Flocs, Squigs, and Other Cheesy Words: Studying Flocculation in Cheesemaking Using Simulated Diffusion-Limited Aggregation

Jacklyn Wyszynski
Dr. Anna Balazs
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Abstract- **A report analyzing the growth of fractals in 2-dimensional and 3-dimensional matrices and their relationship to the essential process of flocculation in the manufacturing of cheese.**

I. Introduction

A food so mysterious that its exact origins are unknown. A delicacy that accounts for one of the most stolen food items in the world [1]. Produced by a process captivating enough to urge its loyal disciples to take sabbaticals all for the chance of uncovering this mystical craft [2]. Humans and mice alike fawn over this creamy delight: Cheese! The art of cheesemaking cannot be fully appreciated without first reflecting upon the historical contexts which early cheesemakers operated in; every piece of cheese is unique.

While the birthdate of the first piece of cheese is unknown, cheese production has been closely related with the dating of the domestication of milk-producing animals in 6000 BC [3]. Through extensive trials of primitive cheesemaking, the early Mesopotamians determined that goat milk was the most effective in the production of cheesy goodness. Among cows, sheep, and buffalo, goats proved themselves as an asset in the creation of nutritious milk with high caloric content. As a result, goat milk has been—and continues to be—the natural choice for cheese due to its unique taste, health benefits, and cultural tradition [4].

Aside from its qualitative attributes, goat milk is also slightly acidic, falling lower on the pH scale than both cow milk and buffalo milk [5]. The Mesopotamians discovered that acidic milk could be converted into acid-curd through a culturing process. According to a myth, cheese was discovered by accident when a nomad storing goat milk within a saddlebag later uncovered that he was toting a pack full of cheese. The desert heat and movement of his travels resulted in the formation of curd and whey [3]. This happy accident spiraled into the growth of cheese production throughout Mesopotamian society.

While this myth is simple speculation, this tale isn't tall. Milk naturally thickens through the coagulation of specific enzymes. Heat and temperature can hasten this process. After coagulation, curd particles flocculate and settle to form aggregate particles. These aggregations, known as flocs, grow through the gradual addition of more curd particles. Ultimately, they age into cheese—a process defining the beginnings of early cheese production.

Today, cheesemaking has been modernized to utilize advanced technology and the principles of food science to improve the quality of dairy products. Biochemists have harnessed lactic acid bacteria to regulate the specific flavors of cheese cultures. Fermentation can be expedited by implementing a substance known as rennet into the initial curd [6]. But these fromagian advances have yet to rewrite the essential process of flocculation; despite revolutionary changes in cheesemaking, flocculation has withstood the test of time to serve as a fundamental step in cheese manufacturing.

Fig. 1 Cheese flocculation (left) and various cheeses (right).
Left image taken from Fermenting for Foodies
https://www.fermentingforfoodies.com/cheesemaking-problems/
Right image taken from Rate MDs
https://www.ratemds.com/blog/study-reveals-cheese-is-as-addictive-as-drugs/

Flocculation is the physical aggregation and separation of sediment particles, often within a liquid solution. Flocs are delicate and flake-like, often held together by nothing more than surface tension [7]. Due to the inherent randomness of solid particle aggregation, every floc contains a unique pattern of sediment [8]. Despite a consistent process involved in cheese production, every wheel of cheese is derived from a distinctive fingerprint—every piece of cheese originates from uniform singularity.

Diffusion Limited Aggregation (DLA) simulations offer a visual representation of the flocculation of curd particles in cheesemaking. These algorithmic displays of particles can be analyzed in both 2-dimensions and 3-dimensions to study the formation of curd fingerprints. 2-D fractals, also known as squigs, demonstrate a top-view model of surface flocculation. 3-D fractals, referred to as flocs, offer a more holistic model for the development of cheese. The collective nature of curd flocculation makes DLA simulations faithful exemplars of this process.

While these systematic mathematical reproductions of curd aggregation offer a baseline understanding of the flocculation process, they fall short of modeling reality. Flocculation can be impacted by a range of variables including ambient temperature, motion, and aging period [9]. While DLA simulations can generate models of flocculation under ideal circumstances with completely random particle movement, additional variables may regulate the movement of aggregate particles. Cheese curds can be impacted by extraneous factors which are not fully considered within a DLA simulation.

The size of squigs and flocs is directly related to the concentration of curd particles within the dairy solution. Over time, the amount of curd increases as more dairy coagulates [4]. The growth of squigs and flocs can be used to analyze the spread of culture in the cheese process; the rate at which flocs form can dictate the cheese yield and formation.

