Bunnies and Foxes and Badgers, Oh My! Exploring Predator Prey Relationships Using Random Walk Algorithms

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***Abstract-* A report investigating the dynamics of predator-prey and predator-predator-prey relationships using the principles of random walk algorithms.**

I. Introduction

In the 1798 book *An Essay on the Principle of Population,* English scholar Thomas Malthus utilized rigid concepts to explain ever changing growth [1]. Malthus examined the inclination of human populations to favor growth over comfort. Later defined as the "Malthusian trap", the philosophy outlined in *An Essay on the Principle of Population* opposed popular media at the time which suggested that European society was something that could be controlled. Malthus argued that society was not perfectible.

Through observation, Malthus claimed that although an increase in food production would temporarily raise the quality of life of society, an abundance of food would lead to stark population growth. This enhanced population size acted to diminish societal wellbeing and would ultimately restore the initial production per capita of food. "The power of population is indefinitely greater than the power in the Earth to produce subsistence for man" [2]. Malthus believed that population growth was a divine truth pertaining to man—this inevitable expansion would chain the lower class to hardship and prevent conditional improvement towards utopian society.

The Malthusian trap sparked immediate controversy. Critics argued that this philosophy ignored the impact of human institutions in the developing populations. The Malthusian hypothesis ignored the capability of an expanded population to utilize technological advancements to bolster food production [3]. The Malthusian was brutish in treating the growth of human populations like the growth of livestock. Many contended Malthus due his opposition to a lavishly promising future [4].

While Malthus' publication was deemed pessimistic and unfavorable at the time, his studies fortified the emergence of mathematical biology. His implementation of exponential population growth juxtaposed against the arithmetic development of food supply was revolutionary in the natural resource analysis [3]. Mathematical biology relies on this formulaic and foundational examination to uncover natural processes [5]. Regardless of the stochastic tendencies of stark increases or drops in population, timely trends could be analyzed to better understand the natural world.

Natural populations are stunningly dynamic. While Malthus' studies on the growth of human populations were limited in their representation of technological advancement, his principles could be more adequately applied to changes in the populations of natural organisms. His ecological stance on human populations could be used to study trends in animal species [6].

With these principles in mind, a particular relationship emerged. Despite variance in environmental conditions, the interactions between predator species and prey species are inherently linked. Mimicking the principles outlined in 18th century Europe, predators unfailingly rely on the food supply of prey to grow.

Born from these conclusions, the Lotka-Volterra predator-prey model was developed in the 1920s by Alfred Lotka and Vito Volterra. The Lotka-Volterra model was revolutionary for its time, and it utilized impulse response functions to study the exponential growth of animal populations. This mathematical model was capable of accounting for the birthrates and deathrates of predators and prey through coupled differential equations to further analyze the rise and fall of species over time [7]. The predator-prey relationship could be used to study the consistent fluctuations of animal species [8].

While the Lotka-Volterra model expanded the avenues for population studies, it remained limited in its ability to account for the holistic nature of population growth. The Lotka-Volterra model was particularly restrictive when analyzing population growth over long periods of time. As species reproduce and continually adapt to their surroundings, natural selection plays a role in the development of population evolving traits [8]. Unfortunately, mathematical equations are confined in their scope to account for predators evolving into better predators, or conversely prey evolving into better prey.

Despite its limitations, the Lotka-Volterra model still provides an adequate representation of population dynamics over shorter lengths of time. The principles of Lotka-Volterra can be applied to the linked interactions between North American bunnies, foxes, and badgers. Each species travels uniquely with respect to its habitat, yet all three species are connected in their perpetual reliance on sustenance. These interactions can be modeled using conditional random walk algorithms. Although not completely stochastic, the relationship between predator-prey and predator-predator-prey can be effectively studied by analyzing the population variance when selective animal species undergo non-restrictive movement.

In my tests, I sought to investigate the relationship between bunnies and foxes in both healthy and diseased contexts. I wanted to first analyze the populations of bunnies and foxes in an ideal predator-prey relationship. I then wanted to consider the impacts of disease and resource scarcity to further inspect the predator-prey dynamics. Finally, I sought to study the implementation of a third species under ideal conditions to examine the effects of competition on predator-predator-prey relationships. I hypothesized that the ideal predator-prey relationship would result in a constant exchange between predator majority and prey majority. Additionally, I predicted that this relationship would be dependent on the success of both species. In other words, a stark decrease in the population of foxes would result in a decline in the population of bunnies. Moreover, I predicted that the introduction of competition into the environment would result in the exhaustion of all three species due to the rapid depletion of prey. I predicted that the competition of foxes and badgers over a diminishing population of bunnies would result in unanimous extinction.

