Dance ‘til Dawn: Navigating the Landscape of Percolation in Square Matrices and its Application to Randomly Spreading Disco Mania

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ENGR 712
January 17, 2024

***Abstract-* A report investigating the existence of a percolation threshold simulated in square matrices of varying dimensions, and its relationship to medieval dancing fever.**

I. INTRODUCTION

In the summer of 1518, a dancing fever claimed upwards of 400 victims. The outbreak began in modern day France when a woman named Frau Troffea inexplicably started dancing with notable zeal. She danced for an entire week straight. Before long, three dozen other dancers joined the performance [1]. The dancing plague was spreading.

While the causes of such a bizarre disease remain unknown, it is theorized that the action measures taken by civic and religious leaders may have exacerbated the spread of the boogie. Council people theorized that the dancing plague may be solved by more dancing, and arranged halls for dancers to gather in. They hired musicians and professional dancers to aid the afflicted citizens to keep dancing [2]. Unfortunately, these measures only resulted in a larger spread of the fever.

Additionally, while cases of the dancing fever are slim in number and often isolated, the spread of such an illness is a considerable subject to be investigated. One possible mathematical model of the spread of contagion may be visualized through percolation theory. Percolation theory studies the behavior of connected networks, and how connections may vary depending on the fraction of intersections occupied [3]. Models based on this approach have been used to explore illness throughout social networks; results have been shown to yield insight into the spread of diseases and the size of outbreaks [4].

Percolation models offer a comprehensive look at the dynamics of disease transmission provoked by uncorrelated, random interactions on the individual level. With modern cases of typical diseases, transmission is dependent on a variety of factors including infection history, individual habits, and sanitation [5]. While percolation theory remains limited in its ability to accurately predict the evolution of active infections, this theory may provide valuable insight into the seemingly indiscriminate spread of the dancing plague of 1518.

Dancing plague outbreaks have been recorded along the Rhine and Moselle rivers, areas connected by water but exhibiting their own unique climates and agriculture [1].
However, in almost all dance breaks, it was common for both infected and their non-partying counterparts to be grouped into halls varying in size. While the abundance of these groups has fluctuated over time, the characteristics of the spread of the dancing plague is synonymous with the simpler case of percolation through square planes.

In my experiment, I investigated the traits of the connected networks that exist at varying fractions of occupied intersections. Through my studies, I sought to discover whether a percolation threshold existed; I wanted to know what fraction of the total number of people would have to be infected to spread the dancing fever throughout the entire population. Furthermore, I investigated whether this threshold was dependent on population size which I simulated through trials on matrices of varying dimensions. I predicted that while a percolation threshold did exist, it would be size dependent. Moreover, I predicted that less than ½ of the population would have to be infected to spread the disease ubiquitously due to the natural tendency of transmission. The factor of randomness alludes to the fact that the disease will be unevenly distributed throughout the population, so there ought to exist clusters of infected individuals even at low infection rates.

II. METHODOLOGY

A. Proving the Existence of a Percolation Threshold

My primary tests were conducted to determine whether there existed a percolation threshold in a 10x10 randomly occupied matrix. An algorithm was used to generate matrices and to count the number of paths formed at each fractional occupancy level. Paths, here, defined as a linear connection between the top and bottom sides of the matrix omitting diagonals and including horizontal variations.

0	1	0	0	1	0	0	1	0	0
0	0	0	1	1	0	0	0	1	0
0	1	0	1	0	0	0	0	1	1
1	1	0	1	1	1	1	1	0	1
1	1	1	0	0	0	1	0	0	0
1	1	0	0	0	1	1	0	0	0
1	0	0	0	1	1	0	0	0	1
0	1	0	0	1	0	0	0	1	1
0	1	0	0	1	0	0	1	1	1
1	1	1	0	1	0	0	1	1	1

Fig. 1 A sample 10x10 randomly occupied matrix with one path.

Inherently, there existed anomalies which had to be accounted for when gathering data. For example, even at a fill percentage as low as 0.15, there existed the possibility for a path to form. While this potential was nearly miniscule, it was still present. To account for the various anomalies that my algorithm would encounter, the volume of data had to be inflated. For the percentages 0.00-1.00, I conducted 1000 trials at every 0.01 percent increase. The 100,000 total path results were then averaged to yield a holistic result.

B. Determining the Value of the Percolation Threshold

To accurately verify the value of the percolation threshold across a range of matrix sizes, I repeated the procedure as described in Section A with a 500x500 matrix. After averaging the values, and graphing my results, I calculated the second derivative along the plotted line for both the 10x10 and 500x500 plots, specifically focusing on the percentage fills of 0.55 to 0.70, where the slope appeared to be the steepest. I analyzed these values to determine the inflection point of each plotted line, and found the correlating percentage fill to be the value of the percolation threshold. Finally, I compared the value of the percolation threshold for the 10x10 grid and the 500x500 grid to determine whether there existed any correlation between the numerical values.