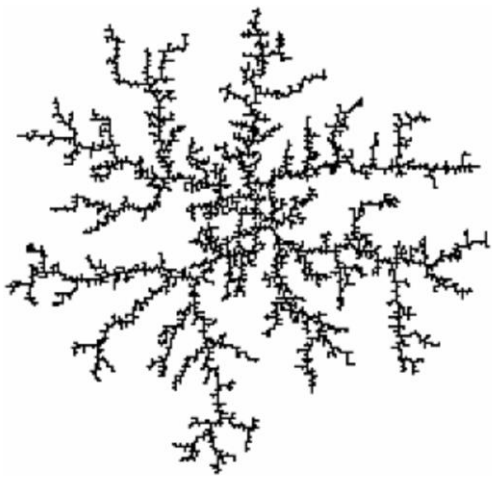

Fig 2. 2-D Fractal (squig) generation using DLA simulation. Image taken from ResearchGate https://www.researchgate.net/figure/Average-fractal-dimensions-for-several-types-of-DLA

In my DLA simulations, I studied the growth of squigs and flocs in 2-dimisions and 3-dimensions. I visualized the expansive network of squigs in solution, and investigated the formation of flocs in solution. Through my experiment, I wanted to determine whether unlike flocs had the potential to grow in like ways. I wanted to create a 2-D model of squig growth across a container. Furthermore, I wanted to create varying flocs utilizing a 3-D DLA simulation to analyze the differing curd fingerprints produced. I wanted to determine if there existed a relationship between the number of curd particles and floc radius. Finally, if this relationship held true, I wanted to determine if there existed a rate at which particle number would increase proportional to radius size which could be applicable across several floc productions. I predicted that while a positive relationship would exist between floc radius and particle count, it would fluctuate greatly due to the stochasticity of particle aggregation. I predicted that increased radius would result in more particles due to the growth of available branches that sediment could attach to. Moreover, I predicted that due to substantial differences in individual forms, all flocs would not grow at the same rate. Since every floc would appear unique, it would be consistent that unlike figures grow at non-identical rates.

II. METHODOLOGY

A. Creating a Model for 2-D Squigs

My first tests utilized a DLA simulation to construct a model for the aerial view of cheese curds. A DLA simulation was performed on a 201x201 matrix with a center seed fixed at 100x100. A particle was generated in an arbitrary location around the perimeter of the grid, and would perform a random walk until it contacted the center seed. If the particle hit a wall, it would continue a random walk with three potential directions of travel accounting for the limitation of the container boundaries. A single particle continued to move until it hit the center seed. No more than one particle would be moving in the grid at a single instance. A total of 1,400 particles were introduced into the 2-D model. Distances of each particle were calculated by saving the particles coordinates in the xy-plane and applying the distance formula. A colormap was applied to the models to judge the trends in distance from the center seed.

B. Comparing 3-D Floc Models

Using a similar process as described in *A*, I expanded my generation to account for a third dimension. I created particles in a 51x51x51 cube and saved the x, y, and z coordinates for each particle. The center seed was placed at 25x25x25. A total of 8,000 particles were introduced to the grid. This simulation proved to be more computationally demanding, but I repeated it several times to view different floc patterns that were generated. For the 3-D representations, boundaries were set in three directions, so particle movement would be restricted by walls like the conditions discussed in *A.*

The x, y, and z coordinates were saved to calculate the distance from each point to the center of the cube. Each floc representation was colored by particle distance from the center of the cube. Spherical points were used in the graphical generation of flocs to imitate the appearance of sediment particles.

C. Particle-Radius Relationships in Flocs

A particle to radius relationship was studied across a range of 3-D flocs. Flocs were generated and subsequently perused to determine the number of particles which existed in every radial increment from the center seed. A total of 100 3-D flocs were generated consistent with prior modeling definitions including particles continuing their random walks towards the center seed. For each floc, the number of particles was counted starting at grid measurements 1x1x1 and terminated at 51x51x51. Radius measurements were increased by one unit for every floc.

The growth of 3-D flocs was measured by taking the average number of particles in each radial increment across all 100 flocs. These points (average number of particles per radius) were initially graphed and demonstrated an exponential relationship. To display the results as a linear relationship, the natural log of both the average number of particles and the grid size was taken and the data was graphed once more. Polyfit was applied to the graph to determine the slope of the particle per radius relationship.