II. METHODOLOGY

A. Ideal Predator-Prey Relationship

My first tests were conducted to analyze the relationship between fox and bunny populations. Taking inspiration from both the random walk algorithm and the Lotka-Volterra model, my primary tests consisted of bunnies and foxes stochastically dispersed within a resource abundant environment. An equal number of bunnies and foxes were randomly generated along with respective coordinates with 20x20 boundary restrictions. At the beginning of each interaction, an animal within the plane would be selected impartially. The selected animal would complete a random walk until it met another animal. The interactions between animals resulted in the following engagements:

I. If both the selected animal and the encountered animal were bunnies, a new bunny was randomly generated into the environment.

II. If both the selected animal and the encountered animal were foxes, the resulting interaction was divided into two equally probable outcomes. The two foxes could reproduce, and a new fox would be randomly generated into the environment. Alternatively, the foxes could fight, and the encountered fox would die. A random number generator was utilized to make the birth versus death decision.

III. If the selected animal and the encountered animal were of different species, the bunny would be eaten by the fox. A new fox would be generated into the environment, thus maintaining a predator-prey dependency.

Time was recorded utilizing the number of steps traveled by the randomly selected an animal. Ten cumulative steps constituted one unit of time. Population data was tallied after every singular unit of time. A total of 100 units of time was recorded for each completed simulation.

To average over a series of results, 100 total simulations were completed. The number of bunnies and foxes recorded at each unit of time was averaged among all trials. The resulting populations were fit to a sine curve to observe the alternating population dominancy as time progressed.

B. Predator-Prey Relationship in a Modified Environment

Using the same 20x20 boundary conditions and interaction principles as described above in *Part A*, I tested whether the implementation of a biased predator reduction and land dependent resource availability would alter the ideal relationship between predator and prey. I started by introducing a disease that solely impacted the predator population. The initial generation of foxes consisted of a 0.3 probability of contagion. Within a fox-to-fox interaction as described in *Part A.II,* if a healthy fox and a contaminated fox were to interact through competition or through reproduction, both foxes involved would be contaminated. Any additional foxes created through contaminated reproduction would carry the disease. Bunny-to-fox interactions as described in *Part A.III.* had no bearing on the spread of the disease, regardless of the health attributes contained by the fox involved in the interaction.

The disease remained dormant in fox species until ten cumulative steps were taken by any animal. Time was redefined in this simulation since restricted computing power did not allow for multiple trials to be averaged. A single unit of time was recorded after a cumulative 100 steps were taken by any species.

In addition to the implementation of a contagious disease, my second simulation also relied on prey-based resource depletion. As total spatial area decreased, as did the food available for bunnies to consume. Food calculations were determined by the availability of open space on the grid. After every ten units of time, the total number of unoccupied spaces was tallied. If the ratio of total bunnies to total available spaces was greater than 0.8, bunnies were randomly killed until the ratio was satisfied. The altered environmental conditions of the simulation were continued for a total of 100 units of time.

C. Introduction of a Second Predatory Species

Using the same 20x20 boundary restrictions as described above, a third species was introduced into the simulation. Badgers were randomly generated mimicking the previous generations of foxes and bunnies. The interactions between species were outlined with the following principles:

I. If both the selected animal and the encountered animal were bunnies, a new bunny was randomly generated into the grid (same as *Part A).*

II. If both the selected animal and the encountered animal were predators, the resulting interaction was divided into two equally probable outcomes. The two predators could reproduce, and a new predator of the respective species would be randomly generated into the environment. Alternatively, the predators could fight, and the encountered predator would die. A random number generator was utilized to make the birth versus death decision. (same as *Part A*)

III. If the selected animal and the encountered animal consisted of a predator species and a prey species, the prey would be killed by the predator. A new predator of the respective species would be randomly added into the matrix. For example, if a badger encountered a bunny, the bunny would be killed, and a new badger would be generated into the environment.

IV. If the selected animal and the encountered animal were both predators of different species, the selected animal would continue random walking until a prey species, or a predator of its own species was encountered.

The predator-predator-prey tests utilized the time measuring principles described in *Part A* with ten steps constituting a single unit of time. Limited computing strength only enabled a single trial to be run with a population measured across 50 units of time.

II. Results

A. Ideal Predator-Prey Relationship

Based upon the data collected, the bunny population and fox population under ideal conditions demonstrated a cyclic relationship. As seen in **Fig. 1**, bunny and fox populations generated sine waves with identical paths. The phase shift in the curve of the fox population is the only noted difference. Fox populations are observed to peak later in time than bunny populations.