C. Does size matter?

Testing the impact of the size of the matrix presented more challenges. Sample volume sizes were inflated due in part to increased trial runs, but also due to the increased matrix dimensions. To simplify the processing of the data, I reduced trial runs to every 0.1 percent increase. I continued to conduct 1000 trials at every percentage, however, and tested matrix sizes ranging from 10x10 to 500x500. I incremented matrix dimensions by 25 units until I reached 100x100, then progressively incremented by 100 units until I reached 500x500.

III. Results

A. 10x10 Trials

Based upon the data collected at every 0.01 percentage increase, a percolation threshold exists at these matrix dimensions as denoted by the sharp changes in the plot.

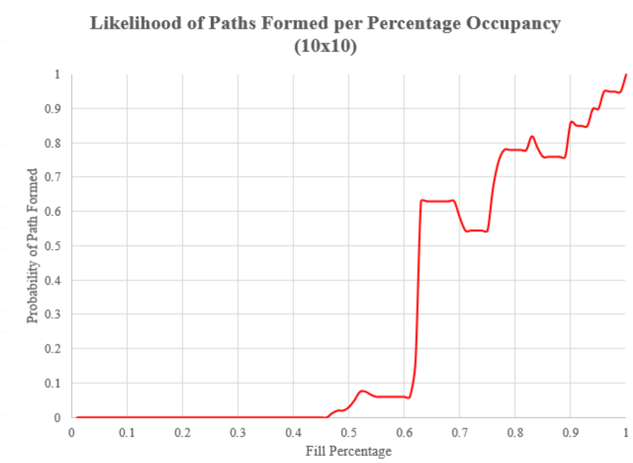

Fig. 2 Data collected from analyzing paths formed at every fractional occupancy (10x10).

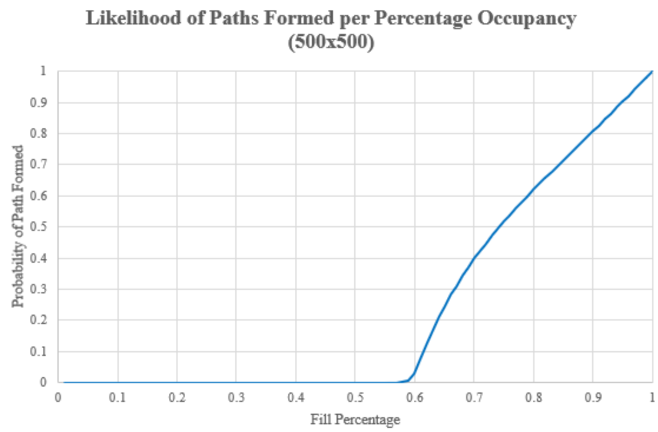

Fig. 3 Data collected from analyzing paths formed at every fractional occupancy (500x500).

B. 10x10, 500x500 Percolation Threshold

The values of the second derivative of the 10x10 and the 500x500 plot indicate that the percolation threshold exists at 0.57 percent occupancy. The plotted line has an inflection point at 0.57 fractional fill, where the values of the second derivative shift from negative to positive, indicating a change from upwards concavity to downwards concavity.

SECOND DERIVATIVE CALCULATIONS 0.55 - 0.7 PERCENT FILL (10X10)					
X	Y	X^3-X	DY/DX	SIGN F'(x)	INFLECTION
0.55	0.06	-0.383625	-0.0759	-1	O
0.56	0.06	-0.384384	-0.0423	-1	O
0.57	0.06	-0.384807	-0.0081	-1	X
0.58	0.06	-0.384888	0.0267	1	O
0.59	0.06	-0.384621	0.0621	1	O
0.6	0.06	-0.384	0.0981	1	O
0.61	0.06	-0.383019	0.1347	1	O
0.62	0.16078921	-0.381672	0.1719	1	O
0.63	0.63	-0.379953	0.2097	1	O
0.64	0.63	-0.377856	0.2481	1	O
0.65	0.63	-0.375375	0.2871	1	O
0.66	0.63	-0.372504	0.3267	1	O
0.67	0.63	-0.369237	0.3669	1	O
0.68	0.63	-0.365568	0.4077	1	O
0.69	0.63	-0.361491	0.4491	1	O
0.7	0.5827023	-0.357	0.4911	1	O

Fig. 4 Table of second derivative values from 0.55 to 0.7 fractional occupancy (10x10).

SECOND DERIVATIVE CALCULATIONS 0.55 - 0.69 PERCENT FILL (500X500)					
X	Y	X^3-X	DY/DX	SIGN F'(x)	INFLECTION
0.55	0	-0.383625	-0.0759	-1	O
0.56	1.1988E-06	-0.384384	-0.0423	-1	O
0.57	6.86563E-05	-0.384807	-0.0081	-1	X
0.58	0.000969655	-0.384888	0.0267	1	O
0.59	0.007544268	-0.384621	0.0621	1	O
0.6	0.028527697	-0.384	0.0981	1	O
0.61	0.073888636	-0.383019	0.1347	1	O
0.62	0.125950937	-0.381672	0.1719	1	O
0.63	0.171028559	-0.379953	0.2097	1	O
0.64	0.209696366	-0.377856	0.2481	1	O
0.65	0.245878259	-0.375375	0.2871	1	O
0.66	0.282015934	-0.372504	0.3267	1	O
0.67	0.310686339	-0.369237	0.3669	1	O
0.68	0.342979071	-0.365568	0.4077	1	O
0.69	0.370022065	-0.361491	0.4491	1	O
0.7	0.400100287	-0.357	0.51	1	O

Fig. 5 Table of second derivative values from 0.55 to 0.7 fractional occupancy (500x500).

