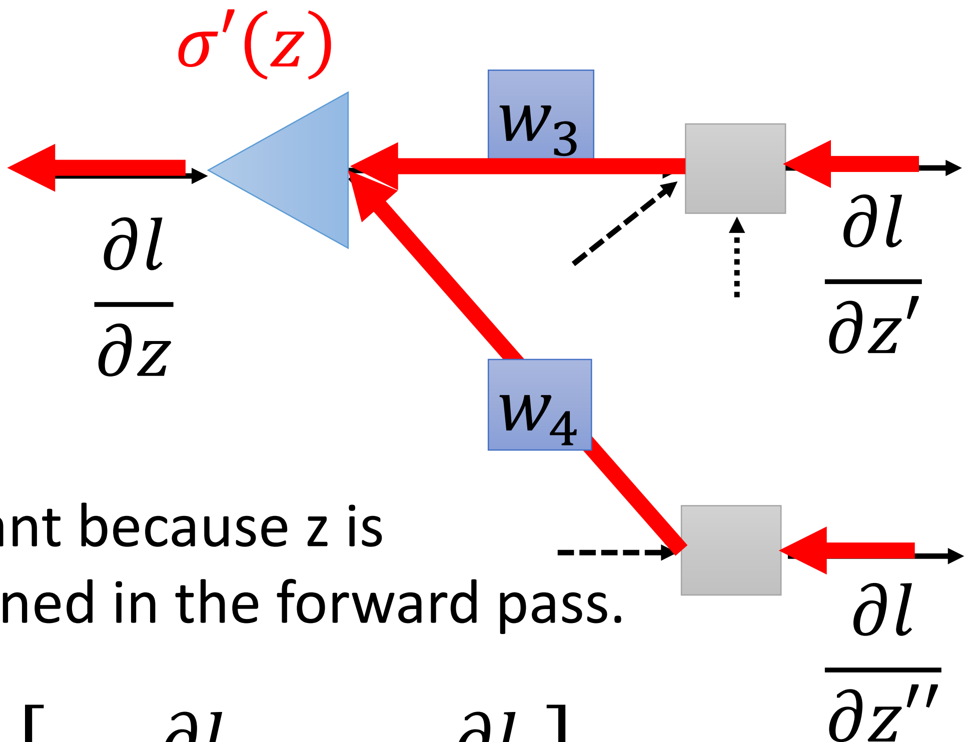


$$\frac{\partial z}{\partial w} = -1$$

$$\frac{\partial z}{\partial w} = 0.12$$

$$\frac{\partial z}{\partial w} = 0.11$$

Backpropagation – Backward pass

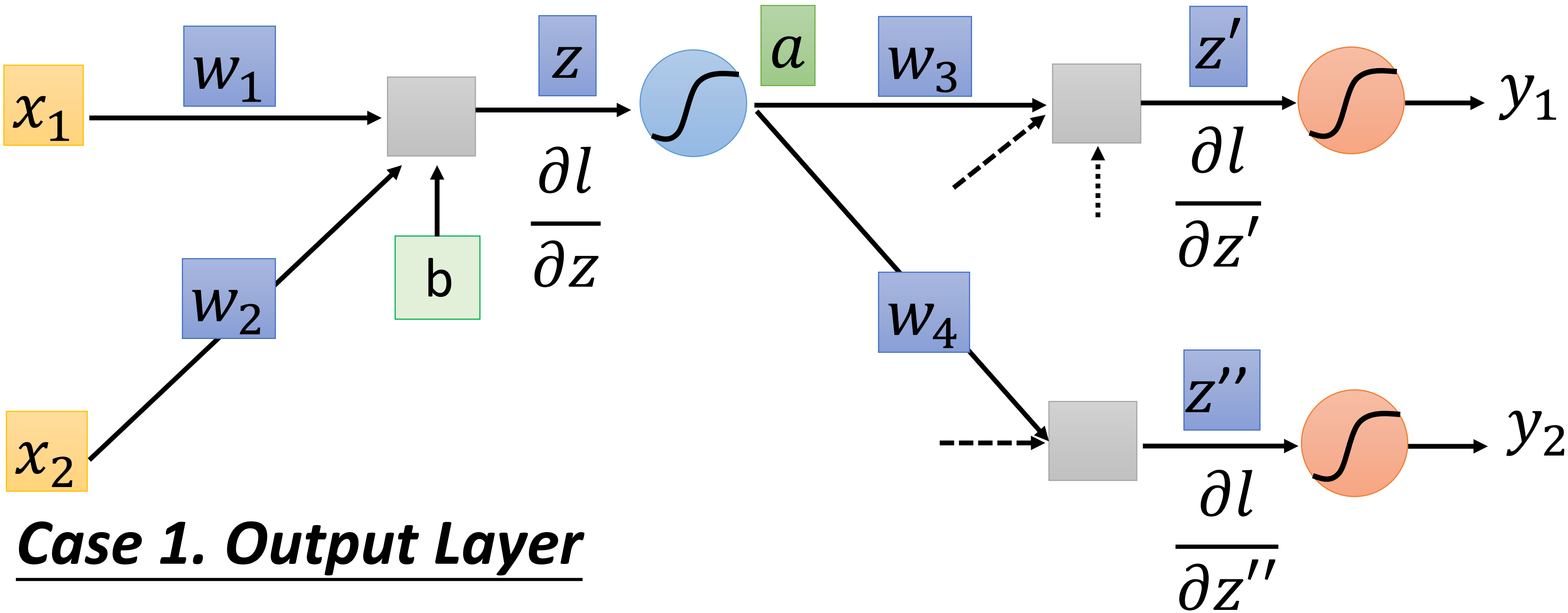


$\sigma'(z)$ is a constant because z is already determined in the forward pass.

$$\frac{\partial l}{\partial z} = \sigma'(z)\left[w_3\frac{\partial l}{\partial z'} + w_4\frac{\partial l}{\partial z''}\right]$$