III. RESULTS

A. Creating a Model for 2-D Squigs

Fig. 3 shows two different squigs generated using the process described in *Section II A* Both squigs are completely unique in shape. Despite singularity across generated figures, both squigs seem to expand primarily in the four cardinal directions. As squig branches extend outwards, the probability of a particle sticking to a point between two branches decreases—it is far more likely for a particle to stick to an outstretched point. This heightened probability results in a similar growth structure across completely independently forming fractals.

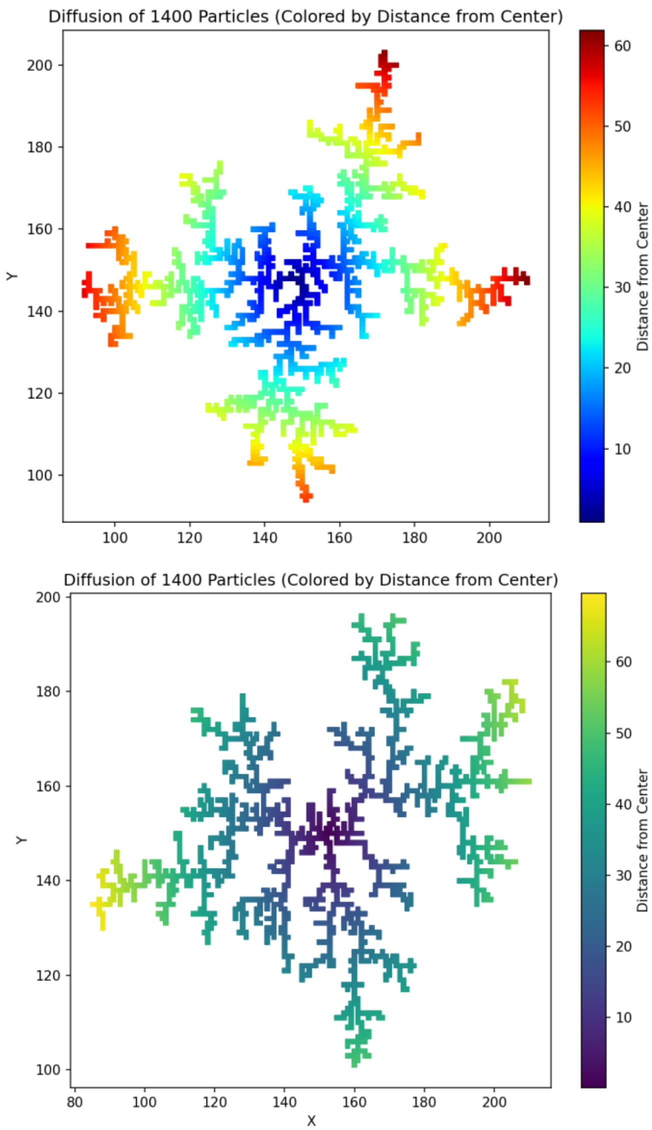

Fig. 3 Squigs produced by the diffusion of 1,400 particles in a 201x201 matrix in jet (top) and viridis (bottom).

In both squigs, the max distance between the furthest outstretched branch and the center seed peaked at about 70 units. This distance is not consistent around all branches of each squig, and is likely due to the possibility of multiple particles being generated around the same side of the container, resulting in repeated aggregation to a specific the closest branch of the squig.

B. Comparing 3-D Floc Models

The 3-dimensional floc models presented greater variation in form than their squig counterparts, likely due to the addition of a z-axis. There exists a high density of sediment particles in each of the three different generations of diffusion. As a result, all the generations retain a cubical form. This shape could be made less geometrical with a reduction in particle number. Within the 3-dimensional simulations, there exists a total of 125,000 open grid spaces for particles to occupy. A simulation with 8,000 introduced particles accounts for about 6.4% of total area available in the grid. Although the percentage fill of aggregate particles falls beneath 10% of the total available area, a clear geometry is still observable.

All three generations show rapid floc growth. Despite less than 7% of total grid space occupied, all generations demonstrate aggregation branches reaching the confines of the cubical container. Each floc representation in **Fig. 4** is colored by particle distance from center with increasing distance denoted by darkening hue. Across all three generations, the maximum distance obtained by each floc exists at the 35–40-unit mark. Additionally, all maximum distances obtained by floc models appear to lie near the vertices of the cube. Consistent with the results discussed in *Part III A*, it is likely that as the floc grows, there is a higher probability for a particle to stick to an outstretched branch.