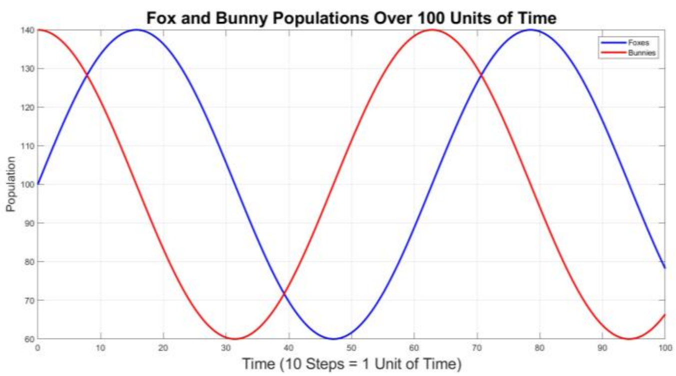

Fig. 1 Sine projection of fox and bunny populations under ideal conditions over 100 units of time with coordinates generated under 20x20 boundary conditions. Note: Ten cumulative steps used to mark one singular unit of time.

Despite a phase shift in the sine curves of bunny and fox populations, both curves show identical amplitudes. The crest and valley of both fox populations and bunny populations reach the same absolute values. At each respective peak, a maximum of approximately 240 total animals existed within the simulation. At each valley, a minimum of an estimated 135 animals existed within the simulation. The maximum density occurred at a population peak and was recorded at 0.60 fill. The minimum density in a population valley was recorded at 0.3375 fill.

There existed several instances in time where the population of foxes and bunnies were equal. At 8 units in time, there existed exactly 127 foxes and 127 bunnies in the simulation. Likewise, at 39 units in time, there existed both 72 bunnies and 72 foxes within the simulation. As seen in **Fig.1**, both sine curves follow this cyclic pattern multiple times throughout the simulation.

B. Predator-Prey Relationship in a Modified Environment

The bunny and fox population varied significantly under the modified environmental conditions compared to the initial tests run in *Part A.* Under ideal conditions, a clear cyclic behavior was observed between the bunny and fox populations. Under the newly implemented modified environment, there existed no ebb and flow between the number of foxes and the number of bunnies in the simulation.

Fig 2. shows a steady decline in the bunny population as time progressed. Due to the addition of contagion in the fox species, the fox population quickly dropped after less than ten units in time. On the other hand, the bunny population steadily declined following a pattern of exponential decay. The bunny population peaked at the minimum decline of the fox population, but slowly decreased and seemingly reached zero based off the trajectory of the bunny population curve.

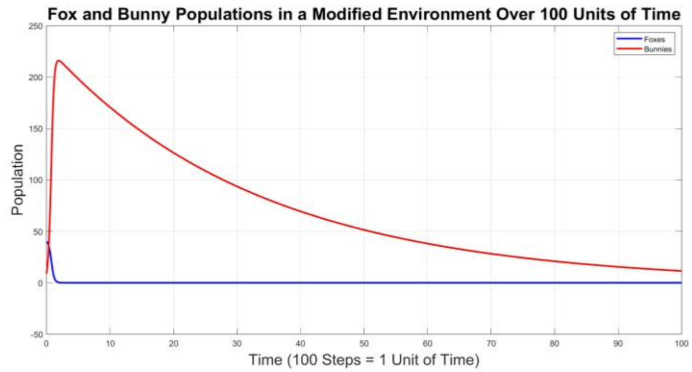

Fig. 2 Graph of fox and bunny populations under modified environment conditions over 100 units of time with coordinates generated under 20x20 boundary conditions. Note: 100 cumulative steps used to mark a singular unit of time.

C. Introduction of a Second Predatory Species

The introduction of the predatory badger into the simulation impacted the population of animals differently from the observations discussed in *Part A.* As seen in **Fig. 3**, the initial generation of bunnies into the simulation decreased nearly immediately with the bunny population decaying to zero after less than five units of time. As the bunny population declined, the population of foxes and badgers followed suit.

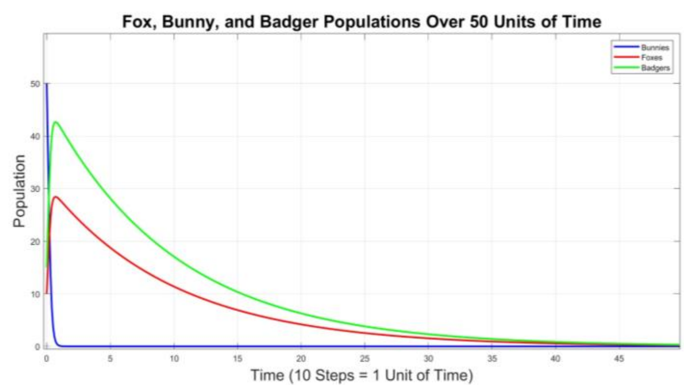

Fig. 3 Graph of fox, bunny, and badger populations over 50 units of time with coordinates generated under 20x20 boundary conditions. Note: Ten cumulative steps used to mark one singular unit of time.