Backpropagation – Backward pass

Compute $\partial l/\partial z$ for all activation function inputs z



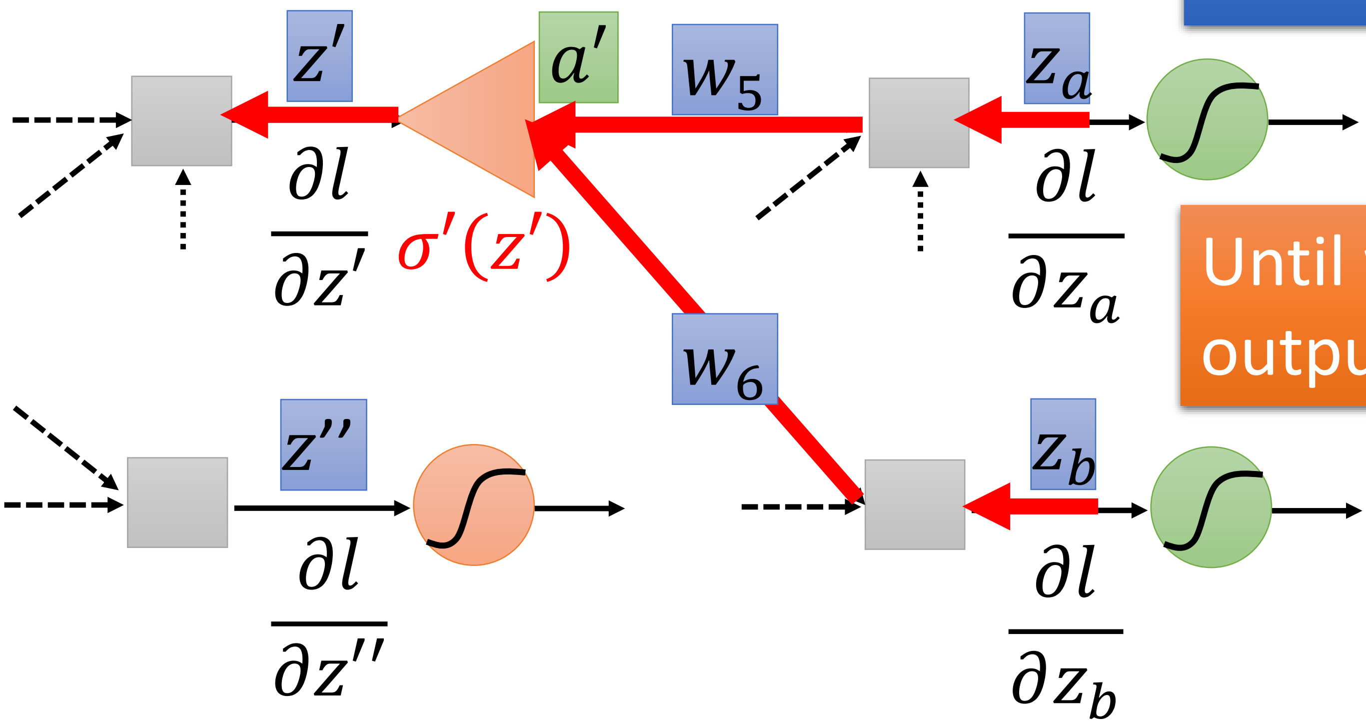
Case 1. Output Layer

$$\frac{\partial l}{\partial z'} = \frac{\partial y_1}{\partial z'}\frac{\partial l}{\partial y_1} \qquad \frac{\partial l}{\partial z''} = \frac{\partial y_2}{\partial z''}\frac{\partial l}{\partial y_2}$$

Backpropagation – Backward pass