This trend is most apparent in the inferno scheme presented in **Fig. 4**. The corner marking (0, 0, 0) as well as the corner marking (0, 50, 50) of inferno show the furthest aggregation development away from the center seed. This result is echoed in the magma scheme, particularly at (0, 0, 50). Unlike the 2-dimensional squigs, the 3-dimensional flocs tend to grow away from the four cardinal directions. Rather, the 3-dimensional flocs favor growth in diagonals, with aggregate branches stretching out until contact is made with the vertices of the cubical container.

The transition from stochastic clumping to defined shape is consistent with that of cheesemaking. As cubical models result in cube-like flocs, spherical models would likely result in sphere-like flocs. Particles will eventually fill the empty spaces available in a container. Hypothetically, an infinite number of sediment particles introduced into the grid would simply form a solid cube of curd. However, through my simulations with controlled numbers of particles, there exists ample empty space. Although the probability for particles to fall within centralized nooks is low, an steep increase in aggregate particles would simply result in denser cheese formation.

C. Particle-Radius Relationships in Flocs

Fig. 5 displays a positive linear relationship between the number of aggregate particles present and increasing radius size. Despite singularity across the 100 flocs generated, all flocs demonstrated rapid growth of branch development as radius size expanded.

As a result of the natural log calculations used in the graphical display, radial measurements from 0 to 5 were truncated to avoid the occurrence of undefined points. Using the truncated data, a slope of approximately 2.549 was derived from the plot.

Fig. 5 demonstrates substantially more variation in the number of particles present with smaller radii. As the radii

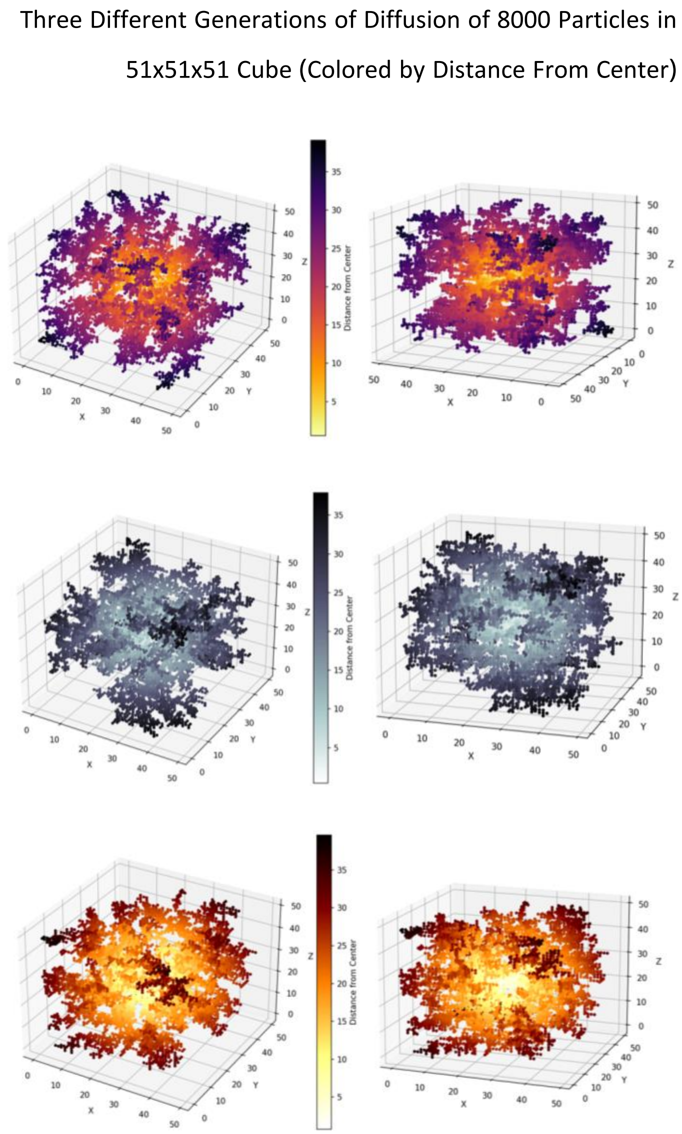

Fig. 4 Three unique floc generations colored by particle distance from center seed. Visualization using inferno (top), bone (middle), and flame (bottom).

increase, the line of 3-D growth matches the linear fit more precisely. This variation is likely due to the higher probability of more evenly dispersed aggregation present at smaller radii. At low radii values—since there exists a smaller number of particles—there also exists less branching. As these branches grow larger, particles can stick more aptly to the growing floc.