The 500x500 plot produced a much smoother curve as opposed to the 10x10 plot. These differences may be largely due to the fact that individual discrepancies between path formations are more significantly represented in the 10x10 matrix since paths occupy a greater percentage of the entire area.

C. Variable Size Trials

Across a variety of matrix dimensions, it seems that the existence of a percolation threshold remains constant across all sizes tested. Additionally, this value remains consistent existing at approximately 0.55 to 0.6 percent occupancy throughout all trials.

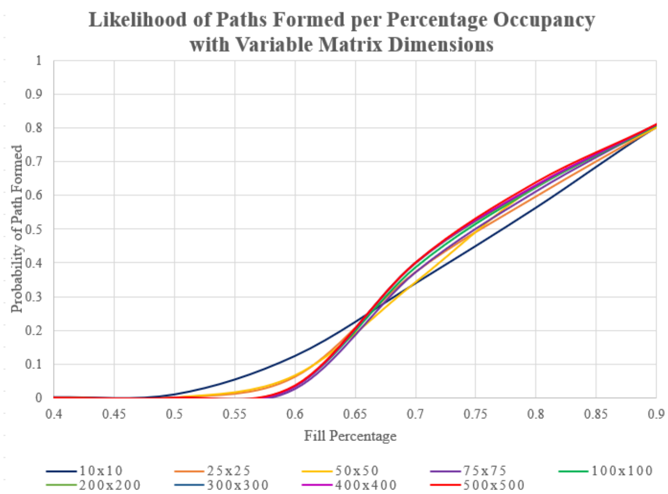

Fig. 6 Tendency of path formation for matrices of varying sizes.

It is important to note that the slopes of the lines are considerably different across the varying matrix dimensions. The 500x500 line is significantly steeper than the 10x10 line. This increased rate of change is likely due to the fact that occupancies in smaller matrix sizes constitute a larger fraction of the total matrix. A single occupancy in a 10x10 matrix represents 0.1% of the total area, while a single occupancy in a 500x500 matrix represents 0.000004% of the total area. The 500x500 matrix will demonstrate a more rapid change in the number of paths formed since at every filled percentage, there exists more occupied spaces in the 500x500 matrix as compared to the 10x10 matrix. Therefore, a single path in the 500x500 matrix constitutes a less significance in comparison to the total matrix area. At lower percentage fills, the 10x10 line is well above the 500x500 line due to these disparities in matrix area.

IV. Conclusion

The concept of percolation bears a curious resemblance to the occurrences of medieval dancing plagues. Just as percolation involves the gradual spread of a substance through interconnected pathways, dancing fevers cascade through ancient communities, drawing more and more individuals into the dance circle. Through my rounds of testing, I have not only confirmed the existence of a percolation threshold, but I have also concluded that this threshold is not dependent on matrix size.

Although I was able to prove the existence of a percolation threshold, inherent errors exist within my simulation of the spread of disease. The simulation that was utilized to assess the value of the percolation threshold is that of an extremely small sample size compared to a realistic community. The smaller sample size is used as a generalization regarding the tendencies for percolation, rather than a larger sample size which would more adequately model the spread of dancing fever throughout a population. Additionally, this model extensively simplifies the spread of the dancing mania. Although it is predicted that dancing mania was transmitted randomly, it cannot be safely assumed that mass psychosis is sheerly stochastic.

In the case of patient zero, Frau Troffea, it is likely that approximately 0.58 of the total town population was infected with the plague resulting in 400 dancers. Based upon the matrix results of the 25x25 and 50x50 matrices, the percolation threshold existed within the 0.55 to 0.60 range, meaning that once this percentage of infection was reached, a total of 400 feverish citizens would be confirmed.

On a much larger scale, the percolation threshold for the 500x500 matrix appeared to be located at roughly 0.57 percent occupancy. If a modern-day dancing plague were to break out, at the estimated percolation threshold for a 250,000-sample size, the number of infected individuals at that percentage could entirely replace the population of Guam [7]. Everybody would dance now.

While percolation is a scientific process with limitations on the psychological factors that may have sparked the dancing plague, the parallels between these two seemingly unrelated anomalies highlight the intricate ways in which both patterns and behaviors propagate through shared networks.

Acknowledgments

I'd like to thank Eliana Crew for the quick project updates before and after classes. I'd also like to thank Alex Hauskrecht for collaborating with me on the path counting algorithm. Finally, I'd like to thank Natan Herzog for helping me debug my code and for expanding my horizons.

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