Both the populations of foxes and badgers experienced exponential decline like that which occurred by bunnies in

Part B. **Fig. 3** shows that the population of the predatory species both decayed to zero at the end of time unit 50. The population of foxes and badgers both peaked at the same moment in time. The maximum recorded values in predator species occurred later in time than the maximum value recorded in the prey species, similar to the observations noted about **Fig. 1**.

Despite both predatory species peaking at the same moment in time, the badger population grew larger than that of the fox population. As a result, the decline in badgers occurred more rapidly than the decline in foxes since both decay curves reached zero at the same time instance.

IV. CONCLUSION

The investigation into predator-prey and predator-predator-prey relationships using the principles of random walk algorithms resonates with the foundational concepts outlined in *An Essay on the Principle of Population* as well as the Lotka-Volterra model. Malthus' observations on the inevitable tension between population growth and resource availability and the pioneering work of mathematical ecology by Alfred Lotka and Vito Volterra provide a framework for understanding the dynamics observed in ecological systems.

Under both ideal and modified environmental conditions, clear relationships were observed between predator and prey species. The initial tests of an ideal predator-prey relationship demonstrated a cyclic pattern of population fluctuations, as noted by the phase shifted sine curves in **Fig. 1**. However, when subjected to modification in the environment such as disease spread and limited resources, the dynamics shifted dramatically. The introduction of contagious disease among the predator species and a decrease in prey resources led to a disruption in the previously observed balance of foxes and bunnies. As my hypothesis predicted, this change resulted in a decline in both populations and a lack of the characteristic cyclic behavior observed in the ideal scenario.

The addition of a second predatory species intensified the dynamics, driving all populations towards decline. Increased reliance on the prey population, punctuated by resource competition by the predator species, ultimately resulted in the extinction of all species involved.

Although this simulation provides a general idea of predator-prey and predator-predator-prey population dynamics, it is overall limited in its ability to replicate nature. All three major tests were conducted with limited trials and simulation sizes due to lack of computing power. Additionally, the spread of contagion and the prey density algorithm implemented in *Part B* superseded the desired biasing effects. While a modified environment clearly showed the dependent relationship of predators and prey, it is unlikely that disease could spread as rapidly as projected in *Part B.* Finally, the interactions between foxes, bunnies, and badgers in *Part C* are not as realistic as they occur in nature. Despite the extinction of the bunny population in **Fig. 3**, both badgers and foxes were able to survive for an extended period. This impact is unlikely if bunnies are the sole means of sustenance in the ecological system. Furthermore, while foxes and badgers do not often fight when they encounter each other, it is unrealistic to omit deaths between the predator species in totality. Despite these sources of error, my projections still provide an adequate representation of natural population growth and decay.

Just as Malthus highlighted the potential for unchecked population to cause damage to society, my findings reveal the balance between predators and prey in natural ecosystems. The ideal predator-prey relationship observed in *Part A* mirrors Malthus' notion of a temporary equilibrium in which population sizes can oscillate within certain bounds. The sinusoidal behavior of both prey and predator resonates with the patterns described by Lotka and Volterra. However, when subjected to environmental stressor such as disease outbreaks and resource scarcity, the simulation in *Part B* illustrates the vulnerability of populations to collapse. These findings not only echo the interdependence of predator and prey highlighted by the Lotka-Volterra model, but they also speak to the fundamental ideas of Thomas Malthus. My results align with the warnings of the Malthusian trap: unchecked growth and the inevitability of population checks are dangerous in the face of limited resources.

Moreover, the introduction of a second predatory species underscores Malthus' recognition of the role of competition in shaping population dynamics. Just as human societies compete for resources, the simulated ecosystems in *Part C* demonstrate how competition between predator species can exacerbate the decline of both predators and prey.

My investigation reaffirms Malthus' insights into the cardinal principles governing population dynamics while simultaneously considering a mathematical approach inspired by Lotka-Volterra predator-prey models. By integrating Malthusian theory and mathematical biology with random walking algorithms, we gain a deeper understanding of the intricate relationships that shape our natural world. This synthesis of diverse population frameworks emphasizes the resilience of ecosystems in the face of environmental change.

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