The variation present at smaller radii values may also be attributed to the smaller sample sizes present. For a new particle to be introduced into the cube, the previous particle needs to be fixed in place. For instance, in a 7x7x7 cube, it is less likely that particle aggregation has been completed since particles constitute a smaller percentage of the total grid space. On the other hand, although the percentage fill is still small, particles within the 51x51x51 cube make up a significantly larger portion of the total volume.

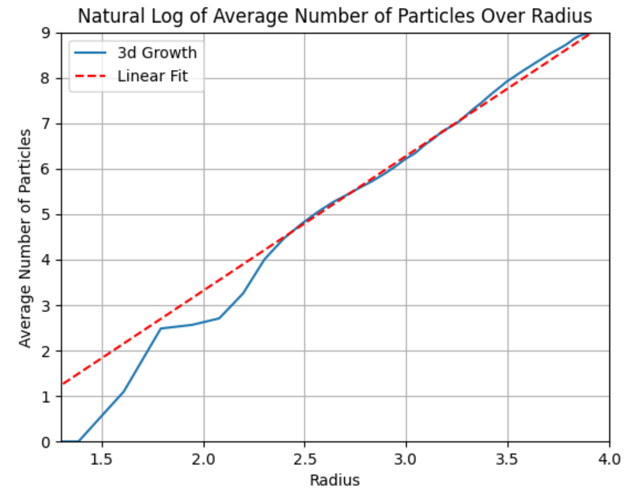

Fig 5. Graph of the natural log of number of particles over natural log of radius averaged across 100 flocs. Natural log data is represented by the blue line '3d Growth' and the polyfitted data is represented by the dashed line 'Linear Fit'.

IV. Conclusion

Cheesemaking is a delicate craft which combines biochemical reactions and physical transformations. The flocculation of milk proteins into larger curd particles plays a fundamental role in determining cheese yield. Understanding the dynamics of flocculation is essential for optimizing cheese production. DLA simulations prove to be powerful tools in exploring the intricacies of complex systems like cheesemaking. These simulations consider the stochastic nature of particle aggregation inertly tied to curd flocculation. Through a combination of modeling in both 2-dimensional and 3-dimensional contexts, the underlying principles of cheesemaking have been uncovered through the methodical implementation of various parameters.

My results align closely with the fundamental principles of cheesemaking. Squig growth is dependent upon itself, meaning a larger squig will grow at a faster rate. A relationship between particle number and radius size is supported through the tests conducted. Similarly, flocculation of cheese curds is dependent on the amount of curd present in a container. Furthermore, the growth of flocs adequately models the development of curd to cheese, and appears consistent across 100 trials. Additionally, as curd ages into cheese, it often adopts the form of the container in which it is processed in. Round cheese wheels are a direct result of cylindrical cheese molds which control the growth of curd flocculation.

These results underscore structural principles of cheesemaking. Square cheese is prone to significantly more breakage than round cheese [10]. This difference in cheese stability is likely due to the number of aggregated particles present in the aging container. As seen in **Fig. 4**, cubical containers produced outgrowing flocs with ample empty space present. A round container would likely result in higher aggregate particle density, which accounts for increased strength and stability in cheese wheels.

While my simulations present a baseline of information regarding curd formation and the growth of cheese formation, my experiment contains inherent constraints. My simulations were produced in small scales with limited floc generation. Ideally, the growth of 3-dimensional flocs would be averaged over larger sample sizes than 100, though I was restrained by my access to computing power. Additionally, my experiments fail to include the impact of pH, temperature, and kinetics within the curdling process. However, my research acts as a stepping-stone for further investigation into more accurate models which may account for increased floc variation due to these factors.

Reflecting the inherent randomness embedded in the flocculation process, every piece of cheese is inherently unique. Just as no two snowflakes are alike, no two cheese wheels will be derived from identical flocs. This result emphasizes the artisanal nature of cheesemaking; this variability adds to both the allure of cheese as a delicacy and fosters an appreciation for the craftsmanship required to master the cheesemaking process. Cheesemaking is more than simply a scientific endeavor—it is a venerable craft that intertwines not only scientific principles, but also tradition and artistry. Cheese is not only a means of nourishment: it is the tangible expression of human ingenuity and creativity.

Acknowledgments

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