Compute $\partial l/\partial z$ for all activation function inputs z

Case 2. Not Output Layer

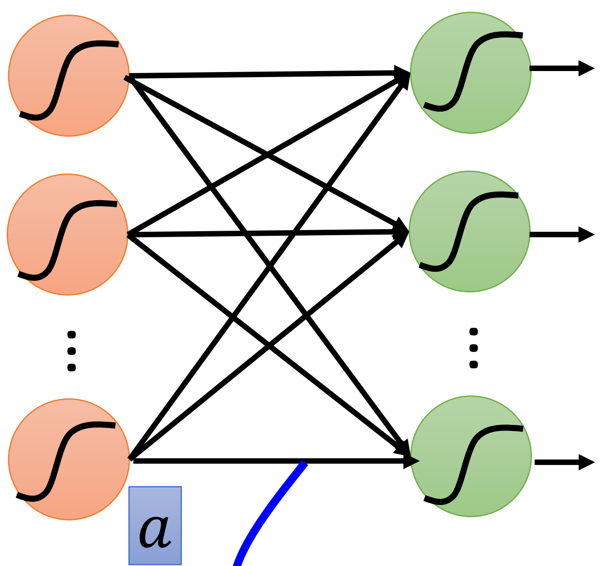


Compute $\partial l/\partial z$ recursively

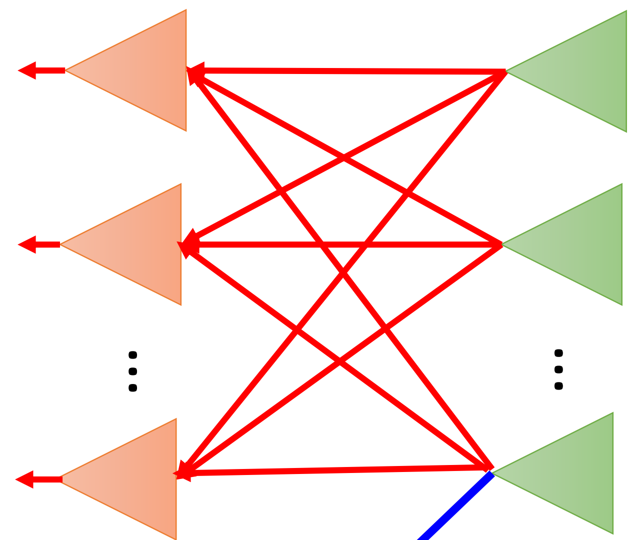
Until we reach the output layer ……

Backpropagation – Summary

Forward Pass



Backward Pass



$$\frac{\partial z}{\partial w} = a \quad \times \quad \frac{\partial l}{\partial z} = \frac{\partial l}{\partial w}$$

for all w